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Suhrawardī's Stance on Modalities and the Logic of Presence

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Abstract. This study examines al-Dīn Suhrawardī's Hikmat al-Ishrāq and explores his logic in the context of his epistemology of presence. The paper aims to address Tony Street's challenges regarding the compatibility of Suhrawardī's critique of Ibn Sīnā with the development of a temporal and modal syllogism that appears similar to that of Ibn Sīnā. Suhrawardī's modalities refer to the different ways a predicate relates to its subject, rather than being propositional operators. His main innovation is the explicit role that presences or actual instances play in his modal-temporal logic, particularly in shaping his notion of contingency, which includes both a generic and an individual or de re form of Plenitude.

Keywords. Suhrawardī, Presence, Dialectic, Modal logic, Temporal Logic.

Introduction

Despite the fact that it has been over 40 years since Henry Corbin and Hossein Ziai observed that the work of Shihāb al-Dīn Suhrawardī, born in 549/1155 in Suhraward, and executed in 587/1191 in Aleppo, had not yet been systematically studied, a thorough research of his work is just at the start. This is surprising since Suhrawardī, father of the School of *Illumination* known as *al-Ishrāq*, was, as pointed out by Frank Griffel,¹ together with Fakhr al-Dīn al-Rāzī, by a wide margin, one the most influential thinkers of the sixth/ twelfth century in Islam. Also, also since we now have reliable editions, translations and numerous studies on his work.²

¹ Cf. F. Griffel, *The Formation of Post-Classical Philosophy in Islam*. Oxford: OUP, 2022), 240.

² Among others, H. Corbin, ed., *Œuvres Philosophiques et Mystiques (Tomes I et II)* (Tehran: Institut d'Etudes et des Recherches Culturelles, 1951 and 2001); Id. Sohravardi et les Platoniciens de Perse (Paris: Gallimard, 1971); H. Ziai, Knowledge and Illumination: A Study of Suhrawardī's Hikmat al-Ishrāq (Atlanta: Scholars Press, 1990); H. Ziai and J. Walbridge, (ed.), The Philosophy of Illumination: A New Critical Edition of the Text of Hikmat al-ishrāq with English Translation, Notes, Commentary, and Introduction (Provo: Brigham young UP, 1999), J. Walbridge, The Leaven of the Ancients: Suhrawardī and the heritage of the Greeks, (Albany: SUNY Press, 2000); Id. The Wisdom of the Mystic East: Suhrawardī and Platonic Orientalism (Albany: SUNY Press, 2001) and M. A. Razavi, Suhrawardi and the School of Illumination (N. York: Routledge, [1997]2013). The most recent include: T. Street, "Suhrawardī on Modal Syllogism", in Islamic Thought in the Middle Ages Studies in Text, Transmission and Translation, in Honour of Hans Daiber, ed. by A. Akasoy and W. Raven (Leiden and Boston: Brill, 2008); S. N. Mousavian, "Did Suhrawardī Believe in Innate Ideas as A Priori Concepts? A Note", Philosophy East and West 64, 2 (2014); J. Walridge, "Suhrawardī's Intimations of the Tablet and the Throne: The Relationship of Illuminationism and the Peripatetic Philosophy", in The Oxford Handbook of Islamic Philosophy, (2019); J. Kaukua, Suhrawardī's Illuminationism, (Leiden: Brill, 2022) and T. Zhang, A Philosophical Enquiry into the Nature of Suhrawardī's Illuminationist Philosophy. Light in the Cave (Leiden: Brill, 2023).

One of the main difficulties for interpreting his work is that Suhrawardī's writings, often develop deep and innovative thoughts that intertwine studies in logic, epistemology, metaphysics with a sharp criticism of the *Peripateticians* but also with Sufi mysticism.³

This puzzled and still puzzles commentators that lead to quite disparate accounts of his thought. In the present paper we attempt to start to revisit these apparent tensions, however, we mainly focus on discussing the elements of his logic as initiated by Tony Street.⁴ According to our view a study linking his epistemology of presence with his stance on logic of modalities can contribute to a more unifying reading of Suhrawardī's fascinating thought.

So far as we know, the logical analysis of Suhrawardī's modalities in the recent literature do not engage yet in three crucial points he explicitly makes,⁵ namely

- (1) modalities, are to be understood as the different ways a predicate *relates* to its subject;
- (2) *proving* that a necessarily contingent relation holds requires relating every instance of the subject with a capacity or potentiality (neither necessarily nor impossibly actualised at a moment occurring in a stretch or periodof time), expressed by the predicate.
- (3) The *dependence* of presences (or more precisely of *abstract witnesses* of such presences) of the predicate-term upon those of the subject-term shape Suhrawardī's logic.

Regarding (1), temporality is integral to the meaning of Suhrawardī's concept of contingency. The experience of time is necessary for the experience of contingency, and the logic of contingency requires the introduction of a period of time during which a capacity is said to actualise. These periods of time should not be understood as binding individual variables occurring in propositional functions. It is claimed that instances or presences, rather than the propositions they verify, are timed. Specifically, the singular act of laughing and the experience of a particular laughing-presence are what is timed, not the type or action or proposition *Humans laugh*. Therefore, if our reconstruction is accurate, according to Suhrawardī's epistemology of presence, the *Zayd laughs at t*₁ is to be understood as *a presence of Zayd's laughter occurs at t*₁.

Concerning (2) and (3), remarkable is the fact that Suhrawardī's proof-methods for modalities and the notion of existence as presence delineated in *al-Ishrāq*, are developed in his discourse on fallacies (*mughālaţa*). These proof-methods are semantical or more accurately dialectical rather than syntactical. Indeed, they follow the so-called dialectical *no-counterexample* interpretation of the quantifiers, by the means of which the search for a counterexample is governed by rules of interaction.

This remind us of Griffel's insightful remark that the marriage of discursive $(bahth\bar{i})$ and taste/intuition $(dhawq\bar{i})$ philosophy – explicitly mentioned in the introduction of *al-Ishrāq* (Provo: Brigham young UP, 1999), section 5, 3 – yields a novel epistemology that grounds Suhrawardī's innovations.⁶

Moreover, it also reminds us of the fact that Suhrawardī's work has been developed during the period when the *logical turn of dialectics* was taking place, during which the twelfth-century-CE eastern school of Radī al-Dīn al-

⁵ cf. *al-Ishrāq* (Provo: Brigham young UP, 1999), 16-7.

³ It has often been pointed out that it is not clear who are the philosophers Suhrawardī calls *Peripateticians* – cf. R. Wisnovsky, "Essence and Existence in the Eleventh- and Twelfth-Century Islamic East (*Mašriq*): A Sketch", in *The Arabic, Hebrew and Latin Reception of Ibn Sīnā 's Metaphysics*, ed. by D. N. Hasse, and A. Bertolacci (Berlin: Walter de Gruyter, 2011), 27–50.

⁴ See T. Street "Suhrawardī on Modal Syllogism", in *Islamic Thought in the Middle Ages Studies in Text, Transmission and Translation, in Honour of Hans Daiber*, ed. by A. Akasoy and W. Raven, (Leiden and Boston: Brill, 2008), 163-178. See too Z. Movahed, "Suhrawardī's Modal Syllogisms", *Sophia Perennis* 2, no.4, Serial Number 21 (2012); who follows the general lines of Street's interpretation – namely the embedding of a *de re* necessary or contingent modality in a *de dicto* necessary modality, though, in contrast to Street, Movahed concludes that this shows the difference rather than the similarities between the modal logic of Suhrawardī and Ibn Sīnā.

⁶ Cf. F. Griffel, *The Formation of Post-Classical Philosophy in Islam* (Oxford: OUP, 2022), 263.

Nīsābūrī (d. 544/1149) students, especially Rukn al-Dīn al-ʿAmīdī (d. 615/1218), and others promoted the fusion of logic and dialectic theory.⁷

As previously stated, Suhrawardī's main contribution to modal logic is the explicit role that presences play in his logic, which is shaped by the logical development of dialectics during his time. The use of modalities in the quoted text demonstrates a logical shift in the argument. However, Suhrawardī did not systematically develop this framework. In the following sections, we will use the Immanent Reasoning framework, which includes verifiers of assertions in dialogical meaning explanations.

I Knowledge is about General Affirmative Laws

The central notion in Suhrawardī's Illuminationist Logic and epistemology is the one of definitely necessary $[al-dar\bar{u}riyya \ al-bat\bar{a}ta]$ propositions:

Since the contingency of the contingent, the impossibility of the impossible, and the necessity of the necessary are all necessary, it is better to make the modes of necessity, contingency, and impossibility parts of the predicate so that the proposition will become necessary in all circumstances. You would thus say, "Necessarily all humans are contingently literate, necessarily animals, or impossibly stones." Such a proposition is called the "definitely necessary." [...]. We can make no definitive and final judgment except concerning that which we know necessarily. Even for that which is only true sometimes, we use the definitely necessary proposition. In the case of "breathing at some time," it would be correct to say, "All men necessarily breathe at some time." That men necessarily breathe at some time is always an attribute of man. That they necessarily do not breathe at some time is also a necessary attribute of a man at all times, even at the time when he is breathing. However, this is different from literacy. While literacy is necessarily contingent, it is not necessary that it be actualised at some time. al-Ishrāq (Provo: Brigham young UP, 1999), 18.

The qualifications *general definite* and *existential indefinite* that Suhrawardī' chooses for universal and existential quantification respectively, point out at his epistemic concerns. Since the general aims of science is to achieve certainty, it is advisable, to always to prioritize:⁸

• affirmative over the negative quality of judgments,

and

• judgements with a universal quantity over an existential one.

This leads Suhrawardī to:

articulate negatives as affirmative assertions with a negation on the predicate, and convert particulars and existentials into universals

Thus, metathetic (i.e. transposed) negations ($ma' d\bar{u}la$) count as affirmations. Affirmations predicate about something, be it mental (like numbers) or extra mental (spatio-temporal entities), whereas *de dicto* negations (that cut the copula), do not.⁹ Here Suhrawardī, different to al-Rāzī (1963, 1: 158) follows Ibn Sīnā who seems to have assumed that existential import is part and parcel of the truth conditions for affirmative judgments, including metathetic ones.

⁷ See W. E. Young, Walter, "The Formal Evolution of Islamic Juridical Dialectic: A Brief Glimpse.", in *New Developments in Legal Reasoning and Logic: From Ancient Law to Modern Legal Systems*, ed by. S. Rahman, M. Armgardt et al. (Cham: Springer, 2021), 83-114; Id., *On the Protocols for Dialectical Inquiry (Ādāb al-Baḥth): A Critical Edition and Parallel Translation of the Sharḥ al-Risāla al-Samarqandiyya by Qutb al-Dīn al-Kīlānī (fl. ca. 830/1427), Prefaced by a Critical Edition and Parallel Translation of its Grundtext: the Risāla fī Ādāb al-Baḥth by Shams al-Dīn al-Samarqandī (d.722/1322).* [Forthcoming] (2021); Id., "On the Logical Machinery of Post-Classical Dialectic: The *Kitāb 'Ayn al-Naẓar* of Shams al-Dīn al-Samarqandī (d. 722/1322)", *Methodos* 22 (2022), https://journals.openedition.org/methodos/8605. See too S. Rahman and W. E. Young, "Argumentation and Arabic Philosophy of Language: Introduction", *Methodos* 22 (2022), https://journals.openedition.org/methodos/8833.

⁸ Cf. *al-Ishrāq* (Provo: Brigham young UP, 1999), 14-5. J. Kaukua, *Suhrawardī's Illuminationism* (Leiden: Brill, 2022), 35-45; provides a lucid discussion of these priorities, though with a slightly different focus than ours.

⁹ Cf. Y. Daşdemir, "The Problem of Existential Import in Metathetic Propositions: Qutb al-Din al-Tahtani contra Fakhr al-Din al-Razi", *Nazariyat* 5/2 (November 2019): 84.

Where Ibn Sīnā and Suhrawardī follow different paths is that the latter restricts the difference between these two forms of negations to the level of elementary propositions. Indeed, according to Suhrawardī at the level of quantified propositions the distinction between *de dicto* and *de re* negation collapses. So, on Suhrawardī's view, the two negatives *No man is stone* and *All men are non-stone* express the same content.¹⁰ However, *Zayd is non-*stone, asserts something about the existing Zayd, but *Zayd is not a stone* does not commit to the existence of Zayd. This stance might be problematic in cases such as *Not all man is literate*. This is clearly a *de dicto* negation and is certainly not equivalent to *No man is literate*. Suhrawardī's solution is extract from *Not all man is literate*, both the subset of those men that do satisfy the attribute and the subset that do not.

If you say, "Not all man is literate," then you may be able to say, "Some man is literate," since the negation only applies to the part. *al-Ishrāq* (Provo: Brigham young UP, 1999), 16.

This reading comes quite close to some uses in natural languages such as when with the assertion *Not all man is literate*, the speaker would also like to convey that *some are not* but *some are*, and perhaps even *most of them are literate*.¹¹

With regard to converting particular and existential propositions into universals, the point amounts to avoiding *indefinite* assertions – a proposition with a singular term – such a *Zayd is literate* is called a *particular proposition*.¹² According to our author, particular propositions, existential propositions and universal propositions, determine a scale of increasing epistemic value – at the top of the scale is Aristotle's perfect syllogism in mood (modalized) *Barbara* of the first figure.¹³ In the case of a particular the idea is that given two (compatible) propositions involving *the same singular term*, we can proceed as follows:

From Zayd is animal and Zayd is man, ------ Infer Something is an animal and Something is a man

Now, we eliminate the indefinite component expressed by the "*Something*". We look if one of the predicates have a "general meaning" with regard to the second – i.e., expresses an attribute that that can be predicated of every instance of the other, and construct a suitable universal. In Suhrawardī's example *animal* satisfies this condition and this yields *All men are animal*. Then we proceed to find another predicate which can be predicated of the subject of the just obtained universal, and, if found, we produce a second universal. In our case, Suhrawardī uses *rational* as predicated of men. This yields *All men are rational*.¹⁴

Indefinite existentials should be converted into universals:

^[...] for example, "Some animals are rational," and "All rational beings are capable of laughter." Let us give a name to the particular without considering the predication of rationality, though rationality accompanies the particular. Let this be D. Thus, *it* can be said, "All D are rational, and all rational beings are so-and-so," according to what we said before. Now we no longer need to say, "Some animals are D" as another premise, because D is the name of that animal, and how can a thing's name be predicated of it? *. al-Ishrāq* (Provo: Brigham young UP, 1999), 22.

¹⁰ Cf. al-Ishrāq (Provo: Brigham young UP, 1999), 21-22.

¹¹ Grice would identify them as a kind of *implicatures*.

¹² Cf. al-Ishrāq (Provo: Brigham young UP, 1999), 14.

¹³ Recall that Arabic logicians changed the order of premises and placed the subject before the predicate. This, of course, does not change the validity of the moods involved but it does not coincide with the Latin medieval denominations of the valid forms of each figure – cf. T. Street, "An outline of Avicenna's syllogistic", *Archiv für Geschichte der Philosophie* 84(2) (2022) and Id., "Avicenna on Syllogism", in *Interpreting Avicenna. Critical Essay*, ed.by P. Adamson (Cambridge: CUP, 2013).

¹⁴ Cf. al-Ishrāq (Provo: Brigham young UP, 1999), 24.

The idea brings to the light some interesting features of Suhrawardī's understanding of quantifiers. Assume Suhrawardī's own example of a syllogism of the first figure in mood *Darii*:

Some animals are rational (beings) All rational beings have the capacity of laughing

The existential claim implies that we can specify a set of animals in such a way that all the elements of that set are rational,

{All those animals who are rational beings}

One possible way to create such a set is to introduce the predicate *human* "without considering the predication of rationality, although rationality accompanies it". For example, as an answer to the question *Who are all those animals that are rational beings*? This answer yields

Every instance of (all) those animals who are human, are rational (beings)

Clearly, the capacity of laughing can be predicated of whatever arbitrary presence d of humans. This verifies the syllogism in *Barbara*

and it also verifies by sub-alternation the conclusion of the syllogism in Darii

Some animals, namely (all) those who are humans, have the capacity of laughing

The procedure indicates how to specify the Subject-Term in the conclusion of the original *Darii* in order to reduce the *uncertainty* expressed by the existential.

Notice that any specification of animals will do the job, provided that the middle term, i.e. *being rational*, can be predicated of every one of the elements of this specification, such as, say, the specifications animals *that read, or that are musicians, etc.* This, certainly reduces the "uncertainty" expressed in conclusion of the particular.¹⁵

As so often in the literature on this form of proof, there is in the texts of Suhrawardī the ambiguity between picking an arbitrary individual d, called *perceptual ecthesis*, which in our case is an instance witnessing the presence of a human, and D as a general term standing for a specification of the set underlying the original existential – in our example D would stand for the set of those animals that are human.¹⁶ However, as pointed out by J. Kaukua, the arbitrary individual d, is always experienced as instantiating a general form: experiencing this particular individual is always experiencing as being a human or a rational being and so on; even if it not first articulated as such.¹⁷ Thus, foremost there is the non-articulated experience of a d, whereby bearer and attribute are not distinguished, then it is articulated as d: D, d being a D. However, even at the articulated abstract level necessary for logic, it is not conceived as separated from the universal it instantiates.

¹⁵ As indicated by Zoe McConaughey in a personal email to S. Rahman this is close to Aristotle's method of, *inventing a term which apply to all things of a certain kind*, in the *Topics*, VIII 2 157a23-26.

¹⁶ T. Street develops a thorough discussion of the uses of ecthesis (*iftirad*) in Ibn Sīnā in his paper "An outline of Avicenna's syllogistic", *Archiv für Geschichte der Philosophie*, 84(2) (2022): 139-142. For a discussion on this ambiguity see M. Crubellier, Z. McConaughey et al. "Dialectic, The Dictum de Omni and Ecthesis", *History and Philosophy of Logic* 40, No. 3 (2019): 207-233.

¹⁷ Cf. J. Kaukua, "Suhrawardī's knowledge as presence in context", Studia Orientalia 114 (2013).

II Toward a Logic of Presence

II.1 Modal Relations, Dispositions and Acquired Capacities.

(19) The relation of the predicate of a categorical proposition to its subject either must exist (in which case it is called "the necessary") or must not exist ("the impossible") or may either exist or not exist ("the possible" or "the contingent"). An example of the first is "Man is animal"; of the second, "Man is stone"; and of the third, "Man is literate." [...]. The contingent is necessary by virtue of that which necessitates it and is impossible on condition of the nonexistence of that which necessitates its existence. When one examines the thing itself in the two states of existence and nonexistence, it is contingent.

[...]

Further, when you say, "All things that move necessarily change," you should know that each and every thing described as moving is not necessarily changing because of its own essence, but because it is moving. Thus, its necessity depends on a condition and *it* is contingent in itself. By "necessary," we mean only that which it has by virtue of its own essence. That which is necessary on condition of a time or state is contingent in itself. *.al-Ishrāq* (Provo: Brigham young UP, 1999), 17.

(23) Conversion is making the entire subject of the proposition the predicate and the predicate the subject while keeping the quality and the truth or falsity of the proposition the same. You know that when you say, "All men are animals," you cannot say, "and all animals are men." The same is true in every proposition whose subject is more specific than its predicate [...]. Then we say, "Necessarily all men are contingently literate," its converse will be "Necessarily something that is contingently literate is a man." The other modes besides contingency also move with the predicate when it is converted." The converse of the necessary definite affirmative proposition is itself a necessary definite affirmative proposition, whatever the mode may be.

If the contingency is part of the predicate of the definite necessary proposition and the negation is with the predicate, the negation will also be moved in conversion, as in the statement "Necessarily all men are contingently nonliterate." Its converse will be the definite affirmative: "Necessarily something that is contingently nonliterate is a man." $al-Ishr\bar{a}q$ (Provo: Brigham young UP, 1999), 19-20.

Thus,

• a *necessarily necessary relation* attributes *actual instances/presences* of the predicate to every presence of the subject, and this relation either

admits *simple conversion*, if there is simple conversion between presences of the subject and presences of the predicate – such as when actual instances of *rational animal* are related to actual instances of *human*, this corresponds to the notion of *definition* of Peripatetics – whereby the predicate expresses the definition of the subject; or

does not admit *simple* conversion – such as when actual instances of *animal* are related to actual instances of *human*, this corresponds to the notion of *genus* of the Peripatetics – whereby the predicate expresses the genus of the subject

• If indeed each presence of the subject **necessarily** bears the **actualised** attribute, then, and only then, the **relation of necessarily necessary exists**.¹⁸

Contingency, is subtler. On one hand, universally quantified predications of contingency, admit both

- the conversion of the subalternate, e.g. from *Necessarily all men are contingently literate* obtain the converse subalternate *Necessarily some contingently literate are men*, and
- and **the simple conversion of the universal**, provided *the subject is not more specific than the predicate* or more precisely when it is *co-extensive* with the predicate– e.g. *Necessarily all men are contingently literate* admits the simple conversion *Necessarily all contingently literate are men*, but *Necessarily all men contingently breath at some time* does not admit simple conversion.

Moreover, as mentioned above, since Suhrawardī gives epistemic priority to universal apodictic assertions, he calls them *definitely necessary propositions*; contingency assertions, should be embedded in propositions within

¹⁸ It might be argued that the second occurrence of the necessity modality has a different meaning from the first: whereas the first occurrence indicates that the subject bears the predicate because of the essence of the subject, the second is about the actualisation of the attributed predicate. Now essential attributes are always actualised, so if the attributed predicate is entailed by the essence of the subject, then that predicate must be actualised. This point, as we shall discuss in the following pages, relates to the embedding of a contingency within the scope of a necessity.

a universal (i.e., necessary) modality. It should be noted that Suhrawardī's approach excludes purely accidental attributes. According to his view, in order to achieve knowledge, we should focus on each attribute that the subject bears because of its essence, rather than because the subject is momentarily bearing another contingent attribute. Therefore, Ibn Sīnā's form of assertions, such as *Every human writing is moving while writing*, falls outside the scope of the epistemological aims Suhrawardī sets for the logic of *Illumination*.¹⁹

Thus, a *necessarily contingent relation* attributes capacities or potentialities to every presence of the subject. Here again the relation of necessarily contingency **exists** iff each instance of the subject **necessarily** bears the attribute but **not necessarily actualised**.²⁰ Such potentialities, can be grouped as follows:

(i) Potentialities that for each actual instance/presence of the subject require this potentiality to be both, sometimes actualised and sometimes not – such as *Zayd's laughing at 12hs* (which is co-extensive with *Human*), and *Zayd not laughing at 12,5hs*, and similarly for *Zayd's breathing* which is not co-extensive with *Human*). This group can be seen as referring to *dispositions* – i.e., *natural not acquired* capacities (this denomination is not explicit in Suhrawardī's texts but the examples strongly suggest such a category.

(ii) Potentialities that do not require this potentiality to be ever actualised for some particular actual instance of a subject, though the potentiality might be actualised for another instance of that subject, such a *literacy*, which might never be actualised for *Zayd* but is actualised for *Imru*' *al-Qays* (this is not explicit either). This form of contingency admits *generic Plenitude*. Here we are dealing with *acquired capacities* rather than with dispositions.

The way a contingent attribute actualises appears to depend on the category to which it belongs. Indeed,

- Dispositions enjoy *individual Plenitude* we might also call it *de re* Plenitude, since they must be actualised at least once **for each individual** instantiating the subject.
- Acquired capacities are, in principle, necessarily contingent on humanity as a whole, not on each individual. For instance, literacy is a human capacity because there is at least one person who is literate, and it is also contingent because there is at least one person who is not. So, we might speak here of *generic Plenitude*.
- Plenitude involving acquired capacities may also take a weak form of actualisation, either hypothetical or counterfactual, and this may apply both generically and individually.

As stated in the quoted text (in fact one among many other texts), Suhrawardī's modalities do not require the development of a logic that appeals to contemporary possible-world frameworks. Syntactically, Suhrawardī's modalities are relations between terms in a syllogism, rather than propositional monadic connectives. Semantically, the modality of contingency requires the satisfaction of either a strong form of Plenitude, which involves producing presences that actualise (at least once) the contingent attribute, or a weak form of Plenitude, which involves presences in a possible course of events rather than possible worlds. Furthermore, it is important to note that since no contingent property can ever be an essential one, the possibility involved is two-sided possibility.

¹⁹ This has been already pointed out by T. Street in his paper "Suhrawardī on Modal Syllogism", in *Islamic Thought in the Middle Ages Studies in Text, Transmission and Translation, in Honour of Hans Daiber*, ed. by A. Akasoy and W. Raven, (Leiden and Boston: Brill, 2008): 167.

²⁰ J. Kaukua, *Suhrawardī's Illuminationism*, (Leiden: Brill 2022), 53; endorses Walbridge's remark that contingency is more about ontological dependence than about realization of potentialities – cf. J. Walbridge, *The Leaven of the Ancients: Suhrawardī and the Heritage of the Greeks* (Albany: SUNY Press, 2000), 153. We agree with the point that conditionality is the main feature of attributions of *bare* or accidental contingency. However, note that the ontological dependence on the subject accounts for the distinction between (bare) necessarily and (bare) contingency, but it fails to elucidate the notion of necessary contingency and how the latter can be distinguished from a necessarily necessary relation. Necessarily necessary relations are distinguished from necessarily contingent ones since the former, but not the latter, require the attributed predicate to be actualised by each instance of the subject.

Clearly, Suhrawardī's classification of the ways a predicate relates to its subject echoes the Aristotelian theory of four Predicables developed in the *Topics* (Top. A 4 101b15-19)),²¹ namely *genus definition*, *proprium* or *accident*. Suhrawardī repeats this point in several parts of *al-Ishrāq*, particularly so when he has to elucidate his view on syllogisms.

However, a word of caution is due: we are not claiming that there is evidence that Suhrawardī ever read or had direct access to the *Topics*. Nevertheless, whatever were the ways he came to know the theory of Predicables, it definitely influenced his own perspective on modalities.

II.2. On Iteration and Simple Conversion:

Usually, those who, under the background of the theory of predicables, understand modalities as affecting the copula rather than as monadic propositional operators, usually reject iteration,²² and the other way-around.

Awkwardly, Ziai claims that Suhrawardī's modal logic is essentially a propositional S5 logic without iteration – or with an iteration that only occurs at the level of the surface grammar.²³ Street contests Ziai's claim and rightly so.²⁴ Indeed, Suhrawardī explicitly writes

(21) Since the contingency of the contingent, the impossibility of the impossible, and the necessity of the necessary are all necessary, it is better to make the modes of necessity, contingency, and impossibility parts of the predicate so that the proposition will become necessary in all circumstances. You would thus say, "Necessarily all humans are contingently literate, necessarily animals, or impossibly stones." Such a proposition is called the "definitely necessary". *al-Ishrāq* (Provo: Brigham young UP, 1999), 18.

If we distinguish the relations *Necessity by definition*, *Necessity by genus*, *Necessity by proprium*, and *Necessary accidens*, as autonomous primitive relations the following table results:

Not iterated Aristotelian Modalities as Predicables	Suhrawardī's Iterated Modalities	
L _δ (δ, definition)	LL (necessarily necessary relation admitting simple conversion)	
Lγ (γ, genus)	LL (necessarily necessary relation NOT admitting simple conversion))	
\mathbf{M}_{π} (π , proprium)	LM (necessarily contingent relation admitting simple conversion)	
$\mathbf{M}_{\boldsymbol{\alpha}}$ (α , accident)	LM (necessarily contingent relation NOT admitting simple conversion)	

²¹ Our references to texts by Aristotle are based in *The Works of Aristotle Translated into English*. Edited by W.D. Ross (Oxford: OCP, 1966).

²² Cf. M. Malink, "A Reconstruction of Aristotle's Modal Syllogistic", *History and Philosophy of Logic* 27 (2006): 96-7. See too M. Malink, M, *Aristotle's Modal Syllogistic* (Cambridge Mass.: Harvard University Press, 2013), 13-17.

²³ Cf. H. Ziai, Knowledge and Illumination: A Study of Suhrawardī's Hikmat al-Ishrāq (Atlanta: Scholars Press, 1990), 70.

²⁴ T. Street, "Suhrawardī on Modal Syllogism", in *Islamic Thought in the Middle Ages Studies in Text, Transmission and Translation, in Honour of Hans Daiber*, ed. by A. Akasoy and W. Raven (Leiden and Boston: Brill, 2008), 169. This is corrected in Walbridge and Ziai, ed., *al-Ishrāq* (Provo: Brigham young UP, 1999), 17, footnote 20.

The second occurrence of L in LL stands for non-contingent predication – i.e. LL stands for actual presences of the predicate for each actual presence of the subject. No other kind of iterations seem to be suitable in such a framework.

II.3 Dialectical Meaning Explanations

II.3.1 Suhrawardī's meaning explanations of modalities

The meaning explanations of Suhrawardī's modal relations are contained in the following short but quite insightful text occurring in the third discourse consecrated to the study of fallacies: .

(48) Know that the universality of a rule stating that something is predicated of something else is disproved by a single instance where that second thing is absent. The universality of a law stating the impossibility of something being predicated of something else is proven/by the existence of that thing in a single case. Thus, if someone asserts that every C is necessarily B but finds a single C that is not B, then the universality of the rule is disproved. Likewise, if someone asserts that it is impossible for any C to be B but then finds a single C which is B, then the law will be disproved. However, if someone asserts that any C may be B, this is disproved by neither the existence nor the absence of instances. Thus, should someone claim that some universal is contingently true of another universal-for example, asserting the "B-ness" of C-then **he need find** only a single instance that is B and another that is not B in order to show that the universal B is not impossible in the nature C (since otherwise no individual C could be described as being B) and that [B] is not necessary [in C) (since in that case no individual C could fail to be B). *al-Ishrāq* (Provo: Brigham young UP, 1999), 38.

Saleh Zarepour, in an email to Rahman, rightly pointed out that in the Arabic source of the quoted text it is indicated that for someone who wants to prove a necessary contingency it is **sufficient** ($2 \le 1$) to find one particular [instance of C] that is B and another particular [instance of C] that is not, rather (s)he **needs** to find such instances, as in Zia and Walbridge's translation above.

Zarepour's point is an Avicennean one: if contingency amounts to **necessarily** finding at least one instance where the potentiality is realized (and one where it is not), then this seems to lead to a strong form of Plenitude: anything possible must be once realized. Ibn $S\bar{n}\bar{a}$'s take on Plenitude is a weaker one: what is required is that such an instance is *conceivable*.²⁵

Nevertheless, it is difficult to decide if Suhrawardī endorse or not a weak form of Plenitude: on one hand the insistence on *presence*, seem to be more ontologically committed than *mere presence in the mind*, on the other there are texts that suggest counterfactual or hypothetical presence, such as in the case of mixed syllogisms.²⁶ The form that a contingent attribute takes appears to depend on the type of potentiality in question – is it a natural or acquired potentiality?. We will come back to this issue further on in our text.

Suhrawardī's text on the rules for justifying assertions involving modalities follow the so-called dialectical *no-counterexample* interpretation of the quantifiers, by the means of which the search for a counterexample is governed by rules of interaction. In such a context, to grasp the meaning of a proposition involved in an assertion amounts to knowing:

- (a) what *requests* or *challenges* are granted to the antagonist by that assertion
- (b) what *commitments* (i.e. *defences*) does the assertion engage to.

That is what we mean when we speak of *dialectical meaning explanation*.²⁷ The dialectical approach constitutes in fact the man background we follow for the reconstruction of Suhrawardī's logic and theory syllogism.

²⁵ This has been admirably discussed by Griffel in his book *Al-Ghazali's Philosophical Theology* (Oxford: OUP, 2009), 167-170. The form that a contingent attribute takes appears to depend on the type of attribute in question. We will come back to this issue further on in our text.

²⁶ Suhrawardī, *Manțiq al-talwīḥāt*, ed., (Tehran: A.A. Fayyāḍ, 1955), 35-36.

²⁷ See S. Rahman, Z. McConaughey, *Immanent Reasoning* (Cham : Springer, 2018) and M. Crubellier, Z. McConaughey et al. "Dialectic, The Dictum de Omni and Ecthesis", *History and Philosophy of Logic* 40, No. 3 (2019): 207-233.

The text clearly set the main features of the dialogical meaning explanations of modalities. In a nutshell

- The dialectical meaning of the two universals, necessity and impossibility, requires the challenger to find an instance of the subject to which she claims the predicate cannot be attributed (or can be attributed in the case of impossibility). The defender must then assert, to the contrary, that the predicate can (or cannot) be attributed for any arbitrary presence the challenger might produce.
- The dialectical meaning of a contingent attribute requires the challenger to ask the defender to find a second instance for each subject instance. One of the members of pair should actualise (hypothetically or counterfactually if we allow cases of weak Plenitude) the attribute and not the other. Note that in the case of dispositions both members of the pair of instances of the subject can be the same, but then, the attribution of the predicate is temporally indexed with different moments. For example, Zaynab laughs at moment t_i and Zavnab does not laugh at t_k .

As mentioned in the introduction, Suhrawardī's main contribution to modal logic is the explicit role that presences play in his logic, which is shaped by the logical development of dialectics during his time. The notions of modalities in the quoted text witnesses the fruitfulness of the intertwining of dialectic and logic.

However, Suhrawardī did not develop this approach into a systematic logical framework. We will undertake this task. To do this, we will use the dialogical theory called *Immanent Reasoning*,²⁸ which incorporates verifiers of propositions into the rules of dialogical meaning explanations of logical constants. On one hand, the verifiers of a proposition (local reasons) can be interpreted as presences, as used in the logic of al-Ishrāq. On the other hand, Suhrawardī's dialectical rules for determining the meaning of modalities suggest a new way of developing modal and temporal logic. We claim that this constitutes an approach that unifies the philosophy, logic, and epistemology within al-Ishrāq.

In order to develop our reconstruction, we will begin by giving the dialogical meaning explanations of nonmodalized universal and existential quantifiers, with particular emphasis on the role that Suhrawardī's gives to presences. We will then proceed to add modalities that satisfy both strong and weak forms of Plenitude, and we will conclude by showing how this closely resembles Suhrawardi's view of modal syllogism.

II.3.2.Dialectical s Meaning Explanations of Non-Modal Universals and Existentials

In order to elucidate the role of presences in the constitution of dialectical meaning let us focus first on non-modal propositions. Despite that the analysis makes use the formal languages of Immanent Reasoning and Aarne Ranta's Type Theoretical Grammar,²⁹ we will also provide a vernacular version. The logical form will be unfolded during the interaction: we call this dynamic encoding.

II.3.2.1 Meaning Constitution and the Experience of Presence

The first semantic level upon which all other depend on concerns establishing links between concepts. This level does not directly render a proposition but rather the semantics conditions out of which a proposition obtains. So, the conceptual link

(the concept) Knowing Being entails (the concept) Living Being

²⁸ Cf. S. Rahman, Z. McConaughey et al., *Immanent Reasoning* (Cham : Springer, 2018). See too N. Clerbout and Z. McConaughey "Dialogical Logic", in Stanford Encyclopedia of Philosophy, https://plato.stanford.edu/entries/logic-dialogical/.

is to be understood as

for any instance (presence) x of *Knowing Being*, an instance of *Living Being* can be obtained by a semantic process that renders instances of the latter out of instances of the former. If we, employ Ranta's Type Theoretical Grammar we obtain³⁰

Linear notation	Vertical Notation
Living Being[x]: prop (x & Knowing Being)	$(x \in Knowing Being)$
	 Living Being[x] ε prop

At a further (connective) level of analysis this either constitutes a propositional implication or a universal quantification such as

if it is knowing then it is living, or also *Every knowing being is a living being.*

The connective/quantifier level presupposes the semantic level. Only when we know how one concept is dependent upon another one, can we render the corresponding logical connective or quantifier. According to our analysis this is one of the most distinctive features of the Epistemology of Illumination and can be declined as the obtaining from the following steps:

- 1. Conceiving the universal *Everything knowing, is living* as a proposition presupposes:
- 2. Every presence of *Living Being* has been experienced as associating presences of *Knowing Being* with presences of *Living Being*. Isolating and identifying the association-procedure itself is production of a further abstraction step. Formally speaking, the association-procedure can be rendered as a function b(x): that takes presences of the antecedent and yields presences of the consequent:

Linear notation	Vertical Notation
$b(x)$: Living Being[x] ε prop (x ε Knowing Being)	(x & Knowing Being)
	 $b(x) \in Living Being[x]: prop$

In short, and expressed as an inference process, now from the simpler to the complex after the experience of the concrete presence has been settled:

Living $Being(x) \in prop \ (x \in Knowing Being)$ $b(x) \in Living Being(x) \ (x \in Knowing Being)$ $(\forall x \in Knowing Being) Living Being(x) \in prop$

This can also be seen as playing a role in Suhrawradī's criticism of, what he calls the *peripatetic* take on definitions (and genus). Universals expressing definitions already assume that their underlying meaning constitution process has been established before. In other words, universals expressing definition and genus assume the formulation of meaning formation rules that encode knowledge gathered by grasping the dependence or interdependence of the actual instances (presences) of the terms involved. Curiously, this seems to echo Ibn Sīnā's notions of implicate

³⁰ In fact, although the notation generally follows that of Ranta's Type Theoretical Grammar, we use we use " $d \in D$ " instead of "d : D". Our notation suggests, on the one hand, its links to the grammatical copula, and on the other to the element-relation in Constructive Type Theory (where the elements are not available without the set/type they instantiate). K. Lorenz and J. Mittelstrass. used this notation in their analysis of Plato's theory of language in their joint paper "On rational philosophy of language: The programme in Plato's Cratylus reconsidered", *Mind* 76 (301) (1967): 1-20.

 $(l\bar{a}zim)$, containment (tadammun) and implication $(iltiz\bar{a}m)$.³¹ Still, Suhrawradī's epistemic point, so we claim, is that the ability of gathering knowledge of the universal by experiencing a presence that instantiates it, should be an explicit part epistemology.

- This suggests that formalized meaning explanations for the forms of statements used in Suhrawradī's epistemology and logic should include presences or witnesses of such presences, in the object language.
- With *witness of a presence* we mean the logical-linguistic construct that is won through a process of abstraction on the experiences of presence.

II.3.2.2 Presences and the Dialectical Meaning of Universals and Existentials

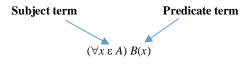
Setting the *dialogical meaning explanation* of the universal quantifier, amounts to the following: If player **X** states a universal quantifier he has to be able to associate a suitable presence of the **consequent** for any arbitrary presence of the **antecedent** chosen by the challenger **Y**.

Statement	Challenge	Defence
$\mathbf{X} ! (\forall x \in A) B(x)$	$\mathbf{Y} ! a \in A$	$\mathbf{X} \mid B(a)$
X states the universal <i>Every A is B</i>	Y chooses an arbitrary presence <i>a</i> of <i>A</i>	X associates <i>a</i> with a presence of $B(a/x)$

This again presupposes those meaning formation rules encoding the knowledge required for producing universal propositions mentioned above:

Statement	Challenge	Defence
\mathbf{X} ($\forall x \in A$) $B(x) \in prop$	Challenge1 Y? IF	Defence1 X $A \in set/prop$
X states that the universal is a proposition	Y asks for the formation of the left (ℒ) component of the universal	X responds that it is a set (or proposition not dependent upon another one)
	Challenge2 Y ? \mathcal{R}_F	Defence2 X $B(x)$: prop ($x \in A$)
	Y asks for the formation of the right (\mathcal{R}) component of the universal	X responds that $B(x)$ is a proposition dependent upon A

This notation comes close to the analysis of a universal assertion in traditional logic whereby the Predicate-Term is said to applies to any instance of the Subject-Term:



The dialogical meaning explanation for an existential, leaves the choice of the presence to the defender:

Statement	Challenge		Defence	
$\mathbf{X} ! (\exists x \in A) B(x)$	Challenge1	$\mathbf{Y} ? \mathcal{L}^{\exists}$	Defence1	$\mathbf{X} a \varepsilon A$
X states the existential Some A is B	Y asks for the left: Wh are <i>E</i>		X respond	s that <i>a</i> is one of those <i>A</i> 's

³¹ Cf. See R. Strobino, "Per se, Inseparability, Containment and Implication: Bridging the Gap between Avicenna's Theory of Demonstration and Logic of the Predicables", *Oriens* 44 (3/4) (2016).

Challenge2Y ? $a/x: A \mathcal{R}^3$ Y asks for the right: Show me that this choice of yours (namely, $a: A$) is indeed a B	Defence2 $X ! B(a)$ X associates the presence of a with a presence of $B(a/x)$
--	---

This analysis allows expressing the Subject-Term as a restricting an underlying domain. Let us take Ibn Sīnā's example

Some poets are good

An assertion, which, as pointed by Ibn Sīnā does not support the inference *There is someone*, say, *Imru' al-Qays*, who is good and a poet.³² Clearly what is asserted is

Some poets are good **as poets** $(\exists x \in Poets) Good(x)$

In other words: Within the domain restricted by the subject, namely, poets, some (of them) are good. Thus, the existential expresses a set, in our example the set of

(All) those presences of poet that are good (as **poets**) $\{x \in poet | Good(x)\}$

Since this set, is what the meaning of the existential amounts to, the dialogical meaning explanation is the same as the one of the existential.³³

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³³ At the strategic level the Proponent, has a winning strategy for a a universal iff for *any* presence of the Subject the challenger can show produce an instance of the Predicate for this choice of the Opponent. The way to implement this, is to allow the Opponent to choose always a **new** instance

Statement	Challenge	Defence
$\mathbf{P} ! (\forall x \in A) B(x)$	$\mathbf{O} \ a \in A$	$\mathbf{P} ! b(a) \in B(a)$
P states that he has a winning strategy for the universal	O chooses the new presence a of A , and requests P to show that it witnesses a presence of $B(x)$	P associates <i>a</i> with a presence of $B(a/x)$

Thus, at the strategic level, the truth of the universal asserted by **P**, requires him to be able to associate presences of the Subject with presences of the Predicate, by means of substituting *x* in *b*(*x*) for any presence of the Subject **O** might choose. In other words, the truth of the universal ($\forall x \in A$) *B*(*x*) requires the Proponent

- a) to build out of the associations triggered by whatever presences of A O might choose, witnesses of B(a/x) an association-procedure b(x),
- b) to build the abstract construct of this procedure, namely $(\lambda x)b(x)$, called the lambda-abstract of b(x). The lambda-abstract indicates that for whatever choice of the Opponent the association-procedure b(x) can be executed in order to yield an instance of B(a/x) for that choice, and
- c) **P**'s execution of b(a/x) is justified by **O** being forced to state himself B(a): this actually is the core of proving the validity (building a winning-strategy) of a syllogism involving universals.

Thus, the canonical form of a winning strategy is: **P** $(\lambda x)b(x) \in (\forall x \in A) B(x)$.

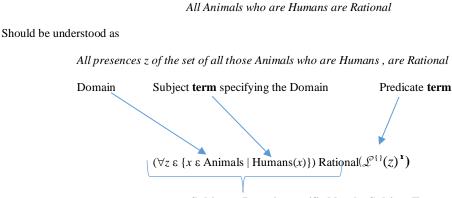
In relation to an existential, such as $(\exists x: A) B(x)$, at the strategic level, the truth of this existential requires **P** to be able to state some presence *a*: *A* chosen by **P** himself, as response to the first challenge, and to state B(a), as a response to the second challenge, by building the association procedure b(a/x).

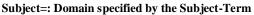
Statement	Challenges			Defence
$\mathbf{P} ! (\exists x \in A) B(x)$	Challenge1	$\mathbf{O} ? \mathcal{L}^{\exists}$	Defence1	$\mathbf{P} a \varepsilon A$

³² Ibn Sīnā, *al-Išārāt wa-l-tanbīhāt bi-Šarḥ al-Ţūsī, al-Manțiq [= Remarks and Admonitions: With Commentary by Tusi]*, ed. by S. Dunyā, (Cairo: Dār al-maʿārif, [thrd. Ed.]1983), Chapter 10.1, 50 .Recall Aristotle's famous example of the *good cobbler* in the *Peri Hermeneias*, XI, 20b35-36 and 21a14-15, and in the *Sophistical Refutations* 20, 177b14-15.

Within a syllogism, premises and conclusion share a common domain, over which the Subject term and the Predicate term have been defined. This is what allows the middle term, to occur as the Predicate in one premise and as Subject in the other.

In a dialectical context, it is useful to indicate that the "testing-instances" for a universal stated by **X**, are chosen by the challenger **Y**. In the example below, it amounts to the indication that **Y**'s choice of an instance witnessing the subject, is *an animal who is human*. Since *Animal* is the **left component** of every instance *z* witnessing the Subject, we adopt the notation " $\mathcal{L}^{\{1\}}(z)^{\mathbf{Y}}$ ", that indicates that **Y** choses some animal who is rational, as his (**Y**'s) case to build a counterexample to the universal.³⁴ Let "Subject" denominate here the Subject term + the domain upon which the Subject term has been defined, then we have:





This renders the Subject-Predicate form of traditional syllogism. Our view is that a rendering of the rules should combine traditional term-notation with explicit use of instances of the quantified expression in the context of the rules that prescribe how to develop the dialectical interaction associated with a quantified assertion. Instances only come to the fore during the development of the argument. Note that this is particularly important when temporality is involved, as it only arises explicitly through the interaction triggered by an assertion of contingency. This process is referred to as dynamic encoding. When a universal is dynamically encoded, the challenger must specify the subject-term that was implicitly present in the previous step of selecting the element of the domain that occurs in that term.

	Reading	Keys	
P states that he has winning-strategy for the existential	Challenge2	\mathbf{O} ? $a/x: A \mathcal{R}^{\exists}$	P responds by choosing one a and that it is one these presences that witnesses A Defence2P $b(a) \in B(a)$ P states that $b(a)$ witnesses $B(a/x)$

- In other words, the truth of the existential requires the Proponent
- a) to state that some presence *a*, chosen by **P** himself witnesses *A*, **and** build out of the association of *a* with a presence of the B(a/x) an association-procedure b(a/x)
- b) to build the complex construct $\langle a, b(x) \rangle$, which results from P's responses to both challenges:
- c) **P**'s choice of some $a \in A$, and the execution of b(a/x) is justified by **O** being forced to state himself both $a \in A$ and B(a): this actually is the core of proving the validity (building a winning-strategy) of a syllogism involving existentials
- Thus, the canonical form of an existential is : $\mathbf{P} ! \langle a, b(x) \rangle \in (\exists x \in A) B(x)$.

³⁴ Cf. M. Crubellier, Z. McConaughey et al. "Dialectic, The Dictum de Omni and Ecthesis", *History and Philosophy of Logic* 40, No. 3 (2019): 207-233.

Encoding	Gloss
X !	agent X claims
DA	the set $\{x \in D \mid A(x)\}$

In the first table below, we write three different versions of the initial statements distributed in three columns. The first columns express the traditional form with a formulation close to natural language. The second column formulates the statement in a more compact (but also vernacular) form. The third column, displays explicitly the logical form of the statement. In the second table, we add in each column the challenge of interlocutor \mathbf{Y} and the response of defender \mathbf{X} .

	Traditional Form Before Interaction	Dynamic Encoding Before Interaction	Explicit Dialectical Encoding Before Interaction
Universals	X ! Every (<i>D</i> who are) <i>S</i> is <i>P</i> X ! No (<i>D</i> who are) <i>S</i> is <i>P</i>	X ! (Every _D S)P X ! (Every _D S)no-P	$\mathbf{X} ! (\forall z \in \{x \in D \mid S(x)\}) P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$ $\mathbf{X} ! (\forall z \in \{x \in D \mid S(x)\}) \sim P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$
Particulars	X ! Some (D who are) S is P X ! Some (D who are) S is not P	(Some _D S)P X ! (Some _D S)no-P	$\mathbf{X} ! (\exists z \in \{x \in D \mid S(x)\}) P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$ $\mathbf{X} ! (\exists z \in \{x \in D \mid S(x)\}) \sim P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$

	Traditional Form During Interaction	Dynamic Encoding During Interaction	Explicit Dialectical Encoding During Interaction
Universals	A form X ! Every (<i>D</i> who are) <i>S</i> is <i>P</i>	A form X ! (Every _D S)P	A form X ! $(\forall z \in \{x \in D \mid S(x)\}) P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$
	Challenge $\mathbf{Y} ! d_i (in D)$ is S, is it a P ?Defence $\mathbf{X} ! d_i$ is F	Challenge $\mathbf{Y} ! DS(d_i)$ Defence $\mathbf{X} ! P(d_i)$ \mathbf{X} response to the challenge must attribute P to the d_i chosen by \mathbf{Y}	Challenge $\mathbf{Y} ! DS(d_i)$ Defence $\mathbf{X} ! P(d_i)$ Given that \mathbf{Y} chooses d_i as the left component of z which is one of the D 's that is an S , i.e. given $\mathcal{L}^{\{1\}}(z)=d_i \in D$
	E form X ! No (<i>D</i> who are) <i>S</i> is <i>P</i>	E form X ! (Every _D S)no-P	E form X ! $(\forall z \in \{x \in D \mid S(x)\}) \sim P(\mathcal{A}^{\{\}}(z)^{\mathbf{Y}})$
	ChallengeDefence $\mathbf{Y} \mid d_i \text{ (in } D) \text{ is } S,$ $\mathbf{X} \mid d_i \text{ is not } F$ is it a P ?	Challenge Y ! $_DS(d_i)$ Defence X ! $\sim P(d_i)$	Challenge $\mathbf{Y} ! DS(d_i)$ Defence $\mathbf{X} ! \sim P(d_i)$ Given $\mathbf{Y} \ \mathcal{L}^{\{\}}(z) = d_i \in D$
Particulars	I form X ! Some (D who are) S is P	l form X ! (Some _D S)P	$\frac{1 \text{ form}}{\mathbf{X} ! (\exists z \in \{x \in D \mid S(x)\}) P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})}$
	Challenge Y which D is S ? Y is d_i (in D) a P ?Defence X ! d_i is S X ! d_i is F		ChallengeDefence \mathbf{Y} ? \mathcal{L}^{\exists} \mathbf{X} ! $S(d_i)$ \mathbf{Y} ? \mathcal{R}^{\exists} \mathbf{X} ! $P(d_i)$
	O form X ! Some (<i>D</i> who are) <i>S</i> is not <i>P</i>	O form X ! (Some _D S)no-P	0 form X ! $(\exists z \in \{x \in D \mid S(x)\}) \sim P(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$
	ChallengeDefenceY which D is S ?X ! di is S	Challenge $\mathbf{Y} ? \mathscr{L}^{\exists}$ Defence $\mathbf{X} ! S(d_i)$	Challenge Y ? \mathcal{L}^\exists Defence X ! $S(d_i)$

	Y is d_i (in D) a P?	VI dis not D	V 2 OF	$\mathbf{V} = \mathbf{D}(\mathbf{A})$	V 9 @F	$\mathbf{V} = D(\mathcal{A})$
	I IS $u_1(\prod D)$ a F ?	\mathbf{A} : u_1 is not \mathbf{F}	1 : 31-	$\mathbf{X} ! \sim P(d_i)$	1 : 31-	$\mathbf{X} ! \sim P(d_i)$

III. The Dialectical Meaning Explanation of Suhrawardī's Modal Relations

How to distinguish a modal assertion from a non-modal one? Suhrawardī, who shares similar views with Ibn Sīnā on this point,³⁵ is primarily interested in modal propositions, especially necessary and universal ones, as, on his view, they are the only ones that generate knowledge. While it is challenging to provide a definitive answer, the following may be applicable:

• Categorical assertions abstract away from the specific modal relationship between the predicate and the subject, as well as the specific domain over which the subject and predicate have been defined. Categorical syllogisms, in a sense, disregard the various forms of internal contentual links present in the premises. However, in mixed syllogisms where some premises are modal and some are not, the implicit modality must be explicitly stated in accordance with the rules of the figure. We will discuss this further later on.

III.1 The Dialectical Meaning Explanation of the Necessarily Necessary Relation

With regard to the necessarily necessary relation, let us recall that

	A necessarily necessary relation amounts to attributing actual instances, i.e. presences/tokens/verifiers of the Predicate term to every presence of the Subject
— th	its (simple) <i>conversion</i> , if there is simple conversion between presences of the subject and presences of the predicate is corresponds to the notion of <i>definition</i> of Peripatetics; or not admit (simple) conversion – this corresponds to the notion of <i>genus</i> of the Peripatetics
	If we combine this with our previous considerations, we obtain
	$(\forall z \in \{x \in D \mid A(x)\})$ LL $B(\mathcal{L}^{\{\}}(z)$ true
Assuming	Subject term $A[x] \in \text{prop} (x \in D)$ Predicate Term $B[x] \in \text{prop} (x \in D)$

Notice that, since it *is better to make the modes of necessity, contingency, and impossibility parts of the predicate* - al-*Ishrāq* (Provo: Brigham young UP, 1999), 16, and, as we mentioned above, we might have syllogisms where the Subject-Term is modalized, e.g. if it is middle term of the major premising of a syllogism of the first-figure, it desirable to have a notation that encodes also these cases. Our proposed notation does exactly that. Indeed, it allows encodings such as the following whereby A is related to D by necessary contingency:

$(\forall z \in \{x \in D \mid \mathbf{LMA}(x)\}) \mathbf{LB}(\mathcal{L}^{\{\}}(z))$

According to our reading of Suhrawardī's text, the point is that the modality prescribes how the defender has to establish the link between the predicate and the subject, given an instance of the subject, claimed by the antagonist to constitute a counterexample to the claim of necessity. This suggests the following rendering of the rules:

Statement	Challenge	Defence

³⁵ See R. Strobino, "Per se, Inseparability, Containment and Implication: Bridging the Gap between Avicenna's Theory of Demonstration and Logic of the Predicables", *Oriens* 44 (3/4) (2016): 263.

Dynamic Encoding	Dynamic Encoding	Dynamic Encoding
\mathbf{X} ! (Every _D A) LL B	$\mathbf{Y} \; ! \; A(d_i)$	$\mathbf{X} \mid B(d_{i})$
All D's that are A, are necessarily B by necessity	Y states $A(d_i)$, whereby $s(h)e$ chooses d_i as being one of those D 's, that are A	X states that d_i witnesses <i>B</i> .
Explicit Dialectical Encoding	Explicit Dialectical Encoding	Explicit Dialectical Encoding
$\mathbf{X} ! (\forall z \in \{x \in D \mid A(x)\}) \mathbf{LLB}(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$	\mathbf{Y} ! $A(d_i)$	$\mathbf{X} b(d_{\mathrm{i}}) \in B(d_{\mathrm{i}})$
	Given: $\mathbf{Y} \mid d_i \in D$	
All <i>D</i> 's that are <i>A</i> , are necessarily <i>B</i> by necessity – whereby " $\mathcal{L}^{\{1\}}(z)^{\mathbf{Y}}$ " stands for an element of <i>D</i> , who is an <i>A</i> , chosen by the adversary Y .	Y states $A(d_i)$, whereby s(h)e chooses d_i as being one of those components of z in D , that are A .	X states that $b(d_i)$ witnesses $B(d_i)$.

We have left by side two issues, namely

- the distinction between modalities that admit simple conversion and those that not, and
- the temporal dimension.

We will deal with simple conversion for both necessary and necessary contingent propositions in a separate section. In relation to the temporal dimension, in the context of Suhrawardī logic, its explicit occurrence is much more relevant for the contingent. Indeed, since according to Suhrawardī temporality is a *condition*, it only involves the contingent: necessarily necessity is always actual in a double sense: it attributes actualised presences of the predicate to actualised presences of the subject. Or, to use Ibn Sīnā's words, the predicate is true all the time that the subject is what it is. Thus, if we assume that presences of the subject can be produced, temporal parameters can be omitted.

Are there no other iterated modalities then? Actually, no. Indeed, in the epistemological framework set by Suhrawardī, only universals constituted by necessarily necessary relations and universals constituted by necessarily contingent relations provide certain knowledge. The first group provides necessary (actual) properties of the subject, the second necessary potentialities of the Subject. In fact, it seems that when Suhrawardī writes a necessity with no second modality under its scope it is an expression of an implicit second necessity. Bare contingency has, to put it in Suhrawardī words, no scientific value.

III. 2 The Necessarily Contingent Relation

III.2.1 Time and the Necessarily Contingent

Suhrawardī's concept of the contingent is based on two main Aristotelian principles concerning change and time. Firstly, time is a logical presupposition of the contingent. This means that if a pair of propositions express two incompatible but contingent attributes of the presence of the same substance – e.g. *Zayd walking* and *Zayd sitting*, their truth must be temporally relativized to avoid contradiction. Secondly, the experience of the contingent is an epistemological presupposition of time. Time can only be perceived by observing how a substance changes from actualising one contingent attribute to another.

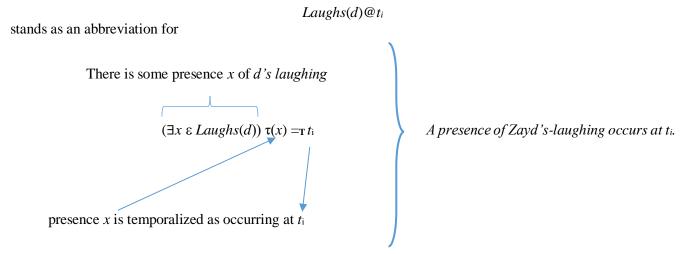
In other words, Suhrawardī's notion of contingency is inherently linked to the temporal dimension. It is evident from Suhrawardī's texts that temporal conditions are neither understood as propositional connectives nor as arguments of a propositional function defined over time moments. The latter would make time a substance, which contradicts well-known metaphysical views on time in ancient and medieval philosophy. Indeed, both approaches

seem to consider moments as bearers of events. Therefore, this analysis may suggest the counterintuitive view that events are properties of moments. In addition, the use of Priorian operators can present difficulties in expressing anaphoric time references in the object-language. For instance, to use a version of an example of Suhrawardī *During his life it happens that Zayd sometimes laughs* – i.e. during the time Zayd is what it is, namely human, it happens that he sometimes laughs – requires a substantial expansion of the object-language of standard Priorian temporal logic.

Suhrawardī's epistemology of presence provides an alternative approach in which temporal conditions contextualise the actualisation of events and actions. This contextualisation can be explicitly stated to *enrich* the content of an assertion, rather than to complete the meaning of a propositional function.³⁶

One way to render this formally, is the following: if "T(ime)" stands for some set of moments, and "@" for a monadic operator that enriches a proposition with elements of T we obtain expressions such as $B(d)@t_i$ whereby "@ t_i " functions an adverb of Time. So, if "d" stands for "Zayd", and "B" for "laughs" and "B(d)" stands for "Zayd laughs", the " $B(d)@t_i$ " expresses the temporally enriched statement "Zayd laughs at (moment/period) t_i ".

In fact, $B(d)@t_i$ encodes the outcome of the *timing* function τ which associates **presences** of B(d) with instants of time: the function τ *temporalizes* presences of laughing. More precisely,



In short, based on our analysis of Suhrawardī's epistemology of presence, we suggest that the statement *Zayd laughs* is temporally enriched by indicating a presence of Zayd's laughter at a specific time.

To apply this analysis within Suhrawardī's modal framework, it is important to note that temporality is only explicitly invoked when a witness for a contingency claim is required. The tables below follow this policy, with the first table displaying the informal formulation and the second displaying the formal one. It is worth noting that the defence of the statement *If there is an instance of the subject which is B, then there must be some instance where its absence is witnessed* – i.e., (Some_DA) $B(d_i) \supset \subset \neg B$ – can be split into two versions. Either the instance *d* of the subject is the same as the instance d_i , and then $\neg B$ actualises at a different moment than when *B* is actualised. Or *d* is different from d_i , and so the moment t_j can be different or equal to t_i . These two possibilities correspond to the division between dispositions and acquired capacities.

Statement	Challenge	Defence
Dynamic Encoding	Dynamic Encoding	Dynamic Encoding
$\mathbf{X} ! (Every_D A) \mathbf{L} \mathbf{M} B$	$\mathbf{Y} \ ! A(d_i)$	\mathbf{X} ! (Some _D A) $B(d_i) \supset \subset \sim B$

³⁶ We owe the expression *enrichment* to F. Recanati, "It Is Raining (Somewhere)", *Linguistics and Philosophy* 30(1) (2007): 123–146.

		X states that d_i witnesses <i>B</i> iff some <i>d</i> (in <i>D</i> which is an <i>A</i>) witnesses its absence.
X ! (Some _D A) $B(d_i) \supset \subset \sim B$	Y Who is that <i>D</i> A?	$\mathbf{X} ! B(d_{\mathbf{i}}) @t_{\mathbf{i}} \supset \subset \sim B(d_{\mathbf{i}}) @t_{\mathbf{j}}$
		Whereby $t_i \neq T t_j$
		Or
		$\mathbf{X} ! B(d_{\mathbf{i}}) @ t_{\mathbf{i}} \supset \subset \sim B(d_{\mathbf{j}}) @ t_{\mathbf{j}}$
		Whereby $d_{j} \neq d_{i} \in DA$

Challenge	Defence
Explicit Dialectical Encoding	Explicit Dialectical Encoding
$\mathbf{Y} ! A(d_{\mathbf{i}})$ $\mathcal{L}^{\{\}}(z) = d_{\mathbf{i}} \in D$	$\mathbf{X} ! b(d_{\mathbf{i}}): (\exists y \in DA) B(d_{\mathbf{i}}) \supset \subset \sim B(y)$
Y states $A(d_i)$, by choosing a d_i which is one of those left components of z , that that are A in D .	X states that d_i witnesses <i>B</i> iff some d_j , chosen by X , witnesses its absence
Y ?∃	$\mathbf{X} ! b(d_{\mathbf{i}}) \in B(d_{\mathbf{i}}) @ \mathbf{t}_{\mathbf{i}} \supset \subset \sim B(d_{\mathbf{i}}) @ \mathbf{t}_{\mathbf{j}}$
	Whereby $\mathcal{L}^{\exists}(y) = d_i \in DA$ and $t_i \neq T t_j$
Y: who is that <i>y</i> ? .	X states that d_i is the y who also witnesses the absence of <i>B</i> , but, of course, at a time different to the one when it witnesses the presence of <i>B</i> .
	Or
	$\mathbf{X} ! b(d_{\mathbf{i}}) \in B(d_{\mathbf{i}}) @ \mathbf{t}_{\mathbf{i}} \supset \subset \mathbf{\sim} B(d_{\mathbf{j}}) @ \mathbf{t}_{\mathbf{j}}$
	Whereby $d_{j} \neq d_{i} \in DA$
	X states that d_j (different to d_i) is the y who witnesses the absence of <i>B</i> . The time of the latter might or not be the same as the one of the former.
	Explicit Dialectical Encoding $\mathbf{Y} : A(d_i)$ $\mathcal{L}^{\{1\}}(z) = d_i \in D$ \mathbf{Y} states $A(d_i)$, by choosing a d_i which is one of those left components of z , that that are A in D . \mathbf{Y} ? \mathbf{Y} ?

III. 2.2 On Weak Plenitude

As previously mentioned, when examining Suhrawardī's examples of the necessary contingent, it appears that he believes there are potentialities that may never actualise, and others that actualise at least once. However, he does not explicitly classify contingencies that enjoy strong Plenitude from those that enjoy a weaker form of it.

However, it looks as if weak Plenitude is more applicable to acquired capacities rather than "natural" ones. For instance, it is implausible to claim that one will never laugh, but it is sensible to claim that no human will ever master 150 languages.

Additionally, when focusing on weak Plenitude in a dialectical context, it can be viewed in two ways: as involving *hypothetical Plenitude* or as *counterfactual Plenitude*. Both shall be discussed below.

III. 2.2.1 Hypothetical Plenitude

In a dialogical setting, there is rule, called by M. Marion and H. Rückert *Socratic Rule* which dictates that certain propositions, referred to as *unanalysable* constituents of the thesis, cannot be stated by the proponent unless the opponent has already stated them.³⁷ These Opponent's unanalysables cannot be challenged. When **P** makes a proposition, he will justify it with the indication "you_n", which expresses that the statement is supported by **O**'s own endorsement of the same statement at move n.

The Socratic rule is based on the idea that proving the conclusion of a syllogism within a dialectical framework involves analysing the premises in a way that results in statements that make up the conclusion. As **O** states the premises and **P** states the conclusion, only the latter is responsible for justifying the conclusion by supporting each of its components with evidence from **O**'s statements that involve the components of the premises.³⁸

This rule is central to the dialectical conception of logic, as seen in Aristotle's *Topics*. It implements the no-counterexample understanding of quantifiers discussed earlier

In standard dialogues, elementary propositions are unanalysable constituents. Suhrawardī's logic requires to include among the category of unanalysables negations of elementary propositions and expressions for unactualized capacities and. This is because the former is necessary to handle negated elementary propositions as affirmative ones and the latter is integral to his understanding of necessary contingent propositions if they enjoy weak plenitude.

In short, if we suppose that the necessarily contingent predicate *B* which occurs in a statement such as **P** ! (Every *DA*)**LMB**, enjoys *hypothetical weak Plenitude*, and the defence of it compels to the proponent to state the bi-implication *B*(*d*_i)@*t*_i ⊃⊂ ~*B*(*d*_j)@*t*_j, then this form of *Plenitude* states that **P**'s bi-implication cannot be challenged if **O** has previously stated it.

Both interlocutors agree that B may actualise if $\sim B$ does too, without committing to the actualisation of either. This is a purely hypothetical statement on the realizability of B.

III. 2.2.2 Counterfactual Plenitude: The Now and the Flow of Time

In the precedent sections we tacitly assumed that the pair witnessing the contingency, occurs at moments occurring on actual or 'real' course of events, even so in the case of what we called hypothetical Plenitude. The main assumption is here that time is linear.

If, on the other hand, we consider that Ibn $S\bar{n}\bar{a}$'s style of counterfactual Plenitude might also be at work in Suhraward \bar{n} 's notion of contingency, this amounts to associating with the meaning of contingency alternative courses of events or histories following the moment of utterance, one of which will materialise. If in the

³⁷ Cf. M. Marion and H. Rückert, "Aristotle on universal quantification: a study from the perspective of game semantics", *History and Philosophy of Logic* 37 (2016): 201–29. See too M. Crubellier, Z. McConaughey et al. "Dialectic, The Dictum de Omni and Ecthesis", *History and Philosophy of Logic* 40, No. 3 (2019): 207-233.

³⁸ In essence, the Socratic rule enables the definition of a winning strategy for **P**, which is the dialectical method of proving validity. This strategy involves a sequence of moves that compel **O** to identify the constituents of the premises that justify the conclusion. **O**'s role is to assist in testing the strength of the thesis by accepting the premises and asking **P** to demonstrate, under this assumption, the conclusion. If the components of **O**'s statements were false, **P** would win immediately and the dialogue would stop. However, the thesis has not been tested under the strongest possible conditions.

materialised history $B(d_i)$ actualises, there is another possible course of events that has not actually materialised, but on which $\sim B(d_i)$ actualises instead, and the other way around.³⁹

Furthermore, it seems reasonable to assert that a witness of the subject term has a potentiality or capacity if it is not actualised now, but will be, rather than if it has been actualised in the past. Perhaps it does not matter for Suhrawardī's general epistemological aims whether the potentiality has been lost at the time of utterance, so that a past actualisation is sufficient for the attribution of contingency. However, we will assume that potentiality is open to the future, although, as far as we can see, nothing depends on such an assumption.

Be that as it may, what is important is to consider a time of utterance t_0 , the "now", in relation to which the direction of the flow of time is defined. In fact, the time of utterance not only shapes the dialectical interaction of claims of contingency, but it is also constitutive of Suhrawardī's view of time, when the present moment of now is introduced as a mental (imaginary) reference point to determine before and after: ⁴⁰

"Before" and "after" are so considered in relation to the instantaneous moment of imagination, and time is that which is around it. *al-Ishrāq* (Provo: Brigham young UP, 1999), section 185, 120.

This yields the following

• Given histories h_i and h_j crossing t_0 , $B(d_i)$, actualises at some moment $t_i > t_0$ on the **real** history h_i , iff $\sim B(d_i)$ actualises at a '**twin**' moment $t_{i^*} > t_0$ on the alternative course of events h_j .

In short, $B(d_i)@t_{i>0}/h_i \supset \sim B(d_i)@t_{i*>0}/h_j$.

The square in the expression h_i indicates this history is the *actual course of events*, and 't₀' indicates the reference point *now*. Of course, the setting also admits the variant in which the negative occurs on the real history.

COUNTERFACTUAL PLENITUDE			
Statement	Challenge	Defence	
$\mathbf{X} ! B(d_{\mathbf{i}}) @ t_{\mathbf{i} > 0} / \overline{h_{\mathbf{i}}} \supset \subset \sim B(d_{\mathbf{i}}) @ t_{\mathbf{i}^* > 0} / h_{\mathbf{j}}$	$\mathbf{Y} \mid B(d_{\mathbf{i}}) @ t_{\mathbf{i} > 0} / \boxed{h_{\mathbf{i}}}$	$\mathbf{X} ! \sim B(d_i) @t_{i^*>0}/h_j$	
The setting also admits the variant in which the negative occurs on the real	Or		
history.	$\mathbf{Y} ! \sim B(d_i) @t_{i^*>0}/h_j$	$\mathbf{X} \mid B(d_{\mathrm{i}}) @t_{\mathrm{i}>0} / h_{\mathrm{i}}$	
	Y challenges the bi-implication by either stating the left or the right of it.	X defends by stating the right if Y states the left and vice versa	

III.3. Simple Convertibility

In order to implement convertibility in the dialectical framework, we will index the notation for modalities in the following way:

³⁹ We deliberately avoid speaking of possible worlds, which we think is quite alien to the notion of modality in Aristotle and some influential Arab post-classical Islamic thinkers, critics of Ibn Sīnā such as Suhrawardī. In fact, the temporal unbundling of the notion of modality in Aristotle, known as the *frequency* or *statistical* interpretation of modalities, is quite well documented – see F. Griffel, *Al-Ghazali's Philosophical Theology* (Oxford: OUP, 2009), 162-172. According to this view, the possibilities represented in a tree of branching histories should be thought of as the real possibilities of our (one and only) world. Our world contains real possibilities for what might be and for what might have been. The basic idea is that the possibilities are of this world rather than other-worldly in the Lewisian sense - cf. N. Belnap and M. Perloff, "Future Contingents and the Battle Tomorrow," *Review of Metaphysics* 64, 3 (2011): 588.

⁴⁰ Because of the flow of time, fixing such a moment a static reference point is an abstract product of the mind.

	Admits Simple Conversion	Does Not Admit Simple Conversion
Necessity	L⇔	L.⇔
Contingency	M⇔	M~⇔

The dialectical meaning explanation will thus allow further challenges, namely

1 requesting to specify the kind of necessity or contingency involved.

2 requesting to show the application or not of simple convertibility

Statement	Challenge 1	Defence1	Challenge2	Defence2
$\mathbf{X} ! (\forall_D A) \mathbf{L} \mathbf{L} B$	$Y ? L^{\Leftrightarrow} \mid L^{\sim \Leftrightarrow}$	$\mathbf{X} ! (\forall_D A) \mathbf{L} \mathbf{L} \Leftrightarrow B$	Y ?⇔	$\mathbf{X} ! (\forall_D B) \mathbf{LL} A$
	Does it admit simple conversion or not?	It admits simple conversion.	Execute the conversion	
		$\mathbf{X} ! (\forall_D A) \mathbf{L} \mathbf{L} \stackrel{\sim}{\leftrightarrow} B$	Y ?~⇔	$\mathbf{X} ! (\exists_D B) \mathbf{L} \mathbf{L} \sim A)$
		It doesn't admit simple conversion.	Show the non- convertibility.	There is at least one <i>B</i> for which <i>A</i> is impossible
$\mathbf{X} ! (\forall_D A) \mathbf{L} \mathbf{M} B$	Y?M⇔∣M~⇔	X ! ($\forall_D A$) LM ⇔ <i>B</i>	Y ?⇔	$\mathbf{X} ! (\forall \mathbf{L} \mathbf{M}_{D} B) \mathbf{L} \mathbf{L} A$
	Does it admit simple conversion or not?	It admits simple conversion.	Execute the conversion	
		$\mathbf{X} ! (\forall_{D} A) \mathbf{L} \mathbf{M} \stackrel{\sim}{\leftrightarrow} B$	Y ?~⇔	$\mathbf{X} ! (\exists \mathbf{L} \mathbf{M}_{D} B) \mathbf{L} \mathbf{L} \sim A$
		It doesn't admit simple conversion	Show the non- convertibility.	There is at least one necessarily contingently B for which A is impossible

IV Syllogism: Dialectical Meaning Explanations at Work

The reconstruction of Suhrawardī's dialectic meaning explanations discussed above renders his own ways to present syllogistic proofs (though his proofs are often quite sketchy). Because of space restrictions on an already too long paper, we will restrict our discussion to the first figure.

On *Manțiq al-talwīhāt* there is a development that indicates how the meaning explanations are to be deployed in a proof. The text involves the controversial *Barbara* mood where the minor premise contains a possibility modality, the major a categorical universal and the conclusion a possibility (usually notated as **XMM** in the Aristotelian notation and **MXM** in the Islamicate one). Moreover, Suhrawardī's argument already prefigures his main *illuminationist rule* for the first figure set in *al-Ishrāq* (Provo: Brigham young UP, 1999), 22-23, whereby **XMM** is reduced to **MMM**):

Know that the conclusion in first-figure syllogisms follows the major in the mixed-premise syllogisms, except when the minor is possible and the major is existential. If we say 'Possibly every J is B' and 'Actually (*bi'l-wujūd*) every B is A', it is known from the nature of possibility that it may never actually occur; so if the J is never described as B, it does not follow that the A comes to it actually, but only potentially, so it's possible. *Manțiq al-talwīhāt* (Tehran: A.A. Fayyād, 1955), 35-36, translated by T. Street.⁴¹

⁴¹ T. Street, "Suhrawardī on Modal Syllogism", in *Islamic Thought in the Middle Ages Studies in Text, Transmission and Translation, in Honour of Hans Daiber*, ed. by A. Akasoy and W. Raven (Leiden and Boston: Brill, 2008), 170.

The point seems to be that if the universal of the major premise *Every B is A* is read as asserting that actual instances of *A* are predicated of actual instances of *B*, then if the conclusion should follow the major premise, then then the major term in the conclusion should be an actualisation of *A*. However, the instances of the middle term *B*, we obtain from the first premise *Possibly every J is B* – i.e. *Every J is possibly B*, might not an actualised *B*, thus the middle term, if univocally understood in both premises, should be thought as standing for a possibility. In such a case, Suhrawardī concludes, *A* can only be predicated potentially. This line of thought leads Suhrawardī to assume that in a proof of such a kind of syllogism the middle and major terms are not actualised potentialities (in both premises) – which in fact amounts to *al-Ishrāq*'s rule for the first figure whereby a term occurring in a syllogism of the first figure is required to have the same modality/non-modality in all the places it occurs. The development of the proof sketched in the text assumes in fact that the categorical universal of the second premise, includes non-actualised capacities in the subject and the predicate – this coincides with our previous remark that the terms involved in a categorical can be read as admitting modalities.

If we present Suhrawardī's argument, sketched in the text quoted above, as a dialogue with a proponent \mathbf{P} and an opponent \mathbf{O} ,⁴² and we assume and focus on an instance of the subject that bears a contingency at one moment and not at another,⁴³ and we also assume that the contingency enjoys hypothetical Plenitude, we get the following:

- 0. **P** ! I can show that every element of domain of discourse D, who is a J is possibly A follows from the premises: I), Every element of D who is J, is possibly B and II) Every element of D who is B is A.
- 1. **O** ! Fine, I give you the premises. Show me now, that the consequent of the conclusion follows from some d_i in the domain of discourse D who is $J : \mathbf{O}$ challenges the conclusion with the move $\mathbf{O} ! J(d_i)$.
- 2. **P** ! What I will do is to show you that the endorsement of the premises will force you to assert this consequent. Let us start with the first premise. Since the first premise states that every element of the domain of discourse who is *J* is possible *B*, and you just chose d_i in the domain of discourse with your first move (sic move 1), that *B* is possible should also hold of d_i , right?: **P** challenges the first premise with the move **P** *you*₁ *J*(d_i); whereby "*you*₁" stands for the indication "you just stated the same at move 1).
- 3 . **O** ! Indeed, I have to assume that d_i is one of those in the domain of discourse that are possibly *B*. However, notice that I neither state herewith that *B* is actualised in d_i , nor do I state that *B* is never actualised. **O** defends the first premise with the move **O** ! $B(d_i)@t_i \supset \subset \sim B(d_i)@t_j$ we skip the steps leading to this answer, whereby the Opponent took the choice to focus on the same individual **O** ! ($\exists y \in DA$) $B(d_i) \supset \subset \sim B(y)$, **P** ? \exists .
- 4. **P** ! Ok, however, premise II states that every element of the domain of discourse who is *B*, is *A*, this must include all those elements in *D* who are non-actualised instances of *B*, so let us again take precisely this d_i that you just conceded with your move 3 as being a non-actualised possible *B*. This possible *B*, must be a possible *A*, right?: **P** challenges the second premise with the move **P** ! you₃ $\varepsilon B(d_i)@t_i \supset \subset \sim B(d_i)@t_j$.
- 5. **O** ! Right. This element of the domain *D* must be possibly *A*: **O** defends the second premise with the move **O** ! $A(d_i)@t_i \supset \subset A(d_i)@t_j$.
- 6. **P** ! But this is exactly what you asked me to show. You just conceded it with your move 5: **P** defends the conclusion with the move **P** $you_5 \in A(d_i)@t_i^* \supset \subset A(d_i)@t_j^*$.

This development is based on the following rules of the already mentioned framework of *Immanent Reasoning*, adapted to syllogism and presented in a simplified form:⁴⁴

1. Starting rule

The player who states the conclusion move 0 is the proponent \mathbf{P} . He states the thesis.

In the context of syllogism, the thesis amounts to the proponent committing to the conclusion provided that the opponent commits to the initial premises.

- 2. Development rule
 - Once the starting rule has been implemented, each player in turn plays a move according to the dialectical meaning explanations for quantifiers, connective, modalities and the other structural moves.
- 3. Socratic rule

 ⁴² The informal presentation below should be sufficient to follow the development of a dialogue. The reader might also consult N. Clerbout and Z. McConaughey's "Dialogical Logic" in the *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/logic-dialogical/.
⁴³ Of course, we read the example of *Manțiq al-talwīḥāt* through the dialectical meaning explanations of the modalities of *al-ishrāq*. This can be seen as highlighting the refinements that Suhrawardī's logic has undergone in developing his *Philosophy of Illuminism*.

⁴⁴ Cf. Z. McConaughey, *Aristotle, Science and the Dialectician's Activity*, chapter 4.2, table 4.9 (Lille: PhD. Thesis, 2021). See too M. Crubellier, Z. McConaughey et al. "Dialectic, The Dictum de Omni and Ecthesis", *History and Philosophy of Logic* 40, No. 3 (2019): 207-233.

Some specific propositions, we call them *unanalysable constituents*, may not be stated by P, unless O stated them before. O can state such propositions when required. When P states such a proposition, he will justify it with the indication *youn*, which indicates that his statement is backed by O's endorsement of it at move *n*, and that he (P) adheres himself to the knowledge conveyed by O's endorsement. In the context of Suhrawardī's logic unanalysable propositions include positive and negative literals (i.e. elementary propositions with and without negation). In order to render *hypothetical Plenitude*, expressions of not actualised capacities such as *B*(*d*_i)@*t*₁ ⊂ ~*B*(*d*_j)@*t*_j will also be treated as unanalysable. **4. Pragmatic coherence rule** (concerns mainly the third figure) When the conclusion Proponent defends is particular and all the premises Opponent defends are universal, Proponent may request the Opponent to instantiate the subject of a premise with the instance *d*_i, chosen by **P**, **provided** *d*_i is **new**: challenge: **P** ?*J*(*d*_i) (for a universal with {*x* ∈ *D* | *J*(*x*)} as subject, and *J* as Subject-Term), (this prevents **O** to state *J*(*d*_i) when he endorsed before some *J**(*d*_i) whereby *J* and *J** are incompatible). **5. Ending rule**The player who states ⊥ give-up, immediately loses. Otherwise, the player who has no available move left at this turn loses.⁴⁵

Let us now present Suhrawardī's argument in the form of a dialogue that implements these rules, focusing on his own example:

All humans are necessarily contingently literate All literate beings are walkers Therefore, all humans are necessarily contingently walkers

If we place a syllogism within a dialogue, then, the idea is that the proponent \mathbf{P} , claims that the conclusion holds if the opponent \mathbf{O} , concedes to state the premises. This yields the notation:

O ! (Every _D J) LM B O ! (Every _D B)A	
P ! (Every <i>DJ</i>) LM A	$\mathbf{P} : (\forall z \in \{x \in D \mid J(x)\}) \mathbf{LMA}(\mathcal{L}^{\{\}}(z)^{\mathbf{O}})^{46}$

Remember that explicit dialectical notation also indicates which player is responsible for substituting the variables. As the opponent states the premises, and they are universals, it is the proponent who will choose the subject for which the predicate is to be stated. The dual is the case of the universal in the conclusion. Since the conclusion is stated by the Proponent, the Opponent will choose the subject of which the predicate is to be stated. Following the Socratic rule, a winning strategy for the Proponent should be to let the Opponent choose first and then copy their choice for their own challenges to the premises. Notice that \mathbf{O} is compelled to choose if he challenges a universal (stated by \mathbf{P}) or defends an existential stated by himself.

N.B. In order to keep close to the text we skip in most of the dialogues the steps $\mathbf{X} (\exists y:A_D) B(d_i) \supset \subset \sim B(y)$, $\mathbf{Y} :=$. Moreover, we will assume that \mathbf{X} 's answer, focuses on individual Plenitude. The resulting variants, triggered by the option of introducing a second individual are pretty straightforward.

	-	Р
	0	
I ! $(\forall z \in \{x \in D \mid J)$	$f(x)$ }) LMB ($\mathcal{L}^{\{\}}(z)$)	! $(\forall z \in \{x \in D \mid J(x)\})$ LMA $(\mathcal{L}^{\{\}}(z)) = 0$

⁴⁵ Two Comments on the Rules.

i) The rationale behind the pragmatic coherence rule concerns the way to deal with ontological assumptions such as those required by *Darapti*.

ii) The prescription on giving up in the ending rule concerns the dialogical interpretation of negation. When challenging a negation such as $\sim A$ stated by player **X**, the challenger **Y** must now overtake the burden of the proof and state *A*. The defender of the negation has two options, either counterattack *A*, or simply give up and concede. The latter is indicated by the move **X** ! \perp give-up. In the Aristotelian texts the move \perp give-up, corresponds to the dialectical use of the term $\dot{\alpha}\delta\dot{\nu}\alpha\tau\sigma\nu$.

⁴⁶ If we deploy the formalization $\mathbf{L}(\forall x (J(x) \supset \mathbf{M}B(x))), \mathbf{L}(\forall x (B(x) \supset \mathbf{M}A(x))) \vdash \mathbf{L}(\forall x (J(x) \supset \mathbf{M}A(x)))$ and assume contemporary modal logic *K*: $\mathbf{L}(A \supset B) \vdash \mathbf{M}A \supset \mathbf{M}B$ will be required which assumes one-sided possibility – see Z. Movahed, "Suhrawardī's Modal Syllogisms", *Sophia Perennis* 2, no.4, Serial Number 21 (2012): 9. But this contravenes Suhrawardī's notion of contingency, whereby a contingency is neither necessary nor impossible.

				Proponent wins	
5	$! A(d_i) @t_i] \supset \subset \neg A(d_i) @t_j]$?II	$! you_3 \in B(d_i)@t_i \supset \subset \sim B(d_i)@t_j$	4
3	$! B(d_i)@t_i \supset \subset \sim B(d_i)@t_i$?I	$\begin{array}{l} \begin{array}{c} J(d_{i}) \\ I \ \mathcal{L}^{\{\}}(z) = d_{i} \in D \end{array}$	2
1	$\begin{array}{c} ! J(d_{i}) \\ ! \mathcal{L}^{\{\}}(z) = d_{i} \in D \end{array}$?0		$! yous \in A(d_i)@t_i' \supset \subset \neg A(d_i)@t_j'$	6
II	$! (\forall z \in \{x \in D \mid B(x)\}) A(\mathcal{L}^{\{}(z))$				

- With move 4 P makes use of the rule that middle term must have the same modality in both premises. That is *every B*, is understood as including also contingent cases of *B*.
- The dialogue ends since 6 is an unanalysable constituent of the conclusion, namely the not actualised capacity $A(d_i)@t_i \supset \subset \sim A(d_j)@t_j$, that cannot be challenged, since **O** himself endorsed it with move 5.
- As mentioned above, if we prefer not to include not actualised capacities among the *unanalysables* and further analyse the biimplication the end-result of the dialogue will not change. It will only a bit longer: as soon as **O** challenges the bi-implication, **P** will do the same. However, this does not seem to be the way Suhrawardī develops syllogisms involving capacities.

	0			Р		
Ι	! (Every _D J)LMB			! (Every _D J) LMA		0
Π	$! (Every_D B)A$					
1	! <i>J</i> (<i>d</i> i)	?0		! yous: $A(d_i)@t_i' \supset \subset \sim A(d_i)@t_j'$	6	
3	$! B(d_i)@t_i \supset \subset \sim B(d_i)@t_j$?I	$\frac{1}{you_1: J(d_i)}$	2	
5	$! A(d_i) @t_i' \supset \subset \neg A(d_i) @t_j'$?II	$! you_3: B(d_i)@t_i \supset \subset \sim B(d_i)@t_j$	4	
				Proponent wins		

P can repeat the same sequence of moves for any arbitrary element of D **O** happens to choose to challenge the universal in the conclusion. In other words, **P** has a winning strategy for this syllogism, and therefore it is valid. The strategy can be seen as a "recapitulation" and generalization that produces an algorithm for winning.⁴⁷ In our case, informally the winning strategy amounts to the following:

- 1) Let **O** choose any arbitrary instance of the universal in the conclusion
- 2) **P** should use exactly this instance, *whichever* this instance chosen by **O** is, to challenge the first premise, and force **O** to predicate *B* of it
- 3) **O** predicates *B* of it, but as a not actualised capacity
- 4) **P** should use exactly this endorsement of **O** (that the non-actualised capacity *B* can be predicated of the instance at stake), to challenge the second premise
- 5) **O** is forced to predicate A of it, but again chooses to endorse it as a not-actualised capacity
- 6) **P** can now use this last endorsement to respond to the challenge to the conclusion
- Apply this sequence for any *d*_i chosen by O at move 1

The winning strategy can be represented as a sequent calculus, where **P**'s assertions are translated as assertions on the right of the turn-style and **O**'s assertions on the left. In fact, the sequent calculus is generated by the winning strategy produced by the dialogue. The interaction between players within a winning strategy puts the dialogical meaning explanation of the modalities at work by fleshing out the meaning of each constituent.

Once more, in the text of the *Manțiq al-talwīḥāt* (Tehran: A.A. Fayyād, 1955), 35-36, quoted above, the subject and the predicate of the major premise (here premise II), are both modalized de facto. Or to put it otherwise the categorical universal include actual and merely possible instances of the subject.

On Suhrawardī's view, the middle and major term of a productive syllogism of the first figure must share the same modality in both premises and conclusion. Suhrawardī explicitly formulates this as a rule in al-Ishrāq (Provo: Brigham young UP, 1999), 22-23, and provides two examples: when the major term relates to its subject by necessity and when the major term relates to its subject by contingency. In his discussion of **MXM** (or **XMM** in

⁴⁷ For the notion of recapitulation in the context of syllogism see E. Kapp, *Greek Foundations of Traditional Logic* (N. York: Columbia University, 1942), 14-16 and 71; M. Crubellier, "Du *sullogismos* au syllogisme", *Revue philosophique*, n° 1 (2011); and Z. McConaughey, *Aristotle, Science and the Dialectician's Activity*, (Lille: PhD. Thesis, 2021).

the Aristotelian notation) in *Manțiq al-talwiḥāt* (Tehran: A.A. Fayyād, 1955), 35-36, the author makes explicit the contingent modality assumed.:

There is no need to multiply the moods of syllogism, rejecting some and accepting others. Further, since the last term leads to the first term by means of the middle, the modes in the definitely necessary proposition are made part of the predicate in one or both of the premises, thus leading to the major. For example, 'All men. are necessarily contingently literate, and all contingently literate beings are necessarily animals by necessity (or contingently walkers), therefore, all men are necessarily animals by necessity (or contingently walkers), therefore, all men are necessarily animals by necessity (or contingently walkers), al-*Ishrāq* (Provo: Brigham young UP, 1999), 22-23.

In the first example, the major term is necessarily necessary, while in the second example, the major term is necessarily contingent. In both examples, the middle term is necessarily contingent, but the modalities of the terms are the same wherever they occur.

Let us lay down the notation for both examples and leave the developments for the diligent reader.

Necessarily Predication of the Subject in the Major

All humans are necessarily contingently literate All (necessarily) contingent literate beings are necessarily animals, by necessity Therefore, all humans are necessarily animals, by necessity

O ! (Every _D J)LMB	O ! $(\forall z \in \{x \in D \mid J(x)\})$ LM $B(\mathcal{L}^{\{\}}(z)^{\mathbf{P}})$		
O ! (EveryLM _D B)LLA	O ! $(\forall z \in \{x \in D \mid \mathbf{LM}B(x)\})$ LL $A(\mathcal{L}^{\{\}}(z)^{\mathbf{P}})$		
P ! (Every _D J) LL A	$\mathbf{P} ! (\forall z \in \{x \in D \mid J(x)\}) \mathbf{LLA}(\mathcal{L}^{\{\}}(z)^{0})$		

Necessarily Contingent Predication of the Subject in the Major

All humans are necessarily contingently literate All (necessarily) contingent literate beings are necessarily contingently walkers Therefore, all humans are contingently walkers by necessity

O ! (Every _D J)LMB O ! (EveryLM _D B)LMA	$ \begin{array}{l} \mathbf{O} ! (\forall z \ \varepsilon \ \{x \ \varepsilon \ D \mid J(x)\}) \ \mathbf{LMB}(\mathcal{L}^{\{\}}(z)^{\mathbf{P}}) \\ \mathbf{O} ! (\forall z \ \varepsilon \ \{x \ \varepsilon \ D \mid \mathbf{LMB}(x)\}) \ \mathbf{LMA}(\mathcal{L}^{\{\}}(z)^{\mathbf{P}}) \end{array} $
P ! (Every <i>DJ</i>) LM A	$\mathbf{P} ! (\forall z \in \{x \in D \mid J(x)\}) \mathbf{LMA}(\mathcal{L}^{\{\}}(z)^{0})$

Negative moods of the first figure are not a problem if we follow Suhrawardī's own formulation where he places the negation before the possibility or more precisely as an impossibility. Thus, Suhrawardī's

All humans are necessarily impossibly stones. al-Ishrāq (Provo: Brigham young UP, 1999), 23

can be encoded as

(EveryHuman_D)L~MStone

VI Conclusion

Does Suhrawardī reject the approach to knowledge of essences of the Peripatetic thinkers of his time? No doubt he does.

Is Suhrawardī's logic compatible with essentialism as claimed by Street ? Yes, definitely so.

However, the logic of illumination has some unique features resulting from the intertwining of the $bahth\bar{i}$ and $dhawq\bar{i}$, which form the basis of all of his innovations namely

- The logic of illumination presupposes that knowledge is primarily derived from the experience of presence. Specifically, it is the experience of self-awareness that leads to the mental construction of the present moment i.e. the *now*.
- Suhrawardī's modal logic is based on a meaning theory that distinguishes various dependent relations between predicate and subject terms. This theory is shaped by dialectical meaning explanations, which provide rules for constructing counterexamples.

The proposed dialectical framework for reconstructing Suhrawardī's logic distinguishes between three temporal aspects: (i) the time involved in constructing knowledge, refers to the succession of moves leading to the justification of the thesis;⁴⁸ (ii) the 'now' during which a presence is grasped by an immediate act of knowledge; and (iii) the timing encoded by the modality at work. Standard temporal quantifiers and Priorian temporal operators do not capture either the first or the latter forms of temporality.

Having said that, there is no explicit systematic development of a complete dialectical framework for his logic in the texts. The formal dialogical framework for the logic of Illumination in the present paper is of course due to our own reconstruction, it is the result of putting together the textual sources where Suhrawardī presents the meaning explanations mentioned above his own logical proofs and his own general aim of developing a philosophy rooted in both discursive and taste/intuition-based knowledge

Our main argument is that Suhrawardī's logic of modalities is shaped by a dialectic perspective on how to justify an assertion involving modalities, which aligns with his philosophy of presence in a straightforward manner. This study serves as a prelude to a deeper exploration of historic and systematic pending issues such as

• Examining Suhrawardī's views modality in the light of his take on causality, whereby 1) "contingency" means to be necessary by a cause, 2) causes are ontologically but not temporally prior to their effects, 3) causes may be composite and include conditions and removal of impediments.

• The study of the legacy of some of his main predecessors such as the works of Abū'l-Barakāt al-Baghdādī (1080-1165), particularly al-Baghdādī's *Book of Evidence* and Sahlān Sāwī's *al-Baṣā'ir*; commentaries to his work by, among others ; Ibn Kammuna, Shahrazuri, Shirazi, Ibn Rizi, al-Albhari, Allamah Hilli, al-Jurjani, Ibn Abi Jumbur Ahsa'i, al-Dawani, al-Dashtaki, Abd am-Razzaq, Mulla Sadra, Isma'il Ankaravi, al-Harawi and Hazin Lahiji.

⁴⁸ Cf. M. Ardeshir, "Brouwer's notion of intuition and theory of knowledge by presence", in *One Hundred Years of Intuitionism, 1907-2007*, ed. by M. van Atten; P. Boldini, M. Bourdeau & Gerhard Heinzmann (Basel/Boston/Berlin: Birkhauser, 2008), 115-130.

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