The statistical origin of the cobweb diagram

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Abstract

The "cobweb theorem" or "cobweb diagram" is one of the first mathematical formalization that deals with the stability of the market process (Ezekiel [1938]). Moore gave intuitively its first formulation (Stigler [1962]), but its mathematical formalization was independently done by Ricci [1930], Schultz [1930] and Tinbergen [1930]. First, the paper shows how the idea of the cobweb mechanism appears in Moore's work in order to solve a statistical problem, namely the statistical estimation of supply and demand curves. "Inadvertently" (Samuelson [1947]) Moore sets out the cobweb idea which mathematical formalization was given by Ricci, Schultz and Tinbergen. Second, this paper is an attempt to trace the links between Moore's work and its followers. It shows that the meaning of the cobweb diagram was different in the authors' thought. It also stress the role of the mathematics in the investigation process of economic phenomena.

Introduction

The history of the simultaneous formulations of the cobweb diagram by Schultz, Ricci and Tinbergen, in 1930, seems rather neglected when compared to the other representations of market processes such as the *tâtonnement* (Ingrao and Israel [1990], Weintraub [1991])[1]. Nevertheless, it deserves to be studied for at least two reasons.

First, it bears witness to the progressive emergence of new investigation processes in economics (Israel [1996], 108). Indeed, these three authors all made use of mathematics to lead their investigations of economic phenomena. If the use of mathematics was not new in economics of that time, its use as an investigation mean was very scarce. This new inquiry mode proceeds by using mathematical rationales first and then by establishing a correspondence between the results and the economic phenomena. Here, the use of mathematics does not rest anymore on a mechanical analogy, as it is usually the case at that time since Walras (Mirowski [1986], Israel [1996]). Indeed, this new process of investigation leads the three authors of the cobweb diagram to follow mathematical consequences of Moore's intuitions and to discover a new way of representing the price formation process.

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[1] For example, Holland [1977], 82 attributed erroneously the origin of the cobweb diagram to Kaldor [1934] and Ezekiel [1938].

Hence, through the history of the cobweb diagram, this paper intends to shed some light on the progressive emergence of a new role for mathematics as inquiry mean in economics.

Second, this new role of mathematics leads to new questions. Indeed, it modifies not only the way of inquiring, but also the questions that economics have to solve. The stability problem emerged directly from the invention of the cobweb diagram and led to Samuelson's works. Before this diagram, the price formation was mainly considered as a procedural problem, like the *tâtonnement* in Walras' thought[2]. As a procedure, it encompassed rules, institutions and behaviours in order to function (Walker [1996]). With Ricci, Tinbergen and Schultz's works, the price formation was considered as a process that converges - or not - to an equilibrium point. This change comes from the economic cycles that cobweb diagram attempted to describe. An economic cycle may indeed be considered as a process, in the sense that it is composed of time ordered phenomena. In this way, the cobweb diagram describes the process followed by economic variables during the time. The aim of this paper is thus to examine how a new topic appeared in economics that deals with the convergence of this process. This topic comes from the use of mathematics as an investigation tool, since the cobweb diagram formulation is the moment when this question arises.

In order to show these two points, our history of the cobweb diagram will begin with Henry Ludwell Moore's works and those of his disciple, Henry Schultz. The paper will show the way of maturation of Moore's intuition that lead to the cobweb diagram (**1. The history of an "oversight"**). Then, the paper will study the way that follow Schultz, Ricci and Tinbergen, to exploit this oversight by using mathematics as an investigation mean that lead them to imagine simultaneously the cobweb diagram (**2. The first formulations of the cobweb diagram**).

1. The history of an "oversight"

In 1947, Samuelson did mention the existence of an "oversight" in Moore's *Synthetic Economics (1929)*. Indeed, the former asserts that the latter established probably inadvertently, with a purely statistical aim, that the quantity supplied depends from the price of the preceding period (Samuelson [1947], 84). Thanks to this oversight, Ricci [1930], Schultz [1930] and Tinbergen [1930] did succeed in establishing - "only incidentally" (Ezekiel [1938], 256) - the first statements of the "cobweb theorem" (Kaldor [1934], 134). Thus, Moore's works constitute the starting point of this history. But, the rather obscure character of these works compelled Schultz, one of his rare disciple, to explain Moore's insights[3]. Hence, the following

[2] Pareto is the most representative author in this perspective. See Lendjel [1999].

[3] According to Mirowski, Schultz will move away from Moore's epistemological and theoretical positions after the death of his mentor (Mirowski [1990], 605).

section will successively present the aspects related to the cobweb analysis in Moore's and Schultz's works[4].

1.1. Moore and the introduction of time in the supply function

Two critiques were mainly addressed against equilibrium theories during Moore's time : their "static character" and their "lack of realism" (Gislain and Steiner [1995], chap. I). Ricci seems to take part to this critical stream, as Moore noticed it ([1929], p. 28)[5] :

> "[t]he whole apparatus gives somewhat the impression of a magic castle, satisfying to the imagination but of little assistance in solving the housing problem. In more prosaic language the theory remains abstract and without grip" (Ricci [1924], 18).

With his "synthetic economics", Moore wishes to prevent these critics in a twofold manner (Moore [1929], 30-31): on the one hand, he intends to lay an empirical foundation of supply and demand equations, following Cournot's project (Moore [1929], 50); on the other hand, he takes into account the historical evolution of economic phenomena and thus of equilibrium.

> "There are three special characteristics which I should like the name *Synthetic Economics* to imply : (1) the use of simultaneous equations to express the *consensus* of exchange, production, capitalization, and distribution ; (2) the extension of the use of this mathematical synthesis into economic dynamics where all of the variables in the constituent problems are treated as functions of time; and (3) the still further extension of the synthesis to the point of giving the equations concrete, statistical forms. With those implications *Synthetic Economics* is both deductive and inductive; dynamic, positive, and concrete" (Moore [1929], 6).

Thus, Moore's tactic consists in dealing with the two critiques together through his synthetic economics. Indeed, the concept of "moving equilibrium" constitutes his global answer :

> "[w]hen these concrete, dynamic functions are substituted in Walras's reasoning for his hypothetical, static functions, the new system of equations [...] determines a moving equilibrium, and Walras's analysis of oscillations about a static general equilibrium becomes available for a description of concrete oscillations about a moving, real, general equilibrium" (Moore [1929], 153).

This debate constituted the relevant context of Moore's seminal idea, which was constitutive of the cobweb analysis, *i.e.* the discontinuous adjustments of output to changes in prices, with a one period lag in supply.

This idea was first formulated in 1925[6]. In this article, Moore explains his attempt to give a "concrete" - *i.e.* statistical - foundation to supply and demand curves (Moore [1925],

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[4] More complete presentation of Moore's works can be find in Stigler [1962], Christ [1985], Epstein [1987], Morgan [1990], Mirowski [1990], Raybaut [1991], Le Gall [1996]. Schultz's works, as others major actors of the history of econometric, are presented in Christ [1985], Epstein [1987] or Hendry and Morgan [1995]. Curiously, Christ does not mention Schultz's contribution to the cobweb analysis.

[5] Moore seems himself to be receptive to this critical stream, as suggest many excerpts in Moore's *Economic cycles : their law and cause* (1914) quoted by Mirowski ([1990], 597-598). See also Epstein [1987], 14.

[6] In fact, this idea can be found as soon as in Moore's 1914 book, but not as a statistical device that permits to estimate simultaneously the supply and demand functions. Notice that earlier formulations of this idea can be find in the history of economic thought. Pareto, for instance, states in his *Cours* : "But, if [the entrepreneur] had bought machines in order to double his production, while the consumption did not rise in the same

358)[7]. The necessity to give a more accurate version of his previous works leads him to investigate the supply side - after its rather debated demand side[8] - of the estimation problem (Epstein [1987], 16)[9]. The task is not that simple since it schematically requires first to find the typical shape of the supply and demand curves for a given commodity - potato, for instance -, second to give them a "concrete" form based on available statistical series[10].

On the first point, Moore considers that supply and demand curves are characterised by the same kind of equation. Hence, he selects three equations (Moore [1925], 357; [1929], 39) :

$$\begin{cases} f\ (x) = a \\ f\ (x) = a + b\ \ x \\ f\ (x) = a + b\ \ x + g\ \ x^2 \end{cases}$$

They are all one variable functions, which suppose that markets can be separately studied in "particular equilibria" (Moore [1929], p. 93)[11]. For statistical convenience, Moore give them a log-linear form afterwards (Raybaut [1991], 919), since "they may easily be fitted to the observations by the method of least squares" (Moore [1929], 100).

On the second point, Moore use the "trend-ratios" method as the first step to get "concrete" functions (Moore [1929], 42). In this statistical method, each variable is measured as a percentage deviation from a polynomial trend (Epstein [1987], 17)[12]. With this method, "the trend-ratios of the prices and quantities of commodities are calculated, and these trend-ratios constitute the raw data to which the typical functions are fitted" (Moore [1929], 69). Moore finds in the *Yearbook of the Department of Agriculture* the data corresponding to the annual production of potatoes and their farm prices in the United States between 1900 and 1913, required to compute their curves of demand and supply.

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proportion, he will not resign to shrink immediately his production and the perturbation of the economic equilibrium will be deeper and lengthier" (Pareto [1896], § 142, p. 19). At last, Hebert [1973] has shown that this idea could be find in Cheysson's 1887 article.

7 Moore reprints this article in his 1929 book, mainly in chapters IV (The Law of Supply) and V (Moving equilibria).

8 Moore's work still gives rise to controversy, even nowadays. See Epstein [1987], Mirowski [1990], Le Gall [1996].

9 According to Epstein, the estimation of the supply curve could rejoin more political stakes, since "[d]erivation of industry supply curves had also been a major problem for price controllers during World War I which they only imperfectly solved [...]" (Epstein [1987], 18).

10 The statistical estimation of supply and demand curves was one of the main topics of these days. It engages at least Holbrook Working's [1925], Elmer Working's [1927], Schultz's [1925], and of course Moore's works.

11 Moore will not integrate many variables in the supply and demand functions before 1929, incorporating thus his "particular equilibria" in a general equilibrium framework.

12 For a critical comment of this method, see Ricci [1930], p. 650.

I Year	II December farm prices (cents per bushel)	III Production (millions per bushels)	IV Price trend-ratio	V Production trend-ratio	VI Price ratio of preceding year	VII Production ratio of current year
1900	43.1	211	0.794	0.988	. . . .	. . . .
1901	76.7	188	1.397	0.810	0.794	0.810
1902	47.1	285	0.850	1.144	1.397	1.144
1903	61.4	247	1.094	0.932	0.850	0.932
[. . .]	[. . .]	[. . .]	[. . .]	[. . .]	[. . .]	[. . .]
1913	68.7	332	1.054	0.945	0.788	0.945
Mean	. . . .	. . . .	1.000	0.998	0.996	0.999

Table 1

These data are twofold reported - in crude and trend-ratio forms - in the table 1 (Moore [1925], 368; [1929], 94). In this table emerge a rather surprising particularity : the time-lag introduced in the price ratio of the sixth column. In 1925, Moore gives no real explanation of it :

> "[t]he law of supply is derived from columns VI and VII. In column VI the price trend ratios that are given in column IV are advanced one year" (Moore [1925], 367).

However, this temporal shift does mean that supply is correlated with price of the preceding period and not with price of the current one anymore, that is to say that the lagged price is expected by the farmers in the current period[13]. Meanwhile, the demand is still related to the current price. This technical trick allows Moore to find two different (and opposite) correlations between observed prices and quantities (r = - 0,95 for demand, and r = + 0,80 for supply) (Moore [1929], 95-96) and thus to found the both curves of demand and supply (the former increasing, the latter decreasing).

Above all, it allows to bypass a major statistical difficulty when trying to estimate two different curves with the same data (Christ [1985], 44)[14]. As Working stated, the problem is

[13] In 1919, Moore puts forward an earlier formulation of this idea : "[w]e would suppose that one important factor leading the farmers to increase or decrease the number of acres planted is the movement of prices in the preceding years. If the price of cotton has been falling, fewer acres will be seeded in that crop; and, on the other hand, if prices have been rising, there is likely to be an increase in the acreage" (Moore [1919], p. 560, quoted by Stigler [1962], 14). But this formulation did not explicitly state that farmers decisions rest on price expected, contrary to what suggests Epstein's comment : "the lagged price was to be interpreted as the expected current price in the behavior of suppliers" (Epstein [1987], 18).

[14] According to Hendry and Morgan [1995], Warning (1906) and Lenoir (1913) were the first to raise this problem. But Elmer J. Working's article is considered as the one that really triggers the identification problem off (Malinvaud [1964], 664, Epstein [1987], 23-28). According to several commentators, neither Moore nor Schultz did succeed in correctly dealing with this problem in their works (Christ [1985], 43-44; Stigler [1962], 13). Only in the particular case where supply reacts with a time-lag to a price variation can this problem be solved (Tinbergen [1930], 670; Malinvaud [1964], 666; Epstein [1987], 19).

that each observed point is theoretically an observed equilibrium, where supply meets demand at a given moment of time. Therefore, one can not observe separately a demand curve from a supply one (Working [1927], 218). Moore's method is thus the exception :

> "[n]o method other than lags was available to allow estimation of two different curves, demand and supply, from the same data" (Epstein [1987], 19).

Indeed, in delaying for a year the price that affects supply, Moore artificially builds a second statistical series in order to solve this problem. Now, he is able to give a concrete form to both functions, respectively p_d = - 1,425x + 2,425 for the demand curve and p_s = 1,222 x - 0, 222 for the supply curve (Moore [1925], 370-371; [1929], 97). But, as it does not formally appear in the mathematical expression of the supply, the time-lag is only implicit[15]. If this equation is empirically based on a time-lag, it discloses nothing on his formal aspect.

Thanks to this method, Moore went to imagine the new concept of "moving equilibrium", that is :

> "[i]n consequence of the graphs of supply and demand passing through the point whose co-ordinates are (1.0, 1.0) the demand for the commodity and the supply of the commodity are in a moving equilibrium about the trends of prices and production. When, for example, the supply price ratio of a given year was unity, the production ratio of the following year was unity; and when the production ratio of that following year was unity the demand price ratio of the same year was unity" (Moore [1925], 370).

Since variables rest on dated values, it becomes possible to conceive a dynamic or "moving" equilibrium[16]. Now this "moving equilibrium" prefigures a rationale that will be used to comment the equilibrium point in the cobweb diagram[17].

A theoretical explanation of the lag-method is given in chapter 2 of *Synthetic Economics*, when Moore deals with the market process topic. Here, the time-dimension of economic phenomena is directly tackled. Indeed, Moore considers that the market process is a time-ordered phenomenon and shall be apprehended as a mechanism where "forces [that affect prices] [...] take time to be brought into play" (Moore [1929], 21). As in a marshallian framework, the "market price" - the market-day clearing price - does not correspond necessarily to the "normal price", *i. e.* the price with which firms have no incentive to change their production decisions, since their production costs corresponds to their price[18]. In a

[15] However, Stigler ([1962], 16) and Christ ([1985], 44) take the liberty to give a dynamical form to these equations - indexing the variable with time - and attributing them to Moore. Both authors suggests that Moore did formulate them, whereas he explicitly did not.

[16] For a comparison between Walras' *tâtonnement* and Moore's moving equilibrium, see Raybaut [1991], 912-914.

[17] One of the major difficulty in Moore's work comes from the gap between textual rationales and mathematical rationales. Indeed, Moore reverses the causal link between variables. In his textual comments, Moore asserts that the current supply depends from the price of the preceding period. In mathematics, we should use a function like $O_t = f(p_{t-1})$. Now, he wrote (in a marshallian fashion) $P_S = f$ (Q), as is the case of the concrete supply function of potatoes : p_S = 1,222 x - 0, 222 (Moore [1929], 97).

[18] On this distinction, see for instance De Vroey [2000].

definite time, say a particular year, the market prices - that give rise to "tentative equilibria" - are established according to the demand side. Indeed,

> "little or no attention is paid to what the costs of production may have been. The resulting prices are prices at *tentative* equilibria. These tentative equilibria involve profits or losses arising out of the differences between prices and costs of production, and these profits and losses become spurs to economic readjustments" (Moore [1929], 22).

Thus, market forces only intervene when arises a discrepancy between market price and normal price. When the former exceeds the later in a given market, the industry increases its production in order to match, in the next period, the demand expected at this market price level. The reverse is also true when the market price is less than the normal price. Hence, following Marshall, Moore distinguishes two kinds of equilibrium related to two kinds of price. The "market price" leads to a "tentative equilibrium", where the "normal price" gives rise to a "final equilibrium" and "offer[s] no inducement to change any factor in the equilibrium" (Moore [1929], 22). This description of the market process requires also explicitly an adjustment period that achieve more or less quickly, depending on whether the stock of machine is considered as given or not[19]. Furthermore, according to Moore, if the supply adjustment rests on a given technical equipment, the reached equilibrium can only be a provisory one (*ibid.*). Hence, the final equilibrium is attained only by altering the instruments of production, with the supposition that no new perturbation occurs (*ibid.*, 23).

This theoretical explanation prefigures in many points the rationale of the cobweb diagram. Indeed, the cobweb diagram requires the distinction between market price and normal price, and its equilibrium point coincides with the concept of "final equilibrium" described by Moore. It rests on Moore's seminal idea of a time-lag in the supply adjustment to a price variation. But Moore himself did neither explicitly imagine such a diagram, nor give a formal account of his time-lag idea[20].

The hazard does not play any part in the simultaneous discover of the cobweb diagram by Schultz, Ricci and Tinbergen. They are all concerned with Moore's work mainly through the economic cycles topic. Indeed, as Moore noticed it, an economic cycle can be generate if new exogenous perturbations arise before the market had cleared the preceding one [21]:

> "the occurrence, or prospect, of new perturbations disturbs the immediate adjustment of the results of earlier changes, with the consequence that the tentative equilibria likewise become ideal goals, whose lines of motion trace to the *oscillations* about the trends of final equilibria" (*ibid.*, 23, 167).

The economic system, never reaching the final equilibrium, is characterised by a "moving equilibrium". Consequently, the time play a considerable role in Moore's explanation of the

[19] According to Raybaut, these adjustments are instantaneous (Raybaut [1991], 913-914).
[20] In the same perspective, Stigler asserts that Moore's equation "system is explosive on usual cobweb analysis, but Moore does not investigate its stability" (Stigler [1962], 16). Christ ([1985], 44) draws the same conclusion.
[21] See Epstein [1987], 19.

market process : it gives a real economic meaning to the statistical trick that allows to estimate the functions of supply and demand.

1.2. Moore's "lag method" explained by Schultz

In his very first contributions, Henry Schultz, Moore's disciple, intends to explain the intuitions of his mentor. In this way, Schultz raises two problems that equilibrium theories have to deal with:

> "it is necessary to pass from a static equilibrium to a moving equilibrium. This problem has two aspects: the purely deductive or mathematical, and the statistical. The first has not as yet been successfully studied, although the signs point to the possibility of the development of a mathematical theory of dynamical economics in the next few decades. The second has recently been solved by Professor Henry Ludwell Moore" (Schultz [1928], viii).

Here, Schultz follows explicitly his mentor's dynamic conceptions ("the moving equilibrium") as much as the role he attributed to statistics. It is not surprising to find in Schultz's first book *Statistical Laws of Demand and Supply* (1928) in range with Moore's works. Schultz tackles thus the estimation problem of the supply curve. His treatment of this problem may shed some light on several obscure points in Moore's rationale, particularly on his statistical trick called "lag method". Indeed, "Professor Moore does not give a detailed explanation of his method" (Schultz [1928], 136). Now, this explanation concern will lead Schultz to the cobweb diagram. We shall thus present Schultz's rationale in order to highlight the "Ariadne' statistical thread" that conducts to the formulation of the cobweb diagram.

We remind that Moore succeeded in estimating the curves (or straight lines) of supply and demand with his "lag method". These curves were justified by important coefficients of correlation between price and quantity variables : r = - 0,95 for the demand, and r = + 0,80 for the supply (Moore [1925], 368). In order to explain this surprising result, Schultz ([1928], 136) builds an ideal situation :

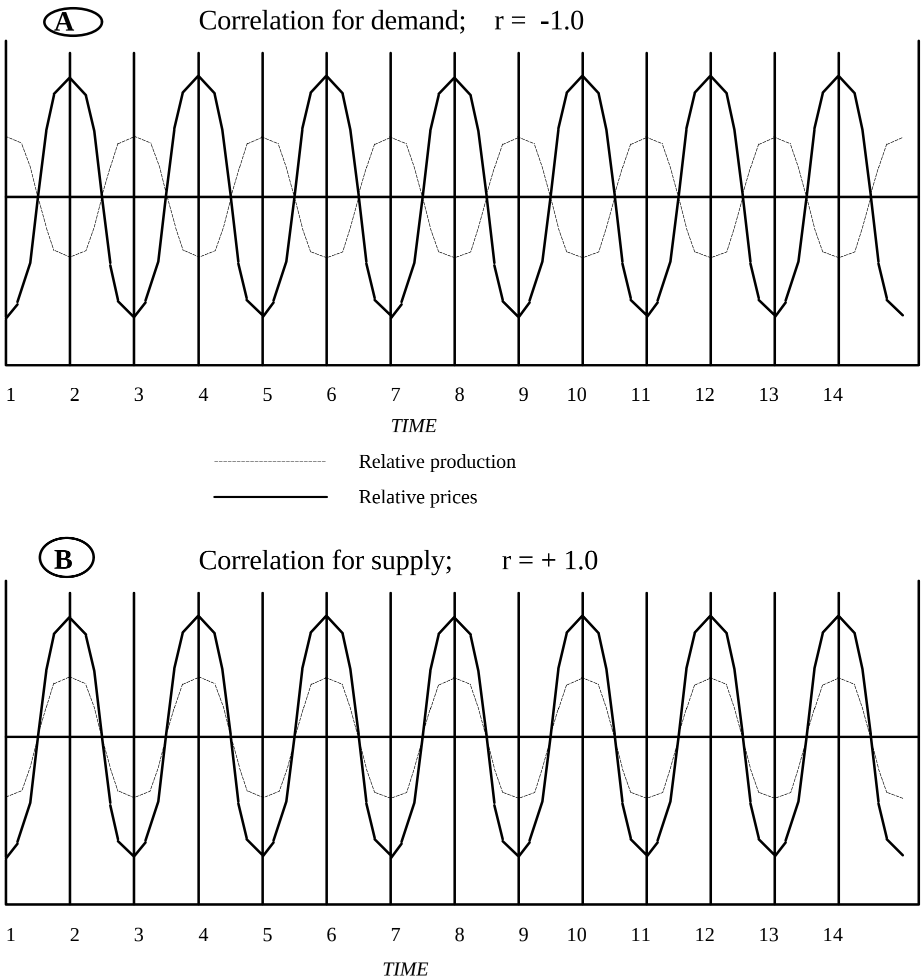
> "[l]et us assume that there is perfect negative correlation (r = - 1.0) between the relative prices and the relative consumption of a commodity for identical "points" in time. From such two perfectly correlated series we can get a perfect demand curve, that is, a negatively inclined curve which describes all the observations, without exception" (*ibid.*, p. 133).

Symmetrically, Schultz supposes a perfect correlation (r = + 1,0) between price of the current period and production of the following period.

> "Since, by assumption, the demand price is also the supply price, this means that the *same* price to which consumers are adjusting their consumption at any given time also "calls forth" a given production at some later time - say, one year hence" (*ibid.*).

Thus, a "perfect supply curve" is obtained.

But, this perfect situation is possible only when the two series are both "periodic functions of time" (*ibid.*). Indeed, Moore's lag method of estimation requires that the variables both describe a regular cycle of the same periodicity[22]. In this case, when the price cycle is delayed for a half-period in comparison to the production cycle, the sign of their correlation is inverted. One goes from a perfect negative correlation (for demand) to a perfect positive correlation (for supply) as illustrated in the following graph (Schultz [1928], 134) :



In graph (A), the two cyclical variables are characterised by the same periodicity, but with opposite phases. When the current level of production is low, the current price of the period is high, and reciprocally. This perfect negative correlation (r = - 1,0) describes a downward-sloping demand curve. In graph (B), the same variables have always the same periodicity, but with a half-period delay (a year). Cycles are perfectly in phase, but with a correlation coefficient of r = + 1,0. When the price of the preceding period falls, so does the production of the current period, but in a lesser extent, and reciprocally in case of price increase. Thus, Schultz succeeds in laying a methodological foundation to Moore's statistical trick.

[22] In this way, Schultz borrows Working's suggestion : "so long as the shifts of the supply and demand curves remain correlated in the same way, and so long as they shift through approximately the same range, the curve of regression of price upon quantity can be used as a means of estimating price from quantity" (Working [1927], 227).

With this work, Schultz intends to highlight Moore's implicit assumption about supply. He recalls that, according to Moore, firms take time to fit their production level to a demand shift. This particularity renders the estimation of the supply curve more difficult than the demand one (Schultz [1928], 130). Indeed, estimating a supply curve supposes to know which price affects the production level: is it the price of the current period, the one of the preceding period or even of the foregoing period ? (Schultz [1928], 131) In fact, there are two ways to get a positive correlation between price and quantity with the lag method : by "lagging" prices behind production or by "lagging" production behind prices (Schultz [1928], 132). One cannot decide on the sole basis of the statistical method. In Moore's example, Schultz considers that the hypothesis of a "lagged" prices - that is a correlation between the production trend ratio of the given year with the price trend ratio of the following year - , is rightly inappropriate :

> "had he "lagged" prices behind production by one year, he would have found that the correlation is only $r = + 0.38$" (Schultz [1928], 137).

But, in general, this assumption cannot be dropped on *a priori* grounds, since it may be appropriate to a non-perishable world-commodity whose production is more or less continuous[23]. Then it is necessary to have recourse to "outside evidences", "*that is,* [statistical results] *must admit of a reasonable explanation in terms of the technology of the industry or the commodity under consideration*" (Schultz [1928], 132). Thus, having explained Moore's implicit hypothesis, Schultz suggests to "lag" production behind prices in case of quickly perishable products, such as foods, and to "lag" prices behind production in case of non-perishable commodities.

It is now clear that Schultz and Moore are concerned with the introduction of time in economics, and not only with a statistical purpose. Since economic phenomena have a time-dimension, their wishes to give an empirical foundation to the curves of supply and demand move Schultz and Moore to integrate time in equilibrium theories. The concept of "moving equilibrium" is then a theoretical concept on one hand, that provides a dynamical analysis of a market process, and a technical concept on the other hand, which gives an empirical or statistical content to supply and demand functions. In order to illustrate this concept, Schultz rests on the case of the sugar market he will take up in 1930 :

> "[w]hen, for example, the supply price relative of a given year was 1.001, the production relative of the following year was 1.043; and when the production relative of that year was 1.043, the demand price relative of the same year was 1.001" (Schultz [1928], 167).

Moore did use a similar example in order to show how a market may stay dynamically in an equilibrium situation. It presents a temporal adjustment scheme of supply and demand that rest on a dynamical conception of actors' behaviours. It gives the general outlines of the cobweb

[23] Again, Schultz follows here Working's argument ([1927], 223).

figure : instantaneous adjustment of demand, a "lagged" adjustment of supply, and the identity of supply price and demand price at any moment[24]. Thus, there is nothing surprising to see Schultz taking up his rationale two years later in order to formulate the cobweb diagram. It suffices to restart it from a disequilibrium situation instead of an equilibrium one. But, his concern will still be mainly statistical.

2. The first formulations of the cobweb diagram

Contrary to frequent comments (Stigler [1962], Christ [1985], Epstein [1987], Mirowski [1990]), the history of the cobweb formulation does not stop with its rough outline in Moore's work. Indeed, Moore's writings are sufficiently obscure to merit some clarifications, as Schultz's works bear witness to it. In particular, the concept of "moving equilibrium" intrigues Schultz's contemporaries. Ricci notices so :

> "[w]hen, the supply price relative of a given year is 1 (*i.e.* when the production relative of that year is 1 and consequently the demand price relative of the same year is 1), the production relative of the following year will be 1, identical to the demand price 1. This is all what Moore indicated in his works on equilibrium; these indications are too succinct" (Ricci [1930], 654).

Thus, the first formulations of the cobweb diagram intend to fill this gap (Schultz [1930], Ricci [1930], Tinbergen [1930]). The fact that these first formulations were simultaneously done confirms that the cobweb diagram was latent in Moore's works. It also reveals what links together these works with the cobweb diagram. The link is above all mathematical, since only one geometric representation emerges from thess variously oriented works. According to us, the cobweb diagram was elaborated by using mathematical rationales first and then by establishing a correspondence between the results and the economic phenomena. In this, Schultz, Ricci and Tinbergen did use of mathematics as an investigation mean of economic phenomena.

The different aims followed by the three authors corroborate this hypothesis. Schultz, as we have seen, tries to restore the coherence of Moore's works. He shows then that the cobweb diagram may give a theoretical justification to the statistical estimation of the supply and demand curves. Ricci seems to provide a critic of Moore's works, which he considers to be one of the multiple avatar of equilibrium theories. According to Ricci, the "explosive" character of the cobweb diagram bears out the "unrealism" of these theories. In this, Ricci is one of the first that raise the stability problem in the equilibrium theories. Tinbergen is moved by more political purposes and aims to afford statistical tools to the socialist planification project. He then follows a similar path as Schultz's. Let us see successively the investigation processes of these authors.

[24] Schultz, just like Moore, uses to all appearances a "marshallian" framework, where quantities figure on the abscissa and prices on ordinate. Later, Schultz will make this framework explicit (Schultz [1938], 73). Tinbergen, as we will see, is the only one to formulate the cobweb diagram in a "walrasian" fashion, where prices are on abscissa and not quantities anymore.

2.1. Schultz and the estimation problem of supply and demand curves

In 1930, Schultz continues his explanation' work of Moore's "implicit" theses (Schultz [1938], 81)[25]. In this article, Schultz intends to show how theoretically two curves may be obtained from the same observed data.

> "Since the statistical data - the time series of prices and quantities - are always taken as the co-ordinates of points of intersection of a demand curve with a supply curve, the foregoing question may be restated as follows: Is it possible to deduce statistically the theoretical demand (or supply) curve when we know only the co-ordinates of the points of intersection of the theoretical (unknown) demand curve with the theoretical (unknown) supply curve at different points of time ?" (Schultz [1930], 28, [1938], 72).

Schultz imagines a solution to this problem by relying on Moore's lag method. The estimation of the both curves turns out to be possible in the following hypotheses' frame : H1/ supply and demand for a given commodity are independent of supply and demand for all the other commodities. H2/ Supply and demand conditions for a given commodity are supposed to be constant through time, *i. e.* supply and demand curves remain constant during the adjustment process. H3/ The conditions of production are such that a high or low price for any year (or other time interval) calls forth a high or low output in the following year, or period. The producer reacts mechanically to a given price modification. His output is optimal if the price remains the same in the following period. H4/ Since the output is given for a year (the instantaneous supply curve is vertical), the prevailing price depends from the demand conditions and coincides thus to the demand price. H5/ The quantity produced of this commodity is entirely consumed in the current year - or period -. H6/ At last, the given market has to be in a disequilibrium state in order to generate a temporal adjustment process susceptible to be observed. Indeed, this situation is the sole mean to obtain observations on supply and demand curves through the adjustment process. Otherwise, the market reproduces the same configuration - the equilibrium point - time after time (Schultz [1930], 33 [1938], 77). If these conditions are met, "it is possible to deduce both of these curves from the same data of prices and quantities" (*ibid.*).

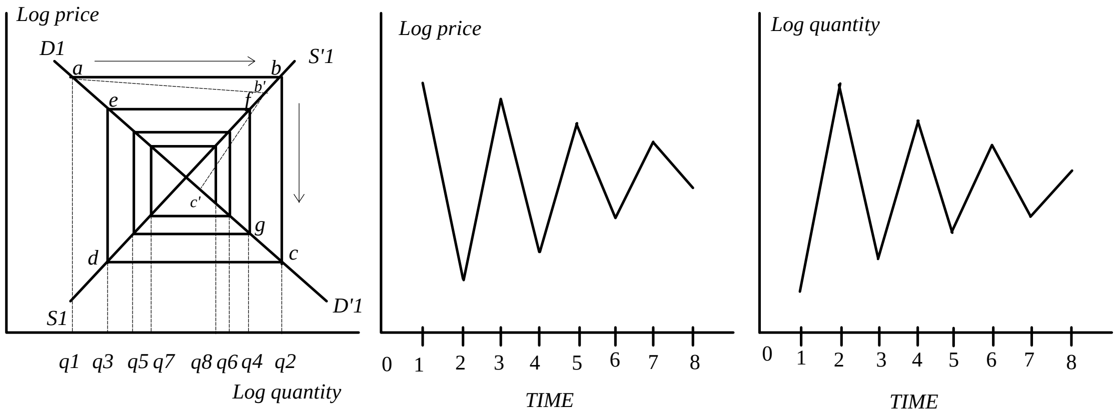
In the first stage of Schultz's demonstration, he puts forward a theoretical scheme - the cobweb diagram - that met these conditions. Schultz retains a formal marshallian framework, in which the curves (in fact, straight lines) remain constant through time, in accordance with hypotheses H1 and H2. He shows that, while meeting the hypotheses H3, H4 and H5, the market process starts from the following disequilibrium situation (H6) :

> "[l]et the quantity produced and available in a given year be oq_1. As this is less than the equilibrium quantity, the demand price will be aq_1. But this high price will, by our assumption, call forth the output oq_2 in the following year. When such an output is placed on the market, the demand price will drop to cq_2. This will reduce the output in

[25] The main passages of Schultz's article [1930] have been translated in English in his 1938 book *The theory and measurement of demand*. In this case, we shall quote the article's page and then the book's one. Ex.: Schultz [1930], 34, [1938], 79. Otherwise, we proceed as usual.

> the third year to oq_3. The price will then rise to eq_3. This process, which is brought into operation as soon as the equilibrium is upset, will continue until the equilibrium has been re-established" (Schultz [1930], 34; [1938], 77).

We have got here one of his rare economic comment of the following mathematical representation of the market process (Schultz [1930], 34, [1938], 79) :



The first diagram shows the path followed by the market to reach the equilibrium situation[26]. The two others exhibit statistical series of price and quantity whose co-ordinates coincide with each step of the process described in the cobweb diagram. These series describe decreasing cyclical oscillations, corresponding to a convergent type cobweb[27]. Here, we have to stress the link between the preceding 1928's graph (see graphs (A) and (B)) and the series exhibited in 1930. The cobweb diagram appears thus to be the last step in Schultz's rationale, making clear his "statistical Ariadne thread" as we will see now.

In the second stage of his demonstration, Schultz points out that the steps of this cobweb like process permits to characterise the curves of supply and demand.

> "The interesting result of this process is that it gives rise to a series of observations on *both* (unknown) curves. Thus the points *a* and *c* are observations on the demand curve, while the points *b* and *d* are observations on the supply curve" (Schultz [1930] 34, [1938], 78).

Indeed, the point *a* has the co-ordinates (q_1; p_1), *b* the co-ordinates (q_2 ; p_1), *c* (q_2 ; p_2), *d* (q_3 ; p_2), *e* (q_3 ; p_3), etc. Points *a, c, e, ...* are characterised by prices and quantities of the same period, unlike points *b, d, f, ...* characterised by a time-lag between prices and quantities. The synchronism of the former corresponds, by hypothesis, to the demand behaviour, whereas the "lagged" character of the latter refers to the supply behaviour. Thus, "when an interval elapses between changes in price and corresponding changes in supply, it is possible to deduce both curves statistically" (Schultz [1930], 35, [1938], 78). He rejoins there the results of his 1928'

[26] Schultz takes also into account the possibility that producers do not react as mechanically as he supposed it in H3. Indeed, they may foresee the likely evolution of price and decide to produce consequently. Instead of following *ab, bc* ... , "[t]he process is more nearly like that indicated by the dotted lines *ab', b'c'*, etc." (Schultz [1930], 35, [1938], 78). In this case, the process converges more quickly to the equilibrium point.

[27] In 1930, Schultz had not yet envisaged the possibility of a divergent cobweb. Schultz will deal with the two other cases ("cyclical" and "explosive") in 1938, but they will still remain secondary (Schultz [1938], 78).

book in which the (same) "lag method" allowed to estimate both curves. Indeed, one traces again the same inversion of the price and quantity cycles (as shown in the preceding graphs) that allowed to inverse the sign of the correlation coefficient of both series, and the same application of this method to the sugar industry. The sole difference is related to the formulation of an economic explanation of this statistical method. It thus confirms the prominent character of the statistical purpose in Schultz's analysis.

If theoretical justifications play a significant part, according to Schultz, the most important lies elsewhere :

> "[t]he importance of such a demand-supply relationship lies in that it admits a straightforward statistical «verification»" (Schultz [1930], 35; [1938], 78).

Schultz stresses this point again, eight years later :

> "whether or not there exists a tendency for an equilibrium to be established between demand and supply, and whether the steps by which equilibrium is established are few or many, are matters which are not nearly so important for present purposes as is the fact that the process of adjustment yields observations on both the demand curve and the supply curve" (Schultz [1938], 78).

Thus, the cobweb diagram allows above all, in Schultz' mind, to estimate supply and demand curves, as it constitutes one of the first formalization of a market process. On looking this primacy, the brevity of his economic comment may amaze. For example, when Schultz assumes that demand matches supply at each step of the cobweb process, he also assumes an instantaneous market clearing which is never evoked. Furthermore, this diagram is not really articulated with its contemporary theoretical framework. Kaldor's article [1934] will later raise this articulation problem. Hence, Schultz's work mainly represents, at least until 1930, a theoretical effort to justify the statistical "lag method".

2.2. Ricci and the critic of the formalization of a market process

Unlike Schultz, Ricci's tone with respect to Moore's works is clearly ironical and critical in his 1930 article[28]. Ricci considers Moore's "synthetic economics" as a synonymous with "mathematical economics", "queen economics", or "rational economics". His will to give a statistical foundation to his results is seen as a "phantasm" in Ricci's opinion (Ricci [1930], 649). In this way, the "moving equilibrium" concept constitutes Ricci's main critical target.

> "That an equilibrium (or disequilibrium) of a given year is linked with the following year's one is a progress for which we have to thank Moore. But - and this is the point we want to stress - Moore's [moving] equilibrium does not stop to be hypothetical. [...] In reality however, these normal values or trend will not never or almost never be reached. Instead, it arises more or less important irregularities" (Ricci [1930], 657-658).

[28] According to Gandolfo, this tone is rather usual in Ricci's writings (Gandolfo [1987], 199). Incidentally, we shall notice that Ricci's article has been translated by Oskar Morgenstern.

This is the way Ricci did use the cobweb diagram to demonstrate the unrealistic character of Moore's conclusions. Indeed, Ricci shows that, following Moore's rationale and equations, the potato industry should be amenable to a "tragic fate" :

> "The American economy, at least for the potato, is then at the mercy of the tragic fate of an increasing disequilibrium. [...] When looking the values in table 1 [Moore's [1929] data], one has certainly not the impression that potato's production and price in the United States would be irrevocably condemn to an increasing disequilibrium [...]" (Ricci [1930], 655, 657).

In fact, Ricci's article is precisely interesting due to his way to present Moore's argumentation. Indeed, by strictly following the rationale of the latter, Ricci allows us to understand how three authors have been led to simultaneously put forward their own version of the cobweb diagram, without any dialogue. In order to apprehend the notion of moving equilibrium, these authors have all examined Moore's estimated equations of supply and demand. Then, they applied his rationale, but with new additional hypotheses, as we have seen with Schultz. Moore's dynamical rationale only explains the equilibrium situation, and not what happens outside it (Ricci [1930], 654). Thus intending to go "farther", Ricci assumes a 20 % lowering of production in order to move from 1 to 0,8. Applying the same rationale and the same equations than Moore's, Ricci finds some values that move more and more away from equilibrium[29]. This rationale deserves to be entirely quoted since it is the sole "economic" comment of the cobweb diagram one can find at this time :

> "[l]et suppose that in a given year the supply (the production) falls from 20% to 0,8. At what price will it be sold ? The equation (1) [of demand] gives a price of 1,285. What will the production for the following year at that price ? The equation (4) [of supply] gives 1,233. Since in the following year the production amounts effectively to 1,233, at what price will it be sold ? The equation (1) gives : 0,668" (Ricci [1930], 654).

However, this comment turns out to be very succinct : he affords no economic explanation, neither on the price formation at each step of the cobweb, nor on the reasons that motive supply to match the production to the level indicated by the price of the preceding period. In fact, the intellectual path leading to the cobweb diagram proves to be essentially mathematical, as Schultz' works already suggested it.

Indeed, Ricci takes Moore's equation, alters its starting values and then examines its consequences. Since the variables have an economic meaning, the mathematical rationale leads to explain or describe economic phenomena. In this, we are not confronted to a purely mathematical investigation exerted by an economist. Mathematics is used there as a mean to obtain theoretical economic results. Until then, the use of mathematics in economics was mainly done through physical analogies. This point has been already noticed by several scholars (Mirowski [1986], 236, Ménard [1990], 101, Israel [1996]). For instance, Ménard considers that classical mechanics and its mathematics "form really the whole structure of the

[29] We recall that Moore's equation were : p_d = - 1,425 x + 2,425, and p_s =1,222 x - 0,222.

conceptualisation of [Walras'] *Eléments d'Economie Politique Pure*" (Ménard [1990], 101). On the opposite, Ricci's cobweb diagram displays a mathematical invention of an economic concept. No physical analogy intervenes in this rationale. However, since this rationale rest on the concept of equilibrium, the question of the stability - raised by this diagram - is borrowed to classical mechanics.

In the theoretical context of this period, the stability of equilibrium did not raise any problem, according to Ricci :

> "[t]he theory we were used until then was valid in a stable equilibrium case. When supply is scarce in a given year, the price increases, provokes extra-profits that stimulate the production, supply increases and price decreases" (Ricci [1930], 655).

According to this theory, the oscillations generated by the market process tend to "an equalisation and to the accomplishment of an equilibrium" (Ricci [1930], 655). With the cobweb hypothesis on the other hand,

> "the equilibrium, once disturbed, is straight lost for ever.[...] This comes from the slope of the straight lines of supply and demand, and the phenomenon arises whatever the starting point hypothesis" (Ricci [1930], 655-656).

This result is paradoxical : while observed cycles are not explosive, estimated curves generate an explosive cobweb (Ricci [1930], 657)[30]. Consequently, Ricci will explore the different configurations of supply and demand in order to find the one that lead to a stable equilibrium, since reality is fundamentally stable in his opinion.

Three possible configurations result from these investigations (Ricci [1930], 656) :

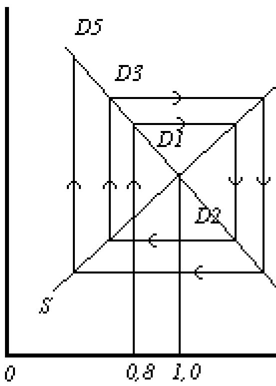


Fig. 1

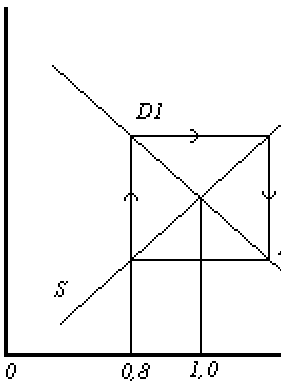


Fig. 2

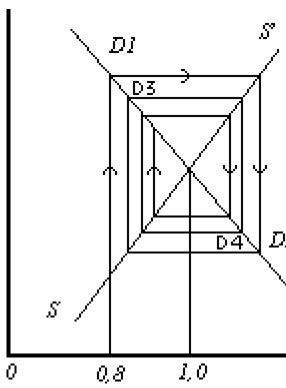


Fig. 3

In a marshallian formal framework, the adjustment process is clockwise[31]. Ricci's economic rationale supposes that the adopted price results at each instant from the quantitative balance of power between supply and demand. The resulting adjustment mechanism obeys thus to the "short-hand" rule. Let start from a produced output less than the equilibrium one; the supply

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[30] The gap between theory and observation will remain for a long time an enigma. Nerlove [1958], for instance, will attempt to solve this problem in putting forward new hypotheses related to agents expectations, but without real success (Pashigian [1987], 463-4).

[31] In the next subsection, Tinbergen uses a walrassian framework, where prices figure on abscissa and quantities on ordinate. The adjustment process is then anticlockwise.

price - that matches the corresponding production costs - is then less than the demand price - the price the demand is ready to pay to obtain this output. The resulting market price for this period expresses this balance of power. Transactions are made according to the demand price. But, at this price level, the entrepreneurs may increase their gains in the following period while raising their output. If "the new price let subsist a profit [...] this means that supply has not raised enough, and [that] the process shall go on" (Ricci [1930], 655). The next period, the entrepreneurs come to the market with an increased given stock of produced commodities and expect to sell it at the price that prevailed in the preceding period. This time, the demand is on the short-side of the market. Thus, "the new price falls under the supply price and brings about a loss, which means that supply increased too much" (Ricci [1930], 655). The process may carry on time after time, converging - or diverging - toward the equilibrium situation according to the slope of supply and demand curves.

In Ricci's opinion, the only empirically conceivable cases are the second and the third one. He thus considers that Moore did not succeed in preventing his theory from the unrealistic case of an unstable situation.

> "This critique shows us how the choice of the slope of supply and demand curves is delicate. [...] Whereas [Moore] made so many hypotheses, displayed so much precautions, and considered [the moving equilibrium] as particularly important, he did not satisfy a more vital requirement, that his equilibrium should be stable" (Ricci [1930], 657).

In fact, this comment sheds light on a new difficulty that arises when using of mathematics as a new investigation mode in economics. Indeed, it assumes that mathematical rationales lead to identify new economic phenomena or, put differently, that mathematical conclusions have an economic meaning. However, as Ricci pointed it out in Moore's works, this identification stage may sometimes be missing[32]. Put in Ricci's words, the instability of equilibrium is only an "unrealistic" mathematical hypothesis without correspondence with any economic phenomena.

Ricci's article constitutes thus a significant step in the history of market processes, since it is one of the first mathematical demonstration of the possibility of their diverging character. But it also shows the emergence of a new use of mathematics as an investigation mean in economics, as Tinbergen's own contribution will confirm it.

2.3. Tinbergen : mathematical investigations and statistical estimation

Like Schultz, Tinbergen mainly intends to solve a statistical problem, *i.e.* the estimation of the supply curve in Holland with help of Moore's method[33]. But his goals were different from Moore's :

> "[i]n an interesting contrast to Moore, Tinbergen saw econometrics as the tool that would make possible effective intervention in the economy to carry out a Socialist

[32] Several scholars have already stressed the inaccuracy of the economic rationale related to the cobweb diagram. See Coase and Fowler [1935], Buchanan [1939] or Waugh [1964].

[33] For a complete review of Tinbergen's [1930] contribution to econometrics, see Epstein [1987], 34-36.

program. His interest was in determining the competitive position of Holland through an analysis of demand and supply elasticities, export propensities, and the cross elasticities of foreign goods. This early work led to a critique of Moore's model of supply because the simple cobweb generated unrealistic cyclical behavior. But it is also notable for the first formal, though extremely limited, discussion of the statistical identification of individual demand and supply equation" (Epstein [1987], 34-35).

Thus, the identification problem is not the only point at stake (Boumans [1993], p. 148). According to Tinbergen, econometrics allows to a Planification Board to dynamically supervise the evolution of the economy (Tinbergen [1930], 678)[34].

The frame of "Determination and interpretation of supply curves : an example" [1930] displays this aim[35]. Tinbergen raises first the estimation problem of supply and demand curves. He then integrates specific theoretical aspects of supply (number of producers in a given industry, production delays, reaction delay of supply to a price shift) into his equations. Finally, he carries out his conclusions to the case of potato flour in order to illustrate how a Planification Board may follow up statistically an industry[36]. If the cobweb diagram appears in fact "only incidentally" in this article (Ezekiel [1938], 256), it however takes part to Tinbergen's project to follow up economic variables through time. Indeed, the cobweb may describe the adjustment of an industry when arises a modification of its aims. Thus, an inquiry of the role of time in economic phenomena appears behind Tinbergen's statistical and planning purposes[37].

His own version of the cobweb arises when he mentions the solution put forward by Moore and Schultz in order to estimate both demand and supply curves. The former assumes that "a [supply] is related to π [price] through a lagged relationship and not a simultaneous one" (Tinbergen [1930], 670). On this basis, Tinbergen sets out his own version of the cobweb that includes four interesting particularities.

First, contrary to Moore, Ricci and Schultz, Tinbergen adopts a "walrassian" formal framework where prices are represented on the abscissa axis and quantities on the ordinate one[38]. This inversion of the axis inverts also the adjustment scheme of economic variables. In this, it restores the coherence between the economic description of this adjustment process and the formalisms used for that purpose. Indeed, Moore assumed that the price paid in one period may influence the quantity that will be produced in the next succeeding period. Tinbergen borrows this formulation and lays it in the following equation[39]:

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[34] Epstein ([1987], 18) relates that the estimation problem of supply curves became all the more acute that its solving may avoid the difficulties of supervising economy met during World War I.

[35] Translated from "Bestimmung und Deutung von Angebotskurven : Ein Beispiel".

[36] A synthetic presentation of this article, and especially the use of his reduced-form method, may be find in Boumans [1993], 147-149. Curiously, Boumans doesn't mention Moore's influence on Tinbergen.

[37] Boumans clearly shows how Tinbergen's reflection on dynamic processes appears from his first works in 1927 and especially in his 1929 PhD thesis (Boumans [1993], 140).

[38] Let recall that Moore's attitude toward the marshallian frame is rather ambiguous, between its explicit reject and its implicit adoption. See Mirowski [1990], 594-6.

[39] We follow Tinbergen's notations.

$$[1] \qquad a^{(t)} = a_0 + a_1\pi^{(t-1)}$$

where a represents the output, π the price, and t the time. The supply at a given moment t depends thus to the price of the preceding period and to an unspecified minimal level of production a_0. Tinbergen adds then a new equation (Tinbergen [1930], 670) :

[2] $a^{(t)} = n^{(t)}$

Each period, demand have to match supply. And finally, the demand function is not timely indexed, as Tinbergen states :

$$[3] \quad n = n_0 + n_1 \pi,$$

where n is the demanded quantity at price π. Thus, Tinbergen gives here the first algebraic formulation of the three equations of the cobweb process in a walrassian frame[40].

Second, Tinbergen's explanation of the cobweb is mainly mathematical, since his rare comments refer to the characteristics of the oscillations generated by the equations. Thus, he draws three graphs (Tinbergen [1930], 671): the first breaks down analytically the phases of the process (Fig. 1), the two others describe diverging (Fig. 2) or converging (Fig. 3) oscillatory processes.

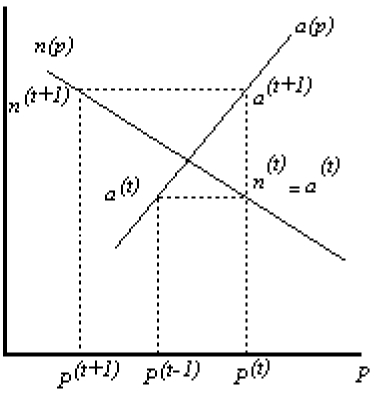

Fig. 1

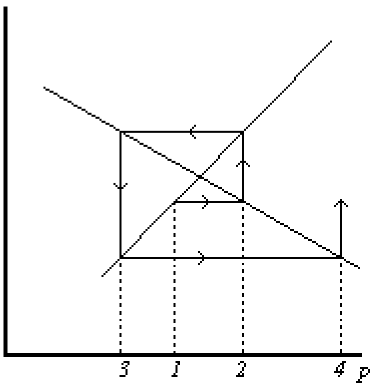

Fig. 2

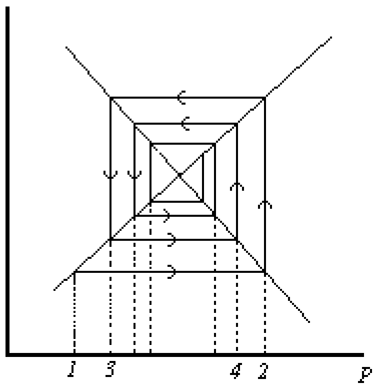

Fig. 3

In the first graph, each step of the process is marked off in a mathematical time and is logically linked to the next step *via* the mathematical relationships. Its economic interpretation - mostly exposed in § 9 - is as succinct - and even similar - as Schultz's one[41]. This mathematical formulation corroborate thus the predominance of mathematics in Tinbergen's investigation process on this problem[42].

[40] Samuelson ([1947], 16) will borrow these equations in the study of one of his dynamic systems.

[41] Contrary to Ricci, Tinbergen affords no economic explanation of an "explosive" or diverging oscillatory process.

[42] Years later, Tinbergen will allude to this kind of inquiry, while dealing with "the dangers of mathematical treatment" in economics : "[i]f the analysis is carried out by people too enthusiastic for the mathematics involved they may either somewhat neglect those [economic] functions or they may make certain basic assumptions because they are easily treated mathematically. If by so doing they choose unrealistic assumptions they are actually not yielding a service to economics. [...] there are example of engineers or physicists hunting for "analogies" between physics and economics and thereby biasing their theories. I would however like to add a remark that might be easily misunderstood and which I therefore hesitate to make: it is not always a disadvantage at first to investigate those cases which, although a little bit unrepresentative, are amenable to

Third, Tinbergen affords also few comments related to the condition of application of the cobweb theorem on the one hand, and to the deciding factors of the kind of generated cycles on the other hand. (i) On the first point, Tinbergen only refers to the statistical conditions to met. The lagged adjustment of supply "is valid only if the sole price of the preceding period determines the supply, without perturbations of any natural factor as it is usually the case of most agricultural products" (Tinbergen [1930], 670). Tinbergen referes here to the objection raised by Elmer Working (1927) that related to the missing variables (Epstein [1987], 34). With this restriction and the choice of the relevant periodicity, the cobweb may be applied to any industry (Pashigian [1987], 463). (ii) On the second point, Tinbergen points out concisely the conditions to get the various type of oscillations :

> "[m]oreover, one easily highlights with Fig. 2 analogue to the Fig. 1, that a permanent movement of prices and turnover is only possible in the cases approaching $a_1 = - n_1$. When a_1 moves strongly over $- n_1$, the indicated mechanism leads either to always increasing oscillations, or to speed up the establishment of an equilibrium situation (Fig. 2 or 3)" (Tinbergen [1930], 670).

The coefficient a_1 and n_1 respectively describe the slope of the supply and the demand curves. When these slopes are equal in absolute values, "the movement of prices and turnover" is characterised by "permanent" and "regular" oscillations[43]. Otherwise, when the curves' slopes are different in absolute values, the oscillatory movement becomes converging ($|a_1| < |n_1|$) or diverging ($|a_1| > |n_1|$). Here, it is interesting to relate that whereas Tinbergen - following Moore - did use these conditions in order to explain the economic cycles, Samuelson [1947] will take them in order to deal with the stability of the economic equilibrium.

Fourth, Tinbergen's version of the cobweb includes one of the very first formulation of the "modern" conception of a dynamic process[44]. Indeed, it consists in

> "a succession of economic periods (time unit) marked by exponents in which economic heights may change values every time" (Tinbergen [1930], 677).

Of course, contrary to Samuelson, Tinbergen makes no use of the mathematical theory of dynamic processes in order to study the evolution of the cobweb[45]. But the geometric analysis makes out to do the job and leads to the same conclusions; especially when the temporal link is rather simple, since it deals with the lagging of one variable behind another one. In

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mathematical analysis; one may make discoveries of a more general character that prove to be useful later" (Tinbergen [1954], 367).

[43] Tinbergen doesn't draw the permanent oscillations case, but he clearly mentioned it, contrary to what asserted Ezekiel ([1938], 256).

[44] Weintraub seems to ignore the Germanic articles related to the stability problem. He states that the first mathematical formulation of a dynamic process can be traced back to Tinbergen 1935's article: "a theory [is] being called 'dynamic' when variables relating to different moments appear in one equation" (Tinbergen [1935], 241; quoted by Weintraub [1991], 24). However, Tinbergen recognises, in this article ([1935], 242, n. 1), that this definition is borrowed to Frisch (1929). Furthermore, Tinbergen provides its first formulation in his PhD thesis, in 1929 (Boumans [1993], 145).

[45] Boumans points out that Tinbergen borrows to physical dynamics its mathematical formalisms (Lagrange's and Hamilon's equations) in order to integrate the time variable in economics (Boumans [1993], 140).

mathematics, the relationship is established not between two variables at the same date, but between two variables with deferred dates. The effect of one variable on the other one is not modified with the introduction of time; it is simply lagged. This "modern" conception of dynamic is thus entirely deterministic, as Israel already stressed it (Israel [1996], 84, 318).

From an economic point of view, the entrepreneur assumes that the prevailing price remain the same in the following period, and decides consequently - and mechanically - to produce according to this price level[46]. But it seems likely that other variables may intervene and affect the price level. Tinbergen knows the reductionism of this mechanical representation of the market process. Then, he introduces, in the individual economic calculus, the ophelimity of actual and "future goods"[47], and even "expectations on the prices evolution"; these factors subvert the implicit determinism of his dynamic equations. The result gives an original representation of the "dynamic evolution of the economy":

> "At the beginning of each period, everybody takes a decision related to its economic activities. He determines at the beginning of the period *t* his supply or his demand on various markets for the period *t, t + 1, t + 2* and so on. The data at his disposal are only partial facts, partial expectations, such as actual prices and expected prices evolution. The principle that lay this determination comes from the agent' maximisation of his ophelimity function that he appreciates for a moment (at the beginning of period *t*). In this ophelimity function intervene all the goods, but also future goods that play a part. The decision taken with this data and following this principle constitutes the economic plan for the time *t*; [...] Only a part from this plan will be realised, *i.e.* the quantities that will be effectively sold. [...] At the beginning of the period *t* + 1 is established a plan in an almost similar fashion, in which a part of the magnitudes established in plan *(t)* occurs as data (only those which are realized) while the others are "revised". Only a determinate part will also be realized from this plan. The system of all these so-called "realized" magnitudes constitute the dynamic evolution of the economy" (Tinbergen [1930], 677).

Thus, the dynamic evolution of the economy, according to Tinbergen, comes from agents inter-temporal optimisation of resources, choices and plans based on expectations and partial realisations of these plans during the next period[48]. Behind this enumeration, Tinbergen sees the possibility to afford a dynamical analysis of the whole economy with the use of cobweb type formalisms. With such analytical tools, Tinbergen could be in a position to defend the dynamic viability of a planned economy. But he gives up this research project since it "will lead him too much away from his aims" (Tinbergen [1930], 679). This is the challenge Samuelson will take up years later.

Tinbergen's work is thus on every point remarkable for its originality, its conciseness and its modernity. His mathematical formulation is the most achieved one in the period, even

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[46] Until then, the economic literature mainly considered entrepreneur's expectations as "static" (Pashigian [1987], 463). Coase and Fowler [1935] will run down this hypothesis.

[47] He follows there Pareto's "pursuit curve" (Pareto [1896], § 41, p. 18).

[48] Boumans rightly relates that the time horizon of agents is only two years (Boumans [1993], 149) (Tinbergen [1930], 678).

compared to Kaldor's [1934]. It is thus curious to see how this article remains the less known, as its lack in Kaldor's famous article [1934] bears witness to.

Conclusion

Two conclusions may be drawn from the foregoing discussion. First, the originators works of the cobweb diagram marked the emergence of a new inquiry mode in economics. This new inquiry mode proceeds by using mathematical rationales first and then by establishing a correspondence between the results and the economic phenomena. Ricci, Schultz and Tinbergen did use of this new process of investigation. It leads them to follow some mathematical consequences of Moore's "statistical trick" and to discover a new way of representing the price formation process. But they do not really understand the economic implications of their discover. Indeed, their main economic purpose was to explain economic cycles, but not to highlight the economic process that - does not - lead to equilibrium. Moreover, in two cases, the authors do not really bother with the economic meaning of their mathematical results. In this, they prefigure the prominent role that will play mathematics in economics, in such an extent that one could ask whether it relegates the economic purpose to the second rank. Hence, this new role of mathematics goes with the rise of a new "danger" in economics, as Tinbergen stated it (Tinbergen [1954], 367). The 1930's constitute then a crucial period to that extent.

Second, we have seen that this new role of mathematics leads to a new question. Indeed, the stability problem emerged directly from the formalization of market processes, and led to Samuelson's works. Put differently, new instruments lead to new questions. Indeed, if Walras did mention the case of unstable equilibria, he attached no real importance to it since the motion should necessarily led to a stable equilibrium[49]. The cobweb diagram puts in light that the formation of economic magnitudes may be problematic. The process may indeed converge to the equilibrium point, but may also move away from it. But, we have to wait for Kaldor's and Samuelson's works in order to have an explicit formulation of what became one of the main difficulties of the general equilibrium theory.

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[49] In mathematical terms, Walras conceived the local stability of equilibrium, and not the global one.

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