Jan Tinbergen and J. G. Koopmans on Multiple Equilibria and Coordination Failures

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Abstract:

In a 1932 policy paper, the Dutch economist Jan Tinbergen suggested that there could be two stable equilibria in a model with two firms acting in two sectors, with one of the equilibria having more production and employment. Arguing on the basis of a diagram that firms could not get out of the "bad" equilibrium on their own, he suggested that this justified government interventions to increase employment. While he referred to the general equilibrium systems of Walras and Cassel, he did not present a fully worked out model but referred to the unpublished study of one J. G. Koopmans. The latter was spurred by Tinbergen's reference to publish his ideas on the problem of multiple equilibria. In his three-part paper, he underlined that a Walrasian system could not show Tinbergen's coordination problem, because there could be no unemployment at an equilibrium point in a Walrasian model. The examples he presented, with three equilibria, showed that they could not be ordered. After this exchange, Tinbergen moved away from Walrasian multiple equilibria, and abandoned the general equilibrium approach to economic modeling in favor of the macrodynamic approach that was developed at the same time by Ragnar Frisch and Michal Kalecki.

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In 1932, at the request of the Board of the Association for Economics and Statistics, Tinbergen was invited to give a talk "to explain the position of socialism with respect to the question of the regulation of production by private individuals."[2] Tinbergen, a member of the board of the Dutch magazine *The Socialist Guide* (*De Socialistische Gids*), had already had several occasions to express his view, which he saw as one possible approach rather than "the" view of socialists. Tinbergen argued that "free competition" of individuals leads to a waste of resources, because he thought that in the absence of a planning device, economic activity was prone to recurrent crises and loss of "productive forces and thus to a lower level of prosperity than would be possible" (Tinbergen, 1932: 50). The problem presented itself even more pointedly when crises were manifesting as "unstable economic processes," making the return to equilibrium difficult or impossible, raising for him the question of instability in economic theory: "are there among the economic-dynamical processes, which theory teaches us, such that show an ever-increasing removal of the equilibrium state?" (Tinbergen, 1932: 58). Although he recognized that the scope of the existing theoretical literature was still limited, he argued that on the basis of various extensions of existing models such as the cobweb model, one may eventually identify "theoretical (as well as experimental) cases of economic processes that do not automatically return to the equilibrium position" (Tinbergen, 1932: 58).

But there was more than a problem of instability, cyclical or not, for Tinbergen: there was a possibility that crises would leave the economy stuck in a position of "bad"

[2] See Knoester and Wellink (1993: 19-20) on this address and the rest of the paper as well as their book on the wider context of the talk, one of five given by Tinbergen in front of the Association, that became the Netherland Economic Association in 1950 and received the "Royal" title in 1987. This talk was published along with the one given by R. van Genechten on the same subject.

equilibrium. From a theoretical point of view, Tinbergen wondered whether a way to represent this idea could lie in the Walrasian theory whose alleged merit was to leave ample room for multiple equilibria to appear. Along that line, he explored the possibility that the severity of the depression of the 1930s may be the result of a coordination failure which would be responsible for the fact that the economy may reach a stable, but "bad" equilibrium (with mass unemployment) from which no individual agent could unilaterally escape. In that context, Tinbergen referred to Walras and Cassel as well as to the unpublished works of J. G. Koopmans, but did not offer a system of equations. He only presented a diagram on the basis of which he described the behavior of two representative firms in an economy with two equilibria, one of which had much more unemployment.

Spurred by Tinbergen's reference to his work, in a three-part paper published in *De Economist* in 1932, J. G. Koopmans recognized that Tinbergen's system was determinate and could lead to non-trivial equilibria, but argued that Tinbergen misrepresented the meaning of the multiple equilibria arising from a Walrasian system. He emphasized that Tinbergen's approach was different from his own. His point was that a Walrasian system was neither able to define a state of unemployment (in the absence of price rigidities), nor to show that an equilibrium may be better than another. Finally, Koopmans touched upon the issue of stability by showing that as in Tinbergen's analysis, one can show that some equilibria may be stable and others unstable. But without specifying a specific dynamic adjustment process, his analysis, as Tinbergen's, remained incomplete.

In the following years, Tinbergen focused on the development of macrodynamic models and moved away from Walrasian systems. It took him several more years to develop a model that he used between 1934 and 1937 to illustrate unstable macroeconomic processes and express the view that a disturbance such as the 1929 stock crash could push the economy over the hill, into an abrupt and bottomless precipice (see Assous and Carret 2020, 2022). On that basis, he continued to emphasize the importance of multiple equilibria and of economic policies and normative choice in the model, using it to justify public interventions to reach a better equilibrium.

In the end, by embracing the development of macrodynamics, Tinbergen managed to provide an answer to the objections raised by Koopmans. This debate sheds new light on the reasons for which Tinbergen decided to move away from Walrasian economics. A first reason, repeatedly claimed in the early 1930s by Tinbergen and Frisch, was that if Walrasian systems were determinate, they suffered from the fact they were static (Frisch, 1933; Tinbergen, 1935). A second reason, underlined here, is that Walrasian theory was an inappropriate framework to address issues related to unemployment and instability, which ultimately led Tinbergen to rethink the role of state interventions through his macrodynamic models.

I. Coordination and economic policy in Tinbergen's pre-advice

To the question "is popular prosperity so great under the system of free competition?" Tinbergen answered with mixed feelings. He underlined the recurrence of "breakdowns in economic life," which he thought should be avoided to obtain a smoother distribution over time of the "joys and burdens of life" (Tinbergen, 1932: 50 ff.). According to him,

fluctuations in economic life, the source of so many ailments, prevented a full use of resources and led to a waste of technical and natural resources. Limiting the uncertainty brought on by the cyclical character of economic life would have the added benefit of eliminating strategic calculations between firms, a mechanism he tried to encapsulate in a diagrammatic representation of the economy. His remarks on the underutilization of productive capacities were illustrated by statistics on the available capacity of industrial enterprises in the United States and in Germany, which showed that even at the top of the boom, the economy was not using its full capacities.

More importantly, Tinbergen noted that in June 1932, the end of the disruption that had begun in 1929 was nowhere in sight;[3] this led him to discuss the possibility that the economic process was unstable and to reflect on the economic policy needed:

> The question has been posed whether we are not dealing here with a movement process that no longer returns automatically to an equilibrium state, or whether we are not dealing here with unstable states. (Tinbergen, 1932: 56)

This led to an entire section on "stable and unstable economic processes".[4] Tinbergen argued that although previous crises were usually followed by a return to equilibrium, there were examples of unstable processes that did not lead to an automatic recovery, such as the inflation that plagued European countries in the early 1920s. This begged the question: "are there among the economic-dynamical processes, which theory teaches us, such that show an ever-increasing removal of the equilibrium

[3] "The crisis which broke out in 1929 led to a disruption of production which did not occur in any previous crisis … And still nothing can be noticed of an end of this disruptive process" (Tinbergen, 1932: 56).
[4] Tinbergen uses here and later the term "labiele" which also exists in French, German and English but English usage has consecrated the term "unstable" which we use in the rest of this paper.

state?" (Tinbergen, 1932: 58). Although such mechanisms were not discussed in the theoretical literature, Tinbergen was confident that he could find dynamic models accounting, for instance, for exponential movements away from the equilibrium.

The discussion was introduced with the hog cycle, in which case he noted that fluctuations usually led back to the equilibrium, but that if the elasticity of production became too great it could lead to fluctuations of increasing amplitude.[5] He also referred to his own model of the "shipbuilding" cycle, published in 1931, where he had observed that its solutions could yield fluctuations of increasing amplitude, depending on the values of certain parameters like the sensitivity of the supply of ships to the price of freight, and that "increasingly violent fluctuations" could arise. Although he was concerned with the shipbuilding cycle in this article, Tinbergen already viewed in 1931 the relevance of such mechanisms for the whole economy:

> The basic problem of any theory on endogenous trade cycles may be expressed in the following question: how can an economic system show fluctuations which are not the effect of exogenous, oscillating forces, that is to say fluctuations due to some "inner" cause? (Tinbergen, 1931: 152).

He concluded his examination of the shipbuilding model by underlining the importance of finding a good dynamic mechanism to approach the problem of stability at the aggregate level:

[5] The hog cycle was first studied by Arthur Hanau of the Berlin Business Cycle Institute, and Tinbergen continued to emphasize the importance of his work almost sixty years later in his interview with Morgan and Magnus (Tinbergen et al., 1987).

> It seems to me that this result is of vital importance to the theory of economic dynamics. … Moreover, it gives us a clue to a method of judging the stability of an economic system in general. (Tinbergen, 1931: 162).

He also referred to his work on market level price dynamics that had been presented in 1931, during the first European meeting of the Econometric Society in Lausanne (published in the first issue of *Econometrica* in Tinbergen [1933]). There, he started again from a simple cobweb model of supply and demand, which he extended first by introducing "speculation," whereby demand depends on the rate of change of prices. This transformed his model into a mixed difference-differential equations, but Tinbergen did not stop there: he introduced a "hyperbolic demand", transforming his difference equation into a nonlinear equation, a relationship between demand and the activity of the industrial sector, a durable consumption good, and finally introduced a second market producing a capital good. Thus Tinbergen had already started to build up toward a more intricate model of the economy and its interdependencies. In all those cases, he was chiefly interested in the periodicity of the oscillations and always used hypotheses on the parameters that would make those oscillations self-sustained.[6]

However, the shipbuilding cycle as well as the cobweb mechanism remained focused only on one market and not the whole economy. Tinbergen pointed out in 1932 that if those models helped to identify new economic processes, they remained of little help to provide an appropriate framework to make sense of the working of the whole economy

[6] This was precisely the approach adopted by Kalecki (1935) that was criticized by Frisch (1935). Tinbergen had prepared the ground for Kalecki's model, both through this approach of endogenous fluctuations, and his 1931 shipbuilding model, which was used by Kalecki to solve his own model. Following Boumans' (2005) idea that Kalecki's model was like a recipe, this shows that the ingredient coming from Tinbergen was more than a mathematical mold. The type of solutions favored by Tinbergen had a direct impact on Kalecki.

and the factors responsible for the ongoing crisis, in large part because they did not explain satisfactorily a mechanism of “removal from the equilibrium state” (Tinbergen, 1932: 58).

In 1932, he claimed that the solution to account for aggregate phenomena may also come from the “Walras-Cassel” model of the economy.[7] According to him, this approach left ample room for multiple equilibria and instability to appear, and he described a possible mechanism illustrated with a diagrammatic representation of the interactions between two representative firms.

In this model with two sectors and two firms, Tinbergen claimed that the economy could get stuck in a “bad” equilibrium with high unemployment. Plotting the “profit lines” of each firm as a function of their employment levels a_1 and a_2, that is, all the combinations of employment for which the profits of firms 1 and 2 are constant, Tinbergen provided the following diagram (Tinbergen, 1932: 62).

[7] “Further investigation of the equilibrium equations (of, for example, Cassel or Walras) which determine the size of production, the level of prices, etc. in a stationary society, shows that there is not one equilibrium (in the sense of economic science), but that in general different equilibria are possible with the same technical and psychological data. It is quite possible, and even most likely, that with some form of utility functions and technical coefficients, two equilibria at least exist, both stable, one for instance with significant unemployment and the other without” (Tinbergen, 1932: 60). Rodenburg (2010: 10) mentions Tinbergen’s argument in the context of his debate with the Dutch economist Goudriaan on dynamics and equilibrium but without contrasting it with the arguments raised by Koopmans.

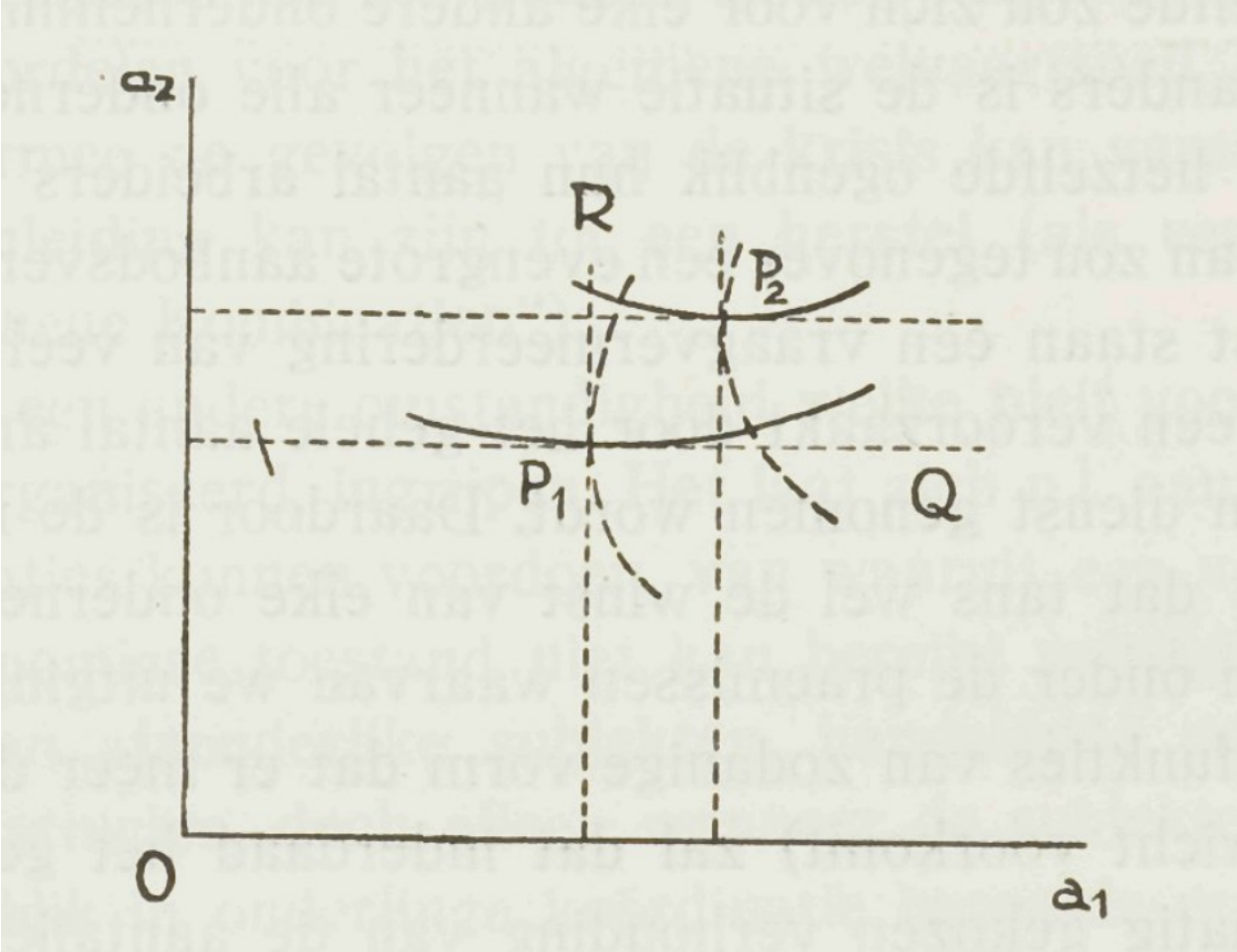

Figure 1: Tinbergen's illustration of a coordination problem (1932: 62)

Tinbergen underlined that an increase of employment from only one firm will correspond to a displacement along a vertical or horizontal line. Because the curves meet at their minimum point, this means that a firm increasing its employment will always see its profits go down if the other firm does not increase employment as well. Tinbergen argued that the configuration where both firms have their profit curves crossing at their minimum points will thus correspond to stable equilibria, because there will be no incentive to deviate alone from such a situation.[8] In Figure 1, there are two

[8] "Presence of two stable equilibrium points means, as can be easily ascertained, that there are two points P1 and P2, in which the profit line of 1 has a horizontal tangent and that of 2 has a vertical tangent.

such equilibria: P_1 and P_2. We can readily see that in the second of these equilibria, there is more aggregate employment than in the first; this begs the question of how to move the economy from the "bad," low equilibrium, to the "better," high equilibrium (Tinbergen, 1932: 61).

If we denote by R and Q the two other points where the profit lines intersect, we can see that the profits of both firms will only increase together in the $P_1\ Q\ P_2\ R$ quadrilateral. But in order for the economy to move in this direction, it is necessary that both firms increase employment at the same time, so that neither has the incentive to reduce employment to go back to a higher level of profits.[9] The bottom line is that a new stable equilibrium can be obtained for a specific ratio between a_1 and a_2, and that only an organized action may help to reach it: "In other words, it has been shown that there are cases in which competition cannot get out of a certain (low) situation, while organized action will make it possible" (Tinbergen, 1932: 61).

Unfortunately, Tinbergen did not explicitly state the set of equations that he might have used to derive the two profit lines, making it difficult to identify the exact mechanisms at work in the move of the economy from one equilibrium to the other. He described some potential mechanisms explaining how such a situation could arise, mainly from the mismatch in the increase of demand and supply when firms act alone,

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This nevertheless means that any change in the number of workers in one's own enterprise results in a reduction in profit: the condition for a stable equilibrium. In the figure, the action of separate companies always means a movement along vertical lines (for firm 2) or horizontal lines (firm 1)" (Tinbergen, 1932: 62).

[9] "Only an organized action - changing the number of workers in both firms at the same time - creates a movement along other lines. Now, from the point P1, one can only reach the point P2 without going through 'regions of lower gain', if one stays within the hatched curvilinear quadrilateral. And again, this is only possible if one does not go along horizontal or vertical lines." (Tinbergen, 1932: 62).

while when they act together this mismatch is construed to have a positive effect on profits because of a potential reduction in costs.

As he pointed out, in the case of a firm acting alone from equilibrium P_1, “the increase in demand which would arise as a result would by no means outweigh the increase in the supply of the commodity produced by that company” (Tinbergen, 1932: 61). But with no other information on the consumption of the workers of the two firms and on the way both firms decide to invest, the argument is a bit enigmatic. The possibility that demand may eventually adjust to supply and reach the “good” equilibrium would in fact be due to the evolution of cost: “Expansion of the number of workers [...] also lowers the unit cost of production” (Tinbergen, 1932: 62), which, for reasons also left unexplained, is assumed to have a positive effect on the demand for both goods.

Whatever the limits of Tinbergen’s argument, it is clear that he was already thinking about stability issues. From that point of view, the response Koopmans published a few months later proves helpful to clarify some features of Tinbergen’s thinking and show how far Tinbergen was departing from the Walrasian approach.

II. J. G. Koopmans’s criticism of Tinbergen’s multiple equilibria

As we have seen, Tinbergen associated the idea of multiple equilibria with a Walrasian system of equations. In a footnote, he admitted that he was puzzled to see that while the idea was mentioned in passing in the literature, it had “not been elaborated anywhere” except in “some unpublished studies of Mr. J. G. Koopmans,” suggesting that his analysis derived from it (Tinbergen, 1932: 60). A couple of months later, Koopmans published a series of three papers with the aim to clarify his position with

respect to Tinbergen's analysis. While he acknowledged in the first paragraph of his paper "the much appreciated help" which "as a non-mathematician" he received from Tinbergen (Koopmans, 1932: 679), he felt the need to underline the differences between his and Tinbergen's analysis. In fact, Koopmans made clear how far Tinbergen's analysis departed from his own approach and this for two main reasons.

First and foremost, Koopmans emphasized that neither in Walras's nor in Cassel's works, one can show that there might be an equilibrium let alone a "stable equilibrium" as far as a "significant degree of unemployment is assumed" (Koopmans, 1932: 682).[10] Accounting for unemployment means that one market equation expressing the equality between demand and supply will have to disappear, leaving the system undetermined with the result that many potential solutions become possible: "this means that the whole complex of economic phenomena, as expressed by the equilibrium equations, becomes undetermined, that is to say, there are not only two or a few, but even, at least in principle, infinitely many alternative solutions of the remaining system of equations" (Koopmans, 1932: 683). In particular, as soon as one assumes that wages are fixed and known, it becomes possible to define a set of equilibria, each parametrized by a specific level of money wage. It is therefore no surprise that with the same technical and psychological data, it is possible to define a single "equilibrium state" for any given level of money wage.

[10] "After all, one of the essential conditions for the existence of a state of equilibrium in the sense of these two authors [Walras and Cassel] is precisely that, at the prices applicable to that situation, the supply of all end products and production factors, including human labor, is fully absorbed by demand. The equations in which this condition is expressed are therefore an indispensable constituent of the whole system of "equilibrium equations" through which, in the thinking of the aforementioned authors, the economic phenomena are examined" (Koopmans, 1932: 682).

Koopmans pointed out that it was unlikely that Tinbergen had understood the issue in this way: “it should be appreciated that, according to the continuation of his argument, the author certainly could not have meant the above statement in such a simple and almost trivial sense. For in reality he also speaks of a system in which the number of equations indeed corresponds to that of unknown quantities” (Koopmans, 1932: 683). The unknowns chosen by Tinbergen are simply not the same as those chosen by Cassel and Walras. The problem however remains similar as long as one can show when a system comprising as many unknowns as equations has multiple solutions. In that case, “the problem can indeed be regarded as “determined” in a similar sense as in the thinking of Walras and Cassel [...] from the formal-theoretical point of view, [the possibility of multiple equilibria] is indeed just as remarkable and of the same fundamental significance and scope as the corresponding possibility in a system of equations built up entirely in accordance with the method of Walras and Cassel, and in which all prices, including wages, figure as unknowns” (Koopmans, 1932: 684).

In addition, Koopmans doubted that on the basis of his graphical illustration, Tinbergen had managed to prove the existence of multiple stable equilibria. If it was clear to him that the two “lines of constant profit” were derived under the assumption that wages are rigid, it remained unclear why both lines could “run according to the starting point adopted by the author in the manner indicated by him in his drawing: a possibility which I do not want to dispute, but of which the proof has not been fully provided here on the spot” (Koopmans, 1932: 684). Koopmans argued that the rigidity of wages acted on the stability of equilibria but in an obscure manner. While he agreed with Tinbergen’s concern with the possibility of defining a stable equilibrium, he

regretted that he did not clarify the mechanisms allowing to identify which equilibrium among all the equilibria defined by a given level of money wage was likely to be reached.

The second issue raised by Koopmans concerned the possibility of ranking equilibria. His point was that the equilibria displayed by Walrasian systems could not be ordered. From the policy point of view, this was an important difference between Tinbergen's and his approach of multiple equilibria, because he claimed that in the absence of nominal rigidities and unemployment there was no way to choose among the different equilibria:

> such an appraisal judgment can no longer be justified with the same certainty when it comes to the comparison between two (or more) alternative equilibrium states … which, after all, is characterized precisely by the fact that in neither of them there is a surplus of unused production factors … The remarkable thing about a double or multiple economic equilibrium in this last sense … consists precisely in the fact *that the various alternative equilibrium states are in every respect, thus also viewed from a normative point of view, completely "equivalent"*, or in others words, that none of these equilibria can be given a "preference" over one or more of the others on the basis of any objective criterion. (Koopmans, 1932: 685, original emphasis)

Thus Koopmans underlined that the different equilibria did not imply that some factors of production were underemployed, and they did not justify a socialist planning policy in order to obtain a higher employment equilibrium. It was necessary to introduce another criterion from which to establish that an equilibrium was better or worse than the other.

Koopmans sought to illustrate this equivalence of different equilibria in a Walrasian system with a numerical example exhibiting three equilibria.

III. Koopmans' multiple equilibria in a Walrasian system

While Koopmans started with a criticism of Tinbergen, he also made it clear that he had a limited knowledge of mathematics and that he relied mainly on others for his mathematical exposition, in particular on Tinbergen himself. But he still felt that he had something important to say about the importance of multiple equilibria in even the simplest exchange economy, and he did not think that Tinbergen had captured his ideas on the subject in his model.

The end of the first part of his paper was dedicated to setting up his model. First, Koopmans pointed out that the issue at hand had nothing to do with "the existence of so-called "indifference zones" generated by the discontinuity of supply and demand curves. It also had nothing to do with firms operating, as a result of a relatively high percentage of "fixed" cost in the production process (Koopmans, 1932: 690), in the decreasing part of the U-shaped supply curve. He acknowledged that both these cases allowed for an exploration of the possibility to "represent a position of 'stable' and one of 'unstable' equilibrium" (Koopmans, 1932: 690).

Koopmans emphasized that his own approach was based on the idea that supply curves could have inverse U-shapes: they started as normal, upward sloping schedules, before bending towards the bottom right in a price-quantity diagram: "which means that, even apart from decreasing costs, etc., as the price per unit increases, the volume of the supply no longer continues to increase beyond a certain point, but on the contrary decreases further and further" (Koopmans, 1932: 690). These supply (and demand)

functions arose from the utility functions used by Koopmans, and the problem he sets up is that of a simple exchange economy with two people and two goods.

There are three types of equations in his system: equilibrium equations relating the ratio of marginal utility of the two goods and their (relative) price, “budget” equations relating the aggregate value of goods given up in the exchange with the aggregate value of goods received, and balancing equations for the total stock of physical units of each goods. The known quantities are endowments (for good I and good II respectively, agent alpha has k and l and agent beta has m and n) and preferences (in the form of linear marginal utility functions with parameters A, a, B, b for alpha and C, c, D, d for beta, respectively for good I and good II). The unknowns of the problem are the final quantity held by participants after the exchange (x and y for alpha, and z and u for beta), and the rate of exchange (price p) between the goods. The equations determining the three equilibria are presented in Table 1.

Agent	Alpha	Beta
Marginal utility	$\frac{A-ax}{B-by} = p$	$\frac{C-cz}{D-du} = p$
Budget	$p(k - x) = y - l$	(determined by the others)
Quantity	$x + z = k + m$	$y + u = l + n$

Table 1: Equilibrium equations of Koopmans’ system

In the case with two agents and two goods, at the equilibrium points the relative utility of each good is equal to the same (relative) price, one budget constraint will be given from the other budget constraint and the physical constraints, so that there are five equations and five unknowns, determining an optimal economic equilibrium. Koopmans

argued that a solution existed because there were as many unknowns as equations in his system. But he also added that this did not mean that there was necessarily a unique solution, and in fact "even with the minimum number of two persons exchanging only two goods, … as a rule not one single, but three different solutions of the equations concerned are possible" (Koopmans, 1932: 694). To prove this, he gave a numerical example and obtained the following equations (Table 2), whose solution exhibits three equilibria, with each equilibrium being established at a different price (Table 3 gives the equilibria solving this system).

Agent	Alpha	Beta
Marginal utility	$\frac{7-x}{10-y} = p$	$\frac{66-12z}{11-u} = p$
Budget	$p(7 - x) = y$	(determined by the others)
Quantity	$x + z = 7$	$y + u = 10$

Table 2: Equilibrium equations of Koopmans' numerical example

	Relative Price I / II	Alpha		Beta	
		Good I	Good II	Good I	Good II
First Equilibrium	1	2	5	5	5
Second Equilibrium	2	3	8	4	2
Third Equilibrium	3	4	9	3	1

Table 3: Equilibrium solutions of Koopmans' numerical example

These three equilibria can be represented in an Edgeworth box diagram, which seems to be what Koopmans had in mind when he described in a footnote the

application of Bowley's and Pareto's "graphical method," which "had to be scrapped due to lack of space" (Koopmans, 1932: 772-773). The following figure produces such a diagram, based on Koopmans' numerical example.

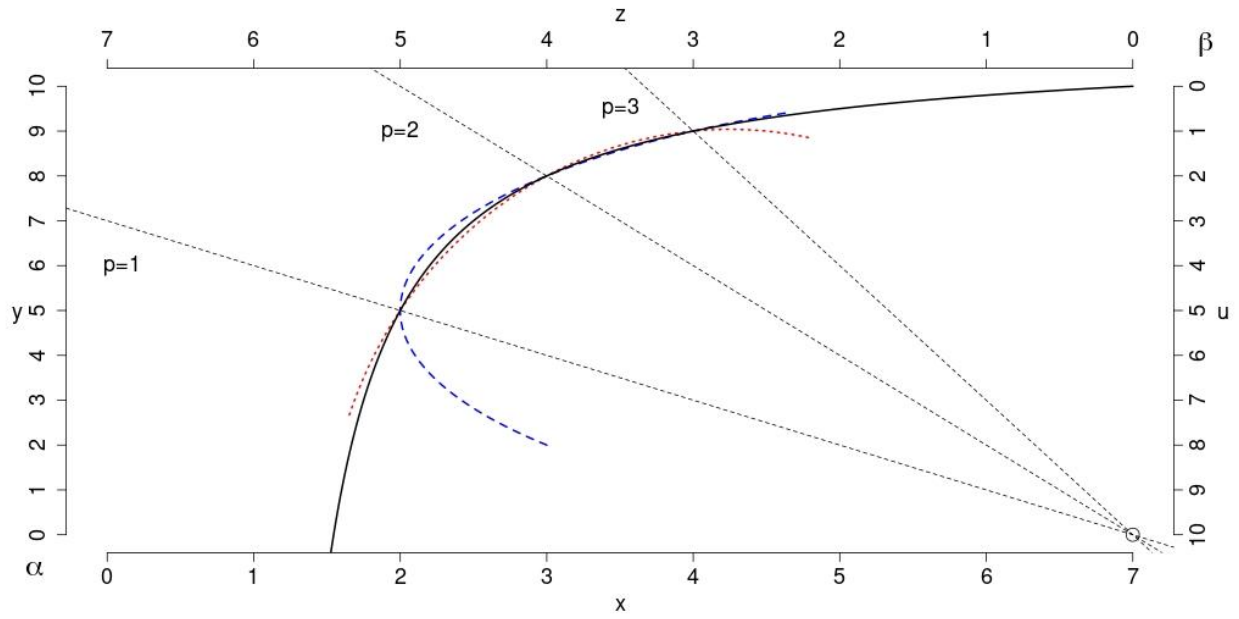

Figure 2: Edgeworth Box derived from Koopmans' numerical example

The thick black line is the Pareto "contract curve," all the points where the indifference curves of both agents are tangent to each other, mathematically expressed as the equality between the ratio of marginal utilities of alpha and beta, $\frac{7-x}{10-y} = \frac{66-12z}{11-u}$. While there are four unknowns in this equation, we also know from the start the physical limitations of the economy: $x + z = 7$ and $y + u = 10$. This subsystem of four unknowns and three equations defines the Pareto curve traced out in the diagram above. The blue (dashed) and red (dotted) lines interlacing around the contract curve are the offer curves of alpha and beta respectively; they are determined by the point of tangency between each agents' indifference curves, and their budget line for different

prices. This budget line is determined once endowments are chosen and a new variable is introduced, the relative price p between the two goods. The adjustment of this price rotates the budget line around the point of initial endowments, which Koopmans called the "zero point of the exchange" (Koopmans, 1932: 773). Economic equilibrium happens when both offer curves are compatible, that is, when both participants are ready to exchange compatible amounts of goods I and II, or in other terms, they have the same relative price for the two goods: the two indifference curves and the budget line are tangent at the same point, which lies on the contract curve.

Three economic equilibria are possible in this system, and they all lie on the contract curve, as is made formal by the first fundamental theorem of welfare economics. This theorem appears trivial in this case, as the economic equilibrium is defined by taking the four unknowns and three equations defining the Pareto curve, and adding one unknown (the price p) and two equations (the equality of price and relative marginal utility $p = \frac{7-x}{10-y} = \frac{66-12z}{11-u}$, and the budget equation $p(7 - x) = y$), so that the system becomes determinate with five unknowns and five equations.

But the fact that the system is determinate does not mean that there is only one equilibrium, as was the point of Koopmans' numerical example; a nonlinear system whose reduced form is a polynomial of degree more than one will in general have more than one solution (Koopmans discussed the problem introduced by complex solutions in the third part of his paper). Koopmans himself gave a form of his system reduced to one equation with the price as an unknown, showing that it was a polynomial of degree three, with three equilibrium solutions. This was underlined by Koopmans to show that

this example had nothing exceptional, or special, but could appear systematically; this led him to wonder why previous writers had not underlined this possibility. It is likely that Tinbergen influenced him in this direction, as Koopmans had underlined his debt towards him for the solution of his model; Tinbergen himself had adopted this approach of reducing his systems to one equation already in Tinbergen (1931), and continued this approach in his later models, such as the 1939 League of Nations model.

Because the marginal utility conditions are fulfilled in each of the three equilibria, Koopmans argued that “there is indeed no reason here to consider one of the three possible equilibrium states with respect to the two others as the ‘best’ or the ‘worst’” (Koopmans, 1932: 702). This was the brunt of his critique of Tinbergen, and he argued in a footnote that the utility of each person in each equilibrium was irrelevant because incomparable.

While Koopmans is convincing in showing the possibility of multiple equilibria for simple, nonpathological preferences, he stops short of analyzing the coordination problem underlying his setup, as a result of the methodology he adopted. The problem of his analysis is that it is fundamentally an equilibrium analysis, and Koopmans says nothing of the way in which the two agents got to one equilibrium in particular, focusing instead on other numerical examples with more goods or more agents. But while interpersonal comparisons are not possible, it is apparent, without even looking at their utility function, that agent alpha prefers the third equilibrium while agent beta will prefer the first equilibrium. This result is a simple consequence of the starting endowments, as alpha owns the whole stock of good I while beta owns the whole stock of good II: in an equilibrium with a lower price for good I, beta will be able to buy more of it while alpha

will not be able to obtain as much good II as in an equilibrium with a higher price for good I. But Koopmans did not say how one equilibrium was chosen instead of the other, even though this seems like a fundamental problem: whoever can influence prices can obtain a favorable equilibrium.

In his original example, Koopmans examined the three situations arising from three different price systems, taking the price as given each time. The only hint toward the dynamic problem underlying the approach of equilibrium is found in the second part of the paper, where Koopmans described a change in price from the starting point of the second, unstable equilibrium: “a slight rise in price beyond this point has the effect of decreasing supply more than demand, so that a shortage arises in the market, the consequence of which is that the price will rise even further” (Koopmans, 1932: 771). Koopmans noted that Walras himself had underlined this possibility of having three equilibria, with the middle one unstable (Walras, 1954: 112).

To prove the instability of the middle equilibrium, Koopmans relied on Walras’s argument which was based on tracing out offer curves in a price quantity diagram, and looking at their intersections with demand curves. It is interesting to note that the implied dynamic analysis is not spelled out in terms of differential or difference equations, something that became one of Tinbergen’s main contributions to the macrodynamic program whose goal was precisely to specify these dynamic adjustment processes (Assous and Carret, 2022, in particular chapter 8).

The model that Koopmans ended up building thus exhibited multiple equilibria that were very different in character from those of Tinbergen: while Tinbergen imagined a

situation where maximality conditions could be satisfied *at different levels of production*, Koopmans built numerical examples showing that *for a given amount of goods* in the economy, different equilibria could be possible. They all satisfied marginal utility conditions, but they were impossible to rank without a normative criterion to choose an equilibrium. This later problem was raised briefly, but left unexplored by Koopmans, who kept his attention on marginal utility functions.

IV. Tinbergen's turn away from Walrasian economics

Tinbergen often claimed he had moved away from the Walrasian system because he regretted that it was static (Tinbergen 1935: 241). This move also coincided with the debate with Koopmans which made clear that Walrasian systems were difficult to reconcile with macroeconomic policy issues, and most importantly with an analysis of unemployment. Nevertheless, he did not abandon the idea that there could exist coordination issues through the existence of multiple equilibria in the economy, and the fact that some of these equilibria may be globally unstable was central to the models he developed during that period (Assous and Carret, 2021, 2022a, 2022b).

This is clear in Tinbergen's 1934 macrodynamic model he built with the aim to account for various economic trajectories, some stable, others unstable and to show how policy could be used to change it. On the basis of an analysis of the determinants of aggregate purchasing power, Tinbergen was led to a nonlinear equation describing the relation between prices, production and the value of consumer goods sold, as well as the movement of employment. As for Walrasian systems, it did not escape him that the possibility of multiple equilibria was here totally natural while any action on the

structural parameter of that equation may change the trajectory of the economy. Using that model in a series of articles published in 1935 and 1936, Tinbergen wondered how a change in initial conditions resulting from temporary shocks may change the trajectory of the economy and started showing concern for policy intervention likely to throw the economy over its stability zone into a region of instability that could potentially trigger a complete collapse of the economy. At the same time, he drew attention to the importance of designing an economic policy likely to prevent such dangerous paths.

It is however only in a 1937 article that he really returned to the idea of coordination failures in the framework of a reflection on Kahn's multiplier and the effect of a rise in public expenditures. His basis was a simplified version of a Dutch model he had presented at the Namur meeting of the Econometric society in 1935, and which he went on to expand while working for the League of Nations in 1936-38. Tinbergen kept the idea central in his 1934 model that there exists a non linear relation between prices, production and the value of consumer goods sold, as well as the movement of employment but introduced a speculative dimension through the effects of the evolution of current profits on expected profits. As in his 1932 pre-advice, he could examine the possibility that the economy may stabilize in two situations of stable equilibria, one with high employment and high output and one with low output and low employment. He also pointed out that an external impulse through a small rise in public expenditures could lead to a higher level of employment and profits, without necessarily maintaining the initial expenditure at the source of the shock.

The idea of multiple equilibria, and how macroeconomic policy may improve the level of production and employment, was also discussed by Tinbergen in works he published

between 1938 and 1943. In his 1938 *Econometrica* article “On the Theory of Business Cycle Control”, he built a small model in which economies could move from one equilibrium to another: “In the case of a nonlinear final equation there exist other equilibrium positions or developments than the one chosen as reference developments and much depends on the situation of these other equilibria, which may be stable or unstable” (Tinbergen 1938b, 33). The main message was that the final position reached by the economy was as important as the trajectory leading to it and that economic policies could play a pivotal role in leading the economy to various stationary states.

This approach of nonlinearities and its consequences on fluctuations was later continued until the publication of an article published in Dutch in 1943. Although Tinbergen examined for the first time the implications of multiple equilibria in a Keynesian cross diagram, his approach consisted in outlining the conditions for which multiple equilibria could occur and remained conceptually the same. Again, the issue was to make clear, depending this time on the slope of the expenditure demand curve relative to the 45° line, when multiple equilibria may appear. In view of the possibility that the expenditure may be non linear and S-shaped, Tinbergen considered the case in which three equilibria may be defined: two stable for extreme values of activity for which the slope of the expenditure curve is lower than 1 and one unstable for which the expenditure line is higher than one. With reference to Goudriaan with whom Tinbergen had debated several times in the 1930s (see Rodenburg 2010), Tinbergen could argue that the importance of the multiplier effect would dramatically change depending on whether public expenditures are increased by the appropriate amount. In particular, Tinbergen could argue that, from a state of low equilibrium, it is only when changes in

expenditures shift the expenditure line so that it becomes tangential with the 45° line that the economy will eventually stabilize in a state of high equilibrium and public expenditures have a full impact on employment.

Conclusion: multiple equilibria, coordination and policy

In Koopmans setup, the policy problem that arose was one of distribution because efficiency was always obtained (preferences were satisfied). Looking at Table 3, we see why it was necessary to have a normative criterion to rank the different equilibria: total production is always the same, but the distribution of this production changes according to the relative price. Such a criterion could for instance be based on notions of justice and equality in the physical distribution of goods: in this respect, the least unequal distribution if the one arising from the first equilibrium, which is actually the worst possible equilibrium for Alpha and the best possible for Beta. The problems raised by this type of analysis made a comeback in the postwar period, especially after Arrow (1951) monograph and the renewal of welfare theory. But in the move to macrodynamics made by Tinbergen, individuals disappeared as the analysis was always carried out at market clearing equilibrium. Coordination problems in macrodynamic models arose from technical issues such as the production length, lags in investment, and other accelerator and multiplier relationships, and led to different policy problems.

The use of such models had its price in the sense that they made it impossible to ground policy recommendations on welfare statements derived from optimizing behaviors. Tinbergen was much much more concerned after 1932 in deriving his

models from aggregate behavioral equations than in developing models based on optimization, as demonstrated by his strong interest in macrodynamics, an interest that was shared with Frisch and other early econometricians.

It took five more decades for macroeconomists to return to the connection between multiple equilibria, normative statements and optimization. This was made possible by the implications of imperfect competition, where the first fundamental theorem of welfare economics no longer applied. The market power wielded by some agents in these models questioned the idea that the public interest could be served without coordination or cooperation. Instrumental in these new developments were the tools of noncooperative game theory, developed after the Tinbergen - Koopmans debate. A major research effort in the early 1980s resulted in the development of a new class of models in which a universally better outcome could, in some cases, be achieved by coordinating the activity of otherwise non-cooperative agents.

Such a result was the outcome of the presence of several equilibria, some of which could be inefficient, and which could be ranked according to Paretian criteria (Silvestre 1993). If one can show that the economy can stabilize in an inefficient equilibrium, then one can conclude on the existence of a coordination failure. The reason is similar to the one outlined by Tinbergen who understood that if everybody simultaneously moved to the better equilibrium, once reached, there would be no incentives to deviate from it. If the only equilibrium is inefficient, then the socially better allocations won't be reached unless agents become cooperative, or if the game is changed as a result of an institutional change. Tinbergen was clearly more concerned with problems of coordination than with problems of cooperation, but instead of turning to imperfect

competition which he surveyed at the same time (Tinbergen, 1934), he turned to the new tools of macrodynamics. In the end, it was in both cases by moving away from Walrasian economics that different research programs were opened.

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