Premature deaths, accidental bequests and fairness

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Abstract

While little agreement exists regarding the taxation of bequests in general, there is a widely held view that accidental bequests should be subjected to a confiscatory tax. We reexamine the optimal taxation of accidental bequests by introducing a concern for compensating individuals for a premature death. Assuming that individuals care about what they leave to their offspring, we show that, whereas the 100 % tax view holds under the utilitarian criterion, the ex post egalitarian criterion (giving priority to the worst-off ex post) implies subsidizing accidental bequests so as to compensate the short-lived. In a second-best setting, compensating the short-lived justifies taxing total bequests at a rate increasing with the age of the deceased. Finally, when the model is extended to an intergenerational setting, accidental bequests cannot be used as a redistributive tool anymore, so that ex post egalitarianism rejoins the 100 % tax view.

Keywords: mortality, accidental bequests, optimal taxation, compensation, OLG models.

JEL classification codes: D63, D64, D91, H31, J10.

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1 Introduction

The taxation of bequests is one of the most debated topics in public finance. Indeed, wealth transmission involves many complex issues: the interests of the deceased, of their offspring, but also the whole distribution of wealth in the society.

These difficulties are illustrated in Mill's early works on inheritance taxation. In his *Principles of Political Economy* (1848), Mill argued that bequests go against the ideal of free competition, since these create an initial - and arbitrary - inequality among competitors. However, according to Mill, when the deceased left a will, one should respect the deceased's wishes.[1] To reconcile this with the ideal of free competition, Mill's solution consists in imposing an absolute limit on the amount of money that a person may inherit.

When bequests are accidental (in case of early death), the second dimension mentioned by Mill is absent: the deceased did not plan to bequeath as much. In such a case, there is stronger support for a confiscatory tax on bequests. This may explain why the conventional view in public finance is that, when there are no inefficiency problems and only redistributive concerns, accidental bequests should be subject to a confiscatory tax (Kaplow 2008).

Two objections have been raised against this widely held view. First, Blumkin and Sadka (2003) argue that a non-confiscatory tax on accidental bequests has the desirable consequence of making the demogrant of an optimal linear income tax system effectively non-uniform. It acts as an additional instrument and increases the efficiency of the tax system. Second, Cremer et al. (2012) observe that bequests, when publicly observable, have informational content if they are correlated with relevant characteristics of taxable agents. This content must be incorporated in the design of the optimal tax.[2]

This paper aims at reexamining the conditions under which the 100 % tax view on accidental bequests prevails, by paying attention to the fact that risk about the timing of death generates inequalities not only among the descendants of the deceased (inequalities in bequests), but also among the deceased themselves (inequalities in consumption and longevity).[3] We reexamine the optimal taxation of accidental bequests in the context of risky and unequal lifetimes, while assuming that (i) individuals have preferences on how lost savings are distributed in case of premature death; (ii) the government cares about the deceased's interests in giving; (iii) the government gives priority to those who will end up being worse-off, even when they may not be identifiable at the time of the decision. Given that assumptions (i) to (iii) play a key role in our analysis, let us explain why our study departs from the literature on these dimensions.

One may argue against assumption (i) that preferences on how lost savings are distributed do not exist, precisely because those bequests are "accidental".

[1] The underlying justification lies in Mill's adherence to the principle of free disposal of one's goods when being alive and even after one's death.

[2] Note that this point applies to all types of bequests.

[3] One could also mention inequalities in time spent with other generations in the family, but we will focus here on the self-centered value of consumption and longevity.

However, it should be stressed that, from the perspective of the deceased, it is inaccurate to talk about "accidental bequest".[4] Death is the surest thing that happens to everyone, only its timing is unknown. Therefore any rational agent should have contingent plans about the use of her wealth after her death, at all times during life. We assume that, in case of premature death, individuals benefit from the fact that their lost savings go to their children —following Mill's idea that the deceased have preferences about what happens to their wealth. This is akin to the "joy of giving" (Andreoni 1990), even if the latter generally concerns individuals who are alive.

Assuming the joy of giving makes parent and child preferences interdependent, this raises the question of whether such interdependent preferences should be taken into account in social evaluation (Boadway 2012). Hammond (1987) and Mirrlees (2007) argue that joy of giving utility should not enter the social welfare function, because it would lead to double counting. On the contrary, Kaplow (1995, 2008) and Farhi and Werning (2013) argue that all aspects of individual preferences deserve consideration when defining social welfare. We adopt the latter view (i.e. assumption (ii)).

This paper also departs from the literature on welfare criteria (assumption (iii)). Besides the - widely used - utilitarian criterion, we also consider the ex post egalitarian criterion, which gives priority to those who end up in the worst position in the final allocation, i.e., the unlucky short-lived. The reason why we use this alternative criterion is that utilitarianism has the unattractive implication of redistributing from short-lived towards long-lived persons, against any intuition for compensation of disadvantage.[5] This is problematic, since longevity differentials are hardly under the control of individuals, but depend on circumstances, and thus deserve compensation (for further discussion and justification, see Fleurbaey et al 2014).[6]

This paper proceeds in three steps. We first consider a static economy composed of *ex ante* identical individuals who face a risky lifetime, and whose initial endowments are equal. We thus abstract from inequalities among the offspring in order to focus on inequalities between the long-lived and the short-lived. We compare the utilitarian optimum with the ex post egalitarian optimum, and study their decentralization. Then, we relax the assumption that the government can tax unconditional and accidental components of bequests at different rates, and consider a second-best setting where bequests are taxed at rates varying with the age of the deceased. Finally, we extend the model to a dynamic overlapping generations (OLG) economy, where individuals enjoy unequal endowments due to longevity inequalities among their ancestors.

Our explorations lead us to three main results. First, the analysis of the static economy shows that, although the conventional 100 % tax view holds

[4]In this paper, we follow the literature and use the term "accidental bequests", in the sense of a bequest that arises only because of the premature death of the donor.

[5]Due to the concavity of temporal utility, the short-lived have a lower capacity to convert resources into utility, and are thus penalized by utilitarianism (Fleurbaey et al 2014).

[6]Genes explain about 25-33 % of longevity inequalities (Christensen et al 2006). Moreover, about 23-40 % of premature deaths are due to environmental factors (Pimentel et al 1998).

under the utilitarian criterion, the decentralization of the ex post egalitarian optimum requires not to tax, but to *subsidize* accidental bequests. The intuition goes as follows. Accidental bequests can be regarded as a way to compensate the unlucky short-lived for their early demise. Thus, when the worst-off include the short-lived, it may be optimal to subsidize their bequests to promote their lifetime well-being.[7]

Second, if the government can only tax total bequests at rates varying with the age of the deceased, utilitarianism implies taxing bequests at a rate that is *decreasing* with the age of the deceased, whereas the ex post egalitarian criterion recommends the opposite, again on the grounds of compensating the short-lived.

Third, when the model is extended to an intergenerational setting, accidental bequests cannot be used as a redistributive tool anymore, and ex post egalitarianism then rejoins the 100 % tax view. The reason is that, in an intergenerational setting, accidental bequests cause inequalities in endowments among the young, some of whom will turn out to be short-lived. Subsidizing accidental bequests would exacerbate well-being inequalities among the short-lived, between those who received accidental bequests and those who did not receive these. In that context, accidental bequests should be taxed at a confiscatory rate, and the compensation of the prematurely dead is then achieved by taxing savings so as to induce consumption profiles decreasing with age.

All in all, our paper shows that, while conditions (i) to (iii) suffice to rule out the conventional 100 % tax view on accidental bequests in a static setting, this does not hold in an intergenerational setting where unequal longevities of individuals affect the endowments of their (potentially short-lived) descendants.

Our paper contributes to the literature on the optimal taxation of inheritance, such as Blumkin and Sadka (2004), Cremer et al (2012), Farhi and Werning (2013) and Piketty and Saez (2013). These papers focus on productivity differentials, whereas our paper concentrates on another source of heterogeneity - the duration of life. Our paper also contributes to the normative literature on the compensation of short-lived persons (Fleurbaey et al 2014, Fleurbaey et al 2016), by considering accidental bequests as an instrument for the compensation of the prematurely dead. Finally, this work is also related to the literature on the annuity puzzle (Yaari 1965, Brown 2004, Davidoff et al 2005, Lockwood 2012, 2018). We show that annuities have a double distributive role: they equalize the endowment of each young individual *ex ante* (by abolishing accidental bequests), but they exacerbate inequalities in lifetime well-being (i.e. *ex post*), by redistributing the savings of the short-lived to the long-lived.

The paper is organized as follows. The static model is presented in Section 2. The utilitarian optimum and its decentralization are studied in Section 3. Section 4 examines the ex post egalitarian optimum and its decentralization. Section 5 studies the second-best optimal taxation of bequests, by allowing for

[7] One might believe that the compensation of the short-lived could instead be achieved through private life insurance. However, as we show in the Appendix, the demand for life insurance at the laissez-faire would be either zero (under perfect annuities) or negative (without annuities). Hence introducing a market for private life insurance cannot decentralize the ex post egalitarian optimum.

a tax rate on total bequests that varies with the age of the deceased. Section 6 extends our model to an OLG economy. Section 7 concludes.

2 The baseline model

Consider a two-period economy with risky lifetime. The population is a continuum of agents of size 1. The first period is the young age, during which individuals receive bequests, supply one unit of labour inelastically, have one child, consume and save for their old days. The second period, i.e. the old age, is a period during which agents consume their savings. Lifetime is risky: the old age is reached with a probability π $(0 < \pi < 1)$.

There exist two kinds of transfers/bequests from parents to children. First, young adults plan to give their children an unconditional bequest b. This is transferred independently of whether the parent reaches the old age or not. This is the "non-accidental" bequest. Second, in case of death before reaching the old age, the child receives the amount d that the parent would have consumed in case of survival. This is the "accidental bequest", and it is conditional on the parent's longevity.[8]

Thus, in case of a long life, an individual leaves b to her child, whereas, in case of premature death, she bequeaths $b + d$.

2.1 Main assumptions

Preferences Individual preferences are additive over time and satisfy the expected utility hypothesis. The utility function has the general form:

$$u(c) + \pi \left(u\left(d\right) + v(b)\right) + (1 - \pi)v\left(d + b\right), \tag{1}$$

where c and d denote first- and second-period consumptions. The period utility function $u(\cdot)$ is increasing and concave (i.e., $u'(\cdot) > 0$, $u''(\cdot) < 0$). Following Becker et al. (2005), we suppose that there exists, in the absence of joy of giving, a consumption level neutral for the continuation of existence $\bar{c} > 0$ such that $u(\bar{c}) = 0$. That consumption level $\bar{c}$ brings the same level of utility as being dead and giving nothing to the descendants.[9]

The function $v(\cdot)$ captures the joy of giving.[10] The modelling of the joy of giving is similar to Glomm and Ravikumar (1992) and Piketty and Saez

[8]Since the individual anticipates how his lost saving will be distributed in case of premature death, the "accidental" bequest is an intentional bequest. However, we use the term "accidental" on the grounds that this part of wealth is transmitted to the child only in case of occurrence of a premature death, but would not be transmitted otherwise.

[9]Assuming a strictly positive threshold $\bar{c}$ can be justified by *reductio ad absurdum*. Without that threshold, it would be the case, in the absence of joy of giving, that either *all* life-periods are worth living, or *no* life-period is worth living, two implausible statements.

[10]The joy of giving is distinct from either pure altruism or truncated altruism. Under pure altruism, parents care about the total well-being of their children. Truncated altruism would be modelled such that parents value the gifts they make to their children through their children's own utility of consumption. Under the joy of giving motive, parents value the bequests they leave to their children through their *own* preferences. Kopczuk and Lupton

(2013).[11] We assume that individuals care about what their children *receive* net of all taxes and transfers. At the laissez-faire, there is equality between what a parent gives and what his child receives: when the individual reaches the old age, the bequest received by the child equals b; when he dies prematurely, the bequest received by his child is $d+b$, since the planned old-age consumption is also transferred to the offspring.[12] However, when public policies are introduced, the equality between what the parent gives and what the child receives does not hold. In their decisions, parents take into account what their child receives net of taxes and transfers, which may differ from the levels of b and d.

Note that the utility function (1) is a numerical representation of individual preferences on lotteries of life, and, as such, does not assign any hedonic value (neither pleasure nor pain) to being alive or dead. The utility function (1) captures the fact that a person cares about the amount of resources that would be transferred to his children in case of premature death. Obviously, a person who is dead has no feeling any more, but this does not imply that the utility function should not incorporate a joy of giving in case of premature death. Forward-looking individuals have life-plans, which include a concern for what would be given to their children in case of premature death.[13]

We assume: $v(0) = 0 > u(0)$, $v(d) > 0$ under $d > 0$, $v'(\cdot) > 0$ and $v''(\cdot) < 0$. Following Hurd (1989), we assume that the marginal utility from giving is always lower than the marginal utility of consumption for a given amount, that is:

$$u'(d) > v'(d) \ \forall d \geq 0. \tag{2}$$

We assume that there exists a level of $d > 0$, denoted $\tilde{d}$, such that $u\left(\tilde{d}\right) = v\left(\tilde{d}\right)$. $\tilde{d}$ is such that, in the absence of unconditional bequest (i.e., $b = 0$), the utility from being alive at the old age and consuming $\tilde{d}$ is exactly equal to the utility from being dead at the old age and giving $\tilde{d}$ to the descendant.

When $d > \tilde{d}$, we have $u(d) > v(d)$, that is, in the absence of unconditional bequest, a person would prefer surviving and consuming d at the old age rather than dying prematurely and giving accidentally d to his offspring. On the contrary, when $d < \tilde{d}$, we have $u(d) < v(d)$, that is, in the absence of unconditional bequest, a person would prefer dying prematurely and giving accidentally d to his offspring rather than surviving and consuming d at the old age. The latter case concerns extremely poor economies. This paper assumes that the economy is sufficiently affluent, so that $d > \tilde{d}$.[14]

(2007, p. 210) prefer to model joy of giving rather than altruism, on the grounds that "the evidence suggests motives other than the maximisation of a dynastic utility function".

[11] On the joy of giving, see also Benhabib and Zhu (2008) and Mian et al (2020).

[12] We assume perfect one-to-one substitutability between the two kinds of bequests within the function $v(\cdot)$. Alternative formulations of $v(\cdot)$ would not strongly affect our results.

[13] Assuming a joy of giving in case of premature death is a simple way to rationalize the low demand for annuitization, in line with the annuity puzzle (Brown 2004, Davidoff et al 2005).

[14] If there were perfect consumption smoothing, affluency would amount to suppose that the hourly wage w satisfies the inequality: $w > (1+\pi)\tilde{d}$. In the absence of perfect smoothing (like in our economy), we cannot provide an explicit condition on w without imposing extra

An important corollary of assuming $d > \tilde{d}$ concerns the comparison of short-lived and long-lived individuals. When $d > \tilde{d}$, it is also the case, under our assumptions on $u(\cdot)$ and $v(\cdot)$, that for any $b > 0$, we have $v(b + d) < v(b) + u(d)$, that is, the prematurely dead are worse-off than the long-lived, which is a reasonable assumption in affluent economies.[15]

Markets It is assumed that the labor market is perfectly competitive, and that workers are paid at a wage rate $w > 0$.[16] Their labor supply is inelastic and their earnings equal w. Regarding the capital market, we suppose that savings bring a return R equal to 1 plus the interest rate. For simplicity, we assume, in the baseline model, that $R = 1$, i.e., a zero interest rate.

When considering economies with risky lifetime, it is common, after Yaari (1965), to assume that the economy includes perfectly competitive annuity markets, yielding an actuarially fair return. The goal of these markets is to insure individuals against the risk of a long life. Given that we would like here to consider the issue of accidental bequests, we will suppose that annuity markets do not exist in the laissez-faire. This assumption is in line with the empirical literature on the so-called annuity insurance puzzle (Brown 2004, Davidoff et al 2005). In the absence of annuities, individuals will then, in case of premature death, transmit the proceeds of their savings to their offsprings. As stated above, this transmission is valued by the donors through the function $v(\cdot)$.[17]

Similarly, our paper assumes that there exists no private life insurance market at the laissez-faire. Here again, we make this assumption to focus on the occurrence of accidental bequests, which is the topic of this paper. It should be stressed, however, that introducing a private life insurance market would not affect our results. Actually, as we show in the Appendix, the demand for private life insurance would be negative in the absence of perfect annuities, and equal to zero in the presence of perfect annuities.[18]

Budget constraints First- and second-period budget constraints are:

$$c = w - s - b + b_0 \quad (3)$$

$$d = s \quad (4)$$

where s denotes savings for future consumption, b denotes the unconditional component of bequests that the person is willing to give to his own child, while b_0 denotes the bequest received from the parents when young.

restrictions on $u(\cdot)$ and $v(\cdot)$. But the assumption $d > \tilde{d}$ is weak: this amounts to suppose that consumption possibilities are sufficiently favorable, so that individuals prefer living long and consuming rather than dying early and leaving the resource as an accidental bequest.

[15] This inequality is obtained by making a first-order Taylor approximation around b of $v(d + b)$. Under concavity, $v(d) + v(b) > v(d + b)$ so that, given $u(d) > v(d)$ for $d > \tilde{d}$, it follows that $u(d) + v(b) > v(d) + v(b) > v(d + b)$.

[16] For the sake of presentation, we assume the absence of heterogeneity in terms of wages. In the Appendix, we examine the robustness of our results to introducing unequal wages.

[17] Our paper is in line with Lockwood (2012, 2018), who regards the bequest motive as a major explanation for the annuity puzzle.

[18] Note that those results are obtained while assuming zero loading costs.

As a consequence of the absence of annuities, a young generation is composed of individuals with unequal endowments, depending on how large the received bequest b_0 is. However, for the sake of presentation, the baseline model examined in the first part of this paper abstracts from inequalities in initial endowments and sets b_0 to 0, in order to focus first on inequalities in lifetime well-being between short-lived and long-lived individuals, and second, on the impact of accidental bequests on those inequalities. Inequalities in initial endowments due to unequal received bequests are examined in Section 6.

2.2 The laissez-faire

The problem of an individual consists of allocating resources along his life, so as to maximize his expected lifetime utility subject to the resource constraints and the market factor prices:

$$\max_{s,b} u\left(w-s-b\right)+\pi\left(u(s)+v\left(b\right)\right)+(1-\pi)v(s+b)$$

The first-order conditions (FOCs) for optimal savings, s and for unconditional bequests b are given by:[19]

$$u'(w-s-b)=\pi u'(s)+(1-\pi)v'(s+b)$$
$$u'(w-s-b)=\pi v'(b)+(1-\pi)v'(s+b)$$

Proposition 1 summarizes the key facts.[20]

Proposition 1 *At the laissez-faire, consumption decreases over the life cycle ($c>d$), and the unconditional bequest is smaller than old-age consumption ($b<d$). The long-lived is better off than the short-lived.*

Proof. $u'(w-s-b)$ is a linear combination of $u'(s)$ and $v'(s+b)$ with $u'(s)>v'(s+b)$ from our assumptions on the forms of $u(\cdot)$ and $v(\cdot)$. Therefore, $u'(w-s-b)<u'(s)$ and $c=w-s-b>d=s$. Comparing the FOCs for s and for b, we obtain that $u'(s)=v'(b)$, which implies that $b<s=d$.

Regarding well-being inequalities, we know that, if $d>\tilde{d}$, the short-lived is worse off than the long-lived,

$$U^{LL}=u(c)+u(d)+v(b)>U^{SL}=u(c)+v(d+b)$$

since $v(b+d)<v(b)+v(d)<v(b)+u(d)$. ■

Proposition 1 states that, at the laissez-faire equilibrium, the long-lived are better off than the short-lived. This inequality arises despite the fact that the

[19] In this paper, we focus on the case where $b>0$, which holds for w sufficiently high.

[20] If individuals do not care about giving to the descendant ($v\equiv 0$), then $b=0$ and one obtains the standard Euler equation: $u'(w-s)=\pi u'(s)$. Note that, even in this case, consumption is not smoothed along the lifecycle, because of the absence of annuities. Alternatively, in the presence of actuarially fair annuities, the return on annuitized savings would be $\frac{1}{\pi}$ and the FOC would be reduced to $u'(w-s-b)=u'(s/\pi)$ implying $c=d$.

prematurely dead derive well-being from having the proceeds of their lost savings being redistributed towards their offsprings. The reason is that, in affluent economies, we have $u(d) + v(b) > v(d + b)$, which implies that the well-being of long-lived persons is larger than the well-being derived by the short-lived from transmitting their lost savings to their offspring.

3 The utilitarian optimum

3.1 Centralized solution

Let us now characterize the utilitarian social optimum. For that purpose, a first thing to notice is that there exists an important difference between the social planning problem and the laissez-faire problem, concerning the level of the accidental bequest. At the laissez-faire, the accidental bequest is equal to the lost savings due to premature death, i.e., d. However, in a social planning problem, there is no reason why the accidental bequest should be equal to what the parent would have consumed in case of a long life. Thus, when writing the social planning problem, we introduce a new variable e for the actual accidental bequest received by the child, and we do not impose that $e = d$.

The problem of the utilitarian social planner, whose goal is to maximize the average lifetime well-being of the population (which is also equal here to the expected lifetime well-being of a representative individual), can be written as:

$$\max_{c,d,b,e} u(c) + \pi \left(u(d) + v\left(b\right)\right) + (1 - \pi)v(e + b)$$
$$\text{s.t. } c + \pi d + (1 - \pi)e + b = w.$$

The FOCs yield:

$$u'(c) = u'(d) = v'(e + b) = v'(b) \tag{5}$$

Hence, consumption is smoothed along the life cycle ($c = d$) and $e = 0$, meaning that accidental bequests are totally eliminated. In addition, one deduces from $u'(d) = v'(b)$ that $b < d$.

A short-lived person, with utility $u(c) + v(b)$, is worse off than a long-lived person whose utility is $u(c) + u(d) + v(b)$, if and only if consumption d is above $\bar{c}$, which is a weak assumption in an affluent economy.

Proposition 2 *At the utilitarian optimum, consumption is smoothed, accidental bequests e are zero, $b < d$, and the short-lived are worse off than the long-lived if and only if $d > \bar{c}$.*

The utilitarian optimum involves a flat consumption profile as well as some positive unconditional bequest, but no accidental bequest.

3.2 Decentralized solution

In order to decentralize the utilitarian optimum, let us now introduce the policy instruments necessary to obtain the above optimal trade-offs.[21] For that purpose, we consider four policy instruments: first, a collective annuitization of savings through public actuarially fair annuities to a fixed degree $\alpha \in [0,1]$, equivalent to a public pension system; second, an actuarially fair public life insurance scheme a, equivalent to a public survivor benefit; third, a subsidy on savings $\sigma \in [0,1]$; fourth, a tax $\theta \in [0,1]$ on accidental bequests (in case of early death).[22] Tax revenues from the taxation of accidental bequests and from annuities are redistributed at the young age through a lump sum transfer T.[23]

Conditional on the levels of those instruments, the agent chooses savings s and unconditional bequest b so as to maximize expected lifetime well-being:

$$\max_{s,b} u(c) + \pi \left[u\left(d\right) + v(b)\right] + (1-\pi)v\left(e+b\right)$$

with $c = w - b - s(1-\sigma) - a + T$, $d = \alpha\frac{s}{\pi} + (1-\alpha)s$ and $e = (1-\alpha)s(1-\theta) + \frac{a}{1-\pi}$ are the additional resources given to the child in case of premature death of his parent. Note that the "joy of giving" term includes what the parent leaves to his child net of the inheritance tax.

FOCs for s and b are now:

$$u'(c)(1-\sigma) = \begin{array}{l} \pi u'\left(d\right)\left(\frac{\alpha}{\pi} + (1-\alpha)\right) \\ +(1-\pi)v'\left(e+b\right)(1-\alpha)(1-\theta) \end{array} \tag{6}$$

$$u'(c) = \pi v'(b) + (1-\pi)v'\left(e+b\right) \tag{7}$$

Let us compare these equations with the utilitarian first-best trade-off, eq. (5) as well as with the optimal first-best level $e = 0$. It is straightforward to show that, if there is no collective life insurance ($a = 0$), full annuitization (i.e., $\alpha = 1$) would decentralize the first-best optimum, leading to $e = 0$ and to the optimal first-best trade-offs, $u'(c) = u'(d) = v'(b)$. In that case, there is no reason to subsidize savings in the first period, i.e. $\sigma = 0$.

Now, let us consider a case where full public annuitization is not possible (i.e., $\alpha < 1$), as it is usually the case in our economies. Still, the utilitarian first-best optimum can be decentralized by, first, fully taxing accidental bequests, i.e. $\theta = 1$ leading to $e = 0$ and to $u'(c) = v'(b)$; second, in order to obtain $u'(c) = u'(d)$, one would need to set a subsidy on savings in the first period equal to:[24]

$$\sigma = (1-\pi)(1-\alpha) > 0.$$

[21] We assume here that the government can distinguish between the unconditional and the accidental parts of bequests. That assumption will also be made in Section 4, but will then be relaxed in Section 5 dedicated to second-best inheritance taxation.

[22] As we show below, those instruments suffice to decentralize the utilitarian social optimum under perfect information. In Section 5, we consider another set of policy instruments which is closer to what is observed in real-world economies.

[23] The lump sum transfer T is taken as given by individuals, but in equilibrium, it is obtained from balancing the government budget constraint.

[24] The optimal level of σ is obtained by replacing (5) into (6).

Our results are summarized in the following proposition:

Proposition 3 *The utilitarian optimum can be decentralized either by introducing full annuitization of savings (i.e., $\alpha = 1$) or, when full annuitization of savings is impossible (i.e., $\alpha < 1$), by introducing a full taxation of accidental bequests (i.e., $\theta = 1$) together with a subsidy on savings (i.e. $\sigma > 0$).*

The decentralization of the utilitarian first-best optimum involves either the full annuitization of savings, or a 100 % tax on accidental bequests (together with a subsidy on savings). The latter instrument coincides with the view (mentioned at the very outset of this paper) that, for pure redistributive reasons, a 100 % tax should be set on accidental bequests.

Interestingly, both collective annuitization and the taxation of bequests contribute, in the presence of joy of giving, to increasing inequalities in lifetime well-being between the long-lived and the short-lived with respect to the laissez-faire equilibrium (see Section 2.2). Indeed, eliminating accidental bequests, i.e. $e = 0$, makes the short-lived worse-off than at the laissez-faire. The underlying intuition is that confiscating accidental bequests or, alternatively, redistributing the savings of the dead to the surviving old, prevents the unlucky short-lived from giving more to their children. As such, those two instruments deteriorate the situation of the unlucky short-lived with respect to the laissez-faire equilibrium.

4 The ex post egalitarian optimum

At the utilitarian optimum, accidental bequests are fully taxed away, comforting the widespread view on this issue. One may wonder whether this result is robust to the social welfare criterion. The use of the utilitarian criterion in a context of unequal lifetimes can be questioned, on the grounds that it pays little attention to ex post well-being inequalities. As we showed above, at the utilitarian optimum, the short-lived are, in an affluent economy, worse off than the long-lived.

Such inequalities in realized lifetime well-being are difficult to justify, since all individuals are identical ex ante, and behave in the exact same way. In our framework, it is a matter of luck whether someone reaches the old age or not. Having a long life or a short life is here a pure circumstance. As formalized in Fleurbaey (2008) and Fleurbaey and Maniquet (2010), the theory of equal opportunities states that governments should intervene so as to abolish well-being inequalities due to circumstances. In the context of unequal lifetimes, this "Principle of Compensation" motivates the compensation of unlucky individuals who turn out to be short-lived.[25]

[25] It is sometimes objected to this analysis that the short-lived are not disadvantaged because they did not forecast their own demise and do not live to contemplate it afterward. But this naive hedonistic argument, which echoes Epicurius' view that death is not bad, overlooks the fact that people have preferences over their lifetime profile, and that a short life is something people generally dislike. Many people whose life ends prematurely do experience the prospect of death in the last weeks of their life and this is not one that is usually welcome.

This section aims at exploring the question of the compensation of the short-lived, and its impact on the optimal taxation of accidental bequests. Several social criteria could do justice to the goal of compensating persons for circumstances, depending on various ethical concerns (see Fleurbaey and Maniquet, 2011 and Fleurbaey, 2008). In order to study the compensation of the short-lived, this section relies on the ex post egalitarian criterion (Fleurbaey et al, 2014), which exhibits two main features: (i) it evaluates social outcomes by adopting an ex post perspective (i.e., focus on *realized* well-being); (ii) it gives priority to the most disadvantaged person.

The ex post approach to social valuation under risk is justified by Fleurbaey (2010). Fleurbaey highlights a distinction between informed preferences (i.e., preferences if individuals knew the state of the world that would be realized) and uninformed preferences (i.e., preferences formed while the state of the world that will be realized is not known). According to Fleurbaey, a social planner should give priority to informed preferences over uninformed preferences. That argument, which supports an ex post approach to social valuation, is most relevant for this paper. No one knows, ex ante, whether one will be long-lived or short-lived. However, only one state of nature will realize: either the individual will be short-lived, or he will be long-lived. Even if individual decisions at the laissez-faire are based on uninformed preferences, there is no reason why, at the normative level, the social planner should base his plans on uninformed preferences. In the presence of risk about the duration of life, informed preferences are a better guide for normative analysis.[26]

Regarding the second feature of the ex post egalitarian criterion, it may, at first glance, seem extreme to give priority to the worst-off individuals. In particular, the ex post egalitarian criterion justifies sacrificing large amounts of welfare for some individuals for the sake of improving a bit the situation of the most disadvantaged. However, maximin criteria can be deduced from intuitive conditions imposed on how the society should rank allocations. In the context of unequal lifetime, Fleurbaey et al (2014) showed that a maximin criterion defined on individual well-being indexes can be derived from assuming that social orderings on allocations satisfy Pareto optimality, an independence criterion (weaker than Arrow's independence), as well as mild transfer axioms involving redistribution towards the most disadvantaged. Thus, although extreme, the ex post egalitarian criterion can be justified on the basis of intuitive properties.[27]

The problem of the ex post egalitarian social planner can be written as:

$$\max_{c,d,e,b} \min \left\{U^{SL}, U^{LL}\right\} \text{ s.t. } c + \pi d + (1-\pi)e + b = w$$

[26] Whereas one might regard the ex post approach as "paternalistic" (since this differs from individuals' ex ante perspectives when making their decision), it should be stressed that the choice between an ex ante or an ex post approach is just about selecting the most relevant informational basis for assessing social situations under risk. If expectations matter less than achievements, adopting an ex post approach is justified. This is the ethical view adopted here.

[27] The ex post egalitarian function exhibits infinite inequality aversion and can be regarded as an extreme form of the strictly concave social welfare function with finite inequality aversion (Leach 2009, Fleurbaey et al 2016). The robustness of the results of this section depends on how large the degree of inequality aversion is, but we always obtain that $e > 0$.

where $U^{LL} \equiv u(c) + u(d) + v(b)$ and $U^{SL} \equiv u(c) + v(e+b)$.

The objective is continuous but not differentiable. That problem can be rewritten as the maximization of the lifetime well-being of the short-lived conditionally on the resource constraint and on an egalitarian constraint that the long-lived are as well-off as the short-lived. This egalitarian constraint is:

$$u(c) + u(d) + v(b) = u(c) + v(e+b) \tag{8}$$

The Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= u(c) + v(b+e) - \mu\left[c + \pi d + (1-\pi)e + b - w\right] \\ &\quad + \lambda\left[u(d) + v(b) - v(e+b)\right] \end{aligned}$$

where μ is the Lagrange multiplier associated with the resource constraint, and λ is the Lagrange multiplier associated with the egalitarian constraint. FOCs yield:

$$u'(c) = \mu = \frac{\lambda}{\pi} u'(d) \tag{9}$$

$$\lambda v'(b) + (1-\lambda) v'(e+b) = \mu = \frac{1-\lambda}{1-\pi} v'(e+b) \tag{10}$$

Assuming that the shadow value of relaxing the egalitarian constraint is low, we have $\pi > \lambda$, so that the consumption profile is decreasing: $c > d$.[28] From the last expression, we deduce:

$$v'(b) = \frac{\mu}{\lambda}\pi \tag{11}$$

Together with $u'(d) = \mu\pi/\lambda$, we obtain that $u'(d) = v'(b)$, so that $d > b$. We then have the full ranking: $c > d > b$.

Expressions (9) and (10) can be rewritten as a Euler type equation:

$$\frac{1}{u'(c)} = \frac{\pi}{u'(d)} + \frac{1-\pi}{v'(b+e)} \tag{12}$$

Since we have proven that $c > d$, we have $u'(c) < u'(d)$, which implies, from the above equation, that $u'(c) > v'(b+e)$. Moreover, since $u'(d) = v'(b) > u'(c)$, we have that $v'(b) > v'(b+e)$ and thus $e > 0$. Finally, let us study the ranking between e and d. Given the egalitarian constraint $u(d) + v(b) = v(e+b)$, and the concavity of $v(\cdot)$, we have $v(e+b) < v(e) + v(b)$. Hence, the egalitarian optimum is characterized by $0 \leq u(d) < v(e)$. Under our assumption $d > \tilde{d}$, $u(\cdot)$ is always above $v(\cdot)$, so that e must be much larger than d to obtain that $u(d) < v(e)$. Accidental bequests must therefore be augmented ($e > d$) in order to provide a compensation to the short-lived.

[28] We focus here on affluent economies, as defined above (see *supra*).

Proposition 4 *In an affluent economy, at the ex post egalitarian optimum, consumption decreases along the life cycle, accidental bequests are higher than old-age consumption, and the short-lived and long-lived are equally well off. For any level of annuitization (i.e., $0 \leq \alpha \leq 1$), that optimum can be decentralized by introducing collective life insurance $a > 0$, a subsidy on accidental bequests $\theta < 0$ together with a tax on savings, $\sigma < 0$.*

Proof. For the first part of Proposition 4, see above. The decentralization is achieved as follows. The levels of σ and θ are obtained by comparing eq. (6) and (7) with the first-best egalitarian trade-offs, eq. (9) and (10). The level of a is fixed so as to satisfy the egalitarian constraint, $u(d) + v(b) = v(e + b)$. ■

The decentralization of the egalitarian optimum requires both linear taxation instruments so as to obtain the optimal trade-offs between consumptions c, d and the joy of giving terms e, b. In addition, so as to obtain that no inequality is left between the long-lived and the short-lived, an adequate level of life insurance a needs to be set. This optimum can be decentralized for different degrees of public annuitization; it would only change the size of the other policy instruments.

Comparing this result with Proposition 3, one can see a first key difference: the utilitarian optimum involves zero accidental bequests, whereas the ex post egalitarian optimum involves accidental bequests that are strictly positive, and strictly higher than old-age consumption.[29] The intuition is that accidental bequests allow the short-lived to give more to their children (in comparison to the long-lived), and, hence, serve to compensate the short-lived. This can be done by promoting life insurance ($a > 0$) and by subsidizing accidental bequests ($\theta < 0$). It is also optimal to tax savings ($\sigma < 0$) so as to preserve the egalitarian first-best consumption trade-off, i.e. $c > d$ and ensure that individuals do not save too much. This is another key difference with Proposition 3, where savings where subsidized and accidental bequests were taxed.

Proposition 4 states that it is optimal to subsidize accidental bequests. It may seem counterintuitive, but it should be reminded that the social goal is to compensate short-lived persons. From that perspective, taxing accidental bequests would be unfair, as it would prevent the short-lived from giving to their descendant, and, as such, would make them worse off. On the contrary, accidental bequests should be subsidized, to make the short-lived better off.[30]

[29] In absence of joy of giving, bequests would disappear altogether and the egalitarian constraint would boil down to $u(c) + u(d) = u(c)$, requiring $d = \bar{c}$, i.e., making old-age consumption low enough to improve the situation of the short-lived as much as possible (see Fleurbaey et al. 2014 for further discussion of this case).

[30] Let us also mention that reducing the degree of annuitization α to zero contributes to reducing inequalities in lifetime well-being, but is not sufficient to achieve the equality $U^{LL} = U^{SL}$. Indeed, under $\alpha = 0$, we have: $U^{LL} - U^{SL} = u(s) + v(b) - v\left(s(1-\theta) + b + \frac{a}{1-\pi}\right)$. That expression reveals that, even with no annuitization possible, for a given level of θ, a collective life insurance is necessary to have $U^{LL} = U^{SL}$.

5 Bequests tax and the age of the deceased

Up to now, we considered a first-best framework, that is, assuming that all fiscal instruments are available. The government could then tax unconditional and accidental bequests at different rates. This section considers a second-best setting where only three instruments are available: a first-period demogrant T, a tax θ_E on bequests left in case of early death, and a tax θ_L on bequests left in case of late death. For simplicity, we abstract from the possibility of public life insurance and collective annuitization.

The underlying motivation for considering these instruments is twofold. First, although it is difficult for a government to impose different tax rates on unconditional and accidental parts of the bequest, the government can use the age of the deceased as an indirect way to tax these two components at different rates. The underlying intuition is that the relative part of the accidental component in the total bequest is decreasing with the age of the deceased.

Second, the public finance literature already considered taxing bequests differently depending on the age of the deceased. Vickrey (1945) argued that the tax on bequests should be increasing with the age gap between the donor and the receiver, in order to prevent fiscal arbitrages. As we will see, this section will provide also an argument for a tax on bequests that is increasing with the age gap between the donor and the receiver, but on egalitarian grounds.

5.1 The individual problem

Individuals choose savings s and unconditional bequest b to maximize utility:

$$\max_{s,b} u(w-s-b+T)+\pi\left[u(s)+v((1-\theta_L)\,b)\right]+(1-\pi)v((1-\theta_E)\,(b+s))$$

FOCs for s and b yield, respectively:

$$\begin{aligned} -u'(w-s-b+T)+\pi u'(s)+(1-\pi)v'(x_E)\,(1-\theta_E) &= 0 \\ -u'(w-s-b+T)+\pi v'(x_L)\,(1-\theta_L)+(1-\pi)v'(x_E)\,(1-\theta_E) &= 0 \end{aligned}$$

where $x_L \equiv (1-\theta_L)\,b$ and $x_E \equiv (1-\theta_E)\,(b+s)$ are the bequest (net of tax) in case of, respectively, late death and early death. Combining those FOCs yields:

$$u'(s) = v'((1-\theta_L)\,b)\,(1-\theta_L) \tag{13}$$

Both the unconditional bequest $b \equiv b\,(T,\theta_E,\theta_L)$ and savings $s \equiv s\,(T,\theta_E,\theta_L)$ are functions of the tax and transfer instruments. Using Cramer's rule, we have:

If $|\varepsilon_{v'(x_E)}| \leq 1$ (resp. >1), $\dfrac{ds}{d\theta_E}, \dfrac{db}{d\theta_E} \leq 0$ (resp. >0),

If $|\varepsilon_{v'(x_L)}| \leq 1$ (resp. >1), $\dfrac{ds}{d\theta_L} \geq 0$ (resp. <0) and $\dfrac{db}{d\theta_L} \leq 0$ (resp. >0),

where $\varepsilon_{v'(x)}$ is the elasticity of marginal utility with respect to x. Two comments are in order. First, the signs of variations of s and b induced by changes in the tax rates depend on the elasticity $\varepsilon_{v'(x)}$. Second, while the sign of variations of bequests left by the long-lived (i.e. b) induced by changes in the tax rates is not ambiguous, the sign of variations of bequests left by a short-lived $(b+s)$ induced by changes in θ_L is ambiguous since b and s vary in opposite directions.[31]

5.2 The second-best utilitarian problem

The utilitarian planner selects a system of taxes and transfers (T, θ_E, θ_L) that maximizes average welfare subject to the revenue constraint. This problem can be written by means of the Lagrangian:

$$\max_{T,\theta_L,\theta_E} \quad \mathcal{L} = \quad u(w-s-b+T) + \pi\left[u(s)+v(x_L)\right] + (1-\pi)v(x_E)$$
$$+\mu\left[\pi\theta_L b + (1-\pi)\theta_E(b+s) - T\right]$$

Using the envelope theorem, we obtain the following FOCs:

$$\frac{\partial\mathcal{L}}{\partial\theta_E} = -(1-\pi)v'(x_E)(b+s) + \mu\left[(1-\pi)(b+s) + \pi\theta_L\frac{\partial b}{\partial\theta_E} + (1-\pi)\theta_E\frac{\partial(b+s)}{\partial\theta_E}\right] = 0$$
$$\frac{\partial\mathcal{L}}{\partial\theta_L} = -\pi v'(x_L)b + \mu\left[\pi b + \pi\theta_L\frac{\partial b}{\partial\theta_L} + (1-\pi)\theta_E\frac{\partial(b+s)}{\partial\theta_L}\right] = 0$$
$$\frac{\partial\mathcal{L}}{\partial T} = u'(c) + \mu\left[\pi\theta_L\frac{\partial b}{\partial T} + (1-\pi)\theta_E\frac{\partial(b+s)}{\partial T} - 1\right] = 0$$

Those FOCs can be used to examine the design of optimal taxes on bequests. For this purpose, let us combine the first two FOCs to write the effect of a compensated change of the tax rate on bequests left by a person who died early ("early bequests"), that is, the effect on the objective function of the social planner, of a marginal change in θ_E when this change is compensated by a change in θ_L, so as to maintain the government's budget balanced:

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}}{\partial\theta_E} &\equiv \frac{\partial\mathcal{L}}{\partial\theta_E} + \frac{\partial\mathcal{L}}{\partial\theta_L}\frac{d\theta_L}{d\theta_E} \\ &= -(1-\pi)(b+s)\left[v'(x_E) - v'(x_L)\right] \\ &+ \mu\left[\pi\theta_L\left(\frac{\partial b}{\partial\theta_E} + \frac{\partial b}{\partial\theta_L}\frac{d\theta_L}{d\theta_E}\right) + (1-\pi)\theta_E\left(\frac{\partial(b+s)}{\partial\theta_E} + \frac{\partial(b+s)}{\partial\theta_L}\frac{d\theta_L}{d\theta_E}\right)\right]\end{aligned} \tag{14}$$

where, from the budget constraint, $\frac{d\theta_L}{d\theta_E} = \frac{-(1-\pi)}{\pi}\left(\frac{s+b}{b}\right)$ and where the terms inside parentheses in the last bracket term are the effects on, respectively, b and $(b+s)$ of compensated changes in θ_E so as to keep the budget balanced.

In the absence of any tax on bequests, this expression collapses to:

$$\left.\frac{\partial\tilde{\mathcal{L}}}{\partial\theta_E}\right|_{\theta_E=\theta_L=0} = -(1-\pi)(b+s)\left[v'(x_E) - v'(x_L)\right]$$

[31] It is also possible to show that s and b increase with the lump sum transfer T.

which is positive, since it is possible to show, from the individual's problem, that in the absence of taxation we have: $v'(x_E) < v'(x_L)$. This expression accounts for the insurance effect of taxation and for the fact that it is optimal, from a utilitarian perspective, to equalize marginal utility of giving across the different states of nature. Hence, starting from a zero tax on bequests, a marginal increase in the tax on early bequests, when compensated by a change in the tax on late bequests so as to maintain the government's budget balanced, increases the welfare of the society. This provides a utilitarian argument for taxing the bequests of individuals who die earlier at a *higher* rate than the bequests of those who die later: $\theta_E > \theta_L$.

The above analysis does not account for revenue effects of increasing θ_E, i.e. the second term in eq. (14). This expression can be written as:

$$\begin{aligned} \frac{\partial \tilde{\mathcal{L}}}{\partial \theta_E} &= -(1-\pi)(b+s)\left[v'(x_E) - v'(x_L)\right] \\ &+ \mu\left[\pi b\left(\frac{\theta_L}{\theta_E}\varepsilon_{b,\theta_E} + \varepsilon_{b,\theta_L}\frac{d\theta_L}{d\theta_E}\right) + (1-\pi)(b+s)\left(\varepsilon_{(b+s),\theta_E} + \frac{\theta_E}{\theta_L}\varepsilon_{(b+s),\theta_L}\frac{d\theta_L}{d\theta_E}\right)\right] \end{aligned} \tag{15}$$

where the signs of the elasticities depend on assumptions regarding $|\varepsilon_{v'(x_E)}|$ and $|\varepsilon_{v'(x_L)}|$ as shown in Section 5.1.

The first line in eq. (15) accounts for the insurance effect described earlier. The second line accounts for the impact of increasing θ_E (compensated by a decrease in θ_L) on the government budget constraint, taking into account individual responses (i.e. how b and $b+s$ adapt to a variation of the tax rates). The sign of this second term is ambiguous, and depends on the sign as well as on the magnitude of the elasticities of early and late bequests to taxation. Without further assumption, it is not clear whether the revenue effect pushes toward higher or lower taxation of early bequests. Yet, assuming that the insurance effect dominates the revenue effect, an increase in early taxation (compensated by a decrease in late taxation) is socially desirable.

5.3 The second-best ex post egalitarian problem

The ex post egalitarian planner selects instruments (T, θ_E, θ_L) that maximize the realized lifetime well-being of the worst-off:

$$\begin{aligned} \max_{T,\theta_L,\theta_E} \min \quad & \{u(w-s-b+T) + u(s) + v((1-\theta_L)\,b), u(w-s-b+T) + v((1-\theta_E)\,(b+s))\} \\ \text{s.t.} \quad & \pi\theta_L b + (1-\pi)\theta_E(b+s) = T \end{aligned}$$

We rewrite this problem as the maximization of the well-being of the short-lived subject to the egalitarian constraint:

$$\begin{aligned} \max_{T,\theta_L,\theta_E} \quad & u(w-s-b+T) + v((1-\theta_E)\,(b+s)) \\ \text{s.t.} \quad & \pi\theta_L b + (1-\pi)\theta_E(b+s) = T \\ & v((1-\theta_E)\,(b+s)) = u(s) + v((1-\theta_L)\,b) \end{aligned}$$

In the absence of taxation, the egalitarian constraint would not be satisfied: since $v(b+s) < u(s)+v(b)$, the short-lived are worse off than the long-lived. To achieve equality, the tax on the bequests left by those who die earlier must be *lower* than the tax on the bequests left by those who have a long life: $\theta_E < \theta_L$. It is optimal to tax more the bequests left by individuals who die at higher ages, on the grounds that lower tax rates on the bequests of those who die earlier favor the compensation of the short-lived. Proposition 5 summarizes our results.

Proposition 5 *Consider a second-best setting with, as available instruments, a demogrant T, and tax rates on bequests θ_E and θ_L based on the age of the deceased. Abstracting from inequalities in endowments across children, we have:*

- *under the utilitarian criterion, the tax on bequests should be decreasing with the age of the deceased ($\theta_E > \theta_L$);*
- *under the ex post egalitarian criterion, the tax on bequests should be increasing with the age of the deceased ($\theta_E < \theta_L$).*

Proof. For utilitarianism, see Section 5.2. For egalitarianism, the constraint is $v((1-\theta_E)(b+s)) = u(s) + v((1-\theta_L)b)$ so that $v((1-\theta_E)(b+s)) > v(s) + v((1-\theta_L)b) > v(s(1-\theta_E)) + v((1-\theta_L)b)$. Using $v''(\cdot) < 0$, we have $v((1-\theta_E)b) + v((1-\theta_E)s) > v((1-\theta_L)b) + v((1-\theta_E)s)$, which is true only if $\theta_E < \theta_L$. ■

How governments should differentiate bequests taxation depending on the age of the deceased is not robust to the underlying social welfare objective. Assigning a large weight to the short-lived leads to a lower tax on the bequests left by those who die earlier, and to a higher tax on bequests left by those who die later on. This compensation argument for a differentiated treatment of bequests depending on the age of the deceased is different, in nature, from Vickrey's (1945) argument against fiscal avoidance, even though both arguments justify a tax rate increasing with the age of the deceased.

6 The OLG economy

In the previous sections, we deliberately ignored inequalities in endowments, in order to focus on inequalities between the short-lived and the long-lived. However, ignoring these inequalities simplifies the picture significantly. As Mill (1848) emphasized, a major argument for taxing bequests lies in the fact that they lead to large inequalities among descendants.

In order to take into account the impact of bequests on next generations, we need to consider a dynamic OLG model, where agents are heterogeneous in terms of the bequests they receive from their parents. This section develops the dynamic extension of the baseline model, and examines the robustness of our results concerning the optimal taxation of accidental bequests.

6.1 The model

Let us consider a two-period OLG model with the same lifecycle structure as in our basic model.[32] The shift from a static to a dynamic model increases intracohort heterogeneity. In our static model, the unique source of intracohort heterogeneity was the duration of life. In this extended model, a second source of heterogeneity lies in individual initial endowments. The individual's endowment depends on the bequest received from his parent and, hence, depends on his parent's duration of life, as well as on the longevity of all his ancestors (which affect the parent's endowment). This leads, after a small number of generations, to a large heterogeneity in endowments. Without any restriction, the extended model would exhibit a *continuum* of types at the stationary equilibrium, making our analysis unnecessarily complex for the purpose at stake.

In order to keep the analysis tractable, we assume a particular structure on preferences, which makes the dynamics of wealth accumulation Markovian, i.e. the endowment of a person born at t only depends on the longevity of his parent born at $t-1$, and not on the longevity of previous ancestors.

Preferences To make the wealth dynamics across generations Markovian, we assume that individuals have quasi-linear preferences:

$$c_t + \pi\big[u\left(d_{t+1}\right) + v(b_{t+1})\big] + (1-\pi)v\left(d_{t+1} + b_{t+1}\right) \tag{16}$$

where c_t is the consumption of a young adult at time t, b_{t+1} denotes the unconditional component of parental bequest, while d_{t+1} is either the old-age consumption for a young adult at time t who survives to the old age, or the accidental bequest that he leaves to his child in case of premature death.

Under these preferences, the endowment received affects only young-age consumption, but has no impact on old-age consumption d_{t+1} and on the unconditional bequest b_{t+1}. Whether a person has a long-lived or a short-lived parent has no impact on the endowment of his child.

Budget constraints Given our assumption of quasi-linear preferences, there are only two groups of young adults at time t:

- Type-E_t individuals: young adults at time t whose parents die early;
- Type-L_t individuals: young adults at time t whose parents die late.

By the law of large numbers, the proportion of individuals of type L_t in the cohort is equal to π, while the proportion of type E_t is equal to $1-\pi$.

The budget constraints faced by an individual of type i_t are:

$$c_t^{i_t} + \tilde{s}_t^{i_t} = w_t + b_t^{i_t} + B_t^{i_t} \tag{17}$$

$$d_{t+1}^{i_t} + b_{t+1}^{i_t} = R_{t+1}\tilde{s}_t^{i_t} \tag{18}$$

[32]We abstract here from the cost of childbearing (the same for all), and thus neglect a potential period 0 (childhood) in which people consume and make no decision whatsoever.

where $\tilde{s}_t^{i_t}$ is total savings (for old-age consumption and unconditional bequests), $b_t^{i_t}$ is the unconditional bequest received from the parents (which is the same for all, whatever the longevity of parents is), and $B_t^{i_t}$ is the accidental bequest, which is conditional on the longevity of the parent. We have $B_t^{E_t} > B_t^{L_t} = 0$. The associated intertemporal budget constraint is:

$$w_t + b_t^{i_t} + B_t^{i_t} = c_t^{i_t} + \frac{b_{t+1}^{i_t} + d_{t+1}^{i_t}}{R_{t+1}} \tag{19}$$

Individuals of type E_t, for whom $B_t^{i_t} > 0$, face better budget conditions than agents of type L_t, for whom $B_t^{i_t} = 0$.

Production Production takes place with labour ℓ_t and capital k_t, according to a production function with constant returns to scale:

$$Y_t = F\left(k_t, \ell_t\right) \tag{20}$$

Under our assumptions, i.e. each young individual supplies 1 unit of labour inelastically, and each cohort being a continuum of size 1, we have $\ell_t = 1$. Using the definition $f\left(k_t\right) \equiv F\left(k_t, 1\right)$, we can write the production process as:

$$Y_t = f\left(k_t\right) \tag{21}$$

If capital fully depreciates after one period, the capital accumulation law is:

$$k_{t+1} = \pi \tilde{s}_t^{L_t} + (1-\pi)\tilde{s}_t^{E_t} \tag{22}$$

Factors are paid at their marginal productivity:

$$w_t = f(k_t) - k_t f'(k_t) \tag{23}$$

$$R_t = f'(k_t) \tag{24}$$

6.2 The laissez-faire

The problem of a type i_t individual is:

$$\max_{c^{i_t}, d_{t+1}^{i_t}, b_{t+1}^{i_t}} \; c_t^{i_t} + \pi\left[u(d_{t+1}^{i_t}) + v(b_{t+1}^{i_t})\right] + (1-\pi)\left[v(d_{t+1}^{i_t} + b_{t+1}^{i_t})\right]$$

$$\text{s.t.} \quad w_t + b_t^{i_t} + B_t^{i_t} = c_t^{i_t} + \frac{d_{t+1}^{i_t} + b_{t+1}^{i_t}}{R_{t+1}}$$

The FOCs of this problem yield $u'(d_{t+1}^{i_t}) = v'(b_{t+1}^{i_t})$, and:

$$R_{t+1}\left[\pi u'(d_{t+1}^{i_t}) + (1-\pi)\left[v'\left(d_{t+1}^{i_t} + v'^{-1}\big(u'(d_{t+1}^{i_t})\big)\right)\right]\right] = 1. \tag{25}$$

This implies that $d_{t+1}^{i_t}$ is independent from the inherited wealth (i.e. the same for $i_t = E_t, L_t$). Let us denote the old-age consumption level satisfying the above expression as $\breve{d}_{t+1}$. Given that $u'(d_{t+1}^{i_t}) = v'(b_{t+1}^{i_t})$, we can see that, whatever the inheritance was, all individuals choose the same level of old-age consumption and unconditional bequest, which only depend on time through the interest rate. The only difference between dynasties concerns what is consumed in the young age, which varies depending on what was received from the parent, which is either $\breve{b}_t$ (for type L_t) or $\breve{b}_t + \breve{d}_t$ (for type E_t).

Let us assume that a stationary equilibrium exists, that it is unique and stable.[33] At the stationary equilibrium, we have:

$$c^E = w + \left(\breve{d} + \breve{b}\right) - \left(\frac{\breve{d}+\breve{b}}{R}\right) > c^L = w + \breve{b} - \left(\frac{\breve{d}+\breve{b}}{R}\right);$$

$$d^E = d^L = \breve{d}, \text{ where } \breve{d} \text{ is such that } \pi u'(\breve{d}) + (1-\pi)\left[v'\left(\breve{d} + v'^{-1}\big(u'(\breve{d})\big)\right)\right] = 1/R;$$

$$b^E = b^L = \breve{b}, \text{ where } \breve{b} = v'^{-1}\big(u'(\breve{d})\big).$$

Proposition 6 summarizes our results regarding the stationary equilibrium.

Proposition 6 *At the stationary laissez-faire equilibrium with quasi-linear utility, old-age consumption and unconditional bequest are independent from initial wealth, and individuals with short-lived parents have higher first-period consumption than individuals with long-lived parents. For a given longevity, individuals of type E are better off than individuals of type L. Within a given type $i = E, L$, the long-lived are better off than the short-lived.*

Proof. See above. ■

A major difference with the baseline model is that inequalities in the parent's duration of life lead to inequalities in endowments at the next generation, and, hence, to well-being inequalities for a given longevity. However, as in the baseline model, it remains true that, within a given type, short-lived individuals are worse-off than long-lived ones.

Survival conditions, summarized by the parameter π, have a key impact on the distribution of wealth. When life expectancy is low, many young individuals benefit from accidental bequests. Also, they are likely to inherit small amounts of accidental bequest, because people facing a large probability of dying save less. On the contrary, when life expectancy is high, fewer young individuals receive accidental bequests, implying that wealth is more concentrated. Moreover, since life expectancy is high, savings are likely to be important so that each of the few heirs receives a higher accidental bequest. Therefore, our framework provides a demographic explanation for the documented rise in wealth inequalities: inequalities would be reinforced by the improvement of survival conditions, which raises the average size of accidental bequests and reduces the proportion of heirs in the population.[34]

[33] Note that, since the heterogeneity is reduced to two groups with time-invariant proportions, this assumption is close to what is usually assumed when considering simple OLG models of capital accumulation with a representative agent.

[34] That explanation is in line with Miyazawa (2006).

6.3 The utilitarian optimum

We consider the problem of a social planner who selects the levels of consumptions, unconditional bequests, accidental bequests and capital so as to maximize the average lifetime welfare prevailing at the stationary equilibrium, while satisfying the steady-state resource constraint:

$$\begin{array}{ll} \max_{c^i,d^i,b^i,e^i,k} & (1-\pi)c^E + \pi(1-\pi)\left[u(d^E)+v(b^E)\right] + (1-\pi)^2 v(b^E+e^E) \\ & +\pi c^L + \pi^2\left[u(d^L)+v(b^L)\right] + \pi(1-\pi)v(b^L+e^L) \\ \text{s.t.} & f(k) = \pi c^L + (1-\pi)c^E + (1-\pi)b^E + \pi(1-\pi)d^E \\ & +(1-\pi)^2 e^E + \pi b^L + \pi^2 d^L + \pi(1-\pi)e^L + k \end{array}$$

The solution to this problem is not unique because of the linearity in c^E and c^L, which makes any allocation of young-age consumption between groups E and L indifferent for a given level of aggregate young-age consumption. Without loss of generality, we will focus here on the case where $c^E = c^L$ to simplify comparisons with the egalitarian optima below.

The FOCs yield $u'(d^E) = u'(d^L)$, so that $d^E = d^L$. Also, for any $i = E, L$ we obtain $\pi v'(b^i) + (1-\pi)v'(b^i + e^i) = v'(b^i + e^i) = 1$. Hence, we necessarily have $e^i = 0$ and $b^E = b^L = v'^{-1}(1)$. Similarly, $u'(d^E) = u'(d^L) = 1$, so that $d^E = d^L = u'^{-1}(1)$. By assumption, $u'^{-1}(1) > v'^{-1}(1)$, so that savings are larger than unconditional bequests. Finally, the optimal intertemporal allocation of resources implies that $f'(k) = 1$. Proposition 7 summarizes these results.

Proposition 7 *At any long-run utilitarian optimum with quasi-linear utility, there is no accidental bequest ($e^i = 0$) and individuals of types E and L benefit from the same unconditional bequests, which are smaller than old-age consumption. The capital stock satisfies the Golden Rule $f'(k) = 1$. There exists a long-run utilitarian optimum such that young-age consumption is the same for individuals of types E and L. Short-lived individuals are worse-off than long-lived ones if and only if $u(d) = u \circ u'^{-1}(1) > 0$.*

The decentralization of that optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule. It also requires either full annuitization of savings (i.e., $\alpha = 1$) or, when full annuitization of savings is impossible (i.e., $\alpha < 1$), full taxation of accidental bequests (i.e., $\theta = 1$) together with a subsidy on savings (i.e. $\sigma > 0$).

Proof. The first part of Proposition 7 follows from the above FOCs of the planner's problem. The decentralization is obtained by comparing the FOCs of the individual's problem in Section 3.2 rewritten at time t for individuals of types E and L with the FOCs of the utilitarian planner's problem. ■

As in the baseline static model, the long-run utilitarian optimum involves zero accidental bequests. Hence, the decentralization of the long-run utilitarian optimum requires a full taxation of accidental bequests (together with the subsidization of savings), or, alternatively, a full annuitization of savings. Thus, the conventional 100 % tax view still holds here, as in the baseline model.

The only difference with respect to the baseline model is that, in addition to those instruments, intergenerational lump-sum transfers are also needed here since individual savings decisions may lead to under- or over-accumulation of capital. Note, however, that since accidental bequests are null at the optimum, there is no heterogeneity in initial endowments received by young individuals, $B_i = e_i = 0$, and thus the decentralization of the long-run utilitarian optimum does not require intragenerational lump-sum transfers.[35]

6.4 The ex post egalitarian optimum

From an ex post egalitarian perspective, shifting from a static model (with no concern for inequalities in endowments among children) to an intergenerational model leads us to think about the compensation of individuals for two kinds of circumstances: on the one hand, inequalities in the duration of life (as in the benchmark model); on the other hand, inequalities in endowments (caused by inequalities in the longevity of parents). We are thus in the presence of two forms of circumstances that are arbitrary, but which affect individual lifetime well-being. Applying the Principle of Compensation in an intergenerational context, we are left with two kinds of circumstances to be compensated.

Let us consider the problem of a social planner maximizing the realized lifetime well-being of the worst-off living at the stationary equilibrium:

$$\max_{\substack{c^E,d^E,b^E,e^E \\ c^L,d^L,b^L,e^L,k}} \min\{U^{ESL}, U^{ELL}, U^{LSL}, U^{LLL}\}$$

$$\text{s.t. } f(k) = \pi c^L + (1-\pi)c^E + \pi(1-\pi)\left(d^E + b^E\right) + (1-\pi)^2(e^E + b^E)$$
$$+\pi^2(d^L + b^L) + \pi(1-\pi)(e^L + b^L) + k$$

where $U^{ESL} = c^E + v(b^E + e^E)$, $U^{ELL} = c^E + u(d^E) + v(b^E)$, $U^{LSL} = c^L + v(b^L + e^L)$ and $U^{LLL} = c^L + u(d^L) + v(b^L)$.

From the egalitarian point of view, there should be an equality of all variables across types E and L, that is, $c^E = c^L = c$, $b^E = b^L = b$, $d^E = d^L = d$ and $e^E = e^L = e$. Moreover, since all children receive the same endowment independently of how long their parent live, we must have also: $v(b) = v(b+e)$. Hence the social planner's problem can be rewritten by means of the Lagrangian:

$$\mathcal{L} = c+v(b+e)+\mu\big[f(k)-c-b-\pi d-(1-\pi)e-k\big]-\lambda\big[u(d)+v(b)-v(b+e)\big]+\xi\left[v(b+e)-v(b)\right].$$

The problem is solved in the Appendix. Proposition 8 summarizes our results.

Proposition 8 *At the long-run ex post egalitarian optimum with quasi-linear utility, there is no accidental bequest ($e = 0$), individuals of types E and L enjoy equal consumptions and unconditional bequests, while the capital stock satisfies*

[35] In case of inequalities in endowments across dynasties at time 0, the equality $c^E = c^L$ is not achieved at $t = 0$. But given the full taxation of accidental bequests, we know that, at the stationary equilibrium, the equality $c^E = c^L$ will prevail. Thus the decentralization of the long-run utilitarian optimum does not require intragenerational lump-sum transfers.

the Golden Rule $f'(k) = 1$. Old-age consumption is fixed to $\bar{c}$ and the short-lived and long-lived are equally well-off.

The decentralization of that optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule. It also requires no collective life insurance, a 100% tax on accidental bequests if full annuitization of savings is not possible, a positive or negative subsidy on savings, and a lump sum tax on old-age consumption.

Proof. See the Appendix. ■

Unlike the static model, the decentralization of the ex post egalitarian optimum in the OLG economy requires a 100 % taxation on accidental bequests. Therefore, shifting from a static model (where there is no concern for inequalities in children's endowments) to a dynamic setting matters for the fair taxation of bequests from an ex post egalitarian perspective. In an intergenerational context, ex post egalitarianism requires equalizing children's endowments, and, hence, does no longer justify subsidizing accidental bequests.

The underlying intuition goes as follows. Parents are interested in what their children actually receive net of all taxes and transfers, so that the equalization of lifetime utilities across children of types E and L requires that the joy of giving terms of all parents become equal, whether these have a long life or a short life (since parents care about children's endowments, which are equalized). Hence, one can no longer rely on accidental bequests to compensate the short-lived. This annihilates the redistributive motive for subsidizing accidental bequests.

Finally, given that the short-lived and the long-lived have the same first-period utility and the same joy of giving term, the equalization of lifetime welfare across them requires old-age consumption d fixed to the critical level for continuing existence $\bar{c}$ (i.e. such that $u(\bar{c}) = 0$). This explains why a lump-sum tax on old-age consumption is required.[36] Thus, in a dynamic setting, the compensation of the prematurely dead cannot be achieved through subsidizing accidental bequests (which would make type L even worse-off than type E), but requires modifying consumption profiles, in line with Fleurbaey et al (2014).

7 Conclusions

We reexamined the optimal taxation of accidental bequests by departing from the literature in two ways. First, we paid attention to the fact that individuals may care about what they would, in case of premature death, leave to their offsprings. Second, we departed from the utilitarian criterion and considered the ex post egalitarian criterion, which gives priority to the worst-off ex post.

Our results can be summarized as follows. In a static setting, when no attention is paid to the endowments of children, accidental bequests should be taxed at a confiscatory rate under utilitarianism, but should be subsidized

[36]The positive or negative subsidy on savings is necessary so as to obtain the optimal trade-offs between consumptions at each period and with the joy of giving term.

under ex post egalitarianism, on the grounds of compensating the short-lived.[37] When considering a second-best setting where the government can only tax total bequests based on the age of the deceased, we find that, whereas utilitarianism implies taxing total bequests at a rate that is decreasing with the age of the deceased, ex post egalitarianism recommends a tax rate increasing in the age of the deceased.[38] However, in an intergenerational setting, not only utilitarianism, but, also, ex post egalitarianism, support the 100 % tax on accidental bequests. Accidental bequests reinforce inequalities among the young, some of whom will be short-lived. As such, accidental bequests do no longer serve fairness, and should be taxed away. Then, compensating the short-lived can only be achieved by taxing savings so as to induce consumption profiles decreasing with age.

These mixed results lead us back to the early study of inheritance taxation by Mill (1848), who faced a dilemma between, on the one hand, respecting the will of the deceased and, on the other hand, avoiding arbitrary inequalities in endowments among descendants. In our paper, accidental bequests - resulting from the deceased's will - are a tool for compensating the prematurely dead, but that tool becomes unattractive when it leads to unequal endowments among persons who may eventually be short-lived. As in Mill (1848), the optimal inheritance tax involves a zero tax below some threshold (corresponding, in our model, to unconditional bequests), and a 100 % tax on the remaining (corresponding here to the accidental bequests). Thus compensating individuals for unequal lifetimes and for unequal endowments (due to the unequal lifetimes of their parents) leads us back to Mill: it is optimal to leave some common bequests for all, and to confiscate any extra bequest. Our explorations thus support Mill's intuitions about a fair inheritance tax.

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[37] Note that assuming unequal productivity would not change our results regarding the taxation of accidental and non-accidental bequests in the first-best.

[38] If differentiated lump-sum taxation is not feasible, introducing unequal productivities could affect the optimal taxation of accidental and non-accidental bequests. But it would still be the case that the ethical treatment of the prematurely dead matters (see the Appendix).

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9 Appendix

9.1 Introducing private life insurance

We can show that introducing private life insurance would not affect our results, because individuals would not, at the laissez-faire, purchase private life insurance in our economy.

Let us first derive the demand for private life insurance in the presence of a perfect annuity market. The problem of the individual is to choose (annuitized) savings s, unconditional bequest b and private life insurance $\tilde{a}$ to maximize:[39]

$$\max_{s,b,\tilde{a}} U = u\left(w - s - b - \tilde{a}\right) + \pi\left[u\left(\frac{s}{\pi}\right) + v(b)\right] + (1-\pi)v\left(b + \frac{\tilde{a}}{1-\pi}\right)$$

FOCs with respect to b and $\tilde{a}$ are:

$$\begin{aligned}
\frac{\partial U}{\partial b} &= -u'(c) + \pi v'(b) + (1-\pi)v'\left(b + \frac{\tilde{a}}{1-\pi}\right) \leq 0 \\
\frac{\partial U}{\partial \tilde{a}} &= -u'(c) + v'\left(b + \frac{\tilde{a}}{1-\pi}\right) \leq 0
\end{aligned}$$

It is impossible that the three conditions hold with equality simultaneously.[40] It must be the case that $\tilde{a} = 0$, that is, a zero demand for private life insurance. The rationale is that in the presence of perfect annuities, there is no accidental bequest, and it is optimal for the individual to invest in the unconditional bequest b, which is given in all states of the world, rather than in life insurance, which is given only in case of premature death. From the perspective of maximizing expected lifetime utility, the unconditional bequest dominates the life insurance, so that the demand for private life insurance is zero.

Consider now the case without perfect annuities, which is the case studied in this paper. The problem of the individual is now to choose (non-annuitized)

[39] We assume here a zero interest rate, as in Section 2.

[40] We can prove this by contradiction. Assume instead that the second and third first-order conditions hold with equality. This is not possible unless $\tilde{a} = 0$.

savings s, unconditional bequest b and private life insurance $\tilde{a}$ to maximize:

$$\max_{s,b,\tilde{a}} U = u(w - s - b - \tilde{a}) + \pi [u(s) + v(b)] + (1-\pi) v\left(b + s + \frac{\tilde{a}}{1-\pi}\right)$$

FOCs with respect to b and $\tilde{a}$ are written as follows:

$$\begin{aligned} \frac{\partial U}{\partial b} &= -u'(c) + \pi v'(b) + (1-\pi) v'\left(b + s + \frac{\tilde{a}}{1-\pi}\right) \leq 0 \\ \frac{\partial U}{\partial \tilde{a}} &= -u'(c) + v'\left(b + s + \frac{\tilde{a}}{1-\pi}\right) \leq 0 \end{aligned}$$

Assuming interior solutions, we obtain from the last FOC that $v'(b) = v'\left(b + s + \frac{\tilde{a}}{1-\pi}\right)$, which implies $\frac{\tilde{a}}{1-\pi} = -s$, that is, a negative demand for private life insurance.

Hence, abstracting from private life insurance does not involve a strong simplification here: a life insurance market would imply, depending on the presence of annuities, either a zero or a negative demand for private life insurance.

9.2 Proof of Proposition 8

In addition to the egalitarian constraint, $u(d) + v(b) = v(b + e)$, the social planning problem now also includes an additional constraint, $v(b) = v(b + e)$ which accounts for the fact that the child obtains the same amount of bequest independently of whether his parents die late or early. The combination of these two equations yields that $e = 0$ and $u(d) = 0$ so that $d = \bar{c}$. Hence, the social planning problem can be rewritten in the following simplified way:

$$\max_{c,b,k} c + u(\bar{c}) + v(b) \text{ s.t. } f(k) = c + b + \pi\bar{c} + k$$

Rearranged FOCs are $v'(b) = 1$ and $f'(k) = 1$. Let us now consider the decentralization of this optimum. Since $e = 0$, it is optimal to set collective life insurance to zero, $a = 0$. When annuitization is incomplete $0 \leq \alpha \leq 1$, in order to obtain $d = \bar{c}$, it is optimal to set a lump sum tax T on old-age consumption if the agent survives such that T satisfies the equality: $(\frac{\alpha}{\pi} + 1 - \alpha)s - T = \bar{c}$. Also, in order to ensure that $e = 0$, it is optimal to set $\theta = 1$ so as to fully eliminate accidental bequests. Note that if annuitization is complete, no accidental bequest remains, and no such taxation is needed.

Using the optimal trade-offs and replacing for $\theta = 1$, equations (6) and (7) of the decentralization can be rewritten as follows:

$$u'(c)(1-\sigma) = \pi\big(\frac{\alpha}{\pi} + 1 - \alpha\big)u'(\bar{c}) \quad \text{and} \quad u'(c) = v'(b) = 1.$$

Combining these equations yields the optimal tax on savings: $\sigma = 1 - \pi\big(\frac{\alpha}{\pi} + 1 - \alpha\big)u'(\bar{c})$, which can be positive or negative.

9.3 Heterogeneous productivity

Under unequal productivity $w_i > 0$, the second-best problem of the utilitarian planner is to find taxes T, θ_E and θ_L that maximize average social welfare subject to the revenue constraint:

$$\max_{T,\theta_L,\theta_E} \mathcal{L} = E\left[\begin{array}{l} u(w-s-b+T)+\pi\left[u(s)+v((1-\theta_L)\,b)\right]+(1-\pi)v((1-\theta_E)\,(b+s)) \\ -\mu\left[T-\pi\theta_L b-(1-\pi)\theta_E(b+s)\right] \end{array}\right]$$

where $E(\cdot)$ is the expectation aggregator.

Using the Envelope Theorem, we obtain the following FOCs:

$$\frac{\partial \mathcal{L}}{\partial T} = Eu'(c)-\mu\left[1-\theta_L E\pi\frac{\partial b}{\partial T}-\theta_E E(1-\pi)\frac{\partial(s+b)}{\partial T}\right]=0 \quad (26)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_L} = -E\pi v'(x_L)b+\mu\left[E\pi b-\theta_L E\pi\frac{\partial b}{\partial \theta_L}-\theta_E E(1-\pi)\frac{\partial(s+b)}{\partial \theta_L}\right]=0 \quad (27)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_E} = -E(1-\pi)v'(x_E)(b+s)+\mu\left[E(1-\pi)\,(b+s)-\theta_L E\pi\frac{\partial b}{\partial \theta_E}-\theta_E E(1-\pi)\frac{\partial(s+b)}{\partial \theta_E}\right]=0 \quad (28)$$

Let us now combine the last two FOCs to write the effect of a compensated change of the tax rate on bequests left by a person who died early, as in Section 5.2:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \theta_E} \equiv \frac{\partial \mathcal{L}}{\partial \theta_E}+\frac{\partial \mathcal{L}}{\partial \theta_L}\frac{d\theta_L}{d\theta_E} = -E(1-\pi)v'(x_E)(b+s)+E\pi v'(x_L)b\frac{E(1-\pi)(s+b)}{E\pi b} +\mu\left[\theta_L E\pi\frac{\partial \tilde{b}}{\partial \theta_E}+\theta_E E(1-\pi)\frac{\partial\left(\tilde{b}+\tilde{s}\right)}{\partial \theta_E}\right]=0$$

where $\frac{\partial \tilde{b}}{\partial \theta_E} \equiv \frac{\partial b}{\partial \theta_E}+\frac{\partial b}{\partial \theta_L}\frac{d\theta_L}{d\theta_E}$ and $\frac{\partial \tilde{s}}{\partial \theta_E} \equiv \frac{\partial s}{\partial \theta_E}+\frac{\partial s}{\partial \theta_L}\frac{d\theta_L}{d\theta_E}$ denote the effects on, respectively, b and s, of compensated changes in the tax rate θ_E.

Taken at the point where both estate taxes are zero yields:

$$\left.\frac{\partial \tilde{\mathcal{L}}}{\partial \theta_E}\right|_{\theta_E=\theta_L=0} = -cov[(1-\pi)(b+s),v'(x_E)]+cov\left[\pi b,v'(x_L)\right]\frac{E(1-\pi)(s+b)}{E\pi b} +E\left[v'(x_L)-v'(x_E)\right]E(1-\pi)(b+s)$$

Under a uniform productivity, the first two terms of the RHS would be canceled, and there would be a gain from an increase in θ_E compensated by a decrease in θ_L, leading to $\theta_E > \theta_L$. However, under heterogeneity in productivity, the sign of the RHS is less clear, since the sign of the first term is ambiguous, and the sign of the second term is negative. Hence it is not as clear as before that, starting from $\theta_E = \theta_L = 0$, a compensated rise in θ_E would be welfare-improving.

The problem of the ex post egalitarian planner can be written as the maximization of the well-being of the short-lived with the lowest productivity level

$w_0 > 0$, subject to the resource constraint and to egalitarian constraints:

$$
\begin{array}{rl}
\max\limits_{T,\theta_L,\theta_E} & u(w_0 - s_0 - b_0 + T) + v((1-\theta_E)(b_0 + s_0)) \\
\text{s.t.} & E\left[\theta_L \pi b + \theta_E (1-\pi)(b+s) - T\right] = 0 \\
\text{s.t.} & u(s_i) + v((1-\theta_L)\, b_i) = v((1-\theta_E)(b_i + s_i)) \forall i \\
\text{s.t.} & u(w_i - s_i - b_i + T) + u(s_i) + v((1-\theta_L)\, b_i)) \\
= & u(w_j - s_j - b_j + T) + u(s_j) + v((1-\theta_L)\, b_j)) \forall i \neq j
\end{array}
$$

If there is no tax on bequests, i.e. $\theta_E = \theta_L = 0$, we see that, for a given productivity w_i, the short-lived is worse-off than the long-lived, since we then have: $u(s_i) + v(b_i) > v(b_i + s_i)$. Hence a decrease in θ_E compensated by a rise in θ_L must improve the situation of the short-lived. We thus obtain that $\theta_E < \theta_L$. That result is robust to introducing heterogeneity in productivity, since it remains true, in that case, that the egalitarian constraint between short-lived and long-lived individuals having the same productivity requires to tax less the bequests left by those who die earlier, and to tax more the bequests left by those who die later on.