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Effect of raw material substitution on the facility location decision under a carbon tax policy

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A B S T R A C T

Current environmental issues that have been made unavoidable by environmental regulations have become new constraints for industrial companies. In this paper, we consider a joint production-location problem for supply chains under a carbon tax policy on transport-related carbon emissions. We characterize the relationship that links the production level to the input quantities by considering a production function, namely, constant elasticity of substitution (CES) function. Our study focuses on the potential impact of increased transportation costs due to carbon taxation on the joint production-location decision. We find that the location-production configuration differs according to the degree of substitutability among the raw material quantities. More importantly, we observe that a higher carbon tax is more likely to cause a significant jump in firm location choice and a considerable change in production decisions when a firm has high flexibility in its ability to substitute among input quantities.

1. Introduction

Despite the many international agreements aimed at reducing greenhouse gas (GHG) emissions, the main cause of global warming, an alarming situation persists (Hoegh-Guldberg et al., 2018). According to the annual report published by the Global Carbon Project (GCP), GHG emissions worldwide in 2018 have been unprecedented for seven years, with growth at more than 2% (Le Quéré et al., 2018). The freight transport sector remains one of the largest and fastest growing emitters of GHG emissions, as shown by an UNFCCC study (UNFCCC, 2018). This is due to the increasing globalization of trade and the geographical dispersion of supply chains (Cristea et al., 2013). Technological innovations related to energy improvements for vehicles and fuels can significantly decrease transport-related carbon emissions but do not counterbalance them (Chapman, 2007; Gaigné et al., 2012). Evidence of such a trend can be found in the latest United States (US) data, which indicate that although transportation energy efficiency has continued to improve from 1975 to 2017, transportation became the largest source of emissions in the US in 2016 and continued to be the largest emitter of greenhouse gases in 2017, exceeding emissions from electricity generation (Bureau of Transportation Statistics, 2018). Hence, it is increasingly likely that public authorities will develop policies that target the growth of emissions generated by transportation and, ultimately, their external effect (Lee et al., 2008; Hoen et al., 2014; Rudi et al., 2016; Das et al., 2020a).

Current environmental regulations such as carbon taxes, which restrict carbon emissions in general and, in particular, those originating from transport activities, may have considerable implications on firms’ strategic, tactical and operational decisions (location, transport demand, industrial processes, etc.) (Waltho et al., 2018). Given the importance of studying the implications of policies that mitigate carbon emissions, we formulate a location model that incorporates a carbon tax on transport-related carbon emissions by considering different types of production technology (expressing the relationship between the production factors of a firm and the produced quantity). Under each technology, we examine the effect of the carbon tax on location choice, production decisions, and the resulting pollution.

Carbon taxes on fossil fuels, such as gas, oil and their derivatives, which emit considerable amounts of CO₂, are a central public instrument in the face of the climate threat, in addition to other instruments such as emissions trading systems (ETSs) (Sumner et al., 2011). The principle behind such taxes is to reach all consumers of fossil energy, including individuals and businesses, to encourage them to change their practices. In addition, the carbon tax rate is expected to increase progressively and regularly to give price signal incentives to reduce...
the use of fossil fuels (Sterner, 2007). For example, in France, the carbon tax started in 2014 and increased from 7 € per ton of CO₂ to 44.60 € per ton in 2018 and will nearly double to 86.2 € per ton in 2022 (De Perthis and Faure, 2018). In this context of regular upward revaluation of the carbon tax, manufacturers, particularly those with a high dependency on transportation, have to adapt, in the more or less long term, their logistics of physical flows and networking in response to changes in the price of raw materials and rising fuel prices (Diabat and Simchi-Levi, 2009; Alhaj et al., 2016).

Our study focuses on the potential impact of increased transportation costs due to carbon taxation on the joint production-location decision. The underlying assumption is that higher freight transportation costs result in an adjustment of production decisions to reduce the transport demand. More specifically, this transport demand reduction occurs in the long run through modification of the spatial organization of production. Wu et al. (2017) reported that although evidence of moving manufacturing facilities due to carbon emissions concerns is not yet conclusive, cases are often observed in practice. They added that such an effect has been contemplated by several industrial businesses according to surveys.

Traditionally, facility location models under carbon regulations describe firm production technology by the bill of materials (BOM), which assumes that the input proportions are fixed (see, e.g., Ramadhin et al., 2010; Chaabane et al., 2012; Liotta et al., 2015). This representation may not fit the production technology of some industrial sectors, where the input proportions can be varied to produce a given level of output.

For example, in the agri-food sector, the input quantities may vary due to regulation, shortages, innovation or even raw material price volatility (Hsu, 1997; Balakrishnan and Geunes, 2000; Ram et al., 2006; Lang, 2010). Therefore, in our framework, we assume that input quantities may be substitutable. This substitution effect is captured through a production function, namely, constant elasticity of substitution (CES), which provides possible substitution schemes among inputs.

We consider three cases: weak, intermediate and high substitutability. We first show that the trade-off between the economic criterion (cost-minimizing location) and environmental criterion (emission-minimizing location) vanishes in the case of (i) weak and intermediate substitution and (ii) the same transport mode for the supply and delivery sides. When substitutability is high, transport-related carbon emissions are significantly reduced compared to those in the case of weak and intermediate substitution. This is due to the fact that the firm chooses the facility location and input proportions that result in a lower level of emissions. High substitutability of inputs enables better environmental and economic performance. Nevertheless, the gains in production cost due to the substitution effect may offset the environmental cost related to carbon taxation. In other words, high substitutability may trigger a conflict between firm economic and environmental objectives and thus induce excess pollution. Our analysis also reveals that a higher carbon tax is more likely to cause a significant jump in a firm’s location choice, a drastic change in a firm’s material selection, and a significant fall in emissions when the firm is free to relocate and has high flexibility in its ability to substitute among input quantities.

The remainder of this paper is structured as follows. Section 2 reviews the related literature. Section 3 develops the mathematical model and the solution procedure. Section 4 presents the results of a numerical study, followed by conclusions in Section 5.

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1 For example, food industries are among the most likely to be affected by rising transportation costs given their high transportation service requirements (Garnett, 2011). Food industries are usually located either where the raw materials are or where consumption is or at an intermediate position (Leistritz, 1992). In all these cases, there are physical flows of goods to be managed upstream (supply), in the industries (processing), and downstream (distribution), which entails a strong dependence on transportation (Bourlakis and Weightman, 2008). In the United States, food is transported in large quantities over long distances in general, and transportation accounts for 11% of the life cycle of GHG emissions (Weber and Matthews, 2008; De Cara et al., 2017).
by considering trade-offs between transportation, facility construction and emissions costs. They analyze the efficiency of different carbon policies. Regarding carbon tax policy, they observe that the supply chain configuration is slightly modified even when the tax rate reaches its maximum level. This is due to the tendency to pay the carbon tax, rather than making operational changes in the supply chain.

Following this review, we note that although the topic of firm location strategy under carbon emission cost has become an area of interest for many researchers, the production technology is assumed, either explicitly or implicitly, to be an assembly supply chain modeled by a BOM, which implies fixed proportions across input quantities. As previously mentioned in the introduction section, this representation may not fit the production technology of some industrial sectors, where the input proportions can be varied to produce a given level of output. The current paper aims at filling this gap and has the objective of studying the relation between carbon tax on GHG emissions, and location decisions under the effect of substitution between raw materials. This operational aspect plays an important role in the strategic decision-making of firm location, as demonstrated in the study by Peeters and Thise (2000), where a small substitution between input sources was shown to lead to substantial jumps in the optimal firm location. However, the environmental issues related to the transport of commodities are left aside on their study. Therefore, we seek to investigate how both a firm’s location and production decisions will be affected by the carbon tax under the effect of substitution.

More recently, Gaïné et al. (2019) studied the role of production technology in the production facility location problem under a carbon tax on transport-related carbon emissions. Using different production technologies (Leontief and Cobb–Douglas functions for the complementarity and substitutability of inputs, respectively), this paper suggests that an increase in carbon taxes does not necessarily cause a reduction in GHG emissions and that the cost-minimizing location may differ from the carbon emission-minimizing location, regardless of the production technology type. However, the analysis was conducted under a line network typology with a limited number of stakeholders (two suppliers and a single end market) and a specific functional form for production technology.

Our research work contributes to the literature by returning to the analysis of Gaïné et al. (2019) and completing it by considering a generalized framework: bidimensional space, several suppliers and final markets, and a general functional form of production technology (CES) to explore and analyze several degrees of substitution among raw materials.

3. Mathematical model

In this section, we formulate the production-location model under a carbon tax policy on transport-related carbon emissions and develop the optimization process.

3.1. Problem description

Consider a single manufacturing company that produces and delivers one output to different downstream markets. The company has to satisfy the sum of all individual exogenous demands. We assume that the downstream demand is fixed to focus on only transport-related carbon emissions. This choice is justified by the fact that commodities shipping accounts for a significant share of GHG emissions (Hoen et al., 2013). Moreover, an increasing number of production companies that are large transport users seek to reduce their carbon emissions beyond the production-related carbon emissions (Hoen et al., 2014). To satisfy the total demand, the company buys its raw materials from different suppliers and the quantities of inputs vary according to the supply costs and technological constraints. Each raw material input is provided by one supplier, and all stakeholders have a fixed and determined location on the Euclidean plane.

The firm chooses its location with respect to its suppliers and final markets, the production constraints and the general objectives of cost minimization. The total cost includes sourcing and transportation costs and transportation emissions costs due to the carbon tax policy. We assume that transport-related carbon emissions are linearly proportional to distance and the quantity of goods transported.

We consider a carbon tax imposed by a regulatory authority and applied to each unit of carbon emissions. The choice of a carbon tax as the carbon policy is mainly explained by the fact that several countries, regions and cities have already adopted this measure or are moving towards it, as indicated by a recent New Climate Economy report (Economy, 2018). In addition, a carbon tax can easily be integrated within the existing taxation system, such as fuel taxes (Hoen et al., 2013). The objective of the firm is to determine its location in a two dimensions Euclidean space and the quantities of inputs purchased from each supplier.

3.2. Notation

For ease of reference, the following notation is used for the parameters and decision variables.

\[
\begin{align*}
F & \quad \text{Production firm;} \\
M_k & \quad \text{Set of markets, } k = \{1, 2, \ldots, K\}; \\
S_i & \quad \text{Set of suppliers, } i = \{1, 2, \ldots, I\}; \\
q_k & \quad \text{Individual demand market;} \\
q^0 & \quad \text{Total demand market, such as } q^0 = \sum_{k=1}^{K} q_k; \\
t_{ik} & \quad \text{Transport cost per unit of distance and per unit of output from } F \text{ to } M_i; \\
a_{ik} & \quad \text{Carbon footprint coefficient per unit of distance and per unit of output from } F \text{ to } M_i; \\
d_{ik} & \quad \text{Market distance from } F \text{ to } M_i; \\
\omega_i & \quad \text{Purchasing price per unit of input;} \\
t_{ij} & \quad \text{Transport cost per unit of distance and per unit of input from } S_j \text{ to } F; \\
a_{ij} & \quad \text{Carbon footprint coefficient per unit of distance and per unit of input from } S_j \text{ to } F; \\
d_{ij} & \quad \text{Supplier distance from } S_j \text{ to } F; \\
\tau & \quad \text{Fee applied to each unit of equivalent CO}_2 \text{ emitted.}
\end{align*}
\]

Decision variables

\[
\begin{align*}
y & \quad \text{Firm location defined by its two coordinates in the plane;} \\
s_i & \quad \text{Quantity of the input purchased from } S_i.
\end{align*}
\]

3.3. Technology specification

The possible firm production techniques are described by a production function \( f : R^I \rightarrow R \) that links the total production level to raw material needs:

\[
q^0 = f(s_1, \ldots, s_I).
\]

We use CES formulation for the production function \( f \). The underlying assumption of the CES is that input quantities can be substituted for each other to achieve the same level of production (Varian, 2014). However, a constant elasticity of substitution (measure of the degree of substitutability) is imposed between the inputs used for production (Holf, 2002). The functional form with \( I \) inputs is defined as follows:

\[
q^0 = f(s_1, \ldots, s_I) = \left( \sum_{i=1}^{I} \frac{f_i s_i^\theta}{\sum_{i=1}^{I} f_i s_i^\theta} \right)^{\frac{1}{\theta}}.
\]
subject to

$$\sum_{k=1}^{K} q_k = f(s_1, \ldots, s_f) \quad (3)$$

$$y \geq 0 \quad (4)$$

$$s \geq 0 \quad (5)$$

Constraint (3) represents the production technology constraint and states that production must meet the total demand. Constraints (4) and (5) are nonnegativity constraints.

Within the cost-minimizing problem described above, there is a production subproblem for determining the set of optimal input quantities $$\mathbf{s}^*$$ at any location $$y$$ (technology design or input mix). This optimal input mix is obtained by minimizing the total upstream cost (production cost noted $$c$$) at each location $$y$$ and noted $$c$$. The result can be expressed as follows:

$$c(y) = \min_s \left[ \sum_{i=1}^{I} (w_i + T_j d_{iy}) s_i \right] \quad (6)$$

with respect to constraints (4) and (5) where $$T_{ij} \equiv t_{ij} + r a_{ij}$$ with $$i = 1, \ldots, I$$ is the total unit transportation cost per unit of distance and per unit of input from $$S_i$$ to $$F$$. Note that $$w_i + T_j d_{iy}$$ represents the delivered price of a shipped input unit, composed of the per unit procurement and total transportation costs. At each location $$y$$, the firm faces different delivered prices for its inputs.

When the production function $$f$$ is assumed to be homogeneous, the production cost function $$c$$ at location $$y$$ can be written as $$c(y) = \psi(y) \cdot q^0$$, where $$\psi(y)$$ is the marginal production cost at location $$y$$, as proved by Hurter and Martinich (1989). Using this result in the case of CES leads to the following expression of the marginal production cost:

$$\psi(y) = \left[ \sum_{i=1}^{I} (w_i + T_j d_{iy}) s_i \right]^{\frac{\rho-1}{\rho}} \quad (7)$$

Then, the demand for each input $$i$$ is given by:

$$s^*_i(y) = \left[ \frac{\psi(y) \cdot \beta_i}{w_i + T_j d_{iy}} \right]^{\frac{1}{\rho}} q^0 \quad (8)$$

These last two analytic results clearly show that the use of inputs is determined by their relative delivered prices. Moreover, the input substitution effect among input quantities results from changes in the relative delivered prices of inputs, which is induced by variation in firm location $$y$$. The carbon tax influences the firm’s input mix through the total unit transport cost $$T_j$$, as the relative delivered prices of the inputs are altered at each level of carbon tax rate. These findings are in line with those obtained by Gaïné et al. (2019) in the case of a linear network, two inputs and a Cobb–Douglas production function, which is a specific case of the CES production function (when $$\phi = 0$$), as mentioned in Section 3.5.

### 3.4. Cost function

The objective of the firm is to minimize the sum of the total purchase cost, transportation cost and environmental impact cost measured by GHG emission level by determining the best location $$y$$ and defining the set of input quantities $$s = (s_1, \ldots, s_f)$$. The firm cost function $$C(y, s)$$ can be expressed as:

$$C(y, s) = \sum_{i=1}^{I} w_i s_i + \sum_{k=1}^{K} l_{dk} d_{sk} q_k + \sum_{i=1}^{I} r j d_{iy} s_i + r E(y, s),$$

where $$\sum_{i=1}^{I} w_i s_i$$ is the input purchase cost; $$\sum_{k=1}^{K} l_{dk} d_{sk} q_k$$ is the downstream transportation cost; $$\sum_{i=1}^{I} r j d_{iy} s_i$$ is the upstream transportation cost; and $$r E(y, s)$$ is the transport-related carbon emissions cost.

The function $$E$$ represents the ecological damage due to upstream and downstream transportation activity. This function, which measures transport-related carbon emissions, is based on distance, the quantity of freight carried, and the carbon intensity of the transportation mode. The carbon footprint $$E(y, s)$$ can be written as:

$$E(y, s) = \sum_{i=1}^{I} a_{i1} d_{iy} s_i + \sum_{k=1}^{K} a_{dk} d_{sk} q_k.$$

The total cost $$C$$ is minimized with the following constrained function:

$$C^* = \min_{y,s} C(y, s) \quad (2)$$

### 3.5. Location choice

Once the optimal set of input quantities has been defined for each location $$y$$, we can determine the cost-minimizing location $$y^*$$ (see Appendix A):

$$C^* = \min_y C(y) = \min_y \sum_{k=1}^{K} \left[ \psi(y) + T_j d_{iy} \right] q_k \quad (9)$$

The minimization of the total cost $$C$$ is equivalent to minimizing the total downstream cost with only one decision variable (location...
choice), as the set of input quantities $s$ is implicitly incorporated within the location-specific marginal production cost $\psi$ at location $y$. We can already notice that the location choice emerges from a trade-off between the production cost $(\sum_{k=1}^{K} \psi(y)q_k)$ on the one hand and the output transportation cost $(\sum_{k=1}^{K} T_{kj}d_{kj}q_k)$ on the other.

We also determine the firm location that minimizes the carbon footprint stemming from transportation, denoted by $y'$ and given by:

$$E^* = \min_y E(y) = \min_y \sum_{k=1}^{K} q_k \psi(y) \frac{\beta_k}{\beta_k + T_{kj}d_{kj}} + T_{kj}d_{kj}$$

The establishment of this ecological benchmark enables us to compare the results of both optimization programs (economic and environmental) to assess situations in which there is a concordance or a discrepancy between the economic objectives of the firm and the ecological objective.

3.6. Optimization methodology

As the production-location problem does not necessarily have a globally convex objective function, classic methods of optimization or numerical analysis do not guarantee a global solution. We solve the following numerical study in Section 4 by discretizing the solution space and implementing the generalized reduced gradient (GRG) algorithm in the case of a nonlinear program with nonnegativity constraints for variable $q_k$. The full description of the algorithm is given by Smeers (1977) and Lee et al. (2004). We choose multiple starting points, including markets, suppliers and center of gravity, to determine an approximate global solution (see Appendix B for the optimization methodology).

Geographic constraints related to the accessibility of suppliers and markets are not considered. In addition, we have assumed that the production and storage capacity of the suppliers are infinite as the objective of this paper is to capture the specific effect of the carbon tax on location and supply decisions. Therefore, the firm’s total demand will be satisfied. Finally, since we have not considered a profit function, we do not have a negative margin problem. Consequently, for these reasons, the model has no conditions for the existence of a solution.

4. Numerical study

In this section, we conduct a numerical study to gain insights from the joint production-location model. Specifically, we investigate how a firm’s location choice, input mix and ecological outcome are affected for the case of a single firm. We use a set of hypothetical data, selected as representative from a large number of experiments performed by Peeters and Thisse (2000). In the following, we describe the study data and then present and discuss the findings.

4.1. Data study description

Consider ten final markets and five input sources with coordinates shown in Tables 2 and 3, respectively. The supplier and market points are depicted in a Euclidean plane and shown in Fig. 1 with a weighting for the market points according to their demand requirements in Table 2. In Fig. 1, the point $G$ (52.84, 44.10) represents the center of gravity, which corresponds to the firm location that minimizes the total transport costs (both supply and distribution). The coordinates of this point are calculated using the weighted average formula of the center-of-gravity method (Love et al., 1988). Note that in this calculation, the suppliers are weighted equally by dividing the total demand market over the suppliers (the weight of each supplier is therefore 28.4). The determination of the center of gravity serves as a reference point to understand and interpret the results, as we will see later.

We assume that the per-unit procurement cost for all inputs is equal to 1. We further assume unit transportation and emissions rates for all inputs and the output. This situation may correspond to a firm that uses a single mode of transportation for supply and delivery.

We consider three values for the elasticity of substitution: $\phi = -5$ (weak substitution); $\phi = -1$ (intermediate substitution); and $\phi = 0.25$ (high substitution). The distribution parameters are identical, $\beta_k = \frac{1}{I}$ with $I = 5$ (which corresponds to the number of suppliers), for all three cases.

4.2. Case of weak substitution

In this case, with $\phi = -5$, the CES approaches the Leontief scenario in which the proportions of the inputs are fixed (see Table 1). This situation may arise, for example, in the case of the cement industry where cement works combine many ingredients in the manufacture of concrete and the proportions of these ingredients are rigidly fixed by the concrete technology and the quality of the product. Any variation in these proportions would result in a change in the nature of the product (Germain, 1969).

The results of this case are reported in Table 4. Fig. 2 reports the quantities of inputs purchased from each source for each level of carbon tax.

When substitutability of input quantities is weak, the cost-minimizing location is situated near the center of gravity $G$ (52.84, 44.10), where both downstream and upstream transportation costs are considered (a central location arises). However, there is an upward pull towards the three supplier points $S_1$, $S_2$, and $S_3$, as the optimal location is northeast of the central position $G$. Therefore, the inputs purchased from these three suppliers are obtained at relatively low delivered prices. This characteristic explains why the input quantities of those three input sources are relatively higher than the other input quantities (see Fig. 2).

From an environmental perspective, the cost-minimizing location exactly matches the pollution-minimizing location after the introduction of a carbon tax. Because of the weakness of substitution, an increase in the tax does not affect the production-location decisions (the changes in input quantities are infinitely small or insignificant). As a result, transport-related carbon emissions are almost independent of the carbon tax $\tau$ (see the right side of Fig. 3 for $\phi = -5$), whereas the total firm cost increases considerably (see the left side of Fig. 3 for $\phi = -5$). As all unit transportation costs and all unit emissions levels are equal, it can be concluded that a carbon tax is not justified for firms that use only one transportation mode for supply and delivery.

This example confirms the theoretical result of Gaïtèné et al. (2019) that a carbon tax is useless in the case of a Leontief technology with a unimodal transportation scheme.
4.3. Case of intermediate substitution

Under this scenario, with \( \phi = -1 \), the production technology is closer to a Cobb–Douglas technology, so we classify this scenario as intermediate substitution. This situation can be found in feed mixture for livestock which requires combining several quantities of cereals (e.g. wheat, barley, soy) to cover livestock nutritional needs. Then, the quantities of the cereals in a food recipe may be varied but to a limited extent to respect the nutritional intake (Gaigné et al., 2019).

The results of this case are summarized in Table 5 for optimal locations and Fig. 4 for optimal input quantities according to carbon tax rate.

When the elasticity of substitution increases from \(-5\) to \(-1\), the change in the optimal firm location is not smooth: there is a significant jump from the neighborhood of the central position \(G\) to supplier \(S_3\),

<table>
<thead>
<tr>
<th>Tax level</th>
<th>Cost minimizing location</th>
<th>Emissions-minimizing location</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–6</td>
<td>(S_1)</td>
<td>(S_3)</td>
</tr>
</tbody>
</table>

The quantities of inputs purchased from suppliers undergo a drastic change, as the jump in the optimal location causes a change in the relative delivered prices of inputs. The firm uses higher quantities from supplier \(S_3\) as the carbon tax increases. Given the presence of supplier \(S_3\) in the vicinity of supplier \(S_1\), the firm complements its excessive purchase of local input with a relatively important amount of the fifth input, obtained at a relatively low delivered price, and maintains the quantities of the other inputs at a low level. Notably,
even when the optimal location remains at supplier $S_3$, regardless of the carbon tax level, the optimal input quantities change because the carbon tax alters the relative delivered prices of the inputs through the total unit transport cost $T_i$. As the delivered prices of inputs (other than local input $s_3$) become more expensive due to the higher environmental cost, the firm slightly diminishes their use, but only to certain extent because substitution is intermediate (i.e., the technological constraint). In return, the use of the local input at $S_3$ increases considerably, for which the firm pays the lowest delivered price (only the purchasing cost), which offsets the reduction in the use of the other inputs.

From an ecological perspective, the optimal location of $S_3$ perfectly matches the emissions-minimizing location, even without the introduction of the carbon tax. Thus, the location is doubly efficient (economically and ecologically beneficial). Indeed, when no carbon tax is introduced (i.e., $\tau = 0$), the total cost falls by 7.36% and the level of emissions decreases by 9.50% compared to the previous case of weak substitution (see the right side of Fig. 3 for $\phi = -1$). Nevertheless, when the carbon tax is introduced and increased, transport-related emissions decline very slightly because the firm slightly decreases the use of inputs other than the local input (emissions from the distribution of the finished products remain constant). Moreover, the firm’s total cost increases significantly (although less than in the case of weak substitution) because the marginal cost of production is greater than the marginal benefit of reducing GHG emissions (see the left side of Fig. 3 for $\phi = -1$).
We now attempt to understand why it is more beneficial for the firm to be located at supplier $S_3$. We consider two situations: the supplier location that minimizes the production cost and the supplier location that minimizes the output transportation cost (distribution cost).

- In the former situation, we consider a Leontief technology in which the suppliers provide the same quantities. This identical amount is the result of the division of total demand by the number of suppliers, as done previously when determining point $G$ (the weight of each supplier is therefore 28.4). We then calculate the marginal production cost at each supplier (we assume that the firm is located at one of the supplier’s locations each time) according to each level of carbon tax. The results are presented in the right side of Fig. 5.

- In the latter situation, we consider a linear technology in which all the raw materials (total demand markets) are supplied by one of the five suppliers. Thereafter, we calculate the output transportation cost for each supplier location according to each level of carbon tax. The results are presented on the left side of Fig. 5.

Supplier $S_3$ is the location that best reduces the production cost regardless of the carbon tax (see the right side of Fig. 5), although the output transportation cost from this location is the second most expensive regardless of the carbon tax (see the left side of Fig. 5). Note that by equally allocating the total market demand over the suppliers, we are assuming a kind of Leontief technology (no substitution is possible), and when we allow substitution among the input quantities, the firm will attempt to more effectively reduce the production cost. Thus, in contrast with the previous case of weak substitution, where a central location arises, in this case of intermediate substitution, the firm searches for an input source location that enables a more efficient reduction in the production cost. In other words, the gains in production cost offset the losses related to the final product distribution cost.

4.4. Case of high substitution degree

In this third case, the value of the substitution parameter $\phi$ is equal to 0.25. The production technology is situated between Cobb–Douglas technology (imperfect substitution) and linear technology (perfect substitution). As in the previous case, feed mixture for livestock can be considered as an example of high substitution degree. It is relatively easy and common to change the supply of raw materials according to market conditions with a relatively high elasticity of substitution between cereals as in the study of Levert et al. (2017).

The results of this case are summarized in Table 6 for optimal locations and Fig. 6 for optimal input quantities according to carbon tax rate.

When the elasticity of substitution increases from $-1$ to 0.25 and $\tau < 3$, we observe a significant jump in the optimal firm location from supplier $S_3$ to supplier $S_1$. This location choice is justified by two reasons. First, when $\tau < 3$, the costs of production of the input sources are relatively similar for supplier locations $S_3$ and $S_1$ (see the right side of Fig. 5). Nevertheless, the advantage of supplier $S_1$ is that it enables a greater reduction in distribution costs (see the left side of Fig. 5). The second reason is the presence of supplier $S_4$ in the immediate neighborhood, which allows the firm to complement its extensive use of the local input ($s_1$) with small quantities of the fourth input, obtained at a relatively low delivered price, and to maintain the quantities of other inputs at extremely low levels (see Fig. 6 when $\tau < 3$). However, the cost-minimizing location does not match the emissions-minimizing location, which is at supplier $S_2$. This is due to the fact that the production combination of inputs at location $S_2$, which favors the local input, makes it economically advantageous to cope with the environmental costs entailed by a lower carbon tax when the latter is strictly lower than 3. Thus, high substitution over input quantities may harm the ecological outcome. Nevertheless, better pursuit of the economic and environmental objectives is ensured compared with the two other technologies (i.e., weak and intermediate substitution). Indeed, the total cost decreases by 53.57% (resp. 49.88%) and the level

![Fig. 5. Production and distribution costs at each supplier’s location according to carbon tax rate.](image)

<table>
<thead>
<tr>
<th>Table 6</th>
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<tbody>
<tr>
<td>Cost and emissions minimizing locations according to carbon tax changes.</td>
</tr>
<tr>
<td>Tax level</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>0–2</td>
</tr>
<tr>
<td>3–6</td>
</tr>
</tbody>
</table>

yielded the following results:
of emissions decreases by 65.78% (resp. 62.19%) compared to the case where \( \phi = -5 \) (resp. \( \phi = -1 \)) when \( r = 0 \) (see Fig. 3 for \( \phi = 0.25 \)). The level of transport-related emissions significantly decreases at a higher carbon tax because the larger weight on emissions (due to the higher carbon tax) in the effective shipping cost shifts the input proportions in the direction that lowers emissions. Thus, the tendency is to substitute other inputs for input 1 (the emissions from the distribution of the finished products remain constant, as in the previous case).

When \( r \geq 3 \), the firm jumps to supplier \( S_2 \) to re-optimize its total cost (the marginal production cost at \( S_2 \) is less than that at \( S_1 \) when \( r \geq 3 \), inducing a new substitution among input quantities and, thus, a new shipping pattern. The initial trend to substitute the other inputs for input 1 is reversed when the firm changes its optimal plant location to supplier \( S_2 \), causing a substantial change in the relative delivered prices of inputs and creating a substitution back toward input 2. This relocation decision causes a discontinuity and a downward jump in carbon emissions. The total rate of reduction of emissions at the critical price (\( r = 3 \)) triggering firm relocation is estimated to be approximately 6.19%. Furthermore, the emission reduction rate after the critical price continues to decline as the optimal amount of input 2 (the least-polluting input per unit of shipped input in this case) increases due to the higher carbon tax.

We now attempt to understand why it is more beneficial for the firm to move towards supplier \( S_2 \). When \( r \geq 3 \), the distribution costs from the location of supplier \( S_1 \) increase, and the gains in production cost (purchasing and input transportation cost) can no longer compensate. The location of \( S_2 \) has the advantage of being the location that minimizes the transportation cost to different final markets (the nearest location to point \( G \)) regardless of the carbon tax level when we assume the existence of only one supplier, as shown on left side of Fig. 5, despite the fact that it is one of the most expensive supplier locations in terms of production cost (see the right side of Fig. 5). The simulation in the case of very high substitution (\( \phi = 0.75 \)) confirms this finding, as the optimal firm location is always at \( S_2 \) regardless of the carbon tax level. Thus, in the case of high substitution, the firm seeks an input source location that enables the most efficient reduction in output transportation cost. In other words, the gains in output transport cost reduction offset the losses related to the production cost.

![Fig. 6. Quantities of inputs purchased at each level of carbon tax.](image)

**Table 7**

<table>
<thead>
<tr>
<th>Substitution parameter</th>
<th>Tax level</th>
<th>Cost minimizing location</th>
<th>Emissions-minimizing location</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5)</td>
<td>0–6</td>
<td>(85.89, 57.01)</td>
<td>(86.09, 56.67)</td>
</tr>
<tr>
<td>(-1)</td>
<td>0–6</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
</tr>
<tr>
<td>0.25</td>
<td>0–6</td>
<td>( S_1 )</td>
<td>( S_1 )</td>
</tr>
</tbody>
</table>

### 4.5. Spatial configuration impact

In this subsection, we examine the effect of the spatial configuration of the suppliers and final markets on the previous results. The spatial representation in Fig. 1 is characterized by a center of gravity \( G \), which is near of the center of gravity of the supplier points. Such a configuration would influence the location choice of the firm. Thus, we modify the spatial configuration such that the suppliers’ points are distant from the center of gravity by placing the supplier points on the right side of the plane and most of market points on the left side (the new spatial coordinates are given in Appendix C). The new spatial configuration is presented in Fig. 7. The point \( G^\prime (61.16, 45.83) \) represents the center of gravity and is calculated using the average weight formula. We repeated the calculations, and the results are summarized in Table 7.

The findings generally confirm and enrich those of previous works. When substitutability is very low, the optimal location is in the neighborhood of \( G^\prime \), but compared to the preceding configuration, there is a clear pull from the trio of suppliers \( S_2, S_3, \) and \( S_5 \), as the optimal location falls in the northeast of the central position \( G^\prime \). Medium substitution favors input source \( S_2 \), which is the location that minimizes the production cost (assuming a Leontief technology). Higher substitution favors input source \( S_1 \), which is the location that minimizes the output transportation cost when we assume a linear technology (see the results of the two benchmarks in Appendix D). We observe no relocation effect when the substitution is high within this new spatial configuration. This can be explained by the fact that the suppliers’ points are more geographically dispersed compared to the previous spatial configuration, where some pairs of suppliers were nearby (see...
Fig. 1). The firm gained the benefit from such proximity, as explained in the previous high substitution case with a low carbon tax.

In order to capture the true impact of a carbon tax on this case, we have also tested the scenario where the markets are identical (same demand). The results were identical to those presented in Table 7.

5. Conclusion

In this paper, we investigated the impact of the substitution elasticity degree across raw material quantities on a firm’s simultaneous production and location decisions under a carbon tax policy for transport-related carbon emissions. The substitution effect has been captured via the CES production function. The analysis of the joint production-location problem through numerical examples showed that the optimal location is considerably influenced by the degree of substitutability. This is because the substitution effect creates a trade-off between the output transportation cost and the production cost. When substitutability is weak, it is more likely that a central location between the upstream and downstream markets arises. When substitutability is intermediate, a firm is more likely to seek an input source location that minimizes the production cost, as gains in production efficiency offset the losses in the distribution cost. When substitutability is high, a firm is more likely to seek an input source location that minimizes the distribution cost because gains in output transportation cost reduction offset the losses in the production cost side.

Another important result is that a marginal increase in the carbon tax may cause a significant jump in the firm’s optimal location, which leads to a noticeable drop in emissions when the elasticity of substitution is high. Increasing the carbon tax changes the relative delivered price of the inputs and makes the delivered price of some inputs extremely high. As the firm can partially control the delivered price of inputs by adjusting its own location, it might be more beneficial economically and environmentally to move toward the source of the input made increasingly expensive due to the carbon tax and to use more of this input.

Gaigné et al. (2019) have shown, based on a specific technology (Cobb–Douglas production function), that a marginal increase in the carbon tax may have the unwanted effect of an increase in emissions when the facility is relocated to re-optimize the cost. The relocation effect can operate in either direction (it may generate a higher level of pollution or cause a large fall in emissions). The unwanted effect of an increase in emissions may occur when we consider a distortion in the transport costs, as in the examples of Gaigné et al. (2019) in a multimodal transportation situation.

Our work is not free of limitations. For example, the environmental impact was limited to transport-related emissions since such emissions account for a considerable share of total emissions, especially for production companies that are large transport users. Thus, production-related carbon emissions were omitted from the analyses. Their consideration would influence the results when input quantities can be substituted. These emissions can be added to the model either as additional constraints or as additional objectives. For example, we can consider that the production techniques within the potential technologies differ in terms of operations costs and output emissions. In other words, each modification of the combination of inputs in the same technology (production technique) induces different operations costs and output emissions. Furthermore, from a methodological perspective, the emissions-minimizing objective may not be a natural benchmark since it disregards the economic benefits of production and transportation. Thus, a more balanced and accepted objective function, such as social welfare maximization, should be considered.

This work presents perspectives that can complete and/or improve our modeling by expanding the boundaries, for example, by considering uncertainty in demand or supply and competition for either the inputs or the outputs. Thus, extending the model to include one or more of these factors may lead to a more interesting scenario with potentially richer insights. This work provides elements for understanding the role of the carbon tax through several numerical illustrations based on artificial data, and under several hypothesis of production function design. Further works could be carried out with real data, which would however require new assumptions (production and storage capacities, ineligible regions such as an ocean, etc.).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Appendix A. Cost-minimizing function

The firm cost function $C(y, s)$ is expressed as:

$$
C(y, s) = \sum_{i=1}^{I} (w_i s_i + \sum_{k=1}^{K} T_{ij} d_{ij} q_k) + \sum_{i=1}^{I} T_{ij} d_{ij} s_i
+ \tau \left( \sum_{i=1}^{I} s_i d_{ij} s_i + \sum_{k=1}^{K} \sum_{i=1}^{I} \alpha_k d_{ij} q_k \right)
$$

Let $T_{ij} = t_{ij} + \tau \alpha_{ij}$, with $i = 1, \ldots, I$ be the total unit transportation cost per unit of distance and per unit of input from $S_j$ to $F$. The firm’s cost can then be rewritten as follows:

$$
C(y, s) = \sum_{i=1}^{I} \left( w_i + T_{ij} d_{ij} \right) s_i + \sum_{k=1}^{K} T_{ij} d_{ij} q_k
$$

The upstream cost (production cost) is given by

$$
c(y) = \sum_{i=1}^{I} \left( w_i + T_{ij} d_{ij} \right) s_i = \psi(y) q^0
$$

Then, the total cost can be rewritten as:

$$
C(y) = \psi(y) q^0 + \sum_{k=1}^{K} T_{ij} d_{ij} q_k
$$

As $q^0 = \sum_{k=1}^{K} q_k$, then

$$
C(y) = \sum_{k=1}^{K} \left( \psi(y) + T_{ij} d_{ij} \right) q_k
$$

Appendix B. Optimization methodology

The methodology to design and evaluate the production-location problem is presented in Fig. B.8. The first step is to consider different starting points, including suppliers, markets, and center of gravity, to avoid a local solution. For each starting point, we consider different degrees of substitution with different levels of carbon tax. Subsequently, we solve each instance by calling the GRG algorithm developed in Microsoft Visual Basic. The program reads the data from a dedicated Excel file. The starting points that generate the best objectives for each degree of substitution and tax level are recovered with a sorting program developed in Microsoft Visual Basic. Finally, we recall the GRG algorithm for the starting points generating the best objectives and record the results (optimal location, optimal inputs quantities, etc.) into a dedicated Excel file.
Appendix C. Data study

For the analysis of Section 4.5, we consider ten final markets and five input sources whose coordinates can be found in Tables C.8 and C.9, respectively. The demand requirements for each market are the same as in Table 2 of Section 4.1.

Appendix D. Benchmarks’ results

To better understand the location choice of the firm in the case of intermediate and high substitution, we develop two benchmarks.

- In the former case, we consider a kind of Leontief technology in which the suppliers are weighted equally. The weight of each supplier is determined by dividing the total demand by the number of suppliers (the weight of each supplier is therefore 28.4). We then calculate the production cost at each supplier according to each level of carbon tax. The results are presented on the right side of Fig. D.9 and indicate that the location of supplier $S_2$ minimizes the production cost regardless of the tax level. This case also explains the limit of nil substitution (i.e., $\phi = -\infty$).

- In the latter case, we consider a linear technology in which all the raw materials are supplied from one of the five suppliers. We calculate the output transportation cost from each supplier according to each level of carbon tax. The results are presented on the right side of Fig. D.9 and indicate that the location of supplier $S_1$ minimizes the distribution cost regardless of the tax level. This case can also explain the limit of infinite substitution (i.e., $\phi = 1$).

References