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To cite this version:

Jiangxingyun Zhang. International Portfolio Theory-based Interest Rate Models and EMU Crisis. Economics and Finance. Université de Rennes, 2017. English. NNT : 2017REN1G011 . tel-01684720

HAL Id: tel-01684720

https://theses.hal.science/tel-01684720

Submitted on 15 Jan 2018

ANNÉE 2017

THÈSE / UNIVERSITÉ DE RENNES 1
sous le sceau de l'Université Bretagne Loire

pour le grade de

DOCTEUR DE L'UNIVERSITÉ DE RENNES 1

Mention : Sciences Économiques

École doctorale Sciences Economiques et sciences De Gestion (EDGE)

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Préparée à l'unité de recherche CREM (UMR CNRS 6211)
Centre de Recherche en Économie et Management
Faculté de Sciences Économiques

International Portfolio Theory-based Interest Rate Models and EMU Crisis

Thèse soutenue à Rennes
le 20 Septembre 2017

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Acknowledgements

First of all, I would like to express my sincere gratitude to my advisor, Prof. Franck Martin. I am amazingly fortunate to have an advisor who gives me the invaluable guidance to my research. In the last four years, Franck has taught me how to conduct good research with his patience, enthusiasm and immense knowledge. Meanwhile, he is also my advisor outside of academia. While travelling together, he always shares with me his wisdom on life, on politics, on football—even I am not a fan. I am proud to have written my dissertation under his supervision. He made me grow as a researcher, but far beyond that.

Secondly, I would like to thank my committee members, Professor Jean-Bernard Chatelain, Professor Georges Prat, especially Professor Jean-Jacques Durand for taking your precious time reading my work. I would like to gratefully and sincerely thank Prof. Fabien Rondeau for his excellent Master lectures which had deeply motivated me. I am also grateful to Prof. Jean-Christophe Poutineau, Prof. Fabien Moizeau, Prof. Pascale Meriot, Prof. Tovonony Razafindrabe and Prof. Yunnan Shi for their various forms of help during my graduate study. I would like to acknowledge my collaborators and mentors for their valuable advice and numerous discussions, in particular Dr. Roberto De Santis, Sylvain Barthélémy, Prof. Hans-Jörg Von Mettenheim, Prof. Guillaume L'Oeillet, Prof. Jean-Sébastien Pentecôte, Prof. Mamy Raoul Ravelomanana and Dr. Guillaume Queffelec.

I am indebted to all the administrative staff of the CREM-CNRS and of the University of Rennes 1, notably Anne l'Azou, Danièle Moret-Bailly, and Cécile Madoulet. Thank you for helping me through all kinds of administrative procedures. And I will never forget the excellent atmosphere given by my dear fellow colleagues: Gabin Langevin, Vincent Malardé, Ewen Gallic, Thi Thanh Xuax Tran, Thibaud Cargoët, May Attalah-Atef Aly Said Ahmed, Clément Dheilly, Romain Gaté and Beaurain Guillaume.

Finally, and most importantly, I would like to thank my loving parents for their unconditional love, support and encouragement. Thank you.

Résumé en français

Modèles de taux d’intérêt basés sur la théorie des choix de portefeuilles internationaux et crise de l’UEM

Depuis la dégradation de la note de la dette grecque en décembre 2009, jusqu’à la mise en œuvre des programmes d’assouplissement quantitatif par la BCE (QE) en mars 2015, la crise dans la zone euro a entraîné des évolutions contrastées des taux d’intérêt à long terme. Les trajectoires des taux à long terme sur les obligations d’État ont alterné les phases de corrélation ou au contraire de dé-corrélation en fonction de l’intensité des risques de contagion et de leurs déterminants : contexte macroéconomique, crédibilité des programmes budgétaires et soutenabilité de la dette, programmes mis en œuvre par la BCE (OMT, BCE). . . Pour la plupart des pays en difficulté, les taux ont augmenté, surtout après 2010, dans un contexte de forte volatilité alors qu’au contraire, les marchés obligataires identifiés comme les plus sains (Allemagne, France) ont rapidement été le refuge pour des stratégies de fuite vers la qualité (flight to quality) dans un climat de faible volatilité des taux et rendements obligataires.

Nous exploitons dans cette thèse le filon théorique selon lequel les considérations de risque de volatilité ont pu subsister pendant la crise de la dette pour alimenter, à côté des réévaluations des risques de défaut, les stratégies d’investissement sur les marchés obligataires et in fine peser sur la formation des prix et des taux longs.

Le chapitre 1 propose une vaste revue de la littérature théorique sur la formation des taux d’intérêt à long terme. Elle a vocation à identifier les variables explicatives pertinentes des taux longs souverains pour nous permettre, dans la suite de la thèse, de proposer une évaluation des composantes fondamentales et non fondamentales des taux d’intérêt et une évaluation des espérances et variances conditionnelles sur les taux longs. L’information sur les variances conditionnelles est essentielle pour apprécier le poids des primes de risque de volatilité dans la formation des taux longs. L’information sur les covariances conditionnelles est elle essentielle pour mettre en œuvre des tests de

contagion ou de flight to quality à la Forbes et Rigobon (2002). Nous constatons dans ce chapitre une grande diversité des modèles explicatifs de la formation des taux longs et la quasi absence de modèles proposant un cadre d'analyse international à plusieurs marchés obligataires. Ce chapitre propose également une revue de la littérature empirique traitant de la mesure des phénomènes de contagion, pure ou fondamentale, et des effets de flight to quality au sein des marchés obligataires de la zone euro.

Le chapitre 2 étudie simultanément les phénomènes de contagion, flight to quality et de transmission de volatilité entre les marchés obligataires de la zone euro durant la période de crise. On souhaite tester en particulier la possibilité d'un transfert de volatilité négatif entre les pays périphériques de la zone euro et les pays pivots (Allemagne, France) pour lesquels la volatilité des rendements obligataires a significativement chuté à partir du début de l'année 2011. L'étude porte sur la période 2008 – 2013 et s'appuie sur une modélisation AR(1)-GARCH(1,1)-VECH tri variée des rendements obligataires de plusieurs trios de pays européens choisis parmi les 7 pays suivants : Grèce, Irlande, Portugal, Espagne, Italie, France, Allemagne. Nos résultats empiriques mettent en avant des mécanismes généralisés de flight to quality au cours de la période de crise, non seulement entre les pays sains et périphériques mais également au sein d'un même groupe de pays. Selon ces premières évaluations, la mise en place de l'OMT à partir de septembre 2012, n'a pas globalement entrainé une remontée significative des corrélations conditionnelles entre rendements obligataires. Enfin ces résultats ne permettent pas globalement de valider l'hypothèse de transfert de volatilité négatif des marchés périphériques vers les marchés pivots.

Le chapitre 3 propose un modèle théorique original pour évaluer le poids des primes de risque de volatilité et de co-volatilité dans la formation des taux longs souverains. Il s'agit d'un modèle de choix de portefeuille à deux pays qui permet de généraliser les propriétés de la théorie traditionnelle de la structure par terme taux d'intérêt (Artus (1987), Shiller (1990)). Nous montrons en particulier que la chronique des covariances anticipées entre taux longs souverains est une composante essentielle de la prime de risque de volatilité. Ainsi par exemple, un scénario de contagion anticipé par les investisseurs (hausse des covariances dans le futur) accroit le risque perçu sur le portefeuille diversifié, diminue la demande d'obligations sur les deux marchés au profit de l'actif sans risque et fait monter les taux longs dans chaque pays. Ce scénario formulé par les investisseurs, à défaut d'être auto réalisateur, vient donc renforcer les mécanismes de contagion préexistants. De la même manière un scénario de flight to quality (baisse des covariances dans le futur), augmente les demandes d'obligations, fait baisser les taux longs dans les deux pays et amplifie ainsi le mécanisme de fuite vers la qualité au profit du pays moins exposé au risque de crédit. Les mécanismes de choix de portefeuille apparaissent au final comme un canal intermédiaire entre la contagion par les fondamentaux et la contagion pure liée aux stratégies spéculatives. Ce modèle théorique fournit également un cadre d'analyse pour évaluer l'impact sur la formation des taux longs des achats de titres de la BCE dans le cadre de sa politique monétaire non conventionnelle d'assouplissement quantitatif (QE). Les achats de titres viennent de manière équivalente accroitre la demande d'obligations ou diminuer l'offre nette d'obligations disponibles à chaque période.

Le chapitre 4 cherche justement à proposer une évaluation empirique de l'impact des programmes de QE de la BCE sur l'équilibre des marchés obligataires de la zone euro. Le modèle théorique du chapitre 4 sert de support pour des équations économétriques des taux longs dans un cadre GARCH in Mean modifié : une variable de primes versées sur les CDS souverains permet d'intégrer explicitement le risque de crédit dans nos évaluations ; les variances et covariances anticipées sur les variations de taux longs (modèle en différences premières) sont des prévisions hors échantillon « one step ahead » issues d'une série d'estimations glissantes de modèles DCC-GARCH bivariés sur les variations de taux. Nous montrons que le rôle des moments d'ordre 2 dans la formation des variations de taux disparait pendant la crise et réapparait avant la mise en place du QE de la BCE et précisément à partir de l'OMT en septembre 2012. On assisterait donc de ce point de vue à une « défragmentation » des marchés obligataires où les mécanismes normaux de choix de portefeuille reprennent progressivement leurs droits. Etant estimés en variation de taux, les modèles économétriques ne permettent pas de chiffrer de manière absolue la contribution de la BCE à la baisse des taux longs sur les différents marchés. Nous montrons en revanche que les corrélations conditionnelles entre taux longs n'ont pas significativement augmenté avec la mise en place du QE, celles-ci étant en moyenne déjà très élevées auparavant. Un test empirique complémentaire montrent que le QE a réduit de près de moitié la sensibilité de spreads de taux par rapport à l'Allemagne aux primes des CDS souverains. On assisterait bien à un écrasement des primes de risque de crédit.

Cette thèse comble finalement un écart dans la littérature entre les modèles de taux d'intérêt et les phénomènes de contagion et de fuite vers la qualité. Elles montrent que les mécanismes de choix de portefeuille constituent sans doute un maillon intermédiaire entre les mécanismes de contagion pure sur les marchés obligataires et les mécanismes de contagion par les fondamentaux.

Contents

General Introduction

The aim of this dissertation is to examine the specific role of volatility risks and co-volatility in the formation of long-term interest rates in the euro area. In particular, a two-country theoretical portfolio choice model is proposed to evaluate the volatility risk premia and their contribution to the contagion and flight to quality processes. This model also provides an opportunity to analyze the ECB's role of asset purchases (QE) on the equilibrium of bond markets. Our empirical tests suggest that the ECB's QE programs from March 2015 have accelerated the "defragmentation" of the euro zone bond markets.

This introduction briefly sketches the main building blocks of this dissertation. Section 1 provides economic context and motivation. Section 2 outlines the evolution of theoretical and econometric approaches of bond pricing. Section 3 presents the Asset Purchase Programmes features. Section 4 provides a survey of the main contributions of the dissertation to the existing literature. Section 5 describes the structure of the dissertation, organized in 4 chapters.

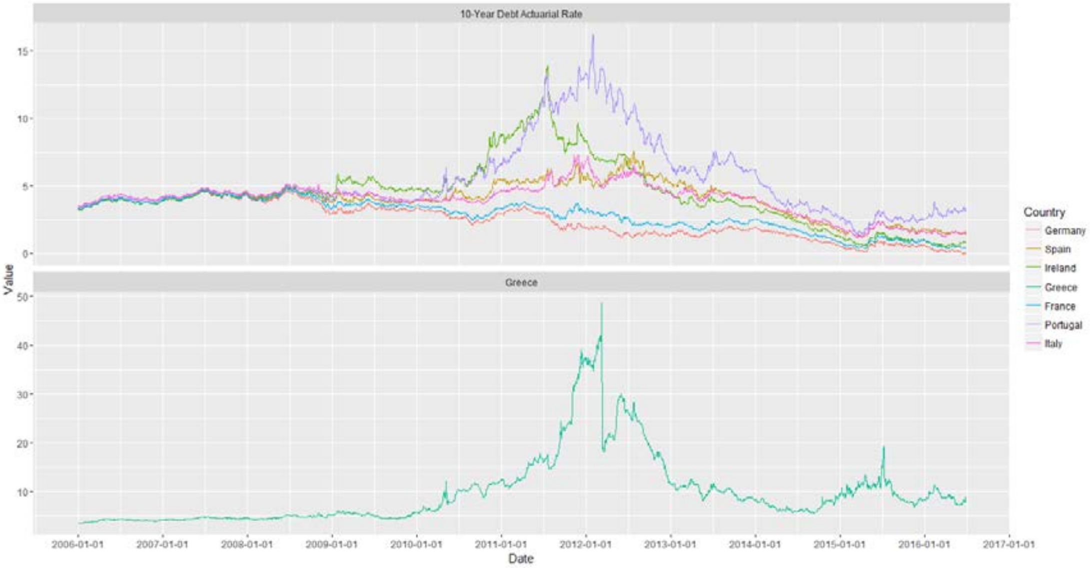
Economic context and motivation

Quickly following the global financial crisis, which erupted in 2007 with the failure of the US sub-prime market and then intensified in September 2008 with the collapse of

Lehman Brothers, a multiple year debt crisis has broke out in the European Union at the end of 2009.

This crisis in the euro zone has led to contrasted evolutions in long-term interest rates. For most countries in difficulty, their rates increase, especially after 2010, in a context of high volatility. On the contrary, the bond markets identified as the healthiest (Germany, France) have quickly been the refuge of flight to quality strategies. We note that the rising of German and French bond markets from the beginning of 2011 takes place in a context of low volatility, unlike the fall of other markets remain turbulent. Both two pivot countries who benefit from processes of flight to quality seem benefit from volatility transfer or negative volatility spillover as well. Investors, without doubt, were also sensitive to high bond rates associated with the process of decreasing rate, which is, triggered in a low volatility context.[1] To be more clearly, in this dissertation we define the risks of volatility as all the second order moments, that is variance and covariance, of the country studied.

FIGURE 1: Actuarial rates (RY, Yield to maturity) of major European countries



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[1] See for example Martin and Zhang (2014).

Figure 2: Return index (RI) of major European countries' bonds

Based on global macroeconomic context, and on the country specific context, such as the credibility of budgetary consolidation programs, the institutional aids or the programs implemented by the ECB (Outright Monetary Transactions and QE), the trajectories of government bonds' long-term rates have alternated the correlation or decorrelation phases according to the intensity of the contagion risks. Since then, abundant literature dealing with the sovereign rate dynamics emerged. They focused mainly on the risk of default and its consequences: role of debt sustainability on rates (Arghyrou and Kontonikas (2012), Costantini et al. (2014)); sensibility of yield spreads to variation of CDS (Bai and Collin-Dufresne (2013), De Santis and Stein (2015)).

Usually, there are two types of interpretations for the evolutions of these interest rates. Fluctuations of interest rates and spreads compared to Germany can be interpreted as rational revaluations of credit risk premiums on sovereign bond issuers. They may also be due to the phenomena of financial contagions related to speculations and potentially self-fulfilling strategies in bond markets who test periodically the sustainability of public debts.

There are some empirical literature which specifically try to distinguish between these two types of contagion mechanisms. Some papers try to identify the contagion effects by applying the methodology proposed by Pesaran and Pick (2007) where rates spreads are explained by global factors and country specific factors. In accordance with Forbes and Rigobon (2002), pure contagion only exists if the crisis leads to a significant increase in the correlation of non-fundamental components of interest rates. Furthermore, Metiu (2012) finds evidences for significant contagion effects among long-term bond yield premium by extending the canonical model of contagion. Arghyrou and Kontonikas (2012) find empirical evidences who prove the existence of contagion effects, particularly among EMU periphery countries. Afonso et al. (2012) find European government bond yield spreads are well explained by macro and fiscal fundamentals over the crisis period.

After doing a vast survey on term structure of interest rates models, surprisingly, we find that there exists a major gap of literature between (i) interest rate models and (ii) the phenomenon of contagion and flight-to-quality, since these two types of literature study the similar problematic, but somehow, they are separated.

By developing on the traditional portfolio theory-based interest rate model of Jones and Roley (1983), Mankiw et al. (1986), Artus (1987) and Artus and Kaabi (1995), Martin and Zhang (2017) design a two-economy model by taking into account the effects of contagion and flight-to-quality, which fills a major gap between these two literature. This model will be presented in chapter 3.

Evolution of bond pricing

Without doubt, interest rate itself and related contingent claims are very widespread and extremely important. That is the reason why numerous interest rate models have been introduced over the years. Some representative findings listed below might give a overall view of this evolution.

From 1960s to 1980s. By analyzing different theories of the term structure of interest rates, Telser (1967) finds that both the expectation theory and liquidity preference theory present advantages and limitations. Merton (1973) proposes a one-factor model assuming that the short-term interest rate process has a constant expected growth rate μ and a constant volatility σ. By examining the forecasting capability of six different econometric interest rate models, Elliott and Baier (1979) conclude that four of them are capable of explaining current rates accurately. However, they lack of forecasting power. Brennan and Schwartz (1979) propose a two-factor model, including long-term and short-term interest rates, for pricing governmental bonds. Unfortunately, no strong evidence can support the existence of significant relation between future values of long-term and short-term interest rates.

From 1980s to 2000s. Cox et al. (1985) develop an intertemporal general equilibrium interest rate model where conditional variance and risk premium vary with the short-term rate and short-term rates are non-negative by design. GARCH model by Bollerslev (1986) generalizes Autoregressive Conditional Heteroscedasticity (ARCH) model proposed by Engle (1982). Although numerous similar models exist, all GARCH-type models share the same idea that is to use values of the past squared observations and past variances to model the current variance. Longstaff and Schwartz (1992) introduce a two-factor general equilibrium model where a representative investor, having a logarithmic utility function, should choose between investing or consuming the only good in the economy. One major advantage of this model is that it allows to get analytical solution for the price of a discount bond and a call option on this bond.

Since 2000. Collin-Dufresne et al. (2001) investigate the determinants of credit spread changes and conclude that monthly credit spread changes are principally driven by local supply or demand shocks. Bali (2003) extend the one-factor BDT term structure model by Black et al. (1990) into a two-factor model. Metiu (2012) extends the canonical model of contagion proposed by Pesaran and Pick (2007) in order to test for contagion of credit events in Euro area sovereign bond markets. Caporin et al. (2013) find the propagation of shocks in euro's bond yield spreads shows almost no presence of shiftcontagion by applying both standard quantile regression and Bayesian quantile regression with heteroskedasticity.

Asset Purchase Programme features

Since the debt crisis, in order to support financial conditions and reduce key interest rates in the euro zone, the ECB has implemented several unconventional policies, among which were two asset purchase programs: Covered Bond Purchase Programme (CBPP) and Securities Market Purchase Programme (SMP). The CBPP was comprised of two sub-programmes: CBPP1 and CBPP2. Under the CBPP1, the ECB committed to purchasing a total of 60 billion during the period from June 2009 to June 2010. Under the CBPP2 the targeted amount of purchases was 40 billion during the period from November 2011 to October 2012.

However, after several years of these programmes, the ECB considered that was not enough. A decision of October 2014 gave an expanded asset purchase programme (APP), which includes all purchase programmes under which private sector securities and public sector securities are purchased to address the risks of a too prolonged period of low inflation. This programme comprises: third covered bond purchase programme (CBPP3), asset-backed securities purchase programme (ABSPP), public sector purchase programme (PSPP), and corporate sector purchase programme (CSPP). Furthermore, the net purchases in public and private sector securities amount is 60 billion on average. They are intended to be carried out until the end of 2017, and in any case, they are intended to be carried out until the end of 2017.

Key contributions of the dissertation

The main methodological contributions can be listed as follows:

1. Development of an original theoretical model based on the optimal portfolio choices of a representative euro-based investor, under the hypothesis that he allocates over a short period its portfolio between two sovereign bonds having default risks and volatility risks, and a risk-free monetary asset.

2. Application of the multiple structural breaks unit root test proposed by Lee and Strazicich (2003). By using the daily Return Index of Greece, we are able to separate the time series into three sub-periods. Another similar application by using two times Zivot and Andrews (2002) test, we have obtained very similar results.

3. Application of the Welch test, proposed by Welch (1951), trying to find out whether the means of conditional variances across period have significantly changed after the implementation of OMT. This test is an approached solution of the Behrens-Fisher problem. The objective of Welch test is to determine whether or not statistically there is an equality of means of two subsamples in the case of their variances are different.

4. Adjustment and application of the flight-to-quality test based on what is proposed by Forbes and Rigobon (2002). We aim to test the hypotheses of structural changes of correlation coefficients across the tranquil and turmoil periods. As pointed out by Forbes and Rigobon (2002), the estimation of the correlation coefficient is biased because of the existence of heteroscedasticity in the return of the bond. More specifically, compared to the estimation during a stable period, the correlation coefficients are over estimated during a turmoil period. In our study, the correlations are conditional and

dynamic. Therefore, we modify the adjustment formula of correlation coefficient proposed by Forbes and Rigobon (2002).

5. Development of a trivariate GARCH-in-mean model which quantifies the transmission of volatility and flight-to-quality phenomenon, by using the specification of the variance-covariance matrix as presented by Bollerslev et al. (1988) , the diagonal VECH model.

6. Development of a one-step-ahead out of sample variances and covariances forecasting model, based on DCC-GARCH model, which allows to simulate the anticipated second order moments by the investors in the market.

7. Development of a two-step econometric model which explains and quantifies the specific roles of global factors, country-specific factors, short-term rates, credit default risks, liquidity risks, and especially the roles of both volatility and co-volatility risks in the formation of long-term interest rates.

The main theoretical results obtained in this dissertation can be summarized as follows:

1. Optimal demand for bonds depends crucially on variances and covariances anticipated by investor. A lower covariance limits the joint risks between two bonds and stimulates the demands of the bonds at the price of risk-free rate. Optimal bond demands are confronted with available bond supply, that is to say not only the market values of sovereign debt stocks but also the ECB's QE programs. These bond purchases of the ECB have effectively reduced the net bond supply in circulation.

2. Some additional equilibrium properties that generalize those of the traditional domestic term structure of interest rates theory (Artus (1987), Mankiw et al. (1986), Shiller and

McCulloch (1987), Jones and Roley (1983)). The expected bond yields and the equilibrium rates dependent, expect for traditional properties, crucially on the anticipated bond yield covariances. These anticipated covariances, along with variances and bond supply, become essential components of volatility risk premium on bond yields. With optimal portfolio choices, we show that the anticipated covariances can indeed amplify or reduce, depending on different scenarios, the mechanism of contagion and Flight-to-quality between markets. To some extent, we propose a new intermediate channel of contagion which is situated between the fundamental contagion and the pure contagion.

The main empirical results obtained in this dissertation can be summarized as follows:

1. Welch test on the comparison of the average conditional variances for the three sub periods (pre-crisis, crisis, post-OMT) clearly show that for Germany and France, the bond markets are less volatile during crisis than before the crisis. Without surprise, the formal adoption of the OMT in September 2012 led to a further decrease in the average level of conditional variances, it's a synonyms for investors to have less risk on bond yields.

2. Results from test of Lee and Strazicich (2003) on the daily Return Index of Greece help us to separate the time series into three sub-periods: pre-crisis (2006/01/01 to 2009/12/01), crisis (2009/12/01 to 2012/08/06), and post-OMT (2012/08/06 to 2016/09/09). It should be noted that the two break dates 2009/12/01 and 2012/08/06 are very close to respectively the Downgrading of Greece sovereign bond and the Implementation of Outright Monetary Transaction.

3. The tests of flight-to-quality between bond markets show that the logic of the flight-to-quality is predominant at the beginning of the sovereign debt crisis. There is systematically a decrease in conditional correlations of bond yields for almost all pairs of markets studied. This is observed in both cross-country correlations of different groups

(periphery to pivot countries) and correlations between countries of the same group. With the exception of the pair country France-Germany, the conditional correlations of the other markets have neither increased significantly from September 2012 nor after the plan OMT by the ECB. Therefore, there isn't a general beginning of a re-correlation of bond markets, but rather a stabilization of conditional correlations at levels close to those estimated in the previous period.

4. Regarding the tests on parameters of volatility two findings emerge from our estimations. There are few evidence that support a global phenomenon of conditional volatility transmission between markets. However, there is a significant relationship between the Greek and Irish bond markets during the crisis period where the increase of the conditional variance in the Greek market has clearly contributed to reduce the risk of volatility seen in the Irish market. Therefore, we could say there is a phenomenon of the eviction of volatility between the two countries.

5. Concerning the one-step-ahead forecasts of variances and covariances, it comes out quite clearly that covariances are playing a more significant and systematic role in the dynamics of sovereign rates. The estimated parameters have most often the expected positive signs. The effects are generally more present in the German and French markets than in the euro zone periphery markets. The parameters associated with the covariance often show a U-shape pattern with lower values over the crisis period.

6. Results from the two-step econometric model show that bond portfolio mechanisms have clearly played a role between Germany, France, Portugal and Spain during pre-crisis period before becoming less important or disappearing during the crisis and again reappear in the post-OMT period. For the second group of countries, Italy, Ireland and, to a lesser extent, Greece, the portfolio mechanisms only appear in the post-OMT phase.

7. Results from estimations of sensitivity of spread to CDS suggest a strong correlation between spreads and CDSs. It clearly shows that the implementation of QE in March 2015 has greatly reduced the spread sensitivity to CDS and thus artificially crushed the credit risk premiums weighing on bond yields. The spread sensitivity to CDS decreases by half in Italy and Spain. It becomes negative for France and almost null for Ireland. Only Portugal, where there is no change in the sensitivity to credit risk, derogates from the rule.

Structure of the dissertation

This dissertation comprises 4 chapters.

Chapter 1, entitled «Introduction to interest rates, financial contagion and flight-to-quality », does a vast survey of the existing interest rate models and financial contagion literature during the EMU crisis, which allows to understand better different determinants in the formation of interest rates. Surprisingly, there exists a major gap of literature between (i) interest rate models and (ii) the phenomenon of contagion and flight-to-quality, since these two types of literature study the similar problematic, but somehow, they are separated. There isn't any interest rate model which takes into account the effects of contagion and flight-to-quality.

Chapter 2, entitled «Correlation and volatility on bond markets during the EMU crisis: does the OMT change the process ?», studies the correlation and volatility transmission between the European sovereign debt markets during the period of 2008-2013. By applying a multivariate GARCH model and a flight-to-quality test, the empirical results support not only the existence of flight-to-quality from the periphery countries (Italy, Portugal, Spain, Ireland and Greece) to the pivot countries (France and Germany), but also the flight within each group. This can be explained by a new phenomenon of speculation in bond markets which didn't exist before the debt crisis. However, the estimations bring little evidence that allow us to generalize it to all markets. It seems that in terms of volatility, the pivot countries are relatively difficult to be influenced by the external turbulence. Although we prefer to believe that Europe has walked out of the sovereign debt crisis after the Outright Monetary Transaction (OMT) plan, this study doesn't bring much support for this point of view.

Chapter 3, entitled «International portfolio theory-based interest rate model », proposes a portfolio choice model with two countries to evaluate the specific role of volatility and co-volatility risks in the formation of long-term European interest rates over the crisis and post-crisis periods with an active role of the European Central Bank. Long-term equilibrium rates depend crucially on the covariances between international bond yields anticipated by investors. Positively anticipated covariances amplify the phenomena of fundamental contagions related to the degradations of public finance and solvency of sovereign debt issuer, while negatively anticipated covariances amplify the phenomena of Flight-to-quality.

Chapter 4, entitled «Impact of QE on European sovereign bond market equilibrium », evaluates the impact of the ECB's QE programs on the equilibrium of bond markets. For this purpose, we firstly summarize the theoretical model to help understanding the formation of long-term sovereign rates in the euro area. Precisely, it's an international bond portfolio choice model with two countries which generalizes the traditional results of the term structure interest rates theory. Particularly, except for traditional properties, long-term equilibrium rates depend as well as on the anticipated variances and covariances, considered as a component of a volatility risk premium, of future bond yields. By using CDS as a variable to control default risks, the model is tested empirically over the period January 2006 to September 2016. We can conclude that the ECB's QE programs beginning from March 2015, have accelerated the "defragmentation process" of the European bond markets, already initiated since the OMT. However, according to the test à la Forbes and Rigobon, it seems difficult to affirm that QE programs have led to a significant increase in the conditional correlations between bond markets. In a supplementary empirical test, we show that QE has significantly reduced the sensitivities of bond yield spreads to the premiums paid on sovereign CDS.

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Chapter 1

Introduction to interest rates, financial contagion and flight-to-quality

> A newcomer to the theory of bond pricing would be struck by the enormous variety of models in use and by the variety of methods used to study them.
>
> *Gibson et al. (2010)*

1.1 Introduction

The interest rate is an essential indicator in the economy. Due to its competencies in determining the cost of capital and in controlling the financial risks, it is considered one of the most important determinant factors for pricing contingent claims. Therefore, numerous researchers have published tons of papers in this field.

The rest of this chapter is organized as follows: section 2 presents major theories of the term structure of interest rates; section 3 gives a brief introduction to some representative models of the term structure and some calibration methods; section 4 gives a brief introduction to the contagion and flight-to-quality phenomenon during EMU crisis; section 5 discusses the relation between interest rate and European sovereign debt crisis.

1.2 Interest Rate Theories

Term structure of interest rates theories describe the relations between bond yields and different terms or maturities. Traditional theories focus on studying the shape of the yield curve and its fundamental determinants. Conventionally, they can be classified into four categories.

1.2.1 Expectation Theory

The expectation theory is one of the oldest term structure of interest rates theories. (Fisher, 1896) is believed to be the first who brought up this hypothesis. Progressively, this theory is well developed by Hicks (1946), Lutz (1940), (Baumol et al., 1966), Meiselman (1962) and Roll (1970). The main assumption behind this theory is that investors do not prefer bonds of one maturity over another. Therefore, only the expected return is taken into consideration. According to the expectation theory, the term structure is driven by the investor's expectations on future spot rates. The forward rate is an unbiased estimator of the future prevailing spot rate. In another word, the spot rate of a long-term bond should be equal to the geometric average of several expected short rates.

$$R(t,T) = \frac{1}{T-t}\int_t^T E_t(r(s))ds \tag{1.1}$$

Where $E_t(r(s))$ presents the expected short rates and the long-term bond matures at time T. However, it doesn't exist a clear boundary between long-term rates and short rates. The choice of boundary should depend on the theoretical framework and objective specialties of the asset studied. Conventionally, people consider that long-term interest rates are rates with a maturity longer than one year.

Empirically, both evidence and critiques are found. Fama (1984b) states that the regressions provide evidence that the one-month forward rate has power to predict the spot rate one month ahead. Mankiw and Miron (1986) find that prior to the founding of the Federal Reserve System in 1915, the spread between long rates and short rates has substantial predictive power for the path of interest rates; after 1915, however, the spread contains much less predictive power. Fama and Bliss (1987) find little evidence that forward rates can forecast near-term changes in interest rates. They state that when the forecast horizon is extended, however, forecast power improves, and 1-year forward rates forecast changes in the 1-year spot rate 2 to 4 years ahead. Froot (1989) confirms what is found by Fama and Bliss , he finds out for short maturities, expected future rates are rational forecasts. The poor predictions of the spread can therefore be attributed to variation in term premium. For longer-term bonds, however, they are unable to reject the expectations theory, in that a steeper yield curve reflects a one-for-one increase in expected future long rates. Campbell and Shiller (1991) find that when the yield spread is high the yield on the longer-term bond tends to fall, contrary to the expectations theory; at the same time, the shorter-term interest rate tends to rise, just as the expectations theory requires. Campbell and Shiller (1983), Fama (1984a), Fama (1984b), Fama and Bliss (1987), Jones and Roley (1983), Mankiw and Summers (1984) test the after-war data. Evidence shows that it exists a huge gap between observed 1-year Treasury rates and predicted rates by expectation hypothesis. Modigliani and Sutch (1966), Modigliani and Shiller (1973), Fama (1976) and Kane and Malkiel (1967) all reached the same conclusion, that is the expectation is one of the most important factors in the formation of interest rates, however, others factors such as liquidity premium play an essential role as well. This explains why the term structure of interest rates are upward sloping.

1.2.2 Liquidity Preference Theory

The liquidity preference theory is firstly proposed and interpreted by Hicks (1946). This theory supposes that financial markets are risky and all investors are risk-averse. Therefore investors tend to prefer short-term maturities and will require a premium to engage in long-term lending. However, borrowers prefer long-term securities in order to ensure themselves having a stable funding source, so they agree to pay this premium. The term structure can be given as follows

$$R(t,T) = \frac{1}{T-t}\Big[\int_t^T E_t(r(s))ds + \int_t^T L(s,T)ds\Big] \tag{1.2}$$

where $L(s,T) > 0$ presents the instantaneous term premium at time t for a bond maturing at time T. Hicks thinks the investment risks rise along with the term, so the liquidity premium L is positively correlated with time T. Thus, we have $L(s,T) > L(s,T-1) > \cdots > L(s,2) > 0$, for all $T \geq 2$.

As many other theories, some concerns related to the liquidity preference theory have been brought up. One concern is that modern portfolio theory considers that bonds have risk premium, and liquidity premium are associated with investors' liquidity preferences, so they argue that the risk of bond price variation also affects liquidity premiums. In another word, liquidity premium is not an exogenous determinant of interest rates. A representative study is conducted by Engle and Ng (1991). They find that the combined effect of the expectation component and the premium component can produce yield curves of the commonly observed shapes. However, the yield curve is more likely to be monotonically increasing when volatility is high. When volatility is low, the premium component is not very important relative to the expectation component.

The liquidity preference theory has been accepted broadly, but there doesn't exist a consensus about the nature of liquidity premium. The debate is mainly manifested in two aspects: first, is liquidity premium always positive? Merton (1973a), Long (1974), Breeden (1979), and Cox et al. (1985) consider that the liquidity premium is time-varying but not necessarily a time-increasing function, which implies that the liquidity premium may be negative. Fama and Bliss (1987) suggest that the liquidity premium are typically nonzero and vary between positive and negative values. This variation seems to be related to business cycle. Liquidity premium are mostly positive during good times but mostly negative during recessions. Second, what are the sizes of liquidity premium? McCulloch (1975), Roll (1970), and Throop (1981) have discussed this issue. Some believe that liquidity premium vary from about 0.54% to 1.56%; others believe that for even a longer period of time, liquidity premium will not exceed 0.5%; some results show that liquidity premium even will decrease with the time.

1.2.3 Market Segmentation Theory

The market segmentation theory states that the bond market with different maturity is completely distinct and segmented.It comes from the non-efficiency of markets and the bounded rationality of investors. It is firstly brought up by Culbertson (1957) who argues that hypothesises of the expectation theory are not validated in reality, so the yield curves of expectations don't hold. In fact, the whole market is composed by bonds of different maturities and they are not substitutable for each other. The term structure is still give by

$$R(t,T) = \frac{1}{T-t}\Big[\int_t^T E_t(r(s))ds + \int_t^T L(s,T)ds\Big] \tag{1.3}$$

but the bond with a given maturity is determined by the supply and demand of the bond in its segment without influenced by the yield of bonds in other segments.

The market segmentation theory argues that borrowers and lenders will limit their transactions to a specific period of time for reasons such as: first, due to certain rules and regulations of the government, borrowers and lenders are required to trade only financial products with a specific maturity; second, in order to avoid certain risks, participants limit their investment to some specific products; third, as what is explained in "A behavioral model of rational choice" by Simon (1955), certain investors optimize the satisfaction rather then the profits.

Among the above three reasons, the regimes and regulations of governments are the strongest constraints. The borrowers and the lenders bound by this reason will not make any substitutions between bonds with different terms, which will form a strong market segmentation. The risk-aversion is the second strong constraint, market segmentation out of this reason is semi-strong. Except for these two reasons above, the rest of the reasons don't have strong constraints for the investors, thus, it forms a weak market segmentation.

The market segmentation theory divides financial markets into short-term, medium-term and long-term markets. The main participants in the short-term market are commercial banks, non-financial institutions and money market funds, they pay more attention to the security of principals. Long-term market participants are mainly those institutions with longer debt maturity structures, such as life insurance companies, pension funds, etc. These institutions have a strong risk aversion, so they pay attention to not only the security of principals but also the payment of coupons. While short-term and long-term market investors have different investment motivations and objectives, participants in both markets are similarly constrained by laws, regulations, and risk aversion requirements. Therefore, the market segmentation of these two markets is strong, investors will not make any substitutions between bonds. In contrast, medium-term market participants are more complex, there doesn't exist a dominant identifiable group, thus market segmentation here is weak.

Many empirical papers have confirm the market segmentation theory. For example , Feroz and Wilson (1992) show that their regression results based on a sample of 119 new municipal bond issues, partitioned on a measure of segmentation (regional vs national underwriter), are consistent with the hypothesis that the association between the quality and quantity of financial disclosure and interest costs is stronger for municipalities that issue bonds in local or regional markets than for those that issue bonds in the national market. Research by Rivers and Yates (1997) verifies differences in the determinants of net interest costs between a sample of small cities and a sample of large cities issuing general obligation debt during 1982. Almeida et al. (2016) find that segmentation alone is able to improve long-horizon term structure forecasts when compared to non-segmented models. Moreover, the introduction of Error Correction Model in latent factor dynamics of segmented models makes them particularly strong to forecast short-maturity yields.

1.2.4 Preferred Habitat Theory

The preferred habitat theory is firstly brought up by Modigliani and Sutch (1966). They argue that different categories of investors have their own habitat preferences, which makes investors generally trade in their own preferred market. However, they do not lock themselves in a particular market segment. As long as another market shows a significantly higher return, they will give up the original investment habits and turn into this more profitable market. According to the preferred habitat theory, investor and borrowers have different specific term-horizons. The term structure is still written as follows

$$R(t,T) = \frac{1}{T-t}\Big[\int_t^T E_t(r(s))ds + \int_t^T L(s,T)ds\Big] \tag{1.4}$$

but the liquidity premium of bonds with different maturities can be positive, negative or zero.

The preferred habitat theory is actually a compromise between the liquidity preference theory and the market segmentation theory. In one hand, it recognizes that markets are segmented that investors have various investment objectives and preferences for bonds with different maturities, so there exists both short-term and long-term investors in the markets. Obviously, this is different from the liquidity premium theory, which argues that, in a natural state, all market participants tend to invest in short-term bonds in order to reduce the risks. In the other hand, the preferred habitat theory suggests that markets are not totally segmented. As long as another market shows a significantly higher return, investors will give up the original investment habits and turn into this more profitable market. This is different from the market segmentation theory that argues no matter what kind of reasons, investors of one market will never be engaged in another market.

In terms of empirical evidence, some debates have been launched. Modigliani and Sutch (1966) consider that the reason why investors switch from a bond to another is due to the fact that they choose a specific bond term, not as a legal constraint or regulation like under the strong market segmentation assumption, but rather out of their own consumption preferences. Nonetheless, Cox et al. (1981) give a different interpretation from that of Modigliani and Sutch. They show that it is not preference for consumption at different times which creates "habitats," but rather the degree of risk aversion. Dobson et al. (1976) analyze eight alternative models, and find the Modigliani-Sutch model is the most flexible of all those considered. Therefore, they imply that the preferred habitat theory is validated.

1.3 Interest Rate Models

Counting the existing interest rate models is like counting stars. This section will only present several well-known and broadly used models or frameworks in the literature. Traditional term structure of interest rates model, also known as Binomial model, is based on a single discrete state variable, the short rate. Modern term structure of interest rates models are more complex, generally they are based on the two fundamental modern financial theory conceptions: equilibrium and arbitrage-free. Under these two hypothesises, vast various models emerged. In order to better understand these models, how to categorize them rightly is essential. Unfortunately, these models are not mutually exclusive but frequently overlapping.

1.3.1 Discrete-time or Continuous-time ?

In fact, most interest rates models were specified in a discrete-time in early times, however, the majority recent models are under continuous-time framework. Without doubt, the power of continuous-time stochastic calculus gives more accurate theoretical solutions but at the price of more sophisticated mathematical skills. Backus et al. (1998) argues that although discrete-time is occasionally less elegant than continuous-time, it makes fewer technical demands on users. As a result, we can focus our attention on the properties of a model, and not the technical issues raised by the method used to apply it. Models such as Vasicek model, CIR model, Ho and Lee model, and HJM model, they are originally presented in continuous-time when firstly brought up, but they can have discrete-time approximations as well.[1]

In terms of empirical research, most models under continuous-time framework need discrete-time approximations. The reason is simple, most empirical tools such as GARCH, VAR, or GMM, they can not handle continuous-time issue, and the frequency of observable data is always daily or above. Therefore, the models originally constructed in discrete-time can be implemented more easily in practice. It would be reasonable to say that continuous-time models are certainly the trend but discrete-time models still have their places.

The models which are going to be introduced in this chapter are either presented in continuous-time or discrete-time, which depends on how these models had been constructed by the original author in the first time. As is discussed above, to some extend, they can be transformed from one to another.

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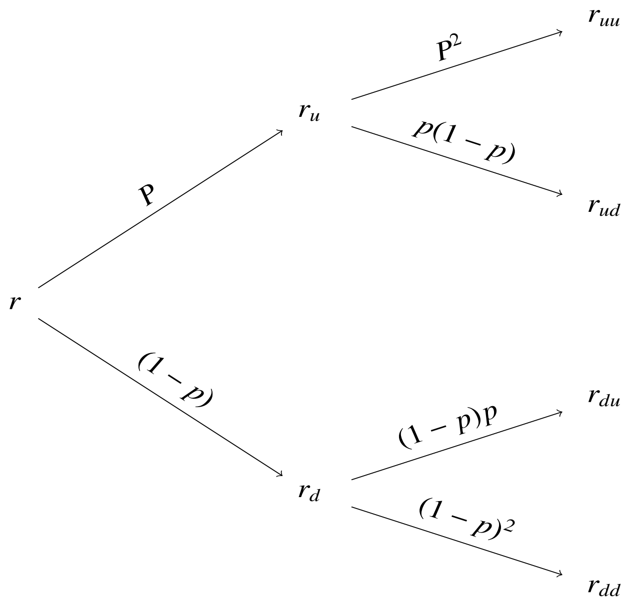
[1] For a large number of detailed discrete-time approximations, see Discrete-time models of bond pricing by Backus et al. (1998).

1.3.2 Binomial Models

Binomial model is used to represent bond price in discrete-time dimension, as the bond prices change randomly and are subject to interest rates. This model is easily interpreted and implemented, that is the reason why it is broadly used in school and industry. In binomial model, the state can either go "up" or "down" over one period of time. In a risk-neutral environment, suppose the interest rate of a zero-coupon r at time t can move with probability p to r_u and probability $1 - p$ to r_d at time $t + 1$, so we can write r_{t+1} as follows

$$r_{t+1} = \begin{cases} r_{tu} \text{ with probability p} \\ r_{td} \text{ with probability 1-p} \end{cases} \tag{1.5}$$

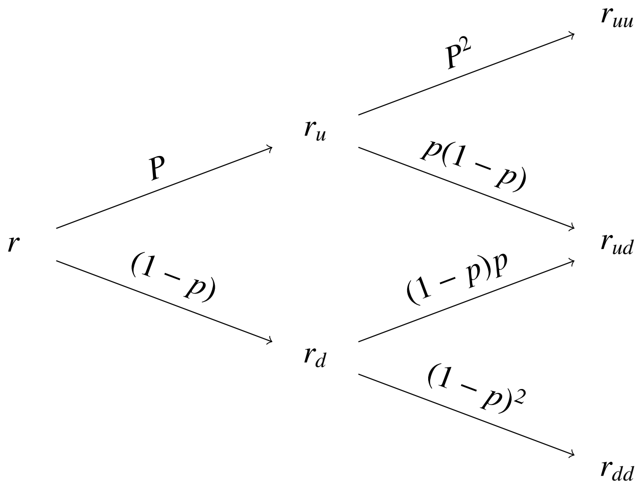
An example of a three-period interest rate tree can be presented as follows



For every single period, the expected interest rate can be presented as follows

$$E_t(r_{t+1}) = pr_{tu} + (1-p)r_{td} \tag{1.6}$$

Generally, in period n, the interest rates can take on 2^n values. Without doubt, it is very time-consuming and computationally inefficient. Therefore, a recombining tree model is proposed. An example of a three-period recombining tree can be presented as follows



In this way, an upward-downward sequence leads to the same result as a downward-upward sequence, which means $r_{ud} = r_{du}$. So we will have only $(n + 1)$ different values at period n, which reduces significantly computational requirements.

1.3.3 Equilibrium Models

A equilibrium model of term structure interest rates is under the framework that in a given economy, a representative investor tries to maximize his utility function, and risk premium and other assets are priced endogenously. The term structure of interest rates are derived assuming the market is at equilibrium.

1.3.3.1 Single factor equilibrium models

In single factor models, it's assumed that all of the information about the term structure at any time is included in a single chosen factor. Most of the time, it is the short-term interest rate $r(t)$. These models are similar in taking three state variables to pilot the dynamic behavior and one control variable to evaluate the risk. With these four components, theories can build relations between long-term rates ($R(t, T)$) and short rates ($r(t)$). Single factor models start by specifying the stochastic differential equation, of which the general form of short-term rate is given below

$$dr(t) = A(r)dt + B(r)dW_t \tag{1.7}$$

where W_t is a Wiener process, modelling the random market risk factor. $A(r)$ denotes the drift term that describes the expected change in the interest rate at that particular time and $B(r)$ presents the diffusion term. $A(r)$ and $B(r)$ depend only on the level of spot rate r, which means they are independent of time t. There are three important single factor models: Merton model, Vasicek model, and CIR model.

Merton model is firstly brought up by Merton (1973b), this model assumes that the short-term interest rate process has a constant expected growth rate μ and a constant volatility σ. The short-term rate can be written as follow

$$dr(t) = \mu dt + \sigma dW_t \tag{1.8}$$

The explicit solution to Merton's model is

$$r(t) = r(s) + \mu t + \sigma \int_s^t dW_s \tag{1.9}$$

And the term structure is give by the sum of short rates and a quadratic function of the time to maturity

$$R(t, T) = r(t) + \frac{(T - t)(\mu - \lambda\sigma)}{2} - \frac{(T - t)^2\sigma^2}{6} \tag{1.10}$$

Where λ is the constant risk premium. Nonetheless, this model is not sufficient realistic. Logically, when the interest rate is higher, the costs of financing rise so the demand declines, therefore the interest rate will drop; conversely, when the interest rate is lower, the demand for financing will increase so the interest rate will rise as a result. That is to say, in long-term, the interest rate should converge at an equilibrium level, this phenomenon is called mean reversion of interest rate. However, Merton's model doesn't converge in the long run due to the assumption in drift term $A(r)$.

Vasicek model is firstly proposed by Vasicek (1977) and this is the very first term structure model which satisfies the property of mean reversion. In this model, the short-term rate follows an Ornstein-Uhlenbeck process, which can be presented as follow

$$dr(t) = a(\theta - r(t))dt + \sigma dW_t \tag{1.11}$$

The explicit solution to Vasicek's model is

$$r(t) = \theta + (r(s) - \theta)e^{-a(t-s)} + \sigma \int_s^t e^{-a(t-u)} dW(u) \tag{1.12}$$

And the term structure is given by

$$\begin{aligned} R(t,T) = &-\frac{1}{T-t}\Big[\frac{1}{a}(e^{-a(T-t)} - 1)r(t) + \frac{\sigma^2}{4a^3}(1 - e^{-2a(T-t)}) \\ &+ \frac{1}{a}(\theta - \frac{\lambda\sigma}{a} - \frac{\sigma^2}{a^2})(1 - e^{-a(T-t)}) - (\theta - \frac{\lambda\sigma}{a} - \frac{\sigma^2}{2a^2})(T-t)\Big] \end{aligned} \tag{1.13}$$

where a, θ, σ λ are positive constants. $a(\theta - r(t))$ represents the expected change in the interest rate at t (drift factor), a is the speed of reversion, θ presents the long-term level of the mean σ denotes the volatility at the time. When $r(t)$ exceeds θ , the expected variation of $r(t)$ becomes negative and then $r(t)$ should return to its equilibrium level θ at the speed of a. An important critic about this model over quite a long time is that, under Vasicek's model, interest rates can theoretically be negative. However, the zero-bond assumption is being constantly challenged since the European sovereign debt crisis.

CIR model is firstly proposed by Cox et al. (1985). The CIR model has a similar structure as Vasicek's model but with differences in the behavior of the diffusion term $B(r)$. In Vasicek's model, the conditional variance and risk premium are supposed constant, while in CIR model, they vary with the short-term rate $r(t)$. More precisely, $\lambda(r,t) = \lambda\sqrt{r(t)}$ which means the short-term rate falls and approaches zero, the diffusion term also approaches zero. In this model, the drift term dominates the diffusion term and pulls the short-term rate back towards its equilibrium level in the long run. This guarantees the short-term rate can not fall below zero. The short-term rate dynamic can be presented as follow

$$dr(t) = a(\theta - r(t))dt + \sigma\sqrt{r(t)}dW_t \tag{1.14}$$

where $W(t)$ is a Q-Brownian motion. The unique positive solution to the dynamic of short-term rate is

$$r(t) = \theta + (r(s) - \theta)e^{-a(t-s)} + \sigma e^{-a(t-s)}\int_s^t e^{a(t-u)}\sqrt{r(u)}dW_u \tag{1.15}$$

Finally, the long-term rate $R(t,T)$ linearly depends on $r(t)$.

$$R(t,T) = \frac{B(t,T)r(t)}{T} - \frac{(\sqrt{(a+\lambda\sigma)^2 + 2\sigma^2} + a + \lambda\sigma)lnA(t,T)R(t,\infty)}{2a\theta T} \tag{1.16}$$

where $B(t,T)$ is the bond price. Szatzschneider (2001) criticize that, in CIR model, the appearance of a local time forces calculation of bonds expiring in exponential time. Moreover, they are hard to be put into practice because solutions are given by complicated formulas. These three important single factor models are summarized in the following table.

TABLE 1.1: Summary of Single Factor Models

Models	Drift Terms A(r)	Diffusion Terms B(r)	General Forms
Merton	μ	σ	$dr(t) = \mu dt + \sigma dW_t$
Vasicek	$a(\theta - r)$	σ	$dr(t) = a(\theta - r(t))dt + \sigma dW_t$
CIR	$a(\theta - r)$	$\sigma\sqrt{r(t)}$	$dr(t) = a(\theta - r(t))dt + \sigma\sqrt{r(t)}dW_t$

1.3.3.2 Multiple factor equilibrium models

In single factor models, the short term rate r is the only explanatory variable. Single factor models have received many critiques which were well summarized by Gibson et al. (2010): (i) the long term rate is a deterministic function of the short term rate; (ii) these models are considered as lacking of accuracy when determining prices since differences between estimated and real prices do exist; (iii) furthermore, standing from a macroeconomic point of view, it is unreasonable to consider that the term structure is solely guided by the short term interest rate; (iv) finally, it would be difficult to obtain accurate volatility structures for the forward rates.

In order to respond to the critiques brought up, multiple factor models have been quickly developed trying to improve and correct possible bias existing in single factor models, which describe better the dynamics of the real interest rates at the cost of sophisticated structures even without analytical solutions. Due to the sophistication of multiple factor models, only representative models and essential results are presented in the following paragraphs.

Brennan-Schwartz model is a two-factor model firstly brought up by Brennan and Schwartz (1979). They think term structure of interest rates should depend not only on short term rates r but also on long term rates l, where the long term rate is defined as follows

$$l(t) = \lim_{T\to\infty} R(t,T) \tag{1.17}$$

The dynamics of short term rates and long term rates are constructed as

$$\begin{cases} dr(t) = \mu_1(r, l, t)dt + \sigma_1(r, l, t)dW_{1,t} \\ dl(t) = \mu_2(r, l, t)dt + \sigma_2(r, l, t)dW_{2,t} \end{cases} \tag{1.18}$$

where $W_{1,t}$ and $W_{2,t}$ are standard Wiener processes; μ_1, μ_2, σ_1, and σ_2 are functions of r, l, and t. The core assumption here is that the level and volatility of short term rates r would have influence on long term rates l.

Fong-Vasicek model is a two-factor model derived by Fong and Vasicek (1991). They developed the old Vasicek model to a two-factor scenario, which the variance of changes in short term rates $v(t)$ becomes the second deterministic factor. The principal reason is that variance of the short rate changes is believed to be a key element in the pricing of fixed-income securities, in particular interest rates derivatives. Under their hypothesis, dynamics of short term rate and its variance are modeled as follows

$$\begin{cases} dr(t) = a(\overline{r} - r(t))dt + \sqrt{v(t)}dW_{1,t} \\ dv(t) = b(\overline{v} - v(t))dt + c\sqrt{v(t)}dW_{2,t} \end{cases} \tag{1.19}$$

where $\overline{r}$ and $\overline{v}$ are respectively the long term means of short term rate and its variance. Clearly, both of them satisfy the property of mean reversion mentioned above, at speed of respectively a and b. The two Wiener processes $W_{1,t}$ and $W_{2,t}$ are correlated.

Longstaff-Schwartz model is firstly proposed by Longstaff and Schwartz (1992). It is a two-factor general equilibrium model where a representative investor should choose between investing or consuming the only good in the economy, whose price is constructed as follows

$$\frac{dP(t)}{P(t)} = (\mu X(t) + \theta Y(t))dt + \sigma\sqrt{Y(t)}dW_{1,t} \tag{1.20}$$

where $X(t)$ and $Y(t)$ are two economy-specific factors. Moreover, the utility function of the representative investor is logarithmic and the two chosen factors are the same as Fong and Vasicek (1991), precisely, the short term rate $r(t)$ and its variance $v(t)$. However, these two factors are not directly associated but presented as a linear combination of the two economic factors $X(t)$ and $Y(t)$ of which the dynamics are given by

$$\begin{cases} dX(t) = (a - bX(t))dt + c\sqrt{X(t)}dW_{1,t} \\ dY(t) = (d - eY(t))dt + f\sqrt{Y(t)}dW_{2,t} \end{cases} \tag{1.21}$$

where $W_{1,t}$ and $W_{2,t}$ are not correlated and a, b, c, d, e and $f > 0$. Longstaff and Schwartz don't give any interpretation for these two economy-specific factors $X(t)$ and $Y(t)$ but states that: (i) they can be related to observable quantities; (ii) they should satisfy the property of mean reversion; (iii) they should be non-negative. Furthermore, the short term rate $r(t)$ and its variance $v(t)$ are constructed as a weighted sum of $X(t)$ and $Y(t)$

$$\begin{cases} r(t) = \mu c^2 X(t) + (\theta - \sigma^2) f^2 Y(t) \\ v(t) = \mu^2 c^4 X(t) + (\theta - \sigma^2)^2 f^4 Y(t) \end{cases} \tag{1.22}$$

where by construction, $r(t)$ and $v(t)$ are non-negative. One major advantage of this model is that it allows to get closed-form solution (analytical solution) for the price of a discount bond and a call option on a discount bond. A discrete-time approximation of the this continuous-time framework can be presented as

$$\begin{cases} r_{t+1} - r_t = \alpha_0 + \alpha_1 r_t + \alpha_2 v_t + \epsilon_{t+1} \\ v_t = \beta_0 + \beta_1 r_t + \beta_2 v_{t-1} + \epsilon_t^2 \end{cases} \tag{1.23}$$

where $\epsilon_{t+1} \sim (0, v_t)$. Here the heteroskedasticity depends on short term rates and volatility follows an AR(1), in another word, its current level depends on its lagged value.

TABLE 1.2: Summary of Multiple Factor Models

Models	Factors	General Forms
Brennan-Schwartz	$r(t), l(t)$	$\begin{cases} dr(t) = \mu_1(r,l,t)dt + \sigma_1(r,l,t)dW_{1,t} \\ dl(t) = \mu_2(r,l,t)dt + \sigma_2(r,l,t)dW_{2,t} \end{cases}$
Fong-Vasicek	$r(t), v(t)$	$\begin{cases} dr(t) = a(\overline{r} - r(t))dt + \sqrt{v(t)}dW_{1,t} \\ dv(t) = b(\overline{v} - v(t))dt + c\sqrt{v(t)}dW_{2,t} \end{cases}$
Longstaff-Schwartz	$X(t), Y(t)$	$\begin{cases} dX(t) = (a - bX(t))dt + c\sqrt{X(t)}dW_{1,t} \\ dY(t) = (d - eY(t))dt + f\sqrt{Y(t)}dW_{2,t} \end{cases}$

1.3.4 Arbitrage-free Models

Arbitrage-free models are, to some extent, quite similar with equilibrium models. In an academic point of view, an arbitrage-free model of term structure of interest rates is under the framework that one or many interest rates present in a market where there is no risk-free strategy which could give a positive return with certainty. The price of all contingent claims are derived assuming that there are no arbitrage opportunities on the market. Standing from the point of view of a practitioner, an arbitrage-free model is a model which allows the theoretical price $P(t,T)$, by construction, to match the observed price $P_{obs}(t,T)$, at the time of calibration t.

Ho-Lee model is a discrete-time multiple period binomial model proposed by Ho and LEE (1986). This approach is built in a binomial framework, however, the implied idea is much more inspiring, that is to add time-dependent parameters allowing users to match observed values. Since the volatility is a key factor to the pricing of interest-rate related contingent claims, the extension of volatility parameter σ based on this idea is critical. After a quite long demonstration,[2] the short term rate is given by

$$r_t = r_{t-1} + (f(0,t) - f(0,t-1)) + log(\frac{\pi + (1-\pi)\delta^t}{\pi + (1-\pi)\delta^{t-1}}) - (1-\pi)log(\delta) + \epsilon_t \qquad (1.24)$$

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[2] See Ho and LEE (1986) for full proof.

where

$$\epsilon_t = \begin{cases} (1-\pi)log(\delta) \text{ if price goes up} \\ -\pi log(\delta) \text{ if price goes down} \end{cases} \tag{1.25}$$

so that $E(\epsilon_t) = 0$, with π presents the probability that the bond price goes up and $f(t, T)$ denotes the spot forward rate at time t maturing at time T.

The equivalent continuous-time version of this model is developed by Dybvig (1988) and Jamshidian (1991), in which the risk-free rate dynamic is designed as follows

$$\begin{cases} dr(t) = \theta(t)dt + \sigma dW_t \\ \theta(t) = \frac{\partial}{\partial T} f(0, T) + \sigma^2 T \end{cases} \tag{1.26}$$

where W_t is a Wiener process under the equivalent martingale measure Q. This extension can be considered as a general version of Merton's model in which θ is constant. The solution for this equation is

$$r(t) = f(0, t) + \frac{1}{2}\sigma^2 t^2 + \sigma W_t \tag{1.27}$$

However, this model does not incorporate any mean reversion feature and possibly leads to explosive or negative values.

Hull-White model proposed by Hull and White (1990) has largely developed the Ho-Lee model in two aspects: (i) now the model is mean reverting; (ii) the impact of diffusion term reduces as time goes by. In other word, the original stochastic process becomes a certain process at long run, which makes the long term rates more predictable than short term rates. The dynamic of short term rate is given by

$$\begin{cases} dr(t) = (\theta(t) + u(t) - r(t))dt + \sigma_1 dW_{1,t} \\ du(t) = -bu(t)dt + \sigma_2 dW_{2,t} \end{cases} \tag{1.28}$$

where $E(dW_{1,t}, dW_{2,t}) = \rho dt$ and $u(t)$ is a component of the mean reversion level and it is mean reverting to 0 at a speed of b with $u(0) = 0$. Due to the complicacy of this model, more explanations and can be found in Hull and White (1994). Nonetheless, this model can still give theoretically negative values.

BDT model is firstly proposed by Black et al. (1990). This model assumes the distribution of short term rates is log-normal. In one hand, this hypothesis allows to avoid the possible negative rates in Hull-White model. In the other hand, it allows volatility to be presented as a percentage, which is consistent with the conventional expression in real markets. With time dependant volatility, this model can be written as

$$dln(r(t)) = (\theta(t) - aln(r(t)))dt + \sigma_r(t)dW_t \tag{1.29}$$

Bali (2003) developed this model into two-factor, and the discrete-time approximation can be described as

$$r_t - r_{t-1} = (a_1 r_{t-1} + a_2 r_{t-1} ln(r_{t-1}) + \frac{1}{2} h_{t-1} r_{t-1}) + r_{t-1}^{\gamma} \epsilon_t \tag{1.30}$$

with $\epsilon_t = \sqrt{h_t z_t}$, $z_t \sim N(0,1)$ and h_t follows a GARCH or TS-GARCH process.[3] Empirically, Bali finds two-factor BDT model outperforms GARCH models due to the fact that GARCH models do not capture the channel between interest rates and volatility.

HJM model is firstly proposed by Heath et al. (1992). It would be more accurate to say that HJM is more a framework rather than a model. It refers to a family of models that are derived by directly modeling the dynamics of instantaneous forward-rates. The main contribution of this framework is to confirm the fundamental relations: (i) between the drift term and volatility parameters; (ii) between the drift term and diffusion term of the forward rate dynamics in a arbitrage-free context. Nonetheless, HJM models are not Markovian.[4] Therefore, discrete-time approximations and Monte-Carlo methods are often implemented by practitioners.[5] The dynamic of forward rates is given by

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[3] See more descriptions about GARCH models in section 1.3.5

[4] If the conditional probability distribution of future states of the process (conditional on both past and present states) depends only upon the present state

[5] The fact that HJM models are not Markovian makes it impossible to use the PDE-based computational approach for pricing derivatives

$$df(t,T) = \mu_f(t,T)dt + \sigma_f(t,T)dW_t \tag{1.31}$$

where W_t is a d-dimensional standard Q-Brownian motion. Here $\mu_f(t,T)$ and $\sigma_f(t,T)$ are adapted processes for each T. The solution to this differential equation is

$$f(t,T) = f(0,T) + \int_0^t \mu(s,T)ds + \int_0^t \sigma(s,T)dW(s) \tag{1.32}$$

next, we set all values at time 0 equals to the observed forward rates $f^*(0,T)$, which means $f(0,T) = f^*(0,T)$. By design, all theoretical values match all observed values so the model is arbitrage-free.

Table 1.3: Summary of Arbitrage-free Models

Models	General Forms	Mean Reverting	Non-negative Rates
Ho-Lee	$dr(t) = \theta(t)dt + \sigma dW_t$		
Hull-White	$dr(t) = (\theta(t) + u(t) - r(t))dt + \sigma_1 dW_{1,t}$	✓	
BDT	dln(r(t))=$(\theta(t) - aln(r(t)))dt + \sigma_r(t)dW_t$	✓	✓
HJM	df(t,T)=$\mu_f(t,T)dt + \sigma_f(t,T)dW_t$	✓	

1.3.5 Volatility Models

Modelling and forecasting volatility has been a key topic of extensive empirical and theoretical research over the last couple of years. Volatility is observed as one of the most important concepts in the field of Finance. It is measured as the standard deviation or variance of returns. Among many volatility models, only one very representative model — GARCH model, will be presented in this section.[6]

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[6] Except for GARCH model, there exists many other volatility models such as ARCH models and TVP-level models, but they are not broadly used as GARCH-type models.

GARCH model stands for Generalized Autoregressive Conditional Heteroscedasticity model, developed by Bollerslev (1986), which generalizes Autoregressive Conditional Heteroscedasticity (ARCH) model proposed by Engle (1982). By the year 2005, Hansen and Lunde (2005) shows there exists already more than 330 different GARCH models. They can be classified into some subcategories such as EGARCH, IGARCH, TGARCH, GJR-GARCH, NGARCH, AVGARCH, APARCH, etc.[7]

Although numerous types exist, all GARCH-type models have the similar set up, because they share the same idea that is to use values of the past squared observations and past variances to model the variance at time t. X_t is called GARCH(q,p) process if

$$X_t = \sigma_t Z_t \tag{1.33}$$

where (Zn) is a sequence of i.i.d. random variables and $Z_t \sim N(0,1)$. Furthermore, the process of σ_t is non-negative given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + ... + \alpha_q X_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_p \sigma_{t-p}^2 \tag{1.34}$$

where $\alpha_i > 0$ for $i = 0, ..., q, \beta_i > 0$ for $i = 1, ..., p$. Therefore, a very basic GARCH(1,1) model can be presented as

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{1.35}$$

Two methods frequently used for GARCH estimations are Maximum Likelihood Estimation (MLE) and Generalized Method of Moments (GMM). For different uses and specifications of models, different estimation methods and algorithms should be applied properly. The selection should also depend on different biases under various sample sizes assumption.[8]

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[7] For more GARCH-type models, see for example Bauwens et al. (2006)

[8] For more detailed selection of estimations methods, see for example PRINC and ŠKOLUDA (2012)

1.3.6 Portfolio Theory-based Models

No one can ignore the portfolio effects when dealing with bond pricing topics. Based on expectation hypothesises and portfolio theory à la Markowitz, a domestic portfolio theory-based framework is proposed by Jones and Roley (1983), Mankiw et al. (1986), Artus (1987) and Artus and Kaabi (1995).

Under this framework, the functioning of the bond market is supposed to depend on several hypothesises regarding investor demand and the evolution of the bond supply. The demand of bonds results from the behavior of risk-averse investors with a one-period investment horizon. In each period , they reallocate their total wealth between a risk-free asset, represented by a short-term rate r_t and a risky asset, represented by a long term bond with a sensitivity σ. The optimization program of the representative investor under mean-variance criterion can be written as

$$\underset{(\alpha_t)}{\text{Max}}\, U = r_t + \alpha_t\Big[E_t(H_t \mid I_t) - r_t\Big] - \frac{\theta}{2}\Big[\alpha_t^2 V_t(H_t \mid I_t)\Big] \tag{1.36}$$

where α_t denotes the share of total wealth invested in the bond market. By design, $1-\alpha_t$ is invested in short term rate r_t. θ presents the absolute risk-aversion. H_t is the return of the bond which depends on the price over this period. Approximately[9], we have

$$H_t = R_t - \sigma(R_{t+1} - R_t) \tag{1.37}$$

and we can easily derive $E_t(H_t) = (1+\sigma)R_t - \sigma E_t(R_{t+1})$ and $V_t(H_t) = \sigma^2 V_t(R_{t+1})$. The solution to this program is

$$\alpha_t = \frac{E_t(H_t \mid I_t) - r_t}{\theta V_t(H_t \mid I_t)} \tag{1.38}$$

The bond demand relative to total wealth is positively related to the expected excess return, meanwhile negatively related to the risk-free rate, the absolute risk-aversion and

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[9] For more detailed explanations, see for example Jones and Roley (1983), Mankiw et al. (1986), Artus (1987) and Artus and Kaabi (1995).

the variance of bond return. The bond supply is supposed exogenous and stochastic.

$$O_t = \bar{O} + \epsilon_t \tag{1.39}$$

where ϵ_t follows a AR(1) type process. The partial demand-supply equilibrium is give by equations 1.38 and 1.39, which can be written as

$$E_t(H_t - r_t) = (\bar{O} + \epsilon_t)\theta V_t(H_t \mid I_t) \tag{1.40}$$

By replacing $E_t(H_t)$ and $V_t(H_t)$, we can obtain

$$R_t = \frac{1}{1+\sigma}\Big[r_t + \sigma E_t(R_{t+1} + \theta\sigma^2)V_t(R_{t+1}(\bar{O} + \epsilon_t))\Big] \tag{1.41}$$

By substitution, the long term equilibrium rate can be described as

$$\begin{aligned} R_t = & \frac{r_t}{1+\sigma} + \frac{1}{1+\sigma}\Big[\sum_{i=1}^{\infty}(\frac{\sigma}{1+\sigma})^i E_t(r_{t+i})\Big] \\ & + \frac{1}{1+\sigma}\Big[\sum_{i=1}^{\infty}(\frac{\sigma}{1+\sigma})^i E_t[\theta\sigma^2 V_t(R_{t+1})(\bar{O} + \epsilon_t)]\Big] \\ & + \frac{1}{1+\sigma}\theta\sigma^2 V_t(R_{t+1})(\bar{O} + \epsilon_t) \end{aligned} \tag{1.42}$$

This is the traditional domestic portfolio theory-based model. The long-term rates depend on the spot short-term rates, anticipated future short-term rates and a risk premium. The risk premium is an increasing function of the following determinants: (i) investor's risk-aversion; (ii) bond sensitivity; (iii) current and future bond supply; (iv) conditional variance of future long-term rates. Recently, Martin and Zhang (2017) generalize this traditional model into a two-economy case, which will be presented in chapter 3.

1.3.7 Forecasting Capability

In terms of forecasting capabilities, among tremendous models, very little evidence shows that one of the models is capable of outperforming others. The choice of model depends on the specific use of the model. However, both Chan et al. (1992) and Bali (2003) conclude that the two most determining factors in an interest rate model are the short-term interest rates and volatility of interest rates variations. That is to say, the most accurate models are those in which interest rates are associated to the volatility.

1.3.8 Calibration Methods

Tremendous attempts have been spent on calibration and estimation of the interest rate models discussed above, whether through the interest rate or through interest-rate related derivatives. All calibration methodologies to be presented in this section has its advantages and disadvantages. When facing the real word problematics, the very strict applicability assumptions of these methods can hardly be fulfilled. Sequentially, some strong simplifications have been made, which could generate big errors.

1.3.8.1 Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is believed to be widely popularized by Ronald Fisher between 1912 and 1922.[10] Through given simple observations, MLE allows to estimate the parameters of a statistical model by calculating the parameter values which can maximize the likelihood. In an independent and identically distributed sample, where X_1, X_2, X_3,..., X_4 have joint density, the likelihood function to maximize is

$$L(\theta; x_1, ..., x_n) = \prod_{i=1}^{n} f(x_i \mid \theta) \tag{1.43}$$

with x_i are simple observations. In practice, logarithm of the likelihood function is more applied due to its computational convenience. This function can be re-written by

$$lnL(\theta; x_1, ..., x_n) = \sum_{i=1}^{n} lnf(x_i \mid \theta) \tag{1.44}$$

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[10] For more evidence, see Aldrich et al. (1997).

By maximizing the objective function, the following maximum likelihood estimator can be obtained

$$\hat{l}(\theta \mid x) = \frac{1}{n} \sum_{i=1}^{n} lnf(x_i \mid \theta) \tag{1.45}$$

where this estimator has the following properties: sufficiency, invariance, consistency, efficiency and asymptotic normality. However, the analytical solutions to many likelihood functions are not available. In this situation, the approximations of the densities are implemented. As a result, the optimality properties of the maximum likelihood estimator are damaged. In terms of optimization algorithms, standard secant updates (DFP and BFGS) and statistical approximations (BHHH) are frequently used in practice.[11]

1.3.8.2 Generalized Method of Moments

Generalized Method of Moments (GMM)[12] proposed by Hansen (1982) is an econometric method broadly used in estimating parameters of the interest rate models, such as in Gibbons et al. (1988), Harvey (1988), Longstaff (1989) and Ait-Sahalia and Kimmel (2010). As this methodology can only deal with discrete-time data set, all models originally constructed in continuous-time need an approximation, which means means in the case of the following stochastic differential equation for the short rate process

$$dr(t) = (a + br)dt + \sigma r^c dW_t \tag{1.46}$$

it needs a discrete-time approximation as

$$r_{t+1} - r_t = a + br_t + \epsilon_{t+1} \tag{1.47}$$

where $E_t(\epsilon_{t+1}) = 0$ and $E_t(\epsilon_{t+1}^2) = \sigma^2 r_t^{2c}$. So the unknown parameters to be defined are $\theta = (a, b, c, \sigma^2)$. Next, we use the equation $E(f_t(\theta)) = 0$ to estimate the parameters

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[11] For more maximum likelihood algorithm discussion, see for example Bunch (1988) and Myung (2003).

[12] For more detailed method introduction, see for example Hansen (2010) and Zsohar (2012).

mentioned above, where

$$f_t(\theta) = (\epsilon_{t+1}, \epsilon_{t+1} r_t, \epsilon_{t+1}^2 - \sigma^2 r_t^{2c}, (\epsilon_{t+1}^2 - \sigma^2 r_t^{2c}) r_t)' \tag{1.48}$$

The GMM consists of (i) replacing $E(f_t(\theta))$ with its sample counterpart

$$g_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} f_t(\theta) \tag{1.49}$$

where N is the number of observations, (ii) choosing parameter estimates to minimize the quadratic form

$$J_N(\theta) = g_N(\theta)' W_N(\theta) g_N(\theta) \tag{1.50}$$

for some positive definite weighting matrix $W_N(\theta)$. Hansen (2010) states that GMM methods are complementary to maximum likelihood methods and their Bayesian counterparts. Their large sample properties are easy to characterize. While their computational simplicity is sometimes a virtue, perhaps their most compelling use is in the estimation of partially specified models or of misspecified dynamic models designed to match a limited array of empirical targets.

1.3.8.3 Markov Chain Monte Carlo

Markov Chain Monte Carlo (MCMC)[13] is a strategy for generating samples, while analyzing the state space using a Markov chain mechanism. Metropolis et al. (1953) are believed to be the first ones who used this method.[14] Several years later, their algorithm is generalized by Hastings (1970). Nowadays, MCMC is frequently used for the pricing of continuous-time multiple factor term structure models with stochastic volatility, such as in Carlin et al. (1992), Broadie and Glasserman (1997) and Glasserman (2013).

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[13] For more detailed method discussion, see for example Brooks (1998) and Geyer (2011).

[14] Hammersley and Handscomb (1964) states that despite a few notable uses of simulation of random processes in the pre-computer era practical widespread use of simulation had to await the invention of computers. Almost as soon as computers were invented, they were used for simulation.

The theoretical base of MCMC is the Hammersley-Clifford theorem by Besag (1974). It implies that the knowledge of two conditional densities $P(X \mid \theta, Y)$ and $P(\theta \mid X, Y)$ allows to fully define the joint distribution $P(\theta, X \mid Y)$. If the analytical solutions to these two conditional densities are available and can be directly drawn from, the algorithm Gibbs sampler can be applied to generate a sequence of random variables called Markov Chain. In other situations, Metropolis-Hastings algorithm will be used. A combination of these phases (Gibbs phases and the Metropolis-Hastings phases) is called the MCMC method.

1.4 Contagion and flight-to-quality during EMU Debt Crisis

Understanding the correlation between of interest rates of different sovereign bond markets is essential for both investors and policy makers. This issue may be particularly important and relevant in European Monetary Union as the governments of the Member States may issue debt, but do not have the ability to monetize or to reduce their excessive long-term debt with inflationary politics.

During the sovereign debt crisis, the correlation of yields between major government bonds has been from positive to negative, and this have changed the behaviors of investors. To be more specific, they have the tendency to increase their allocation to government bonds of the pivots countries, like France, Germany, given by a lower perceived risk, and decrease their allocation to government bonds of periphery countries like Greece, Spain. Unfortunately, most research focuses on either the correlation between different stock markets or between stock markets and bond markets rather than between different bond markets. Two important antagonist concepts of explaining the correlation are contagion and flight-to-quality.

1.4.1 Contagion

1.4.1.1 Contagion definitions

Although the importance of contagion analyses in financial market, there doesn't exist a universal agreement on their definitions in the literature. Masson (1998) defines contagion as a phenomenon where an economy has the potential for both good and bad equilibrium, and an external event – a crisis elsewhere – triggers a move from the first equilibrium to the second. Kodres and Pritsker (2002) define contagion in their model quite generally as a price movement in one market resulting from a shock in another market. Forbes and Rigobon (2002) define the contagion as a significant increase in cross-market linkages after a shock to one country (or group of countries). Corsetti et al. (2005) define contagion as a structural break in the international transmission of financial shocks. Bauwens et al. (2006) consider contagion as a movement in the same direction characterized by strongly increasing correlations in falling stock markets across asset classes.

Nonetheless, numerous definitions of contagion can still be classified into two major categories: fundamental contagion and pure contagion. However, the border is porous between these two notions. Fundamental contagion is often considered as the transmission of shocks across national borders through real or financial linkages. To be more specific, when investors believe one economy could have bad fundamentals, they will reevaluate the riskiness of their investments leading to a portfolio rebalance which may push this economy into crisis. Pure contagion often refers to the international capital flows that are not obviously due to the changes in fundamentals but related to the changes in risk appetites of investors.

1.4.1.2 Proof of bias in correlation coefficient during turmoil period

During a financial crisis, in addition to the contagion, usually the volatility is abnormally high. Forbes and Rigobon (2002) point out that due to the heteroscedasticity over high volatility period, the classic correlation coefficient estimation is biased and inaccurate. Supposing we have two assets X_t and Y_t with respectively stochastic returns x_t and y_t. The relation between theses two returns is given by

$$y_t = \alpha + \beta x_t + \epsilon_t \tag{1.51}$$

where $E(\epsilon_t) = 0$ and $(\epsilon_t^2) = c < \infty$, with c is a constant and $E(x_t\epsilon_t) = 0$. Defining (l) represents the low volatility period and (h) represent high volatility period. Since $E(x_t\epsilon_t) = 0$, OLS estimates of equations 1.51 are consistent for both $\beta^h = \beta^l$. By their definition, we can obtain the following relation

$$\beta^h = \frac{\sigma_{xy}^h}{\sigma_{xx}^h} = \frac{\sigma_{xy}^l}{\sigma_{xx}^l} = \beta^l \tag{1.52}$$

By design, we know $\sigma_{xx}^h > \sigma_{xx}^l$. Therefore, we can obtain $\sigma_{xy}^h > \sigma_{xy}^l$ which implies the covariance is higher in the high volatility period and the increase in both covariance and variance should be proportional. To be more specific, their relation should be linear. However, from equation 1.51, we know

$$\sigma_{yy}^2 = \beta^2\sigma_{xx} + \sigma_{\epsilon\epsilon} \tag{1.53}$$

Since the variance of the residuals is positive ($\sigma_{\epsilon\epsilon} > 0$), the increase in variance of y is less proportional than the increase in variance of x. Therefore,

$$\left(\frac{\sigma_{xx}}{\sigma_{yy}}\right)^h > \left(\frac{\sigma_{xx}}{\sigma_{yy}}\right)^l \tag{1.54}$$

By definition, we know

$$\rho = \frac{\sigma_{xy}}{\sigma_x\sigma_y} = \beta\frac{\sigma_x}{\sigma_y} \tag{1.55}$$

Next, combined with equation 1.54, we can finally obtain $\rho_h^* > \rho_l^*$. This implies the estimated correlation coefficient will increase when entering a turmoil period even when the real correlation remains constant. So the classic method of correlation estimation is biased and misleading when dealing with contagion related problems.

1.4.1.3 Correction of bias in correlation coefficient during turmoil period

As already proved above, according to Forbes and Rigobon (2002), the estimates of correlation coefficient (ρ^*) are over-estimated during a high volatility period and they are conditional on the variance of x. More precisely, if we continue to maintain the hypothesises that $E(\epsilon_t) = 0$ and $E(x_t\epsilon_t) = 0$, this conditional correlation can be written as

$$\rho^* = \rho\sqrt{\frac{1+\delta}{1+\delta\rho^2}} \tag{1.56}$$

where ρ^* is the conditional correlation and ρ is the unconditional correlation, with $\delta = \frac{\sigma_{xx}^h}{\sigma_{xx}^l} - 1$. Equation 1.56 implies over the turmoil period, even when the unconditional correlation remains constant as over the tranquil period, when the variance of x increases, the conditional correlation will increase. This is quite worrying because when a test on contagion shows a significant increase in correlation, in other word a contagion is detected, could be simply an increase in market volatility but nothing else. Thus, the correction is needed and we can find it by resolving equation 1.56, which gives

$$\rho = \frac{\rho^*}{\sqrt{1+\delta[1-(\rho*)^2]}} \tag{1.57}$$

We should note that this correction is under the assumptions that there are no omitted variables ($E(\epsilon_t) = 0$)nor endogeneity between markets ($E(x_t\epsilon_t) = 0$). In the real world, these assumptions may not be fulfilled, however, there isn't any solution to adjust the correlation without making these two hypotheses.

1.4.1.4 Contagion test

Collins and Biekpe (2003) propose a simple contagion test based on the conditional correlation coefficient introduced by Forbes and Rigobon (2002), but slightly different. While Forbes and Rigobon are using estimated sample correlations , Collins and Biekpe use actual sample correlations. Therefore, the contagion is defined as a significant increase in the adjusted correlation over a turmoil period compared with the tranquil period. The following hypotheses is then tested

$$\begin{cases} H_0 : \rho_1 - \rho_2 = 0 \\ H_0 : \rho_1 - \rho_2 > 0 \end{cases} \tag{1.58}$$

The T-stat is constructed as follows

$$t = (\rho_1 - \rho_2)\sqrt{\frac{n_1 - n_2 - 4}{1 - (\rho_1 - \rho_2)^2}} \tag{1.59}$$

with $t_{(0.05, n_1 - n_2 - 4)}$.

1.4.1.5 Contagion related empirical research

Some empirical research related to contagion. Dungey et al. (2006) quantify the contribution of contagion to the bond spreads of the crisis following the Russian bond default in August 1998, by using a latent factor model and a new data set spanning bond markets across Asia, Europe and the Americas. Jorion and Zhang (2007) find strong evidence of dominant contagion effects for Chapter 11 bankruptcies and competition effect for Chapter 7 bankruptcies by analyzing the US Credit Default Swaps (CDS) and stock markets. Beirne and Fratzscher (2013) show that over EMU crisis, both (i) a deterioration in countries' fundamentals and fundamentals contagion and (ii) a sharp rise in the sensitivity of financial markets to fundamentals, are the main explanations for the rise in sovereign yield spreads and CDS spreads, not only for euro area countries but globally. Claeys and Vašíček (2014) find during EMU crisis, the contagion has been a rather rare phenomenon limited to a few well defined moments of uncertainty on financial assistance packages for Greece, Ireland and Portugal.

1.4.2 Flight-to-quality

1.4.2.1 Flight-to-quality definition

In contrast to the contagion, the same definition of flight-to-quality is widely accepted. It can be defined as a process has both a decrease in the correlation between markets over a given period, and a reallocation of portfolios to healthier markets, with little default risks. In terms of its mechanism in bond markets, it can be described by a decline in bond prices on markets in difficulty and higher prices on markets supposed healthier, which in turn leads to a decrease in the correlation and covariance between markets.

1.4.2.2 Flight-to-quality related empirical research

Some empirical research related to flight-to-quality. Bauwens et al. (2006) find examples of flight-to-quality are in the Asian and Russian crisis 1997 and 1998, and contagion is found after September 11. Furthermore, stock market volatility contributes to flight-to-quality and bond volatility to contagion. Beber et al. (2009) conclude that the credit quality matters for bond valuation but that, in times of market stress, investors chase liquidity, not credit quality. Choudhry and Jayasekera (2014) find during the crisis, the level and amount of spillover from the major economies increase. But now there is also clear evidence of spillover from smaller EU economies to the major economies, this is especially true for Germany and the UK.

1.5 Conclusion

In this chapter, the four major theories of term structure of interest rates: the expectation theory, the liquidity preference theory, the market segmentation theory and the preferred habitat theory, are reviewed and discussed.

Furthermore, a number of interest rate models are reviewed and classified. Unfortunately, there doesn't exist a clear boundary between different groups of models, most of the time they are overlapping.

In terms of forecasting capabilities, very little evidence shows that one of the models is capable of outperforming others. However, most accurate models are believed to be those in which interest rates are associated to the volatility.

As for the calibration methods, they all have obvious pros and cons. Users should choose the most suitable one depending on the specific use of the model. In practice, the very strict applicability assumptions of these methods can hardly be fulfilled. Sequentially, some strong simplifications have been made, which could generate big errors.

Since the EMU crisis, the contagion and flight-to-quality phenomenon on bond markets become one of the core concerns for investors and policy makers. A lot of literature tries to distinguish the fundamental contagion from the pure contagion. However, the border is still porous between these two notions.

At the end of this survey, it's not hard to find that there exists a major gap of literature between (i) interest rate models and (ii) the phenomenon of contagion and flight-to-quality, since these two types of literature study the similar problematic, but somehow, they are separated. There isn't any interest rate model which takes into account the effects of contagion and flight-to-quality. By developing on the traditional portfolio theory-based interest rate model of Jones and Roley (1983), Mankiw et al. (1986), Artus (1987) and Artus and Kaabi (1995), Martin and Zhang (2017) design a two-economy model by taking into account the effects of contagion and flight-to-quality, which fills a major gap between these two literature. This model will be presented in chapter 3.

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Chapter 2

Correlation and volatility on bond markets during the EMU crisis: does the OMT change the process?

> Financial crisis is not fearing as long as we know when it begins and when it ends. The problem is we don't.
>
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>
> *Jiangxingyun Zhang*

2.1 Introduction

[1] Understanding the volatility transmission and flight-to-quality phenomenon between sovereign debt markets is important for investors and policy makers. The transmission of volatility between different government bonds affects directly the evolution of their risk premium. For the policy makers, it can influence the cost of public debt as well as the economic decisions. This issue may be particularly important and relevant in European Monetary Union as the governments of the Member States may issue debt, but do not have the ability to monetize or to reduce their excessive long-term debt with inflationary politics. During the sovereign debt crisis, the correlation of yields between major government bonds has been from positive to negative, and this have changed the behaviors of investors. To be more specific, they have the tendency to increase their allocation to government bonds of the pivots countries, like France, Germany, given by a lower perceived risk, and decrease their allocation to government bonds of periphery countries like Greece, Spain. Therefore, an accurate modeling of this flight-to-quality phenomenon is helpful to investors for better portfolio diversification.

Flight-to-quality and contagion are two antagonist concepts for explaining the correlation between markets. Forbes and Rigobon (2002) define the contagion as a significant increase in cross-market linkages after a shock to one country (or group of countries). In this study, we define the flight-to-quality as a significant decrease in cross-market linkages after a shock to a group of countries. Furthermore, in accordance with Baur et al. (2006), we define positive and negative contagion as well as flight-from-quality in the Table 2.1.

This chapter has three objectives. First of all, it proposes a formal test of the phenomenon of flight-to-quality among major bond markets in the euro zone. By using

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[1] This chapter is published as Martin, F., Zhang, J. (2014). Correlation and volatility on bond markets during the EMU crisis: does the OMT change the process?. Economics Bulletin, 34(2), 1327-1349.

this test and a trivariate AR(1)-VECH-GARCH(1,1) model, it examines whether it exist a significant decline of conditional correlations between bond yields of the countries in crisis and those who were identified by investors as refuge. Next, it questions about the aspect of speculation during the flight-to-quality. More precisely, it examines the interdependence of conditional variances between bond yields in different markets by adding two lagged effects of the source market to the original trivariate AR(1)-VECHGARCH(1,1) model. We could expect particularly an increase in the perception of risk (volatility) and conditional variances on the markets in crisis, and also a decrease in conditional variances on the refuge markets where the following scenario that an unavoidable decline of the high return yields of the bonds becomes increasingly clear in the eyes of investors. Specifically, it is to test a possible change of sign on the parameters associated with the transmission of volatility in the trivariate AR(1)-VECH-GARCH(1,1) model. Finally we examine the impact of the OMT decided in September 2012, on both the variances and conditional correlations between bond markets and parameters of transmission of volatility. The central question here is whether this decision has marked the end of the phenomenon of flight-to-quality, whether it is the beginning of a recorrelation of the markets and somehow whether it is the end of the sovereign debt crisis for the majority of investors. Therefore, we test in the other words the faith of the investors in the efficiency of a "Draghi Put" offered by the OMT.

TABLE 2.1: Overview flight-to-quality, flight-from-quality and contagion

	Correlation falling	Correlation rising
Periphery Countries' Bond markets falling	Flight-to-quality	(Negative) Contagion
Periphery Countries' Bond markets rising	Flight-from-quality	(Positive) Contagion
Pivot Countries' Bond markets falling	Flight-from-quality	(Negative) Contagion
Pivot Countries' Bond markets rising	Flight-to-quality	(Positive) Contagion

This chapter is organized as follows. Section 3.3 describes the data and presents the trivariate VECH models which support the different tests of hypotheses. Section 3.4 presents not only the test of flight-to-quality and contagion but also the tests of conditional variances of bond yields: the tests of comparison in each sub period and the tests on the parameters of volatility transmission. Section 2.4 summarizes our results and present our vision on the process of recovery from crisis and efficiency of "Draghi Put".

2.2 Data and model

This chapter uses daily time series from January 2008 to September 2013. We use the total return index of 10-years government bond (Source: Datastream) of seven major countries in the European Monetary Union including France (FR), Germany (GER), Italy (IT), Portugal (PT), Spain (ES), Ireland (IR), and Greece (GR). Moreover, France and Germany are classified as pivot countries, and Italy, Portugal, Spain, Ireland and Greece are classified as periphery countries. We choose two important dates to separate the time series into three sub-periods, December 8th 2009, the day when the government bond of Greece was downgraded to BBB by Fitch, which is always considered as the beginning of the sovereign debt crisis, and September 12th 2012, when the OMT plan was approved by the EMU members which could be, to some extent, regarded as the end of the European debt crisis.

The econometric model that serves to support the empirical evaluation based on the following two assumptions. The process followed by the daily bond yields (variations of log RI) is of type AR(1) with the error term noted $\epsilon_{i,t}$ of type GARCH. In order to stay in the frame of EMH (Efficient Market Hypothesis), we suppose that the interdependence between bond yields goes only through second conditional moments. The variance-covariance matrix is apprehended by a parsimonious VECH formulation and it is sufficient for the implementation of the tests of contagion / flight to quality and those on the volatility interdependence (tests of volatility spillover).

$$R_{i,t} = \mu + \phi R_{i,t} + \epsilon_{i,t} \tag{2.1}$$

The status and interpretation to error term $\epsilon_{i,t}$ is essential. Note first that the autoregressive form of the conditional mean equation of bond yields allows us to consider a gradual diffusion but not an instantaneous positive or negative shocks to bond yields. So we have next two possible and complementary readings of the variable $\epsilon_{i,t}$.

In a context of information efficiency, the variable $\epsilon_{i,t}$, reflects in principle all important "news" to anticipate rationally the bond yields over a period beginning at the current time t and ending at a future date corresponding to a horizon for each investor. The expected returns depend on the expected future price and the probable value of future payments (coupon or principal). The rational investors should actually anticipate all future equilibrium in the bond market. They are therefore sensitive to any information on future demand for securities, including those from the central bank (Securities Markets Programme, Outright Monetary Transactions).

The variable $\epsilon_{i,t}$, should also include all the information relating to funding needs and the present and future supply of securities. The most critical information are probably those that explicitly focus on the solvency of the sovereign issuer, especially all the variables involved in the mechanisms of debt sustainability, such as future nominal growth, primary balance, debt to GDP ratio, institutional rescue plan. We should also understand that the prices and bond yields should also integrate a risk premium of volatility the same as it is determined by the market equilibrium.

In contrast, the $\epsilon_{i,t}$, in our model may also reflect some more speculative behaviors such as formation of temporary bubbles and the triggering by mimetic behaviors (noise trading). It may be related to non-rational expectations as well.

We use a trivariate GARCH (1,1) model to quantify the transmission of volatility and flight-to-quality phenomenon. Bollerslev et al. (1988) present one simplified formulation of the multivariate GARCH model, the diagonal VECH model. Based on this formulation, we estimate the coefficient of volatility transmission by adding four parameters d_{11},d_{12},d_{31} and d_{32} which take into account the effects of lagged conditional variances of the source country. Therefore, in this trivariate diagonal VECH model, the conditional variance equations become:

$$\begin{aligned} h_{11,t} &= c_1 + a_1\epsilon^2_{1,t-1} + b_1h_{11,t-1} + d_{11}D_1h_{22,t-1} + d_{12}D_2h_{22,t-1} \\ h_{22,t} &= c_2 + a_2\epsilon^2_{2,t-1} + b_2h_{22,t-1} \\ h_{33,t} &= c_3 + a_1\epsilon^2_{3,t-1} + b_3h_{33,t-1} + d_{31}D_1h_{22,t-1} + d_{32}D_2h_{22,t-1} \\ h_{ij,t} &= c_{ij} + a_{ij}\epsilon_{i,t-1}\epsilon_{j,t-1} + b_{ij}h_{ij,t-1} \end{aligned} \tag{2.2}$$

where D_1 and D_{21} are two dummy variables that separate the two sub estimated periods, D1=1 and D2=0 if before the rupture date, $D_1 = 0$ and $D_2 = 1$ otherwise, $h_{ii,t}$ is the conditional variance of each market at time t, $\epsilon_{ii,t-1}$ is the one period lagged ARCH factor, $h_{ii,t-1}$ is the one period lagged GARCH factor, $h_{22,t-1}$ is the one period lagged conditional variance of market 2 (source market).d_{11},d_{12},d_{31} and d_{32} are four estimations of the volatility transmission from market 2 to market 1 and 3 in two different sub periods. $\epsilon_{i,t}$ is a white noise that $E_t(\epsilon_{i,t}) = 0$ and $V_t(\epsilon_{i,t}/I_{t-1}) = h_{ii,t}$ The rest elements are the same as presented above. The correlation coefficient is defined as follows:

$$\rho_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{ij,t}}\sqrt{h_{jj,t}}} \tag{2.3}$$

where $i, j = 1, 2, 3$ and $i \neq j$, $\rho_{ij,t}$ is the essential factor in this methodology because it represents the conditional correlation between returns of different government bonds. The parameters of the trivariate GARCH model are estimated by the method of maximum log-likelihood. Precisely, with algorithm of Simplex and some guessing values, we stop the calculation at the fifteenth iteration. Next, with the values obtained from this pre-calculation, we use the method of BHHH to estimate the GARCH model. This calculation is programmed in Winrats version 8.1.

2.3 Principals of tests on second conditional moments

To test whether the means of conditional variances across period have significantly changed, we apply the test of Welch. Published by Welch (1951), this test is an approached solution of the Behrens-Fisher problem. The objective of Welch test is to determine whether or not statistically there is an equality of means of two subsamples in the case of their variances are different. In this sense, it is a more robust alternative then the student test when the condition on the variances is not respected. Therefore, the hypothesis is constructed :

$$\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{cases}$$

The statistic of this test proposed by Welch is:

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}}} \tag{2.4}$$

Where μ_i is the mean of the simple, S_i the variance and N_i the number of observations. The degree of freedom ν associated with this variance estimate is approximated using the Welch–Satterthwaite equation:

$$\nu \approx \frac{\left(\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}\right)^2}{\frac{\left(\frac{S_1^2}{N_1}\right)^2}{N_1-1} + \frac{\left(\frac{S_2^2}{N_2}\right)^2}{N_2-1}}$$

(2.5)

It's important to notice that the degree of freedom is associated with the i^{th} variance estimate. The statistic t follows the distribution of student with the degree of freedom ν.

In the flight to quality test, we test the hypotheses of structural changes of correlati on coefficients across the tranquil and turmoil periods. As pointed out by Forbes and Rigobon (2002), the estimation of the correlation coefficient is biased because of the existence of heteroscedasticity in the return of the bond. More specifically, compared to the estimation during a stable period, the correlation coefficients are over estimated during a turmoil period. In our study, the correlations are conditional and dynamic. Therefore, we modify the adjustment formula of correlation coefficient proposed by Forbes and Rigobon into the following formula:

$$\rho^*_{i,p} = \frac{\rho_{i,p}}{\sqrt{1 + \delta(1 - \rho^2_{i,p})}} \tag{2.6}$$

Where $\delta = \frac{h^T}{s^S} - 1$ is the relative increase in the variance of the source country across stable period and turmoil periods. $\rho_{i,p}$ is the average of dynamic conditional correlations during period $p, p = (s, t)$, while s and t indicate the stable period and turmoil period. We should note that the stable period is a relative concept. It will be presented as pre-crisis period and the post-OMT period in our text. With the adjusted correlation coefficients, we apply the test proposed by Collins and Biekpe (2003) to detect the existence of flight-to-quality across stable period and turmoil period.
The Student test is:

$$\begin{cases} H_0 : \rho^*_S = \rho^*_T \\ H_1 : \rho^*_S > \rho^*_T \end{cases}$$

Where ρ^*_T is the adjusted correlation coefficient in turmoil period and ρ^*_S is the adjusted correlation coefficient in stable period. The statistic of the student test applied by Collins and Biekpe is:

$$t = (\rho^*_S - \rho^*_T)\sqrt{\frac{n_S + n_T - 4}{1 - (\rho^*_S - \rho^*_T)^2}} \tag{2.7}$$

where $t \sim T_{n_S + n_T - 4}$.

If we accept H_1, it means that the correlation coefficient across two periods has significantly decreased during the turmoil period, that is an evidence of the flight-toquality phenomenon.

2.4 Results

Before the presentation of the results of the tests, Table 2.2 and Table 2.3 present the statistics of the cumulative yields and the conditional variances in the three sub periods that we analyze. We note that the conditional variances which are presented in Table 2.3 are obtained from a univariate GARCH model.

The principal results from our different statistical tests can be summarized and interpreted as follows. Welch test on the comparison of the average conditional variances for the three sub periods (pre-crisis, crisis, post-OMT) clearly show that for Germany and France, the bond markets are less volatile during crisis than before the crisis. Without surprise, the formal adoption of the OMT in September 2012 led to a further decrease in the average level of conditional variances, it's a synonyms for investors to have less risk on bond yields.

Conversely, for Greece, the conditional variance of returns increases sharply during the crisis. It starts to drop from the implementation of the OMT. However, it doesn't find its pre-crisis levels. We find some evolution profiles for Spain, Italy and Portugal. The situation of the Irish bond market is unique among the seven cases studied. The conditional variance of returns increases sharply during the crisis, and after the OMT, we find this level of risk is even lower than the pre-crisis period.

TABLE 2.2: Cumulative Period Yield and Ranking

Market	Pre-crisis Period	Crisis Period	Post-OMT Period
France	0.1809 (3)	0.2346 (2)	0.0257 (6)
German	0.1892 (1)	0.2541 (1)	0.0083 (7)
Spain	0.1699 (5)	0.0352 (5)	0.1819 (4)
Greece	0.0739 (7)	-0.7120 (7)	1.4148 (1)
Italy	0.1873 (2)	0.1029 (3)	0.1123 (5)
Portugal	0.0929 (6)	0.1019 (4)	0.1898 (3)
Irland	0.1773 (4)	-0.1056 (6)	0.2207 (2)

TABLE 2.3: Average (m) Conditional Variances and Ranking

Market	Pre-crisis (m1)	Crisis (m2)	Post-OMT (m3)
France	0.15425 (1)	0.14609 (1)	0.09631 (1)
German	0.20356 (5)	0.16743 (2)	0.11961 (2)
Spain	0.18955 (4)	0.57506 (3)	0.41559 (4)
Greece	0.94434 (7)	4.63899 (7)	2.75178 (7)
Italy	0.16967 (2)	0.62821 (4)	0.46088 (5)
Portugal	0.17390 (3)	1.65644 (6)	1.32545 (6)
Iralnd	0.26755 (6)	1.08241 (5)	0.19276 (3)

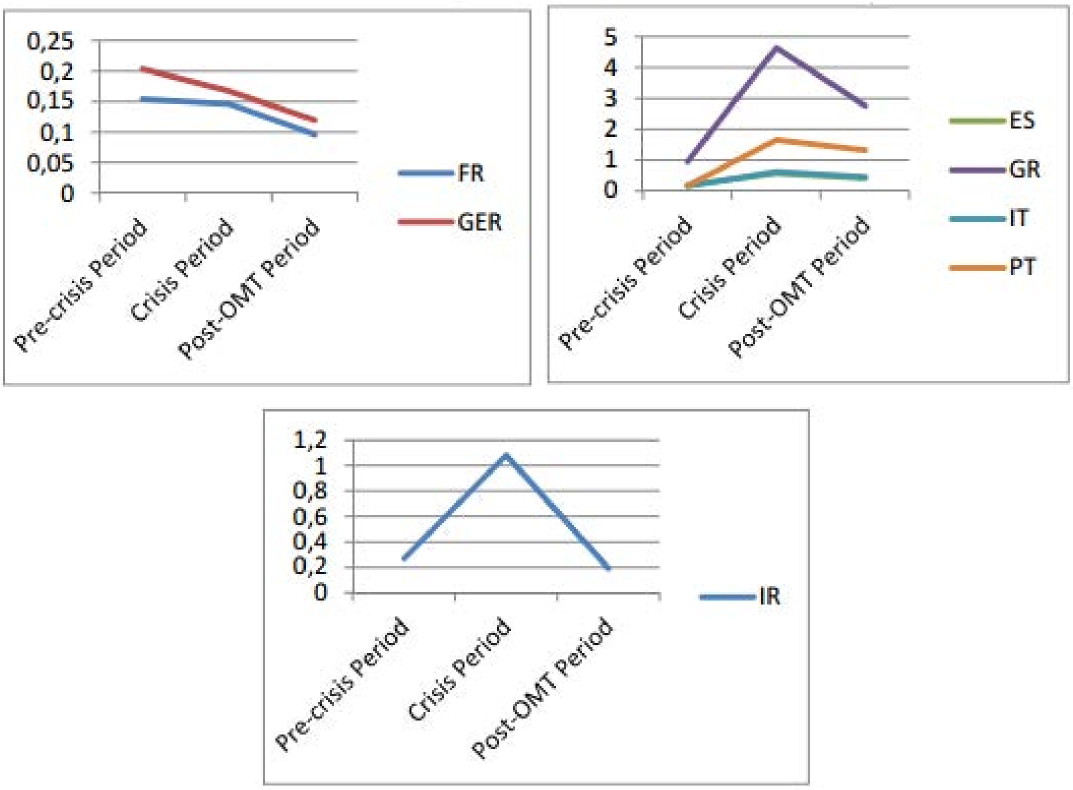

FIGURE 2.1: Three Patterns of Average Conditional Variances of Daily Returns

TABLE 2.4: Results of tests of Welch

Market	Pre-crisis (m1) with Crisis (m2)	Crisis (m2) with Post-OMT (m3)	Pre-crisis (m1) with Post-OMT(m3)	Conclusion
France	m1>m2**	m2>m3**	m1>m3**	m1>m2>m3
German	m1>m2**	m2>m3**	m1>m3**	
Spain	m1<m2**	m2>m3**	m1<m3**	m2>m3>m1
Greece	m1<m2**	m2>m3**	m1<m3**	
Italy	m1<m2**	m2>m3**	m1<m3**	
Portugal	m1<m2**	m2>m3**	m1<m3**	
Iraland	m1<m2**	m2>m3**	m1>m3**	m2>m1>m3

Notes: ** and * indicate statistically significance at the 5% and 10% level, respectively.

Regarding the tests on parameters of volatility transmission (d_{11}, d_{12},d_{31} and d_{32} in Equation 2.2) two findings emerge from our estimations. There are few evidence that support a global phenomenon of conditional volatility transmission between markets. However, there is a significant relationship between the Greek and Irish bond markets during the crisis period where the increase of the conditional variance in the Greek market has clearly contributed to reduce the risk of volatility seen in the Irish market. Therefore, we could say there is a phenomenon of the eviction of volatility between the two countries.

The tests on the evolution of conditional covariance between bond markets show that the logic of the flight to quality is predominant at the beginning of the sovereign debt crisis (period 2 in our estimations).There is systematically a decrease in conditional correlations of bond yields for almost all pairs of markets studied. This is observed in both cross-country correlations of different groups (periphery to pivot countries) and correlations between countries of the same group.

With the exception of the pair country France-Germany, the conditional correlations of the other markets have neither increased significantly from September 2012 nor after the plan OMT by the ECB. Therefore, there isn't a general beginning of a re-correlation of bond markets, but rather a stabilization of conditional correlations at levels close to those estimated in the previous period (period 2 in our study).

TABLE 2.5: Average Level of Conditional Correlations in Different Periods

Market Trio	Market Pair	Pre-crisis to Crisis		Crisis to Post-OMT	
FR-GR-GER	FR-GR	0.56908	0.09207	0.01156	0.01399
	FR-GER	0.95018	0.70936	0.68354	0.78483
	GR-GER	0.43865	0.00683	-0.10285	-0.09220
ES-GR-GER	ES-GR	0.49877	0.15961	0.30176	0.25930
	ES-GER	0.85981	0.06770	-0.10759	-0.12006
	GR-GER	0.33690	0.03126	-0.12537	-0.12193
IT-GR-GER	IR-GR	0.80993	0.31709	0.24036	0.14662
	IR-GER	0.72849	0.08425	-0.02687	-0.03691
	GR-GER	0.49295	-0.03772	-0.10493	-0.10292
PT-GR-GER	PT-GR	0.73662	0.24290	0.31767	0.33931
	PT-GER	0.53073	0.03304	-0.08860	-0.15920
	GR-GER	0.40280	-0.00796	-0.11682	-0.10372
IR-GR-GER	IR-GR	0.74533	0.35567	0.22491	0.24333
	IR-GER	0.75001	0.03396	-0.02555	-0.04147
	GR-GER	0.62031	-0.09997	-0.12810	-0.12208

TABLE 2.6: Results of Flight-to-Quality (FTQ) Tests for Major Country Pairs

	Pre-crisis to Crisis			Crisis to Post-OMT		
	Alternative Hypothesis	P-value	Result	Alternative Hypothesis	P-value	Result
FR-GR	$\rho_S^* > \rho_T^*$	0.01	FTQ	$\rho_S^* < \rho_T^*$	0.90	Still Crisis
FR-GER	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.01	Out of Crisis
GR-GER	$\rho_S^* > \rho_T^*$	0.04	FTQ	$\rho_S^* < \rho_T^*$	0.62	Still Crisis
ES-GR	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.17	Still Crisis
ES-GER	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.43	Still Crisis
IT-GR	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.48	Still Crisis
IT-GER	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.32	Still Crisis
PT-GR	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.49	Still Crisis
PT-GER	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.27	Still Crisis
IR-GR	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.45	Still Crisis
IR-GER	$\rho_S^* > \rho_T^*$	0.00	FTQ	$\rho_S^* < \rho_T^*$	0.48	Still Crisis

2.5 Conclusion

Our multivariate GARCH modeling of bond yields of major countries in the euro area over the period January 2008 - September 2013 brings several important and original results.

Concerning the conditional variance of returns, that is to say, the evaluation of risks perceived by investors, three patterns of evolution appear clearly. As for France and Germany, their bond yields are less volatile since the beginning of the sovereign debt crisis (period 2 in modeling) and the implementation of the OMT has accentuated this trend. For the Greek, Spanish, Italian and Portuguese bond markets, the conditional variances and perceived risks rise sharply during the crisis before falling with the implementation of the OMT. However, they don't return to the pre-crisis period levels. The Irish market has an intermediate evolution pattern with a peak of volatility during the crisis and then finish by a risk level lower than pre-crisis period.

The results also show that there is little evidence of volatility spillover between markets, with the exception of the link between the Greek market and the Irish market spotted during crisis.

The tests on conditional correlations of returns clearly show that the investors have followed a generalized logic of flight to quality since the beginning of the debt crisis. Therefore, we find a decrease in conditional correlations of bond yields. The implementation of the OMT and "Draghi put" had only the effect of blocking this process of decline of the correlations between markets. The French and German markets are the only two who return to their pre-crisis correlation level.

Our empirical results tend to support the argument that during the sovereign debt crisis the German and French bond markets have made an ideal investment haven for investors. The logic of the flight to quality and the protection against sovereign risk have contributed to a decline in interest rate the same as an increase of bond return.Without doubt, the decrease in conditional variances of these returns also fueled more speculative strategies based on optimization of the return-risk pair in bond portfolios management.

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Chapter 3

International portfolio theory-based interest rate model

> Since the short term interest rate is the opportunity cost of holding money, it is widely believed that the Federal Reserve has more direct control over short term than over long term interest rates in the United States....The term structure of interest rates thus appears central to the monetary transmission mechanism. Unfortunately, the determinants of the term structure remain poorly understood.
>
> *Mankiw et al.* (*1986*)

3.1 Introduction

[1] The articles focusing on the dynamics of sovereign interest rates during the debt crisis in euro area mainly seek to identify the episodes of contagion and Flight-to-quality between bond markets. These literature focus primarily on the default risks of sovereign issuers. Either they try to evaluate the solvability of issuers by relevant variables, most importantly by entering the debt sustainability variables into the models(Afonso et al. (2012), Arghyrou and Kontonikas (2012), Gómez-Puig and Sosvilla Rivero (2014), Ludwig (2013)), or directly by the premium paid on sovereign CDS (De Santis and Stein (2016), Longstaff et al. (2011), Claeys and Vašíček (2014)). Certain articles also explain the credit risks by extreme events on sovereign bond markets (Metiu (2012)). One of the challenges met by these research is how to distinguish a phenomenon of fundamental contagion that declines in bond markets are associated with a downgrade of sovereign credit ratings, from pure contagion that declines in markets and rising rates are the consequences of speculative strategies. The border is porous between these two notions of contagion since pure contagion can, through the rise of the rates, lead to an objective downgrade of sovereign debt issuers and switch into a regime of fundamental contagion. In our opinion, this explains why empirical literature has difficulty in distinguishing between these two types of contagion. Most of the time, articles conclude on the coexistence of the two contagion regimes.[2]

In the contrast to contagion, the episodes of Flight-to-quality are defined as a decrease in the correlation between bond markets over a given period. It is conceived as a reallocation of bond portfolios to healthier markets, with little default risks. The mechanism of Flight-to-quality is therefore a decline in bond prices on markets in difficulty and higher prices on markets supposed healthier, which in turn leads to a decrease in the correlation and covariance between markets.

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[1] A complete version of this chapter is published as Martin, F., Zhang, J. (2017). Modelling European sovereign bond yields with international portfolio effects. Economic Modelling, 64, 178-200.

[2] We can find a comprehensive literature review in Silvapulle et al. (2016).

This type of sequence has been clearly observed in the European bond markets, for example from the beginning of 2011, where we see the scissor-type patterns of the price trajectories of German and French bond market. It is legitimate to ask, once this process of flight-to-quality has been triggered, whether it has been connected, by a traditional logic of optimal portfolio allocation, to the effects of covariances, or more generally to the second order moments of the bond portfolio yields. It is precisely this question we want to address in this chapter. In addition to the credit risk effects, we try to explain the formation of the European sovereign interest rates over the debt crisis period in the euro zone, by rehabilitating the portfolio choice theory and the volatility and joint volatility risks. Moreover, we are interested as well as in the post-crisis period, with particularly the implementation of the ECB's QE and the associated asset purchase programs.

We propose an original theoretical model based on the optimal portfolio choices of a representative euro-based investor. He allocates over a short period its portfolio between two sovereign bonds having default risks and volatility risks, and a risk-free monetary asset. Optimal demand for bonds depends crucially on variances and covariances anticipated by investor. A lower covariance limits the joint risks between two bonds and stimulates the demands of the bonds at the price of risk-free rate. Optimal bond demands are confronted with available bond supply, that is to say not only the market values of sovereign debt stocks but also the ECB's QE programs. These bond purchases of the ECB have effectively reduced the net bond supply in circulation.

The equilibrium properties of this model generalize those of the traditional domestic term structure of interest rates theory (Artus (1987), Mankiw et al. (1986), Shiller and McCulloch (1987), Jones and Roley (1983)). The expected bond yields and the equilibrium rates dependent, expect for traditional properties, crucially on the anticipated bond yield covariances. These anticipated covariances, along with variances and bond supply, become essential components of volatility risk premium on bond yields. With optimal portfolio choices, we show that the anticipated covariances can indeed amplify or reduce, depending on different scenarios, the mechanism of contagion and Flight-to-quality between markets. To some extent, we propose a new intermediate channel of contagion which is situated between the fundamental contagion and the pure contagion.

The theoretical model is empirically validated by an econometric model based on daily European 10-year bond yields over the period January 2006 to September 2016. Precisely, we work on 21 bond country pairs in a bivariate GARCH framework. All variables are first differenced. The mean equations integrate, in addition to a CDS premium to control sovereign credit risks, the anticipated variances and covariances anticipated for the next period.

The difficulty here is a realistic representation of risk anticipations (variance and co-variance). For this purpose, we apply an additional bivariate DCC-GARCH model with first-differenced interest rates to simulate the investors' anticipations. They are simulated by 500-day rolling windows. Therefore, risk anticipations are the out of sample one-step-ahead forecasts of variances and covariances changes in bond yields.

The rest of the article is organized as follows. Section 2 presents our theoretical model and its equilibrium properties focusing on the relation between additional properties and the traditional properties of term structure of interest rates theory. It focuses in particular on the new mechanisms of contagion and Flight-to-quality and on the impact of QE on long-term equilibrium rates. Section 3 presents the principle of econometric modeling. Section 4 presents and analyze the empirical results. Section 5 summarizes the chapter and concludes.

3.2 Portfolio choice and bond demand

In the view of a representative investor who wants a diversified portfolio, we build a three assets model with: two sovereign bonds of 2 different countries ($i = 1, 2$) but of the same maturity, designating the benchmark of the investor's point of view of its exposure to interest rates risk; a risk-free monetary asset. In the rest of the chapter, country 1 is the healthy country with low exposure to credit risks, while country 2 represents the country in difficulty with a higher credit risk. The investment horizon is one period, between two dates t and $t+1$. One period return ($H_{i,t}$) for a zero-coupon obligation is given by price evolution[3]

$$H_{i,t} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}} \tag{3.1}$$

Using the sensitivity indicator (S)[4] and actuarial rates ($R_{i,t}$) rather than prices, the one period return can be approximated, with close convexity bias, by the evolution of interest rates during the period[5]

$$H_{i,t} \cong R_{i,t} - S\,(R_{i,t+1} - R_{i,t}) \tag{3.2}$$

The bond yield over a period is, therefore, given by the actuarial yield to maturity increased by the effect of an added or subtracted value, which is determined by both the variation in rates over the period and the sensitivity of the bond. The expected returns can be formed as follows

$$\mu_i = E_t(H_{i,t}) = (1+S)R_{i,t} - S\,[E_t(R_{i,t+1})] \tag{3.3}$$

And variance of return

$$\sigma_i^2 = V_t(H_{i,t}) = S^2 V_t(R_{i,t+1}) \tag{3.4}$$

With $E_t(.)$ and $V_t(.)$ as rational expectations operators. The anticipated returns depend on expectations about future long-term rate $E_t(R_{i,t+1})$ and the conditional variances of returns depend on the uncertainty of the future long-term rate $V_t(R_{i,t+1})$. Similarly, the covariance between bond yields is presented as

$$\sigma_{12} = Cov_t(H_{1,t}, H_{2,t}) = S^2 Cov_t(R_{1,t+1}, R_{2,t+1}) \tag{3.5}$$

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[3] For a more detailed demonstration proof of this model, see in Appendix H.

[4] We assume that the two bonds have the same sensitivity.

[5] Strictly speaking, if interest rates are annualized, the yield formula is relevant for a horizon of one year.

It reflects well covariance of future long-term rates. With standard notations, the expected return (μ_p) and risk (σ_p^2) of the portfolio are

$$\mu_p = \alpha_1\mu_1 + \alpha_2\mu_2 + (1 - \alpha_1 - \alpha_2)r \tag{3.6}$$

$$\sigma_p^2 = \alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12} \tag{3.7}$$

Where α_1, α_2 represent the percentages of total wealth (W_t) invested in each obligation,while r represents the risk-free rate. By maximizing the traditional risk-return criterion (with r_a as the parameter of absolute risk aversion) first order conditions give us a 2-2 system with respect α_1 and α_2. The solution is the optimal demand of bonds which can be expressed as follows

$$\alpha_1^* = \frac{(\mu_1 - r)}{r_a[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} - \frac{\sigma_{12}(\mu_2 - r)}{r_a\sigma_2^2[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} \tag{3.8}$$

$$\alpha_2^* = \frac{(\mu_2 - r)}{r_a[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} - \frac{\sigma_{12}(\mu_1 - r)}{r_a\sigma_1^2[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} \tag{3.9}$$

Here, we focus on our interpretation of the effects of covariance σ_{12} on the optimal demands of obligations. This covariance represents the capacity to offer a hedge for the investment on the other bond market. It plays an essential role for the rest of chapter.

If $\sigma_{12} = 0$, the demand of bonds is identical to those obtained in an autarkic framework where investors allocate their wealth between risk-free asset and domestic obligations, that is to say $\alpha_i^* = \frac{(\mu_i - r)}{r_a\sigma_i^2}$ for $i = 1, 2$.

If $\sigma_{12} < 0$, the demand of bonds in each market is greater than the demand in an autarkic scenario. Two effects co-determine the optimal demand. The relevant risk in the first component of the demand function is the residual risk ($[\sigma_i^2 - (\frac{\sigma_{12}}{\sigma_j})^2]$), that is to say the risk which is not covered by the position on the other market. Hedging demand (in the second term of the demand function) is even greater while σ_{12} is low (highly negatively correlated), and the other investment is assumed profitable (($\mu_j - r$) big) with low risks (σ_j^2 small).

If $\sigma_{12} > 0$, the demand of bonds is less than which is in an autarkic situation, that is to say the investors would prefer risk-free asset.

Finally, note that if the correlations are perfect between two bonds, whether negative (perfect complement) or positive (perfect substitute), the residuals are zero so the optimal demand of obligation takes extreme values. If the correlation is perfect negative, it is possible to build a zero free portfolio by combining the two obligations. This portfolio becomes a substitute risk-free asset when profitability is greater than r. The risk-free asset is thus abandoned. If the correlation is perfect positive, the optimal portfolio combines the risk-free asset and the more profitable one between the two obligations. The less profitable one is abandoned.

3.3 Bond market equilibrium

The equilibrium conditions of supply and demand on bond markets for both countries ($i = 1, 2$) are given by

$$\alpha^*_{1,t}[\mu_1(R_{1,t}), \mu_2(R_{2,t})]W_t = \varepsilon^S_{1,t}W_t \tag{3.10}$$

$$\alpha^*_{2,t}[\mu_1(R_{1,t}), \mu_2(R_{2,t})]W_t = \varepsilon^S_{2,t}W_t \tag{3.11}$$

$\varepsilon^S_{i,t}$ is the supply of bonds (in percentage of total wealth) considered as a random process, and a primary source of risk in the model. A more sophisticated version of the model integrates a random Noise-Trader demand ($\varepsilon^{NT}_{i,t}$) and a random European Central Bank demand ($\varepsilon^{ECB}_{i,t}$) to take into account the purchases of bonds through unconventional monetary policy. Therefore, the net supply of bonds is given by

$$\Sigma_{1,t} = \varepsilon^S_{1,t} - \left(\varepsilon^{NT}_{1,t} + \varepsilon^{ECB}_{1,t}\right) \tag{3.12}$$

$$\Sigma_{2,t} = \varepsilon^S_{2,t} - \left(\varepsilon^{NT}_{2,t} + \varepsilon^{ECB}_{2,t}\right) \tag{3.13}$$

The bond demand of Noise-Trader and ECB decreases the net bond supply

3.3.1 Expected equilibrium return

Market equilibrium conditions give a 2-2 system with respect μ_1 and μ_2. Solving the system, we find the equilibrium expected return (rational expectations) of each obligation. Solutions are given by

$$\mu_1^* = r + \frac{\sigma_2^2[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]}{[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} r_a \left(\Sigma_{1,t} + \frac{\sigma_{12}}{\sigma_1^2} \Sigma_{2,t} \right) \tag{3.14}$$

$$\mu_2^* = r + \frac{\sigma_1^2[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]}{[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} r_a \left(\Sigma_{2,t} + \frac{\sigma_{12}}{\sigma_2^2} \Sigma_{1,t} \right) \tag{3.15}$$

Or a simplified version

$$\mu_1^* = r + \sigma_1^2 r_a \left(\Sigma_{1,t} + \frac{\sigma_{12}}{\sigma_1^2} \Sigma_{2,t} \right) \tag{3.16}$$

$$\mu_2^* = r + \sigma_2^2 r_a \left(\Sigma_{2,t} + \frac{\sigma_{12}}{\sigma_2^2} \Sigma_{1,t} \right) \tag{3.17}$$

To our knowledge, relations (4.7) and (4.8) are new results. The expected equilibrium returns are expressed as the risk-free rate plus a risk premium which depends crucially on the total bond supply, which measures the quantity of risks in the portfolio. The bond supply of country 2 has an impact on μ_1^* depending on the ratio covariance on variance $(\frac{\sigma_{12}}{\sigma_1^2})$ which plays as a beta factor.

3.3.2 Long-term equilibrium rate

By taking the definition of expected returns, the solutions are obtained in form of actuarial rate of return on bonds, which are the true endogenous variables of the model.

$$R_{1,t}^* = \frac{1}{1+S} \left[r_t + S E_t(R_{1,t+1}) + S^2 V_t(R_{1,t+1}) r_a \left(\Sigma_{1,t} + \frac{Cov_t(R_{1,t+1}, R_{2,t+1})}{V_t(R_{1,t+1})} \Sigma_{2,t} \right) \right] \tag{3.18}$$

$$R_{2,t}^* = \frac{1}{1+S} \left[r_t + S E_t(R_{2,t+1}) + S^2 V_t(R_{2,t+1}) r_a \left(\Sigma_{2,t} + \frac{Cov_t(R_{1,t+1}, R_{2,t+1})}{V_t(R_{2,t+1})} \Sigma_{1,t} \right) \right] \tag{3.19}$$

The third term in bond rates appears as a one period risk premium with (net) supply effects depending on covariance regime. If $Cov_t(R_{1,t+1}, R_{2,t+1}) = 0$, we have

$$R^*_{i,t} = \frac{1}{1+S}\left[r_t + S\,E_t(R_{i,t+1}) + S^2 V_t(R_{i,t+1}) r_a(\Sigma_{i,t})\right] \tag{3.20}$$

i.e. standard Euler's equation of long-term rate in a domestic term structure model according to the SHILLERian tradition. By resolving Euler's equation with respect to $R^*_{i,t}$ by forward substitutions on $R_{i,t+1}$ and by supposing S is constant in time, we find

$$\begin{aligned} R^*_{1,t} &= \tfrac{r_t}{1+S} + \tfrac{1}{1+S}\left[\textstyle\sum_{i=1}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t(r_{t+i})\right] \\ &+ \tfrac{1}{1+S}\left[\textstyle\sum_{i=0}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t\left[S^2 V_{t+i}(R_{1,t+i+1}) r_a\left(\Sigma_{1,t+i} + \tfrac{Cov_{t+i}(R_{1,t+i+1},R_{2,t+i+1})}{V_{t+i}(R_{1,t+i+1})}\Sigma_{2,t+i}\right)\right]\right] \end{aligned} \tag{3.21}$$

$$\begin{aligned} R^*_{2,t} &= \tfrac{r_t}{1+S} + \tfrac{1}{1+S}\left[\textstyle\sum_{i=1}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t(r_{t+i})\right] \\ &+ \tfrac{1}{1+S}\left[\textstyle\sum_{i=0}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t\left[S^2 V_{t+i}(R_{2,t+i+1}) r_a\left(\Sigma_{2,t+i} + \tfrac{Cov_{t+i}(R_{1,t+i+1},R_{2,t+i+1})}{V_{t+i}(R_{2,t+i+1})}\Sigma_{1,t+i}\right)\right]\right] \end{aligned} \tag{3.22}$$

The equilibrium rates of rational expectations include forecasts of all future equilibrium of the two bond markets. Covariance forecasts play an essential role.

One complete resolution of this model, under the assumption of rational expectations, is to explain the equilibrium variance and covariance with interest rates, that is to say the second order moments which are consistent with the equilibrium conditions between supply-demand and any assumptions made, in terms of variance and covariance, with uncertainties weighing on the principal exogenous variables in the model: bond supply, obligation demand of both the ECB and Noise-Traders, and short-term rates.

We choose to keep an intermediate resolution of the model, under a semi-reduced form, maintaining the principle of an exogeneity of conditional second order moments. The idea here is that these second order moments can also obey other determinants as those introduced in the model.

3.4 Principal theoretical results

Long-term equilibrium rate given by (4.12) and (4.12) integrate both the traditional properties of the yield curve and additional mechanisms deriving from enlargement of the framework: 2 countries, purchases of bonds by the ECB.

3.4.1 Traditional properties

We can still find the effects of present monetary policy (r_t) and the anticipated policy for the future ($E_t(r_{t+i})$) as key determinants of long-term rates. The impacts of anticipated short-term rates decrease with the horizon of forecasts ($t + i$) and the term values $\left(\frac{S}{1+S}\right)^i$ associated with the sensitivity S.

The second term of (4.12) and (4.13) could be interpreted as an intertemporal risk premium on long-term rates. This risk premium compensates the anticipated uncertainty about future bond yields. So it's fundamentally a volatility risk premium but not a credit risk premium. It depends crucially on sensitivity of bonds (S), on absolute risk aversion of investors (r_a), on chronic anticipated variances on future long-term rates ($V_{t+i}(R_{1,t+i+1})$) and, through the term $E_t(\Sigma_{1,t+i})$, on chronic expectations on bond supply for future periods.

One complete resolution of the model, under the assumption of rational expectations, is to explain the equilibrium variance and covariance with long-term rates ($V_{t+i}(R_{1,t+i+1})$). We can show that when in a stationary scenario where $V_t(R_{1,t+1}) = V_{t+i}(R_{1,t+i+1})$[6] , the equilibrium variance of long-term rates depends on the uncertainty of future short-term rates and on the uncertainty about future bond supply. So it is the uncertainty of the conventional monetary policy and the future trajectory of public finances which determine in fine the equilibrium risk premium on long-term rates.

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[6] See for example Martin (2001).

3.4.2 Additional properties: in an Open economy and Quantitative Easing

The additional properties of the model are given by the presence of the term

$$\frac{Cov_{t+i}(R_{1,t+i+1}, R_{2,t+i+1})}{V_{t+i}(R_{1,t+i+1})}\Sigma_{2,t+i}$$

in equation (4.12) and its equivalent in equation (4.13). Based on this, we can find 3 following results.

Additional property 1: Impacts of anticipated covariance
Higher anticipated covariance leads to higher interest rates in both two countries; bond demands are lower because the hedging opportunities are lower as well. This means, in particular, that a contagion scenario between two markets, defined as a raising of both correlation and covariance between rates and with a scenario of future increases in long-term rates, is amplified. A “belief” that long rates will rise in both countries leads immediately (as soon as the portfolios are re-optimized) higher present rates.

We use the term "amplified" to describe the fact that the results of a contagion on interest rates, found in this mechanism of optimal portfolio choice, is to enhance the existing contagion. Conversely, scenario of end of the crisis with lower long-term rates in the country ($i = 1, 2$) and a decreasing covariance between markets is not amplified but on the contrary reduced. The scenario with higher covariance in the future reduces the possibilities of hedging, limits the bond demand and leads to rising rates in both countries.

We can also evaluate the nature of "amplified" in a scenario that covariance declines between two markets. In this case, a Flight-to-quality will favor one of the two markets. This scenario brings down rates in both two countries, therefore it’s amplified only in the market which is supposed to benefit from the Flight-to-quality mechanism. These results are summarized in Table 4.4.

Table 3.1: Impacts of covariance regime on long-term rates

Covariance regime	Anticipated evolution of rates	
Cov(R_1,R_2)$\nearrow$ Contagion or End of crisis	$R_1 \nearrow$,$R_2 \nearrow$ Contagion is amplified	$R_1 \searrow$,$R_2 \searrow$ End of crisis is reduced
Cov(R_1,R_2)$\searrow$ FTQ	$R_1 \rightarrow$ or $\searrow$, $R_2 \nearrow$ Amplified in the country which benefits from FTQ	$R_1 \nearrow$, $R_2 \rightarrow$ or $\nearrow$

Additional property 2: Impacts of news on public finance

Present and future conditions on bond supply (i.e. bond issue and debt amount) of each country have an impact on the bond market equilibrium of the other country. This impact fundamentally depends on the covariance regime anticipated by investors. For example, bad news about deficits and debt in Greece ($i = 2$) lead to higher interest rates in Greece ($i = 2$) but lower in Germany ($i = 1$) if the covariances are assumed negative. This mechanism becomes a component of the Flight-to-quality process.

The opposite case in a regime of positive covariance, the bad news about Greece's public finances lead long-term rates to rise in both countries. The scenario of contagion is here again amplified.

Additional property 3: Impacts of ECB's QE

The model also gives some lights on the impact of unconventional monetary policy on long-term rates in different countries. For example, in a regime of positive covariance, when the ECB buys (QE) or announces that it will buy (OMT and QE) Greek bonds ($i = 2$), this leads to a decline of interest rates in both Greece and Germany ($i = 1$). On the contrary, this leads to higher rates in Germany if the covariances are supposed negative. In either case, this leads to a lower rate in Greece.

The willingness of the ECB, which aims to drive down long-term rates in the countries in difficulty, may therefore be reduced by the beliefs of investors. Remember that the variances here are exogenous, but the ambition of the ECB with the QE will trigger a joint process of falling rates. To achieve this, it would be right to make balanced purchases of bonds, which means not only in country 2 but also in country 1, in reality, on all the bond markets. For investors, the fact of knowing that purchases are joined and strongly correlated will clearly to increase the level of anticipated covariance, and thus enhance the efficiency of QE.

Finally, note that bond purchases by the ECB have an impact on the volatility premium and equilibrium long-term rates, which could be considered as a partial debt cancellation with an explicit modelling of the credit risk premium. The fact that the ECB puts on its balance sheet a portion of the debt of countries in difficulty means, for investors, a disappearance of the bonds purchased by the ECB , and therefore a lower potential volatility in portfolios. The impact on rates is analogous to a partial debt cancellation along with a disappearance of certain quantity of credit risks.

3.5 Conclusion

This chapter proposes a general concept framework to understand the formation of long-term interest rates in the euro area over the periods of crisis and post-crisis with an active role of the ECB. We take the framework of portfolio theory to examine the specific role of short-term risks, perceived by investors, in the formation of sovereign bond yields. Our study is different from most of the recent papers which focuses on the impact of credit risk, on changes in CDS premiums, on the formation of interest rates and on the measurement of the phenomenon of contagion and Flight-to-quality between bond markets (De Santis and Stein (2016), Metiu (2012)), Afonso et al. (2012), Arghyrou and Kontonikas (2012), Krishnamurthy and Vissing-Jorgensen (2011), Silvapulle et al. (2016)).

Credit risk remains well integrated in our modeling, however the main theoretical purpose here is to determinate in which way the trajectory of sovereign bond yields in different countries could be influenced by the variances and covariances anticipated by investors. Did German and French markets have benefited from the decrease in anticipated variances on bond yields in the early beginning of the crisis since 2011 (Martin and Zhang (2014))? And on the contrary, were high volatility in peripheral markets able to feed the rise of rates? Has the Flight-to-quality process in favor of France and Germany since 2011 been amplified by the decrease in observed and anticipated covariances? Between these two markets and peripheral markets, did the decreasing covariance potentially feed German and French markets on additional demand?

The theoretical model proposes a portfolio framework with three assets: a risk-free monetary rate and two sovereign bonds including default risks and volatility risks when the holding period is less than the maturity of the bond. The market equilibrium of each country results from the global demand for obligations and the supply of bonds, that is to say the available bond stock. The demand is given by optimal portfolio choices and purchase programs by the ECB in the context of Quantitative Easing. The bond purchases of ECB actually reduce the net bond supply and limit the volatility risks in the international monetary and bond portfolios. The anticipated variances and covariances play a key role in the future trajectories of long-term equilibrium sovereign bond rates.

In particular, the anticipated covariances constitute a channel likely capable of amplifying the mechanisms of contagion and Flight-to-quality between markets. A downgrade of public finance in a country leading to new bond issues not only raises rates in this country but also in neighbor countries if the anticipated covariances are positive (amplified contagion). The bad news about public finances, on the contrary, decrease rates in neighbor countries if the anticipated covariances are negative (amplified Flight-to-quality). Our theoretical model also suggests that the bond purchases programs of the ECB in the framework of Quantitative Easing should not be targeted to a single market in difficulty but rather on several diversified markets in order to trigger a joint decreasing rate process.

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Chapter 4

Impact of QE on European sovereign bond market equilibrium

> Within our mandate, the European Central Bank is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.
>
> Mario Draghi, President of ECB
> 26 July 2012

4.1 Introduction

[1] The ECB's QE programme, launched in March 2015, has proved an essential, but paradoxical experience. It is essential because it is the first such experiment conducted in the euro zone, while the United States, Japan, and the United Kingdom already have some experience with this type of unconventional monetary policy. In the euro zone, the QE1 programme (from March 2015) and QE2 programme (from March 2016) are the only available economic policies to fight deflation risks in a context in which fiscal policies are restrictive and focused on reducing public deficits and debts following the implementation of the new European fiscal compact on January 1, 2013.

The QE experience in the euro zone is also paradoxical, because brokers seem to be convinced that the large-scale government bond purchase programme allows the ECB effective control of long-term sovereign rates. Therefore, we have switched to a fixed-rate regime, guided by short-term rates via the REFI rate, and by long-term rates through ECB bond purchases. When fears of "tapering" are stronger, the occasional increases in long-term interest rates show that market operators are also convinced of the ECB's capacity to maintain sovereign bond yields close to zero. The paradox comes precisely from the fact that the academic literature is much more nuanced than brokers' opinions on the effectiveness of QE programmes. This includes the first transmission mechanism, where the effective control of long-term interest rates determines the financial costs of sovereign states.

Our contribution to the debate on the impact of QE on the bond market equilibrium is summarized as follows. Firstly, we use an original theoretical model developed by Martin and Zhang (2017) to identify the normal mechanisms of bond markets and the potential effects of asset purchases by the ECB. This is an international bond portfolio choice model with two countries: one country with few default risks (core countries in the zone), and a more vulnerable country (periphery countries). The optimal bond

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[1] This chapter is published in "Handbook of Global Financial Markets: Transformations, Dependence, and Risk Spillovers", World Scientific Press. Forthcoming September 2017.

demand is not only met by the supply of bonds, based on the evolution of public deficit and debt, but also by the ECB's purchases, which effectively reduce the net supply of bonds in circulation. The properties of the equilibrium model thus generalize the traditional term structure of interest rates theory (Mankiw et al. (1986), Shiller and McCulloch (1987), Walsh (1985), Jones and Roley (1983), and Artus (1987)). Long-term equilibrium rates depend crucially on the variances and anticipated covariances of the bond yields of the two countries. Therefore, the expression of a volatility risk premium is enriched by a covariance effect, that is, the joint risks between markets. Thus, from a theoretical point of view, we note that purchases by the ECB affect the long-term equilibrium rates, which depend on the sign of the covariances anticipated by investors.

Secondly, the model is estimated econometrically over the period January 2006 to September 2016 using daily data. The anticipated variances and covariances of the bond yields are simulated using a bivariate DCC-GARCH model, with a 500-day rolling window. Risk forecasts are given by the one-step-ahead forecasts of variances and covariances. The default risk is controlled by introducing the premium of sovereign credit default swaps (CDS). A global uncertainty variable, the Vstoxx index, and short-term rates in the euro zone rated AAA complete the set of explanatory variables of sovereign bond rates. We estimate 21 pairs of European countries in a framework of conditional heteroscedasticity with a VECH specification matrix of the variance-covariance for innovations. Estimations are performed over several periods, plus a series of estimations with 500-day rolling windows in order to evaluate possible deformation of the market mechanisms over the post-crisis period with the implementation of the OMT and APP by the ECB.

Finally, by applying the test of Forbes and Rigobon (2002) and Pesaran and Pick (2007), we examine whether the implementation of the APP has led to a significant increase in the correlations between non-fundamental or residual components of interest rates. From our point of view, this tests an indirect way of identifying whether QE led to a significant decrease in sovereign bond yields. We further propose a complementary test, centered on the impact of QE on credit risk premiums to evaluate possible deformations since the implementation of QE, of the sensitivity of credit spreads to the premiums paid on the sovereign bonds.

The results of this chapter can be summarized as follows. Firstly, if we look at the significance of variances and covariances anticipated by investors, it seems that the mechanism of optimal portfolio allocation among the euro zone bond markets disappeared during the sovereign debt crisis and reappeared closely after the implementation of the OMT in September 2012. From this point of view, the ECB's QE from March 2015 had only prolonged the defragmentation process, which is initiated by the OMT, in the bond markets. Secondly, the formal test à la Forbes and Rigobon shows the QE has led to a clear rebound in the correlation between non-fundamental determinants of long term rates. However, since the correlation levels were already high on average before the implementation of the QE, it would be inaccurate to conclude that the QE had led to a significant and general increase in the correlations between markets. It would be improper to say that the QE had played a determining role over this falling rate period. Finally, the regressions of interest rate spreads on sovereign CDSs show that QE had significantly crushed the credit risk premiums of long term rates. With the exception of Portugal, the sensitivity of interest rate spreads to CDSs had decreased by about half since the implementation of the QE in March 2015.

The remainder of the chapter is organized as follows. Section 2 reviews recent literature about different QE impacts. Section 3 presents the practical terms of the QE programmes. Section 4 shows the main assumptions and the results of the theoretical model of bond yields. Section 5 presents the different econometric methods and the construction of the correlation structural break test. Section 6 presents both the econometric and test results. Lastly, Section 7 gives a general conclusion.

FIGURE 4.1: 10-Year Bond Rates

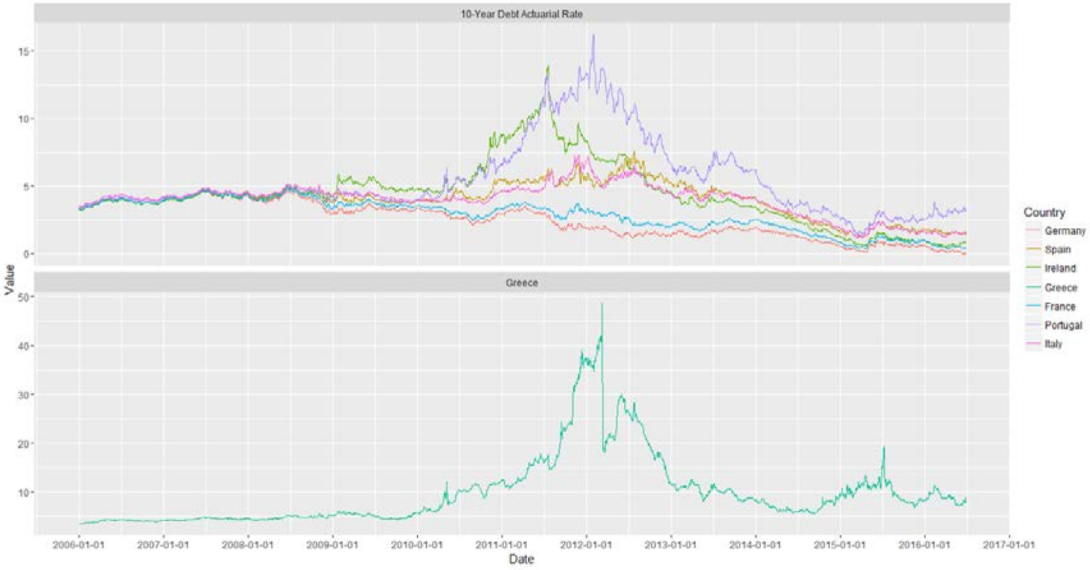

FIGURE 4.2: 10-Year Debt Return Index

4.2 Literature about QE impacts

Most of the empirical literature on asset purchases by the Federal Reserve (FED) and Bank of England (BoE) has focused on their effects on government bond markets. Doh (2010) uses a preferred-habitat model that explicitly considers the zero bound for nominal interest rates. The analysis suggests that purchasing assets on a large scale can effectively lower long-term interest rates. By using an event-study methodology, Krishnamurthy and Vissing-Jorgensen (2011) conclude that it would be inappropriate to focus only on US Treasury rates as a policy target, for the reason that QE impacts pass through several channels that affect particular assets differently. Gagnon et al. (2011) show some evidence that the Federal Reserve's purchases led to economically meaningful and long-lasting reductions in longer-term interest rates on a range of securities, including securities that were not included in the purchase programs. Meaning and Zhu (2011) estimate that the lasting reduction in bond supply via Federal Reserve asset purchases lowered government bond yields significantly and the its new maturity extension programme (MEP) should have an effect on longer-term Treasury bond yields comparable to that of the outright asset purchases under the Large-Scale Asset Purchase programme (LSAP). D'Amico and King (2013) find that yields within a particular maturity sector responded more to changes in the amounts outstanding in that sector than to similar changes in other sectors. This phenomenon was responsible for a persistent downward shift in yields averaging about 30 basis points over the course of the program. Empirical research by d'Amico et al. (2012) indicates that LSAP-style operations mainly impact longer term rates via the nominal term premium; within that premium, the response is predominantly embodied in the real term premium. Li and Wei (2012) show that the first and the second LASP programs and the Maturity Extension program have a combined effect of about 100 basis points on the 10-year Treasury yield. De Santis (2016) points out that studies on the US bond market estimate that bond purchases from the Federal Reserve (FED) between December 2008 and March 2010 have contributed to decreasing long-term government rates by about 90 basis points.

Many papers confirm that comparable purchases by the BoE in the United Kingdom have reduced long-term rates between March 2009 and January 2010. Meier (2009) states that tentative evidence on the BoE's quantitative easing is moderately encouraging, although the strategy is neither guaranteed to succeed nor as perilous as some of its detractors claim. Joyce et al. (2011) find that asset purchases financed by the issuance of central bank reserves may have depressed medium to long-term government bond yields by about 100 basis points, with the largest part of the impact coming through a portfolio balance effect. Joyce and Tong (2012) provide evidence of local supply and duration risk effects consistent with imperfect asset substitution, which has implications beyond the financial crisis for how we think about price determination in the gilt market. Meaning and Zhu (2011) find the Bank of England's Asset Purchase Facility (APF) had a significant impact on financial markets when the first stages were announced, but the effects became smaller for later extensions of the programmes. Breedon et al. (2012) estimate that the impact of the UK's initial 2009–10 QE programme has lowered government bond yields through the portfolio balance channel—by around 50 or so basis points. Christensen and Rudebusch (2012) prove declines in US yields mainly reflected lower expectations of future short-term interest rates, while declines in UK yields appeared to reflect reduced term premiums. McLaren et al. (2014) find that changes in expected QE purchases had a significant effect on gilt yields following each announcement and local supply effects may account for around half of the total impact on gilt yields in these events, and are passed through to yields of related assets.

In terms of the APP by ECB, Altavilla et al. (2015) find the impact of the APP on asset prices is sizeable albeit the program was announced at a time of low financial distress, which appears puzzling in light of existing literature that finds a large impact of asset purchases only in periods of high financial distress. De Santis (2016) considers that the monetary policies of the ECB have reduced the GDP-weighted European long-term rate by 63 basis points, with more pronounced effects on the most vulnerable countries by evaluating the impact of monetary policy announcements of by the ECB, relayed by Bloomberg between September 2014 and February 2015. Valiante (2016) considers that ex-ante effects are as important as ex-post effects, once asset purchases are conducted.

Using the Double Difference Method (DDIF) on a market panel that has undergone solely QE treatment, he estimates that the contribution of the APP to the decrease in euro zone bond rate is about 1 percent. Andrade et al. (2016) confirm that the January 2015 announcement of the programme has significantly and persistently reduced sovereign yields on long-term bonds and raised the share prices of banks that held more sovereign bonds in their portfolios.

Overall, whether in the USA, the UK, or the euro zone, we find that the contribution of QE to the decline in sovereign rates is not highly significant, and even more modest for the euro zone between the announcement of QE in September 2014 and September 2016 . This might suggest that other mechanisms have played a role in this process, such as the effects of deflation risks on nominal rates (Fisher effect), the significant improvement of sovereign issuers, and so on.

4.3 Asset Purchase Programme (APP) and Public Sector Purchase Programme (PSPP) in practice

The ECB's QE programmes mainly correspond to the Public Sector Purchase Programme (PSPP) launched on 9 March 9, 2015. It is part of a broader framework of the ECB's Assets Purchase Programmes (APP) initiated internally with the Securities Markets Programme (SMP, Table 4.1). Purchases made in the framework of the PSPP mainly concern sovereign bonds (government bonds), but marginally on those of agencies and supranational organizations. Since June 2016, the QE programmes began focusing on investment-grade corporate bonds (Table 4.1).

The ECB has set several constraints on these purchasing programs: (1) purchases are for securities with a maturity between 2 and 30 years; (2) purchases may not relate to securities whose returns are below the deposit rates of the ECB (-0.2 percent by 03/2015, and -0.4 percent by 03/2016); and (3) the ECB may not hold more than 25 percent of securities from the same issue, and no more than 33 of the same issuer.[2]

The proposed purchase amounts were 60 billion euros per month in March 2015. This was raised to 80 billion in March 2016, and then to 70 billion in December 2016. The distribution of purchases between countries is based on the weight of each country in terms of the capital of the ECB (Table 4.2). The important issues here are to evaluate the effect of these purchases on the equilibrium of the bond markets, and to be able to compare the amounts purchased with the available quantity of securities in each market. These quantities are given by a country's debt stock at market value. They evolve with new issues and the amortization of capital. Claeys et al. (2015) shows that these purchases are, for most countries, quite significant in terms of available securities. They show, for example, that the cumulative purchases by the ECB would saturate the constraints of the 25 percent by May 2017 for Germany and by January 2017 for Portugal. The constraint will already be saturated for Livia (May 2015), Cyprus (June 2015), and Ireland (November 2016). There would be more margin for Austria, Belgium, Ireland, France, Italy, and Spain.

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[2] In March 2015, the ECB's assets in Greece were, as a result of the aid program, in excess of the national debt limit of 33 percent. This is why Greece was excluded since the beginning of ECB's purchase program.

TABLE 4.1: Asset purchase programmes in practice

History of cumulative purchases under the APP.
End of month, in euro millions.
Holdings are end-of-month book value at amortised cost. Source: European Central Bank

		Monthly net,purchases at book value				Quarter-end, amortisation adjustment	Quarter-end, amortisation adjustment	Quarter-end, amortisation adjustment	Quarter-end, amortisation adjustment	Holdings			
		Asset-backed securities purchase programme	Covered bond purchase programme 3	Corporate Sector purchase programme	Public sector purchase programme	Asset-backed securities purchase programme	Covered bond purchase programme 3	Corporate Sector purchase programme	Public sector purchase programme	Asset-backed securities purchase programme	Covered bond purchase programme 3	Corporate Sector purchase programme	Public sector purchase programme
2014	October	0	4.768	0	0	0	0	0	0	0	4.768	0	0
	November	368	13.033	0	0	0	0	0	0	368	17.801	0	0
	December	1.376	11.885	0	0	0	-54	0	0	1.744	29.632	0	0
2015	January	582	10.623	0	0	0	0	0	0	2.326	40.255	0	0
	February	1.137	10.953	0	0	0	0	0	0	3.463	51.209	0	0
	March	1.158	12.587	0	47.383	0	-190	0	-27	4.622	63.606	0	47.356
	April	1.163	11.464	0	47.701	0	0	0	0	5.785	75.07	0	95.057
	May	1.42	10.039	0	51.622	0	0	0	0	7.205	85.108	0	146.679
	June	1.59	10.215	0	51.442	1	-326	0	-592	8.796	94.997	0	197.53
	July	943	9.006	0	51.359	0	0	0	0	9.739	104.003	0	248.889
	August	1.348	7.459	0	42.826	0	0	0	0	11.087	111.462	0	291.715
	September	1.928	10.11	0	51.008	0	-422	0	-1.261	13.015	121.151	0	341.462
	October	1.563	9.993	0	52.175	0	0	0	0	14.577	131.144	0	393.637
	November	601	6.869	0	55.105	0	0	0	0	15.178	138.013	0	448.742
	December	145	5.803	0	44.309	-1	-476	0	-1.836	15.322	143.34	0	491.215
2016	January	2.26	7.197	0	52.956	0	0	0	0	17.582	150.537	0	544.171
	February	989	7.784	0	53.358	0	0	0	0	18.571	158.321	0	597.529
	March	418	7.819	0	53.059	1	-503	0	-2.565	18.991	165.638	0	648.022
	April	-16	6.615	0	78.499	0	0	0	0	18.975	172.253	0	726.521
	May	84	5.556	0	79.673	0	0	0	0	19.059	177.809	0	806.194
	June	526	6.098	6.401	72.072	0	-530	-3	-3.064	19.585	183.377	6.398	875.201
	July	783	3.258	6.816	69.65	0	0	0	0	20.368	186.634	13.214	944.852
	August	-226	3.504	6.707	50.513	0	0	0	0	20.142	190.139	19.921	995.364
	September	530	4.731	9.872	69.972	0	-566	-72	-4.093	20.672	194.304	29.722	1.061.244
	October	589	3.437	8.422	72.974	0	0	0	0	21.261	197.741	38.144	1.134.218
	November	1.253	4.993	9.036	70.145	0	0	0	0	22.514	202.734	47.18	1.204.362
	December	317	1.369	4.042	55.032	-1	-588	-152	-4760	22.83	203.516	51.069	1.254.635

TABLE 4.2: Part 1 of Breakdown of debt securities under the Public Sector Purchase Programme: Monthly net purchases and WAM of PSPP portfolio holdings

Monthly net purchases

	31/03/2015	30/04/2015	31/05/2015	30/06/2015	31/07/2015	31/08/2015	30/09/2015	30/10/2015	30/11/2015	31/12/2015	31/01/2016	29/02/2016
Austria	1.216	1.205	1.314	1.31	1.363	1.064	1.279	1.324	1.411	1.155	1.348	1.359
Belgium	1.528	1.53	1.656	1.657	1.642	1.362	1.633	1.678	1.764	1.446	1.709	1.707
Cyprus	0	0	0	0	98	0	0	90	97	0	0	0
Germany	11.07	11.148	12.144	11.97	11.975	9.926	11.851	12.195	12.903	10.443	12.347	12.44
Estonia	0	0	0	5	15	10	8	3	7	0	6	7
Spain	5.447	5.471	5.909	5.915	5.891	4.882	5.789	6.042	6.334	5.137	6.107	6.125
Finland	774	786	841	836	850	687	825	844	909	734	865	879
France	8.757	8.624	9.485	9.426	9.465	8.087	9.485	9.95	10.221	8.267	9.969	9.989
Ireland	722	735	775	784	771	639	824	809	840	684	782	803
Italy	7.609	7.585	8.228	8.164	8.248	6.719	8.234	8.365	8.876	7.181	8.559	8.499
Lithuania	39	83	123	133	126	143	125	114	117	104	109	109
Luxembourg	183	205	84	261	80	86	138	6	30	42	169	110
Latvia	75	177	205	46	22	23	19	63	31	23	41	30
Malta	5	53	85	66	24	11	18	11	2	7	60	21
the Netherlands	2.487	2.527	2.667	2.663	2.657	2.213	2.603	2.721	2.883	2.191	2.84	2.794
Portugal	1.074	1.084	1.174	1.164	1.16	966	1.148	1.184	1.248	1.018	1.197	1.214
Slovenia	209	219	231	228	232	192	227	245	248	197	252	280
Slovakia	506	522	529	546	442	423	467	377	533	277	575	554
Supranationals	5.682	5.748	6.173	6.267	6.3	5.393	6.335	6.153	6.65	5.403	6.021	6.437
Total	47.383	47.701	51.622	51.442	51.359	42.826	51.008	52.175	55.105	44.309	52.956	53.358

WAM of PSPP portfolio holdings

	31/03/2015	30/04/2015	31/05/2015	30/06/2015	31/07/2015	31/08/2015	30/09/2015	31/10/2015	30/11/2015	31/12/2015	31/01/2016	29/02/2016
Austria	7.79	7.99	7.84	7.74	7.92	8.03	8.01	8.11	8.25	8.52	8.37	8.43
Belgium	8.8	9.1	9.13	9.09	8.84	8.8	9	9.21	9.62	9.51	9.53	9.59
Cyprus					5.41	5.32	5.24	5.43	5.91	5.82	5.74	5.66
Germany	8.12	7.9	7.11	6.87	6.91	7.09	6.96	6.98	7.02	7	7	6.96
Estonia				3.05	2.96	2.87	2.78	2.71	2.63	2.54	2.46	2.38
Spain	11.66	9.73	9.71	9.82	9.66	9.68	9.68	9.66	9.74	9.7	9.77	9.75
Finland	7.26	7.15	7.16	7.24	7.25	7.33	7.39	7.51	7.61	7.59	7.6	7.72
France	8.22	7.84	7.83	7.83	7.93	7.87	7.84	7.83	7.81	7.73	7.69	7.69
Ireland	9.43	9.14	9.61	9.55	9.77	9.66	9.1	8.98	9.34	9.4	9.48	9.49
Italy	9.07	8.41	8.68	8.83	9.03	9.12	9.27	9.29	9.28	9.27	9.33	9.34
Lithuania	6.46	5.22	6.11	5.9	5.69	5.55	5.4	5.36	5.64	6.01	6.24	6.47
Luxembourg	7.01	6.88	6.71	6.41	6.29	6.26	6.29	6.15	6.13	6.07	6.08	6.1
Latvia	6.43	5.93	6.3	6.23	6.13	6.07	5.96	6	5.88	5.85	5.7	5.58
Malta	10.37	8.47	11.05	12.01	11.03	10.49	9.99	9.7	9.6	9.62	9.58	9.53
the Netherlands	6.71	6.97	6.85	6.82	6.72	6.64	6.6	6.56	6.59	6.51	6.51	6.7
Portugal	10.96	10.77	10.84	10.61	10.59	10.77	10.86	10.64	10.57	10.36	10.22	10.12
Slovenia	6.33	7.92	7.5	7.57	7.21	7.73	8.04	8.05	8.13	7.97	7.74	7.57
Slovakia	9.49	9.26	9.29	9.21	8.85	8.66	8.66	8.47	8.54	8.58	8.41	8.33
Supranationals	7.26	8.05	7.8	7.43	7.19	7.05	6.88	6.9	7.05	6.97	6.91	7.03
Total	8.56	8.25	8.07	8	8	8.03	7.99	8	8.06	8.02	8.02	8.03

TABLE 4.3: Part 2 of Breakdown of debt securities under the Public Sector Purchase Programme: Monthly net purchases and WAM of PSPP portfolio holdings

Monthly net purchases

	31/03/2016	30/04/2016	31/05/2016	30/06/2016	31/07/2016	31/08/2016	30/09/2016	31/10/2016	30/11/2016	31/12/2016	31/01/2017	28/02/2017	31/03/2017
Austria	1.353	2.06	2.14	1.849	1.878	1.354	1.884	1.959	1.88	1.495	1.97	1.888	1.958
Belgium	1.71	2.612	2.695	2.341	2.368	1.704	2.377	2.477	2.372	1.867	2.489	2.37	2.398
Cyprus	-16	0	0	0	0	0	-21	0	0	0	0	0	0
Germany	12.411	18.985	19.573	16.888	17.247	12.368	17.188	18.016	17.29	13.568	17.713	16.96	16.977
Estonia	0	5	0	0	0	0	0	0	0	0	0	0	0
Spain	6.111	9.318	9.619	9.238	8.453	6.112	8.487	8.827	8.461	6.656	8.789	8.367	8.459
Finland	871	1.324	1.366	1.224	1.197	871	1.205	1.258	1.206	939	777	726	730
France	9.852	14.933	15.398	13.683	13.569	9.769	13.609	14.16	13.48	10.689	14.24	13.558	13.707
Ireland	808	1.078	1.112	1.085	986	697	982	1.022	979	648	547	557	565
Italy	8.53	12.998	13.442	12.772	11.867	8.476	11.808	12.323	11.707	9.417	12.302	11.785	11.89
Lithuania	125	104	113	105	72	48	73	125	99	75	70	65	75
Luxembourg	144	35	16	26	6	0	10	3	75	34	10	74	67
Latvia	44	76	58	90	27	72	45	90	39	16	39	56	65
Malta	60	60	66	37	12	11	7	37	99	55	29	49	30
the Netherlands	2.759	4.224	4.355	3.781	3.834	2.915	3.842	3.996	3.831	3.041	4.021	3.825	3.869
Portugal	1.213	1.405	1.451	1.438	958	722	1.022	1.021	1.023	726	688	656	663
Slovenia	237	236	242	254	223	153	219	225	216	168	160	152	150
Slovakia	433	329	323	233	221	123	133	164	230	216	334	301	294
Supranationals	6.413	8.717	7.706	7.028	6.732	5.117	7.102	7.271	7.157	5.425	7.185	6.818	6.919
Total	53.059	78.499	79.673	72.072	69.65	50.513	69.972	72.974	70.145	55.032	71.362	68.208	68.814

WAM of PSPP portfolio holdings

	31/03/2016	30/04/2016	31/05/2016	30/06/2016	31/07/2016	31/08/2016	30/09/2016	31/10/2016	30/11/2016	31/12/2016	31/01/2017	28/02/2017	31/03/2017
Austria	8.51	8.71	8.7	8.92	9.11	9.23	9.32	9.38	9.32	9.26	9.26	9.31	9.31
Belgium	9.74	9.79	9.78	9.65	9.79	9.92	10	10.07	9.98	10.04	10.11	10.03	10.14
Cyprus	5.52	5.44	5.36	5.27	5.19	5.1	5.09	5.01	4.92	4.86	4.76	4.67	4.59
Germany	7.05	7.18	7.31	7.44	7.6	7.69	7.88	8	8.06	8.16	8.15	7.88	7.65
Estonia	2.28	2.2	2.13	2.04	1.96	1.87	1.78	1.71	1.62	1.56	1.46	1.37	1.29
Spain	9.68	9.61	9.66	9.73	9.71	9.64	9.57	9.47	9.37	9.27	9.13	8.94	8.82
Finland	7.67	7.6	7.58	7.53	7.55	7.56	7.61	7.65	7.44	7.38	7.39	7.29	7.26
France	7.67	7.68	7.68	7.65	7.71	7.7	7.75	7.8	7.73	7.71	7.68	7.66	7.64
Ireland	9.42	9.47	9.38	9.38	9.31	9.25	9.38	9.35	9.19	9.19	9.07	9.03	8.92
Italy	9.35	9.36	9.31	9.29	9.24	9.2	9.15	9.07	8.95	8.92	8.85	8.79	8.72
Lithuania	6.7	6.61	6.57	6.62	6.5	6.42	6.41	6.6	6.61	6.59	6.69	6.69	6.79
Luxembourg	6.16	6.09	6.01	5.98	5.9	5.81	5.74	5.66	5.52	5.47	5.39	5.52	5.54
Latvia	5.45	5.37	5.66	6.62	6.56	6.59	6.57	7.32	7.29	7.26	7.51	7.53	7.66
Malta	10.4	10.15	10.57	10.87	10.89	10.75	10.81	11.12	11.54	11.57	11.52	11.36	11.3
the Netherlands	6.86	7.15	7.4	7.57	7.71	7.76	7.9	8.01	8.01	8.02	7.99	7.93	7.83
Portugal	10.18	10.17	10.21	10.14	10.07	9.91	9.84	9.8	9.68	9.53	9.37	9.28	9.19
Slovenia	7.74	8.09	8.32	8.27	8.23	8.25	8.41	8.56	8.71	8.9	8.89	8.93	9.2
Slovakia	8.24	8.24	8.11	8.02	7.92	7.85	7.86	7.83	7.79	7.9	7.93	8.04	8.09
Supranationals	6.92	6.79	6.83	6.86	7.06	7.12	7.27	7.33	7.25	7.31	7.32	7.36	7.36
Total	8.05	8.08	8.13	8.18	8.26	8.28	8.35	8.38	8.33	8.33	8.3	8.2	8.11

4.4 Theoretical background: an international portfolio model

In order to understand the formation of bond yields, we use the theoretical model proposed by Martin and Zhang (2017).

4.4.1 Hypothesis

The model is a three-asset bond portfolio choice model: a risk-free monetary asset at rate r and two zero coupon bonds ($i = 1, 2$) of the same maturity, including mainly interest rate risks. The sensitivity of the bond is denoted by S. The optimal demand of bonds, following Markowitz, can be written as follows:

$$\alpha_1^* = \frac{(\mu_1 - r)}{r_a[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} - \frac{\sigma_{12}(\mu_2 - r)}{r_a\sigma_2^2[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} \tag{4.1}$$

$$\alpha_2^* = \frac{(\mu_2 - r)}{r_a[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} - \frac{\sigma_{12}(\mu_1 - r)}{r_a\sigma_1^2[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} \tag{4.2}$$

where μ_1, μ_2, σ_1, σ_2 and σ_{12} represent the first and second order moments of bond yields over one period.

The supply-demand equilibrium conditions in each market are given by

$$\alpha_{1,t}^*[\mu_1(R_{1,t}), \mu_2(R_{2,t})]W_t = \varepsilon_{1,t}^S W_t \tag{4.3}$$

$$\alpha_{2,t}^*[\mu_1(R_{1,t}), \mu_2(R_{2,t})]W_t = \varepsilon_{2,t}^S W_t \tag{4.4}$$

where $\varepsilon_{i,t}^S$ is the supply of bonds (in percentage of total wealth W_t), considered as a random process and a primary source of risk in the model. If we introduce a random European Central Bank demand ($\varepsilon_{i,t}^{ECB}$) to take into account the purchases of bonds through unconventional monetary policy, the net supply of bonds is given by

$$\Sigma_{1,t} = \varepsilon^{S}_{1,t} - \varepsilon^{ECB}_{1,t} \tag{4.5}$$

$$\Sigma_{2,t} = \varepsilon^{S}_{2,t} - \varepsilon^{ECB}_{2,t} \tag{4.6}$$

The bond demand of ECB decreases the net bond supply.

4.4.2 Equilibrium properties

The market equilibrium conditions give a 2-2 system, with respect to μ_1 and μ_2. Solving the system, we find the equilibrium expected return (rational expectations) of each obligation. The solutions are given by

$$\mu_1^* = r + \sigma_1^2 r_a \left(\Sigma_{1,t} + \frac{\sigma_{12}}{\sigma_1^2}\Sigma_{2,t}\right) \tag{4.7}$$

$$\mu_2^* = r + \sigma_2^2 r_a \left(\Sigma_{2,t} + \frac{\sigma_{12}}{\sigma_2^2}\Sigma_{1,t}\right) \tag{4.8}$$

The expected equilibrium returns are expressed as the risk-free rate plus a risk premium, which depends crucially on the total bond supply, which, in turn, measures the quantity of risk in the portfolio. The bond supply of country 2 has an impact on μ_1^*, depending on the covariance on variance ($\frac{\sigma_{12}}{\sigma_1^2}$), which plays the role of a beta factor.

Using the definition of expected returns ($\mu_i = E_t(H_{i,t}) \simeq (1+S)R_{i,t} - S E_t(R_{i,t+1})$ with $H_{i,t} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}}$), the solutions are obtained in the form of an actuarial rate of return on bonds, which are the true endogenous variables of the model.

$$\begin{aligned} R_{1,t}^* = & \frac{1}{1+S}[r_t + S E_t(R_{1,t+1}) \\ & + S^2 V_t(R_{1,t+1}) r_a (\Sigma_{1,t} + \frac{Cov_t(R_{1,t+1}, R_{2,t+1})}{V_t(R_{1,t+1})}\Sigma_{2,t})] \end{aligned} \tag{4.9}$$

$$\begin{aligned} R^*_{2,t} =& \frac{1}{1+S}[r_t + S E_t(R_{2,t+1}) \\ &+ S^2 V_t(R_{2,t+1}) r_a(\Sigma_{2,t} + \frac{Cov_t(R_{1,t+1}, R_{2,t+1})}{V_t(R_{2,t+1})}\Sigma_{1,t})] \end{aligned} \tag{4.10}$$

The third term in bond rates appears as a one period risk premium with (net) supply effects depending on covariance regime. If $Cov_t(R_{1,t+1}, R_{2,t+1}) = 0$, we have

$$R^*_{i,t} = \frac{1}{1+S}\left[r_t + S E_t(R_{i,t+1}) + S^2 V_t(R_{i,t+1}) r_a(\Sigma_{i,t})\right] \tag{4.11}$$

That is i.e. the standard Euler's equation of long-term rate in a domestic term structure model, according to the SHILLERian tradition. By resolving Euler's equation with respect to $R^*_{i,t}$, by forward substitutions on $R_{i,t+1}$, and by supposing S is constant in time, we find

$$\begin{aligned} R^*_{1,t} &= \tfrac{r_t}{1+S} + \tfrac{1}{1+S}\left[\textstyle\sum_{i=1}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t(r_{t+i})\right] \\ &+ \tfrac{1}{1+S}\left[\textstyle\sum_{i=0}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t\left[S^2 V_{t+i}(R_{1,t+i+1}) r_a\left(\Sigma_{1,t+i} + \tfrac{Cov_{t+i}(R_{1,t+i+1},R_{2,t+i+1})}{V_{t+i}(R_{1,t+i+1})}\Sigma_{2,t+i}\right)\right]\right] \end{aligned} \tag{4.12}$$

$$\begin{aligned} R^*_{2,t} &= \tfrac{r_t}{1+S} + \tfrac{1}{1+S}\left[\textstyle\sum_{i=1}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t(r_{t+i})\right] \\ &+ \tfrac{1}{1+S}\left[\textstyle\sum_{i=0}^{\infty}\left(\tfrac{S}{1+S}\right)^i E_t\left[S^2 V_{t+i}(R_{2,t+i+1}) r_a\left(\Sigma_{2,t+i} + \tfrac{Cov_{t+i}(R_{1,t+i+1},R_{2,t+i+1})}{V_{t+i}(R_{2,t+i+1})}\Sigma_{1,t+i}\right)\right]\right] \end{aligned} \tag{4.13}$$

The equilibrium rates of rational expectations include forecasts of all future equilibrium of the two bond markets. Covariance forecasts play an essential role.

Compared to the traditional properties of term structure of interest rates theory, additional properties of our model are given by the presence of the term

$$\frac{Cov_{t+i}(R_{1,t+i+1}, R_{2,t+i+1})}{V_{t+i}(R_{1,t+i+1})}\Sigma_{2,t+i}$$

in equation (4.12) and its equivalent in equation (4.13). Based on this, we can find 3 following results, which are summarized in Table 4.4.

Additional property 1: Impacts of anticipated covariance

Higher anticipated covariance leads to higher interest rates in both two countries. This implies that a "belief" that long-term rates will increase in both countries leads immediately (as soon as the portfolios are re-optimized)to higher present current rates. The result of this optimal portfolio choice mechanism will enhance the existing contagion, or make the crisis last longer.

Additional property 2: Impacts of news on public finance

Present and future conditions on bond supply of each country have an impact on the bond market equilibrium of the other country. Bad news about deficits and debt in country 1 lead to higher interest rates in country but lower in country 2 if the covariances are assumed negative. This mechanism becomes a component of the Flight-to-quality process. The opposite case in a regime of positive covariance, the bad news about public finances of country 1 lead long-term rates to rise in both countries. The scenario of contagion is here again amplified.

Additional property 3: Impacts of ECB's QE

The model also gives some lights on the impact of unconventional monetary policy on long-term rates in different countries. In a regime of positive covariance, when the ECB buys bonds from country 1 which is supposed as the country in difficulty, this leads to a decline of interest rates in both countries 1 and 2. In a regime of negative covariance, this leads to lower rates in country1 but higher rates in country 2. In either case, this leads to a lower rate in country 1 . The objective of the ECB, which is to drive down long-term rates in the countries in difficulty, might be reduced by the beliefs of investors. As the variances here are exogenous, the ambition of the ECB will trigger a joint process of falling rates. To achieve this, it would be right to make balanced purchases of bonds, which means not only in country 2 but also in country 1. Finally, note that bond purchases by the ECB have an impact on the volatility premium and equilibrium long-term rates, which could be considered as a partial debt cancellation with an explicit modelling of the credit risk premium.

TABLE 4.4: Impacts of covariance regime on long-term rates

Covariance regime	Anticipated evolution of rates	
Cov(R_1,R_2)$\nearrow$ Contagion or End of crisis	$R_1 \nearrow$,$R_2 \nearrow$ Contagion is amplified	$R_1 \searrow$,$R_2 \searrow$ End of crisis is reduced
Cov(R_1,R_2)$\searrow$	$R_1 \rightarrow$ or $\searrow$, $R_2 \nearrow$	$R_1 \nearrow$, $R_2 \rightarrow$ or $\nearrow$
FTQ	Amplified in the country which benefits from FTQ	

4.5 Empirical modelling

4.5.1 Data

The daily time series used in this chapter are from January 2006 to September 2016 (Source: DataStream). As in many related literature, we use the Return Yield ($R_{i,t}$) of 10-year government bonds [3] of seven major countries in the European Monetary Union including France, Germany, Italy, Portugal, Spain, Ireland and Greece; Vstoxx index (global factor); 3-month AAA Bond Rate (who represents the risk-free rate); as well as 5-year Credit Default Swaps (indicator of default risks).[4]

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[3] Most of the recent papers (Ehrmann and Fratzscher (2017), Silvapulle et al. (2016), Costantini et al. (2014), Gómez-Puig and Sosvilla-Rivero (2016)) use 10-year sovereign bond yields as benchmark. However, De Santis and Stein (2016) propose to use 5-year sovereign bond yield as a benchmark for the reason that aggregate demand is typically affected by long-term interest rates, and therefore the correlation between long-term sovereign yields and the risk-free rate is a key economically relevant question.

[4] The market for CDS spreads used to measure the price of the credit risk is more liquid at 5-year maturity De Santis and Stein (2016).

In the dynamics of interest rates, there exists obviously different phases. By applying the unit root test proposed by Lee and Strazicich (2003) on the daily Return Index of Greece, we are able to separate the time series into three sub-periods: pre-crisis (2006/01/01 to 2009/12/01), crisis (2009/12/01 to 2012/08/06), and post-OMT (2012/08/06 to 2016/09/09). We should notice that the two break dates 2009/12/01 and 2012/08/06 are very close to respectively the Downgrading of Greece sovereign bond and the Implementation of Outright Monetary Transaction[5].

4.5.2 Modelling choice

We pay attention to equations (4.7) and (4.8) of the theoretical model. It describes expected market equilibrium returns. By introducing a time index on the risk-free rates and both variances and covariances of yields, the equilibrium relation for country i is written

$$\mu^*_{i,t} = r_t + \sigma^2_{i,t} r_a \left(\Sigma_{i,t} + \frac{\sigma_{ij,t}}{\sigma^2_{i,t}} \Sigma_{j,t} \right) \tag{4.14}$$

With

$$\mu_{i,t} = \frac{E_t(P_{i,t+1}) - P_{i,t}}{P_{i,t}} \tag{4.15}$$

This relation assumes that the bond price $P_{i,t}$, or the associated yield $R_{i,t}$, is daily adjusted so that the expected return $\mu_{i,t}$ given the anticipated future price $E_t(P_{i,t+1})$ is consistent with the risk-free rate and the equilibrium risk premium. Since the absolute risk aversion coefficient (r_a) and the net bond supply ($\Sigma_{i,t}$, $\Sigma_{j,t}$) are not directly observable, so the equilibrium relation can be considered as a linear relation between the expected yield ($\mu_{i,t}$) the risk-free rate (r_t) and the variance ($\sigma^2_{i,t}$), and covariance ($\sigma^2_{ij,t}$) anticipated one-step-ahead.

To estimate this equilibrium relation from equations (4.7) and (4.8), it is necessary to do it, as for the estimation of CAPM models by replacing the expected returns by the

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[5] In a previous version of this chapter, by using two times Zivot and Andrews (2002) test, we have obtained very similar results. See more descriptions and discussions about these two tests, see in Appendix G

observed daily returns in the sample. If the anticipations of investors especially on tomorrow's prices are rational, the difference between the two series should be a white noise. If these anticipations are not rational, there exists some inconvenience.

We use rather actuarial yields $R_{i,t}$, which also reflects the necessary adjustment on prices $P_{i,t}$, in order to respect the equilibrium relation. Due to the non-stationarity of actuarial rate series, finally the first differenced actuarial rates are explained by the first differenced risk-free rate, first differenced variances and covariance, and two first differenced control variables presented before, Vstoxx and CDS.

The second concern for this modelling is the definitions and estimations of variables and covariances anticipated by investors. A simple solution consists in estimating and evaluating the impact of the second order moments on the yield with a single step in the framework of a bivariate GARCH-in-mean model. We prefer to perform the estimations in two steps rather than a bivariate GARCH-in-mean model, because this auxiliary model is more manipulable and it provides actual out-of-sample risk predictions.

4.5.3 Model and estimation

In this empirical approach, we try to verify the results found in the theoretical model by applying two different methods: a two-step bivariate GARCH model and a two-step[6] rolling linear regression model.

4.5.3.1 Two-step GARCH model

In this two-step GARCH model, first step aims to forecast conditional variances and covariances which can be described as investors' anticipations of second order moments

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[6] As already been discussed in many literature, especially by Murphy and Topel (2002), this two-step procedure fails to account for the fact that imputed repressors are measured with sampling error, so hypothesis tests based on the estimated covariance matrix of the second-step estimator are biased, even in large sample. However we consider in our case, this sampling error is under control with a limited bias.

in our theoretical model. In the second step, estimations will be performed by using the forecast values of both variances and covariances from the first step, in order to understand the role of portfolio effects in the formation of sovereign bond yields.

Step 1: Investors' anticipations are simulated by using rolling window of classic bivariate DCC(1,1)-GARCH(1,1) model proposed by Engle (2002), and widely applied by for example Jones and Olson (2013) and Celık (2012).

$$H_t = D_t R_t D_t, where \quad D_t = Diag(\sqrt{h_{i,t}}) \tag{4.16}$$

where R_t is a 2×2 matrix of time-varying correlations. D_t is a 2×2 diagonal matrix of time-varying standard deviations of residual returns. The variances are obtained with univariate GARCH(1,1) processes. Specifically,

$$h_t = c + a\varepsilon_{t-1}^2 + bh_{t-1} \tag{4.17}$$

One single sample of 500 days will give us a one-step-ahead prediction of conditional variance and covariance.[7]. By using a rolling window of 500 days, we obtain some of one-step-ahead forecast values which can be considered as investors' anticipations of conditional variances and covariances.

Step 2: In this step, we try to determine the role of portfolio effects in the formation of sovereign bond yields by applying a bivariate GARCH Model. We have two countries in each estimation. [8]For $i = 1$ and 2, the mean equations are presented as follows.

$$\begin{aligned} \Delta R_{i,t} =& \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) \\ &+ \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t} \end{aligned} \tag{4.18}$$

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[7] For example, a sample from t to t+500 will be able to give a forecasting for t+501.

[8] In order to avoid the non stationary problem of the series, all the variables used in this model are first differenced. It's also the same case for the first step.

With $R_{i,t}$ Actuarial Rate (Yield to maturity) of each bond, $E_t(V_{i,t+1})$ anticipated variance, $E_t(Cov_{ij,t+1})$ anticipated covariance[9], $Vstoxx_{t-1}$ Vstoxx Index, r_t 3-month AAA Government Bond Rate. In accordance with the canonical contagion approach of Pesaran and Pick (2007), equation (4.16) tries to explain bond yields with global and specific factors. $Vstoxx_{t-1}$ denotes the implied volatility risks and also a global factor as discussed by Afonso et al. (2012), Arghyrou and Kontonikas (2012), and Metiu (2012). The specific factors are the Credit Default Swaps $CDS_{i,t}$ which denotes credit default risks, and both anticipated variances $E_t(V_{i,t+1})$, anticipated covariances $E_t(Cov_{ij,t+1})$ denote portfolio effects described in our theoretical model.

Under the hypothesis of conditional normal distribution of disturbances, the parameters of the model are estimated by the method of maximum likelihood. The log Likelihood function which should be optimized is given as follows

$$L_T(\theta) = -\frac{1}{2}\sum_{t=1}^{T}[ln[det(H_t(\theta))] + \varepsilon_t(\theta)' H_t(\theta)^{-1}\varepsilon_t(\theta)] \tag{4.19}$$

With $H_t = V(\epsilon_t/I_{t-1}) = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$ and $\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$

The matrix of variance-covariance is based on a diagonal VECH specification. Therefore, the conditional variance and covariance are expressed by

$$h_{ii,t} = c_{ii} + a_{ii}\varepsilon^2_{i,t-1} + b_{ii}h_{ii,t-1} \tag{4.20}$$

$$h_{ij,t} = c_{ij} + a_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + b_{ij}h_{ij,t-1} \tag{4.21}$$

where c_{ij}, a_{ij} and b_{ij} are parameters and $\varepsilon_{i,t-1}$ is the vector of errors from the previous period. This specification supposes that the current conditional variances and covariances are determined by their own past and past shocks. Precisely, with algorithm Simplex

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[9] Both anticipated variance and covariance simulate the investors' expectations as described in our theoretical model. We use a DCC-GARCH model with a rolling window of 500 daily data to forecast these two terms using one-step-ahead method.

and some guessing values, we stop the calculation at the fifteenth iteration. Next, with the values obtained from this pre-calculation, we use the method BHHH to estimate the GARCH model.

4.5.3.2 Two-step rolling linear regression

In this part, in order to estimate the evolution of amplifying factor discussed in section above, we perform a rolling linear regression model.

Step 1: It is exactly the same method here to generate a set of forecast conditional variances and covariances as performed in ***Step 1*** in section 4.5.3.1. Therefore, we obtain a set of one-step-ahead forecast conditional variances and covariances which can be considered as investors' anticipations of these second order moments.

Step 2: The linear model is same as the mean equation (4.18) presented above in the GARCH framework. For easy reading, we rewrite it here:

$$\begin{aligned}\Delta R_{i,t} =& \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) \\ &+\beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}\end{aligned}$$

To be more specific, in accordance with the forecasting of conditional variance and covariance with DCC-GARCH model, rolling window here is also 500 days. Each estimation of this linear model gives us several fitted coefficients, thus, the rolling window generates the dynamics of fitted coefficients which can be presented as sensibilities to the return yields.

4.5.4 Principals of correlation test

In this section, we test the hypotheses of structural changes of correlation coefficients across the pre-QE and post-QE periods. As pointed out by Forbes and Rigobon (2002), the estimation of the correlation coefficient is biased because of the existence of heteroscedasticity in the return of bond. More specifically, compared to the estimation over a stable period, the correlation coefficients are over estimated over a turmoil period. In our study, we consider the pre-QE period as the turmoil period and post-QE as the stable period. Because of the correlations are conditional and dynamic in our model, so we need to modify the adjustment formula of correlation coefficient proposed by Forbes and Rigobon into the following formula:

$$\rho^*_{ij,p} = \frac{\rho_{ij,p}}{\sqrt{1 + \delta(1 - \rho^2_{ij,p})}} \tag{4.22}$$

Where $\delta = \frac{h^{pre}}{s^{post}} - 1$ is the relative increase in the variance of the source country across pre-QE period and post-QE period. $\rho_{i,p}$ is the average of dynamic conditional correlations over period p, where $p = (pre, post)$, while *pre* and *post* indicate the pre-QE period and post-QE period. With the adjusted correlation coefficients, we apply the test proposed by Collins and Biekpe (2003) to detect the existence of structural breaks across pre-QE period and post-QE period.

The Student test is:

$$\begin{cases} H_0 : \rho^*_{post} = \rho^*_{pre} \\ H_1 : \rho^*_{post} > \rho^*_{pre} \end{cases}$$

Where ρ^*_{post} is the adjusted correlation coefficient over post-QE period and ρ^*_{pre} is the adjusted correlation coefficient over pre-QE period. The statistic of the student test applied by Collins and Biekpe is:

$$t = (\rho^*_{post} - \rho^*_{pre}) \sqrt{\frac{n_{post} + n_{pre} - 4}{1 - (\rho^*_{post} - \rho^*_{pre})^2}} \tag{4.23}$$

where $t \sim T_{n_{post}+n_{pre}-4}$. If we accept H_1, it means that the correlation coefficient across two periods has increased significantly over the post-QE period, which is evidence of the impact of QE.

Finally, note that the test proposed here refers to residues, in other words, the non-fundamental components of an explanatory model. In accordance with the recommendations of Pesaran and Pick (2007), we have global variables that explain both countries (Vstoxx, r, covariance) and country-specific variables for each bond market (variance, CDS).

4.5.5 Sensitivity of spread to CDS

It is logical to expect that the ECB's massive purchases of sovereign bonds will, ceteris paribus, raise the prices of bonds and, thus, reduce the credit risk premiums on bond yields and the spreads of bond yields. Under this assumption, interest rate spreads for Germany should be less sensitive to changes in premiums paid on sovereign CDSs. This represents market appreciation on credit risks. We test this hypothesis by estimating the following model over different sub-periods:

$$Spread_{i,t} = \beta_{0,i} + \beta_{1,i} CDS_{i,t} + \epsilon_{i,t}. \tag{4.24}$$

With $spread_{i,t}$: spread of country i compared to Germany; $CDS_{i,t}$ premiums paid on CDS[10]. The model will be estimated over a Pre-QE period and a Post-QE period.

4.6 Empirical results

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[10] The CDS premium is purged of the effects related to the overall uncertainty variable $Vstoxx$ as in De Santis and Stein (2016).

4.6.1 Dynamics of price and yields and patterns of anticipated variances and covariances

The patterns of variance and covariance evolutions anticipated by investors (Appendix B and C) are inextricably related to the dynamics of sovereign bond prices and the associated actuarial rates (Figures 4.1 and 4.2). Recall that the important phases of the market dynamics have implications in terms of covariances and volatility.

The first bond price collapse took place on October 16, 2009, accompanying the beginning of the fiscal crisis in Greece. Only the Greek obligations are affected. The downgrade of Greece's debt by Moody's on December 8, 2009, amplified the collapse of the Greek bond market. The first effects of the contagion occurred in March 2010 in the Irish, Spanish, and Portuguese markets. In contrast, the German and French markets kept improving. According to our estimations, the first increase in anticipated volatility occurred in March 2010, which affected all seven bond markets analyzed.

The contagion reached a historical level at the end of 2010, where, for the first time, the bond markets of seven countries began suffering significantly, and showed simultaneous downward movements (Figure 1). The short-term downward trend for the German and French markets began to increase again from March 2011 in a low volatility context. In fact, in addition to a decrease in anticipated volatility in these two markets, our estimations show a pronounced decrease in the covariances between the German, French, and other markets. Clearly, the beginning of the flight-to-quality phenomenon favors the two countries considered to be the healthiest, because they represent a lower credit risk. This phase is the first nodal point in the bond market trajectory during the sovereign debt crisis. The second nodal point appears in 2011, on December 8, when the ECB announced the implementation of an exceptional three-year refinancing programme for euro zone banks (LTRO), and Mario Draghi implied that the ECB would not make massive purchases of sovereign bonds. Then, December 8, 2011, represents a peak of anticipated volatility for the German and French markets. Since this date, Italy, Spain, and Ireland have joined Germany and France in a stable phase in terms of obligation yields. For the former three countries, the anticipated covariances with Germany and France step out of their lowest level, and gradually rebound. This signified the end of the crisis. Then, Portugal joined this group in early 2012. It was not until July 26, 2012, when Mario Draghi gave the "whatever it takes speech" and announced the implementation of the OMT in the euro zone, that Greece stepped into the new post-crisis phase.

The anticipated variances of the 10-year bond yield variations show strong disparities in the observed average levels, ranging from 0.04 for Germany to 2.5 for Greece. The evolution patterns over time present more similarities. In all seven markets, the volatility anticipated by investors began to rise from the beginning of 2010. However, the profiles are different in the peripheral countries of the euro zone, where the volatility rose significantly from early 2010, reaching a peak at the end of 2011. The date of this volatility peak is December 8, 2011, and is associated with the decision by the ECB to grant three-year special financing to banks in the euro zone (LTRO). This event, which is favorable for the resolution of the sovereign debt crisis, was counterbalanced by Mario Draghi claiming that the ECB did not intend to carry out massive purchases of sovereign bonds. Therefore, December 8, 2011, is a break point in the history of the interest rate trajectory in the euro zone.

4.6.2 Two-step GARCH results

The results of 21 bivariate model estimations are shown in Tables D.1. With regard to the one-step-ahead forecasts of variances and covariances, the results quite clearly show that covariances are playing a more significant and systematic role in the dynamics of sovereign rates. Most of the estimated parameters have the expected positive signs. In general, the effects are more present in the German and French markets than they are in the euro zone peripheral markets. The parameters associated with the covariance often show a U-shape pattern, with lower values over the crisis period. For example, we observe the following sequence. The impacts of the Germany–Portugal covariance on German yields are 1.62 (pre-crisis), 0.48 (crisis), and 1.86 (post-OMT), and 0.90 (crisis) and 4.84 (post-OMT) for the impacts of the France–Ireland covariance on French yields. In general, we find that in terms of covariance impacts on the evolution of sovereign bond yields, the French yields are twice as sensitive as the German yields. The impacts of the one-step-ahead forecasts of variances are higher at the end of the sample (i.e., over the post-OMT period), particularly in the German and French markets. The sensitivity of the first-differenced yield to variance variation is 2.97 for Germany (Germany–France in Table D.1) and 1.82 for France (France–Italy in Table D.1). Note that the same coefficients may vary widely from one estimation to another, depending on the selected pairs and, therefore, depend on the covariances included in the regression model.

The variable Vstoxx reflects an overall uncertainty that should impact the yields of risky assets in general, as well as sovereign bonds. This mechanism has been well supported by our estimates, with generally positive coefficients for this variable. The obtained coefficients are again higher for Germany (0.013), France (0.021), and Ireland (0.014). The risk-free rate also shows the expected positive sign. Throughout the overall period, the estimated coefficients range from 0.12 to 0.20 for Germany, France, and Greece. This coefficient is never significant for Ireland. In terms of the influence of the CDS (cleaned by Vstoxx), we obtain similar results to those obtained for the covariances. The fitted coefficients are higher and more significant for Germany and France, with slightly higher values over post-OMT period. The coefficients of the CDS are also higher in the Ireland bond market, according to the regression results. Finally, we note that the parameters of the variance–covariance H_t of the VECH model are almost always significant at the 5% level, with the usual positive signs.

4.6.3 Two-step rolling regression results

Next, we examine the patterns of coefficient evolutions associated with variances and covariances obtained from the 500-day rolling window regressions. The results are given in Appendix E. For each country, the curve shows the estimated coefficient ($\hat{\beta}_{1,i}$) associated with changes in the variances described in equation (18), which explains the first-differenced yields. There are as many coefficients as estimated pairs, that is, six for each country. As emphasized above, the estimated coefficients ($\hat{\beta}_{1,i}$) may be affected by the selected pairs and, therefore, by the selected covariances in equation (18). For each country, the graphic also shows rolling estimates of the coefficients ($\hat{\beta}_{2,i}$) associated with different variations of covariances. Again, there are as many coefficients as there are pairs of covariances. Note that all coefficient estimates ($\hat{\beta}_{1,i}$ and $\hat{\beta}_{2,i}$) are modulated by considering the uncertainty of the estimates (i.e., the estimated standard deviation ($\hat{\sigma}_{\hat{\beta}_i}$)). Each graphic reports the central value of the estimate, a lower bond ($\hat{\beta}_i - 1.96\hat{\sigma}_{\hat{\beta}_i}$), and an upper bond ($\hat{\beta}_i + 1.96\hat{\sigma}_{\hat{\beta}_i}$) for the estimate. A coefficient β_i is considered significant at the 5 percent level if neither the lower nor the higher bonds of the estimate cross zero ($\hat{\beta}_i = 0$).

In order to qualify the different phases of our sample between pre-crisis, crisis, and post-OMT periods, we adopt the following principle. A coefficient $\hat{\beta}_i$ is considered specific to a particular phase if the majority of dates included in the estimations (i.e., more than 250 days) are within this range. Using this principle, the phases associated with the coefficients $\hat{\beta}$ correspond to the historical ranges with a lag of 251 days, which, in practice, corresponds to a lag of 11 months. Thus, from the point of view of the coefficients $\hat{\beta}$ associated with the covariances and variances anticipated by investors, the crisis period begins on 11/01/2010, and the post-OMT period begins on 08/04/2013.

More specifically, with regard to coefficients $\hat{\beta}_2$ associated with the covariances, these rolling windows make it possible to give a much more precise pattern of estimates than in the case of the sub-period estimations (Tables D.1), where a U-shape emerged, especially for Germany and France.

We highlight the following results. The pattern of coefficients $\hat{\beta}_2$ with a U-shape is confirmed globally for Germany and France. To a lesser extent, this is also true of Portugal (covariances with Italy, France, and Spain) and Spain (covariances with Germany, Ireland, and France). In some cases, the U-shape curve during the crisis period is accompanied by the non-significance of the coefficient β, which is sometimes even negative.

Italy, Ireland, and Greece show atypical profiles, where bond yields become sensitive to covariances from the beginning of the post-OMT period only: the covariances of Italy with Germany, Ireland, and France; the covariances of Ireland with Italy, Spain, and France; and the covariance of Greece with Ireland. Again, the negative impacts of the covariances on the long-term rates need to be highlighted: the covariance between Italy and Portugal on Italian rates, and the covariance between Greece and Germany on the Greek rates.

Overall, these results show that bond portfolio mechanisms have played a clear role between Germany, France, Portugal, and Spain during the pre-crisis period, before becoming less important, or disappearing during the crisis, and reappearing in the post-OMT period. For the second group of countries, Italy, Ireland, and to a lesser extent, Greece, the portfolio mechanisms only appear in the post-OMT phase.

Furthermore, it is possible to quantify the effects of the first-differenced anticipated covariance on the formation of interest rates. For illustrative purposes, we evaluate the contribution of the decrease in bond yield covariances between Germany and France and other European countries on the trajectory of German and French rates during the period of the sovereign debt crisis. The tables below summarize the cumulative effects on the German and French rates, as shown in the graphics in Appendix E, with the means of the coefficients $\hat{\beta_2}$ used in the first phase of the estimations.

TABLE 4.5: Example of covariance effects on German market

Germany			
Pair with	Cumulative ΔCov	$\hat{\beta}_2$ mean	Cumulative effects
Italy	-0.05	7	-0.35
Portugal	-0.06	2.5	-0.15
Spain	-0.06	7	-0.42
Ireland	-0.06	2.5	-0.15
Greece	-0.12	2	-0.24
			Total effects: -1.31

TABLE 4.6: Example of covariance effects on French market

France			
Pair with	Cumulative ΔCov	$\hat{\beta}_2$ mean	Cumulative effects
Italy	-0.06	15	-0.90
Portugal	-0.06	10	-0.60
Spain	-0.06	13	-0.78
Ireland	-0.08	5	-0.40
Greece	-0.07	7	-0.49
			Total effects: -2.81

The cumulative declines in the covariance between the German and French yields with those of the other five countries are very similar. On the other hand, the sensitivity of the yields to covariances is significantly higher in the French market, where the coefficients vary between 5 (covariance with Ireland) and 15 (covariance with Italy), with most being twice as big as the coefficients $\hat{\beta}_2$ associated with the German yields.

Overall, being aware of the transitory nature of the covariance effects on the yields due to the estimation of the model in variation, we estimate that the decrease in covariances at the beginning of the crisis period (beginning of 2011) potentially contributed to the decreases in French and German yields of 131 and 281 bps, respectively. Finally, we find some evidence supporting the mechanism of an amplified flight-to-quality process, led by anticipated covariances and portfolio effects, as presented in Table 4.4.

Finally, we emphasize that, in contrast to the post-OMT period, the phase beginning 11/03/2015 with the implementation of the QE by the ECB is associated with a sensitive increase in the German and French yields and those of the other five countries. In the same period, the sensitivity of bond yields to the covariances rises as well. We deduce that the portfolio effects reduce the process of decreases in German and French bond yields. This time, we illustrate a process of decreasing yields and exiting the crisis, reducing the covariance and portfolio effects, as presented in Table 4.4.

Finally, we examine the effects of anticipated variances on the yield variations, because they are provided by the two-step rolling linear regression estimations. With the exception of France and Greece, it is always possible to identify at least a sub-period where the coefficient $\hat{\beta}_1$ is significantly different from zero and positive. Unlike the results obtained on the covariances, the periods with significant coefficients generally do not cover the three phases of pre-crisis, crisis, and post-OMT. In fact, the $\hat{\beta}_1$ coefficients can only be significant in an occasional way over a short period.

As in the case of Germany, several estimations agree on a coefficient $\hat{\beta}_1$ being close to 5 over a short period of one year, beginning from mid-2015. A similar result is obtained for Ireland with a coefficient close to 8. For Italy and Portugal, the coefficients are also significant from mid-2015, with some values close to 1 and 0.5, respectively. Finally, for Spain, there is a U-shape pattern for the coefficient $\hat{\beta}_1$, with the following values: close to 3 till 2012, 0 till mid-2015, and 1 afterwards.

4.6.4 Higher correlations between yields since APP ?

The correlations between the residuals of the first-differenced bond yields are reported in Appendix F.1. These clearly show that, beginning in March 2015, there is a very strong rebound in dynamic conditional correlations estimated by the bivariate GARCH model. Therefore, this is where the "visible hand" of the APP appears. However, Table 4, which reports the statistics of the T-test, à la Forbes and Rigobon, shows that it is not possible to conclude that all correlations are higher after the APP, as a general result.

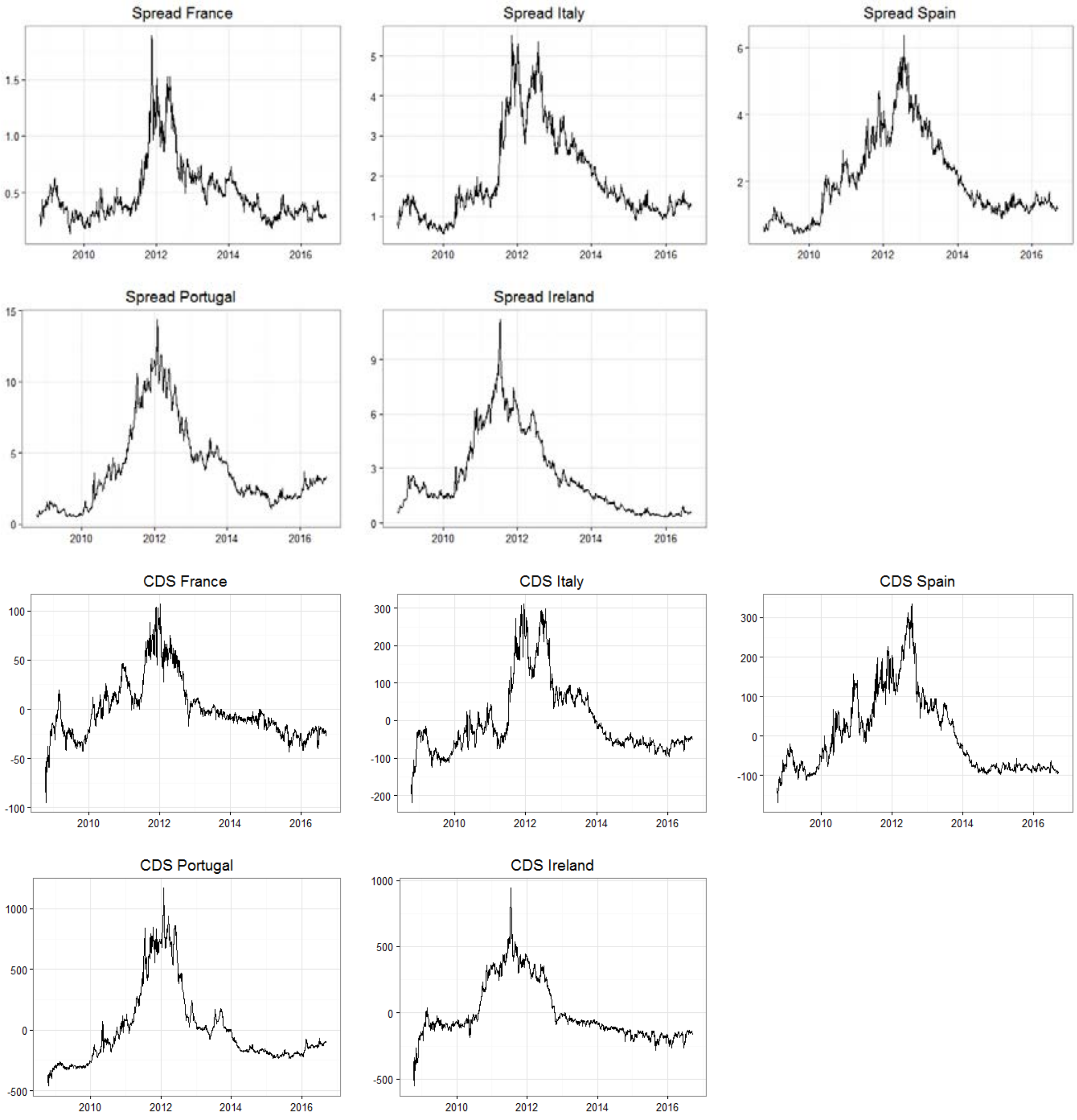
TABLE 4.7: Results of correlation structural break tests à la Forbes and Rigobon

Country Pair	T-stat	Sig.	Country Pair	T-stat	Sig.
Germany-France	-1.250	0.211	France-Germany	-1.207	0.227
Germany-Italy	0.089	0.928	Italy-Germany	0.118	0.905
Germany-Portugal	0.028	0.977	Portugal-Germany	0.028	0.977
Germany-Spain	-0.889	0.373	Spain-Germany	-0.923	0.355
Germany-Ireland	0.004	0.996	Ireland-Germany	0.008	0.993
Germany-Greece	-0.083	0.933	Greece-Germany	0.168	0.866
France-Italy	-0.325	0.744	Italy-France	-0.505	0.613
France-Portugal	**-2.003**	**0.045**	Portugal-France	**-2.005**	**0.045**
France-Spain	-1.126	0.260	Spain-France	-1.266	0.205
France-Ireland	0.406	0.684	Ireland-France	0.814	0.415
France-Greece	-0.136	0.891	Greece-France	-0.202	0.839
Italy-Portugal	-0.589	0.555	Portugal-Italy	-0.451	0.651
Italy-Spain	**-1.777**	**0.075**	Spain-Italy	-1.434	0.151
Italy-Ireland	0.054	0.956	Ireland-Italy	0.073	0.941
Italy-Greece	1.425	0.154	Greece-Italy	1.421	0.155
Portugal-Spain	-0.375	0.707	Spain-Portugal	-0.393	0.694
Portugal-Ireland	0.028	0.977	Ireland-Portugal	0.058	0.953
Portugal-Greece	**-1.902**	**0.057**	Greece-Portugal	**-2.308**	**0.021**
Spain-Ireland	0.595	0.551	Ireland-Spain	1.095	0.273
Spain-Greece	-0.857	0.391	Greece-Spain	-0.977	0.328
Ireland-Greece	0.272	0.785	Greece-Ireland	0.186	0.852

4.6.5 Lower sensitivity of Spread to CDS since APP ?

The following charts reproduce the sovereign bond spreads with Germany and the CDSs for the period 2008—2016, suggesting a strong correlation between spreads and CDSs. The following table shows the results of bond spread regressions on the CDSs. They clearly show that the implementation of QE in March 2015 greatly reduced the spread

Figure 4.3: CDSs and Spreads



sensitivity to the CDS and, thus, artificially crushed the credit risk premiums weighing on bond yields.

The spread sensitivity to the CDS decreases by half in Italy and Spain, and becomes negative for France, and almost null for Ireland. Only Portugal, where there is no change in the sensitivity to credit risk, is an exception to the rule.

TABLE 4.8: Sensitivities of spreads to CDSs over different periods

Regression model[11]: $Spread_{i,t} = \beta_{0,i} + \beta_{1,i} CDS_{i,t} + \epsilon_{i,t}$

Country	Period	$\bar{R}^2$	β_1	T-stat	Sig.
France	Full sample	0.6112	0.0071	57.0186	0.0000
	Before QE	0.5882	0.0073	48.9736	0.0000
	During QE	0.3471	-0.0049	-14.4181	0.0000
Italy	Full sample	0.9237	0.0109	158.2638	0.0000
	Before QE	0.9186	0.0109	137.6247	0.0000
	During QE	0.2588	0.0059	11.6981	0.0000
Spain	Full sample	0.8528	0.0115	109.4617	0.0000
	Before QE	0.8490	0.0120	97.1619	0.0000
	During QE	0.0842	0.0067	6.0651	0.0000
Portugal	Full sample	0.9560	0.0093	212.1261	0.0000
	Before QE	0.9543	0.0093	187.3026	0.0000
	During QE	0.8682	0.0133	50.6342	0.0000
Ireland	Full sample	0.9439	0.0096	186.6597	0.0000
	Before QE	0.9441	0.0093	168.3740	0.0000
	During QE	0.0101	-0.0004	-2.2300	0.0263

4.7 Conclusion

This study analyzes the impact of the ECB's APP on the European bond market equilibrium and, particularly, on its contribution to the decline in sovereign rates. Therefore,

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[11] Spreads are calculated by the difference between the 10-year government bond yield of the country studied and that of Germany. Variable CDS_t takes residual values ($\hat{u}_{i,t}$) from following auxiliary regression: $CDS_{i,t} = \beta_{0,i} + \beta_{1,i} Vstoxx + u_{i,t}$

we use a conceptual framework to understand the formation of long-term interest rates over the pre-crisis and post-crisis periods with the implementation of the ECB's QE. We adopt the portfolio theory framework to examine the specific role of short-term risks, as perceived by investors, in the formation of sovereign bond yields. Our study is different to most existing works, which focus on the impact of credit risk on changes in CDS premiums, the formation of interest rates, the measurement of the contagion, and the flight-to-quality between bond markets (De Santis and Stein (2016), Metiu (2012)), Afonso et al. (2012), Arghyrou and Kontonikas (2012), Pesaran and Pick (2007), Krishnamurthy and Vissing-Jorgensen (2011), and Silvapulle et al. (2016)).

The theoretical model proposes a portfolio framework with three assets: a risk-free monetary rate and two sovereign bonds, including default risks and volatility risks when the holding period is less than the maturity of the bond. The market equilibrium of each country results from the global demand for obligations and the supply of bonds (i.e., the available bond stock). Demand is given by the optimal portfolio choices and the purchase programmes by the ECB in the context of QE. Bond purchases by the ECB actually reduce the net bond supply, and limit the volatility of risks in the international monetary and bond portfolios. The anticipated variances and covariances play a key role in the future trajectories of long-term equilibrium sovereign bond rates.

In particular, the anticipated covariances constitute a channel that is likely able to amplify the mechanisms of contagion and the flight-to-quality between markets. A downgrade of public finance in a country leading to new bond issues not only raises rates in this country but also in neighboring countries if the anticipated covariances are positive (amplified contagion). In contrast, bad news on public finances decreases the rates in neighboring countries if the anticipated covariances are negative (amplified Flight-to-quality). Our theoretical model also suggests that the bond purchase programmes of the ECB in the QE framework should not be targeted to a single market, but should rather target several diversified markets in order to trigger a joint decreasing rate process.

The empirical approach is based on daily data for the period January 2006 to September 2016, and is integrated in the first step of a bivariate DCC-GARCH model. Then, to simulate the series, we estimate it using 500-day rolling windows in order to simulate the variances and covariances anticipated by investors. In the second step, bivariate GARCH models with a VECH specification for the variance-covariance matrix are proposed, and estimations are performed on 21 country pairs over both the whole period October 2008 to September 2016, and three sub-periods qualified as pre-crisis, crisis, and post-OMT. All variables used in the models are first-differenced variables. The mean equation explains the bond yields by the variances and covariances anticipated by investors, the short-term interest rate of euro zone issuers rated AAA, the VSTOXX index, and the premium paid on CDS as an essential determinant of the default risk premium required on sovereign bonds. In the third step, a linear version of this model, without ARCH effects, is estimated using 500-day rolling windows to analyze and identify possible dynamics of the coefficients by taking into account the matrix of variance–covariance anticipated by investors.

The estimates over the whole period, sub-periods, and, more importantly, those using rolling windows show that bond markets do not evolve solely as a result of changes in default risks and CDS premiums. The bond yields of the seven European countries are also sensitive to the volatility risks of covariances implied in bond portfolios. All the results can be refined as follows.

(i) German and French bond yields are more sensitive to the volatility risks of covariances than those of peripheral countries. **(ii)** The covariance effects are stronger than those of variances, that are often present in an occasional way. **(iii)** The decrease in covariance between the German and French markets at the beginning of the crisis period significantly reduced the risk premiums required by investors, and contributed to the decrease in yields by 131 bps for Germany and 281 bps for France. **(iv)** Overall, the short-term risk premium intensity on portfolios and bond yields declines sharply during the crisis, and reappear later over the post-OMT and post-QE periods. Everything happens as if the mechanism of international bond portfolio allocation ceased during the sovereign debt crisis and then reappeared. These results are consistent with those of other recent studies (De Santis and Stein (2016), Ehrmann and Fratzscher (2017)), highlighting the hypothesis of possible fragmentation on the European bond markets during the crisis.

Note too that the implementation of the ECB's QE on actual purchases from March 2015 contributed to a process of returning to normal or defragmenting the bond markets. According to our evaluations, this defragmentation was initiated before the implementation of the QE, before being strengthened by the APP. Finally, according to the test of à la Forbes and Rigobon (2002), it seems difficult to affirm that QE programmes have had a significant increase on correlations between bond markets, simply because the correlations were already high before the implementation of QE. The complementary tests of the regression of the CDS on spreads show that the QE significantly reduced their sensitivity to credit risk premiums.

By construction, our dynamic econometric model, estimated in first difference, does not allow us to propose an absolute quantification of the QE's impact on the bond rates of each country. However, the combination of results obtained from econometric estimates, particularly the rolling model, the conditional correlation test (à la Forbes and Rigobon), and the CDS spread sensitivity tests, suggest that the impact of QE on bond market equilibrium is not as strong as expected. Therefore, the probable cessation of QE from 2018 would not, from this point of view, lead to a significant increase in interest rates.

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General Conclusion

In this thesis, firstly, we briefly introduce the interest rate models and the financial contagion during the EMU crisis by doing a survey of the existing papers. Surprisingly, we find there exists a major gap of literature between (i) interest rate models and (ii) the phenomenon of contagion and flight-to-quality, since these two types of literature study the similar problematic, but somehow, they are separated. There isn't any interest rate model which took into account the effects of contagion and flight-to-quality.

Secondly, while thinking about how to fill the gap mentioned above, by applying a trivariate GARCH model and a flight-to-quality test, we study the correlation and volatility transmission between the European sovereign debt markets during the period of 2008-2013. The empirical results support not only the existence of flight-to-quality from the periphery countries (Italy, Portugal, Spain, Ireland and Greece) to the pivot countries (France and Germany), but also the flight within each group. It seems that in terms of volatility, the pivot countries are relatively difficult to be influenced by the external turbulence.

Thirdly, in order to fill the literature gap, we propose a portfolio theory-based interest model with two countries to evaluate the specific role of volatility and co-volatility risks in the formation of long-term European interest rates over the crisis and post-crisis periods, with an active role of the European Central Bank. We find that the Long-term equilibrium rates depend crucially on the covariances between international bond yields anticipated by investors. Positively anticipated covariances amplify the phenomena of fundamental contagions related to the degradations of public finance and solvency of sovereign debt issuer, while negatively anticipated covariances amplify the phenomena of Flight-to-quality.

Finally, we evaluate the impact of the ECB's QE programmes on the equilibrium of bond markets by a two-step econometric approach over the period January 2006 to September 2016 analyzing 21 European market pairs in a bivariate GARCH framework. Empirical results show that the decline in German and French long-term rates from March 2011 is partially due to the decrease in both risk premium and covariances with periphery countries. These declines actually amplify the mechanisms of Flight-to-quality. Finally, a lower sensitivity of rate to volatility and co-volatility risks during the crisis period gives credit to the hypothesis of an occasional fragmentation of the European sovereign bond markets. By using CDS as a variable to control default risks, we can conclude that the ECB's QE programs beginning from March 2015, have accelerated the "defragmentation process" of the European bond markets, already initiated since the OMT. However, according to the test à la Forbes and Rigobon, it seems difficult to affirm that QE programs have led to a significant increase in the conditional correlations between bond markets. In a supplementary empirical test, we show that QE has significantly reduced the sensitivities of bond yield spreads to the premiums paid on sovereign CDS.

Results tend to support the argument that during the sovereign debt crisis the German and French bond markets have made an ideal investment haven for investors. The logic of the flight-to-quality and the protection against sovereign risk have contributed to a decline in interest rate the same as an increase of bond return. Without doubt, the decrease in conditional variances of these returns also fueled more speculative strategies based on optimization of the return-risk pair in bond portfolios management. German and French bond yields are more sensitive to the volatility risks of covariances than those of peripheral countries. The covariance effects are stronger than those of variances, that are often present in an occasional way. The decrease in covariance between the German and French markets at the beginning of the crisis period significantly reduced the risk premiums required by investors, and contributed to the decrease in yields by

131 bps for Germany and 281 bps for France. Overall, the short-term risk premium intensity on portfolios and bond yields declines sharply during the crisis, and reappear later over the post-OMT and post-QE periods. Everything happens as if the mechanism of international bond portfolio allocation ceased during the sovereign debt crisis and then reappeared. These results are consistent with those of other recent studies which highlight the hypothesis of possible fragmentation on the European bond markets during the crisis.

This thesis fills a gap in the literature between interest rate models and the phenomenon of contagion and flight-to-quality. Meanwhile, it provides some empirical evidence which helps investors and policy makers understand better the formation of interest rates during EMU crisis, which allows them to make better decisions.

Future research may focus on following tow points. First, although the anticipated co-volatility of investor is theoretically proved important in the formation of long-term interest rates, it is hard to simulate them correctly in the real world. Second, to some extent, we know how to react when facing financial contagion, but we need to know when it comes and when it ends. Therefore, trustworthy indicators should be constructed.

As discussed at the beginning of this thesis, the interest rate is an essential indicator in the economy, due to its competencies in determining the cost of capital, in controlling the financial risks, and in pricing contingent claims. Among thousands of models, which one should use? Unfortunately, there is no simple answer. It depends on the main goal of the model, the economic context, the pricing and risk management effectiveness and many other user-specific conditions.

Appendix A

Descriptive Statistics

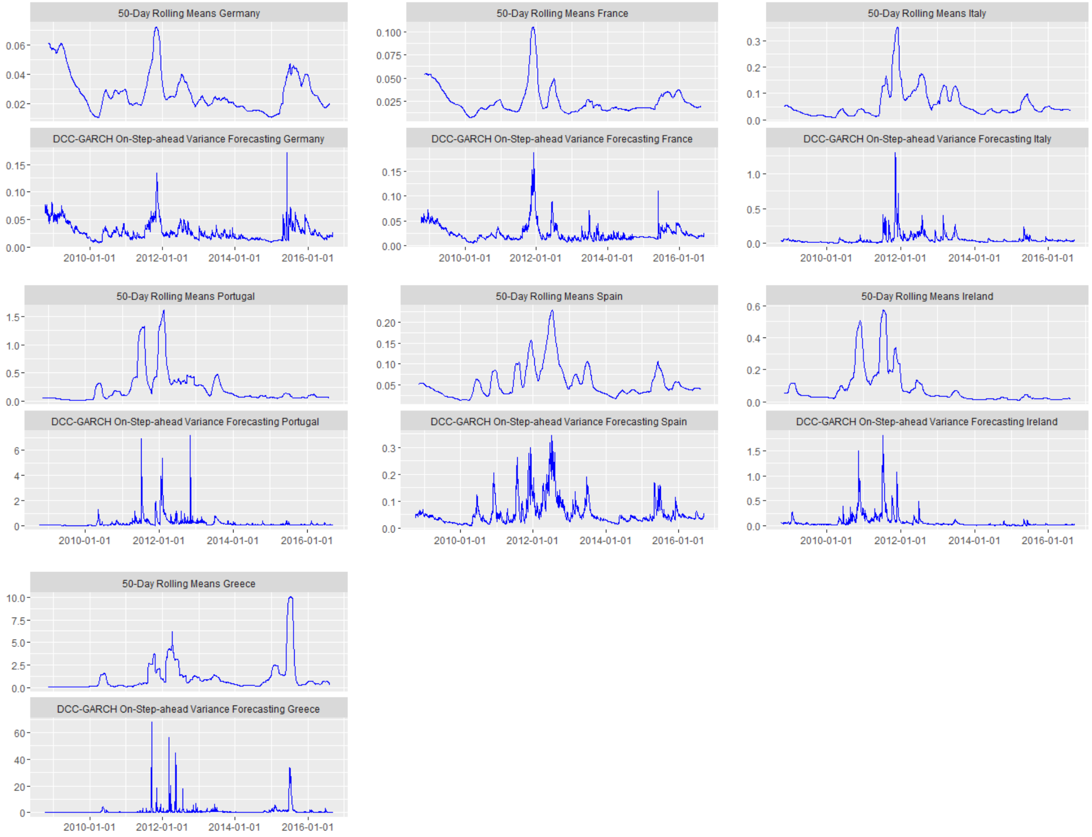
TABLE A.1: Descriptive statistics

Statistic	N	Mean	SD	Min	Max
German	2,070	1.826	1.078	−0.187	4.117
Spain	2,070	3.858	1.529	0.983	7.590
Ireland	2,070	4.408	2.779	0.338	13.895
Greece	2,070	11.645	7.564	4.393	48.602
France	2,070	2.323	1.105	0.100	4.346
Portugal	2,070	5.752	3.110	1.368	16.211
Italy	2,070	3.772	1.434	1.049	7.288
AAA	2,070	0.230	0.573	−0.682	3.637
CDSES	2,070	155.339	100.322	45.420	492.070
CDSFR	2,070	49.722	32.422	14.006	171.560
CDSIR	2,070	228.386	226.935	29.280	1,191.158
CDSDE	2,070	25.017	16.671	6.640	92.500
CDSGR	2,070	9,209.199	6,805.436	66.500	14,911.740
CDSIT	2,070	163.326	99.781	48.000	498.660
CDSPT	2,070	346.660	314.960	37.000	1,521.450
Vstoxx	2,070	26.048	9.553	12.713	87.513

Appendix B

One-step-ahead Variance Forecasts

FIGURE B.1: One-step-ahead Variance Forecasts



Appendix C

One-step-ahead Covariance Forecasts

FIGURE C.1: One-step-ahead Covariance Forecasts

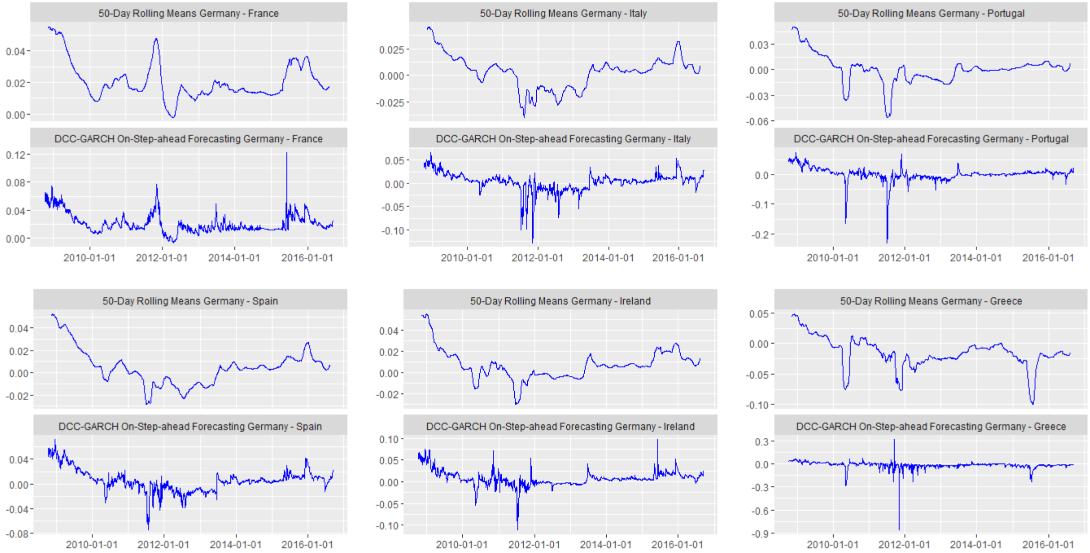
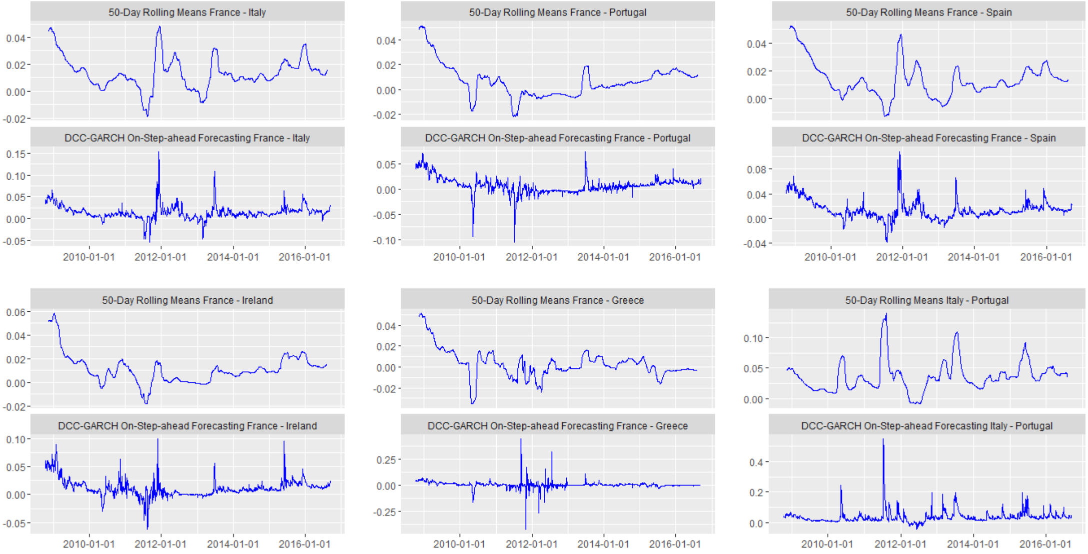


Figure C.2: One-step-ahead Covariance Forecasts



Appendix D

Two-step GARCH Models

TABLE D.1: Parameter estimates of the GARCH models

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

	Germany-France				Germany-Italy				Germany-Portugal			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	**-0.003**		**0.008**	**-0.005**	**-0.003**		**0.006**	**-0.006**	**-0.003**	-0.003	0.003	**-0.005**
	(0.015)		**(0.000)**	**(0.002)**	**(0.006)**		**(0.012)**	**(0.000)**	**(0.006)**	(0.375)	(0.282)	**(0.001)**
β_{02}	-0.001		**0.051**	**-0.010**	**-0.007**		**-0.006**	**-0.008**	**-0.007**	-0.006	0.002	**-0.010**
	(0.775)		**(0.000)**	**(0.001)**	**(0.000)**		**(0.005)**	**(0.000)**	**(0.000)**	(0.112)	(0.403)	**(0.000)**
β_{11}	0.584		**-2.332**	**2.976**	0.010		**-2.637**	0.413	0.116	**-2.837**	**-2.203**	**0.496**
	(0.121)		**(0.020)**	**(0.000)**	(0.953)		**(0.003)**	(0.128)	(0.491)	**(0.005)**	**(0.000)**	**(0.005)**
β_{12}	**-2.669**		-0.220	-0.696	0.035		0.065	**0.210**	0.006	0.715	0.000	0.010
	(0.000)		(0.913)	(0.484)	(0.328)		(0.222)	**(0.011)**	(0.456)	(0.479)	(1.00)	(0.189)
β_{21}	0.139		0.274	-0.975	**0.266**		**-2.191**	**1.868**	**0.704**	**1.629**	**0.487**	**1.879**
	(0.795)		(0.842)	(0.111)	**(0.039)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.044)**	**(0.000)**	**(0.000)**
β_{22}	**3.464**		**-4.812**	**3.609**	**1.197**		0.346	**2.28**	**0.430**	-0.270	0.021	**1.319**
	(0.000)		**(0.096)**	**(0.004)**	**(0.000)**		(0.206)	**(0.000)**	**(0.092)**	(0.774)	(0.96)	**(0.012)**
β_{31}	**0.005**		**0.011**	**0.013**	**0.008**		**0.013**	**0.016**	**0.005**	**-0.005**	**0.008**	**0.013**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	**(0.000)**	**(0.000)**
β_{32}	**0.004**		**0.006**	**0.021**	**-0.010**		**-0.010**	**-0.011**	**0.003**	-0.001	**0.003**	**0.008**
	(0.000)		**(0.017)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.800)	**(0.026)**	**(0.000)**
β_{41}	**0.122**		**-0.148**	0.054	0.070		**-0.119**	-0.072	**0.106**	**0.161**	-0.089	0.055
	(0.005)		**(0.003)**	(0.595)	(0.122)		**(0.067)**	(0.488)	**(0.025)**	**(0.021)**	(0.248)	(0.620)
β_{42}	**0.201**		-0.178	-0.139	**0.188**		0.078	-0.135	**0.125**	**0.159**	-0.089	0.098
	(0.023)		(0.329)	(0.453)	**(0.000)**		(0.332)	(0.105)	**(0.012)**	**(0.086)**	(0.371)	(0.424)
β_{51}	**0.004**		**0.005**	**0.006**	**0.006**		**0.006**	**0.007**	**0.003**	-0.001	**0.003**	**0.006**
	(0.000)		**(0.000)**	**(0.003)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.540)	**(0.000)**	**(0.001)**
β_{52}	**0.002**		**0.002**	**0.010**	**-0.003**		**-0.002**	**-0.004**	**0.000**	0,000	**0.000**	**0.001**
	(0.000)		**(0.029)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.006)**	(0.612)	**(0.078)**	**(0.000)**
c_{11}	**0.000**		**0.000**	**0.000**	**0.000**		**0.000**	**0.000**	**0.000**	**0.005**	**0.000**	**0.000**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	**(0.000)**
c_{22}	**0.003**		**0.004**	**0.004**	**0.000**		**0.001**	**0.001**	**0.001**	**0.002**	**0.001**	**0.003**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
c_{12}	**0.005**		**0.001**	**0,000**	**0.000**		0.000	**0.002**	**0.000**	**0.004**	**0.000**	**0.000**
	(0.000)		**(0.039)**	**(0.066)**	**(0.002)**		(0.117)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.002)**
b_{11}	**0.905**		**0.697**	**0.916**	**0.895**		**0.634**	**0.942**	**0.879**	**-0.927**	**0.833**	**0.840**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
b_{22}	**0.457**		**0.398**	**0.195**	**0.692**		**0.532**	**0.220**	**0.478**	**0.271**	**0.626**	**0.236**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.004)**	**(0.000)**	**(0.001)**	**(0.000)**	**(0.000)**
b_{12}	**-0.992**		**0.557**	**0.895**	**0.935**		0.084	**-0.977**	**0.783**	**-0.938**	**0.771**	**0.870**
	(0.000)		**(0.005)**	**(0.000)**	**(0.000)**		(0.867)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
a_{11}	**0.072**		**0.333**	**0.060**	**0.094**		**0.380**	**0.041**	**0.077**	0.022	**0.165**	**0.077**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.121)	**(0.000)**	**(0.000)**
a_{22}	**0.568**		**0.855**	**0.678**	**0.202**		**0.238**	**0.347**	**0.379**	**0.446**	**0.400**	**0.366**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
a_{12}	0.002		**0.147**	**0.027**	**0.029**		**0.110**	-0.005	**0.080**	**0.019**	**0.188**	**0.024**
	(0.127)		**(0.031)**	**(0.056)**	**(0.002)**		**(0.082)**	(0.473)	**(0.000)**	**(0.028)**	**(0.000)**	**(0.038)**

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

	Germany-Spain				Germany-Ireland				Germany-Greece			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	**-0.004**		0.002	**-0.005**	**-0.003**		**0.004**	**-0.006**	**-0.002**	-0.003	**0.005**	
	(0.000)		(0.285)	**(0.001)**	**(0.012)**		**(0.061)**	**(0.001)**	**(0.044)**	(0.358)	**(0.032)**	
β_{02}	**-0.011**		-0.003	**-0.010**	0.004		**0.016**	-0.006	**-0.005**	-0.005	-0.003	
	(0.000)		(0.550)	**(0.000)**	(0.300)		**(0.010)**	(0.442)	**(0.000)**	(0.286)	(0.219)	
β_{11}	0.067		**-2.073**	**0.640**	-0.028		**-1.923**	0.353	-0.006	-1.476	**-1.918**	
	(0.704)		**(0.007)**	**(0.014)**	(0.890)		**(0.009)**	(0.321)	(0.974)	(0.218)	**(0.034)**	
β_{12}	**-0.775**		**-1.170**	0.000	0.080		0.079	**3.531**	-0.001	**1.410**	0.000	
	(0.000)		**(0.000)**	(1.00)	(0.344)		(0.435)	**(0.027)**	(0.195)	**(0.066)**	(0.496)	
β_{21}	0.080		**-1.250**	**2.190**	**2.241**		**1.126**	**4.311**	-0.010	-0.635	0.010	
	(0.683)		**(0.001)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	(0.797)	(0.325)	(0.911)	
β_{22}	**2.207**		**2.335**	**1.847**	-0.359		-0.759	-3.634	0.032	**-1.251**	0.093	
	(0.000)		**(0.009)**	**(0.000)**	(0.658)		(0.485)	(0.317)	(0.519)	**(0.073)**	(0.233)	
β_{31}	**0.005**		**0.011**	**0.011**	**0.006**		**0.011**	**0.016**	**0.009**	**-0.004**	**0.014**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.014)**	**(0.000)**	
β_{32}	-0.001		0.004	-0.001	**0.016**		**0.010**	**0.108**	**-0.005**	**0.580**	-0.006	
	(0.196)		(0.171)	(0.246)	**(0.000)**		**(0.008)**	**(0.000)**	**(0.036)**	**(0.002)**	(0.134)	
β_{41}	0.070		**-0.177**	0.020	**0.079**		**-0.213**	-0.004	0.048	**0.117**	**-0.199**	
	(0.135)		**(0.002)**	(0.857)	**(0.095)**		**(0.000)**	(0.970)	(0.260)	**(0.064)**	**(0.000)**	
β_{42}	0.094		**-0.491**	-0.048	0.161		-0.262	0.326	**0.203**	**0.216**	**0.210**	
	(0.146)		**(0.022)**	(0.616)	(0.320)		(0.359)	(0.595)	**(0.000)**	**(0.043)**	**(0.004)**	
β_{51}	**0.003**		**0.004**	**0.004**	**0.004**		**0.005**	**0.007**	**0.006**	0.001	**0.007**	
	(0.000)		**(0.000)**	**(0.076)**	**(0.000)**		**(0.000)**	**(0.001)**	**(0.000)**	(0.704)	**(0.000)**	
β_{52}	**0.001**		**0.002**	0.000	**0.002**		**0.002**	**0.014**	0.000	**-0.002**	0.000	
	(0.000)		**(0.000)**	(0.389)	**(0.000)**		**(0.000)**	**(0.000)**	(0.924)	**(0.002)**	(0.859)	
c_{11}	**0.000**		**0.000**	**0.000**	**0.000**		**0.000**	**0.000**	**0.000**	0.000	**0.000**	
	(0.000)		**(0.018)**	**(0.000)**	**(0.000)**		**(0.039)**	**(0.000)**	**(0.000)**	(0.711)	**(0.054)**	
c_{22}	**0.001**		**0.003**	**0.001**	**0.003**		**0.009**	**0.004**	**0.001**	**0.003**	**0.000**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	**(0.005)**	
c_{12}	**0.000**		**0.000**	**0.000**	**0.000**		**0.001**	0.003	**0.000**	**0.002**	0.000	
	(0.000)		**(0.011)**	**(0.027)**	**(0.000)**		**(0.000)**	(0.111)	**(0.001)**	**(0.035)**	(0.173)	
b_{11}	**0.899**		**0.792**	**0.918**	**0.908**		**0.779**	**0.899**	**0.889**	**0.982**	**0.779**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	
b_{22}	**0.653**		**0.560**	**0.240**	**0.547**		**0.294**	**0.713**	**0.620**	0.147	**0.869**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.429)	**(0.000)**	
b_{12}	**0.876**		**0.815**	**0.943**	**0.787**		**0.545**	-0.346	**0.930**	-0.022	**0.929**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	(0.656)	**(0.000)**	(0.965)	**(0.000)**	
a_{11}	**0.090**		**0.238**	**0.057**	**0.084**		**0.265**	**0.074**	**0.102**	0.013	**0.266**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.125)	**(0.000)**	
a_{22}	**0.358**		**0.438**	**0.564**	**0.672**		**1.345**	**0.308**	**0.221**	**0.188**	**0.102**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.017)**	**(0.000)**	
a_{12}	**0.066**		**0.100**	**0.024**	**0.072**		**0.181**	**-0.043**	**0.030**	**-0.083**	**0.047**	
	(0.000)		**(0.009)**	**(0.006)**	**(0.000)**		**(0.020)**	**(0.095)**	**(0.000)**	**(0.048)**	**(0.021)**	

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

	France-Italy				France-Portugal				France-Spain			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	-0.002		**0.014**	**-0.007**	-0.002		**0.014**	**-0.008**	**-0.004**		**0.013**	**-0.007**
	(0.156)		**(0.000)**	**(0.005)**	(0.120)		**(0.000)**	**(0.001)**	**(0.010)**		**(0.000)**	**(0.004)**
β_{02}	**-0.007**		**-0.006**	**-0.009**	**-0.008**		0,000	**-0.011**	**-0.011**		0.008	**-0.011**
	(0.000)		**(0.003)**	**(0.000)**	**(0.000)**		(0.911)	**(0.000)**	**(0.000)**		(0.137)	**(0.000)**
β_{11}	0.495		**-1.976**	**1.828**	**0.905**		**-2.967**	0.212	**-1.378**		**-3.983**	-0.380
	(0.308)		**(0.028)**	**(0.005)**	**(0.023)**		**(0.000)**	(0.745)	**(0.015)**		**(0.000)**	(0.626)
β_{12}	**0.063**		0.049	**0.148**	-0.001		-0.020	0.005	-0.235		**-0.565**	0.062
	(0.013)		(0.211)	**(0.051)**	(0.879)		(0.112)	(0.483)	(0.103)		**(0.057)**	(0.619)
β_{21}	0.375		-0.798	-0.204	0.003		**-1.209**	**2.379**	**3.468**		0.260	**5.231**
	(0.208)		(0.205)	(0.567)	(0.982)		**(0.000)**	**(0.000)**	**(0.000)**		(0.767)	**(0.000)**
β_{22}	**0.747**		0.023	**1.562**	**0.987**		0.386	**2.192**	**2.747**		0.764	**1.444**
	(0.000)		(0.937)	**(0.000)**	**(0.000)**		(0.472)	**(0.000)**	**(0.000)**		(0.486)	**(0.000)**
β_{31}	**0.009**		**0.004**	**0.024**	**0.007**		0.003	**0.017**	**0.005**		0.002	**0.021**
	(0.000)		**(0.025)**	**(0.000)**	**(0.000)**		(0.174)	**(0.000)**	**(0.000)**		(0.369)	**(0.000)**
β_{32}	**-0.010**		**-0.011**	**-0.011**	**0.003**		0.001	**0.009**	**-0.002**		-0.001	-0.001
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		(0.351)	**(0.000)**	**(0.008)**		(0.534)	(0.167)
β_{41}	**0.160**		-0.040	-0.099	**0.155**		-0.010	-0.192	0.095		-0.056	-0.236
	(0.066)		(0.743)	(0.497)	**(0.035)**		(0.942)	(0.175)	(0.254)		(0.716)	(0.105)
β_{42}	**0.161**		**0.170**	-0.018	**0.142**		0.128	0.032	**0.138**		-0.190	0.021
	(0.000)		**(0.028)**	(0.808)	**(0.005)**		(0.179)	(0.806)	**(0.034)**		(0.347)	(0.803)
β_{51}	**0.006**		**0.006**	**0.008**	**0.003**		**0.005**	0.002	**0.003**		**0.003**	**0.006**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	(0.202)	**(0.000)**		**(0.000)**	**(0.000)**
β_{52}	**-0.003**		**-0.002**	**-0.004**	**0.001**		**0.001**	**0.001**	**0.001**		**0.001**	0.000
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**		**(0.014)**	(0.323)
c_{11}	**0.000**		0.000	**0.000**	**0,000**		**0.000**	**0.000**	**0.000**		**0.001**	**0.000**
	(0.000)		(0.597)	**(0.000)**	**(0.000)**		**(0.035)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**
c_{22}	**0.001**		**0.001**	**0.001**	**0.001**		**0.000**	**0.002**	**0.000**		**0.010**	**0.001**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.001)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**
c_{12}	**0.001**		0.001	**0.001**	**0,000**		**0.001**	**0.001**	**0.000**		**0.003**	**0.001**
	(0.000)		(0.103)	**(0.032)**	**(0.000)**		**(0.014)**	**(0.031)**	**(0.000)**		**(0.000)**	**(0.001)**
b_{11}	**0.856**		**0.696**	**0.824**	**0.847**		**0.681**	**0.856**	**0.859**		**0.674**	**0.821**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**
b_{22}	**0.564**		**0.557**	**0.280**	**0.670**		**0.625**	**0.363**	**0.702**		**0.113**	**0.232**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**		**(0.009)**	**(0.000)**
b_{12}	**-0.746**		**-0.890**	-0.391	**0.830**		-0.197	**0.714**	**0.847**		**0.496**	0.156
	(0.000)		**(0.000)**	(0.539)	**(0.000)**		(0.660)	**(0.000)**	**(0.000)**		**(0.000)**	(0.486)
a_{11}	**0.167**		**0.565**	**0.132**	**0.171**		**0.574**	**0.095**	**0.162**		**0.522**	**0.125**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**
a_{22}	**0.259**		**0.240**	**0.350**	**0.308**		**0.464**	**0.359**	**0.301**		**0.768**	**0.565**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**
a_{12}	**0.060**		0.048	0.048	**0.102**		**0.206**	**0.049**	**0.090**		**0.297**	**0.144**
	(0.004)		(0.176)	(0.165)	**(0.000)**		**(0.020)**	**(0.017)**	**(0.000)**		**(0.000)**	**(0.000)**

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

	France-Ireland				France-Greece				Italy-Portugal			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	-0.003		**0.015**	**-0.007**	0.000		**0.011**		**-0.003**		**-0.003**	**-0.003**
	(0.109)		**(0.000)**	**(0.004)**	(0.200)		**(0.000)**		**(0.000)**		**(0.042)**	**(0.008)**
β_{02}	0.002		**0.018**	-0.011	**-0.010**		0.000		**-0.008**		**-0.001**	**-0.010**
	(0.668)		**(0.009)**	(0.188)	**(0.000)**		(0.111)		**(0.000)**		**(0.042)**	**(0.000)**
β_{11}	-0.484		**-2.066**	-0.481	0.380		-1.130		-0.007		-0.023	**0.213**
	(0.243)		**(0.017)**	(0.545)	(0.390)		(0.322)		(0.776)		(0.354)	**(0.000)**
β_{12}	0.048		0.070	0.956	0.000		0.000		**-0.020**		**-0.033**	-0.011
	(0.621)		(0.539)	(0.608)	(0.130)		(0.717)		**(0.001)**		**(0.075)**	(0.116)
β_{21}	**1.738**		**0.906**	**4.848**	**0.330**		**0.341**		**0.117**		0.028	0.104
	(0.000)		**(0.006)**	**(0.000)**	**(0.000)**		**(0.000)**		**(0.002)**		(0.699)	(0.142)
β_{22}	-0.590		-1.008	2.715	0.050		0.021		**0.957**		**1.263**	**0.760**
	(0.346)		(0.283)	(0.458)	(0.150)		(0.734)		**(0.000)**		**(0.000)**	**(0.000)**
β_{31}	**0.005**		0.002	**0.021**	**0.010**		**0.000**		**-0.010**		**-0.011**	**-0.010**
	(0.000)		(0.322)	**(0.000)**	**(0.000)**		**(0.015)**		**(0.000)**		**(0.000)**	**(0.000)**
β_{32}	**0.014**		**0.010**	**0.076**	**-0.010**		**-0.012**		**0.005**		0.002	**0.011**
	(0.000)		**(0.021)**	**(0.000)**	**(0.040)**		**(0.072)**		**(0.000)**		(0.111)	**(0.000)**
β_{41}	0.124		-0.069	-0.158	**0.140**		-0.101		**0.137**		**0.102**	0.037
	(0.108)		(0.638)	(0.302)	**(0.080)**		(0.423)		**(0.000)**		**(0.011)**	(0.483)
β_{42}	0.092		-0.138	0.104	**0.200**		**0.247**		**0.135**		**0.234**	0.050
	(0.603)		(0.684)	(0.858)	**(0.000)**		**(0.000)**		**(0.015)**		**(0.008)**	(0.687)
β_{51}	**0.003**		**0.004**	**0.006**	**0.010**		**0.018**		**-0.002**		**-0.002**	**-0.003**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**		**(0.000)**		**(0.000)**	**(0.000)**
β_{52}	**0.002**		**0.002**	**0.009**	0.000		0.000		**0.001**		**0.001**	**0.003**
	(0.000)		**(0.000)**	**(0.002)**	(0.940)		(0.971)		**(0.000)**		**(0.000)**	**(0.000)**
c_{11}	**0.000**		**0.000**	**0.000**	**0.000**		0.000		**0.000**		**0.000**	**0.000**
	(0.000)		**(0.021)**	**(0.000)**	**(0.000)**		(0.493)		**(0.000)**		**(0.049)**	**(0.001)**
c_{22}	**0.003**		**0.011**	**0.005**	**0.000**		**0.000**		**0.000**		**0.000**	**0.001**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.012)**		**(0.000)**		**(0.000)**	**(0.000)**
c_{12}	**0.001**		**0.006**	**0.001**	**0.000**		**0.000**		**0.000**		**0.000**	**0.000**
	(0.000)		**(0.000)**	**(0.049)**	**(0.000)**		**(0.053)**		**(0.000)**		**(0.046)**	**(0.003)**
b_{11}	**0.842**		**0.674**	**0.815**	**0.840**		**0.711**		**0.943**		**0.907**	**0.692**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**		**(0.000)**		**(0.000)**	**(0.000)**
b_{22}	**0.565**		**0.294**	**0.714**	**0.500**		**0.863**		**0.692**		**0.563**	**0.497**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**		**(0.000)**		**(0.000)**	**(0.000)**
b_{12}	**0.724**		-0.427	**0.813**	**0.750**		**-0.636**		**0.892**		**0.462**	**0.715**
	(0.000)		(0.111)	**(0.000)**	**(0.000)**		**(0.023)**		**(0.000)**		**(0.040)**	**(0.000)**
a_{11}	**0.176**		**0.569**	**0.130**	**0.200**		**0.560**		**0.037**		**0.052**	**0.107**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**		**(0.000)**		**(0.002)**	**(0.000)**
a_{22}	**0.618**		**1.266**	**0.293**	**0.300**		**0.126**		**0.296**		**0.530**	**0.325**
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**		**(0.000)**		**(0.000)**	**(0.000)**
a_{12}	**0.146**		**0.265**	**0.053**	**0.090**		**0.144**		**0.049**		**0.118**	**0.067**
	(0.000)		**(0.012)**	**(0.029)**	**(0.000)**		**(0.057)**		**(0.000)**		**(0.002)**	**(0.008)**

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

	Italy-Spain				Italy-Ireland				Italy-Greece			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	**-0.003**		**-0.003**	**-0.003**	**-0.002**		**-0.002**	**-0.003**	**-0.002**	-0.002	-0.002	
	(0.001)		**(0.050)**	**(0.003)**	**(0.003)**		**(0.073)**	**(0.003)**	**(0.034)**	(0.581)	(0.201)	
β_{02}	**-0.010**		-0.003	**-0.012**	0.003		**0.017**	-0.007	**-0.004**	-0.004	-0.003	
	(0.000)		(0.609)	**(0.000)**	(0.456)		**(0.015)**	(0.408)	**(0.000)**	(0.380)	(0.243)	
β_{11}	**-0.088**		**-0.082**	-0.033	-0.049		**-0.056**	**-0.253**	-0.024	-1.035	-0.041	
	(0.011)		**(0.041)**	(0.686)	(0.456)		**(0.002)**	**(0.001)**	(0.204)	(0.455)	(0.108)	
β_{12}	-0.169		0.071	-0.210	0.131		**0.232**	3.531	0.000	0.694	0.000	
	(0.259)		(0.876)	(0.447)	(0.228)		**(0.068)**	(0.154)	(0.472)	(0.319)	(0.798)	
β_{21}	**0.401**		0.177	**0.691**	**0.710**		**0.324**	**2.806**	-0.018	**1.337**	-0.014	
	(0.000)		(0.120)	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	(0.160)	**(0.096)**	(0.466)	
β_{22}	**0.506**		-0.719	**0.613**	0.281		**-1.638**	0.157	-0.024	0.236	-0.018	
	(0.000)		(0.117)	**(0.012)**	(0.407)		**(0.015)**	(0.946)	(0.349)	(0.819)	(0.703)	
β_{31}	**-0.010**		**-0.011**	**-0.010**	**-0.010**		**-0.011**	**-0.009**	**-0.009**	**-0.008**	**-0.011**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	
β_{32}	**-0.001**		0.000	**-0.002**	**0.018**		**0.012**	**0.146**	**-0.005**	0.179	**-0.009**	
	(0.025)		(0.931)	**(0.004)**	**(0.000)**		**(0.005)**	**(0.000)**	**(0.000)**	(0.345)	**(0.013)**	
β_{41}	**0.135**		**0.108**	0.057	**0.165**		**0.140**	**0.075**	**0.147**	**0.180**	**0.099**	
	(0.000)		**(0.005)**	(0.267)	**(0.000)**		**(0.000)**	**(0.097)**	**(0.000)**	**(0.016)**	**(0.011)**	
β_{42}	**0.184**		**-0.355**	0.070	0.128		-0.264	0.328	**0.218**	0.232	**0.222**	
	(0.004)		**(0.096)**	(0.417)	(0.436)		(0.467)	(0.569)	**(0.000)**	(0.025)	**(0.003)**	
β_{51}	**-0.002**		**-0.002**	**-0.002**	**-0.002**		**-0.001**	**-0.002**	**-0.001**	**-0.001**	**-0.001**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.021)**	**(0.000)**	
β_{52}	**0.003**		**0.003**	**0.001**	**0.003**		**0.002**	**0.020**	0.000	-0.001	0.000	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	(0.327)	(0.334)	(0.688)	
c_{11}	**0.000**		**0.000**	**0.000**	**0.000**		**0.000**	**0.000**	**0.000**	0.000	**0.000**	
	(0.000)		**(0.053)**	**(0.000)**	**(0.000)**		**(0.056)**	**(0.000)**	**(0.000)**	(0.207)	**(0.034)**	
c_{22}	**0.000**		**0.002**	**0.001**	**0.002**		**0.009**	**0.003**	**0.001**	**0.003**	**0.000**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.002)**	
c_{12}	**0.000**		0.001	**0.000**	**0.000**		0.000	0.000	**0.002**	**0.003**	**0.000**	
	(0.000)		(0.213)	**(0.000)**	**(0.043)**		(0.241)	(0.527)	**(0.000)**	**(0.052)**	**(0.044)**	
b_{11}	**0.946**		**0.890**	**0.637**	**0.948**		**0.879**	**0.723**	**0.839**	**0.891**	**0.902**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	
b_{22}	**0.718**		**0.616**	**0.318**	**0.556**		**0.291**	**0.735**	**0.339**	0.146	**0.876**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.403)	**(0.000)**	
b_{12}	**0.905**		0.249	**0.770**	**0.777**		0.403	**-0.973**	-0.379	-0.116	**0.926**	
	(0.000)		(0.659)	**(0.000)**	**(0.000)**		(0.131)	**(0.000)**	(0.237)	(0.843)	**(0.000)**	
a_{11}	**0.036**		**0.054**	**0.124**	**0.036**		**0.063**	**0.124**	**0.048**	0.031	**0.049**	
	(0.000)		**(0.008)**	**(0.000)**	**(0.000)**		**(0.005)**	**(0.000)**	**(0.000)**	(0.232)	**(0.002)**	
a_{22}	**0.316**		**0.388**	**0.568**	**0.682**		**1.353**	**0.284**	**0.294**	**0.201**	**0.102**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.005)**	**(0.000)**	
a_{12}	**0.047**		**0.106**	**0.060**	**0.054**		**0.174**	**0.014**	**-0.033**	**-0.057**	**0.034**	
	(0.000)		**(0.019)**	**(0.010)**	**(0.006)**		**(0.018)**	**(0.023)**	**(0.007)**	**(0.074)**	**(0.000)**	

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

	Portugal-Spain				Portugal-Ireland				Portugal-Greece			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	**-0.004**		0.000	**-0.005**	**-0.003**		0.001	**-0.005**	**-0.003**	-0.004	0.000	
	(0.000)		(0.794)	**(0.001)**	**(0.002)**		(0.420)	**(0.003)**	**(0.002)**	(0.154)	(0.807)	
β_{02}	**-0.010**		-0.003	**-0.009**	0.002		**0.017**	-0.006	**-0.005**	-0.004	-0.003	
	(0.000)		(0.635)	**(0.000)**	(0.540)		**(0.011)**	(0.439)	**(0.000)**	(0.410)	(0.162)	
β_{11}	**-0.055**		**-0.029**	**-0.053**	-0.007		0.005	**-0.037**	-0.002	-1.992	**0.017**	
	(0.000)		**(0.006)**	**(0.000)**	(0.250)		(0.639)	**(0.000)**	(0.407)	(0.128)	**(0.000)**	
β_{12}	**0.351**		**-1.159**	0.188	0.143		0.078	1.850	-0.001	0.451	0.000	
	(0.000)		**(0.000)**	(0.236)	(0.391)		(0.589)	(0.256)	(0.253)	(0.569)	(0.670)	
β_{21}	**1.671**		**1.138**	**1.732**	**0.087**		**0.058**	**2.166**	**0.107**	**2.814**	**0.097**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.035)**	**(0.000)**	**(0.000)**	**(0.004)**	**(0.000)**	
β_{22}	**0.163**		**0.525**	**0.312**	-0.047		0.021	1.366	0.021	0.853	0.000	
	(0.091)		**(0.000)**	**(0.021)**	(0.853)		(0.941)	(0.234)	(0.301)	(0.388)	(0.994)	
β_{31}	**0.005**		**0.005**	**0.009**	**0.005**		**0.005**	**0.011**	**0.006**	0.000	**0.005**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.951)	**(0.000)**	
β_{32}	**-0.001**		0.000	**-0.001**	**0.018**		**0.012**	**0.128**	**-0.004**	**0.799**	**-0.008**	
	(0.032)		(0.986)	**(0.047)**	**(0.000)**		**(0.004)**	**(0.000)**	**(0.063)**	**(0.000)**	**(0.053)**	
β_{41}	**0.132**		0.031	**0.139**	**0.111**		-0.022	**0.177**	**0.106**	0.074	-0.015	
	(0.001)		(0.496)	**(0.083)**	**(0.002)**		(0.656)	**(0.032)**	**(0.003)**	(0.311)	(0.746)	
β_{42}	**0.135**		-0.333	0.082	0.116		-0.203	0.318	**0.203**	**0.261**	**0.221**	
	(0.029)		(0.129)	(0.303)	(0.466)		(0.531)	(0.591)	**(0.000)**	**(0.016)**	**(0.002)**	
β_{51}	**0.001**		**0.001**	**0.001**	**0.001**		**0.001**	**0.002**	**0.001**	0.001	**0.001**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.165)	**(0.000)**	
β_{52}	**0.001**		**0.003**	0.000	**0.002**		**0.002**	**0.017**	0.000	**-0.002**	0.000	
	(0.000)		**(0.000)**	(0.638)	**(0.000)**		**(0.000)**	**(0.000)**	(0.743)	**(0.000)**	(0.965)	
c_{11}	**0.000**		**0.000**	**0.000**	**0.000**		**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.001)**	**(0.001)**	**(0.000)**	**(0.096)**	**(0.001)**	
c_{22}	**0.000**		**0.003**	**0.001**	**0.002**		**0.010**	**0.004**	**0.001**	**0.003**	**0.000**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	**(0.003)**	
c_{12}	0.002		0.001	**0.002**	**0.000**		**0.002**	0.001	**0.000**	**0.003**	0.000	
	(0.626)		(0.230)	**(0.000)**	**(0.029)**		**(0.036)**	(0.274)	**(0.000)**	**(0.000)**	(0.144)	
b_{11}	**0.885**		**0.630**	**0.890**	**0.898**		**0.589**	**0.907**	**0.901**	**0.967**	**0.658**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	
b_{22}	**0.668**		**0.545**	**0.169**	**0.550**		**0.282**	**0.710**	**0.489**	0.136	**0.847**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	(0.496)	**(0.000)**	
b_{12}	-0.736		0.404	**-0.733**	**0.809**		-0.199	-0.287	**0.926**	-0.452	**0.940**	
	(0.836)		(0.397)	**(0.000)**	**(0.000)**		(0.653)	(0.781)	**(0.000)**	(0.234)	**(0.000)**	
a_{11}	**0.100**		**0.477**	**0.059**	**0.094**		**0.564**	**0.065**	**0.091**	**0.022**	**0.450**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.015)**	**(0.000)**	
a_{22}	**0.358**		**0.450**	**0.608**	**0.687**		**1.365**	**0.316**	**0.282**	**0.194**	**0.117**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	**(0.018)**	**(0.000)**	
a_{12}	-0.001		0.101	**-0.043**	**0.046**		0.138	-0.040	**0.028**	**-0.064**	**0.033**	
	(0.947)		(0.242)	**(0.068)**	**(0.018)**		(0.161)	(0.292)	**(0.000)**	**(0.012)**	**(0.051)**	

Mean equation: $\Delta R_{i,t} = \beta_{0,i} + \beta_{1,i}\Delta E_t(V_{i,t+1}) + \beta_{2,i}\Delta E_t(Cov_{ij,t+1}) + \beta_{3,i}\Delta Vstoxx_t + \beta_{4,i}\Delta r_t + \beta_{5,i}\Delta CDS_{i,t} + \varepsilon_{i,t}$
Columns without estimates represent the periods during which estimations haven't converged.

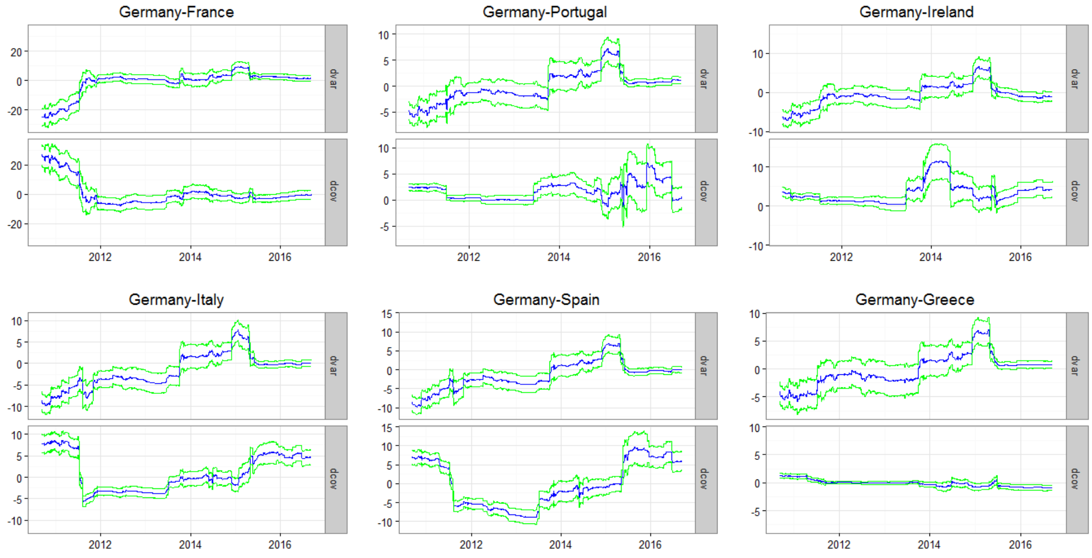
	Spain-Ireland				Spain-Greece				Ireland-Greece			
Coefficient	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT	Full sample	Pre-crisis	Crisis	Post-OMT
β_{01}	**-0.003**		-0.002	**-0.005**	**-0.003**	-0.004	0.000		0.001	-0.002	0.006	
	(0.013)		(0.566)	**(0.001)**	**(0.007)**	(0.183)	(0.908)		(0.787)	(0.631)	(0.187)	
β_{02}	0.004		**0.012**	-0.008	**-0.006**	-0.006	-0.004		**-0.005**	-0.003	**-0.004**	
	(0.248)		**(0.069)**	(0.367)	**(0.000)**	(0.188)	(0.118)		**(0.000)**	(0.475)	**(0.093)**	
β_{11}	**0.778**		**-0.547**	0.336	**0.735**	1.247	**-0.472**		0.013	-0.291	0.068	
	(0.000)		**(0.017)**	(0.111)	**(0.000)**	(0.382)	**(0.034)**		(0.878)	(0.678)	(0.545)	
β_{12}	**0.137**		0.037	1.742	-0.001	**1.683**	0.000		0.000	**-1.796**	0.000	
	(0.073)		(0.806)	(0.553)	(0.183)	**(0.023)**	(0.575)		(0.605)	**(0.071)**	(0.839)	
β_{21}	**0.855**		**0.376**	**2.435**	-0.018	**2.108**	**-0.117**		**0.975**	**3.961**	**0.966**	
	(0.000)		**(0.009)**	**(0.000)**	(0.683)	**(0.014)**	**(0.098)**		**(0.000)**	**(0.000)**	**(0.000)**	
β_{22}	**-0.558**		-0.486	2.465	0.046	0.356	0.030		**0.076**	**4.197**	0.045	
	(0.046)		(0.248)	(0.477)	(0.213)	(0.630)	(0.636)		**(0.010)**	**(0.000)**	(0.245)	
β_{31}	0.000		-0.002	**0.002**	0.000	-0.001	-0.001		**0.014**	**0.007**	**0.004**	
	(0.850)		(0.470)	**(0.006)**	(0.509)	(0.156)	(0.706)		**(0.000)**	**(0.006)**	**(0.059)**	
β_{32}	**0.014**		**0.008**	**0.122**	**-0.005**	**0.858**	**-0.008**		**-0.005**	**0.961**	**-0.007**	
	(0.000)		**(0.052)**	**(0.000)**	**(0.032)**	**(0.000)**	**(0.049)**		**(0.046)**	**(0.000)**	**(0.079)**	
β_{41}	0.064		-0.310	0.046	0.077	**0.118**	-0.304		0.175	0.016	**0.567**	
	(0.284)		(0.110)	(0.637)	(0.207)	**(0.098)**	(0.102)		(0.053)	(0.877)	**(0.000)**	
β_{42}	0.122		-0.412	0.419	**0.189**	**0.192**	**0.246**		**0.238**	**0.242**	**0.264**	
	(0.451)		(0.143)	(0.471)	**(0.000)**	**(0.068)**	**(0.000)**		**(0.000)**	**(0.032)**	**(0.000)**	
β_{51}	**0.003**		**0.003**	**0.002**	**0.003**	**0.002**	**0.004**		**0.002**	**0.002**	**0.002**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	
β_{52}	**0.002**		**0.002**	**0.016**	0.000	**-0.002**	0.000		0.000	**-0.003**	0.000	
	(0.000)		**(0.000)**	**(0.000)**	(0.917)	**(0.000)**	(0.921)		(0.864)	**(0.000)**	(0.925)	
c_{11}	**0.000**		**0.001**	**0.001**	**0.000**	**0.002**	**0.001**		**0.000**	**0.000**	**0.002**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.004)**		**(0.001)**	**(0.090)**	**(0.000)**	
c_{22}	**0.002**		**0.009**	**0.004**	**0.001**	**0.003**	**0.000**		**0.001**	**0.003**	**0.000**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	**(0.004)**		**(0.000)**	**(0.001)**	**(0.002)**	
c_{12}	**0.003**		**0.003**	0.000	**0.000**	**0.002**	0.000		**0.000**	0.001	0.001	
	(0.000)		**(0.005)**	(0.104)	**(0.000)**	**(0.005)**	(0.805)		**(0.001)**	(0.177)	(0.103)	
b_{11}	**0.876**		**0.718**	**0.472**	**0.892**	-0.046	**0.788**		**0.696**	**0.937**	**0.448**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	(0.535)	**(0.000)**		**(0.000)**	**(0.000)**	**(0.000)**	
b_{22}	**0.549**		**0.296**	**0.707**	**0.471**	0.224	**0.845**		**0.558**	0.182	**0.816**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	(0.160)	**(0.000)**		**(0.000)**	(0.296)	**(0.000)**	
b_{12}	**-0.658**		**0.524**	**0.680**	**0.899**	-0.187	-0.407		**0.489**	**0.555**	0.071	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	(0.552)	(0.279)		**(0.000)**	**(0.063)**	(0.848)	
a_{11}	**0.121**		**0.240**	**0.151**	**0.110**	**0.480**	**0.191**		**0.539**	**0.058**	**1.322**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**		**(0.000)**	**(0.002)**	**(0.000)**	
a_{22}	**0.691**		**1.307**	**0.324**	**0.294**	**0.280**	**0.125**		**0.271**	**0.271**	**0.144**	
	(0.000)		**(0.000)**	**(0.000)**	**(0.000)**	**(0.003)**	**(0.000)**		**(0.000)**	**(0.001)**	**(0.000)**	
a_{12}	**0.139**		**0.263**	**0.095**	**0.053**	**0.135**	**0.116**		**0.171**	**0.083**	**0.200**	
	(0.000)		**(0.000)**	**(0.005)**	**(0.000)**	**(0.062)**	**(0.044)**		**(0.000)**	**(0.052)**	**(0.036)**	

Appendix E

Two-step Rolling Linear Regression Estimates

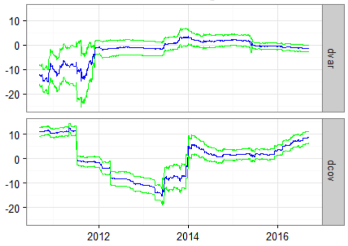
(*dvar* represents the sensitivity of interest rates to conditional variances. *dcov* represents the sensitivity of interest rates to conditional covariances.)

Figure E.1: Two-step Rolling Linear Regression Estimates

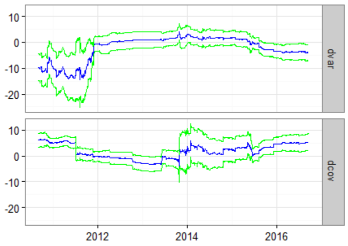


France-Germany

France-Portugal



France-Ireland



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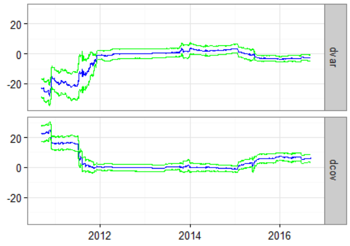
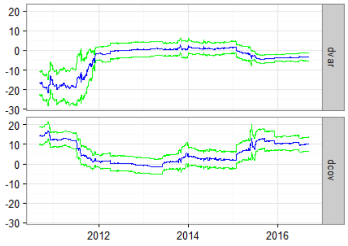
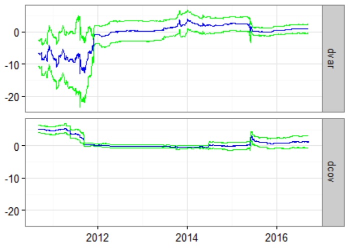


France-Spain



France-Greece



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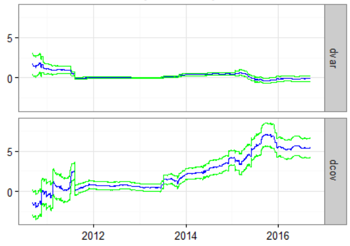

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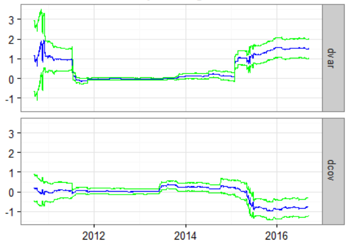

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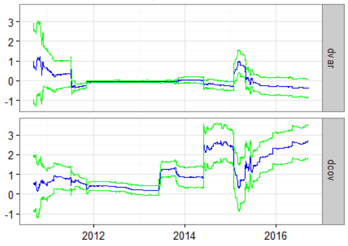
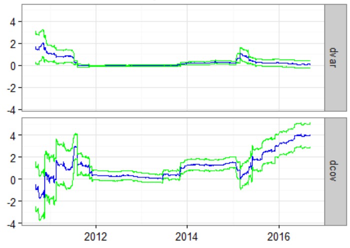
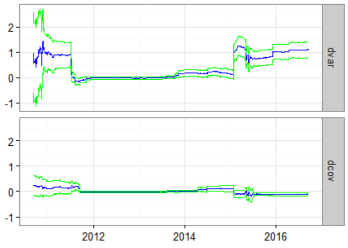


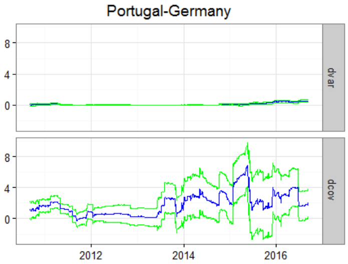
Italy-France

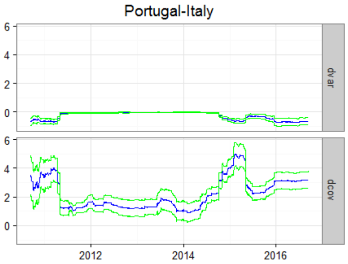


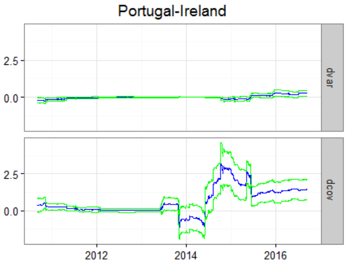
Italy-Spain

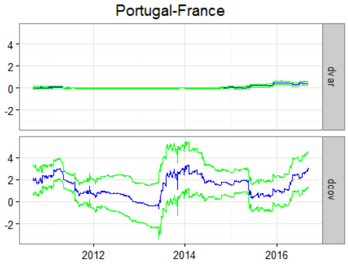
Italy-Greece




Portugal-Germany
8
4
0
dvar
8
4
0
dcov
2012
2014
2016


Portugal-Italy
6
4
2
0
dvar
6
4
2
0
dcov
2012
2014
2016


Portugal-Ireland
2.5
0.0
dvar
2.5
0.0
dcov
2012
2014
2016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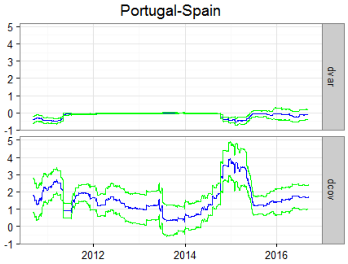
0
-2
dvar
4
2
0
-2
dcov
2012
2014
2016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dvar
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4
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2
1
0
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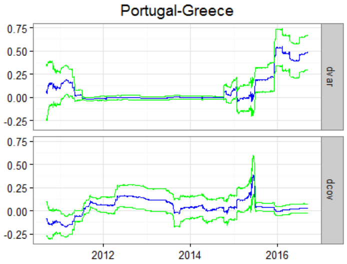
dcov
2012
2014
2016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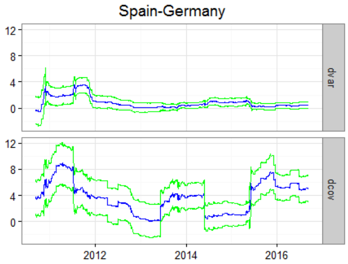
0.50
0.25
0.00
-0.25
dvar
0.75
0.50
0.25
0.00
-0.25
dcov
2012
2014
2016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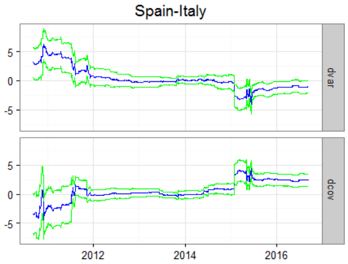
0
dvar
12
8
4
0
dcov
2012
2014
2016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5
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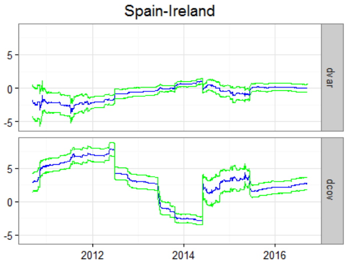
-5
dvar
5
0
-5
dcov
2012
2014
2016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5
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-5
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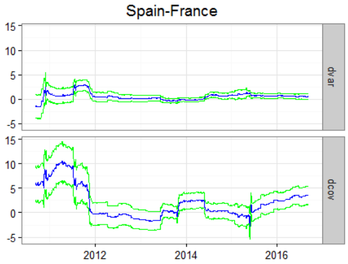
5
0
-5
dcov
2012
2014
2016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5
0
-5
dcov
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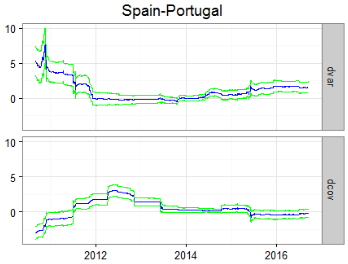
2014
2016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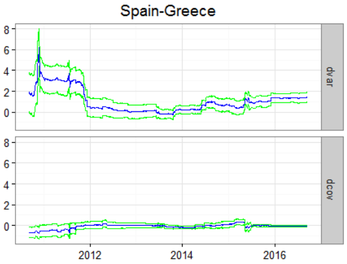
dvar
10
5
0
dcov
2012
2014
2016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6
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dvar
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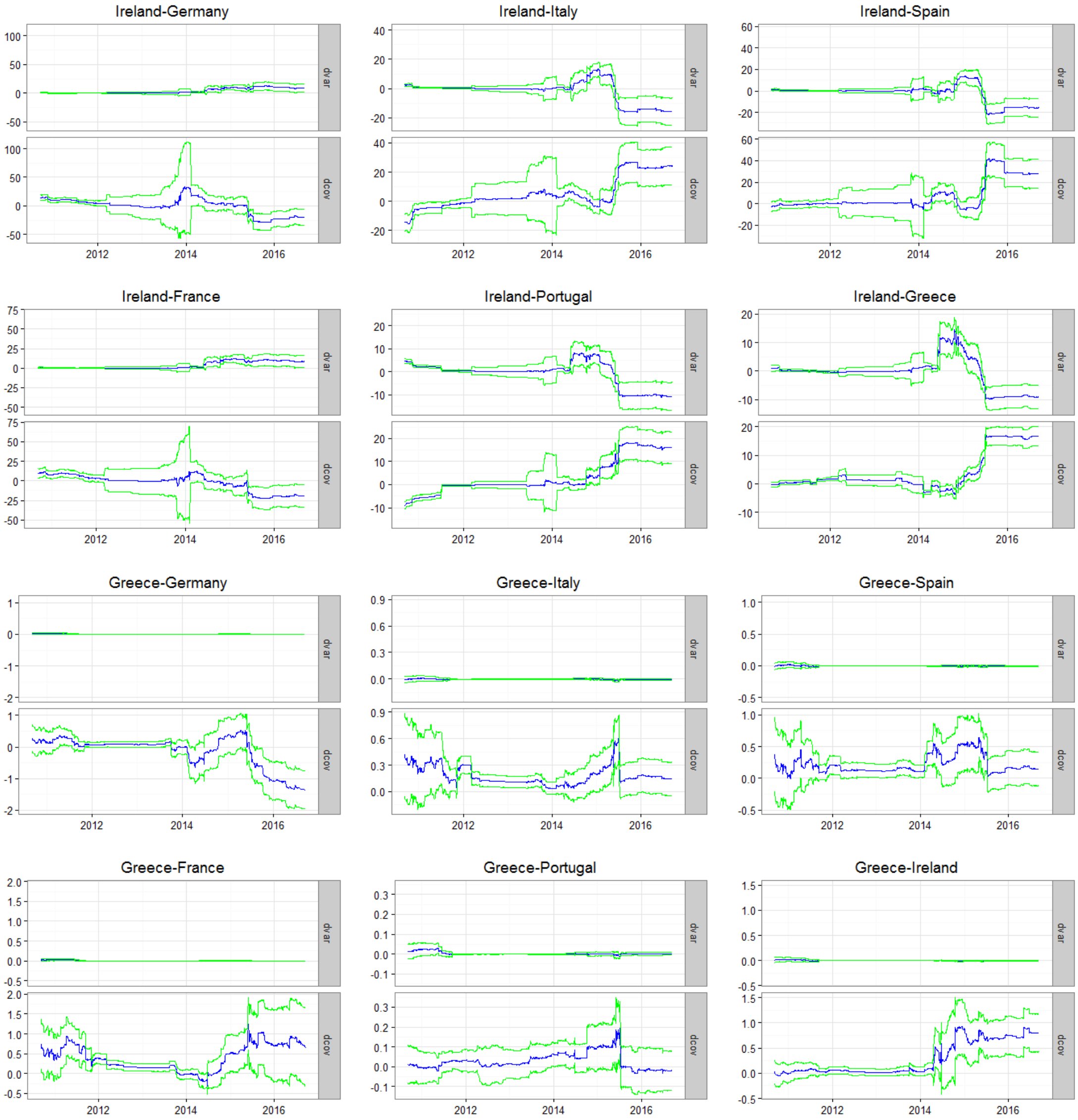
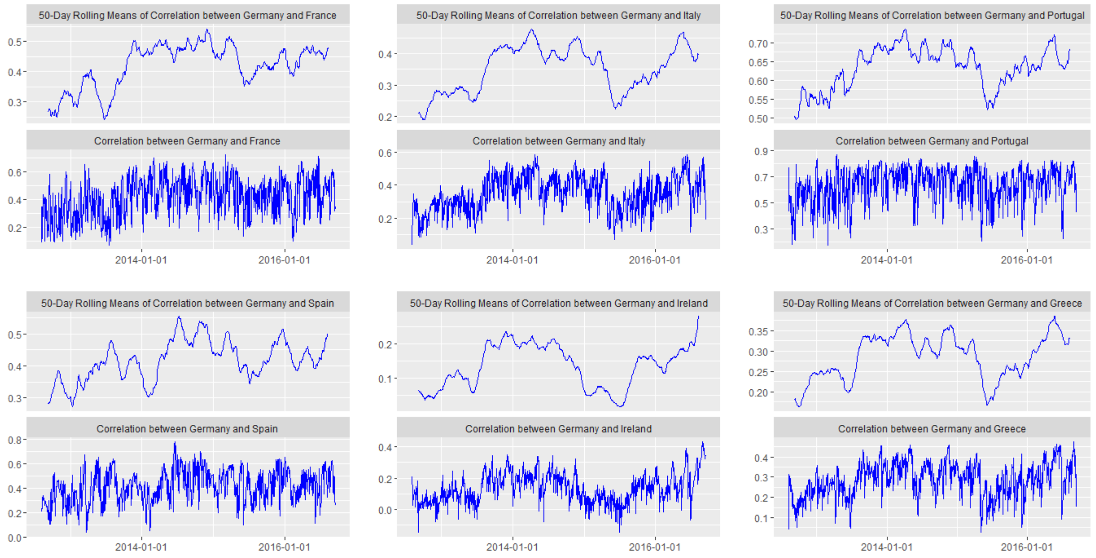
6
4
2
0
dcov
2012
2014
2016

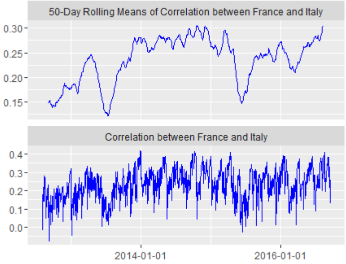

Ireland-Germany
Ireland-Italy
Ireland-Spain
Ireland-France
Ireland-Portugal
Ireland-Greece
Greece-Germany
Greece-Italy
Greece-Spain
Greece-France
Greece-Portugal
Greece-Ireland
dvar
dcov
2012
2014
2016

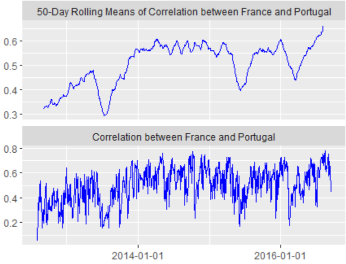
Appendix F

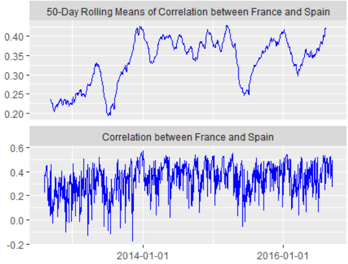
Conditional Correlation Dynamics

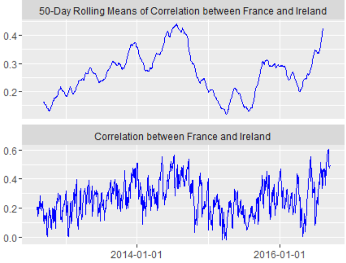
FIGURE F.1: Conditional Correlation Dynamics




50-Day Rolling Means of Correlation between France and Italy
0.30
0.25
0.20
0.15
Correlation between France and Italy
0.4
0.3
0.2
0.1
0.0
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between France and Portugal
0.6
0.5
0.4
0.3
Correlation between France and Portugal
0.8
0.6
0.4
0.2
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between France and Spain
0.40
0.35
0.30
0.25
0.20
Correlation between France and Spain
0.6
0.4
0.2
0.0
-0.2
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between France and Ireland
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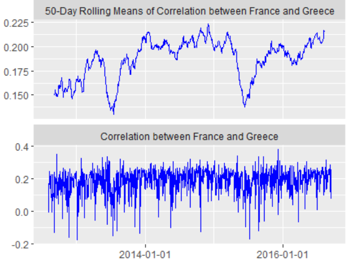
0.3
0.2
Correlation between France and Ireland
0.6
0.4
0.2
0.0
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between France and Greece
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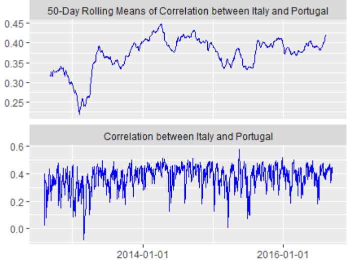
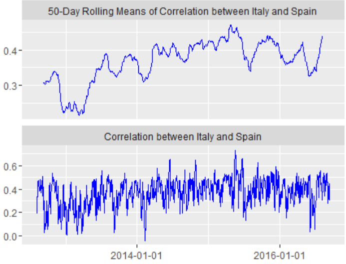
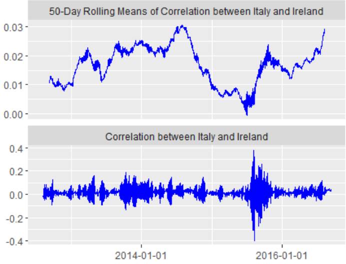
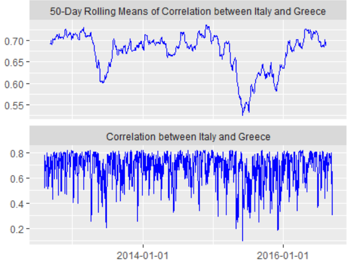
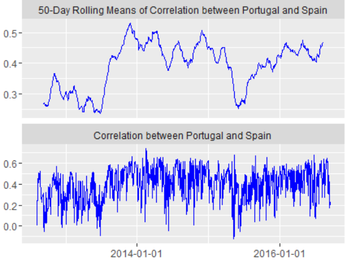
0.200
0.175
0.150
Correlation between France and Greece
0.4
0.2
0.0
-0.2
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Italy and Portugal
0.45
0.40
0.35
0.30
0.25
Correlation between Italy and Portugal
0.6
0.4
0.2
0.0
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Italy and Spain
0.4
0.3
Correlation between Italy and Spain
0.6
0.4
0.2
0.0
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Italy and Ireland
0.03
0.02
0.01
0.00
Correlation between Italy and Ireland
0.4
0.2
0.0
-0.2
-0.4
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Italy and Greece
0.70
0.65
0.60
0.55
Correlation between Italy and Greece
0.8
0.6
0.4
0.2
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Portugal and Spain
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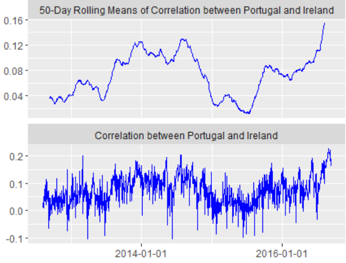
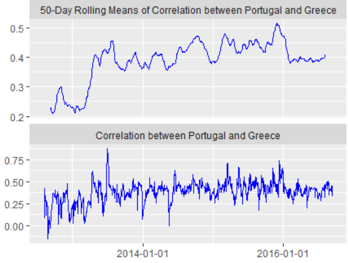
0.4
0.3
Correlation between Portugal and Spain
0.6
0.4
0.2
0.0
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Portugal and Ireland
0.16
0.12
0.08
0.04
Correlation between Portugal and Ireland
0.2
0.1
0.0
-0.1
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Portugal and Greece
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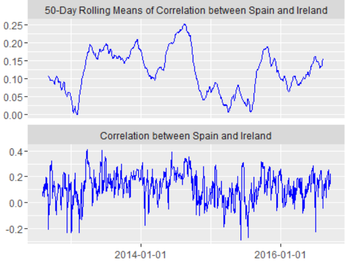
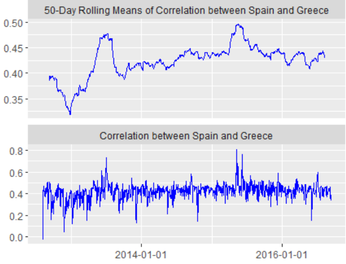
0.4
0.3
0.2
Correlation between Portugal and Greece
0.75
0.50
0.25
0.00
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Spain and Ireland
0.25
0.20
0.15
0.10
0.05
0.00
Correlation between Spain and Ireland
0.4
0.2
0.0
-0.2
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Spain and Greece
0.50
0.45
0.40
0.35
Correlation between Spain and Greece
0.8
0.6
0.4
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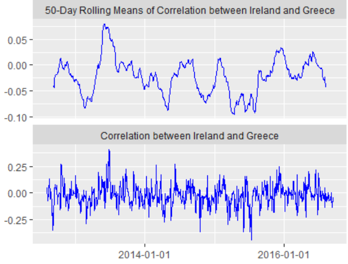
0.0
2014-01-01
2016-01-01


50-Day Rolling Means of Correlation between Ireland and Greece
0.05
0.00
-0.05
-0.10
Correlation between Ireland and Greece
0.25
0.00
-0.25
2014-01-01
2016-01-01

Appendix G

Structural Break Tests

Zivot and Andrews (2002) endogenous structural break test is a sequential test which utilizes the full sample and uses a different dummy variable for each possible break date. The single regression model is like a traditional Augmented Dickey and Fuller (1979) test, which can be written as

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} ... + \delta_p \Delta y_{t-p} + \epsilon_t \tag{G.1}$$

where y_t is the variable of interest, Δ is the first difference operator, t is the time index, δ and γ are tow coefficients, and ϵ_t is the error term. A unit root is present if $\gamma = 0$. The test statistic which can be compared to the relevant critical value for the Dickey–Fuller Test is given by

$$DF = \frac{\hat{\gamma} - 1}{SE(\hat{\gamma})} \tag{G.2}$$

The break date is selected from a sequential possible statistics of this ADF test, among which the minimum (most negative) is selected.

Lee and Strazicich (2003) test allows for two endogenous breaks both under the null and the alternative hypothesis. They show that the two-break Lagrange Multiplier unit root test statistic which is estimated by the regression according to the LM principle will not spuriously reject the null hypothesis of a unit root. Following the their notation, the LM unit root test can be obtained from the regression:

$$\Delta y_t = d'\Delta Z_t + \phi \tilde{S}_{t-i} + \sum_{i=1}^{p} \gamma_i \Delta \tilde{S}_{t-i} + \eta_t \tag{G.3}$$

where $\tilde{S}_t$ are detrended series, Z_t is a vector of exogenous variables defined according to the testing specification required. In case of the two-break Model (C) with breaks in level and trend proposed by Perron (1989), Z_t is given by $Z_t = [1, t, D_{1t}, D_{2t}, DT^*_{1t}, DT^*_{2t}]'$, where D_{ij} and DT^*_{ij} are dummy variables. The unit root hypothesis is tested by the t-statistic of ϕ, denoted as $\tilde{\tau}$. The LM test statistic LM_τ is given by

$$LM_\tau = inf \tilde{\tau}(\lambda) \tag{G.4}$$

where λ is the break fraction. Note that it can take a very long time for two or more breaks if the simple is big (observations > 500). The calculation time increases at the speed of T^{n+1}, with n is number of breaks. That is to say, with 1000 data points, a two-break test is 1000 times as long as a one-break test.

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Dickey, D. A. and Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*, 74(366a):427–431.

Lee, J. and Strazicich, M. C. (2003). Minimum lagrange multiplier unit root test with two structural breaks. *Review of Economics and Statistics*, 85(4):1082–1089.

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis. *Econometrica: Journal of the Econometric Society*, pages 1361–1401.

Zivot, E. and Andrews, D. W. K. (2002). Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. *Journal of business & economic statistics*, 20(1):25–44.

Appendix H

Proof of the international portfolio-based interest rate model

Bond return is defined as

$$H_{i,t} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}} \tag{H.1}$$

Approximation of this bond return can be written as

$$H_{i,t} \cong R_{i,t} - S(R_{i,t+1} - R_{i,t}) \tag{H.2}$$

Therefore, bond return expectation and variance are given by

$$\begin{aligned} \mu_i = E_t(H_{i,t}) &= E_t[R_{i,t} - S(R_{i,t+1} - R_{i,t})] \\ &= E_t(R_{i,t}) - E_t[S(R_{i,t+1} - R_{i,t})] \\ &= R_{i,t} - E_t(SR_{i,t+1}) + SR_{i,t} \\ &= (1 + S)R_{i,t} - E_t(SR_{i,t+1}) \end{aligned} \tag{H.3}$$

and

$$\begin{aligned}\sigma_i^2 = V_t(H_{i,t}) &= V_t[R_{i,t} - S(R_{i,t+1} - R_{i,t})] \\ &= V_t(R_{i,t}) + V_t[S(R_{i,t+1} - R_{i,t})] \\ &= V_t(SR_{i,t+1}) \\ &= S^2 V_t(R_{i,t+1})\end{aligned} \tag{H.4}$$

Covariance is given by

$$\begin{aligned}\sigma_{12} &= Cov_t(H_{1,t}, H_{2,t}) \\ &= E_t(H_{1,t}H_{2,t}) - E_t(H_{1,t})E_t(H_{2,t}) \\ &= E_t[(R_{1,t} - S(R_{1,t+1} - R_{1,t}))(R_{2,t} - S(R_{2,t+1} - R_{2,t}))] \\ &- [(1+S)R_{1,t} - E_t(SR_{1,t+1})][(1+S)R_{2,t} - E_t(SR_{2,t+1})] \\ &= S^2 E_t(R_{1,t}, R_{2,t}) - S^2[E_t(R_{1,t})E_t(R_{2,t})] \\ &= S^2 Cov_t(R_{1,t}, R_{2,t})\end{aligned} \tag{H.5}$$

Expected return of the portfolio is

$$\begin{aligned}\mu_p &= E_t(H_{p,t}) = E_t[\alpha_{1,t}H_{1,t} + \alpha_{2,t}H_{2,t} + (1 - \alpha_{1,t} - \alpha_{2,t})r_t] \\ &= \alpha_1\mu_1 + \alpha_2\mu_2 + (1 - \alpha_1 - \alpha_2)r\end{aligned} \tag{H.6}$$

Variance of the portfolio is

$$\begin{aligned}\sigma_p^2 &= V_t(H_{p,t}) = V_t[\alpha_{1,t}H_{1,t} + \alpha_{2,t}H_{2,t} + (1 - \alpha_{1,t} - \alpha_{2,t})r_t] \\ &= V_t[\alpha_{1,t}H_{1,t} + \alpha_{2,t}H_{2,t}] \\ &= \alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}\end{aligned} \tag{H.7}$$

Optimization program of the representative investor is

$$\begin{aligned}\max_{\alpha_1,\alpha_2} U &= \max_{\alpha_1,\alpha_2}[E_t(H_{p,t}) - \frac{r_a}{2}V_t(H_{p,t})] \\ &= \max_{\alpha_1,\alpha_2}[\alpha_1\mu_1 + \alpha_2\mu_2 + (1-\alpha_1-\alpha_2)r - \frac{r_a}{2}(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12})]\end{aligned} \tag{H.8}$$

First order conditions of this optimization program are

$$\frac{\partial U}{\partial \alpha_1} = \mu_1 - r - r_a\alpha_1\sigma_1^2 - r_a\alpha_2\sigma_{12} = 0 \tag{H.9}$$

$$\frac{\partial U}{\partial \alpha_2} = \mu_2 - r - r_a\alpha_2\sigma_2^2 - r_a\alpha_1\sigma_{12} = 0 \tag{H.10}$$

After solving the 2*2 system, we can obtain the optimal allocation for each bond

$$\alpha_1^* = \frac{(\mu_1 - r)}{r_a[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} - \frac{\sigma_{12}(\mu_2 - r)}{r_a\sigma_2^2[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} \tag{H.11}$$

$$\alpha_2^* = \frac{(\mu_2 - r)}{r_a[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} - \frac{\sigma_{12}(\mu_1 - r)}{r_a\sigma_1^2[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} \tag{H.12}$$

The bond supplies are defined as

$$\alpha_1^* W = \Sigma_1 W$$

$$\alpha_1^* = \Sigma_1$$

$$\frac{(\mu_1 - r)}{r_a[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} - \frac{\sigma_{12}(\mu_2 - r)}{r_a\sigma_2^2[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} = \Sigma_{1,t} \tag{H.13}$$

$$\alpha_2^* W = \Sigma_2 W$$

$$\alpha_2^* = \Sigma_2$$

$$\frac{(\mu_2 - r)}{r_a[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} - \frac{\sigma_{12}(\mu_1 - r)}{r_a\sigma_1^2[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} = \Sigma_{2,t} \tag{H.14}$$

After solving the 2*2 system, we obtain the equilibrium expected return (rational expectations) of each bond

$$\begin{aligned}\mu_1^* &= r + \frac{\sigma_2^2[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]}{[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]} r_a \left(\Sigma_{1,t} + \frac{\sigma_{12}}{\sigma_1^2} \Sigma_{2,t} \right) \\ &= r + \sigma_1^2 r_a \left(\Sigma_{1,t} + \frac{\sigma_{12}}{\sigma_1^2} \Sigma_{2,t} \right) \end{aligned} \tag{H.15}$$

$$\begin{aligned}\mu_2^* &= r + \frac{\sigma_1^2[\sigma_2^2 - (\frac{\sigma_{12}}{\sigma_1})^2]}{[\sigma_1^2 - (\frac{\sigma_{12}}{\sigma_2})^2]} r_a \left(\Sigma_{2,t} + \frac{\sigma_{12}}{\sigma_2^2} \Sigma_{1,t} \right) \\ &= r + \sigma_2^2 r_a \left(\Sigma_{2,t} + \frac{\sigma_{12}}{\sigma_2^2} \Sigma_{1,t} \right) \end{aligned} \tag{H.16}$$

Combined with the definition in equation (H.2), we have the following relations

$$\mu_1^* = (1+S)R_{1,t}^* - E_t(SR_{1,t+1}) = r + \sigma_1^2 r_a \left(\Sigma_{1,t} + \frac{\sigma_{12}}{\sigma_1^2} \Sigma_{2,t} \right) \tag{H.17}$$

$$\mu_2^* = (1+S)R_{2,t}^* - E_t(SR_{2,t+1}) = r + \sigma_2^2 r_a \left(\Sigma_{2,t} + \frac{\sigma_{12}}{\sigma_2^2} \Sigma_{1,t} \right) \tag{H.18}$$

Then, we obtain the actuarial rates of bond returns

$$R_{1,t}^* = \frac{1}{1+S} \left[r_t + SE_t(R_{1,t+1}) + S^2 V_t(R_{1,t+1}) r_a \left(\Sigma_{1,t} + \frac{Cov_t(R_{1,t+1}, R_{2,t+1})}{V_t(R_{1,t+1})} \Sigma_{2,t} \right) \right] \tag{H.19}$$

$$R_{2,t}^* = \frac{1}{1+S} \left[r_t + SE_t(R_{2,t+1}) + S^2 V_t(R_{2,t+1}) r_a \left(\Sigma_{2,t} + \frac{Cov_t(R_{1,t+1}, R_{2,t+1})}{V_t(R_{2,t+1})} \Sigma_{1,t} \right) \right] \tag{H.20}$$

By forward substitutions, we can finally solve the Euler's equations and obtain

$$
\begin{aligned}
R^*_{1,t} &= \frac{r_t}{1+S} + \frac{1}{1+S}\left[\sum_{i=1}^{\infty}\left(\frac{S}{1+S}\right)^i E_t(r_{t+i})\right] \\
&+ \frac{1}{1+S}\left[\sum_{i=0}^{\infty}\left(\frac{S}{1+S}\right)^i E_t\left[S^2 V_{t+i}(R_{1,t+i+1}) r_a\left(\Sigma_{1,t+i} + \frac{Cov_{t+i}(R_{1,t+i+1}, R_{2,t+i+1})}{V_{t+i}(R_{1,t+i+1})}\Sigma_{2,t+i}\right)\right]\right]
\end{aligned}
\tag{H.21}
$$

$$
\begin{aligned}
R^*_{2,t} &= \frac{r_t}{1+S} + \frac{1}{1+S}\left[\sum_{i=1}^{\infty}\left(\frac{S}{1+S}\right)^i E_t(r_{t+i})\right] \\
&+ \frac{1}{1+S}\left[\sum_{i=0}^{\infty}\left(\frac{S}{1+S}\right)^i E_t\left[S^2 V_{t+i}(R_{2,t+i+1}) r_a\left(\Sigma_{2,t+i} + \frac{Cov_{t+i}(R_{1,t+i+1}, R_{2,t+i+1})}{V_{t+i}(R_{2,t+i+1})}\Sigma_{1,t+i}\right)\right]\right]
\end{aligned}
\tag{H.22}
$$

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Modèles de taux d’intérêt basés sur la théorie des choix de portefeuilles internationaux et crise de l’UEM

L’objectif de cette thèse est d’étudier à côté du risque défaut, le rôle spécifique des risques de volatilité et de co-volatilité dans la formation des taux longs dans la zone euro. On propose en particulier un modèle théorique de choix de portefeuille à deux pays permettant d’évaluer la contribution des primes de risque de volatilité aux processus de contagion et de fuite vers la qualité dans différents épisodes de la crise de la dette souveraine. Ce modèle permet également d’analyser le rôle des achats d’actifs (QE) de la BCE sur l’équilibre des marchés obligataires. Nos tests empiriques suggèrent que les programmes QE de la BCE à partir de mars 2015 n’ont fait qu’accélérer « une défragmentation » des marchés obligataires de la zone euro, apparue plus tôt dans la crise, dès la mise en place de l’OMT. .

Mots clés : Structure à terme des taux d’intérêt; Modèles GARCH; Contagion; Flight-to-quality; Test Forbes et Rigobon; Théorie du portefeuille; Impact QE; OMT.

International PortfolioTheory-based InterestRate Models and EMUCrisis

This dissertation examines the specific role of volatility risks and co-volatility in the formation of long-term interest rates in the euro area. In particular, a two-country theoretical portfolio choice model is proposed to evaluate the volatility risk premia and their contribution to the contagion and flight to quality processes. This model also provides an opportunity to analyze the ECB’s role of asset purchases (QE) on the equilibrium of bond markets. Our empirical tests suggest that the ECB’s QE programs from March 2015 have accelerated the "defragmentation" of the euro zone bond markets.

Keywords : Term structure of interest rates; GARCH models; Contagion; Flight-to-quality; Forbes and Rigobon test; Portfolio theory; QE impact; OMT.