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Suhrawardī's Stance on Modalities and his Logic of Presence

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Abstract

Abstract. The present study, focused on Shihāb al-Dīn Suhrawardī *Ḥikmat al-Ishrāq*, develops some preliminary explorations on his remarkable *epistemology of presence*, in light of his postulate of the priority of the experience of unity by presence. Furthermore, the paper should pave the way for responding to the challenges of Tony Street and others on the compatibility of Suhrawardī's critique of Ibn Sīnā with the development of a temporal and modal syllogism that seems quite close to that of Ibn Sīnā. Suhrawardī's modalities are to be understood as the different ways a predicate relates to its subject rather than as propositional operators. Necessarily necessary modality relates unconditionally the subject-term with the predicate-term; in contrast, necessarily contingent modality relates conditionally these terms. Moreover, necessarily necessary predication either admits simple conversion (corresponds to *definition*) or not (corresponds to *genus*). Necessarily contingent predication can also be declined in admitting simple conversion (corresponds to *proprium*) and not admitting this conversion (corresponds to *accident*). Necessarily contingent predication includes acquired potentialities or capacities such as literacy; and *natural* or not-acquired, such as breathing. Whereas predicating a not-acquired capacity of an actual individual involves time as focusing in one particular individual – e.g. laughing/breathing is necessarily but contingently said of humans since there must be at least one time when laughing is present, and one when it is absent concerning **each individual**; predicating an acquired capacity amounts to asserting such a contingency with regard to the **whole genus** – e.g., predicating literacy of humans assumes that at least one individual in at least one time actualizes literacy iff there is another human who does not – there are some variants related to commitment to either strong or Weak Plenitude (only acquired capacities seem to admit Weak Plenitude). We will put this at work in Suhrawardī's own (sketchy) framework for syllogism.

Introduction

Despite the fact that it has been over 40 years since Henry Corbin and Hossein Ziai pointed out that the work of Shihāb al-Dīn Suhrawardī had not yet been systematically studied, a thorough research of his work is just at the start. This is surprising since Suhrawardī born in 549/1155 in Suhraward, Northeastern Iran and executed in 587/1191 in Aleppo, was one of the most influential thinkers after Ibn Sīnā – see Griffel (2011, p. 46) – and also since we now have reliable editions, translations and commentaries initiated by among others Corbin (1952, 1976, 1986, 2001), Ziai (1990, 2007), Walbridge & Ziai (1999), John Walbridge (2001, 2017), and continued by scholars such as Aminrazawi (1997), Street (2008), Mousavian (2014a,b), Zhang (2018).

One of the main difficulties to approaching the work of Suhrawardī, father of the School of *Illumination* known *as al-ishrāq*, is that his writings, that explicitly target in his criticisms the work of what he calls the *Peripateticians*,[1] often display deep and innovative thoughts that span epistemology, metaphysics and Sufi mysticism composed in a complex intertwined form.

This puzzled and still puzzles commentators that lead quite disparate accounts of his thought: some describe him as

- mystical thinker (as for example – Corbin (1971, 1976, 1986),
- Neo-platonist – Ziai (1990),
- anti-essentialist – Walbridge and Ziai (1999),
- combining empiricism Aristotelism, Platonism and gnosticism – Ziai (1990), Aminrazawi (1997), Walbridge (2014), Zhang (2018),
- subscribing to innate ideas – Ziai (1990), Aminrazawi (1997), Marcotte (2012), Aminrazawi (1997) – and prioritizing essence over existence – Mullā Sadrā, Ziai (1990), Wisnovsky (2005)
- prioritizing existence over essence, Wisnovsky (2005), Walbridge (2017), Zhang (2018),
- follower rather than opponent of Ibn Sīnā – Aminrazawi (1997),
- one who is neither a Cartesian innatist – Mousavian (2014a,b), Zhang (2018) – or not in all senses of innate or *fiṭrī* – Walbridge (2014), nor an anti-essentialist nor one that gives priority to either essence or existence but launches a kind phenomenological presence epistemology (*ḥuḍūr*) based on intellectual intuition that mirrors mystic intuition (*dhawq*) – Rizvi (2008) – and as a sharp opponent of Ibn Sīnā's epistemology and logic – Walbridge and Ziai (1999, pp xxiv-xxv),[2]
- dismissing the importance of the discussion on logic and its foundations – Ziai (1990, p. 69),
- giving priority to propositional over predicate logic – Walbridge (2000, p. 155),
- developing some logical notions that distinguish the logic of *Philosophy of Illumination* of that of the followers of the Peripatetic School , presented in *Intimations* – Ziai (1990, pp 41-45, 51-56 and 73-75),
- shaping a logic that constitutes basically a further, in some points even more extensive - development of particular features of the one of Ibn Sīnā – Street (2008).

In fact, as pointed out by most of the commentators it is difficult, or even fruitless, to reduce Suhrawardī's Illuminationist theory and illuminationist epistemology to existing categories as empiricism, rationalism and so on.

[1] It has often been pointed out that it is not clear who are, besides Ibn Sīnā, the philosophers Suhrawardī calls *Peripateticians*. For insightful discussion on this issue see Wisnovsky (2011). .

[2] Relevant to the discussion on the ontological status of *fiṭrī* is Zarepour's (2020) paper on synthetic a priori in Ibn Sīnā.

In the present paper we attempt to start to revisit these apparent tensions, taking as point of departure the logic discussed by Street (2008) and Movahed (2012).[3]

We will to propose a new reading of Street's (2008) Avicennean interpretation of Suhrawardī's logic and in one of the last sections of our paper we explore very briefly the possibility to extend Ardeshir's (2008) proposal to link Browerian intuition with Suhrawardī's notion of presence.

According to our view, an approach that explores the consequences of his epistemology of presence for the foundations of logic provides some perspective which might invite to a more unifying reading of Suhrawardī's fascinating thought.

We are certainly aware that this methodology that hinges on binding Suhrawardīs' logic with his Illuminationism might raise eyebrows among many respected scholars, since the discussion and systematic development of logical rules does not seem to take such a predominant place in his work. Particularly so, because Suhrawardī considers the deployment of a system of logical rules to involve "mental" objects rather than objects of epistemic acquaintance, and that the study of logical rules from a purely syntactic point of view is not in principle a constituent of philosophy or at least not a crucial one – cf. Ziai (1990, pp. 47-60). According to Suhrawardī logic requires a higher-order level perspective where predicative differentiation has taken over the more fundamental, lower level of epistemic acts of presence. It is this so to say lower level that constitutes the core of his philosophy of Illumination.

However, notice that the main philosophical issue of Suhrawardī against the Peripateticians, is precisely their definition by essences, the discussion of which Suhrawardī himself includes into the field of logic. Indeed, the criticism of the alleged explicative and epistemic power of notion of definition by genus and specific difference is part and parcel of his epistemology of presence. For sure, Suhrawardī, does not seem to be particularly interested in discussing inferential rules, however, his remarks on *ecthesis* (*iftirāḍ*) and on the epistemic priority of universal forms over particular ones are, so we claim, intimately linked to his epistemology and metaphysics. For short, our proposal is to distinguish within Suhrawardī's logic the level of *constitution of meaning* and the level one where the *inference rules* have been already gathered. Both levels, require, the notion of *presence*, upon which the inferential rules for his own Illuminationist rules for modalities are dependent.

Moreover, we think, that there are sufficient textual remarks that indicate how to bind both levels. According to our view the key to the conjugation of the two levels is the role Suhrawardī bestows to *a* dialectical interpretation of necessity that allows putting into action the unity of the act of experience of presence..

These and other considerations involving Suhrawardī's conception of time lead us to a reconstruction of his *logic of illumination*.

More precisely, on our view, the *logic of illumination* is an attempt to cast some of the features of the Avicennean framework into a logical system where the notion of presence, conceived as a personal

[3] As discussed below, Movahed's paper can be seen as following the general lines of Street's (2008) interpretation – namely the embedding of *de re* modalities in a *de dicto* necessary modality, though, in contrast to Street, Movahed concludes that this shows the difference rather than the similiarities between the modal logic of Suhrawardī and Ibn Sīnā (and the Peripateticians of that time).

experience of knowledge, leads, through higher-order processes, to new insights on central epistemological notions such as definition, necessity, contingency and plenitude – see Kaukua (2013, p. 322). These processes have been thoroughly discussed by Tianyi Zhang (2018). Our task will be to examine the logical result of these processes and provide, so we hope, additional elements to Tony Street's (and others) discussions on the compatibility of Suhrawardī's critique of Ibn Sīnā with his own *Iluminationist* take on modalities.

So far as we know, the logical analysis of Suhrawardī's modalities in the recent literature do not engage in two crucial points he explicitly makes concerning the meaning of modalities, namely

(1) modalities, are to be understood as the different ways a predicate *relates* to its subject (*al-Ishrāq* (1999, p. 16, p. 17);
(2) whereas, proving a necessarily necessary relation requires relating *actual presences* or *instances* of the subject *unconditionally* with instances of the predicate; proving a necessarily contingent relation requires relating instances of the subject *conditionally* with a capacity or potentiality (not necessarily actualized at every time for every presence of the subject), expressed by the predicate – *al-Ishrāq* (1999, p. 38).

Temporality is part and parcel of the meaning underlying Suhrawardī's notion of modality. More precisely, time measures the change from the potential state to its actuality, its duration and in the context of syllogism it applies to the presences of the terms constituting the premises. Time instants (or intervals), formally speaking , should not be understood as some sort of individual variables – this contravenes the (Aristotelian and Post-Aristotelian) view that time is not a substance. Time instants are not metalogical indexes for evaluating propositions modified by temporal operators either – such as in the standard truth-functional semantic for Prior's temporal logic.

Actually, we claim, it is the instances rather than the propositions they verify, that are *timed*: this particular action of mine, this singular act of laughing is in fact what is timed, not the type (i.e., not the proposition) *Humans laugh.* More generally, time (also in combination with other conditions) has the role to shape Suhrawardī's epistemology of presence, where knowledge is understood as knowledge by the experience of the now and here.

One final general historical but crucial point that, in our view, needs to be considered when analysing Suhrawardī's logic, is that the notion of modalities, their proof-methods and the germane notion of existence as presence delineated in *al-Ishrāq* are developed in his discourse on fallacies (*mughālaṭa*). Moreover, these proof-methods are semantical and or more accurately dialectical rather than syntactical. Indeed, they follow the so-called dialectical *no-counterexample* interpretation of the quantifiers, by the means of which the search for a counterexample is governed by rules of interaction.

This recalls not to forget that Suhrawardī's work has been developed during the period when the *logical turn of dialectics* was taking place, during which the twelfth-century-CE eastern school of Raḍī al-Dīn al-Nīsābūrī (d. 544/1149)) students, especially Rukn al-Dīn al-ʿAmīdī (d. 615/1218), and others promoted the fusion of logic and dialectic theory also in syllogism – see Walter E. Young (2021a, section 2.2). The dialectical approach constitutes in fact the man background we follow for the reconstruction of Suhrawardī's logic and theory syllogism.

I- A Note to Suhrawardī on Definition

All the different interpretations of Suhrawardī's Illuminism notwithstanding, it is clear that the criticism of the notion of definition of the Peripatetic philosophy and its alleged explicative role in the acquisition of knowledge constitutes a major motivation for the development of his epistemology of presence.[4]

We will not examine here the fascinating thoughts of Suhrawardī displayed by his criticism of the Peripatetic notion of knowledge by definition. Let us quote instead the insightful summary of Ardeshir (2008, p. 120) which is relevant for our own aims:

> In his critiques of Ibn Sīnā's theory of definition, he explains that *formal* characterization of an object is *not* sufficient to define the object, and moreover, it does not lead us to *know* it. He then argues that *to know* an object we need to found it on certain bases. These bases are what Suhrawardī called *fiṭrī* [Rahman&Seck] and they have their origin in self-awareness […]. So he believes that in the process of defining any object, some kind of *fiṭrī* [Rahman&Seck] is prior to any formal characterization. This very important observation on the theory of definitions led him to establish his general theory of knowledge. In his theory of knowledge, Suhrawardī believes that 'experience' of the self is the most valid type of knowledge. He believes that this experience is the basis
> of any philosophical reasoning.
> In a primary division, knowledge can be divided into two types, *knowledge by presence* or *immediate knowledge* (al-ilm al-ḥudūrī), or *non-representational knowledge* and *knowledge by correspondence*, or *knowledge by representation* (al-ilm al-ḥ usūlī). In 'knowledge by presence', as it stands, the object is *present* to the subject, and that is not the case for 'knowledge by correspondence'.

In fact, in this text Ardeshir translates *fiṭrī* as 'innate knowledge', though in order to avoid identifying it with Cartesian pre-given knowledge we left the original term – this identification has been pointed out by Mousavian (2014a,b) and further endorsed by Zhang (2018, p. 153).

Indeed, as briefly delineated in section V, where we further discuss some suggestions of Ardeshir (2008) *fiṭrī*, should be linked to knowledge by presence.

Actually, what is given as present to us are epistemic unities such as *Aristotle-being-a-man*, *Socrates-being-a-man,* that will be linguistically articulated by mental activity into the composites *Aristotle is a man*, *Socrates is a man. Man exists* expresses the mental result of articulated assertions (such as *Aristotle is a man),* that in their turn are also the result of extramental acts of presential knowledge

Presence provides also the primordial ground of existence and is not a being of reason. The further propositionally articulated notion of existence, called univocal (*bi-l-tawāṭu'*) – cf. see Zhang (2018, chapter 3.3.1) is not a property

Univocal Existence, is a relation that can be substituted by a copula, as in " Animality **exists(can be found)/is** in this man", "Zayd **exists (can be found)/is** in the house".

> (60) The followers of the Peripatetics argue that we can think of man without existence, but we cannot think of him without a relation to animality. Yet the relation of animality to

[4] For an insightful discussion on who is the precise target of Suhrawardī criticisms see Wisnovsky (2011).

> humanity means nothing except its being existent in him, either in the mind or in concrete reality.
>
> […]
>
> "Existence" can be said of relations to things, as when one says that something is existent in the house, in the market, in the mind, in concrete reality, in a time, or in a place. Here the word "existence" occurs with the word "in" with the same meaning *in* all of these. "Existence" may be used as a copula, as when one says, "Zayd exists writing". It may be said of the reality and essence, as when one says, "The essence of the thing and its reality, the existence of the thing, its concreteness, and its self." These are taken as beings of reason and are applied to external quiddities. This is what most people understand by "existence," but the Peripatetics give it another meaning, for they are in the habit of explaining it in their arguments, overlooking the fact that they had also assumed that it is the most evident of things, not definable by anything else. *al-Ishrāq* (1999, p. 47).

Univocal Existence does not add to the subject of assertions of essence or concreteness or identity such as *Man **is/exists (as)** a rational animal*, *Zayd **is/exists(as)** this (concrete) individual, Zayd **is/exists (as)** Zayd*, since this would trigger the further question about the existence of, say the existent Zayd . This comes very close to the celebrated dictum of Kant (*Critique of Pure Reason*, B626–27): *Existence is not a real predicate* – a predicate that some scholars ascribe to Ibn Sīnā:[5] Moreover, if we recall, as pointed by Zhang,[6] that in Arabic existence is related to finding, Kant's example that the concept of hundred thalers (to be found) in my pocket is not enlarged by thinking of them as existent; the closeness of Suhrawardī's text just quoted, to Kant's take on existence is frankly striking.

This form of existence is highly relevant for the meaning explanation of universal assertions involving necessarily necessary modalities discussed in our section III.

II Suhrawardī's Remarks on Peripatetic Logic

A brief overview of Suhrawardī's arguments against the epistemic power of definitions – where some evidently carry more weight than others – indicate that the philosophy of Illumination proposes a kind of epistemological inversion in relation to the one Suhrawardī ascribes to Ibn Sīnā and his followers: knowledge, and in particular certainty, is gathered by acts of presence that in a subsequent (mental) elaboration can be expressed by definitions, not the other way round. Particularly interesting in this context is his general rejections of the mirror image, form does not amount to passive reflection by but by illumination – that is by active revelation.[7]

> (104) Know that the form is not in the mirror, for otherwise your view of the thing would not change with your point of view. Moreover, if with your finger you touch a mirror a cubit's distance from your face, you find that the distance· between the form of your finger where it meets the glass and the form of your face is greater than the thickness of the mirror. Were the form actually in the mirror, it would be in its visible surface, since that is its polished part; but this is not so. *al-Ishrāq* (1999, p. 72).

[5] For thorough discussions on Ibn Sīnā's distinction between essence and existence see Morewedge (1972) Bartolacci (2012)

[6] In a personal email to S. Rahman.

[7] Contemporary expressionists like Robert Brandom, would be delighted with this remark of Suhrawardī, compare with the following quote of Brandom who subscribes to expressionism of German Romantic philosophers: *To the Enlightenment picture of mind as mirror, Romanticism opposed an image of the mind as lamp.Broadly cognitive activity"was to be seen not as a kind of passive reflection but as a kind of active revelation* – Brandom(2008, p.8).

> […].
>
> (145) Since you know that vision is not by the imprinting of the form. of its object in the eye nor by something emerging from the eye, it can only be by the illuminated object being opposite a sound eye-nothing more. Imagination and images in mirrors will be explained later, for they have great importance. Being opposite amounts to the. absence of a veil between that which sees and that which is seen. *al-Ishrāq* (1999, p. 96).
> […].
>
> (225) You know that forms cannot be imprinted in the eye and that, for similar reasons, they cannot be imprinted somewhere in the brain.
> The truth is that the forms in mirrors and the imaginative forms are not imprinted. […] These forms are suspended and are in neither a place nor a locus. The imaginative faculty is the locus in which the forms of the imagination are made evident and are suspended. *al-Ishrāq* (1999, p. 138).

In the next section we will attempt to suggest how this *expressionism* at work in his epistemology, can be related to some particular features of his take on logic, which constitutes the main subject of the present section.

In relation to the logical constants deployed by the Peripatetic thinkers of his time, the texts show that he has good knowledge of the standard propositional connectives (whereby his understanding of disjunction is the exclusive version of it). In relation to quantifiers and negation he stresses nuances that prefigure some features of his own epistemological views that conjugate with his Illumination theory . We will discuss these features in the next section, but let us examine two of his main tenets on term logic –cf. *al-Ishrāq* (1999, section 16, pp. 12-14).

The qualifications *general definite* and *existential indefinite* that Suhrawardī' chooses for universal an existential quantification respectively, indicate his epistemic concerns:

> We call the general definite proposition the "universal proposition." We call a proposition whose judgment is specified by "some" the "existential indefinite proposition." *al-Ishrāq* (1999, p. 14).

Since the general aims of science is to achieve certainty, it is advisable, to always to prioritize:

- judgements with a universal quantity over an existential one – *al-Ishrāq* (1999, p. 14-15)., and
- affirmative over the negative quality of judgments – *al-Ishrāq* (1999, p. 15).

> If things are done in accordance with what we are saying, then only universal propositions will remain, for the particular propositions are not investigated in the sciences. At the same time, the rules governing propositions will become fewer, clearer, and easier·. *al-Ishrāq* (1999, p. 14).

The first point amounts to avoiding *indefinite* assertions that also seem to yield priority to term logic over propositional logic – a proposition with a singular term – such a *Zayd is literate* is called a *particular proposition* – cf. *al-Ishrāq* (1999, p. 14). Actually, according to our author, particular propositions, existential propositions and universal propositions, establish an order from indefinite to definite that furnishes at the same time a degree of certainty: they determine a scale of increasing

epistemic value – at the top of the scale is Aristotle's perfect syllogism in mood (modalized) *Barbara* of the first figure – as many others Suhrawardī rejects the fourth figure – cf. *al-Ishrāq* (1999, p. 21, 22).[8]

Suhrawardī's understanding of universal quantification, his preferred form of quantification, is fine-grained and he carefully distinguished the distributive from the non-distributive use. Moreover, he clearly distinguishes between *in sensu diviso* and in *sensu composito*:[9]

> (20) Know that when we say, "All C are B", it means that each and every thing described by C is described by B. This is because when you say, "All C are B", you will understand that the concept C is a general meaning. You then make explicit that it refers to the particulars in C by saying "each and every," because the meaning is not C as a whole: You can say, "Any man can fit in a single house," but you cannot say, "All men can fit in a single house." When, for example, you see a proposition such as "All sleepers may awaken," you will realize that the meaning of "all sleepers" is not "one who sleeps qua being asleep," because in such a case it is not possible to imagine someone actually sleeping being described as awake. Rather, what is meant here is that every individual who is described as asleep may sleep and wake up. […]. Further, when you say, "All things that move necessarily change," you should know that each and every thing described as moving is not necessarily changing because of its own essence, but because it is moving. Thus, its necessity depends on a condition and *it* is contingent in itself. By "necessary," we mean only that which it has by virtue of its own essence. That which is necessary on condition of a time or state is contingent in itself. ·. *al-Ishrāq* (1999, p. 16-17).

Notice that though in this passage Suhrawardī mentions Ibn Sīnā's descriptional form *All things that move necessarily change (so long as they move),* [10]descriptional forms are not in the focus of his interest. Indeed, since Suhrawardī considers the universal necessity form, which is constituted from essences, to have the highest epistemic value, Ibn Sīnā's descriptional forms, that confine the predicate to an accident of the subject, as pointed out by Street (2008, section 3), take a marginal place, if any, in his logic. This might suggest that essences have a central part in Suhrawardī's epistemology – as pointed out by Street (p. 174). However, as we will discuss further on, universal (necessarily) necessary propositions express the knowledge of essences that has been won by experiencing that every presence (i.e. every actual instance) of the subject leads to a presence of the predicate and vice-versa. In other words, the knowledge of essences encoded by universal (necessarily) necessary propositions is result of a knowledge by presence, not its origin.

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[8] Recall that Arabic logicians changed the order of premises and placed the subject before the predicate. This makes it closer to the logic form of contemporary quantification and of course does not change the validity of the moods involved but it does not coincide with the Latin medieval denominations of the valid forms of each figure. A lucid presentation of this point has been provided by Street (2008), let us quote his excellent summary: *The first mood of the first figure is Barbara, given by Aristotle in the form: A belongs to all B (major premise), B belongs to all C (minor premise), therefore A belongs to all C (conclusion). The major premise is so-called because it provides the predicate of the conclusion, whereas the minor premise provides the subject. Arabic logicians stated Barbara differently in two respects. First, they put the subject of the premise before the predicate, and secondly, they put the minor premise before the major: every C is B, every B is A, therefore every C is A, which is just as obvious—or perfect—an inference as it is when stated in Aristotle's fashion.* […]. *The second mood, Celarent, has different vowels to show that the major premise and the conclusion are E-propositions, that is, of the form 'no C is B'. But now the order of the premises as stated in the Arabic will be out of step with the vowels in the Latin mood name: every C is B (a-proposition), no B is A (e-proposition), therefore no C is A (e-proposition). Still, we should refer to this as Celarent because we can then compare it easily to the analyses of the same inference by Aristotle and the medieval Latin authors.* Street (2008, pp. 176-177).

[9] Cf. Rahman & Zarepour (2021, pp. 416-417).

[10] Cf Street (2002 ; p. 133), where he quotes Avicenna in *Pointers and Remainders*, p. 265.

In this context, what perhaps might be seen as puzzling, is that though, according to Suhrawardī, knowledge is the primary experience of a presence, universal laws have, in his philosophy, epistemological priority over the particular and even the singular This might had led to the anti-essentialist reading of Walbridge (2000, pp. 149-150) (and others), contested by Street (2008). One possible way out of the tension is to restrict Suhrawardī's endorsement of universal laws as paradigm of knowledge to a way of presenting t Peripatetic thought. However, the epistemological priority of universal laws is clearly part of the *illuminationist epistemology*. Further on, when discussing the meaning of Suhrawardī's modalities, we will propose some way out this tension, based on the idea that scientific knowledge, is the outcome of experiencing the interdependence of one presence or actual instance of a concept over the presence of another.

The epistemological priority of universal laws brings us back to the two points mentioned above:

convert particulars and existentials into universals, and
articulate negatives as affirmative assertions with a negation on the predicate.

Let us, discuss these points separately.

- If a particular conditional proposition should constitute the premise of a syllogism, then it should be turned into a universal:

> if it is said, "If Zayd is in the sea, then he is drowning," let it be specified and thus made universal. It should then be said, "Whenever Zayd is in the sea and does not have a boat and does not know how to swim, then he is drowning." *al-Ishrāq* (1999, p. 14-15).

- Indefinite existentials should be converted into universals, by means of *ecthesis* (*iftirāḍ*).

> There is also indefiniteness in "some," for the individual things may be many. Let that "some" in a syllogism be given a name – C, for example. Thus, it can be said, "All C is such and such," and the proposition will become definite, so removing the misleading indefiniteness. The existential proposition is not useful except in certain cases of conversion and the contradictory. *al-Ishrāq* (1999, p. 14-15).

> Should the first premise be particular, then we will make it exhaustive, as mentioned before-as, for example, "Some animals are rational," and "All rational beings are capable of laughter." Let us give a name to the particular without considering the predication of rationality, though rationality accompanies the particular. Let this be D. Thus, *it* can be said, "All D are rational, and all rational beings are so-and-so," according to what we said before. Now we no longer need to say, "Some animals are D" as another premise, because D is the name of that animal, and how can a thing's name be predicated of it? . *al-Ishrāq* (1999, p. 22).

> (50) The Peripatetics establish the validity of conversion by ecthesis and *reductio ad absurdum,* though *reductio ad absurdum* in conversion is itself based on ecthesis. If we say that necessarily no C is B, then likewise no B is C, since otherwise it would be true that some B is C . Let us, then, posit a specific thing, which we will call D, which is both C and B. Therefore, something that is described by C is also described by B. However, it has already been said that necessarily no C is B. [The Peripatetics] may establish the validity of the conversion of both the universal affirmative proposition and the particular affirmative proposition by ecthesis, or they may establish their validity by *reductio ad absurdum,* which, in another place, is itself based on ecthesis. *al-Ishrāq* (1999, p. 40).

The idea is a bit laborious but simple and brings to the light some interesting features of Suhrawardī's understanding of quantifiers. Assume Suhrawardī's own example of a syllogism of the first figure in mood *Darii*:

Some animals are rational (beings)
All rational beings have the capacity of laughing

The existential clearly entails that there is a way to specify the set of animals in a such way that all of the elements of specified set are rational,

{All those animals that are rational beings}

Substitute *rational* with *humans* "without considering the predication of rationality, though rationality accompanies"

{All those animals that are humans}

create the universal:

Every instance of those animals who are humans, are rational (beings)

In short,

All humans are rational

Clearly, the capacity of laughing can be predicated of whatever arbitrary presence *d* of humans. This verifies the syllogism in *Barbara*

All (those animals who are) humans are rational (beings)
All rational beings have the capacity of laughing

All (those animals who are) humans have the capacity of laughing

and it also verifies by sub-alternation the conclusion of the syllogism in *Darii*

Some animals, namely those who are humans, have the capacity of laughing[11]

In fact, the procedure indicates how to specify the Subject Term in the conclusion of the original *Darii*

Some animals are rational (beings)
All rational beings have the capacity of laughing
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[11] This seems to contests Ziai's (1990, p. 69, f. 3) scepticism towards M.T. Dānesh-Pazūh (1958, p. 21) comments (in his edition of ibn Sahlan's *Tabṣira*) regarding the link of Suhrawardī's method with Aristotle's remark in his *Analytica Priora* A1-23 that all inferences, including those involving existentials, can be obtained by *Barbara* and *Celarent.*

Some animals have the capacity of laughing

Notice that any specification of animals will do the job, provided that the middle term, i.e. *being rational*, can be predicated of every one of the elements of this specification, such as, say, the specifications animals *that read, or that are musicians, etc.*

This, certainly reduces the "uncertainty" expressed in conclusion of the particular:

> Which are the animals referred in *Some animals have the capacity of laughing?*
> Humans are.

As so often in the literature on this form of proof, there is in the texts of Suhrawardī the ambiguity between picking an arbitrary individual *d* , called *perceptual ecthesis*, which in our case is an instanced witnessing the presence of a human, and *D* as a general term standing for a specification of the set underlying the original existential – in our example *D* would stand for the set of those animals that are human.[12]

However, in Suhrawardī, the arbitrary individual *d*, is always experienced as instantiating a general form: experiencing this particular individual is always experiencing as being a man, or a rational being and so on; even if it not yet articulated as such. Thus, instead of d the experienced presence is *d*: *D, d being a B.*

The key is to *invent a term*, as *representative of a kind*, that makes the premises true. Clearly, from a purely logical point of view, the method is not general enough , we cannot always assume that the relevant set (the Σ-type) can be specified in a suitable way – see Movahed (2010, p. 15). Actually, this stresses the fact that Suhrawardī's logic assumes a fully interpreted language – we will come back to this point when we discuss Suhrawardī's take on the third figure.[13]

Contemporary notation for fully interpreted languages as the one of Ranta's '1994) *Constructive Type Theoretical Grammar* (CTG),[14] helps to makes the point explicit, since *presences* can be expressed at the object language. We will discuss such a notation in the following sections.

This brings us to the second point: metathetic (i.e. transposed) negations (*ma'dūla*) count as affirmations. The general point of affirmations is that they predicate about something, be it mental (like numbers) or extra mental (spatio-temporal entities), whereas *de dicto* negations (that cut the copula), do

[12] Street (2002, pp. 139-142) provides a thorough description of the uses of ecthesis *(iftirad)* in Avicenna. For a discussion on this ambiguity see Crubellier (2014, pp. 277-280) and Crubellier, McConaughey, Marion & Rahman (2019).

[13] As indicated by Zoe McConaughey in a personal email to S. Rahman this is close to Aristotle's method of , *inventing a term which apply to all things of a certain kind,* in the *Topics*, VIII 2 157a23-26. However, Aristotle does not make explicit his method, and Suhrawardī's particular way of delimiting the indefiniteness of an existential in order to achieve certainty is probably of his own. The epistemic point of such a reduction is clear, assertions of affirmative universals have a definiteness that existential lacks.

[14] Ranta (1994, pp. 43-44).

not. [15]. Here Suhrawardī, different to al-Rāzī (1963, 1:158)[16] – cf. Daşdemir (2019, p. 102) – follows Ibn Sīnā who seems to have assumed that existential import is part and parcel of the truth conditions for affirmative judgments, including metathetic ones. According to this view, if an attribute of a subject is said to exist (i.e. its predication is true), then the subject must also exist – cf. Daşdemir (2019, pp. 89-95).

> Negative propositions are those in which negation cuts the copula. In Arabic, negation must precede the copula to negate it, as when they say, "Zayd not he literate." However, if the negation is connected to the copula in such a way as to become part of either the subject or the predicate, the affirmative nature of the copula will remain. Thus, when it is said in Arabic, "Zayd is nonliterate," the affirmative copula remains and the negation has become part of the predicate. Such affirmative propositions are called ~~infinite~~ ma'dūla.
> In languages other than Arabic, whether or not the negative particle precedes the copula may not determine affirmation or negation. Instead, so long as there is a copula and the negation is part of the subject or predicate, then the proposition itself will remain affirmative, unless the negation cuts the copula. When you say, "All non-even numbers are odd," then oddness has been affirmed of every number described as non-even and, thus, the proposition will remain affirmative. A mental affirmative judgment can only apply to something established in the mind. An affirmative proposition concerning something that exists outside the mind must likewise apply to something that exists outside the mind. *al-Ishrāq* (1999, p. 15).

> *An Illuminationist doctrine {on negations}*
> (25) Know that the difference between negation in an affirmative proposition and the negation that severs the relation of affirmation is that the first cannot apply to the non-existent, since the affirmation must apply to something that can- be affirmed. In the second, the denial may apply to that which can be denied. *al-Ishrāq* (1999, p. 21-22).

Walbridge and Ziai (1999, p.15) translate ma'dūla for *infinite*, which is Kant's terminology. They are well aware of this and in footnote 16 they warn that the Arabic texts speaks of *ma'dūla sentences.* This kind of sentences, that in the specialized literature of nowadays are called *metathetic* sentences, have its origin in Aristotle's *indefinite names* (*ónoma aóriston*) (closely related to privatives, such as *this man is non-sighted*, which presupposes that men can see) and has been largely discussed in the Arabic tradition, by al-Fārābī, Ibn Sīnā and the post Avicennean scholars. As pointed out by Daşdemir's (2019, p. 84), on Ibn Sīnā's view, de dicto negation can cancel a metathetic one - presumably there is no double negation of metathetics. Similar stance adopts Suhrawardī.

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[15] Daşdemir's (2019) paper offers a useful and thorough discussion on the subject, let us quote some paragraphs relevant to our objectives: *According to Avicenna, "the proposition, then, whose predicate is an indefinite name or an indefinite verb, is called metathetic (maʿdūla/maʿdūliyya) and modified (mutaghayyira)."*[…] *the simplest form of a proposition is its "binary" (thunāʾī) form, which consists of a subject and a predicate, in which the copula is latent. When the copula is made explicit, the proposition becomes "ternary" (thulāthī). The word or particle of negation is usually inserted into the ternary proposition in two ways: before the copula or before the predicate. In the first case, the proposition then becomes negative, for the negation attached to the copula does away with the relation between the subject and the predicate, thereby implying the absence of any relation between them. In the second case, the proposition is still positive, but with a negative predicate, that is, metathetic. To continue with Avicenna's examples, of the following two propositions Zayd is not just. Zayd is not-just. the first one is negative whereas the second is metathetic, for the predicate is not "just" but the composite term of "not-just," which goes with the prefix "not- (ghayr)" that expresses metathesis (ʿudūl). Therefore, the term "not-just," composed of the word "just" and the prefix "not-," is predicated of the subject, Zayd, in the affirmative way. The metathetic proposition can still be negated by inserting the element of negation into the sentence once more, but this time before the copula in order to disconnect the meaning of "not-just" from the subject: "Zayd is not not-just."*. Daşdemir's (2019; p. 84).

[16] The point of al-Rāzī is that if *mental existence* is included as complying with existential import, then the difference between metathetic and de dicto negations collapses – cf. Daşdemir (2019, p. 110).

If a negation is negated, the result will be affirmative. *al-Ishrāq* (1999, p. 16).

As mentioned above, according to Suhrawardī (and Ibn Sīnā) metathetic negations carry existential import and therefore they have in an epistemology of presence a priority status. Where Ibn Sīnā and Suhrawardī seem to follow different paths is that the latter restricts the difference between these two forms of negations to the level of elementary propositions, at the level of quantified propositions the distinction collapses. So, according to Suhrawardī, the two negatives *No man is stone* and *All men are non-stone* express the same content:

> However, this distinction only applies to propositions about individuals and does not apply to universal propositions or other quantified propositions. When you say, "All men are nonstone," or "No man is stone," you are making a judgment about each and every thing that can be described as "man" in the propositions, whereas the negation applies only to stoniness. Thus, all individuals that can be described as being "man" must exist for the description to be correct. *al-Ishrāq* (1999, p. 21-22).

This might produce some problems if we rely on the existential import for distinguishing between both. Indeed, according to Suhrawardī, whereas, say, *(this) goat-stag is not stone* and *(this) goat-stag is non-stone* differ (the first is true, but the second is false since *goat-stag* is non-existent), *All goat-stags are non-stones* and *No goat-stag is stone* do not vary on truth-value. Presumably, since affirmations carry always existential import, both will be false: the metathetic existential import will be carried from the elementary to the universals. But how to infer an elementary proposition with de dicto negation that carries no existential import from a universal that carries existential import? Perhaps, the kind of reduction mentioned above is needed that converts *(this) goat-stag is not stone* into *Always this (this) goat-stag is not stone.*

Be that as it may, a second problem lurks. What about *Not all man is literate*? This is clearly a de dicto negation and is certainly not equivalent to *No man is literate*.

> If you say, "Not all man is literate," then you may be able to say, "Some man is literate," since the negation only applies to the part. *al-Ishrāq* (1999, p. 16).

So far, we understood, the point is to confine *Not all man is literate*, to those cases where *All man is literate* and its contrary *No man is literate* are both false. In such a case, it will follow that *Some man is not literate* but we still can assert *Some man is literate*. This restricts the use of the negation of an affirmative universal to those cases where the universal affirmative and the universal negative are both false.

Notice that this reading is quite close to some uses in natural languages such as when with the assertion *Not all man is literate,* the speaker would also like to convey that *some are not* but *some are,* and more precisely *most of them are literate*– Grice would identify them as a kind of *implicatures* – e.g.:

> Questioner: *Are all men literate*?
> Respondent: *Not all are*.

Movahed (2010, 7-14) gives a useful summary of the *innovations* of the logical rules for conversion underlying the philosophy of Illumination. Let us mention them as a conclusion to this section:

1) Reducing all negative categorical propositions to affirmatives by obversion of their predicates (and obtain in this manner their metathetic versions)

2) Reducing particular categorical premises to universals by ecthesis

3) Apply two rules for the second figure, namely:

a)

(26) Whenever there are two universal propositions with different subjects, and the predicate of one cannot be affirmed of the other in all respects, or in some one respect, then it is known .with certainty that if one of the two subjects could be conceived to be included under the other, it would have been possible for the predicate of the other to be predicated of it. Therefore, it is impossible for one of the two to be described by the other, regardless of which one is made the subject of the conclusion and which one is made the predicate. *al-Ishrāq* (1999, p. 23).

b)

The conclusion will be definite and necessary, asserting the impossibility of predicating its predicate [on the other subject] or the necessity of negation therein. *al-Ishrāq* (1999, p. 23).

As indicated by Movahed (2010, p. 14) These rules validate

Every *F* is a *G*
Every *H* is a non-*G*

No *F* is an *H*

Caveat: In fact, Suhrawardī's syllogisms are all modal, but for the moment we leave this feature out.

4) Apply one rule for the third figure. This rule is the one we mentioned above concerning elementary affirmative involving individuals

(27) When we find a single thing described by two predicates, we know that at least one thing from one of the predicates is necessarily described by the other predicate. For example, if "Zayd is animal," and "Zayd is man," then we know that "Some animal is man," and "Some man is animal," whatever else may be the case. Should this specific thing be a general meaning, then we will make the proposition exhaustive, as in "all men are animal, and all men are rational." *al-Ishrāq* (1999, p. 24).[17]

The matters get more interesting when modalities occur. This is the subject of the next section, but before discussing Suhrawardī's approach to modal syllogism let us spend some brief lines on contemporary formal analyses of syllogism

III Towards a Logic of Presence

III. 1 Brief General Remarks on Formal Analyses of Syllogism

Łukasiewicz (1957) formal reconstruction of Aristotle's assertoric syllogism, describes syllogism as a propositional formula (to wit, an implication constituted by a conjunction in the antecedent

[17] Movahed (2010, pp. 14-17) mentions two other innovations, though these seem to be incorrect.

– rendering the premises – and a formula in the consequent – rendering the conclusion), governed by the rules of a propositional logic that, according to this view, Aristotle failed to make explicit. Since then, two main perspectives arose, that contested Lukasiewicz's (very) dubious axiomatic interpretation and helped to render a unifying analysis of Aristotle's work:

1) the proof-theoretical reading that yields syllogisms as what they are, namely inferences. This work was initiated by Ebbinghaus (1964) in the framework of Lorenzen's (1955) *Operative Logik*, and by Corcoran (1974), Smiley (1973) and Thom (1981) who choose the Gentzen-style natural deduction for their reconstruction. Different to Ebbinghaus, Corcoran and Smiley understood syllogism as a syntactic system with an underlying semantics, that, according to their view, (again) Aristotle failed to make explicit. Ebbinghaus (1986) resorted to Lorenzen's notion of *admissibility*,[18] which yields proof-theoretical meaning explanations when coupled with a dialogical reading of the *dictum de omni.* This takes us to the next point.
   Joining an old German tradition, such as Brandis (1833), Kapp (1942, 1975), Ebbinghaus (1964) and Fritz (1984),[19] and Brunschwig's (1967, xxxix) view of syllogism as a *machine a faire des premises a partir d'une conclusion donnee*, that gave the dialectical games of the *Topics* a central role in the emergence of the rules of syllogism, recent interpretations witness a dialogical turn that combines the proof-theoretical reading at the level of validity with the dialectical insights on meaning in order to acknowledge the unity of the work of Aristotle – see Marion & Rückert (2016), Crubellier (2011, 2014, 2017), Crubellier et al. (2019), McConaughey (2021).

With regard to Aristotle's modal logic, the initial many-valued interpretations, inspired by the work of Łukasiewicz (1953), such as the one of McCall (1963), were quite rapidly substituted by the world-wide spreading of possible-worlds semantics of Kripke and Hintikka. The result was quite unsatisfactory and gave a rather confusing picture of Aristotle's modal logic – a confusion for which Aristotle's himself was blamed. Perhaps, the most thorough of these attempts is the one of Nortmann (1996), though he also gives up on rendering a coherent picture of Aristotle's modal syllogism – cf. Nortmann (1996, p. 133, pp. 266-288, 376).

A new perspective towards the understanding of Aristotle's modal syllogism is the work of Malink (2006, 2013). Malink defends the coherence of Aristotle's views on modalities by pushing forward the project of Patterson (1995), who rejects the possible-world interpretation and acknowledges the central role of the *Topics* for unifying Aristotle's logic and more precisely the theory of predicables.

Malink's rejection of possible-world interpretation of Aristotle, is on our view convincing and the emphasis on the *Topics*, certainly adequate. However, Malink neglects the dialectical meaning explanations underlying the *Topics.* Actually, it seems that Malink follows the contemporary syntactic + underlying semantics conception after all– quite foreign to the framework of the *Topics* – and develops therefore a kind of mereological semantics, called *preorder semantics,* that "interprets" the syntactic system.[20] The neglect of the dialectical component does, on our view, blur the point that it is the interactive stance on meaning and knowledge of the *Topics*, under the background of a fully interpreted language, that shapes the unity of logic, epistemology and metaphysics within the Aristotelian framework. Another neglect of Malink is Aristotle's approach to the temporal dimension of events. Moreover, in further work in collaboration with Jacob Rosen he claims that modal syllogistic is

[18] For a discussion comparing Corcoran's (1974) approach to the one of Ebbinghaus (1964), see Lion & Rahman (2018).
[19] We owe the references to the German tradition to Marion & Rückert (2016, p.204, footnote 17).
[20] This approach of Malink seem to be an ideal candidate of what Andrade-Lotero & Dutilh Novaes (2012) call *squeezing*.

independent of a second modal system of inferences based on Aristotle's Possibility Principle Aristotle, according to which given the premise that *A* is possible, and given a deduction of *B* from *A*, that *B* is possible can be inferred.[21] Oddly enough, this second system, as acknowledged by Malink & Rosen (2013) themselves, is the one that Aristotle deploys the most when developing demonstrations within epistemology and metaphysics.

The different contemporary formal reconstructions of syllogism within the Arabic tradition harbour at the same time some approaches that are still in the shadow of Lukasiewicz propositional view with those influenced by contemporary possible-worlds semantics, where both alethic and temporal modalities are conceived either as propositional monadic operators and temporal operators in the style of Prior (1955) – see for example Rescher and vander Nat (1974).and Street (2002), or temporal quantifiers – see Hodges (2016, p. 159) and Hasnawi & Hodges (2016, p. 48)– or combining time quantifiers with quantifiers over situations – see Chatti (2019). Many of the logical reconstructions are syntactic, some of them assume an underlying possible worlds semantics – see, e.g. El-Rouayheb (2016). However, if modalities are conceived as propositional operators that admit both an alethic and a temporal reading without distinguishing explicitly past and future (rather than a bidimensional modal-temporal reading) and are associated to the possible world semantics for S5 a notion of temporal order will result that is not the one assumed in Arabic logic – recall that the semantics of S5 assumes reflexivity, symmetry and euclideanity of the accessibility relation. Moreover, quantifying over time instances assumes that time is discrete and a substance. The latter, it is not suitable for measuring changes in a same substance, but it is rather constituted by a sequence of different temporal situations, all which seems to go against the philosophical and epistemological views on time of most of the Islamic thinkers.

Thom (2008, 2012) is one of the few scholars that adopts a proof-theoretical interpretation of Arabic (both, apodictic and assertoric) syllogism – he might be called the Ebbinghaus-Corcoran of Arabic Logic. Moreover, Thom (2008) proposed to read Ibn Sīnā's quantifiers as a kind of *de re-de dicto* modifiers of the predicate – this fruitful insight of Thom (2008) has also been adopted by Street (2013) who also implements it in his paper on Suhrawardī – see Street (2012). Strobino (2015, 2016) who also develops proof-theoretical reconstructions, particularly in Ibn Sīnā, links inferential approaches to modality with the theory of predicables of the *Topics* – cf. Strobino (2016 – but again, so far as we know, Strobino does not delve into the dialectical feature that shapes the notion of demonstration within the *Topics*.

Notice that in the context of Arabic thought the study and development of patterns of argumentation, within transmitted sciences, ʿUlūm Naqliyya, informs the ones in the rational sciences, ʿUlūm ʿAqliyya, and vice-versa – such as the inception of methods for testing claims of causal necessity. Indeed, as pointed out by Ahmad Hasnawi (2009, 2013), in the Islamicate tradition one of the links between dialectics and syllogism is rooted in al-Fārābī (1971) idea that dialectics is the theory on how to constitute a question or problem to be answered by some deductive means. According to al-Fārābī every form of syllogism is to be seen as developing an answer to a *maṭlūb* (quaesitum) formulated as a disjunction between, either contraries, contradictories or a combination of both. Important researches in this direction are on one hand Alexander Lamprakis ongoing translation and commentary of al-Fārābī's *Kitāb al-Amkina al-mughalliṭa*, which intertwines Aristotle's theory of syllogism with syllogistic with

[21] This formulation follows Fine (2011) and Malink and Rosen (2013) though the authors develop only a syntactic interpretation of it. Aristotle's own formulation is: *Now that this has been shown, it is clear that if something false but not impossible is hypothesized, what follows because of the hypothesis will be false but not impossible. For example, if A is false but not impossible, and if when A is B is, then B will also be false but not impossible. Pr. An,* I, 15, 34ª 25-9.

the, abductive perspective of the *Topics* and, on the other Walter E. Young's (2019) work on causation, notably in al-Samarqandī's theory of causality in the *Kitāb ʿAyn al-Naẓar*, that is dialectical in nature, it is assumed to apply to transmitted and rational sciences and develops further theories that emerged independently of the Aristotelian dialectics – if not in open contraposition to Aristoteles's views on the matter – cf. Young (2021a,b,c). The work of Young has been preceded by the one of Larry Miller (2020, reprint of 1984), who was the first one in suggesting to use the formal instrument of Lorenzen-Lorenz dialogical logic in order to analyse Islamic disputation theory, and particularly so in the context of the work of al-Samarqandī.

Be that as it may, Suhrawardī's conception of syllogism is an apodictic one where the meaning explanations of modalities are closely related to the theory of predicables, and which he also links it with the dialectical theories on fallacies, though he seems to rejects the *qiyās* framework for juridical argumentation.

III.2 The Ontological Ways the Predicate Relates to Its Subject

If the Stoic propositional approach to temporal and alethic modalities should have impacted Arabic logic,[22] there is no evidence, or at least not clear one, for this in Suhrawardī conception of modalities– besides his use of the standard propositional connectives, conjunction, (exclusive) disjunction, implication and negation, mentioned above.

Actually, it very much looks as Suhrawardī's proposes an approach to modality and modal syllogism suitable to his epistemology of presence and that conjugates the syllogistic framework with the pair potential-actual as applied to the Subject-Predicate relation.

The following texts in *al-Ishrāq* provide both, his approach to modalities and what me might call the (dialectical) meaning explanations underlying his view on modal syllogisms. Let us start by discussing the first texts that define modalities as qualifying the different ways the predicate relates to its subject.

III.2.1. Suhrawardī's Modal Relations

The following texts defines convertible and non-convertible modal relations

Rule three
[On modalities in propositions]
(19) The relation of the predicate of a categorical proposition to its subject either must exist (in which case it is called "the necessary") or must not exist ("the impossible") or may either exist or not exist ("the possible" or "the contingent"). An example of the first is "Man is animal"; of the second, "Man is stone"; and of the third, "Man is literate." […]. The contingent is necessary by virtue of that which necessitates it and is impossible on condition of the nonexistence of that which necessitates its existence. When one examines the thing itself in the two states of existence and nonexistence, it is contingent.
[…]

[22] Moreover, it is not that clear that even for the Stoics modalities should be thought as the propositional operators of contemporary modal logic: *Stoic modalities seem to be properties of propositions (rather than operators), just as truth and falsehood; and, if one follows Boeth. in /nt. 234 and Epict. Diss. 2.19.1-5 this could be true of Diodorus' and Philo's modalities as well*. Bobzien (1993, p. 66). For a contemporary development of modalities as predicates see Stern (2016).

> Further, when you say, "All things that move necessarily change," you should know that each and every thing described as moving is not necessarily changing because of its own essence, but because it is moving. Thus, its necessity depends on a condition and *it* is contingent in itself. By "necessary," we mean only that which it has by virtue of its own essence. That which is necessary on condition of a time or state is contingent in itself. .*al-Ishrāq* (1999, p. 16, p. 17).
>
> **Rule five**
> On conversion
> (23) Conversion is making the entire subject of the proposition the predicate and the predicate the subject while keeping the quality and the truth or falsity of the proposition the same. You know that when you say, "All men are animals," you cannot say, "and all animals are men." The same is true in every proposition whose subject is more specific than its predicate […]. Then we say, "Necessarily all men are contingently literate," its converse will be "Necessarily something ·that is contingently literate is a man." The other modes besides contingency also move with the predicate when it is converted." The converse of the necessary definite affirmative proposition is itself a necessary definite affirmative proposition, whatever the mode may be.
> If the contingency is part of the predicate of the definite necessary proposition and the negation is with the predicate, the negation will also be moved in conversion, as in the statement "Necessarily all men are contingently nonliterate." Its converse will be the definite affirmative: "Necessarily something that is contingently nonliterate is a man." *al-Ishrāq* (1999, p. 19-20)

Summing up,

- a *necessarily necessary relation* amounts to attributing *actual instances*, i.e. *presences/tokens/verifiers* of the predicate to every presence of the subject, and this relation either

  admits ***simple conversion*****,** if there is simple conversion between presences of the subject and presences of the predicate – such as when actual instances of *rational animal* are related to actual instances of *human*, this corresponds to the notion of *definition* of Peripatetics; or
  does not admit ***simple*** **conversion** – such as when actual instances of *animal* are related to actual instances of *human*, this corresponds to the notion of *genus* of the Peripatetics

Moreover, it seems that a consequence of his criticism of, what he calls the *peripatetic* take on definitions (and genus), is that such universals already assume that their underlying meaning constitution process has been established before. In other words, universals expressing definition and genus assume the formulation of meaning formation rules that encode knowledge gathered by grasping the dependence or interdependence of the actual instances of the terms involved.[23]

In fact, contingency, is subtler. Since Suhrawardī gives priority to universal apodictic assertions, he calls them ***definitely necessary propositions***, contingency assertions, in order to have epistemic value, should be embedded in necessary ones:

> You would thus say, "Necessarily all humans are contingently literate, necessarily animals, or impossibly stones." Such a proposition is called the "definitely necessary." In the sciences we investigate the contingency or impossibility of things as part of what we are investigating. We can

[23] This might be linked to Ibn Sīnā's notions of implicate (*lāzim*), containment (*taḍammun*) and implication (*iltizām)*, thoroughly discussed by Strobino (2016), though, as we shall suggest further on in the paper, Suhrawardī seems to build the *lāzim* -link between concepts out of the dependence (or interdependence) of the instances of the concepts involved.

> make no definitive and final judgment except concerning that which we know necessarily. Even for that which is only true sometimes, we use the definitely necessary proposition. In the case of "breathing at some time," it would be correct to say, "All men necessarily breathe at some time." That men necessarily breathe at some time is always an attribute of man. That they necessarily do not breathe at some time is also a necessary attribute of a man at all times, even at the time when he is breathing. However, this is different from literacy. While literacy is necessarily contingent, it is not necessary that it be actualized at some time. *al-Ishrāq* (1999, p. 16, p. 18).

So

- a *necessarily contingent relation* amounts to attributing capacities or potentialities to every presence of the subject. Such potentialities, might be grouped as follows

> (i) potentialities that for each actual instance of the subject require this potentiality to be **both,** sometimes actualized and sometimes not – such as *laughing* (which is co-extensive with *Human*)[24] and *breathing* which is not co-extensive with *Human*).
> (ii) potentialities that do not require this potentiality to be ever actualized for some particular actual instance of the subject, though the potentiality might be actualized for another instance of the subject, such a *literacy*; or, if it actualizes **for no actual** instance of the subject, such as Avicenna's famous example of a heptagonal house, given some non-actual conditions it can be at least asserted as hypothesis (i.e. the assumption that an actualization is not contradictory).

Whereas the first group can be seen as referring to "natural" or *not acquired* capacities (this terminology is not Suhrawardī's) the second group of potentialities concerns *acquired* capacities, which require some condition or learning (*education* e.g. in the case of literacy or being a musician).[25]

Notice that in the first group, time allows focusing on the contingency of one particular individual: this individual has the contingent capacity of laughing since sometimes it laughs and sometimes not.

Acquired capacities are in *general*, necessarily contingent said of the humanity as a whole, not of each individual: literacy is a human capacity since there is at least one human who is literate and at least who is not. Interestingly, Suhrawardī indicates the rule for predicating (of what we call) acquired capacities as a sufficient general rule for proving necessary contingent propositions.

Our caveat *in general*, indicates that, as we will discuss below, it is not that clear if Suhrawardī's conceptual framework leaves room for the particular case of Avicennean example of the heptagonal house mentioned above, or if Suhrawardī is committed to a stronger notion of plenitude than one that allows merely hypothetical actuality.

[24] Aristotle calls such a capacity an *idion* (*proprium*).

19

[25] Notice that, according to our reading, a not iterated necessity (occurring in a universal proposition) only indicates that every actual instance of the subject can be related to an instance of the predicate, but it does not prescribe that the latter instance must be an (actualized) presence. A contingency that does not occur within the scope of a necessity indicates a pure accident – i.e; a potentiality that is neither co-extensional to the subject nor a potentiality of (instances of) the genus occurring in the definition of its subject. Regarding non-modal premises, Suhrawardī deals with them in principle as if they are open to be understood as potentialities or actualities; however, the rules for the syllogistic figures, establish that terms must have the same modality in all the premises they occur: this makes of most of the mixed syllogisms modal ones.

Furthermore, according to our reconstruction, Suhrawardī's rules for modal syllogism in *al-Ishrāq* (1999, p. 16, p. 17) admit both a weaker and a stronger reading of plenitude, that, so far as we can see do not lead to different sets of valid inferences.

According to our understanding, Suhrawardī's modalities require neither syntactically nor semantically a modal logic in the style of contemporary possible-world framework. Indeed, syntactically seen, Suhrawardī's modalities, are relations between the terms occurring in a syllogism, rather than propositional monadic connectives; and semantically they require either actual presences of the terms they relate or hypothetical ones (i.e. open assumptions on presences of a term), rather than possible worlds.

Not only does the text quoted above makes this explicit, Suhrawardī repeats this point in several parts of *al-Ishrāq*, particularly so when he has to elucidate his view on syllogisms, such as in the following passage on the second figure of syllogism:

> Likewise, if the predicate of one definite proposition has a contingent relation and [the predicate of] the other·a necessary relation, then a, necessary relation is impossible for the first and contingency is impossible for the other. Likewise, if the predicate of one has a contingent relation and [the predicate of] the other a relation of impossibility, it *is* as we had said before. *al-Ishrāq* (1999, p. 24).

The attentive reader has surely already associated Suhrawardī's classification of the ways a predicate relates to its subject with the Aristotelian theory of Predicables developed in the *Topics*.

Let us recall Aristotle's (Top. A 4 101b15-19)) four predicables: *genus (*with differentia)*, definition*, *proprium* or accident. Further in the text (Top. An. 8 103b6-19 Aristotle reduces the four predicables to two basic relations:

> If *B* is predicated of *A*, then *A* can be predicated of *B*. This concerns definition – *Human is Rational Animal* – and proprium *Human has the capacity of laughing*:
>
> If *B* is predicated of *A*, then *A* cannot be predicated of *B*. This concerns *bare* genus (genus without differentia) – *Human is Animal* – and accidens *Human is awake.*

Moreover, either *B* is predicated essentially of *A* or not. In the first case, *B* is the definition or genus of *A*, in the latter, its proprium or accident.[26]

However, a big caution is due: we are not claiming that there is evidence that Suhrawardī ever read or had direct access to the *Topics*. Nevertheless, whatever were the ways he came to know the theory of predicables, it is likely that this influenced his take on modalities.

The key feature of Suharwardī's approach is that these relations concern presences of the subject with presences of the predicate, where the latter might be constituted by the presences of a capacity or potentiality

[26] Cf. Malink (2006, p. 97).

III.2.2 On Iteration:

It will have not escaped the reader's notice that those who, under the background of the theory of predicables, understand modalities as affecting the copula rather than as monadic propositional operators, usually reject iteration, See for example Malink (2006):

> Logically, necessary and possible propositions are not obtained by enriching assertoric propositions with modal sentential operators, but by applying a necessity […] or a possibility copula instead of an assertoric one to two terms .[…]
> Aristotelian modality is an integrated and inseparable part of the copula. Copula modalities are applied to two terms and yield a proposition, with the result that they cannot be iterated as they yield an entity of entirely different type than the two entities they are applied to. – cf. Malink (2006, p. 96)

Awkwardly, Ziai (1990, p. 70) claims that Suhrawardī's modal logic is essentially a propositional S5 logic without iteration – or with an iteration that only occurs at the level of the surface grammar. This is corrected in Walbridge and Ziai (1999, p 17, footnote 20), whereby Suhrawardī's modalities are interpreted both as propositional connectives and affecting the copula. Street (2008), contests Ziai's (1990) claim and rightly so, and observes:

> […] it's clear from his account of the proposition and its contradiction that one of the modalities (be it necessity, possibility or 'at one time') belongs to the predicate, while the second belongs to the proposition as a whole. In short, the proposition with its modalized predicate is embedded in the first 'necessarily'. Street (2008, p.169).

Suhrawardī explicitly writes

> (21) Since the contingency of the contingent, the impossibility of the impossible, and the necessity of the necessary are all necessary, it is better to make the modes of necessity, contingency, and impossibility parts of the predicate so that the proposition will become necessary in all circumstances. You would thus say, "Necessarily all humans are contingently literate, necessarily animals, or impossibly stones." Such a proposition is called the "definitely necessary". *al-Ishrāq* (1999, p. 16, p. 18).

Street's (2008) idea is to integrate the patent iteration present in Suharwardī's description of definitely necessary propositions:

- with the point that modalities are to be understood as ways to qualify the copulative relations between subject and predicate, and
- Suharwardī's own suggestion to move the "inner" modality to the predicate :

Further on in the text Street (2008, p. 173) suggests to adapt the mixed de dicto/de re reading of the Avicennean modalities proposed by Thom (2008) to Suhrawardī's framework. Another context to be examined is how iteration works in Suhrawardī's take on syllogism.

Notice that having a *de dicto* necessity has the consequence that a premise of a syllogism with a de re possibility in the predicate, assumes one-sided possibility, which is not compatible with Suhrawardī's general take on contingency .This line of thought has been pursued by, among others, Movahed (2012),

who allows the passage from the necessary to the possible in his axiomatic reconstruction of Suhrawardī's modal syllogism, that follows Street's (2008) reading of iteration by necessity.[27]

We will not pursue this path. On our view, whereas the iteration of necessity indicates that the predicable requires actual presences of both subject and predicate – to put in Aristotelian terms, the relation is by *definition* or *genus*; the embedding of a contingent modality within a necessary one indicates the predicable requires presences of a capacity – to put in Aristotelian terms, the relation is by *accident* or *proprium*.

If we distinguish the relations *Necessity by definition*, *Necessity by genus*, *Necessity by proprium*, and *Necessary accidens*, as autonomous primitive relations -– as Malink (2006) does – we can draw the following table that suggests that Suhrawardī's form of iteration is compatible with approaches such as the one proposed by Patterson (1995) for interpreting Aristotelian modalities:

Not iterated Aristotelian Modalities as Predicables	Suhrawardī's Iterated Modalites
$\mathbf{L}_{\delta}$ (δ, definition)	**LL** (convertible necessarily necessary relation)
$\mathbf{L}_{\gamma}$ (γ, genus)	**LL** (non-convertible necessarily necessary relation)
$\mathbf{M}_{\pi}$ (π, proprium)	**LM** (convertible necessarily contingent relation) (reciprocal relation)
$\mathbf{M}_{\alpha}$ (α, accident)	**LM** (non-convertible necessarily contingent relation) (reciprocal relation)

The second occurrence of **L** in **LL** stands for non-contingent predication – i.e**. LL** stands for actual presences of the predicate. No other kind of iterations seem to be suitable for the framework.

III.3 Dialectical Meaning Explanations

III.3.1 Suhrawardī's meaning explanations

In order to gather a better understanding of Suhrawardī's view on modalities we should have a look at his own meaning explanations of definitely necessary assertions. These are contained in the following short but quite insightful text occurring in the third discourse consecrated to the study of

[27] See too El-Rouayheb (2016, p. 79), who assumes a possible-world semantic in his semantic tableau for an *essentialist* reading of the controversial Barbara $\mathbf{L}(\forall x\,(J(x) \supset \mathbf{M}B(x))), \mathbf{L}(\forall x\,(B(x) \supset \mathbf{L}A(x))) \vdash \mathbf{L}(\forall x\,(J(x) \supset \mathbf{M}A(x)))$ adopted by many post-Avicennean thinkers.

fallacies, where he sets how to refute claims involving the different kinds of definitely necessary propositions:

> (48) Know that the universality of a rule stating that something is predicated of something else is disproved by a single instance where that second thing is absent. The universality of a law stating the impossibility of something being predicated of something else is proven/by the existence of that thing in a single case. Thus, if someone asserts that every C is necessarily B but finds a single C that is not B, then the universality of the rule is disproved. Likewise, if someone asserts that it is impossible for any C to be B but then finds a single C which is B, then the law will be disproved. However, if someone asserts that any C may be B, this is disproved by neither the existence nor the absence of instances. Thus, should someone claim that some universal is contingently true of another universal-for example, asserting the "B-ness" of C-then **he need find** only a single instance that is B and another that is not B in order to show that the universal B is not impossible in the nature C (since otherwise no individual C could be described as being B) and that [B] is not necessary [in C) (since in that case no individual C could fail to be B). *al-Ishrāq* (1999, p. 38).

Saleh Zarepour pointed out in an email to Rahman, and rightly so, that in the Arabic source for someone seeking to prove a necessary contingency it **suffices (كفاه)** *to find a particular [instance of C] that is B and another particular [instance of C] that is not* rather him **needing** to find such instances , as in Zia and Walbrdige's translation above.

Zarepour's point is an Avicennean one: if contingency amounts to **necessarily** finding at least one instance where the potentiality is realized (and one where it is not), then this seems to lead to a strong form of plenitude: anything possible must be once realized. Ibn Sīnā's take on plenitude is a weaker one: what is required is that such an instance is *conceivable*. This has been admirably discussed by Griffel (2009):

> For Avicenna, however, "what neither holds always nor holds never" refers to predications about things in the outside world as well as those that exist only in the mind. The "heptagonal house" (*al-bayt al-musabba*), for instance, may never exist in the outside word but will at one point in time exist in a human mind and is therefore a possible being. For Avicenna, the principle of plentitude is valid for existence in the mind (*fī l-dhihn*) but not for existence *i n re* (*fī l -a'yān*), that is, in the outside world. It is contingent that some houses, or all houses, are heptagonal, since the combination of "house" and "heptagonal" is neither necessary nor impossible. Here Avicenna clearly divorces modality from time. The possibility of a thing is not understood in terms of its actual existence in the future but in terms of its mental conceivability. […]
> In principle, Avicenna does not part with the Aristotelian statistical understanding of the modalities. In order to be possible, something must exist for at least one moment in the past or future. Mental existence (*al-wujūd fī-l-dhihn*), however, is one of the two modes of existence in Avicenna's ontology. Whether something exists in our minds depends upon whether it is the subject of a predication. There is no ontological difference between whether a thing exists in reality or merely in the human mind. F. Griffel (2009, pp. 167-168).

Though the terminology is about *being in the mind*, arguably, *conceivability* does not need to be interpreted as a psychological understanding of possibility, but close to Aristotelian notion of possibility as non-contradictory.[28] Nevertheless, is this what Suhrawardī has in mind? It is difficult to answer to this

[28] See e.f . *Pr. An,* I, 15, 34[a] 25-9. In fact, as discussed by Rosen and Malink (2012, p. 186) this notion of possibility is ubiquitous in Aristotle's work.

question: on one hand the insistence on *presence,* seem to be more ontologically committed than *mere presence in the mind*, [29] on the other there are texts that might suggest this, such as

> If we say 'Possibly every J is B' and 'Actually (*bi'l-wujūd*) every B is A', it is known from the nature of possibility that it may never actually occur; so if the J is never described as B, it does not follow that the A comes to it actually, but only potentially, so it's possible. *Manṭiq al-talwīḥāt* (1955, p. 35-36), quoted and translated in Street (2008, p. 170).[30]

Now, this case is of a mixed syllogism, where the categorical non-modal universals is understood here as a possibilist quantifier, that admits too a reading in which the middle-term is affected by contingency.
Nevertheless, Suhrawardī accepts existence in the mind.

> (68) Therefore, all attributes may be divided into two classes. The first is the concrete attribute, which also has a form in the intellect such as black, white, and motion. The second is the attribute whose only concrete existence is its existence in the mind and which has no existence at all except in the mind. *al-Ishrāq* (1999, p. 50)

In fact, as thoroughly discussed by Zhang (2018) Suhrawardī's notion of existence is complex and sophisticated and it is unlikely that it leads to the kind of Weak Plenitude defended by Ibn Sīnā. In point of fact, the ontological commitments engaged by the logic of "presences" makes it implausible to read Suhrawardī's necessarily contingent modality as compatible with Weak Plenitude.[31] Moreover, Suhrawardī commitment to Srong Plenitude can be seen as constituting a part of his criticism to the take on Plenitude defended by the Peripatetic thinkers of his time.

Be that as it may, if, as we do below, introduce presences into the object language, distinguishing between "actual" and "hypothetical" presences is quite straightforward and the logical result is not really different after all – at least from the point of view of describing the set of valid syllogisms. Having said, it can be argued that, according to the main epistemological tenets of Suhrawardī's Illumination Philosophy, the epistemic force conveyed by a hypothetical actualization is weaker than of a categorical one.

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[29] Movahed (2012, p. 14-15), who assumes a S5 possible-worlds semantic, distinguishes between the potentiality *B* of some non-actual reference of the individual constant *a,* and the potentiality attributed to the actual individual constant *a* (in fact he takes *a* to be a definite description), i.e. $\Diamond(B(a))$, and $(\Diamond B)(a)$, whereby the capacity *B* is expressed of the actual individual constant *a*. An accurate rendering of this idea can be found in Fitting & Mendelsohn (1998, p. 195), where lambda notation is introduced in order to distinguish $\Diamond\langle\lambda x.\ B(x)\rangle(a)$ from $\langle\lambda x.\ \Diamond B(x)\rangle(a)$. Indeed, one way to read this is that whereas the first expression encodes the idea that the reference of the individual constant *a* actualizes *B* at some possible world, the second indicates that the actual reference of the individual constant *a* might actualize *B* (i.e. actualizes *B* in at least some possible world).

[30] In this context the following remarkable lines are relevant, where he discusses the case of knowing the meaning of *Phoenix*, for someone who never had acquaintance with such a bird before, though he knows from others that such a bird exists.

> Suppose that someone knows for certain that a bird called "phoenix" exists but has not seen it and seeks to know it specifically. He will know only the general attributes of the bird – that it flies, for example. Only through many people telling him that the other attributes of the .bird called "phoenix" are such-and-such can he know it so well that he can say that the attributes mentioned by someone describing it to him all belong to what he seeks and to nothing else. *al-Ishrāq* (1999, p. 37).

Thus, according to Suhrawardī, given these circumstances, one can have knowledge of *Phoenix*, provided he gathers enough information of others who describe this animal as having such-and-such properties so that these attributes belong to this animal and nothing else. This recalls the contemporary notion of definite description. Is Suhrawardī suggesting that a definite description, rather than a definition, is enough for grasping the meaning of *Phoenix*? It looks very much so, however, the text assumes that existence its known by the transmission of others.

[31] Rahman had some interchange on the issue with Jari Kaukua (Jyväskylä).

Coming back to Suhrawardī's text on the rule for proving assertions involving modalities they follow the so-called dialectical *no-counterexample* interpretation of the quantifiers, by the means of which the search for a counterexample is governed by rules of interaction.

Notice that the notion of modality, the germane proof-methods (discussed in the quote) and the notion of existence as presence, occur in his third discourse, the discourse on fallacies (*mughālaṭa*), in *al-Ishrāq* (pp. 42-51). As pointed out by Walbridge and Ziai (1999, introduction p. 15) in this section, central to his criticism of the Peripatetic doctrines, Suhrawardī examines the positions of his opponents in the form of disputes on natural philosophy and metaphysics.

This recalls us not to forget that Suhrawardī's work has been developed during the period when the *logical turn of dialectics* was taking place, during which the twelfth-century-CE eastern school of Raḍī al-Dīn al-Nīsābūrī (d. 544/1149) students, especially Rukn al-Dīn al-ʿAmīdī (d. 615/1218), and others promoted the fusion of logic and dialectic theory also in syllogism – see Walter E. Young (2021a, section 2.2) and Rahman&Young (2022, Introduction to the special *Methodos* volume *Argumentation and Arabic Philosophy of Language*). [32]

. In such a context, challenge and answer, provide the meaning of the expression involved. In other words, to grasp the meaning of a proposition involved in an assertion by **X** amounts to know

> (a) what *rights* does the antagonist **Y** have in the context of a dialectical interaction triggered by that assertion – whereby *requests* or *challenges* put *rights* into action, and
> (b) what *commitments* does the assertion engages to – whereby *defences* are the ways to make use of those *commitments*.

That is what we mean when we speak of d*ialectical meaning explanation* – see Rahman et al. (2018, chapter 3) and Crubellier et al. (2019).

The dialectical approach constitutes in fact the man background we follow for the reconstruction of Suhrawardī's logic and theory syllogism.

In any case it is important to observe that in such a dialectical framework there are two different, intertwined levels of analysis for the links between antecedent and consequent involved in a universal proposition and/or implication:

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[32] Noteworthy is the fact that the fusion of dialectic and logic shares some common features with Aristotle's own take on the meaning of quantifiers as formulated in the *Topics* Θ 157^{a}34-37:

> When it happens that, after you have induced from many cases, someone does not grant the universal, then it is your right to ask him for an objection. However, when you have not stated that it does hold of some cases, you have no right to ask "of which cases does it not hold? For you must previously carry out an induction to ask for an objection in this way." *Top.* VIII 2, 157a34–37;

and in *Prior Analytics* A 24^{b}28–29:

> We use the expression 'predicated of every' when none of the subject can be taken (*λαβειν*) of which the other term cannot be said.

We owe to Marion & Rückert (2016, p. 206) the denomination *no-counterexample* interpretation of quantifiers, who argue that a dialectical interpretation is closer to the Aristotelian conceptual framework than a truth-functional. For a further thorough dialectical interpretation of the syllogism see McConaughey (2021). However, the highly innovative way the Islamicate thinkers conjugated logic with a dialectic that included all the distinctions and richness of disputation theory originated in Islamic Law, goes far beyond the Aristotelian framework.

a semantic link involving the meaning dependences of one concept upon the other, and the link established by the universal quantifier and the implication.

In order to elucidate the intertwining of these levels let us focus first on non-modal proposition.

III.3.2. Dialectical s Meaning Explanations of Non-Modal Universals and Existentials

The semantic level concerns the links between concepts, such as we find between *Living Being* and *Knowing Being*, to adapt a common example in the literature. This level does not directly render a proposition but rather the semantics conditions out of which a proposition obtains. So, the conceptual link

Knowing Being entails Living Being

is to be understood as

> for any instance (presence) *x* of *Knowing Being,* an instance of *Living Being* can be obtained by a semantic process that renders instances of the latter out of instances of the former.

In other words, at this level of analysis the semantical link between *Living Being* and reconceived not merely as a consequent to a propositional function defined over *Knowing Being*. If we, once more employ CTG, where instances (presences) can be expressed at the object language level we we have

Linear notation	**Vertical Notation**
Living Being[*x*]: *prop* (*x*: *Knowing Being*)	(*x*: *Knowing Being*)
	...
	Living Being[*x*]: *prop*

At a further (connective) level of analysis this either constitutes a propositional implication such as

> *if it is knowing then it is living*, or also
> *Every knowing being is a living being*.

Note that the connective/quantifier level presupposes the semantic level. Only when we know how one concept is dependent upon another one, can we render the corresponding logical connective or quantifier.

Placing both levels of analysis, in a dialogical setting, the following points become clear:

(a) If interlocutor **X** asserts that something is knowing, then the antagonist **Y** can ask him to further assert that it is living.

(b) Asserting *Everything knowing, is living*, presupposes that the semantic link between *knowing* and *living* has been previously endorsed.

- If such an endorsement took place, it was also the outcome of a previous act of knowledge, by the means of which **presences of the consequent** (in our case *Knowing Being*) are experienced as being dependent upon **presences of the antecedent** (in our case *Living Being*).

According to our analysis this is one of the most distinctive features of the Epistemology of Illumination and can be declined as the obtaining from the following steps:

1. The proposition *Everything knowing, is living*, presupposes an abstraction process over the construction:

   Living Being(*x*): *prop* (*x*: *Knowing Being*)

   In other words, conceiving the universal as a proposition presupposes

2. Every presence of *Living Being* has been experienced as associating presences of *Knowing Being* with presences of *Living Being*. Isolating and identifying the association-procedure itself is production of a further abstraction step. Formally speaking, the association-procedure can be rendered as a function *b*(*x*): that takes presences of the antecedent and yields presences of the consequent:

Linear notation	**Vertical Notation**
b(*x*): *Living Being*(*x*): (*x*: *Knowing Being*)	(*x*: *Knowing Being*) … *b*(*x*)*: Living Being*(*x*)

3. The previous step is product of an abstraction over acts of experiencing that presences of *Knowing Being* witness presences of *Living Being.*
   In other words, if for any *a*: *Knowing being,* we experience this *a* as witnessing a presence of *Living Being*, an association-procedure *b*(*x*) can be isolated, such that *b*(*a/x*): *Living Being*.

In short, and expressed as an inference process, now from the simpler to the complex after the experience of the concrete presence has been settled:

Living Being(*x*): *prop* (*x*: *Knowing Being*)
---
b(*x*)*: Living Being*(*x*) (*x*: *Knowing Being*)

(∀*x: Knowing Being*) *Living Being*(*x*): *prop*

Coming back to the dialectical setting the *dialogical meaning explanation* of the universal quantifier, where the presences are made explicit in the object language, amounts to the following

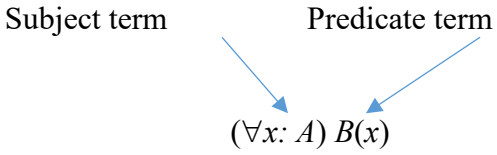
If player **X** states a universal quantifier he has to be able to associate presences of the consequent for any arbitrary presence of the consequent chosen by the challenger **Y**

Statement	Challenge	Defence
X ! $(\forall x{:}\ A)\ B(x)$ **X** states the universal *Every A is B*	**Y** $a{:}\ A$ **Y chooses** an arbitrary presence a of A	**X** ! $b[a]{:}\ B(a)$ **X** associates a with a presence of $B(a/x)$

This again presupposes suitable semantic formation rules -the semantic level mentioned above

Statement	Challenge	Defence
X $(\forall x{:}\ A)\ B(x){:}\ prop$ **X** states that the universal is a proposition	**Challenge1** **Y** ? $\mathcal{L}_F$ **Y** asks for the formation of the **left** ($\mathcal{L}$) component of the universal --- **Challenge2** **Y** ? $\mathcal{R}_F$ **Y** asks for the formation of the **right** ($\mathcal{R}$) component of the universal	**Defence1** **X** $A{:}\ set/prop$ **X** responds that it is a set (or proposition not dependent upon another one) --- **Defence2** **X** $B(x){:}\ prop\ (x{:}\ A)$ **X** responds that $B(x)$ is a proposition dependent upon A

Notice that this notation follows the idea of traditional logic that the predicate term, states a predicate of presences of a subject:



The dialogical meaning explanation for an existential, leaves the choice of the presence to the defender:

Statement	Challenge	Defence
X ! $(\exists x{:}\ A)\ B(x)$ **X** states the existential *Some A is B*	**Challenge1** **Y** ? $\mathcal{L}^{\exists}$ **Y** asks for the left: Which/ Who of the A's are B? --- **Challenge2** **Y** ? $\mathcal{R}^{\exists}$ **Y** asks for the right: Show me that this choice of yours is indeed a B	**Defence1** **X** $a{:}\ A$ **X** responds that a is one of those A's --- **Defence2** **X** $b(a){:}\ B(a)$ **X** associates the presence of a with a presence of $B(a/x)$

Notice that this allows to have the same subject-predicate structure as the one for the universal quantifier. Moreover, it allows expressing the subject term as a restricted domain. Let as in

Some poets are good

Which, as pointed by Ibn Sīnā in the *al-Išārāt* (1983, Chapter 10.1, pp. 501-502) does not allow to infer that there is someone, say Imraʾa al-Qays, who is good and a poet.[33] Clearly what is asserted is

Some poets are good ***as poets*** $(\exists x{:}\ Poets)\ Good(x)$

In other word: *Within the restricted domain of subject, namely, poets, some are good.*

As mentioned in our discussion on ecthesis, the existential expresses a set, in our example the set of

Those presences of poet that are good (as **poets**) $\{x{:}\ poet \mid Good(x)\}$

Since this set, is what the meaning of the existential amounts to, the dialogical meaning explanation is the same as the one of the existential:

Statement	**Challenge**	**Defence**
X ! $\{x{:}\ A \mid B(x)\}$ **X** states that the set of those *A*'s that are *B* can be defined over *A*	**Challenge1** **Y** ? $\mathscr{L}^{\{\}}$ **Y** asks for the left: Which/ Who of the *A*'s are *B*? --- **Challenge2** **Y** ? $\mathscr{R}^{\{\}}$ **Y** asks for the right: Show me that this choice of yours is indeed a *B*	**Defence1** **X** $a{:}\ A$ **X** responds that *a* is one of those *A*'s --- **Defence2** **X** $b[a]{:}\ B(a)$ **X** associates the presence of *a* with a presence of $B(a/x)$

In the appendix we describe the rules for the other usual standard logical constants. Once the logical constants have been constituted out of its dialectical meaning explanations, those moves can be identified, allowing a higher level – namely, the strategic level.

At the strategic level the Proponent, has a winning strategy for a a universal iff for *any* presence of the Subject the challenger can show produce an instance of the Predicate for this choice of the Opponent. The way to implement this, is to allow the Opponent to choose always a **new** instance

Statement	**Challenge**	**Defence**
P ! $(\forall x{:}\ A)\ B(x)$ **P** states that he has a winning strategy for the universal	**O** $a{:}\ A$ **O chooses** the **new** presence *a* of *A,* and requests **P** to show that it witnesses a presence of $B(x)$	**P** ! $b(a){:}\ B(a)$ **P** associates *a* with a presence of $B(a/x)$

[33] In fact, by Ibn Sīnā's precise counterexample is that from the premises *Imraʾa al-Qays is good* and *Imraʾa al-Qays is a poet*, *Imraʾa al-Qays is a good poet* does not follow.

Thus, at the strategic level, the truth of the universal requires **P** to be able to associate presences of the Subject with presences of the Predicate, by means of substituting x in $b(x)$ for any presence of the Subject **O** might choose.

In other words, the truth of the universal $(\forall x{:}\ A)\ B(x)$ requires the Proponent

a) to build out of the associations triggered by whatever presences of A **O** might choose, witnesses of $B(a/x)$ an association-procedure $b(x)$,
b) to build the abstract construct of this procedure, namely $(\lambda x)b(x)$, called the lambda-abstract of $b(x)$. The lambda-abstract indicates that for whatever choice of the Opponent the association-procedure $b(x)$ can be executed in order to yield an instance of $B(a/x)$ for that choice, and
c) **P**'s execution of $b(a/x)$ is justified by **O** being forced to state himself $B(a)$: this actually is the core of proving the validity (building a winning-strategy) of a syllogism involving universals.

Thus, the canonical form of a winning strategy for a universal has the form:

$$\mathbf{P}\ (\lambda x)b(x){:}\ (\forall x{:}\ A)\ B(x)$$

Any c witnessing the universal is a construct equal to the lamda-abstract $(\lambda x)b(x)$, built out of plays whereby **P** can execute the association-procedure $b(a/x)$ witnessing $B(a/x)$, for any $a{:}\ A$ chosen by **O**. Notice the proviso, **new**, implements the idea that the winning-strategy is not dependent upon one particular choice of **O**. Indeed, if the winning-strategy is a robust one, it will yield the same result for any alphabetic variant of presences of the antecedent.

<table>
<tr><th>Statement</th><th>Challenge-Defence</th><th>Strategic object</th></tr>
<tr><td rowspan="2">P ! $(\forall x{:}\ A)\ B(x)$

P states that he has a winning strategy for the universal</td><td>Challenge
O $a{:}\ A$

O chooses the new presence a of A, and requests P to show that it witnesses a presence of $B(x)$</td><td rowspan="2">P $(\lambda x)b(x){=}c{:}\ (\forall x{:}\ A)\ B(x)$</td></tr>
<tr><td>Defence
P ! $b(a){:}\ B(a)$

P associates a with a presence of $B(a/x)$, and O is forced to state himself $B(a)$</td></tr>
</table>

In relation to an existential, such as $(\exists x{:}\ A)\ B(x)$, at the strategic level, the truth of this existential requires **P** to be able to state some presence $a{:}\ A$ chosen by **P** himself, as response to the first

challenge, and to state $B(a)$, as a response to the second challenge, by building the association procedure $b(a/x)$.

Statement	Challenges	Defence
P ! $(\exists x{:}\ A)\ B(x)$ **P** states that he has winning-strategy for the existential	**Challenge1** **O** ? $\mathscr{L}^{\exists}$ --- **Challenge2** **O** ? $\mathscr{R}^{\exists}$	**Defence1** **P** $a{:}\ A$ **P** responds by choosing one a and that it is one these presences that witnesses A --- **Defence2** **P** $b(a){:}\ B(a)$ **P** associates the presence of a with a presence of $B(a/x)$

In other words, the truth of the existential requires the Proponent

a) to state that some presence a, chosen by **P** himself witnesses A, **and** build out of the association of a with a presence of the $B(a/x)$ an association-procedure $b(a/x)$
b) to build the complex construct <a, $b(x)$>, which results from P's responses to both challenges:
c) **P**'s choice of some $a{:}\ A$, and the execution of $b(a/x)$ is justified by **O** being forced to state himself both $a{:}\ A$ and $B(a)$: this actually is the core of proving the validity (building a winning-strategy) of a syllogism involving existentials

Thus, the canonical form of a winning strategy for a universal has the form:

P $\langle a, b(x)\rangle{:}\ (\exists x{:}\ A)\ B(x)$

So, any c witnessing the existential can be analysed as the pair <a, $b(x)$>.

Statement	Challenges-Defences	Strategic Object
P ! $(\exists x{:}\ A)\ B(x)$ **P** states that he has winning-strategy for the existential	**Challenge1** **O** ? $\mathscr{L}^{\exists}$ **Defence1** **P** $a{:}\ A$ --- **Challenge2** **O** ? $\mathscr{R}^{\exists}$ **Defence2** **P** $b[a]{:}\ B(a$	**P** $\langle a, b(x)\rangle{=}c{:}\ (\exists x{:}\ A)\ B(x)$

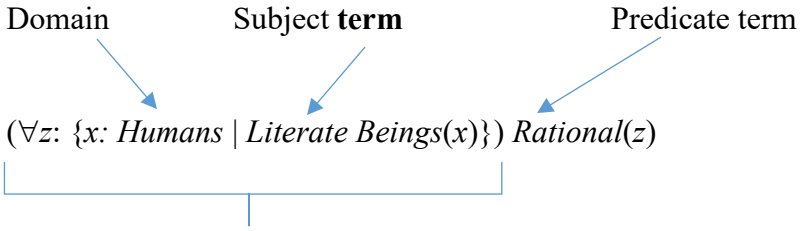
Let us now observe that within a syllogism, premises and conclusion share a common domain, over which the Subject term and the Predicate term have been defined. This is what allows the middle term, to occur as the Predicate in one premise and as Subject in the other. Moreover, since as we will see

when we introduce modalities into the universal; the Subject term can be also be modalized, we need a first term to build the modal relation : as in *All Humans who are necessarily Contingently literate Beings, are necessarily necessary Rational*. Let for the moment ignore the modality in order to stress the underlying structure of the Subject – the "Subject" denominates here the Subject term + the domain upon which the Subject term has been defined:

All Humans who are Literate Beings are Rational

Should be understood as

All presences z of the set of all those Humans who are Literate Beings , are Rational



Subject =: Domain + Subject Term

Actually, sometimes it is useful to be more precise and indicate that the presences *z* chosen by the challenger are animals, but exactly those animals that, to put it Suhrawardī's words, *always accompany Being Human*:

$$\mathbf{X}\ !\ \forall z\text{: } \{x\text{: } Animal \mid Human(x)\}\ Rational(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}}$$

From this perspective the semantic and logical analysis is a consequence of the dialectical interaction underlying a debate on meaning and its possible extensions.

Many contemporary reconstructions that appeal to relevant, modal, or other forms of so-called non-classical logics, seem to overlook the fact that it is the dialectical constitution of meaning that leads the way in the logical shaping of dialectics. As dialogicians often put it: the dialectical conception of logic is not logic plus dialectical interaction, but it is dialectical interaction that shapes both meaning and logical reasoning.

III.3.3 The Dialectical Meaning Explanation of Suhrawardī's Modal Relations

A good question is how to distinguish the modal assertion from a no-modal one. Clearly, Suhrawardī, who is here very close to Ibn Sīnā views, is mostly, if not exclusively interested in modal propositions, since they are, on his view, the only ones that have epistemic value. Particularly so in the context of syllogism, where only necessary propositions are to be considered, a stance, that, so far we understood, is also at the heart of Ibn Sīnā's view on scientific truths that should ultimately express essential connections between terms– see Strobino (2016, p. 263).

It is in fact difficult to give a definitive answer because of this reluctance to speak about non-modal propositions, or when he speaks of them it very much looks as if Suhrawardī believes that all assertions are implicitly modal.

In principle we might attempt to distinguish between categorical and modal logic in the following way:

- Categorical assertions, abstract away from both the specific (modal) way the predicate is related to the subject and the specific domain over which subject and predicate have been defined. In other words, according to this perspective categorical syllogisms are constituted by inferences obtained independently of the kind of predicables and specific domain occurring in the assertions involved. In some sense, categorical syllogisms are *blind* to the diverse forms of internal contentual links at work in the premises involved. Thus, the Subject term and the Predicate term might **implicitly** express potentialities, whose verification requires bringing forward both a positive and a negative instance (of the terms involved); or necessities (whose verification requires positive instances of the terms involved), however if the whole syllogism is to examined as a categorical one, the inner structure of the predicate is ignored. Another way to put it, close to Suhrawardī 's own sketchs of proofs (as we will discuss in section IV), is that categorical universals are to be understood as possibilist quantifiers that include not only actualities but also potentialities.[34]

III.3.3.1 The Dialectical Meaning Explanation of the Necessarily Necessary Relation

With regard to the necessarily necessary relation, let us recall that

- a *necessarily necessary relation* amounts to attributing *actual instances*, i.e. *presences/tokens/verifiers* of the Predicate term to every presence of the Subject, and this relation either

  admits (**simple**) ***conversion*****,** if there is simple conversion between presences of the subject and presences of the predicate – this corresponds to the notion of *definition* of Peripatetics; or

  does not admit (**simple**) **conversion** – this corresponds to the notion of *genus* of the Peripatetics

If we combine this with our previous considerations, we have:

$$(\forall z\colon \{x\colon D \mid A(x)\})\ \mathbf{LL}B(\mathcal{L}^{\{\}}(z)\ \text{true}$$

[34] Perhaps, this amounts to taking premises of a categorical syllogism to be *epistemic assumptions*, to use the happy formulation of Göran Sundholm (1997), that is, premises *assumed to be known*, rather than *known to be true* as in modal necessity. *Logical truth* obtains by going over to function types. Thus, the generalization "true in all propositional instances" draws upon function *types* formed from the types of propositions (i.e., $(x)\beta\colon (\xi\colon \alpha)\beta$) – cf. Sundholm (2013) – whatever the propositions and domains (over which these propositions have been formed) are chosen and whatever form of predictability might be involved.

Assuming

Subject term $A[x]$: prop $(x: D)$
Predicate Term $B[x]$: prop $(x: D)$

Notice that, since it *is better to make the modes of necessity, contingency, and impossibility parts of the predicate – al-Ishrāq* (1999, p. 16), and, as we mentioned above, we might have syllogisms where the subject-term is modalized, e.g. if it is middle term of the major premising of a syllogism of the first-figure, it desirable to have a notation that encodes also these cases. Our proposed notation does exactly that. Indeed, it allows encodings such as whereby A is related to D by necessary contingency:

$$(\forall z\colon \{x\colon D \mid \mathbf{LM}A(x)\})\ \mathbf{L}\ B(\mathcal{L}^{\{\}}(z)\text{true}$$

A rendering of the rules that follows the traditional formulations of syllogism; should, on our view, combine

- the term-notation with one where
- the use of instances of the quantified expression is only made explicit in the context of the rules that prescribe how to develop the dialectical interaction associated with a quantified assertion. It is only during the proof that instances come to the fore. Notice that, as we will discuss below this is particularly salient in relation to the temporal dimension which also only comes to the object language, as result of the interaction triggered by an assertion of contingency.

In this context it is useful to recall Ranta's (1994, pp. 7-9) notion of *sugaring*, a procedure by the means of which the fully explicit formal encoding undergoes some transformations until the natural language expression results. On our view, sugaring processes have to be built upon some specific contextual background: in this case argumentative interaction, that we call *dynamic encoding*.

We shall not make explicit the steps that takes us from the full formalized notation to the dynamic encoding. However, the table will show the initial step (the fully formalized notation) and the final step (the resulting dynamic encoding).

Let us however, provide the following reading keys

"**X** ! …" stands for the "agent **X** claims …",

"A_D" stands for the set $\{x\colon D \mid A(x)\}$.

N. B: The exclamation sign disappears when the instance of the proposition is made explicit. The point is that with **X** ! *A,* the speaker claims that there is some instance verifying his claim, and with **X** *a: A* the instance *a* is brought forward. The initial statement has an exclamation sign, and thus, no explicit, proof object: this must be constructed during the interaction

According to our reading of Suhrawardī's text, the point is that the modality prescribes how the defender has to establish the link between the predicate and the subject, given an instance of the subject, claimed by the antagonist to constitute a counterexample to the claim of necessity. This suggests the following rendering of the rules:

Statement	Challenge	Defence
Explicit Dialectical Encoding **X** ! $(\forall z: \{x: D \mid A(x)\})$ **LL**$B(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$ All D's that are A, are necessarily (necessary) B – whereby "$\mathcal{L}^{\{\}}(z)^{\mathbf{Y}}$" stands for an element of D, who is an A, chosen by the adversary **Y**.	**Explicit Dialectical Encoding** **Y** ! $A(d_i)$ Given: $\mathcal{L}^{\{\}}(z)=d_i : D$ **Y** states $A(d_i)$, whereby he chooses d_i as being one of those left components of z, that that are A in D.	**Explicit Dialectical Encoding** **X** $b(d_i)$: $B(d_i)$ **X** associates d_i with a presence of $B(d_i/\mathcal{L}^{\{\}}(z))$.
Dynamic Encoding **X** ! (EveryA_D)**LL**B	**Dynamic Encoding** **Y** ! $A(d_i)$ The challenge makes explicit the presence d_i: D; chosen by **Y** to test the universal	**Dynamic Encoding** **X** ! $B(d_i)$ The defence makes explicit the presence d_i ; as of the instances of the Subject of which the predicate can be stated.

We have left by side two issues, namely

- the distinction between modalities that admit simple conversion and those that not, and
- the temporal dimension

In relation to the first, we will deal with simple conversion for both necessary and necessary contingent propositions in a separate section.

In relation to the temporal dimension, in the context of Suhrawardī logic, its explicit occurrence is only relevant for the contingent. Indeed, since according to Suhrawardī temporality is a condition, it only involves the contingent: necessarily necessity is always actual.

Are there no other iterated modalities then? Actually, no, at least not with regard to achieving scientific knowledge. Indeed, in the epistemological framework set by Suhrawardī, only universals constituted by necessarily necessary relations and universals constituted by necessarily contingent relations provide certain knowledge. The first group provides necessary properties of the subject, the second necessary potentialities of the Subject. Bare necessity, expresses either a tacit necessity or a lack of contingency. Bare contingency has, to put in Suhrawardī words, no scientific value.

Let us deal now with the case of the necessarily contingent relation.

III. 3.3.2 The Dialectical Meaning Explanation of the Necessarily Contingent Relation

III.3.3.2.1 Time and the Necessarily Contingent

As already mentioned, the necessarily contingent relation can also be declined in one admitting simple conversion (corresponds to the *proprium* of Peripatetic logic) and one that does not (corresponds to *accident*).

Moreover, necessarily contingent predication includes acquired potentialities or capacities such as literacy; and *natural* or not-acquired, such as breathing. Whereas predicating a not-acquired capacity of an actual individual involves time as focusing in one particular individual – e.g. laughing/breathing is necessarily but contingently said of humans since there must be at least one time when laughing is present, and one when it is absent concerning **each individual**; predicating an acquired capacity amounts to asserting such a contingency with regard to the **whole genus** – e.g., predicating literacy of humans assumes that at least one individual in at least one time actualizes literacy iff there is another human who does not. Furthermore, it might be useful to distinguish commitments to either strong from commitments to Weak Plenitude.

Thus, the temporal dimension is constitutive of the meaning of Suhrawardī's notion of contingency. Moreover, as mentioned in our introduction; according to our view, in the framework of Suhrawardī's epistemology and logic it is the instances rather than the propositions they verify, that are *timed*.

Indeed, ff we have a close look at Suhrawardī's texts it seems quite clear that temporal conditions are not here understood here in a propositional way, neither as implications nor as indexes that saturate a propositional function. Temporal conditions are contextual parameters that can be made explicit in order to *enrich* a proposition that has already content,[35] rather than in order to complete the meaning of a propositional function. Thus if "*T (ime)"* stands for some set of instants, and "@" for an operator that enriches a proposition with elements of *T* – "@" can be thought as standing for some kind adverbial – we obtain

$$\begin{array}{l} B\,(d)\text{: } prop \\ t\text{: } T \\ \text{----------} \\ B(d)@t\text{: } prop \end{array}$$

Whereby~

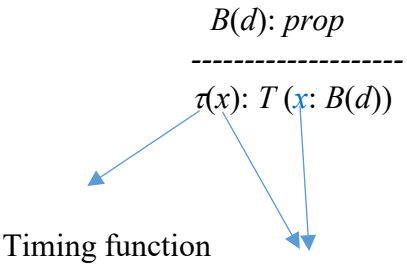
$$B(x)\text{: } prop\ (x\text{: } A) \qquad d\text{: } A$$

- In other words, "*@t"* occurring in *B*(*d*)*@t* stands for an operator, the formal counterpart of an adverbial, that can be added to a proposition. It is **not** in principle to be understood as resulting from the propositional function *B*(*x,y*): *prop* (*x*: *T, y: A*(*x*)).[36]

In fact, $B(d)@t_i$ is the result of sugaring the outcome of the *timing* function τ which associates presences of *B*(*d*) with instants of time – e.g. the function *times* presences of laughing:

[35] We owe the expression *enrichment* to Recanati (2017).

[36] There is an alternative were a suitable propositional function can be defined, if we assume that "presences" do not occur in *B,* namely *b*(*x,y*): *B* (*x*: *T, y: A*), which assumes both *A* and *B* to be independent propositions, though their presences condition *B.* Moreover, *T* might have its own ontological status different from the one of *A* and the one of *B.* This alternative requires making explicit at the object language level the function *b*(*x,y*), which brings up a higher level: we need function-types of the form (*x*: *C*) *D*, such as *f*: ((*t*$_i$/*x*: *T*, *d*$_i$/*y*: *A*) *B*) (*t*$_j$/*x*: *T*, *d*$_i$/*y*: *A*) ~*B* & *g*: ((*t*$_j$/*x*: *T*, *d*$_i$/*y*: *A*) ~*B*) (*t*$_i$/*x*: *T*, *d*$_i$/*y*: *A*) *B*). Though technically possible it is still philosophically, unclear: Time indeed can be said to be part of the meaning of the contingency of *breathing*, but is it a condition such as a cause?



Given b: $B(d)$ and $\tau(b) = t_i$: T

The assertion *d laughs at* t_i, will be first encoded as

$$\mathbf{X}\ !\ (\exists x\colon B(d))\ \tau(x) = {}_T t_i$$

And then sugared as
$\mathbf{X}\ !\ (\exists x\colon B(d)\ \ \tau(x) =_T t_i \Rightarrow \mathbf{X}\ !\ B(d)@t_i \Rightarrow$ *d laughs at* t_i

Or more generally
$\mathbf{X}\ !\ (\exists x\colon B(d)\ (\exists t\colon T)\ \tau(x) =_T t \Rightarrow$ *d laughs at some time t*

Since in Suhrawardī's framework the explicit appeal to temporality comes to the fore when the counterpart of the individual witnessing the subject has been identified, we will make this enrichment process explicit once such an identification has taken place.

Statement	Challenge	Defence
Explicit Dialectical Encoding $\mathbf{X}\ !\ (\forall z\colon \{x\colon D \mid A(x)\})\ \mathbf{LM}B(\mathcal{L}^{\{\}}(z)^{\mathbf{Y}})$ All D's that are A, are necessarily contingently B – whereby "$\mathcal{L}^{\{\}}(z)^{\mathbf{Y}}$" stands for an element of D, who is an A, chosen by the adversary **Y**.	**Explicit Dialectical Encoding** $\mathbf{Y}\ !\ A(d_i)$ $\mathcal{L}^{\{\}}(z)=d_i : D$ **Y** states $A(d_i)$, whereby he chooses d_i as being one of those left components of z, that that are A in D.	**Explicit Dialectical Encoding** $\mathbf{X}\ b(d_i)\colon (\exists y\colon A_D)\ B(d_i) \supset\subset \sim B(y)$ **X** states that d_i witnesses B iff some d_j, chosen by **X**, witnesses its absence and makes the function performing the association function b explicit.
Dynamic Encoding $\mathbf{X}\ !\ (\text{Every}A_D)\mathbf{LM}B$	**Dynamic Encoding** $\mathbf{Y}\ !\ A(d_i)$ The challenge makes explicit the presence $d_i : D$ chosen by **Y** to test the universal	**Dynamic Encoding** $\mathbf{X}\ !\ (\text{Some}A_D)\ B(d_i) \supset\subset \sim B$ **X** states that d_i witnesses B iff some $d$witnesses its absence.

Statement	Challenge	Defence
Explicit Dialectical Encoding **X** b: $(\exists y: A_D)\ B(d_i) \supset\subset \sim B(y)$.	**Explicit Dialectical Encoding** **Y** $?_{\exists}$ **Y:** who is that y? . -	**Explicit Dialectical Encoding** **X** $b(d_i)$: $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$ Whereby $\mathcal{L}^{\rho\exists}(y)=d_i : A_D$ and $t_i \neq_T t_j$ **X** states that d_i is the y who also witnesses the absence of B, but, of course, at a time different to the one when it witnesses the presence of B. It might be also the case that d_i never witnesses B. This will be handled in our section on Plenitude. **Or** **X** $b(d_i)$: $B(d_i)@t_i \supset\subset \sim B(d_j)@t_j$ Whereby $d_j \neq d_i: A_D$ **X** states that d_j (different to d_i) is the y who witnesses the absence of B. The time of the latter might or not be the same as the one of the former.
Dynamic Encoding **X** ! (SomeA_D)$B(d_i) \supset\subset \sim B$	**Dynamic Encoding** **Y** Who is that A_D?	**Dynamic Encoding** **X** ! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$ Whereby $t_i \neq_T t_j$ **Or** **X** ! $B(d_i)@t_i \supset\subset \sim B(d_j)@t_j$ Whereby $d_j \neq d_i: A_D$

III. 3.3.2.2 Plenitude

Strong or Weak Plenitude Commitments come to the fore when the bi-implication is further analysed. If both sides commit to actual presences the assumed plenitude is strong, if at least one of the sides only commits to a hypothetical judgment Weak Plenitude is assumed.

STRONG PLENITUDE **Dynamic Encoding**		
Statement	**Challenge**	**Defence**
X ! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$	**Y** ! $B(d_i)@t_i$	**X** ! $\sim B(d_i)@t_i$

Whereby $t_i \neq_T t_j$	**Or** **Y** ! $\sim B(d_i)@t_j$ **Y** challenges the bi-implication by either stating the left or the right of it.	---------- **X** ! $B(d_i)@t_j$ **X** defends by stating the right if **Y** states the left and vice versa
X ! $B(d_i)@t_i \supset\subset \sim B(d_j)@t_j$ Whereby $d_j \neq d_i$: A_D	**Y** ! $B(d_i)@t_i$ **Or** **Y** ! $\sim B(d_j)@t_j$	**X** ! $\sim B(d_i)@t_i$ ---------- **X** ! $B(d_j)@t_j$

Weak Plenitude amounts to endorsing the strong statement that the potentiality involved might never by actualized by instances of the subject. Moreover, it amounts to stating that at least one of the sides of the bi-implication constitutes a hypothetical judgment – a judgment based on a non-actualized witness of the predicate at some putative time – that is a set of instants T^* defined in relation to the satisfaction of the condition H.

The CTT-notation allows expressing hypotheticals at the object-language level: .

Hypothetical/non-actualized presence *x* of the Hypothesis

$f(x)$: $P[x]$ $(x: H)$

function **non-actualized Predicate** **hypothesis**

non-actualized presence of *P*

Whereby the square brackets, as usual, indicate that x might or not occur in P

In fact, "purely hypothetical" reasoning, can represented by assuming a possibly infinite set of Hypotheses H_1, …H_n.. In such a case the verification function f is a multi-function, and we obtain $f(x_1, \ldots, x_n)$: $P[x]$ $(x_1: H_1, \ldots, x_n: H_n)$. In the literature, quite often it carries the abbreviated notation $\boldsymbol{f}$: $P[\boldsymbol{x}]$ $(\boldsymbol{x}: \boldsymbol{H})$. This notation is used to provide a proof-theoretical notion of modality – see Ranta (1991, pp. 86-88; 1994, pp. 145-150).[37]

[37] To put it bluntly, Ranta's basic idea is that given a ***yes*** or ***no*** question in relation to the say the proposition A, one might posit that A ($\sim A$) is true *within* a *context* **H**. A context is a set of dependent assumptions A_i.:

$x_1: A_1, \ldots, x_n: A_n(x_{1, \ldots, x_{n-1}})$

In order to avoid notational complexity involved in deploying such kind of hypotheticals, we will use the following dialectical rule that pragmatically leaves challenge and defence at a purely non-actualized level: It amounts to stating that the proposition $P[\boldsymbol{x}]$ can be formed ("conceived"), provided the list of hypotheses $\boldsymbol{H}$.

Statement	Challenge	Defence
X $P[\boldsymbol{x}]$: *prop* ($\boldsymbol{x}$: $\boldsymbol{H}$)	**Y** $\boldsymbol{x}$: $\boldsymbol{H}$	**X** $P[\boldsymbol{x}]$: *prop*
X states: the proposition $P[\boldsymbol{x}]$ can be formed ("conceived"), out of the list of hypotheses $\boldsymbol{H}$.	**Y** let us assume such a list	**X** under this assumption the proposition $[\boldsymbol{x}]$ results.

If we apply this notation to necessary contingency we obtain the following (we only display the dynamic encoding) dialectical meaning explanation of Weak Plenitude:

WEAK PLENITUDE **Dynamic Encoding**		
Statement	**Challenge**	**Defence**
X ! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$ Whereby t_j might not be different to t_i. This commits to Weak Plenitude: *B* might be both absent and present,	**Y** ! $B(d_i)@t_i$ **Or**	**X** ! $\sim B(d_i)@t^*_j$: *prop* ($\boldsymbol{x}$: $\boldsymbol{H}$) Whereby " t^*_j" stands for an instant, within the set T^*, dependent upon $\boldsymbol{H}$. ----------

The context can be read as a list of *approximations by a progressive specification* to answers to a question on the holding of *A* – cf. Ranta (1994, pp. 86-88). In other words, the answer if *A* holds or not is made dependent upon answers to some list of questions (not yet answered) – see Dango (2016). This approach naturally leads to belief contexts – see Ranta (1994, pp. 153-157). Consider an agent **a** stating judgements one after another, later ones possibly making reference to earlier ones. Let us assign to **a** a belief context, i.e. the beliefs or hypotheses he has adopted at some occasion.

$$\mathbf{H^a} = x_1\text{: } A_1, \ldots., x_n\text{: } A_n(x_{1, \ldots, x_{n-1}}).$$

Given a proposition $A(x_{1, \ldots, x_n})$ we obtain

a believes $A(x_{1, \ldots, x_n})$ is *true* or $A(x_{1, \ldots, x_n})$ is *in* the set of beliefs of **a.**

i.e.,

$$a(x_{1, \ldots, x_n})\text{: } A(x_{1, \ldots, x_n})\ (\mathbf{H^a});$$

which is a hypothetical judgement. The agent **a**, might move to another context $\mathbf{H^a_i}$ where he modified some of his beliefs. In fact, **H**, might encode not only one progressive list but several ones, constituting different hypotheses. Each of the lists can be interpreted as a list of *epistemic alternatives* to some initial list of beliefs associated with the claim – cf. Ranta (1994, pp. 147-150). Assume for example that *A* is made dependent upon, among other assumptions, of the truth of a disjunction, then, there might be a list of assumptions that extends the first list assuming that the first member of the disjunction holds, and a second one where it does not. The point is that each list of assumptions constituting a belief claim is not taken to be exhaustive and leaves room for further explorations: that is why possible truth is hypothetical rather than categorical. Notice that whereas the specification *within* each $\mathbf{H_i}$ is unidimensional, in the sense that it indicates *one way* the specification of the precedent assumption might take, the specification displayed by the alternative is multidimensional, i.e., it indicates *different ways* a specification can be conceived, or to use Ranta's (1994, pp. 145-147) own words different ways *to extend a context*. So, if $\mathbf{H^a_k}$ stands for some initial context, or shared context of beliefs, but at some stage two alternative extensions of this initial context develop in such a way that some new proposition *B* dependent upon the hypotheses in $\mathbf{H^a_k}$, is part of the hypotheses in $\mathbf{H^a_i}$ but not $\mathbf{H^a_j}$, where the new proposition *C* is present (but not in $\mathbf{H^a_j}$):

$$\mathbf{H^a_i} = \mathbf{H^a_k},\ y_1\text{: } B_1(x_{1, \ldots, x_n}), \ldots, y_m\text{: } B_1(x_{1, \ldots, x_n, y_{1, \ldots, x_{m-1}})$$

$$\mathbf{H^a_j} = \mathbf{H^a_k},\ z_1\text{: } C_1(x_{1, \ldots, x_n}), \ldots, z_k\text{: } C_k(x_{1, \ldots, x_n, z_{1, \ldots, z_{k-1}})$$

but one of the sides will be actual and the other hypothetical.	**Y** ! $\sim B(d_i)@t_n$ Whereby "t_n" stands for *any* instant t including t_i **Y** challenges the bi-implication by either stating the left or the right of it. If **Y** is committed to the absence of B, he might choose an arbitrary time or even state the stronger statement that B is **never** actualized by d_i.	**X** ! $B(d_i)@t^*_j$: *prop* (x: ***H***)
X ! $B(d_i)@t_i \supset\subset \sim B(d_j)@t_j$ Whereby $d_j \neq d_i$: A_D	**Y** ! $B(d_i)@t_i$ **Or** **Y** ! $\sim B(d_j)@t_n$	**X** ! $\sim B(d_i)@t^*_i$: *prop* (***x***: ***H***) ---------- **X** ! $B(d_j)@t^*_j$: *prop* (***x***: ***H***)

Notice that the hypotheses ***H*** can be conceived as the condition for the actualization of the B. For instance, if B stands for the capacity of *Literacy*, then ***H***, can stand for the condition *Education* (and some relevant further conditions). So, whereas Whereby $\sim B(d_i)@t_n$ states that d_i never actually acquired the capacity of *Literacy*, $B(d_i)@t^*_j$ (***x***: ***H***) states that this capacity can still be predicated of him, though only hypothetically – i.e. provided at some putative time t_{j*} it can be assumed that d_i enjoys *Education*

Three final remarks concerning Weak Plenitude:

- In principle, Weak Plenitude seems to make more sense when applied to acquired capacities rather than to "natural" ones – it seems implausible to claim that I will never laugh, but it is sensible to claim that no human will actually ever master 150 languages.
- Though logically speaking, the case with two different presences of A_D seems not to be necessary, we included it because it looks as if Suhrawardī considers this case to constitute the most important form of contingency.
- Suhrawardī does explicitly mention neither Strong nor Weak Plenitude, he prefers instead the more cautious move to assume that there are potentialities that might never actualize (at least, for an individual), and others that do actualize at least once.

III.3.4 Simple Convertibility

In order to implement convertibility, we will index the notation for modalities with the indexes distributed according to the following configurations.

	Admits Simple Conversion	**Does Not Admit Simple Conversion**
Necessity	**$L^{\Leftrightarrow}$**	**$L^{\sim\Leftrightarrow}$**
Contingency	**$M^{\Leftrightarrow}$**	**$M^{\sim\Leftrightarrow}$**

The dialectical meaning explanation will thus allow further challenges, namely

1 requesting to specify the kind of necessity or contingency involved.
2 requesting to show the application or not of simple convertibility

Statement	Challenge 1	Defence1	Challenge2	Defence2
X ! $(\forall A_D)$**LL***B*	**Y ? L**$^{\Leftrightarrow}$ | **L**$^{\sim\Leftrightarrow}$ Does it admit simple conversion or not?	**X** ! $(\forall A_D)$**LL**$^{\Leftrightarrow}$*B* It admits simple conversion.	**Y** ?$^{\Leftrightarrow}$ Execute the conversion	**X** ! $(\forall B_D)$**LL***A*
		X ! $(\forall A_D)$**LL**$^{\sim\Leftrightarrow}$ *B* It doesn't admit simple conversion.	**Y ?**$^{\sim\Leftrightarrow}$ Show the non-convertibility.	**X** ! $(\exists B_D)$**LL**~*A*) There is at least one *B* for which *A* is impossible
X ! $(\forall A_D)$**LM***B*	**Y ? M**$_{\Leftrightarrow}$ | **M**$_{\sim\Leftrightarrow}$ Does it admit simple conversion or not?	**X** ! $(\forall A_D)$**LM**$^{\Leftrightarrow}$*B* It admits simple conversion.	**Y** ?$^{\Leftrightarrow}$ Execute the conversion	**X** ! $(\forall$**LM***B_D*$)$**LL***A*
		X ! $(\forall A_D)$**LM**$^{\sim\Leftrightarrow}$*B* It doesn't admit simple conversion	**Y ?**$^{\sim\Leftrightarrow}$ Show the non-convertibility.	**X** ! $(\exists$**LM***B_D*$)$**LL**~*A* There is at least one necessarily contingently *B* for which *A* is impossible

IV Syllogism: Dialectical Meaning Explanations at Work

Suhrawardī's relational view on modalities developed within a framework of dialectical meaning explanations discussed above shapes his take on syllogisms. Let us now test our interpretation by examining his take on syllogism. We will develop proofs within a dialogical framework that follows Suhrawardī's own ways to present his proofs (though they are often quite sketchy) based on what we called his dialectic meaning explanations of modalities.

IV.1 First figure

On *Manṭiq al-talwīḥāt* there is a development that indicates how the meaning explanations are to be deployed in a proof. The text involves the controversial *Barbara* mood where the minor premise contains a possibility modality, the major a categorical universal and the conclusion a possibility (usually notated as **XMM** in the Aristotelian notation and **MXM** in the Islamicate one). Moreover, Suhrawardī's argument already prefigures his main *illuminationist rule* for the first figure set in *al-Ishrāq* (1999, pp. 22-23), whereby **XMM** is reduced to **MMM**):

> Know that the conclusion in first-figure syllogisms follows the major in the mixed-premise syllogisms, except when the minor is possible and the major is existential. If we say 'Possibly every J is B' and 'Actually (*bi'l-wujūd*) every B is A', it is known from the nature of possibility that it may never actually occur; so if the J is never described as B, it does not follow that the A comes to it actually, but only potentially, so it's possible.
> *Manṭiq al-talwīḥāt* (1955, p. 35-36), quoted and translated in Street (2008, p. 170).

The point seems to be that if the universal of the major premise *Every B is A* is read as asserting that actual instances of *A* are predicated of actual instances of *B*, then if the conclusion should follow the major premise, then then the major term in the conclusion should be an actualization of *A*. However, the instances of the middle term *B*, we obtain from the first premise *Possibly every J is B* – i.e. *Every J is possibly B,* might not an actualized *B*, thus the middle term, if univocally understood in both premises, should be thought as standing for a possibility. In such a case, Suhrawardī concludes, *A* can only be predicated potentially. This line of thought leads Suhrawardī to assume that in a proof of such a kind of syllogism the middle and major terms are not actualized potentialities (in both premises) – which in fact amounts to *al-Ishrāq*'s rule for the first figure whereby a term occurring in a syllogism of the first figure is required to have the same modality/non-modality in all the places it occurs.

The development of the proof sketched in the text assumes in fact that the categorical universal of the second premise, includes non-actualized capacities in the subject and the predicate – this coincides with our previous remark that the terms involved in a categorical can be read as admitting modalities .

If we display Suhrawardī's argument sketched in the text quoted above, as a dialogue with a proponent **P** and an Opponent **O**,[38] we obtain:

0. **P** ! I can show that every element of domain of discourse *D*, who is a *J* is possibly *A* follows from the premises: I), Every element of *D* who is J*,* is possibly *B* and II) Every element of *D* who is *B* is *A*.
1. **O** ! Fine, I give you the premises. Show me now, that the consequent of the conclusion follows from some d_i in the domain of discourse *D* who is *J* : **O** challenges the conclusion with the move **O** ! $J(d_i)$.
2. **P** ! What I will do is to show you that the endorsement of the premises will force you to assert this consequent. Let us start with the first premise. Since the first premise states that every element of the domain of discourse who is *J* is possibly *B*, and you just chose d_i in the domain of discourse with your fist move (sic move 1), that *B* is possible should also hold of d_i , right?: **P** challenges the first premise with the move **P** $you_1\ J(d_i)$; whereby "you_1" stands for the indication "you just stated the same at move 1).
3 . **O** ! Indeed, I have to assume that d_i is one of those in the domain of discourse that are possibly *B*. However, notice that I neither state herewith that *B* is actualized in d_i, nor do I state that *B* is never actualized. **O** defends the first premise with the move **O** ! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$ – we skip the steps leading to this answer, whereby the Opponent took the choice to focus on the same individual **O** ! $(\exists y{:}A_D)B(d_i)\supset\subset \sim B(y)$, **P** $?_\exists$.
4. **P** ! Ok, however, premise II states that every element of the domain of discourse who is *B*, is *A*, this must include all those elements in *D* who are non-actualized instances of *B*, so let us again take precisely this d_i that you just conceded with your move 3 as being a non-actualized possible

[38] The informal presentation below should be sufficient to follow the development of a dialogue. In the appendix we provide a short overview of rules for dialogical logic. The reader might also consult Clerbout & McConaughey's entry Dialogical Logic in the Stanford Encyclopedia of Philosophy.

B. This possible *B*, must be a possible *A*, right?: **P** challenges the second premise with the move **P** *you*$_3$: $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$.

5. **O** ! Right. This element of the domain *D* must be possibly *A*: **O** defends the second premise with the move **O** ! $A(d_i)@t_{i'} \supset\subset \sim A(d_i)@t_{j'}$.
6. **P** ! But this is exactly what you asked me to show. You just conceded it with your move 5: **P** defends the conclusion with the move **P** *you*$_5$: $A(d_i)@t_{i'} \supset\subset \sim A(d_i)@t_{j'}$.

N.B. Notice that if at move 5, instead of answering, **O** decides that he wishes to state after all that d_i is never *B* and challenge **P**'s move 4, **P** can copy-cat the same move and challenge himself move 3 of **O**. If **O** answers the latter challenge with the move corresponding to Strong Plenitude, then **P** can emulate the same answer for his response to this new version of **5**. Similar sequence of moves will trigger **O**'s endorsement to Weak Plenitude. Once this has been carried out the play will resume with the original moves 5 and 6. However, it looks as if Suhrawardī way to proof such kind of syllogisms involving contingency, does not commit to any side of the bi-implication.

This development is based on the following rules, that we take from Rahman et al. (2018, p. 62), adapted to syllogism in McConaughey (2021, chapter 4.2, table 4.9) and present in a simplified form:

1. Starting rule
The player who states the conclusion move 0 is the proponent **P.**

2. Development rule
Once the starting rule has been implemented, each player in turn plays a move according to the dialectical meaning explanations for quantifiers, connective, modalities and the other structural moves.

3. Socratic rule
Some specific propositions, we call them *unanalysable constituents,* may not be stated by **P**, unless **O** stated them before. **O** can state such propositions when required. When **P** states such a proposition, he will justify it with the indication *you*$_n$, which indicates that his statement is backed by **O**'s endorsement of it at move *n*, and that he (**P**) adheres himself to the knowledge conveyed by **O**'s endorsement.

In the context of Suhrawardī's logic unanalysable propositions include positive and negative literals (i.e. elementary propositions with and without negation). In order to shortcut the length of a play, expressions of unactualized capacities such as $B(d_i)@t_i \supset\subset \sim B(d_j)@t_j$ will also be treated as unanalysable. However, if, for some philosophical reason, needed these expressions can be further analysed. Defining such expressions as unanalysable meets purely logical aims. Unanalysable constituents cannot be challenged (since **O** is allowed to state them when required and **P** only states them after **O** endorsed them by stating them before).

4. Pragmatic coherence rule (concerns mainly the third figure)
When the conclusion Proponent defends is particular and all the premises Opponent defends are universal, Proponent may request the Opponent to instantiate the subject of a premise with the instance d_i, chosen by **P**, **provided** d_i is **new**: challenge: **P** $?_{J(di)}$; defence: **O** ! $J(d_i)$ (for a universal with $\{x: D \mid J(x)\}$ as subject, and *J* as subject-term), (this prevents **O** to state $J(d_i)$ when he endorsed before some $J^*(d_i)$ whereby *J* and J^* are incompatible).

5. Ending rule
The player who states ⊥ give-up, immediately loses. Otherwise, the player who has no available move left at this turn loses.

- Comments on the Rules.
  i) The rationale behind the *Socratic rule* is that proving the conclusion of syllogism within a dialectical framework amounts to analysing the premises in such a way that the resulting statements are those that constitute the conclusion. Since the premises are stated by **O** and the conclusion by **P** the later but not the former is committed

to justify the conclusion by justifying each of the constituents of the conclusion, by grounding them on statements of **O** involving constituents of the premises. In other words, the use of the Socratic rule allows defining a winning strategy for **P** (the dialectical way to proof validity) as a sequence of moves that force **O** to state those constituents of the premises that should provide a justification of the conclusion.
In standard dialogues *unanalysable constituents* are elementary propositions. In Suhrawardī's logic we need to add expressions of not actualized capacities and also metathetic negations added to elementary propositions, since the former is part and parcel of his take on necessary contingent propositions and the latter concerns his proposal to deal with negated elementary propositions as affirmative ones.

ii) The rationale behind the pragmatic coherence rule, will be commented when we discuss the third figure: it concerns the way to deal with ontological assumptions such as those required by *Darapti.*

iii) The prescription on giving up in the ending rule concerns the dialogical interpretation of negation. When challenging a negation such as $\sim A$ stated by player **X** , the challenger **Y** must now overtake the burden of the proof and state A. The defender of the negation has two options, either counterattack A, or simply give up and concede. The latter is indicated by the move **X** ! ⊥ give-up. In the Aristotelian texts the move ⊥ give-up, corresponds to the dialectical use of the term ἀδύνατον.

Let us now present Suhrawardī's argument in the form of a dialogue that implements these rules, focusing on his own example:

All humans are necessarily contingently literate
All literate beings are walkers
Therefore, all humans are necessarily contingently walkers

If we place a syllogism within a dialogue, then, the idea is that the proponent **P**, claims that that the conclusion holds if the opponent **O**, concedes to state the premises. This yields the notation:

O ! (EveryJ_D)**LM**B	**O** ! $(\forall z\colon \{x\colon D \mid J(x)\})$ **LM**$B(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
O ! (EveryB_D)A	**O** ! $(\forall z\colon \{x\colon D \mid B(x)\})$ $A(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
---	---
P ! (EveryJ_D)**LM**A	**P** ! $(\forall z\colon \{x\colon D \mid J(x)\})$ **LM**$A(\mathcal{L}^{\{\}}(z))^{\mathbf{O}}$ [39]

- Recall that explicit dialectical notation, also indicates which player is in charge of the substituting the variables. Since the premises are stated by the opponent, and these are universals, it is the proponent who will choose the presence of the subject of which the predicate is requested to be stated. The dual is the case of the universal in the conclusion: since the conclusion is stated by the Proponent, it is the Opponent who will choose the presence of the subject of which the predicate is requested to be stated. In fact, a consequence of the Socratic rule is that a winning strategy for **P** should follow the idea to leave **O** choose first and the copy-cat this choice for his own challenges to the premises. Notice that **O** is forced to choose if he challenges a universal (stated by **P**) or defends an existential stated by himself.

- In order to keep close to the text we skip and in most of the dialogues the steps **X** $(\exists y{:}A_D)$ $B(d_i)\supset\subset \sim B(y)$, **Y** $?_\exists$. These steps will become important in the second figure because of negation.

[39] If we deploy the formalization **L**$(\forall x\ (J(x) \supset \mathbf{M}B(x)))$, **L**$(\forall x\ (B(x) \supset \mathbf{M}A(x))) \vdash$ **L**$(\forall x\ (J(x) \supset \mathbf{M}A(x)))$ and assume contemporary modal logic K: **L**$(A\supset B) \vdash \mathbf{M}A\supset\mathbf{M}B$ will be required – see Movahed (2012; p. 9), which assumes one-sided possibility. But this contravenes Suhrawardī's notion of contingency, whereby a contingency is neither necessary nor impossible.

Moreover, we will assume that **X's** answer, focuses on the presence and absence of *B* in the same individual. The resulting variants, triggered by the option of introducing a second individual are pretty straightforward.

O		**P**		
I	! $(\forall z$: $\{x$: $D \mid J(x)\})$ **LM**$B(\mathcal{L}^{\{\}}(z))$		! $(\forall z$: $\{x$: $D \mid J(x)\})$ **LM**$A(\mathcal{L}^{\{\}}(z))$	0
II	! $(\forall z$: $\{x$: $D \mid B(x)\})$ $A(\mathcal{L}^{\{\}}(z))$			
1	! $J(d_i)$ $\mathcal{L}^{\{\}}(z)=d_i$: D	?0	you_5: $A(d_i)@t_i$, $\supset\subset \sim A(d_i)@t_j$,	6
3	! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_i$		?I ! $J(d_i)$ $\mathcal{L}^{\{\}}(z)=d_i$: D	2
5	! $A(d_i)@t_i$, $\supset\subset \sim A(d_i)@t_j$, (possibilist interpretation of the universal in II)		?II you_3: $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$ (possibilist interpretation of the universal in II)	4
			Proponent wins	

- With move 4 **P** deploys a possibilist interpretation of the universal categorical in the second premise. That is *every B*, is understood as including also contingent cases of *B*.
- The dialogue ends since 6 is an unanalysable constituent of the conclusion, namely the not actualized capacity $A(d_i)@t_i$, $\supset\subset \sim A(d_j)@t_j$, that cannot be challenged, since **O** himself endorsed it with move 5.
- As mentioned above, if we prefer not to include not actualized capacities among the *unanalysables* and further analyse the bi-implication the end-result of the dialogue will not change. It will only a bit longer: as soon as **O** challenges the bi-implication, **P** will do the same. However, this does not seem to be the way Suhrawardī develops syllogisms involving capacities.

O		**P**		
I	! (EveryJ_D)**LM**B		! (EveryJ_D) **LM**A	0
II	! (EveryB_D)A			
1	! $J(d_i)$	?0	you_5: $A(d_i)@t_i$, $\supset\subset \sim A(d_i)@t_j$,	6
3	! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$		?I you_1: $J(d_i)$	2
5	! $A(d_i)@t_i$, $\supset\subset \sim A(d_i)@t_j$,		?II you_3: $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$	4
			Proponent wins	

P can repeat the same sequence of moves for any arbitrary element of discourse **O** happens to choose to challenge the universal in the conclusion. In other words, **P** has a winning strategy for this syllogism, and therefore it is valid.

The strategy can be seen as a "recapitulation" and generalization that produces an algorithm for winning.[40] In our case, informally the winning strategy amounts to the following:

1) Let **O** choose any arbitrary instance of the universal in the conclusion
2) **P** should use exactly this instance, *whichever* this instance chosen by **O** is, to challenge the first premise, and force **O** to predicate *B* of it
3) **O** predicates *B* of it, but as a not actualized capacity
4) **P** should use exactly this endorsement of **O (**that the non-actualized capacity *B* can be predicated of the instance at stake), to challenge the second premise

[40] As pointed out by McConaughey (2021, p. 140), Kapp (1942, pp. 14-16 & 71) points out the importance of two stages in a dialectical context, anticipation and recapitulation. These are indeed, the elements that allow to build a winning strategy. See too Crubellier (2011) who indicates that one of the first meanings of *syllogismos* is precisely recapitulation.

5) **O** is forced to predicate *A* of it, but again chooses to endorse it as a not-actualized capacity
6) **P** can now use this last endorsement to respond to the challenge to the conclusion

- **Apply this sequence for any d_i chosen by O at move 1**

The emergent winning strategy can be represented as a sequent calculus, where **P**'s assertions are to translated as assertions at the right of the turn-style and **O**'s assertions at the left – Rahman, Seck, Drissi (forthcoming). However, the point is that, this sequent calculus has been generated by the winning strategy produced by the dialogue: it is the interaction between players that puts the dialogical meaning explanation of the modalities at work, by fleshing out the meaning of each constitutent. The sequent-calculus is only the abstract outcome based on a *pensée aveugle,* to put in Leibniz's words.

Once more, in the text *Manṭiq al-talwīḥāt* (1955, p. 35-36) quoted above, the subject and the predicate of the major premise (here premise II), are both modalized de facto. Or to put it otherwise the categorical universal include actual and merely possible instances of the subject.

The point is that, on Suhrawardī's view the middle and major term of a productive syllogism of the first figure must share the same modality in both of its premises and its conclusion. In *al-Ishrāq* (1999, pp. 22-23), Suhrawardī explicitly formulates this as a rule and provides two examples, namely, when the major term relates to its subject by necessity and when the major term relates to its subject by contingency. The latter amounts to making explicit the contingent modality assumed in his discussion of **MXM** (or **XMM** in the Aristotelian notation) in *Manṭiq al-talwīḥāt* (1955, p. 35-36):

> There is no need to multiply the moods of syllogism, rejecting some and accepting others. Further, since the last term leads to the first term by means of the middle, the modes in the definitely necessary proposition are made part of the predicate in one or both of the premises, thus leading to the major. For example, 'All men. are necessarily contingently literate, and all contingently literate beings are necessarily animals by necessity (or contingently walkers), therefore, all men are necessarily animals by necessity (or contingently walkers). *al-Ishrāq* (1999, pp. 22-23).

In one of the examples the middle term is necessarily contingent and the major term is necessarily necessary also in both premises. In the second example the middle term is necessarily contingent as in the first example, but the major term is necessarily contingent. In both examples the modalities of the terms are same wherever they occur.

Let us work out the dialogues for both examples, but we will only display the informal encoding:

Necessarily Predication of the Subject in the Major

All humans are necessarily contingently literate
All (necessarily) contingent literate beings are necessarily animals, by necessity
Therefore, all humans are necessarily animals, by necessity

O ! (EveryJ_D)**LM***B*	**O** ! $(\forall z\colon \{x\colon D \mid J(x)\})$ **LM***B*$(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
O ! (Every**LM***B*$_D$)**LL***A*	**O** ! $(\forall z\colon \{x\colon D \mid$ **LM***B*$(x)\})$ **LL***A*$(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
----------	----------

P ! (EveryJ_D)**LL**A **P** ! ($\forall z$: {x: D | $J(x)$}) **LL**$A(\mathcal{L}^{\{\}}(z))^{\mathbf{O}}$

O			**P**		
I	! (EveryJ_D)**LM**B			! (EveryJ_D)**LL**A	0
II	! (Every**LM**B_D)**LL**A				
1	! $J(d_i)$	?0		you_5: $A(d_i)$	6
3	! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$		?I	you_1: $J(d_i)$	2
5	! $A(d_i)$		?II	you_3: $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$	4
				Proponent wins	

- The dialogue ends since 6 is an unanalysable constituent of the conclusion, namely the elementary proposition $A(d_i)$, that cannot be challenged, since **O** himself endorsed it with move 5.

Necessarily Contingent Predication of the Subject in the Major

All humans are necessarily contingently literate
All (necessarily) contingent literate beings are necessarily contingently walkers
Therefore, all humans are contingently walkers by necessity

O ! (EveryJ_D)**LM**B **O** ! ($\forall z$: {x: D | $J(x)$}) **LM**$B(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
O ! (Every**LM**B_D)**LM**A **O** ! ($\forall z$: {x: D | **LM**$B(x)$}) **LM**$A(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
---------- ----------
P ! (EveryJ_D)**LM**A **P** ! ($\forall z$: {x: D | $J(x)$}) **LM**$A(\mathcal{L}^{\{\}}(z))^{\mathbf{O}}$

O			**P**		
I	! (EveryJ_D)**LM**B			! (EveryJ_D)**LM**A	0
II	! (Every**LM**B_D)**LM**A				
1	! $J(d_i)$	?0		you_5: $A(d_i)@t_i, \supset\subset \sim A(d_i)@t_j$	6
3	! $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$		?I	you_1: $J(d_i)$	2
5	! $A(d_i)@t_i, \supset\subset \sim A(d_i)@t_j$		?II	you_3: $B(d_i)@t_i \supset\subset \sim B(d_i)@t_j$	4
				Proponent wins	

- The dialogue ends since 6 is an unanalysable constituent of the conclusion, namely the not actualized capacity $A(d_i)@t_i, \supset\subset \sim A(d_i)@t_j$, that cannot be challenged, since **O** himself endorsed it with move 5.

Negative moods of the first figure are not a problem if we follow Suhrawardī's own formulation where he places the negation before the possibility or more precisely as an impossibility. Thus, Suhrawardī's

> *All humans are necessarily impossibly stones.*
> *al-Ishrāq* (1999, p. 23)

Can be encoded as

(Every$Human_D$)**L~M**$Stone$

However, this seems to produce a difficulty in the second figure.

IV.2 Second Figure

If the rule that terms must be the "same" wherever they occur is generalized for the three figures there might be a difficulty, for the second figure in those moods where the middle term is negated in one of the premises and not negated in the other. In the following text, where Suhrawardī stresses the relational nature of modalities, the solution proposed is to allow this difference.

> (26) Whenever there are two universal propositions with different subjects, and the predicate of one cannot be affirmed of the other in all respects, or in some one respect, […]. The conclusion will be definite and necessary, asserting the impossibility of predicating its predicate [on the other subject] or the necessity of negation therein. So affirmations and negations will be made parts of the predicate, as in: "All men are necessarily contingently literate," and "All stones are necessarily impossibly literate." We thus know that " All men necessarily are impossibly stones." **So, in this specific mood, it is not a condition that the predicates *be* the same in every aspect.** They need only be *the* same in that which they share apart from the mode that is made part of the predicate, and it is thus permissible for the two modes of the two premises to be different. They differ from the first figure in that these two statements are propositions such that what is impossible for the subject of one of the two is possible for the subject of the other. For each of the propositions, what is possible for the subject of one is impossible for the subject of the other. Their two subjects are necessarily incompatible, yielding the conclusion that these two statements are propositions whose subjects are necessarily different. Likewise, if the predicate of one definite proposition has a **contingent relation** and [the predicate of] the other a necessary relation, then a **necessary relation** is impossible for the first and contingency is impossible for the other. Likewise, if the predicate of one has a **contingent relation** and [the predicate of] the other a relation of impossibility, it *is* as we had said before.). *al-Ishrāq* (1999, pp. 23-24) – our highlighting.

The example that provides the main pattern for this figure is

> All humans are necessarily contingently literate
> All stones are necessarily impossibly literate
> Therefore, all humans are necessarily impossibly stones

Given the premises, whoever human we choose, this human has the potential of being literate, and whichever stone we choose, it is impossible this stone to have the potential of being literate. Thus, it is impossible for any human to become a stone.

Now, in order this argument to go through, we need to allow the subject of the second premise to include all those objects that have the potential of becoming a stone, precisely the potential that occurs as predicate of the conclusion. So here again, the subject of the second premise is assumed to include both actual stones and those objects having the potential to become stone.[41]

The development of the dialogical proof for this mood of the second figure makes these two features apparent, namely, (a) the negative and not negative occurrence of the middle term, and (b) the "possibilist" interpretation of the universal quantification in the second premise. In relation to the second feature, the point is that not (necessarily) actualized presences of the capacity of becoming a stone, that instantiate the predicate of the

[41] Street (2008, p. 171) points out that this argument is very close to Ibn Sīnā's own defence of modal *Camestres.*

conclusion, are to be included in the subject of the second premise. This will lead to a contradiction that makes settles the incompatibility of both of the subjects of the premises.[42]

O ! (EveryJ_D)**LM***B*	**O** ! ($\forall z$: {x: D | $J(x)$}) **LM***B*($\mathcal{L}^{\{\}}(z)$)$^{\mathbf{P}}$
O ! (EveryA_D) **L~M***B*	**O** ! ($\forall z$: {x: D | $A(x)$}) **L~M***B*($\mathcal{L}^{\{\}}(z)$)$^{\mathbf{P}}$
----------	----------
P ! (EveryJ_D)**L~M***A*	**P** ! ($\forall z$: {x: D | $J(x)$}) **L~M***A*($\mathcal{L}^{\{\}}(z)$)$^{\mathbf{O}}$

O			**P**		
I	! (EveryJ_D)**LM***B*			! (EveryJ_D) **L~M***A*	0
II	! (EveryA_D) **L~M***B*				
1	! $J(d_i)$	?0		! ~($A(d_i)@t_{i'} \supset\subset$ ~$A(d_j)@t_{j'}$)	2
3	! $A(d_i)@t_{i'} \supset\subset$ ~$A(d_j)@t_{j'}$				
5	! (SomeA_D) $B(d_i) \supset\subset$ ~B		?I	! you_1: $J(d_i)$	4
7	! ~((SomeA_D) $B(d_i) \supset\subset$~B))		?II	you_3: $A(d_i) \supset\subset$ ~$A(d_j)$ (possibilist interpretation of the universal in II)	6
13	! ⊥ give-up		?7	you_5: (SomeA_D) $B(d_i) \supset\subset$ ~B	8
9	Who is that A_D?	?8		$B(d_i)@t_i \supset\subset$ ~$B(d_j)@t_j$	12
11	$B(d_i)@t_i \supset\subset$ ~$B(d_j)@t_j$		?5	Who is that A_D?	10
				Proponent wins	

- The negation in 7 forces **P** to state the dual of this statement with move 8. **O**'s challenge in move 9 requests P to choose an individual who instances not *B*. P will delay his choice until **O** did his before. Once this has been accomplished he can force **O** to contradict himself. For the sake of variation, we considered the case where **O** chooses here a different individual. It is this possibility of **P** delaying his choice that makes it desirable to make explicit the moves that elucidate why the bi-implication in 5 and 7 share the same timing.
- The dialogue ends since **O** gives up with move 13, after **P** challenges moves 7 that contradicts **O**'s own endorsement of $B(d_i) \supset\subset$ ~$B(d_j)$ at move 5. Notice that when, with move 3, ! $A(d_i)@t_{i'} \supset\subset$ ~$A(d_j)@t_{j'}$, **O** challenged **P**'s negation stated at move 2, **P** did not give up. He rather used **O**'s endorsement of $A(d_i)@t_{i'} \supset\subset$ ~$A(d_j)@t_{j'}$ to challenge the second premise.

IV.3 Third Figure

The third figure has some difficulties linked to negations in the premises, the existential in the conclusion and the fact that the example does not seem to mention modalities or not that clearly so

> And if both premises contain negations, then the two negations should be made part of the two predicates. One should say, "All men are non-birds, and all men are non-horses." The conclusion will be affirmative: "Something described as a non-bird is a non-horse."
> […]
> Therefore, it is not necessary that each single one of the two be described by the other, but it will be necessary that something of the one be described by the other.

[42] Proofs of this syllogism in a framework that makes use of contemporary modal logic, may need K4 (transitvity axiom) and K5 axiom or K5 and Barcan – see Movahed (2012, p. 13).

> […]
> The validity of the syllogism will depend only on this: the certainty that one thing is described by two things. It differs from the first figure in that the two statements are propositions within each of which is something described by each predicate and within which is something described by both predicates. Therefore, an individual described by one predicate is also described by the other. This is all there is to these two propositions, and we can dispense with lengthy discussions. *al-Ishrāq* (1999, pp. 25).

The example mentioned seems to be a non-modal syllogism, that at a first looks seems to challenge Aristotle's dictum that there is no conclusion from two negatives premises. However, Suhrawardī's method of metathetic negation is to integrate the negation into predicate, deal with them as affirmative forms and obtain a case of what is known in the Aristotelian tradition as *Darapti*:

> All humans are non-birds
> All humans are non-horses
> Therefore, some non-birds are non-horses

The ensuing elucidation, lead to some interpreters as Movahed (2012, p14) to assume a necessarily necessary relation that can later on be eliminated.

> Therefore, it is not necessary that each single one of the two be described by the other, but it will be necessary that something of the one be described by the other. *al-Ishrāq* (1999, pp. 25).

In our framework such a reading yields

$\mathbf{O}\,!\,(\text{Every}J_D)\sim B$	$\mathbf{O}\,!\,(\forall z\colon \{x\colon D \mid J(x)\})\sim B(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
$\mathbf{O}\,!\,(\text{Every}J_D)\sim A$	$\mathbf{O}\,!\,(\forall z\colon \{x\colon D \mid J(x)\})\sim A(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$

Now, keeping the modality tacit, the point of these few lines is that it suggests that the validation of the syllogism follows from a kind of *Ars inveniendi*, whereby the conclusion is constructed, by picking one individual who witnesses both properties expressed by the predicate-terms in the premises, and then creating out this individual the set of all those of these individuals that enjoy that both properties.

Indeed, the resulting set is what the existential some non-birds are non-horses amounts to. In other words, the (convertible) existential

$$(\exists z\colon \{x\colon \textit{Animals} \mid \sim \textit{Birds}\ (x)\})\ \sim \textit{Horse}(\mathcal{L}^{\{\}}(z))$$

amounts to building the set of those animals-non-birds, that are non-horses) – more precisely the intensional Σ-type:

$$\{z\colon \{x\colon \textit{Animals} \mid \sim \textit{Birds}\ (x)\} \mid \sim \textit{Horses}(\mathcal{L}^{\{\}}(z))\}$$

The middle term in the two premises tell us which of the animals that are neither birds nor horses to choose to specify the set :

> choose those animals who are humans (rather than insects).

Now, the existential in the conclusion involves two main issues that have to be dealt with

1) the usual ontological assumption required by *Darapti*
2) should we keep the subject-predicate structure or rather link in the conclusion the minor and major terms with a conjunction?

In relation to the first issue let us recall that Suhrawardī's take on definitely necessary propositions involves the assumption that instances of the subject must be actual – in short, the subject must be present.

The usual way to implement this is to add as a third premise an existential that assures the non-emptiness of the subject of the minor and the major premises – see for example Movahed (2012, p. 14), or instead of a premise the non-emptiness of the subject-term, is added as an Existential occurring in the consequent of the universals – e.g. Hodges (2016, p. 159).

However, this does not render justice to the fact that ontological commitment is to understood as part of the contextually bounded meaning of the definitely necessary propositions.

Stating a necessary universal commits **O** to presences of the subject, however, in a context where **O** stated two universals with the same Subject-terms, then once **O** endorsed that one individual witnesses, the subject-term, **P** can now use this endorsement in order to challenge the second universal: **O** stated after all that every witness of the Subject-term enjoys the property expressed by the Predicate-term.

Moreover, the precise specification of the set that constitutes the conclusion, has to be constructed out of **O**'s endorsements during the dialogical interaction. Time here is of the essence, but, as we will briefly discuss in the next section, here the point is the time involved in the construction – as pointed out by Ardeshir (2008), rather than the (resulting) time attached to the presences of the terms.

There are two ways to implement this approach to Suhrawardī's take on *Darapti,* namely

- deploying Suhrawardī's method of ecthesis in the conclusion in order to *universalize away* the existential – as in the case of *Darii* discussed above
- deploying the so-called *pragmatic coherence rule,* which stresses the dynamics of the dialectical framework

DARAPTI UNIVERSALIZED AWAY

This seems to be the solution envisaged by Suhrawardī, because of his stance on the epistemic priority of universals. According to this we specify the set of non-birds and separate the set of, say, those non-birds who are rational beings: Accordingly, we substitute the term *non-birds* with (*non-birds* who are) *rational beings* and obtain

All humans are (non-birds who are) rational beings

All humans are non-horses

The first premise admits simple conversion and we obtain the syllogism in *Barbara*

All (animals who are non-birds who are) rational beings are humans
All (animals who are) humans are non-horses
Therefore, all (non-birds who are) rational beings are non-horses

$(\forall u\colon \{z\colon \{x\colon \mathit{Animals} \mid \sim \mathit{Birds}\ (x)\} \mid \mathit{Rational}(\mathcal{L}^{\{\}}(z))\})\ \mathit{Humans}(u)$
$(\forall z\colon \{x\colon \mathit{Animals} \mid \mathit{Humans}\ (x)\})\ \sim \mathit{Horses}(z)$

$(\forall w\colon \{z\colon \{x\colon \mathit{Animals} \mid \sim \mathit{Birds}\ (x)\} \mid \mathit{Rational}(\mathcal{L}^{\{\}}(z))\})\ \sim \mathit{Horses}(w)$

N.B. for the sake of notational simplicity, we did not display the further logical structure of *u*, *z*, and *w* in the consequences of the universals

Clearly, the existential *Some (non-birds who are) rational beings are non-horses* of the original conclusion obtains by sub-alternating the conclusion of the *Barbara*:

Something described as a non-bird is a non-horse. al-Ishrāq (1999, pp. 25)

That is

Something (namely, *rational beings*)*, described as a non-bird is a non-horse.*

Since we can use the same procedure for non-horses, we obtain indeed

Therefore, an individual described by one predicate is also described by the other. al-Ishrāq (1999, pp. 25)

As mentioned in our discussion on *Darii* choosing how to specify non-birds, is arbitrary provided, here, the result converts with human.

As pointed out by Movahed (2010, p. 15) the method is from the point of logic not general enough. Clearly, we cannot always assume that the relevant set (the Σ-type) can be specified in a suitable way – see Movahed (2010, p. 15). Actually, this stresses the fact that Suhrawardī's logic assumes a fully interpreted language.

Nevertheless, let us point out that the introducing a way to deal with ontological commitment is not a logical move either, i.e., it is not syntactically motivated either.

Moreover, the *logic of presence*, at least in our reconstruction, seems to prefigure many of the tenets of constructivism, whereby the assertion of an existential commits to exhibiting an instance. Suhrawardī's method adds a further note to this request, the exhibited *presence*, might seen as a representative of a set that specifies the original Σ-type expressed by the existential. A specification enough for universalizing away the existential in the context of an inferential process. True, it is not always clear how the specification should look and if one suitable can always be found: this must be left to the context.

DARAPTI AND THE PRAGMATIC COHERENCE RULE

Let us recall here the *pragmatic coherence rule* introduced by Zoe McConaughey (2021, chapter 4.2, table 4.9), that implements both, Aristotle's and Suhrawardī's view that ontological commitments are contextually-bounded commitments:

> **Pragmatic Coherence Rule**:
> When the conclusion Proponent defends is particular and all the premises Opponent defends are universal, Proponent may request the Opponent to instantiate the subject of a premise with the instance d_i, chosen by **P**, **provided** d_i is **new**: challenge: **P** $?_{J(di)}$; defence: **O** ! $J(d_i)$ (for a universal with $\{x: D \mid J(x)\}$ as subject, and J as subject-term), (this prevents **O** to state $J(d_i)$ when he endorsed before some $J^*(d_i)$ whereby J and J^* are incompatible)

In relation to the second question, logically speaking we can adopt both of the options. However, the conjunction-analysis seems to contravene Suhrawardī's own formulation of the conclusion, namely:

> The conclusion will be affirmative: "Something described as a non-bird is a non-horse." *Ishrāq* (1999, pp. 25).

Thus, the conclusion of this negative form of *Darapti* is encoded as:

	Explicit Encoding	**Dynamic Encoding**
Without explicit modality	$(\exists z: \{x: D \mid\sim A\ (x)\})\ \sim B(\mathcal{L}^{\{\}}(z))$	$(\text{Some}\ \sim A_D)\sim B$
With explicit modality	$(\exists z: \{x: D \mid \mathbf{LL}\sim A\ (x)\})\ \mathbf{LL}\sim(\mathcal{L}^{\{\}}(z))$	$(\text{Some}\ \mathbf{LL}\sim A_D)\mathbf{LL}\sim B$

Putting all together we obtain for Suhrawardī's own example for the third figure:

> All humans are necessarily necessary non-birds
> All humans are necessarily necessary non-horses
> Therefore, some necessarily necessary non-birds are necessarily necessary non-horses

O ! $(\text{Every}J_D)\mathbf{LL}\sim B$	**O** ! $(\forall z: \{x: D \mid J(x)\})\ \mathbf{LL}\sim B(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
O ! $(\text{Every}J_D)\ \mathbf{LL}\sim A$	**O** ! $(\forall z: \{x: D \mid J(x)\})\ \mathbf{LL}\sim A(\mathcal{L}^{\{\}}(z))^{\mathbf{P}}$
---	---
P ! $(\text{Some}\ \mathbf{LL}\sim A_D)\mathbf{LL}\sim B$	**P** ! $(\exists z: \{x: D \mid \mathbf{LL}\sim A\ (x)\})\ \mathbf{LL}\sim B(\mathcal{L}^{\{\}}(z))^{\mathbf{O}}$

O			P		
I	! (EveryJ_D)**LL**~B			! (Some **LL**~A_D)**LL**~B)	0
II	! (EveryJ_D) **LL**~A				
1	? $\mathcal{L}^{\exists}$	?0		*you*$_5$: ~A(d_i)	8
3	! J(d_i)		?II	?$_{J(d_i)}$ (use of the pragmatic coherence rule)	2
5	! ~A(d_i)		?II	*you*$_3$: J(d_i)	4
7	! ~B(d_i)		?I	*you*$_3$: J(d_i)	6
9	? $R^{\exists}$	?0		*you*$_7$: ~B(d_i)	10
				Proponent wins	

- The dialogue follows the steps for the construction of the conclusion suggested by Suhrawardī: Instead of choosing immediately one animal that is not a bird, the Proponent, deploys the pragmatic coherence rule and asks **O** to endorse that d_i is one of the humans who are no birds. Then **P** asks, **O** to endorse that this very human is also no-horse. Only after these endorsements, has **P** now the knowledge of the presence of a particular animal, namely a human; that witness both being a non-bird and non-horse.
- The dialogue ends since 10 is an unanalysable constituent of the conclusion, namely the negative literal ~B(d_i), that cannot be challenged, since **O** himself endorsed it with move 7.
- Notice that if we prefer not to distinguish negative literals from other negations and treat them as usual negations the end-result of the dialogues will not change. It will only a bit longer: as soon as **O** challenges ~A(d_i), **P** will do the same, similarly with ~B(d_i). However, this does not seem to be what Suhrawardī has in mind when he deploys metathetic negations to produce affirmations.

Movahed (2012, p.14) suggests other modal configurations for the third figure and prefers to use the conjunctive form for the conclusion. The latter contravenes the point that the minor term is purported to be the subject and the major the predicate of the conclusion. Concerning the former we did not find textual backup in *al-Ishrāq*. However, Movahed might have in mind some other sources. Be that as it may, the framework presented above can deal with several of these combinations.

V Time, Intuition and Dialectical Construction. Very Brief Remarks

In his excellent paper "Brouwer's notion of intuition and theory of knowledge by presence", Ardeshir (2008) discusses how *intuition* in the sense of Browerian a priori construction can be enriched with Suhrawardī's notion of *knowledge by presence*:

> The title of this philosophy [*al-Ishrāq*] is from the Arabic root *SHARQ*, meaning 'rising', in particular, 'rising of the sun'. The term is also linked to the Arabic word for'East', which represents a specifically Eastern form of philosophical thought, form of thought in contrast with cognitive reason, based on intuitive and immediate knowledge.
> We argue that the theory of intuition in Brouwer's intuitionism can be interpreted in the *expanded* theory of 'knowledge by presence'. Ardeshir (2008, pp. 116-117).

Furthemore, Ardeshir (2008, p. 118) proposes the further interesting problem: It is well known that Abū'l-Barakāt al-Baghdādī's *Book of Evidence,* had an important influence on Suhrawardī. Now, al-Baghdādī rejects Aristotle's and Ibn Sīnā's notion of time as measure of change and defends an *a priori* conception of it. Suhrawardī seems to hold both, one attached to knowledge by presence: the *a priori* experience of the *now*, that is immediate and not-linguistic and the other linked to the knowledge by

representation and correspondence, which is the one related to the Aristotelian notion of time. So, my own experience as a contingent being, has both sides: my experience of me being now, and the history of these experiences constituted by timed experiences presence.

In his fascinating paper Ardeshir (2008) develops an interpretation of how both aspects of time conjugate in Suhrawardī's *Hikmat al-Ishrāq*. A detailed discussion of this reading in the context of the dialectical framework developed herewith must be left for further work. Let us make some suggestions that might lead the way to this future work.

As pointed out in the introduction the most relevant meaning explanations of modalities occur in the third discourse of *Hikmat al-Ishrāq*, which is advocated to the study of dialectical issues. This motivated the dialectical interpretation of his logic, where presences, which are witnessing a proposition are distinguished from the proposition they witness. Nevertheless, they are to be thought together. This can perhaps be understood as rendering the 'illuminative relation' called *al-iḍāfat al-ishrāqīyah*, which Ardeshir (2008, p. 123) indicates as the way knowledge by presence and correspondence/representative knowledge are related in Suhrawardī's work.

This is what the Constructive Type Theoretical reconstructions contributes with: the presence or witnessing instance, is made explicit at the object language level. But both, experiencing *presence* and experiencing what is experienced are grasped at the same time, probably first in unarticulated way.

Furthermore, as mentioned above, the use of witnessing instances of the quantified expression is only made explicit during the rules that prescribe how to develop the dialectical interaction associated with a quantified assertion. It is only during the proof that instances come to the fore.

Moreover, also the temporal dimension of the presence that verifies a proposition only comes to the object language, as a result of the interaction triggered by an assertion

Now, the dialectical framework proposed, adds the self-awareness of the presence as special move, namely what we called the Socratic rule which prescribes the use of you_n, which indicates that **P**'s statement is backed by **O**'s endorsement of it at move *n*, and that he (**P**) adheres himself to the knowledge conveyed by **O**'s endorsement. Or even better that he makes its own: the mediated knowledge becomes now **immediate knowledge of the unanalysable presences**.

In other words, al-Baghdādī's *a priori* time, is the time that regulates the succession of moves during the interaction. The Opponent fixes the time of change attached to the potentialities defining a potentiality and the *now*, is the move where the Proponent experiences the presence by appropriating, the knowledge won during the dialectical interaction by triggering an immediate act of awareness: the inner enlightening conviction of knowing.

VI Conclusion

> Does Suhrawardī reject the approach to knowledge of essences of the Peripatetic thinkers of his time?
> No doubt he does.

Is Suhrawardī's logic compatible with essentialism as claimed by Street (2008)?
Yes, definitely so.

However, the logic of illumination has some interesting original features of its own, namely

- the logic of illumination shapes knowledge as emerging from the experience of presence – logic presupposes knowledge, knowledge is experience of presence, experiences of presence is experience of the now;
- the meaning theory underlying Suhrawardī's logic makes explicit the experience of presence by making them part of the object language – this meaning theory is shaped by rules that prescribe how to build a counterexample (we call these rules *meaning explanations*) ;
- scientific knowledge, expressed by universal laws, encode the generalization process by the means of which actual instances of a type (the presences of a type) are experienced as being dependent upon the actual instances of another one. When this dependence amounts to interdependence, essences are grasped: the meaning explanation of the universal necessarily necessary reciprocal law, prescribes the steps leading to the constitution of the interdependence at work in that law.

Moreover, we claim that a dialectical framework brings to the fore all these features of Suhrawardī's logic. Certainly, it can also, by further generalization on winning strategies, be formulated as an inference system, for example, in the form of a sequent calculus .

Nevertheless, it is the play-level, where presential knowledge is constituted by the interaction of concrete players who bring forward moves based on presences that either justify or refute the main thesis: meaning explanations are dialogical meaning explanations.

The dialectical framework also allows distinguishing **the a-priori time** of construction of knowledge – studied by Ardeshir's (2008) approach to Suhrawardī's epistemology – that is the succession of moves leading to the justification of the thesis, the **now** during which a presence is grasped by an immediate act of knowledge, and the **timing** encoded by the modality at work. Neither of these forms of temporality are captured by standard temporal quantifiers or Priorian temporal operators.

The upshot of this reconstruction is that the mediated knowledge of a complex expression and even of a syllogistic inference, is grounded ultimately in the **immediate, intimate knowledge of the unanalysable presences** the dialectical interaction brings to the fore.

Having said that, though, Suhrawardī's formulations admit indeed a pretty straightforward dialectical reading, and despite the fact that his meaning explanations for modalities occur in discourse on fallacies and disputations on natural philosophy and metaphysics, there is no explicit systematic development of a complete dialectical framework for his logic in the texts.

The formal dialogical framework for the logic of illumination in the present paper is of course due to our own reconstruction, it is the result of putting together the textual sources where Suhrawardī presents the meaning explanations mentioned above and his own logical proofs. Suhrawardī distinguishes dialectics by the kind of premises it deals with, but what we claim is that the dialectic methodology shapes his understanding of modalities.

The present study should be read as a kind or prelude to a further deeper exploration involving historic and systematic pending issues such as

The study of the legacy of some of his main predecessors such as the works of Abū'l-Barakāt al-Baghdādī (1080-1165), particularly in al-Baghdādī's *Book of Evidence* – cf. Ziai (1990, pp. 19-20 and Street (2008, p. 166) and Sahlān Sāwī's *al-Baṣā'ir* (*The Insights*); commentaries to his work by, among others ; Ibn Kammuna,Shahrazuri, Shirazi, Ibn Rizi, al-Albhari, Allamah Hilli, al-Jurjani, Ibn Abi Jumbur Ahsa'i, al-Dawani, al-Dashtaki, Abd am-Razzaq, Mulla Sadra, Isma'il Ankaravi, al-Harawi and Hazin Lahiji.

Coordinating Suhrawardī's views on modality and causality, with his framework for modal syllogism. As stressed by Walbridge and Ziai (1999, introduction p. XXV), Suhrawardī points out in the third discourse of the first part of his *al-Ishrāq* that

1) "contingency" means to be necessary by a cause
2) that a cause is ontologically but not temporally prior to its effect
3) that a cause may be composite and include conditions and removal of impediments

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APPENDIX

DIALOGICAL MEANING EXPLANATIONS I: LOCAL MEANING[43]

	LOCAL MEANING I: SYNTHESIS		
	Move	**Challenge**	**Defence**
Conjunction	**X** ! $A \wedge B$	**Y** ? $L^{\wedge}$ or **Y** ? $R^{\wedge}$	**X** $p_1 : A$ (resp.) **X** $p_2 : B$
Existential quantification	**X** ! $(\exists x : A)B(x)$	**Y** ? $L^{\exists}$ or **Y** ? $R^{\exists}$	**X** $p_1 : A$ (resp.) **X** $p_2 : B(p_1)$
Disjunction	**X** ! $A \vee B$	**Y** ? $^{\vee}$	**X** $p_1 : A$ or **X** $p_2 : B$
Implication	**X** ! $A \supset B$	**Y** $p_1 : A$	**X** $p_2 : B$
Universal quantification	**X** ! $(\forall x : A)B(x)$	**Y** $p_1 : A$	**X** $p_2 : B(p_1)$
Negation	**X** ! $\neg A$ Also expressed as **X** ! $A \supset \perp$	**Y** $p_1 : A$	**X** $p_2 : \perp$

	LOCAL MEANING II: ANALYSIS		
	Move	**Challenge**	**Defence**
Conjunction	**X** $p : A \wedge B$	**Y** ? $L^{\wedge}$ or	**X** $L^{\wedge}(p) \ : A$ (resp.)

[43] Cf. Rahman (2018 et al. Chapter 4)

		$\mathbf{Y}\ ?\ R^{\wedge}$	$\mathbf{X}\ R^{\wedge}(p)\ \ :B$
Existential quantification	$\mathbf{X}\ p\colon (\exists x\colon A)B(x)$	$\mathbf{Y}\ ?\ L^{\exists}$ or $\mathbf{Y}\ ?\ R^{\exists}$	$\mathbf{X}\ L^{\exists}(p)\ \ :A$ (resp.) $\mathbf{X}\ R^{\exists}(p)\ \ :B(L^{\exists}(p)\ \ )$
Disjunction	$\mathbf{X}\ p\ \ :A \vee B$	$\mathbf{Y}\ ?^{\vee}$	$\mathbf{X}\ L^{\vee}(p\ \ )\ :A$ or $\mathbf{X}\ R^{\vee}(p\ \ )\ :B$
Implication	$\mathbf{X}\ p\colon A \supset B$	$\mathbf{Y}\ L^{\supset}(p)\ \ :A$	$\mathbf{X}\ R^{\supset}(p)\ \ :B$
Universal quantification	$\mathbf{X}\ p\colon (\forall x\colon A)B(x)$	$\mathbf{Y}\ L^{\forall}(p)\ \ :A$	$\mathbf{X}\ R^{\forall}(p)\ \ :B(L^{\forall}(p)\ \ )$
Negation	$\mathbf{X}\ p\colon \neg A$ Also expressed as $\mathbf{X}\ \ p\colon A \supset \bot$	$\mathbf{Y}\ L^{\neg}(p)\ \ :A$ $\mathbf{Y}\ L^{\supset}(p)\ \ :A$	$\mathbf{X}\ R^{\neg}(p)\ \ :\bot$ $\mathbf{X}\ R^{\supset}(p)\ \ :\bot$
Falsum	$\mathbf{X}\ p\ \ :\bot$ **(**given **Y** ! ***C*****)**	$\mathbf{Y}\ you\ gave\ up\ (n)\colon C$[44]	—

DIALOGICAL MEANING EXPLANATIONS II: STRUCTURAL RULES OR GLOBAL MEANING ON HOW TO DEVELOP A DIALOGUE[45]

	GLOBAL MEANING
STARTING RULE	The player who states the thesis move 0 is the proponent **P.** *In syllogistics, the thesis is a complex statemen by the means of which the proponent commits to the conclusion provided that the opponent commits to the initial premises as recapitulation for a justification).* The play starts with the opponent **0** stating these premises (moves *l.n* for *then* premises). The proponent **P** then states the conclusion at move 2

[44] In fact, since falsum involves an elementary proposition rather than a connective it should be regulated by structural rules. See structural rule SR7 below. The reading of stating **falsum** as giving up stems from (Keiff, 2007).

[45] These rules are essentially the ones developed for by McConaughey (2021, chapter 4.2.2) for Aristotle's Syllogism. We adapted it slightly to our context: namely the Socratic Rule.

DEVELOPMENT RULE	Once the starting rule has been implemented, each player in turn plays a move according to the challenge and defence rules prescribed by the local meaning of the expression in play, and according too the other structural rules specified below.
SOCRATIC RULE	The proponent **P** may not state an *unanalysable* expressions if it is not explicitly backed by the indication $you_{i,}$ where i is the move at which the opponent **O** has stated that very same expression: $you_{i,}$ indicates that his statement is backed by **O**'s endorsement of it at move n, and that he (**P**) adheres himself to the knowledge conveyed by **O**'s endorsement. In the context of Suhrawardī's logic unanalysable expressions include positive and negative literals (i.e. elementary propositions with and without negation). In order to shortcut the length of a play, expressions of unactualized capacities such as $B(d_i)@t_i \supset\subset \sim B(d_j)@t_j$ will also be treated as unanalysable. However, if, for some philosophical reason, needed these expressions can be further analysed. Defining such expressions as unanalysable meets purely logical aims. Unanalysable constituents cannot be challenged (since **O** is allowed to state them when required and **P** only states them after **O** endorsed them by stating them before).
PRAGMATIC COHERENCE RULE	When the conclusion Proponent defends is particular and all the premises Opponent defends are universal, Proponent may request the Opponent to instantiate the subject of a premise with the instance d_i, chosen by **P**, **provided** d_i is **new**: challenge: **P** $?_{J(d_i);}$ defence: **O** ! $J(d_i)$ (for a universal with $\{x: D \mid J(x)\}$ as subject, and J as subject-term), (this prevents **O** to state $J(d_i)$ when he endorsed before some $J^*(d_i)$ whereby J and J^* are incompatible).
ENDING RULE	The player who states ⊥ give-up, immediately loses. Otherwise, the player who has no available move left at this turn loses.

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