Mathematical hygiene

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The goal of this talk is to bring together research from two apriori unrelated academic domains: the study of purity of methods in the philosophy of mathematics and the study of verbal hygiene in sociolinguistics.

Verbal hygiene [Cameron, 1995]

Verbal hygiene is, broadly speaking, "the urge to meddle in matters of language".

- The set of normative ideas that language users have about which linguistic practices should be preferred, and the ways in which they go about encouraging or enforcing others to adopt their preference.

Introducing. . .

Mathematical hygiene
The set of normative discourses regulating mathematical practices.

To clarify the proposal:

A case study of mathematical hygiene
Descartes' algebraic geometry and Newton's subsequent criticism of it.
- We will show that both Descartes and Newton's attitudes can be seen as shaped by their social context.

Comparing verbal and mathematical hygiene
Both Descartes' and Vaugelas' normative discourses can be seen as emanating from a common socio-political project: the development of a new bourgeois intellectual class.
- This class is often associated with personae (identities/stereotypes) [Agha, 2003, Eckert, 2008, among others], such as the *honnête homme.*

Plan

1 Introduction

2 Verbal hygiene

3 Mathematical hygiene (purity of methods)

4 Case study: 17th century France
- Descartes vs Newton (mathematical hygiene)
- Descartes vs Vaugelas (mathematical vs verbal hygiene)

5 Conclusion

A linguistic universal [Cameron, 1995, Curzan, 2014]

As soon as two ways of saying (roughly) the same thing become available to language users, some of those users will start formulating opinions about whether one grammatical variant is better than the other, and encouraging others to use what they consider to be the superior one.

- Although the shape of those opinions varies according to languages, time periods and social contexts.

Prescriptivism

When faced with two grammatical options, prescriptivists identify one of the two as superior and promote the use of the "better" variant over its alternative.

- Linguistic preferences/arguments are not arbitrary: almost always linked to the prescriptivist's politics
  [Joseph, 1987, Cameron, 1995, Curzan, 2014, Ayres-Bennett, 2020, among others]

Explicit link between language and politics
Non-sexist language attempts to reduce stereotyping and discrimination against women in language through, for example, proscribing the use of masculine expressions with general reference in favour of gender neutral/inclusive expressions.

(1)   Every candidate should submit ~~his~~/their/his or her CV.

With most verbal hygiene, the link is more subtle. . .

Standard language prescriptivism

Complaints about non-standard language play role in "legitimiz[ing] the norms of formal registers of Standard English rather than the norms of everyday spoken English" [Milroy and Milroy, 1999, 30],

- This in turn legitimizes discrimination against English speakers who do not master or regularly employ Standard English.

[Milroy and Milroy, 1999] identify two kinds of arguments promoting the Standard language:

Legalistic arguments

"implicitly legalistic and which are concerned with correctness, attack 'mis-use' of specific parts of the phonology, grammar, vocabulary of English (and in the case of written English 'errors' of spelling, punctuation, etc.)" [Milroy and Milroy, 1999, 30-31].

Moralistic arguments

"do not devote themselves to stigmatizing specific errors in grammar, phonology, and so on. They accept the fact of standardisation in the written channel, and they are concerned with clarity, effectiveness, morality and honesty in the public use of the standard language" [Milroy and Milroy, 1999, 38].

Example of a legalistic argument

> the our Language is extremely *imperfect*; that its daily Improvements are by no means in proportion to its daily *Corruptions*; and the Pretenders to polish and refine it, have chiefly multiplied *Abuses* and *Absurdities*; and, that in many Instances, it offends against every Part of Grammar

Jonathan Swift. (1712). *A Proposal for Correcting, Improving and Ascertaining the English Tongue.* cited by [Milroy and Milroy, 1999].

Example of a moralistic argument

Each of these passages has faults of its own, but quite apart from avoidable ugliness, *two qualities are common to all of them. The first is* staleness of imagery; *the other is* lack of precision. *The writer either has a meaning and cannot express it, or he inadvertently says something else, or he is almost indifferent as to whether his words mean anything or not. This mixture of* vagueness *and sheer* incompetence *is the most marked characteristic of modern English prose, and especially of any kind of political writing. As soon as certain topics are raised, the concrete melts into the abstract and no one seems able to think of turns of speech that are not* hackneyed

George Orwell. (1946). *Politics and the English Language.* cited by [Milroy and Milroy, 1999].

Honesty

Words like phenomenon, element, individual (as noun), objective, categorical, effective, virtual, basis, primary, promote, constitute, exhibit, exploit, utilize, eliminate, liquidate, are used to dress up simple statements and give an air of scientific impartiality to biased judgments.

George Orwell. (1946). *Politics and the English Language*.

Purity

Foreign words and expressions such as cul de sac, ancien regime, deus ex machina, mutatis mutandis, status quo, gleichschaltung, weltanschauung, are used to give an air of culture and elegance. Except for the useful abbreviations i.e., e.g., and etc., there is no real need for any of the hundreds of foreign phrases now current in English.

George Orwell. (1946). *Politics and the English Language*.

Functional arguments

Appeal to effective communication.

- "One form of language should be preferred over another because it facilitates effective communication" is widespread.
- As [Cameron, 1995] observes, it is "a theme harped on by verbal hygienists across the cultural and political spectrum."

Examples of functional arguments

It is the cliché trotted out by the Queen's English Society whenever its members meet to extol the virtues of standard English: if we didn't abide by a single standard communication would break down. It is the major argument of the racist and xenophobic US English language movement, which campaigns to outlaw languages other than English in public domains: we need a common language or the nation will fragment. Conversely, it is the theme of Orwell's 'Politics and the English language', according to which totalitarianism succeeds by preventing us from communicating our thoughts clearly, so that in time we cease to have any clear thoughts at all.

Deborah Cameron. (1995). *Verbal Hygiene*, Routledge.

Prescriptivism vs Descriptivism

Descriptivism (anti-prescriptivism)

The view that all grammatical variants are equal, and, as such, there should be no preferences between them.

- According to descriptivists, the variant identified as the *standard* one is not inherently better than the *non-standard* one, and it is nonsensical, and even problematic, to encourage the use of one over the other.

The typical attitude to [prescriptivism] among linguists runs the gamut from despair at prescribers' ignorance to outrage at their bigotry, and it is aptly if apocalyptically summed up in the title of a 1950 book by Robert Hall Leave Your Language Alone. [Cameron, 1995, 3]

Anti-prescriptivism as politics

• Linguists adopt the descriptivist perspective because they are opposed to the elitist, conservative political project that prescriptivism about the standard language supports, see for example [Labov, 1970].

• According to [Cameron, 1995, 5], the prescriptive/descriptive "binarism sets the parameters of linguistics as a discipline. The very first thing that any student of linguistics learns is that 'linguistics is descriptive not prescriptive' – concerned, in the way of all science, with objective facts and not subjective value judgments".

In a mathematical context. . .

Verbal hygiene (about language)

Normative discourses about the language of mathematics.

- How do language ideologies in mathematical writing line up with language ideologies in other domains?

Mathematical hygiene (not (obviously) about language)

Normative discourses about the practice of mathematics.

- What do the ideologies about mathematical practice look like?
- How to ideologies about mathematical practice line up with ideologies about linguistic practice?

See appendix for example where verbal and mathematical hygienes seem to meld together. . .

Examples of mathematical hygiene: Purity of methods

Roughly, a solution to a problem, or a proof of a theorem, is pure if it draws only on what is "close" or "intrinsic" to that problem or theorem.

Other common language: avoids what is "extrinsic", "extraneous", "distant", "remote", "alien" or "foreign" to the problem or theorem.

Examples: coordinate proofs in elementary geometry, complex analysis in number theory

Proposal
We find both prescriptivism (legalistic, moralistic, functional arguments) for/against purity, as well as descriptivsm.

Legalistic and moralistic arguments in purity of methods

Newton argues for purity based on 'correctness' and 'simplicity/elegance'.

Newton, *Lucasian Lectures on Algebra*, 1680s
Equations are expressions belonging to arithmetical computation and in geometry properly have no place except in so far as certain truly geometrical quantities (lines, surfaces and solids, that is, and their ratios) are stated to be equal to others. Multiplications, divisions and computations of that sort have recently been introduced into geometry, but the step is ill-considered and contrary to the original intentions of this science: for anyone who examines the constructions of problems by the straight line and circle devised by the first geometers will readily perceive that geometry was contrived as a means of escaping the tediousness of calculation by the ready drawing of lines. Consequently these two sciences [arithmetical computation and geometry] ought not to be confused. The Ancients so assiduously distinguished them one from the other that they never introduced arithmetical terms into geometry; while recent people, by confusing both, have lost the simplicity in which all elegance of geometry consists.

Functional arguments in purity of methods

Epistemic value seems to be to mathematical hygiene what communication is to verbal hygiene.

- (Im)pure proofs help one understand the theorem better.

Jean Dieudonné, *Linear algebra and geometry*, 1969

It may well be that some will find this insistence on "purity" of the various lines of reasoning rather superfluous and pedantic; for my part, I feel that one must always try to understand what one is doing as well as one can and that it is good discipline for the mind to seek not only economy of means in working procedures but also to adapt hypotheses as closely to conclusions as is possible.

In contrast, Bourbaki (1948) view impurity/unity as contributing to the "profound intelligibility of mathematics".

Descriptivism in purity of methods

Nathanson, *Elementary Methods in Number Theory*, 2000

The theorems in this book are simple statements about integers, but the standard proofs require contour integration, modular functions, estimates of exponential sums, and other tools of complex analysis. This is not unfair. In mathematics, when we want to prove a theorem, we may use any method. The rule is "no holds barred." It is OK to use complex variables, algebraic geometry, cohomology theory, and the kitchen sink to obtain a proof. But once a theorem is proved, once we know that it is true, particularly if it is a simply stated and easily understood fact about the natural numbers, then we may want to find another proof, one that uses only "elementary arguments" from number theory. Elementary proofs are not better than other proofs, nor are they necessarily easy. Indeed, they are often technically difficult, but they do satisfy the aesthetic boundary condition that they use only arithmetic arguments.

According to Descartes, geometrical problem solving consisted of two steps.

- Analysis: reduce a geometrical problem to an algebraic equation

- Synthesis: construct the roots of that equation

Descartes, letter to Mersenne, December 1637

In the *Optics* and the *Meteorology* I merely tried to show that my method is better than the usual one; in my *Geometry*, however, I claim to have demonstrated this. Right at the beginning I solve a problem which according to the testimony of Pappus none of the ancients managed to solve.

Pappus problem in 4 lines. Given 4 lines L_i in the plane and 4 angles θ_i. For any point P in the plane, let d_i be the length of the segment from P to L_i making the angle θ_i with L_i. Find the locus of points P such that $d_1 d_2 : d_3 d_4$.
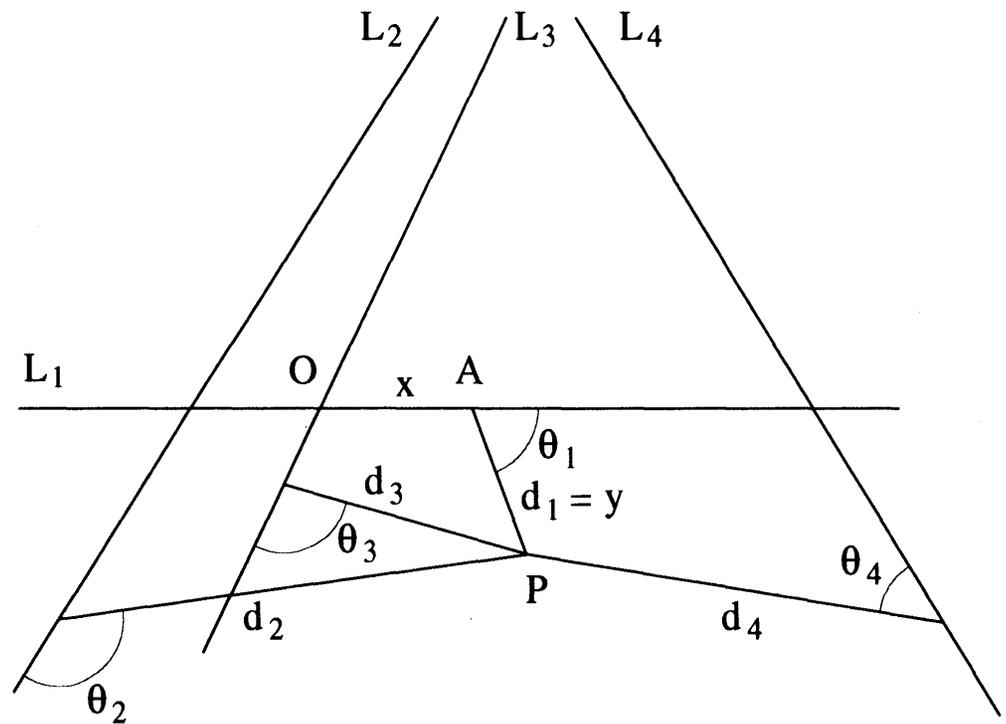
Descartes, *La géométrie*, 1637

Here I beg you to note in passing that the hesitation of the ancients to use arithmetic terms in geometry, which could only proceed from not seeing their relation clearly enough, caused a great deal of obscurity and difficulty in the way they explained themselves.

Descartes, letter to Mersenne, 9 February 1639

Those who know the conjunction that is between geometry and arithmetic, cannot doubt that all one can do by arithmetic can also be done by geometry; but to want to make this understood to whose who conceive of them as completely distinct sciences, this would be a waste of time [*oleum et operam perdere*].

Descartes, letter to Mersenne, 31 March 1638

But the good thing is, touching on this question of Pappus, that I have only put into the construction and the entire demonstration, without including all the analysis, that which they imagine themselves that I have put in: of which they testify that they understand very little. But what deceives them is that I build it, as architects do buildings, only prescribing everything that needs to be done, and leaving the work of the hands to the carpenters and masons. They also do not know my demonstration, because I speak in it by a, b. This does not however make it in any way different from those of the ancients, except that by this way I can often put in a line what for them fills several pages, and for this reason it is incomparably clearer, easier and less prone to error. than theirs. For the analysis, I have omitted part of it, in order to leave some work for clever minds; for if I had given it to them, they would have boasted of having known it before for a long time, whereas now they cannot say anything about it until they discover what they do not know.

Newton, *Veterum Loca Solida Restituta*, 1670s

Descartes in regard to his accomplishment of this problem makes a great show as if he had achieved something so earnestly sought after by the Ancients and for whose sake he considers that Apollonius wrote his books on conics. With all respect to so great a man I should have believed that this topic remained not at all a mystery to the Ancients.... To reveal that this topic was no mystery to them, I shall attempt to restore their discovery by following in the steps of Pappus' problem.

Newton, *Principia*, Book I, Lemma 19, Corollary 2

And thus there is exhibited in this corollary not an [analytical] computation but a geometrical synthesis, such as the ancients required, of the classical problem of four lines, which was begun by Euclid and carried on by Apollonius.

Newton, *Veterum Loca Solida Restituta*, 1670s
To be sure, their [the Ancients'] method is more elegant by far than the Cartesian one. For he [Descartes] achieved the result [the solution of the Pappus problem] by an algebraic calculus which, when transposed into words (following the practice of the Ancients in their writings), would prove to be so tedious and entangled as to provoke nausea, nor might it be understood. But they accomplished it by certain simple proportions, judging that nothing written in a different style was worthy to be read, and in consequence they were concealing the analysis by which they found their constructions.

Guisnée, *L'Histoire de l'Académie Royale des Sciences de 1705*

From this it follows that if an indeterminate problem of second degree can be resolved by two or more of the four curves, it is necessary to prefer the most simple one. This greater simplicity of the solution is a part of what one calls its elegance, the rest consists in drawing more immediately on what is given in the question, and bringing to it a minimum quantity of foreign and auxiliary principles.

Guicciardini, *Reading the Principia*, 1999

It should be noticed furthermore that, from the mid-1670s, Newton began looking at ancient texts not only for mathematical interests. . . It is striking that in the same years Newton began attributing to Jews, Egyptians and Pythagoreans a lost knowledge concerning alchemy, God and mathematics. It is plausible that in Newton's mind the restoration of the lost books of the ancient geometers of Alexandria was linked to his attempt to re-establish a *prisca sapientia*. . . . He viewed the history of mankind as a regress, a process of corruption, rather than a progress. In his alchemical, theological and chronological works, which he began composing in secrecy in the 1670s, the 'Moderns' are always depicted as inferior to the 'Ancients'. Newton's rejection of Cartesian algebra, his distancing himself from the analytical method of fluxions, and his interests in the geometrical works of Apollonius and Pappus are in resonance with other facets of his intellectual endeavour.

Newton stresses:

- that algebra is only analysis, discovery, and that only synthesis gives a proof, from first principles, for establishing the result with certainty (classical account of synthesis)

- that algebraic analysis cannot simply be reversed to give the synthesis, which is the most simple method for getting syntheses; but that his geometrical analysis can (classical account of analysis)

- the elegance of pure geometry, in its simplicity, and its opposite, the tediousness of algebra

- do what the ancients did, because it's the lost truth, and they didn't do algebra

Descartes stresses:

- the power to solve all problems in bringing together algebra and geometry, which the ancients did not have—overstated, but he does get Pappus for n lines

- the lack of method among the ancients, and the methodicalness of his algebraic techniques, which can promote learning among well-versed students (part of creating a new elite against the old warrior caste)

Both Descartes and Newton thought both synthesis and analysis were valuable.

Descartes acknowledges the importance of proving results synthetically, but thinks it's best left as an exercise for "clever minds".

You have to find the solution somehow, and there's reason to think that Newton used algebra to at least help with doing that sometimes.

But he did not publish his analyses (though he sends them in correspondence sometimes).

So it is at least in part an issue of what gets written, and what is left to the reader.

There's an agenda there: Descartes is more inclined to reform the reader in getting her to do the work of discovery, and Newton is not.

This agenda corresponds with Descartes' interest in creating among the new bourgeoisie and courtesans an intellectual elite, active in the exercise of their own reason rather than imitating the ancients, who were not just the warrior elite of the past.

By contrast, Newton believed on religious grounds in a elect people who alone could know things rightly—reflecting this, he did not publish but instead instructed small groups of disciples around him.

Descartes, Second Objections and Replies, *Meditations*, 1641

Analysis shows the true way by means of which the thing in question was discovered methodically and as it were a priori, so that if the reader is willing to follow it and give sufficient attention to all points, he will make the thing his own and understand it just as perfectly as if he had discovered it for himself. . .

[Synthesis] is not as satisfying as the method of analysis, nor does it engage the minds of those who are eager to learn, since it does not show how the thing in question was discovered.

It was synthesis alone that the ancient geometers usually employed in their writings. But in my view this was not because they were utterly ignorant of analysis, but because they had such a high regard for it that they kept it to themselves like a sacred mystery.

Descartes, *La géométrie*, 1637

I have given these very simple [means] to show that it is possible to construct all the problems of ordinary geometry by doing no more than the little covered in the four figures that I have explained. This is one thing that I believe the ancient mathematicians did not observe, for otherwise they would not have put so much labor into writing so many books in which the very sequence of the propositions shows that they did not have the true method of finding all, but rather gathered together those propositions that they had happened to encounter.

Descartes, *La géométrie*, 1637

But I shall not stop to explain this in more detail, because I should deprive you of the pleasure of mastering it yourself, as well as of the advantage of training your mind by working over it, which is in my opinion the principal benefit to be derived from this science. Because I find nothing here so difficult that it cannot be worked out by any one at all familiar with ordinary geometry and with algebra, who will consider carefully all that is set forth in this treatise.

La géométrie is called an "essai", written in the first-person, with an autobiographical aspect.

He invites readers to work with the methods themselves, as competent geometers, not as disciples.

He quotes at length Commandino's translation of Pappus to show how cumbersome their writing is, then reprises the Pappus problem in French quite briefly.

Descartes, *La géométrie*, 1637
I believe that I have in this way completely accomplished what Pappus tells us the ancients sought to do, and I will try to give the demonstration in a few words, for I am already wearied by so much writing.

This stress on brevity is a mark of the honnête homme.

The honnête homme emerges as an ideal member of the court of Louis XIV. He is intelligent but also courageous and generous, without artifice. He aims to be a master of conversation, to entice others with his elegance and wit. He is a generalist so that he can converse with anyone about anything, able to adapt to the needs of his audience, never boring them with pedantry.

Examples: Cléonte (Molière, *Le bourgeois gentilhomme*, 1670), Hippolyte (Racine, *Phèdre*, 1677)

The honnête homme must above all be master of himself, in control of his passions.

Example: Descartes' calm public response to Fermat's attack on his method of tangents (1638).

Descartes, *La recherche de la vérité par la lumière naturelle*, 1640s

An *honnête homme* is not required to know Greek or Latin any more than the languages of Switzerland or Brittany, or the history of the Empire any more than that of the smallest state in Europe. He ought simply to take care to employ his leisure in good and useful occupations, and equip his memory only with the most necessary knowledge.

Claude Favre de Vaugelas (1585-1650)

Influential grammarian, known particularly for his *Remarques sur la langue française, utiles à ceux qui veulent bien parler et bien écrire* (1647).

- *Remarques* was hugely inspirational for generations of grammarians, called les Remarqueurs [Caron and Ayres-Bennett, 2019].

Remarques laid out a method for learning the speech of the Court, i.e. the home of the *honnête homme.*

- What Descartes wanted to do by means of mathematics, Vaugelas et cie. wanted to do by means of language.

Ultimately then the linguistic is subsumed by the sociolinguistic, Vaugelas's theory of language depending on the desire to make communication as quick and easy as possible in order not to cause the listener any displeasure or make the interlocutor appear ridiculous, the worst possible fate to befall the honnête homme. Avoiding such ridicule would especially be the concern of the nouveau riche or the upper middle class man trying to rise in society and hoping to be accepted socially by the former nobility, and there are indications that such men did indeed read the Remarques as an aid to self-betterment.

Wendy Ayres. 1983. *Vaugelas and the Development of the French Language*. PhD Thesis, St Hilda's College, Oxford.

Nouvelles remarques de M. de Vaugelas sur la langue françoise (1690), posthumous work
I say again that an infinity of people write like this; however, it's a mistake against the purity of the language, which wants us to say *La philosophie sainte & la Philosophie profane défendent*, or instead, *La Philosophie saint & la profane défendent*. But I insist that the first is better: because we must always remember that our Language loves word repetitions very much, something that also contributes a lot to the clarity of language, which the French language shows to all the languages of the world. Also it generally does not delete anything: which is nevertheless a great elegance among the Greeks and Romans, but which often creates obscurity and equivocation.

In this passage, we find similar themes in Vaugelas' verbal hygiene as in Descartes' mathematical hygiene:

- Valuing clarity.
- Improving on the Ancients.

Conclusion

Studying normative discourses on mathematical practice, in the way that sociolinguists study normative discourses on language, can help us better understand mathematical practice itself.

- Brings to the forefront the social and cultural aspects of mathematical practice.
- Parallels with verbal hygiene can help us understand how language and math are related, at least from the point of view of human cognition and ideology.
  - Both mathematical and linguistic practice seem to be guided by *personae*, ideological constructions such as the *honnête homme*.

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Mathematical hygiene as verbal hygiene?

Weierstrass, letter to Schwarz, 1875

The more I ponder over the principles of the theory of functions—and I do this incessantly—the stronger grows my conviction that it must be built upon the foundation of algebraical truths, and that, therefore, to employ for the proof of simple and fundamental algebraical theorems the "transcendental", if I may say so, is not the correct way, however enticing *prima vista* the considerations may be by which Riemann has discovered many of the most important properties of the algebraical functions. It is a matter of course that every road must be open to the searcher as long as he seeks; it is only a question of the systematic demonstration.

Mathematical verbal hygiene

Gregory's memorandum of a visit to Newton in May 1694

To this edition the author will attach two treatises, one about the geometry of the ancients where the errors of the moderns about the mind of the ancients are detected, on the problem proposed by Pappus and repeated by Descartes, on loci plane and solid, on analysis and porisms, on the neatness of the ancients' method of investigating and writing, where it will be shown that our specious algebra is fit enough for making the discoveries but quite unfit to [give them] literary [form] and to bequeath them to posterity.