# Rigid Wages, Endogenous Job Destruction, and Destabilizing Spirals

Euiyoung Jung

PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

**WORKING PAPER N° 2021 – 27**

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AGENCE NATIONALE DE LA RECHERCHE
ANR
INVESTIR L'AVENIR
Funded by a French government subsidy managed by the ANR under the framework of the Investissements d'avenir programme reference ANR-17-EURE-001

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**Abstract**

This paper studies the theoretical link between real wage rigidity and the destabilizing mechanism driven by the countercyclical precautionary saving demand against unemployment risk. First, I analytically show that destabilizing supply-demand feedback is a general equilibrium outcome of rigid labor cost adjustments. Second, the calibrated wage rigidity consistent with empirical labor market dynamics suggests that real wages are less likely to be sufficiently rigid to cause the destabilizing mechanism. Finally, the way we model job destruction dynamics can have a fundamental impact on the range of real wage rigidity consistent with empirical labor market dynamics and, thus, economic dynamics. Therefore, in contrast to the presumption of many researchers, assuming exogenous job destruction is not innocuous.

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[*]Euiyoung JUNG: Paris School of Economics, 48A Boulevard Jourdan, Paris, France / eyjung@psemail.eu

# 1 Introduction

After the Great Recession of 2008, macroeconomists became increasingly interested in destabilizing spirals driven by households' precautionary saving behavior against uninsured earning risk, particularly unemployment risk (Werning 2015, Ravn and Sterk 2017, Den Haan et al. 2018, Challe 2019). The intuition of the mechanism is as follows. Suppose a contractionary shock that reduces labor demand hits an economy. If asset market friction prevents households from insuring themselves properly against unemployment risk, then households respond to an increase in unemployment risk by increasing savings for precautionary purposes. The shortage in aggregate demand that originates from households' precautionary behavior induces firms to reduce employment. This reinforces precautionary saving demand further, which leads to an additional drop in labor demand. This endogenous amplification mechanism can be so powerful that even tiny recessionary shocks can bring about deep recessions. The destabilizing interactions between the labor market and frictions in goods and financial markets help illustrate the collapse in employment and aggregate demand during the Great Recession.

The well-known Shimer critique (2005) notes that a standard matching model is prone to underestimating labor market volatility when wages are determined by Nash bargaining. In this line of models, real wages are generally so flexible that prices rather than quantity mostly absorb business cycle disturbances. Introducing real wage rigidity is one way to improve the empirical relevance of the model economy. Furthermore, the aforementioned destabilizing mechanism casts subtlety about the level of wage rigidity. As real wage rigidity increases, labor demand fluctuates more extensively and procyclically through both job destruction (countercyclical layoffs) and creation (procyclical vacancies) margins. This implies that countercyclical unemployment risk fluctuates more extensively as wage rigidity increases, which intensifies destabilizing precautionary saving responses over business cycles. This link between real wage rigidity and precautionary saving behavior hints that business cycle behavior in the labor market and the magnitude of the (de)stabilizing impact of supply-demand feedback fundamentally rely on the extent of real wage rigidity.

Based on this idea, the current study explores the theoretical link between real wage rigidity and the destabilizing mechanism driven by uninsured unemployment risks. Concretely, I show that the destabilizing process driven by countercyclical precautionary saving demand against unemployment risk is a general equilibrium outcome of rigid adjustments in labor costs. Furthermore, this paper shows that when we endogenize job destruction dynamics, the calibrated wage rigidity consistent with empirical labor market dynamics or empirical wage stickiness is generally not sufficiently rigid to set off the destabilizing mechanism. Under adequate calibrations, saving demand fluctuates procyclically due to households' saving incentive for consumption smoothing, and consequently, demand feedback endogenously stabilizes an economy rather than amplifying business cycle fluctuations.

This paper develops a model with various frictions, such as labor market matching friction, costly separation and idiosyncratic match-specific shocks, an incomplete asset market with a borrowing constraint, real wage rigidity, and Rotemberg price adjustment costs. Firms have to make endogenous layoff decisions due to idiosyncratic match-specific productivity and costly separation. Therefore, the model completely endogenizes the likelihood of unemployment (the job destruction rate and the job finding probability). Prices should be rigid to cause demand responses that affect the supply side over business cycles, which induces endogenous supply-demand feedback to be relevant. Family insurance is not allowed, and asset market friction impedes households from insuring themselves against unemployment risks. Consequently, households' saving demand is affected by the precautionary saving motive against unemployment risks, which may cause the destabilizing mechanism. More specifically, the saving demand in an incomplete asset market is affected by the two saving motives of intertemporal substitution (consumption smoothing) and precautionary saving against unemployment risk. The precautionary saving demand against unemployment risk tends to fluctuate countercyclically due to countercyclical unemployment risk fluctuations. In contrast, the consumption smoothing motive tends to fluctuate procyclically due to procyclical income fluctuations. The cyclical responses in the net saving demand rely on which motive dominates.

Through an analytical analysis based on a simple ad hoc wage rule under which wages are responsive only to exogenous productivity shocks, I show that demand feedback endogenously amplifies the initial impact of business cycle shocks on the real economy when wages are either excessively rigid or flexible. In contrast, demand feedback is stabilizing when wages are moderately rigid. Intuitively, labor demand declines significantly in response to recessionary shocks if wages are very rigid. A sizable decrease in labor demand stimulates a strong precautionary saving demand against unemployment risks. This precautionary saving demand dominates the household saving incentive to smooth consumption. As a result, the net saving demand increases in response to recessionary shocks (more generally, saving demand is countercyclical), which causes aggregate demand to decline. The decline in aggregate demand reduces labor demand further and deepens recessions. In contrast, if wages are sufficiently flexible, then a sizeable wage cut in response to recessionary shocks induces labor demand to decrease a little or to even increase. The wage decline leads workers to reduce savings today according to the consumption smoothing motive. Moreover, if wages are sufficiently flexible to raise labor demand in response to recessionary shocks, a decline in unemployment risk depresses precautionary saving behavior. The net saving demand declines unambiguously, and the consequent increase in aggregate demand leads to a further increase in labor demand. Likewise, demand feedback amplifies economic fluctuations in this case. In contrast, when wages are moderately rigid, unemployment rises in response to recessionary shocks due to rigid wage adjustments, but procyclical wage adjustments curb the decline in labor demand. As unemployment risk increases to a lesser extent, an increase in precautionary saving demand is restricted compared to the case of sufficiently rigid real wages. However, a relatively large downward wage adjustment in response to recessionary shocks depresses households' saving demand as they pursue consumption smoothing. In equilibrium, net saving demand fluctuates procyclically as the procyclical consumption smoothing motive dominates the countercyclical precautionary saving demand against unemployment risks over business cycles, which generates demand feedback that offsets the impact of business cycle shocks on the real economy.

The intuition from the analytical analysis implies that the degree of real wage rigidity is the critical factor that determines the business cycle behavior in the labor market and, consequently, the (de)stabilizing impact of demand feedback. I numerically extend this idea to a more general wage setting in the literature, namely, that of Nash wage bargaining. To measure the plausible rigidity of labor costs, I estimate business cycle statistics by using CPS and JOLTS data and calibrate the model with Nash wage bargaining to match salient business cycle features in the U.S. labor market. Under the calibrated real wage rigidity consistent with empirical labor market dynamics, procyclical labor demand fluctuations are not extensive enough to produce potent countercyclical precautionary saving reactions. Demand feedback is generally stabilizing because the procyclical consumption smoothing motive primarily drives cyclical saving demand responses. The role of households' precautionary saving demand is restricted to undermine the stabilizing effect of demand feedback. In contrast, the degree of wage rigidity that causes the (temporal) destabilizing process results in labor market dynamics in stark contrast with empirical evidence. The results imply that countercyclical precautionary saving demand driven by uninsured unemployment risk might not suffice to account for the destabilizing mechanism. Instead, to bolster the theory of destabilizing spirals driven by the precautionary saving behavior against unemployment risk, we may have to identify additional factors such as rich household heterogeneity, cyclical policy reactions, or additional frictions and investigate their interactions with incomplete labor and asset markets.

This paper also suggests an interesting modeling caveat. The earlier works that address the destabilizing supply-demand feedback tend to simplify endogenous unemployment risk dynamics. In particular, the literature treats job destruction dynamics exogenously. Indeed, it is currently a common practice in the macroeconomic modeling of labor markets to rule out endogenous job destruction dynamics. The seminal work by Shimer (2012) serves as a rationale for this modeling simplification. The paper contends that acyclic and stable job destruction rates contribute little to unemployment fluctuations. Instead, unemployment fluctuations are mostly accounted for by volatile and procyclical fluctuations in job finding probability. Therefore, researchers usually presume that assuming an exogenous job destruction rate is an innocuous modeling shortcut, which simplifies a model without loss of a great deal of generality. This paper tackles this modeling practice and shows that how we address job destruction dynamics in a model can cause a nontrivial difference in the implication of the destabilizing mechanism. Particularly, endogenous job destruction imposes a stricter calibration restriction on the plausible range of real wage rigidity compared to the case that excludes endogenous job destruction. If we suppose that all job destructions occur exogenously at a constant arrival rate, then it is generally possible to calibrate the model to be consistent with empirical labor market dynamics even under sufficiently rigid real wages that result in the destabilizing spirals. This is because the exogenous job destruction assumption rules out potentially extensive countercyclical fluctuations in job destruction under rigid labor costs. For this reason, the exogenous job destruction assumption is no longer innocuous. Rather, it can have a critical impact on the range of wage rigidity consistent with the empirical evidence and, therefore, economic dynamics. Thus, the way we model job destruction could play a significant role when studying labor market dynamics and further the magnitude of the (de)stabilizing impact of households' precautionary saving demand against unemployment risk.

**Related literature**

Werning (2015) shows that countercyclical earning risks can substantially destabilize an economy by stimulating households to have a strong countercyclical precautionary saving motive. Ravn and Sterk (2016, 2017) emphasize that the general equilibrium interactions between asset and labor market frictions can cause destabilizing supply-demand feedback led by countercyclical precautionary saving demand against unemployment risk. Den Haan, Rendahl, and Riegler (2018) indicate that the interaction of incomplete markets and sticky nominal wages magnifies business cycle fluctuations through a similar destabilizing process. Challe (2019) studies the optimal monetary policy in this type of economy, and McKay and Reis (2016) analyze automatic stabilizers when destabilizing demand feedback matters for aggregate demand dynamics. I extend the prior works by allowing for endogenous job destruction and reassess the destabilizing mechanism under the real wage rigidity calibrated to the labor market in the United States.

Shimer (2012) argues that the separation rate is relatively stable and almost acyclical and contributes little to unemployment fluctuations. In contrast, Elsby et al. (2009) find a relatively significant role of the separation rate in unemployment fluctuations. Fujita and

Ramey (2009) also suggest that the separation rate may account for between forty and fifty percent of unemployment fluctuations, depending on the way that researchers address the data and metrics. Although a vast body of literature explores the empirical behavior of entries into and exits from unemployment, theoretical explanations of observed labor market volatility are scarce. This paper contributes to this line of debate by illustrating the theoretical link between wage rigidity and labor market volatility.

One of the key elements of the modeling in this paper is endogenous job destruction. Mortensen and Pissarides (1994, 1999) analyze the impact of employment protection on labor market dynamics in a deterministic equilibrium. Trigari (2009) and Walsh (2005) incorporate endogenous job destruction into a new Keynesian framework. Thomas (2006) and Veracierto (2008) analyze the impact of employment protection on economic volatility in a real business cycle model. Similarly, Zanetti (2011) investigates how employment protection affects labor market volatility in a new Keynesian economy. Blanchard and Landier (2002) study the impact of partial labor market reform in an economy associated with dual labor markets. The model in the current study resembles Krause and Lubik (2007). They investigate the necessity of real wage rigidity to improve the empirical relevance of the new Keynesian model with labor market matching friction. Fujita and Ramey (2012) assess the quantitative effects of various modeling approaches on the job destruction margin in a search and matching model. The current study that endogenizes job destruction dynamics focuses on the general equilibrium interaction between frictional labor and asset markets and its impact on economic volatility.

This paper proceeds as follows. Section 2 describes the model. Section 3 analytically illustrates the link between wage rigidity and the impact of demand feedback. Section 4 discusses the empirical features of the U.S. labor market and the calibration strategy. Section 5 presents quantitative analyses and suggests a modeling caveat. Section 6 concludes.

# 2 The model

## 2.1 The final good producer

A representative firm produces a final output good in a competitive market. The sector combines intermediate goods according to the following production function:

$$Y_t = \Big[ \int_0^1 Y_t(z)^{\frac{\eta_t - 1}{\eta_t}} dz \Big]^{\frac{\eta_t}{\eta_t - 1}}$$

where $Y_t(z)$ is the input of the $z$th intermediate good and $\eta_t$ is the elasticity of substitution across intermediates. There are shocks to the elasticity of substitution, which generates exogenous disturbances in the desired markups. The representative producer takes as given the final good price $P_t$ and pays $P_t(z)$ for the $z$th input good. The optimal decision of the representative producer is

$$Y_t(z) = \Big(\frac{P_t(z)}{P_t}\Big)^{-\eta_t} Y_t$$

$$P_t = \Big[ \int_0^1 P_t(z)^{1-\eta_t} dz \Big]^{\frac{1}{1-\eta_t}}$$

The first equation is a demand function of an input good $Y_t(z)$ produced by a firm $z$ that sets a nominal price of the good at $P_t(z)$. The second equation describes the aggregate price index implied by the zero-profit condition.

## 2.2 Intermediate good producers

### 2.2.1 The environment

A unit continuum of risk-neutral entrepreneurs manages firms that produce intermediate goods in a monopolistic-competitive market. Entrepreneurs face a series of frictions, such as quadratic price adjustment costs, labor market matching friction, and costly separation. Risk-neutral entrepreneurs discount the future surplus from ongoing matches by the constant discount rate instead of the stochastic discount factor determined by the saving demand among households.

A firm possesses plenty of matches, and each match is subject to a match-specific idiosyncratic productivity shock. Idiosyncratic productivity is *i.i.d*: productivity is newly drawn each period from the time-invariant distribution with the cumulative distribution function $F(a)$ defined over $(0, \infty)$ and the density function $F'(a)$, and idiosyncratic productivity is not autocorrelated.

I assume that risk-neutral entrepreneurs provide insurance to risk-averse workers against the wage dispersion caused by idiosyncratic productivity shocks. Thus, wages are homogeneous, regardless of idiosyncratic match productivity or employment duration.

### 2.2.2 The labor market

There is a unit mass of households who are either employed $(n_t)$ or unemployed $(u_t)$. The labor market has a Diamond-Mortensen-Pissarides (DMP) structure. I suppose a standard Cobb-Douglas matching technology:

$$M_t = \chi \widetilde{u}_t^{\gamma} v_t^{1-\gamma}$$

where $\widetilde{u}_t = u_{t-1} + F_t n_{t-1}$ is the number of job seekers at the beginning of period $t$, and $v_t$ is the number of vacancies. The labor market tightness is defined as

$$\theta_t \equiv \frac{v_t}{\widetilde{u}_t}$$

Incumbent workers whose match-specific idiosyncratic productivity is revealed to be unproductive at the beginning of the period lose their jobs, but discharged workers can participate in the labor market in the same period and may find a new job immediately. Unemployment fluctuates as follows:

$$u_t = (1 - f_t)(u_{t-1} + F_t n_{t-1}) \tag{1}$$

$F_t$ and $f_t$ are the separation rate (job destruction rate) for incumbent workers and the job finding probability, respectively. All matches, including newly formed matches, are subject to idiosyncratic productivity shocks. Firms dissolve new matches associated with low idiosyncratic productivity before production begins. Therefore, the job finding rate, $f_t$, and the vacancy filling rate, $q_t$, depend on the threshold separation productivity, along with labor market tightness:

$$q_t = \frac{M_t}{v_t}(1 - \widehat{F}_t) = \chi\theta_t^{-\gamma}(1 - \widehat{F}_t) \tag{2}$$

$$f_t = \frac{M_t}{\widetilde{u}_t}(1 - \widehat{F}_t) = \chi\theta_t^{1-\gamma}(1 - \widehat{F}_t) \tag{3}$$

where $\widehat{F}_t$ is the separation rate for newly formed matches. The vacancy filling rate $q_t$ should be distinguished from the match filling rate $\tilde{q}_t \equiv \frac{M_t}{v_t}$. Finally, inflows to and outflows from unemployment in each period can be written as $F_t n_{t-1}$ and $f_t\widetilde{u}_t$, respectively.

### 2.2.3 The entrepreneur's problem

Each entrepreneur takes as a given the match filling rate, $\tilde{q}_t$. Idiosyncratic match productivity is well distributed at each firm: a firm that hires $n_t$ workers has $F'(a)n_t$ matches of idiosyncratic productivity $a$. Layoffs incur purely wasteful separation costs. For simplicity, I assume that the separation of new employees and incumbent workers incurs the same amount of separation cost, $\Omega$. It causes the job destruction rates in both the job creation and destruction margins to be identical, i.e., $F_t = \hat{F}_t$.[1] A risk-neutral entrepreneur $i$ maximizes the intertemporal profits subject to the law of motion of labor, production technology, and the demand function of an input good $Y_t(i)$. A detailed description of the entrepreneur's problem is presented in Appendix A.3.2. The equilibrium conditions are as follows:

$$\Psi(\Pi_t - 1)\Pi_t = 1 - \eta_t + \eta_t\mu_t + \beta E_t\Big[\Psi(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\Big] \tag{4}$$

$$\lambda_t = w_t - \beta E_t\Big[\mu_{t+1}A_{t+1}\int_{\bar{a}_{t+1}} a dF(a) - (1 - F_{t+1})\lambda_{t+1} - F_{t+1}\Omega\Big] \tag{5}$$

$$\frac{\kappa}{\tilde{q}_t} = \mu_t A_t \int_{\bar{a}_t} a dF(a) - (1 - F_t)\lambda_t - F_t\Omega \tag{6}$$

[1] The separation cost for newly formed matches is likely to be smaller than the cost for incumbent workers. In this case, firms require higher marginal productivity for new employees because of homogeneous wages and less costly separation. The main results in this paper are robust to this homogeneous separation cost assumption.

$$\mu_t A_t \bar{a}_t = \lambda_t - \Omega \tag{7}$$

$$n_t = (1 - F_t)(n_{t-1} + M_t) \tag{8}$$

$$Y_t = A_t n_t \frac{\int_{\bar{a}_t} a dF(a)}{1 - F_t} \tag{9}$$

Equation (4) is the Phillips curve *à la* Rotemberg quadratic price adjustment costs. $\Psi \geq 0$ denotes the quadratic price adjustment cost. Prices are flexible if $\Psi = 0$ and are rigid otherwise. $\mu_t$ denotes the real marginal cost of production that equals the real marginal revenue in equilibrium.

Equation (5) describes the shadow cost of labor, $\lambda_t$. Employment incurs wage spending, but each match produces a continuation value captured by the expectation term. When the continuing match is associated with low idiosyncratic productivity with probability $F_{t+1}$, a firm will separate the match and pay the separation cost. If the match remains productive with probability $1 - F_{t+1}$, the firm will obtain the net match surplus on average of $\mu_{t+1} A_{t+1} \frac{\int_{\bar{a}_t} a dF(a)}{1-F_t} - \lambda_{t+1}$. Using equation (6), the continuation value can be simplified as follows.

$$\lambda_t = w_t - \beta \frac{\kappa}{\chi} E_t[\theta_{t+1}^{\gamma}] \tag{10}$$

That is, the continuation value of a match is high (low) when firms expect high (low) labor market tightness, i.e., high (low) vacancy posting costs, in the following period.

Equation (6) is the job creation condition. A firm posts vacancies until the average vacancy posting cost equals the expected net surplus from a filled vacancy. With the rate of $\tilde{q}_t$, a vacancy is matched with a job candidate. Due to the idiosyncratic productivity shock, the vacancy is eventually filled with probability $1 - F_t$. Similarly, a firm obtains $\mu_t A_t \frac{\int_{\bar{a}_t} a dF(a)}{1-F_t} - \lambda_t$ from a filled vacancy on average. However, if a new match is not sufficiently productive with probability $F_t$, the firm would pay the separation cost and terminate the contract before production begins.

Equation (7) is the job destruction condition that determines the threshold level of productivity for separation. By replacing $\lambda_t$ using equation (5), we can obtain

$$\mu_t A_t \bar{a}_t - w_t + \beta E_t\Big[-\Omega F_{t+1} + \mu_{t+1}A_{t+1}\int_{\bar{a}_{t+1}} a dF(a) - (1-F_{t+1})\lambda_{t+1}\Big] = -\Omega$$

The right-hand side is the separation cost. The left-hand side describes the surplus from a marginal match. If the match-specific idiosyncratic productivity is lower than $\bar{a}_t$, then a firm dissolves the match because the match surplus is smaller than the separation cost. The job creation and destruction conditions suggest that labor demand is affected by the shadow cost of labor, not merely wages.

Equations (8) and (9) describe the law of motion of labor and the aggregate production function, respectively. Equation (9) suggests that entrepreneurs face a trade-off between the quantity and quality of labor. They can enhance the average labor productivity by sorting out unproductive matches. Due to the stricter productivity criteria, however, fewer matches survive.

## 2.3 Households

### 2.3.1 The environment

Households maximize lifetime welfare in which periodic utility is determined by a standard *CRRA* utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}-1}{1-\sigma}\right] \quad \text{where} \quad \sigma > 1$$

where $c_t$ is the amount of consumption, and the risk-aversion parameter $\sigma$ is strictly greater than 1. Neither working nor job searching yields disutility, and working hours are indivisible. Since family insurance is inaccessible, households are heterogeneous depending on their employment status.

I impose strong assumptions to ensure model tractability. Namely, households cannot go into debt, and the government should strictly balance the budget in each period. As labor is the sole production factor, the borrowing constraint degenerates the wealth distribution in equilibrium. Put differently, the bond held by a household should be zero regardless of the employment status [2]. Although households cannot save in equilibrium, household saving demand still affects the supply side through the price channel.

#### 2.3.2 Workers

Due to the *i.i.d* idiosyncratic productivity, the borrowing constraint, and the homogeneous wage, workers are symmetric in equilibrium. The worker's problem can be described in a recursive way. The value function can be specified as

$$V^W(d_{t-1}|\mathbf{S_t}) \equiv \max_{c_{w,t},d_t} u(c_{w,t}) + \beta E_t\Big[P_{w,t+1}V^W(d_t|\mathbf{S_{t+1}}) + (1-P_{w,t+1})V^U(d_t|\mathbf{S_{t+1}})\Big] \quad (11)$$

subject to the budget constraint and the zero-lower bound bond holding constraint.

$$c_{w,t} + \frac{d_t}{R_t} = w_t + \frac{d_{t-1}}{\Pi_t} - \alpha\frac{T_t}{n_t}$$
$$d_t \geq 0$$

$d_{t-1}$ denotes the amount of bonds purchased in period *t-1*. $w_t$ is the real wage, and $c_{w,t}$ is the consumption of a worker in period $t$. $\alpha\frac{T_t}{n_t}$ denotes the lump-sum tax per worker. $\mathbf{S_t}$ represents the vector of state variables in period $t$. $P_{w,t} \equiv 1 - F_t + F_t f_t$ is the transition rate from employment to employment in period $t$. The nominal bond price is $\frac{1}{R_t}$, where $R_t$ is the gross nominal interest rate of the one-period bond held from $t$ to *t+1*.

#### 2.3.3 The unemployed household

The value function of the unemployed $V^U(d_{t-1}|\mathbf{S_t})$ is specified as

$$V^U(d_{t-1}|\mathbf{S_t}) \equiv \max_{c_{u,t},d_t} u(c_{u,t}) + \beta E_t\Big[(1-P_{u,t+1})W(d_t|\mathbf{S_{t+1}}) + P_{u,t+1}V^U(d_t|\mathbf{S_{t+1}})\Big] \quad (12)$$

[2]This way of asset market specification, called a *maximally tight asset market*, was analyzed extensively in Krusell et al. (2011) and employed in many subsequent works, such as McKay and Reis (2016), Ravn and Sterk (2017), and Challe (2019).

subject to the budget constraint and the borrowing constraint.

$$c_{u,t} + \frac{d_t}{R_t} = b_t + \frac{d_{t-1}}{\Pi_t}$$
$$d_t \geq 0$$

$b_t$ denotes unemployment benefits. I assume that $b_t = \tilde{r} c_{w,t}$, where the constant replacement rate $(\tilde{r})$ is strictly smaller than 1. Similarly, unemployed households are symmetric in equilibrium. $P_{u,t} \equiv 1 - f_t$ denotes the transition rate from unemployment to unemployment in period $t$.

### 2.3.4 Decisions

Due to the borrowing constraint, the asset market reaches autarky: neither borrowing nor lending across households is feasible. Households consume all cash-in-hand in each period. Since households are symmetric conditional on employment status, the envelope conditions and autarky budget constraints give the Euler equation of each household as follows.

$$-c_{w,t}^{-\sigma} + \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} (P_{w,t+1} c_{w,t+1}^{-\sigma} + (1 - P_{w,t+1}) b_{t+1}^{-\sigma}) \Big| \mathbf{S_{t+1}} \right] + R_t \mu_{w,t} = 0$$
$$-b_t^{-\sigma} + \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} ((1 - P_{u,t+1}) c_{w,t+1}^{-\sigma} + P_{u,t+1} b_{t+1}^{-\sigma}) \Big| \mathbf{S_{t+1}} \right] + R_t \mu_{u,t} = 0$$

where $\mu_{w,t}$ and $\mu_{u,t}$ are the Lagrange multipliers on the borrowing constraint of employed and unemployed individuals, respectively.

### 2.3.5 Asset pricing

In the appendix, I show that $0 \leq \mu_{w,t} < \mu_{u,t}$ always holds in equilibrium. Although any allocation with $\mu_{w,t}$ that satisfies $0 \leq \mu_{w,t} < \mu_{u,t}$ can be consistent with the equilibrium, a particular case such that $\mu_{w,t} = 0$ for all $t$ deserves a note because it is the constrained Pareto-optimal allocation, given $R_t$. Hereafter, I suppose $\mu_{w,t} = 0$ for all $t$. This enables us to identify the equilibrium bond price, which clears the excess demand for a bond.

**Proposition 2.1** *The equilibrium nominal gross interest rate, $R_t^N$, is determined as*

$$\frac{1}{R_t^N} = \beta E_t \left[ \frac{1}{\Pi_{t+1}} \Big( \frac{c_{w,t+1}}{c_{w,t}} \Big)^{-\sigma} (P_{w,t+1} + (1 - P_{w,t+1})\tilde{r}^{-\sigma}) \Big| \mathbf{S_{t+1}} \right] \tag{13}$$

*Proof) See Appendix A.3.1.*

Workers are indifferent at zero bond holding in equilibrium. The equilibrium bond price is determined by workers' saving demand driven by the two saving motives of consumption smoothing and the precautionary saving demand against unemployment risks. The consumption smoothing motive is captured by the term $\left(\frac{c_{w,t+1}}{c_{w,t}}\right)^{-\sigma}$. If workers expect their current income to be lower than their future income, then they reduce savings to smooth consumption. The real interest rate should rise to clear the bond market. The endogenous wedge, $P_{w,t+1} + (1 - P_{w,t+1})\tilde{r}^{-\sigma}$, captures the precautionary saving motive against unemployment risk. Since $\tilde{r} < 1 < \sigma$, an increase in unemployment risk (a decrease in $P_{w,t+1}$) lowers the real interest rate. If workers expect that they are more likely to be unemployed tomorrow than today, then they raise savings for a precautionary reason. The real interest rate should fall to clear the bond market. The endogenous wedge disappears if unemployment benefits are complete ($\tilde{r} = 1$), if households are risk neutral ($\sigma = 0$), or if workers never become unemployed ($P_{w,t} = 1 \ \forall t$).

## 2.4 Policies and market clearing conditions

### 2.4.1 Fiscal policy

Under a strict balanced budget constraint, the government imposes a lump-sum tax on workers and entrepreneurs to finance unemployment insurance benefits.

$$T_t = b_t u_t \tag{14}$$

$\alpha$ percent of the total tax is imposed on workers, and entrepreneurs pay the rest.

### 2.4.2 Monetary policy

The monetary authority commits to a *Taylor rule* that responds to inflation. I address the case in which shocks are not too large, which prevents nominal interest rates from binding the zero-lower bound.

$$R_t^N = R\Big(\frac{\Pi_t}{\Pi}\Big)^{\gamma_R} \tag{15}$$

where $\Pi$ and $R$ are the steady-state inflation and the nominal interest rate target, respectively.

### 2.4.3 Market clearing conditions

Entrepreneurs consume the net profits after paying all production costs and the lump-sum tax:

$$c_t^e = \Big(1 - \frac{\kappa v_t}{Y_t} - \frac{\Omega F_t(n_{t-1} + M_t)}{Y_t} - \frac{\Psi}{2}(\Pi_t - 1)^2\Big)Y_t - w_t n_t - (1-\alpha)T_t \tag{16}$$

The final output net of production costs equals the aggregate consumption in equilibrium.

$$C_t = \Big(1 - \frac{\kappa v_t}{Y_t} - \frac{\Omega F_t(n_{t-1} + M_t)}{Y_t} - \frac{\Psi}{2}(\Pi_t - 1)^2\Big)Y_t \tag{17}$$

$$C_t \equiv c_{w,t} n_t + b_t u_t + c_t^e \tag{18}$$

## 2.5 Shocks

### 2.5.1 The idiosyncratic match-specific productivity shock

I suppose that idiosyncratic productivity follows a log-normal distribution:

$$F(x) = \Phi\Big(\frac{ln(x) - \mu_a}{\sigma_a}\Big) \tag{19}$$

$\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. $\mu_a$ and $\sigma_a$ are the mean and standard deviation of the associated normal distribution, respectively.

### 2.5.2 Aggregate shocks

The exogenous productivity shock, $A_t$, follows

$$A_t = A^{1-\rho_A} A_{t-1}^{\rho_A} e^{\sigma_A \epsilon_t^A} \tag{20}$$

where $A$ is the deterministic steady-state productivity normalized to 1. The markup shock follows

$$\eta_t = \eta^{1-\rho_\eta} \eta_{t-1}^{\rho_R} e^{\sigma_\eta \epsilon_t^\eta} \tag{21}$$

The white noise terms, $\epsilon_t^A$ and $\epsilon_t^\eta$, are independent and follow the standard normal distribution.

## 2.6 Wage determination: Nash bargaining

The standard matching model with Nash wage bargaining tends to be limited in its ability to satisfactorily reconstruct empirical labor market dynamics. How to improve the empirical capability of the model is still an open research question [3]. Following Hall (2005), I assume that the equilibrium wage is a weighted average between the Nash bargaining solution and a long-run wage target that equals the deterministic steady-state wage. Formally,

$$w_t = w^{\gamma_w} w_{n,t}^{1-\gamma_w}$$

[3]Some researchers assume ad hoc wage stickiness (Shimer (2004), Hall (2005), Blanchard and Gali (2010), McKay and Reis (2016), Ravn and Sterk (2017)). Gertler and Trigari (2009) formalize Calvo wage rigidity. Hagedorn and Manovskii (2008) suggest an alternative calibration that produces sufficient fluctuations in unemployment. Hall and Milgrom (2008) and Christiano et al. (2016) consider an alternative wage bargaining scheme.

where the degree of wage inertia, $\gamma_w$, lies in the interval $[0, 1]$. $w_{n,t}$ is the equilibrium wage under Nash bargaining conditional on $\gamma_w = 0$ [4]. Proposition 2.2 describes the equilibrium wage under Nash bargaining in the absence of wage inertia.

**Proposition 2.2** *The Nash bargaining solution, $w_{n,t}$, is determined by*

$$w_{n,t} = \Big(\frac{\zeta}{1-\zeta}\frac{\frac{\mu_t Y_t}{n_t} - \lambda_t + \Omega}{\Delta V_t}\Big)^{\frac{1}{\sigma}} + \alpha\frac{T_t}{n_t} \tag{22}$$

*Proof) See Appendix A.3.3.*

$\frac{\mu_t Y_t}{n_t} - \lambda_t + \Omega$ represents the firm's average match surplus, and $\Delta V_t \equiv V_t^w - V_t^u$ denotes the worker's surplus from employment. $\zeta$ denotes the worker's bargaining power.

# 3 Understanding the dynamics: An analytical approach

## 3.1 Overview

This section analytically illustrates how real wage rigidity affects labor market volatility and the role of demand feedback. For simplicity, I normalize the matching function scale parameter and the vacancy posting cost to 1, $\chi = \kappa = 1$. The steady-state labor market tightness is supposed to be 1, and I rule out fiscal redistribution across households, i.e., $\alpha = 0$. Technology shocks are supposed to be the only source of business cycle fluctuations. The baseline wage-setting assumption, Nash bargaining, overcomplicates the analytical approach. Alternatively, following Blanchard and Gali (2010), I assume a simple wage rule given by

$$w_t = \bar{w}(\frac{A_t}{A})^{\gamma_w} \quad \text{where} \quad \gamma_w \geq 0$$

[4]This wage specification induces wages to generally remain in a bargaining set. If exogenous shocks are not too large, then the steady-state wage would be in the wage bargaining set. Namely, $w$ can constitute an equilibrium wage; therefore, the weighted average of $w$ and $w_{n,t}$ can qualify for an equilibrium wage. One can refer to Hall (2005) for further discussion.

where $\gamma_w$ determines the degree of wage rigidity. Now, refer to the following system of equations:

$$\lambda_t = w_t - \beta E_t[\theta_{t+1}^{\gamma}]$$
$$\theta_t^{\gamma} = \mu_t A_t \Big( \int_{\bar{a}_t} a dF - (1 - F_t)\bar{a}_t \Big) - \Omega$$
$$w_t = \bar{w} \Big( \frac{A_t}{A} \Big)^{\gamma_w}$$
$$\lambda_t = \mu_t A_t \bar{a}_t + \Omega$$
$$\frac{1}{R_t^N} = \beta E_t \left[ \frac{1}{\Pi_{t+1}} \Big( \frac{w_{t+1}}{w_t} \Big)^{-\sigma} (1 - F_{t+1} + F_{t+1} f_{t+1} + F_{t+1}(1 - f_{t+1})\tilde{r}^{-\sigma}) \Big| \mathbf{S_{t+1}} \right]$$
$$R_t^N = R \Big( \frac{\Pi_t}{\Pi} \Big)^{\gamma_R}$$
$$\Psi(\Pi_t - 1)\Pi_t = 1 - \eta_t + \eta_t \mu_t + \beta E_t \Big[ \Big( \Psi(\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \Big) \Big]$$
$$\log(A_t) = \rho_A \log(A_{t-1}) + \sigma_A \epsilon_t^A$$

Up to a first-order approximation, the variables $\{\lambda_t, \theta_t, \bar{a}_t, \Pi_t, R_t^N, \mu_t, w_t\}$ respond only to technology shocks and are independent of the endogenous state variable, $n_{t-1}$. This enables us to write the percentage deviation of a variable $x_t \in \{\lambda_t, \theta_t, \bar{a}_t, \Pi_t, R_t^N, \mu_t, w_t\}$ from its deterministic steady-state ($\hat{x}_t \equiv \frac{x_t - x}{x}$) in response to the percentage deviation of the productivity shock ($\hat{A}_t \equiv \frac{A_t - A}{A}$) such that $\hat{x}_t = \Phi_x \hat{A}_t$, where $\Phi_x$ represents the policy rule. A variable $x_t$ increases (decreases) in response to positive technology shocks if $\Phi_x$ is positive (negative), and the response would be more extensive as the absolute value of $\Phi_x$ increases.

## 3.2 Dynamics without demand feedback

To comprehend labor market dynamics in response to technology shocks, it is constructive to begin with the dynamics in the separation threshold, labor market tightness, and the shadow cost of labor. By using the first-order approximation and the method of undetermined coefficients, we can write the policy rules of the variables as follows:

$$\Phi_{\bar{a}} = -C_{\bar{a}} + \Phi_{\bar{a}}^{\gamma_w}\gamma_w - \Phi_{\bar{a}}^{\mu}\Phi_{\mu} \quad where \quad C_{\bar{a}} > 0,\ \Phi_{\bar{a}}^{\gamma_w} > 0,\ and \quad \Phi_{\bar{a}}^{\mu} > 0 \tag{23}$$

$$\Phi_{\lambda} = -C_{\lambda} + \Phi_{\lambda}^{\gamma_w}\gamma_w - \Phi_{\lambda}^{\mu}\Phi_{\mu} \quad where \quad C_{\lambda} > 0,\ \Phi_{\lambda}^{\gamma_w} > 0,\ and \quad \Phi_{\lambda}^{\mu} > 0 \tag{24}$$

$$\Phi_{\theta} = +C_{\theta} - \Phi_{\theta}^{\gamma_w}\gamma_w + \Phi_{\theta}^{\mu}\Phi_{\mu} \quad where \quad C_{\theta} > 0,\ \Phi_{\theta}^{\gamma_w} > 0,\ and \quad \Phi_{\theta}^{\mu} > 0 \tag{25}$$

The dynamics without demand feedback indicate when prices are flexible. When prices are flexible while the technology shock is a sole source of business cycle fluctuations, the marginal revenue of production, $\mu_t$, is constant, i.e., $\Phi_\mu = 0$. Equations (23) to (25) suggest that the policy rules of the separation threshold and the shadow cost of labor are linear and increasing functions of $\gamma_w$. In contrast, the policy rule of labor market tightness is a decreasing function of $\gamma_w$. Proposition 3.1 summarizes the dynamics of the variables depending on real wage rigidity in a flexible price economy.

**Proposition 3.1** *Thresholds $\gamma_\lambda$, $\gamma_{\bar{a}}$ and $\gamma_\theta$ exist in $\mathbb{R}$, which satisfies*

1. $0 < \gamma_\lambda < \gamma_{\bar{a}} < \gamma_\theta$
2. $\Phi_x = 0 \quad \text{if} \quad \gamma_w = \gamma_x \quad \text{for} \quad x \in \{\lambda, \bar{a}, \theta\}$
3. $\frac{d\Phi_\lambda}{d\gamma_w} > 0$, $\frac{d\Phi_{\bar{a}}}{d\gamma_w} > 0$, *and* $\frac{d\Phi_\theta}{d\gamma_w} < 0$

*Proof) See Appendix A.3.5.*

I take an example of labor market dynamics in response to adverse technology shocks. The opposite case is symmetric. First, suppose that wage rigidity $\gamma_w$ is lower than $\gamma_\lambda$. The proposition suggests that the shadow cost of labor rises in response to an adverse technology shock. Intuitively, if the real wage falls little when an adverse technology shock occurs, then firms raise the separation threshold because the match surplus decreases significantly. As the separation threshold increases, each vacancy is less likely to be filled. Combined with the adverse impact of the technology shock on the match surplus, firms decrease vacancies as the surplus from job creation declines. Because layoffs increase while vacancies decline, labor market tightness unambiguously falls. Since tightness declines in response to a persistent adverse technology shock, firms expect the continuation value of a match to decline in the following periods. A decrease in the continuation value raises the current shadow cost of labor, which offsets the impact of the initial wage decline on the shadow cost of labor. If real wage rigidity equals $\gamma_\lambda$, then these two forces exactly cancel out, and $\lambda_t$ is invariable to changes in $A_t$. If $\gamma_w < \gamma_\lambda$, then the shadow cost of labor would rise because the decline in the continuation value would be larger than the wage decline[5]. Provided that $\gamma_w < \gamma_\lambda$, the economy would be more volatile as real wage rigidity rises ($\gamma_w$ decreases).

When $\gamma_\lambda < \gamma_w$, the shadow cost of labor declines in response to the adverse technology shock due to a sufficient drop in the wage. If $\gamma_w$ is in the interval $[\gamma_\lambda, \gamma_{\bar{a}}]$, then the separation rate still rises because the decrease in the shadow cost of labor is not enough to offset the decline in the marginal match surplus. Labor market tightness falls as layoffs increase. Similarly, labor market tightness is expected to decline, which increases the shadow cost of labor through the continuation value channel. Because the decline in the wage is larger than the decrease in the continuation value, the shadow cost of labor falls in equilibrium.

When $\gamma_w$ lies in the interval $[\gamma_{\bar{a}}, \gamma_\theta]$, the wage falls enough to result in a sufficient decline in the shadow cost of labor. Due to the relatively large decrease in the shadow cost of labor, separation becomes a more expensive choice for firms than maintaining marginal workers. Thus, firms lower the separation rate. In contrast, although a vacancy is more likely to be filled as the separation threshold is lowered, the adverse productivity shock diminishes the surplus from new matches. Consequently, firms curtail vacancies, and tightness falls.

When $\gamma_w$ is higher than $\gamma_\theta$, wages are sufficiently flexible, and the shadow cost of labor drops considerably after the adverse technology shock. Firms lower the threshold productivity, which results in a decrease in inflows to unemployment. However, the sizable decrease in the shadow cost of labor raises the net surplus from job creation. Although the number of job seekers in the labor market declines because fewer layoffs occur, firms may post more vacancies or decrease them a little in response to the adverse technology shock. Consequently, labor market tightness rises in equilibrium. Conditional on $\gamma_\theta < \gamma_w$,

[5]The intuition implies that an amplification process exists if $\gamma_w$ is sufficiently low. If the wage is considerably rigid, firms sharply raise the separation rate in response to a drop in the match surplus. The increased threshold lowers labor market tightness, which further raises the shadow cost of labor. As the shadow cost of labor increases, firms raise the threshold productivity further, and so on. The zero vacancy corner solution case is the extreme case of this amplification process that results from sufficiently rigid wages.

the labor market would be more volatile as wages become more flexible ($\gamma_w$ increases). [6]

Unemployment fluctuates according to $u_t = (1 - f_t)(u_{t-1} + F_t n_{t-1})$. The policy rule of unemployment is a function of the deviation of $n_{t-1}$ from its steady-state, along with the deviation of the technology shock. The percentage deviation of unemployment from its steady-state can be described as

$$\hat{u}_t = \Phi_u^A(\gamma_w)\hat{A}_t + \Phi_u^n \hat{n}_{t-1}$$

$\Phi_u^n$ is independent of real wage rigidity, while the policy rule attached to $\hat{A}_t$ is a linear function of real wage rigidity. $\Phi_u^A(\gamma_w)$ denotes the policy rule of unemployment conditional on $n_{t-1}$. The unemployment dynamics in response to technology shocks conditional on $n_{t-1}$ can be summarized as follows:

**Corollary 3.1.1** *There exists $\gamma_u \in \mathbb{R}$ such that $\gamma_{\bar{a}} < \gamma_u < \gamma_\theta$ and satisfies*

$$\text{If } \gamma_w \leq \gamma_u, \quad \Phi_u^A(\gamma_w) \leq 0 \quad \text{and} \quad \frac{|\partial \Phi_u^A(\gamma_w)|}{\partial \gamma_w}\Big|_{n_{t-1}} < 0$$
$$\text{If } \gamma_w \geq \gamma_u, \quad \Phi_u^A(\gamma_w) \geq 0 \quad \text{and} \quad \frac{|\partial \Phi_u^A(\gamma_w)|}{\partial \gamma_w}\Big|_{n_{t-1}} > 0$$

*Proof) See Appendix A.3.5.*

Conditional on $n_{t-1}$, unemployment rises in response to the adverse technology shock insofar as $\gamma_w$ is lower than $\gamma_u$. Two cases are possible. If real wages are sufficiently rigid, then the shadow cost of labor rises or declines little in response to the shock. Therefore, the separation rate increases, while labor market tightness declines. Unemployment unambiguously rises because layoffs increase while job creation declines. In contrast, if real wages are not rigid enough, then the job destruction rate may decline in reaction to the adverse technology shock because the shadow cost of labor falls sufficiently. Nonetheless, unemployment rises because firms largely curtail vacancies. That is, outflows from

[6] Likewise, there is an amplification process. Since the shock is persistent, firms expect a higher tightness, which lowers the current shadow cost of labor and stimulates firms to further lower the separation threshold. Firms reduce separations and expand recruitment. If $\gamma_\theta << \gamma_w$, then this expansionary amplification process continues until the higher average vacancy posting cost that comes from fewer job seekers cancels out the increase in the net surplus from job creation. The no-separation equilibrium would be the case of excessively flexible wages.

unemployment decrease more than the reduction in inflows to unemployment.

When $\gamma_w$ is higher than $\gamma_u$, unemployment falls in response to the adverse technology shock. Wages are so flexible that the shadow cost of labor declines considerably in response to the shock. It causes firms to decrease layoffs. In addition, firms reduce vacancies a little or even increase them. This is because flexible wage arrangements and the lowered separation threshold neutralize the impact of the adverse technology shock on the firm's surplus from job creation. Unemployment falls as the reduction in inflows to unemployment dominates the decrease in outflow, or outflows from unemployment may even rise simultaneously.

Corollary 3.1.1 implies that when wages are very flexible, an adverse technology shock can even lead to an increase in output. Although the gross match surplus falls due to the adverse technology shock, firms may attain a higher net match surplus as the shadow cost of labor declines significantly due to flexible wage adjustments. Despite the adverse technology shock, they may reduce layoffs and increase recruitment. Although the average productivity declines due to the adverse technology shock and the lowered threshold productivity, output may rise because employment increases. The corollary also implies that the degree of wage rigidity causes a nonmonotonous impact on economic volatility. Both overly flexible and overly rigid wages lead to higher volatility in the real economy, while the economy becomes relatively stable under moderately rigid real wages.

The unemployment risk dynamics are worth mentioning. When $\gamma_w$ is smaller than $\gamma_{\bar{a}}$, unemployment risk unambiguously rises in response to the negative technology shock: the job destruction rate rises, and the job finding probability falls. In contrast, when $\gamma_w$ is greater than $\gamma_\theta$, the job destruction rate falls, while the job finding probability rises. Unemployment risk unambiguously falls in this case. When $\gamma_w$ lies between $\gamma_{\bar{a}}$ and $\gamma_\theta$, the nonmonotonous elasticity of $\bar{a}_t$ and $\theta_t$ suggests that unemployment risk would rise (fall) in response to the negative technology shock if $\gamma_w$ is close to $\gamma_{\bar{a}}$ $(\gamma_\theta)$. Notably, the likelihood of unemployment can fluctuate countercyclically in the two different cases. When wages are very rigid, unemployment risk rises while output falls in response to an adverse technology shock. If wages are sufficiently flexible, then unemployment risk declines while output rises when a negative technology shock occurs.

## 3.3 Dynamics with demand feedback

Demand feedback affects labor demand through the price channel in the model economy. Concretely, the equilibrium real interest rate is adjusted to clear the bond market. The nominal interest rate and inflation vary accordingly, which leads to fluctuations in firms' marginal revenue ($\mu_t$) through the Phillips curve. As match surplus varies, labor demand is affected, which endogenously amplifies or dampens employment and output fluctuations. By solving the worker's Euler equation, the Taylor rule, and the Phillips curve, we can find the policy rule of marginal revenue.

**Proposition 3.2** *The policy rule of $\mu_t$ is written as*

$$\Phi_\mu = -\underbrace{\frac{\Psi(1-\beta\rho_A)}{\eta\mu(\gamma_R-\rho_A)}}_{\text{Positive coefficient}}\Big[\underbrace{\sigma(1-\rho_A)\gamma_w}_{\text{Consumption smoothing}} - \underbrace{\frac{(\tilde{r}^{-\sigma}-1)\rho_A F}{P_w+(1-P_w)\tilde{r}^{-\sigma}}\Big((1-F)(1-\gamma)\Phi_\theta - 2F'(\bar{a})\bar{a}\Phi_{\bar{a}}\Big)}_{\text{Precautionary saving}}\Big]$$

*Proof) See Appendix A.3.5.*

The policy rule indicates that demand feedback through precautionary saving demand disappears if $\tilde{r}=1$ or $\sigma=0$. The consumption smoothing motive is irrelevant if $\sigma=0$ or wages are constant, i.e., $\gamma_w=0$. By using policy rules (23) and (25) , we can solve the policy rule of $\mu_t$.

**Corollary 3.2.1** *The policy rule of $\mu_t$ can be described as*

$$\Phi_\mu = C_\mu - \Phi_\mu^{\gamma_w}\gamma_w \quad \textit{where} \quad C_\mu > 0 \quad \textit{and} \quad \Phi_\mu^{\gamma_w} > 0 \tag{26}$$

*Proof) See Appendix A.3.5.*

Under the plausible parameter restrictions [7], the intercept ($C_\mu$) and slope ($\Phi_\mu^{\gamma_w}$) of the policy rule are strictly positive. Likewise, a level of wage rigidity ($\gamma_\mu$) that yields $\Phi_\mu=0$ exists. That is, $\gamma_\mu=\frac{C_\mu}{\Phi_\mu^{\gamma_w}}$. Specifically, $\gamma_\mu$ should be lower than $\gamma_\theta$ because unemployment risk declines in response to the adverse technology shock when $\gamma_w \geq \gamma_\theta$. As both consumption smoothing and precautionary saving motives are depressed, saving demand

[7] Refer to Assumption 1 in Appendix A.3.5.

should fall unambiguously. The consequent downward adjustment in real interest rates or the equivalent increase in aggregate household consumption demand raises the profit margin of firms in equilibrium, i.e., $\mu_t$ increases. When $\gamma_w = \gamma_\mu$, saving demand is invariable to shocks because consumption smoothing and precautionary saving motives cancel out.

Two opposing cases help us comprehend how demand feedback propagates to the supply side. First, suppose that wages are constant ($\gamma_w = 0$). By plugging equation (26) into equations (23) and (25) , the policy rule of the separation threshold and tightness under rigid prices can be written as follows:

$$\Phi_{\bar{a}} = \underbrace{-C_{\bar{a}}}_{negative} - \underbrace{\Phi^{\mu}_{\bar{a}} C_{\mu}}_{positive}$$

$$\Phi_{\theta} = \underbrace{C_{\theta}}_{positive} + \underbrace{\Phi^{\mu}_{\theta} C_{\mu}}_{positive}$$

The negative term $-\Phi^{\mu}_{\bar{a}} C_{\mu}$ and the positive term $\Phi^{\mu}_{\theta} C_{\mu}$ show that demand feedback endogenously amplifies fluctuations in the separation threshold and labor market tightness. The intuition is straightforward. Due to constant wages, the consumption smoothing motive disappears. On the other hand, the constant wage causes a substantial decrease in labor demand in response to adverse technology shocks. Unemployment risk rises, and household saving demand increases in response to an increase in unemployment risk. The real interest rate should fall to clear the bond market. The downward pressure on the real interest rate pushes down the current inflation or raises the expected inflation rate. The marginal revenue from production declines according to the Phillips curve, while the shadow cost of labor rises as the marginal revenue declines. As a result, demand feedback amplifies an increase in layoffs and a decrease in recruitment. This scenario is consistent with the destabilizing spirals highlighted in Ravn and Sterk (2016, 2017).

Now consider the case of $\gamma_w > \gamma_\theta$, where wages are sufficiently flexible. In this case, we have

$$\Phi_{\bar{a}} = \underbrace{-C_{\bar{a}} + \Phi_{\bar{a}}\gamma_w}_{positive} - \underbrace{\Phi^{\mu}_{\bar{a}}(C_{\mu} - \Phi_{\mu}\gamma_w)}_{negative}$$
$$\Phi_{\theta} = \underbrace{C_{\theta} - \Phi_{\theta}\gamma_w}_{negative} + \underbrace{\Phi^{\mu}_{\theta}(C_{\mu} - \Phi_{\mu}\gamma_w)}_{negative}$$

Likewise, policy rules show that demand feedback amplifies the responses of the variables. Flexible wage arrangements lead to a considerable wage cut in response to the adverse technology shock, which depresses workers' saving motive to smooth consumption. Furthermore, flexible wage arrangements translate into increased labor demand when adverse technology shocks occur, which lowers unemployment risk. As a result, precautionary saving demand declines. Saving demand unambiguously falls, and the equilibrium real interest rate should increase to clear the bond market. The ensuing inflationary pressure (or an expected decline in prices) raises the marginal revenue from production. This raises labor demand further. Consequently, demand feedback amplifies fluctuations in the real economy.

Proposition 3.3 summarizes the impact of demand feedback on the output dynamics depending on real wage rigidity. Demand feedback is defined as destabilizing if the output responses to the shock conditional on the endogenous state variable $n_{t-1}$ are more extensive under rigid prices than its flexible price counterpart. Otherwise, it is defined as stabilizing. Figure 1 depicts the output policy rule, which illustrates the impact of demand feedback on output fluctuations depending on the level of wage rigidity [8].

**Proposition 3.3** *There exist $\underline{\gamma_w}$ and $\overline{\gamma_w}$ such that*

1. $0 < \underline{\gamma_w} < \overline{\gamma_w}$
2. *Demand feedback is stabilizing if $\underline{\gamma_w} \leq \gamma_w \leq \overline{\gamma_w}$. Otherwise, it is destabilizing.*

*Proof) See Appendix A.3.5.*

[8] The figure depicts the case such that $\gamma_Y$, which makes the output policy rule conditional on $n_{t-1}$ invariable to shocks, is assumed to be greater than $\gamma_{\mu}$. The other case is discussed in the appendix

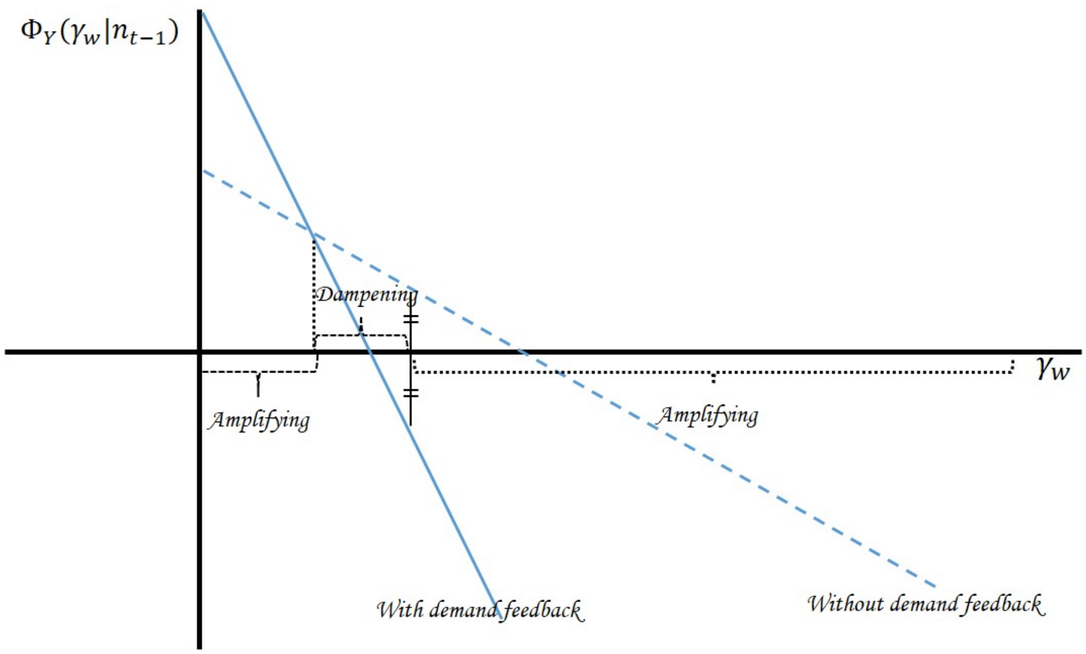


Figure 1: The output policy rule with and without demand feedback

When $\gamma_w < \underline{\gamma_w}$, real wages are so rigid that precautionary saving demand becomes a primary determinant of saving demand. If wages are sufficiently rigid, then labor demand decreases significantly in response to recessionary shocks. Precautionary saving demand increases in response to a sizable rise in unemployment risk. Aggregate demand declines, and, thus, labor demand falls further. Consequently, recessions deepen. Therefore, when wages are sufficiently rigid, the destabilizing demand feedback that originates from countercyclical precautionary saving demand amplifies procyclical output fluctuations.

When $\gamma_w > \overline{\gamma_w}$, demand feedback is expansionary. Two situations are possible. First, wages may be flexible enough to raise labor demand in response to adverse technology shocks in the absence of demand feedback. Since both the current income and unemployment risk decline, saving demand falls unambiguously. A decrease in saving demand generates expansionary demand feedback, which amplifies the impact of shocks on employment and output. On the other hand, wages may not be sufficiently flexible, and downturns occur in response to adverse technology shocks if prices are flexible. However, the expansionary demand feedback driven by the consumption smoothing motive converts recessions to expansions in general equilibrium.

When $\underline{\gamma_w} < \gamma_w < \overline{\gamma_w}$, demand feedback is stabilizing. Wages are rigid enough to cause unemployment risk to rise under flexible prices in response to adverse technology shocks. Despite an increase in precautionary saving demand, the equilibrium real interest rate rises because the consumption smoothing motive dominates. Demand feedback that increases labor demand neutralizes the impact of business cycle shocks on employment and output. Expansionary demand feedback can even result in expansions in response to adverse technology shocks.

**Summary and discussion**

We discuss that the (de)stabilizing impact of demand feedback is closely related to the degree of real wage rigidity. Some comments are worth noting. First, demand feedback alters the threshold wage rigidity that makes each variable invariable to exogenous shocks. I call the new threshold shifted by demand feedback *the threshold under rigid prices*. Second, if we allow fiscal or profit redistribution across agents or suppose an alternative wage-setting assumption such as Nash bargaining, the labor market variables should be affected by the endogenous state variable, i.e., $n_{t-1}$. That is, the labor market condition affects firms' firing and hiring decisions. Finally, policy rules are a function of shock persistence. This is why the dynamics would be shock-dependent once we allow diverse business cycle shocks.

# 4 Empirical evidence and calibration

This section discusses some stylized facts about labor market dynamics in the United States and the calibration strategy. I begin by summarizing the salient features of the labor market in the United States. After that, I discuss calibration in detail. The data and estimation details are presented in the appendix.

## 4.1 Stylized facts on the labor market in the United States

I estimate the time series of job finding probability and job destruction rates using Current Population Survey (CPS) gross flows and unemployment duration data. In addition, the empirical moments of the vacancy-unemployment ratio are computed by using the Job Openings and Labor Turnover Survey (JOLTS).

Figure 2 displays the fluctuations in unemployment and the estimated transition probabilities. The average quarterly job finding probability and the net separation rate are approximately 0.33 and 0.03, respectively, over the sample periods. During the Great Recession of 2008, the unemployment rate sharply rose because of a large drop in the job finding probability and an increase in the job destruction rate. The unemployment rate has steadily declined since 2010 as the job finding probability has increased, while the job destruction rate has declined. Although the job finding probability shows clear negative comovements with unemployment, the correlation between unemployment and the job destruction rate seems to be elusive. The average vacancy-unemployment ratio from 2001 to 2019 is approximately 0.55. In particular, the vacancy-unemployment ratio fluctuated roughly around 0.65 before the Great Recession, but it sharply fell to below 0.2 during the Great Recession. It started to rebound in 2011 and remained above one after 2018 until the recent outbreak of the COVID-19 pandemic.

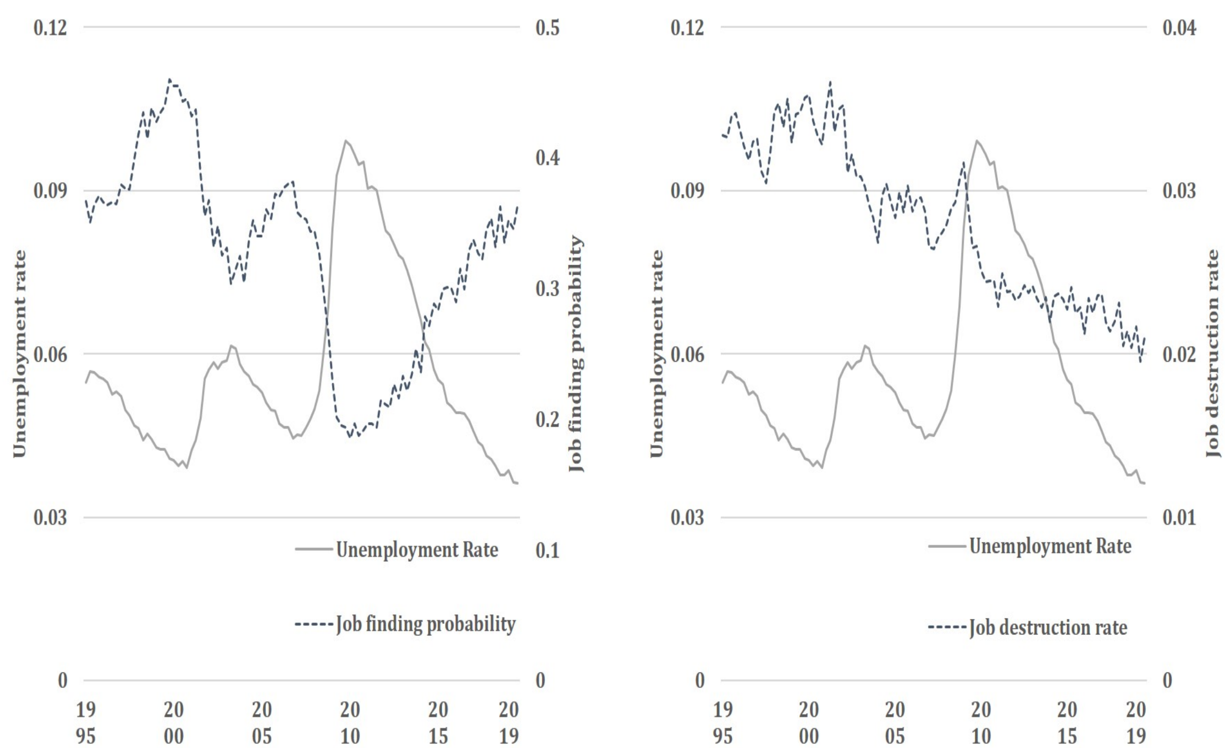


Figure 2: Unemployment and transition probabilities

Table 1: Quarterly summary statistics, U.S. labor market

| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $Y$ |
|---|---|---|---|---|---|---|---|
| Standard deviation | | 0.107 | 0.120 | 0.229 | 0.090 | 0.046 | 0.010 |
| Quarterly autocorrelation | | 0.961 | 0.912 | 0.942 | 0.905 | 0.539 | 0.926 |
| Correlation matrix | $u$ | 1 | -0.906 | -0.960 | -0.945 | -0.032 | -0.910 |
| | $v$ | - | 1 | 0.983 | 0.837 | -0.042 | 0.877 |
| | $v/u$ | - | - | 1 | 0.902 | 0.012 | 0.892 |
| | $f$ | - | - | - | 1 | 0.274 | 0.859 |
| | $F$ | - | - | - | - | 1 | -0.106 |
| | $Y$ | - | - | - | - | - | 1 |

*Note*: All variables are reported in logs as a deviation from the HP trend with a smoothing parameter of 1,600. Due to data availability, the statistics related to the vacancy and the vacancy-unemployment ratio are computed from Q1 2001 to Q3 2019. The other statistics are based on data that range from Q1 1995 to Q3 2019.

Table 1 summarizes the business cycle statistics of selected variables, namely, unemployment $(u)$, vacancy $(v)$, vacancy-unemployment ratio $(v/u)$, job finding probability $(f)$, job destruction rate $(F)$, and real GDP per capita $(Y)$. The estimation results are consistent with the well-known features of the labor market in the United States. For example, the vacancy-unemployment ratio is twice as volatile as unemployment and vacancy. The job finding probability is 2.5 times more volatile than the job destruction rate. Unemployment and vacancy show a strong negative correlation. The job finding probability is highly procyclical, while the job destruction rate is nearly acyclical and stable. The correlation between the job finding probability and the job destruction rate is weakly positive. The business cycle properties imply that unemployment risk (i.e., the likelihood of unemployment) fluctuates countercyclically. Furthermore, countercyclical fluctuations in unemployment risk are primarily driven by procyclical fluctuations in the job finding probability.

## 4.2 Calibration

I solve the model using a first-order perturbation method. The model period is one quarter. External calibration refers to the estimates in the literature. Some parameters are jointly computed to attain designated steady-state targets. The remaining parameters are internally calibrated through a stochastic simulation to match target second-order moments in the U.S. labor market. Table 2 summarizes the calibration results.

Table 2: Calibration targets and parameter values

| | | |
|---|---|---|
| Steady-state targets | | |
| $\Pi$ | 1 | Gross inflation rate |
| $R^4 - 1$ | 0.04 | Annual nominal interest rate |
| $F$ | 0.032 | Job destruction rate |
| $f$ | 0.378 | Job fining rate |
| $v/u$ | 0.65 | Vacancy-Unemployment ratio |
| $c_u/c_w$ | 0.89 | Consumption loss upon unemployment (11%) |
| $\eta/(\eta-1)-1$ | 0.143 | Steady-state markups ($\approx 15\%$) |
| $\kappa/\Omega$ | 0.448 | Hiring cost relative to separation costs ($\approx 50\%$) |
| | 4 | Average price duration (Calvo equivalent, Quarters) |
| Parameter values | | |
| $\sigma$ | 2 | Coefficient of relative risk aversion |
| $\beta$ | 0.985 | Time discount rate |
| $\gamma$ | 0.5 | Matching function elasticity |
| $\alpha$ | 0.4 | Tax on personal income / Total tax revenue |
| $\kappa$ | 0.04 | Vacancy posting cost |
| $\Psi$ | 80 | Price adjustment cost |
| $\gamma_R$ | 1.5 | Interest rate rule on inflation |
| $\eta$ | 8 | Elasticity of substitution between intermediates |
| $\tilde{r}$ | 0.89 | Relative consumption upon unemployment |
| $\Omega$ | 0.0894 | Separation cost |
| $\zeta$ | 0.6171 | Worker bargaining power |
| $\chi$ | 0.6141 | Matching function scale parameter |
| $\rho_A$ | 0.9 | Autocorrelation of technology shocks |
| $\rho_\eta$ | 0.75 | Autocorrelation of markup shocks |
| $\sigma_A$ | 0.0038 | Standard deviation of technology shocks |
| $\sigma_\eta$ | 0.31 | Standard deviation of markup shocks |
| $\mu_a$ | -0.186 | Idiosyncratic productivity shock parameter (average) |
| $\sigma_a$ | 0.103 | Idiosyncratic productivity shock parameter (standard deviation) |

**External parameters**

I set the relative risk aversion, $\sigma$, to 2, which lies in the mid-range of the empirical estimates in the literature (Mankiw et al. 1985, Attanasio and Weber 1995, Crump et al. 2015, Havranek 2015). Because households cannot save in equilibrium but consume all of their income, $\tilde{r}$ should not be interpreted merely as an unemployment insurance (UI) replacement rate. As in Ravn and Sterk (2017), I suppose that households cut their consumption by 11 percent upon unemployment, $\tilde{r} = 0.89$. [9]. The elasticity of substitution between intermediates, $\eta$, is set to 8 to indicate that the desired steady-state markups are 15 percent. Following a conventional choice in the new Keynesian literature, the Taylor rule parameter, $\gamma_R$, is set to 1.5. The matching function elasticity to job seekers, $\gamma$, is set to 0.5, which lies in the estimation range suggested in Petrongolo and Pissarides (2001). The Rotemberg price adjustment cost is calibrated to match a price adjustment frequency of 4 quarters, which gives $\Psi = 80$ [10]. Finally, the vacancy posting cost, $\kappa$, is calibrated so that the dismissal of a worker is roughly twice as expensive as the cost of posting a vacancy. The results are robust to this choice of the vacancy posting cost.

**Steady-state targets and indirectly calibrated parameters**

I calibrate the steady-state job finding rate, job destruction rate, and vacancy-unemployment ratio to their pre-crisis averages, specifically, $F = 0.032$, $f = 0.378$ and $\frac{v}{u} = 0.65$, respectively. The separation cost, worker bargaining power, and scale parameter for the matching function are computed to be consistent with these targets. As taxes on personal income account for roughly 40 percent of the total tax revenue in the United States [11], I suppose that $\alpha = 0.4$ [12]. The model is approximated at the zero inflation steady-state, and the steady-state annual nominal interest rate is supposed to be 4 percent. The discount rate is computed accordingly with the Euler equation. The idiosyncratic productivity distribution approximates the empirical cumulative distribution function of the net compensation to employees in the United States for 2012. The estimation gives $\mu_a = -0.186$ and $\sigma_a = 0.103$.

[9] This target is consistent with the estimates in the empirical literature on the impact of unemployment on household spending. One can refer to Hurd and Rohwedder (2010) or Chodorow-Reich and Karabarbounis (2016) for a discussion of the impact of unemployment on household consumption in the United States.

[10] I derived the log-linearized Phillips curve implied by the model and adjusted the slope attached to the marginal cost to be consistent with the Phillips curve under Calvo price rigidity in which the expected duration of the nominal price is four quarters.

[11] OECD (2020), Tax on personal income (indicator).

[12] Under this value, corporate consumption accounts for 10 percent of aggregate consumption in the steady-state.

**Internally calibrated parameters**

I calibrate the degree of wage inertia and the shock processes to match the empirical labor market business cycle features in the United States as closely as possible. The primary target moments are the following.

1. The cyclicality and volatility of the job destruction and finding rates
2. The volatility of unemployment $(u)$, vacancy $(v)$, and the $v/u$ ratio
3. The auto-correlation and volatility of output

Considering output autocorrelation, I fix the persistence of technology shocks at 0.9. I also suppose the autocorrelation of markup shocks to be 0.75, which suggests that the half-life of the shock is roughly two and a half quarters. Next, the shock size and the degree of wage inertia are jointly adjusted to reconstruct the second-order target moments. Concretely, the degree of wage inertia controls the cyclicality and the relative volatility across variables. Conditional on the wage inertia, the absolute volatility of the variables is adjusted by the size of business cycle shocks. I find that $\sigma_A = 0.0038$, $\sigma_\eta = 0.31$, and $\gamma_w = 0.43$ give reasonably good matches to the target moments. The markup shock accounts for approximately 25 percent of the total output fluctuations, conditional on the calibrated wage inertia.

# 5 Quantitative analysis: Nash wage bargaining

In this section, I quantitatively examine the baseline case with Nash wage bargaining. To clarify the impact of real wage rigidity, I examine three cases subject to different levels of wage inertia: $\gamma_w \in \{0, 0.43, 0.88\}$. The case of $\gamma_w = 0$ corresponds to the economy where the equilibrium wage equals the Nash bargaining solution. The case of $\gamma_w = 0.43$ is the calibrated case that matches the empirical labor market dynamics. $\gamma_w = 0.88$ is the highest possible wage inertia up to the second decimal point, which ensures a locally unique and stable solution.

## 5.1 Model evaluation

Table 3: The model statistics *w.r.t* the different levels of wage inertia

| Data | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $Y$ | |
| Standard deviation | | 0.11 | 0.12 | 0.23 | 0.09 | 0.05 | 0.01 | |
| Correlation | $u$ | 1 | -0.91 | -0.96 | -0.95 | -0.03 | -0.91 | |
| | $Y$ | -0.91 | 0.88 | 0.89 | 0.86 | -0.11 | 1 | |
| **Without wage inertia** | | | | | | | | |
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.02 | 0.09 | 0.08 | 0.03 | 0.07 | 0.02 | 0.01 |
| Correlation | $u$ | 1 | 0.80 | 0.72 | 0.72 | 0.81 | 0.78 | 0.64 |
| | $Y$ | 0.64 | 0.24 | 0.15 | 0.14 | 0.30 | 0.25 | 1 |
| **Calibrated wage inertia** | | | | | | | | |
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.11 | 0.12 | 0.21 | 0.08 | 0.01 | 0.02 | 0.01 |
| Correlation | $u$ | 1 | -0.63 | -0.89 | -0.89 | 0.81 | -0.87 | -0.63 |
| | $Y$ | -0.63 | 0.48 | 0.61 | 0.61 | -0.11 | 0.66 | 1 |
| **Highest wage inertia** | | | | | | | | |
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.90 | 0.61 | 1.16 | 0.46 | 0.55 | 0.00 | 0.05 |
| Correlation | $u$ | 1 | -0.16 | -0.86 | -0.86 | 0.86 | -0.47 | -0.98 |
| | $Y$ | -0.98 | 0.12 | 0.82 | 0.82 | -0.83 | 0.47 | 1 |

*Note*: The variables are reported in logs. The standard deviation of the variables measures the deviation from its deterministic steady-states. The simulated business cycle statistics are based on 1,000 simulations over a 100-quarter horizon.

Table 3 reports the simulated second-order moments. First, the model cannot reproduce basic business cycle properties without imposing wage inertia. The procyclical job destruction rate is nearly two times more volatile than the job finding probability. Both the job destruction rate and the job finding rate show positive comovements with unemployment. Vacancy and unemployment are positively correlated, and the volatility of vacancies, unemployment, and the vacancy-unemployment ratio is small. The job destruction rate is more volatile than labor market tightness (and, thus, job finding probability). According to the intuition from the previous section, labor costs are not sufficiently rigid; therefore, vacancies and unemployment are quite stable. The result echoes Shimer (2005) and Krause and Lubik (2007) and indicates that a standard matching model with Nash wage bargaining may need to include real wage rigidity to improve the empirical performance of the model unless we consider additional market frictions such as convex hiring costs or alternative calibration [13].

The calibrated case (i.e., $\gamma_w = 0.43$) shows fairly good matches to the data. Unemployment is highly countercyclical, and vacancies are procyclical. The vacancy-unemployment correlation is reasonably negative. The job destruction rate is stable and weakly countercyclical. In contrast, the job finding probability is procyclical and much more volatile than the job destruction rate. Following the metric in Shimer (2012), I observe that the job destruction rate accounts for approximately 5 percent of the aggregate unemployment fluctuations under this level of wage inertia. This result is consistent with Shimer's estimate.

Under the highest level of wage inertia, the economy becomes extremely unstable. The shadow cost of labor is almost constant and yields strongly countercyclical unemployment fluctuations. The job destruction rate is even more volatile than the job finding probability and is strongly countercyclical. Massive job destruction, which lowers labor market tightness, produces incentives for firms to post vacancies during recessions. It neutralizes the adverse impact of shocks on job creation. Consequently, vacancies show weak procyclicality and a slightly negative correlation with unemployment.

When wages are very rigid, the model violates the Blanchard-Khan condition due to indeterminacy. As Ravn and Sterk (2017) point out, indeterminacy under substantial wage rigidity is a typical feature in this frictional environment. Rigid wage arrangements cause sizable procyclical fluctuations in labor demand, which translate into an effective countercyclical precautionary saving demand. Potent destabilizing supply-demand feedback operates, and consequently, the economy is involved in self-fulfilling boom-bust cycles.

## 5.2 Impulse-response analysis

Figures 3 to 5 illustrate the impulse responses conditional on the different levels of wage inertia. The figures display the dynamics in response to the shocks that diminish the

[13]One can refer to Merz and Yashiv (2007) for a discussion on convex hiring costs or Hagedorn and Manovskii (2008) for a discussion on alternative calibration.

supply-side capacity in a standard new Keynesian model[14]: adverse technology shocks and markup shocks that raise firms' monopolistic power. Appendix A.4 provides the supplementary impulse responses of other variables.

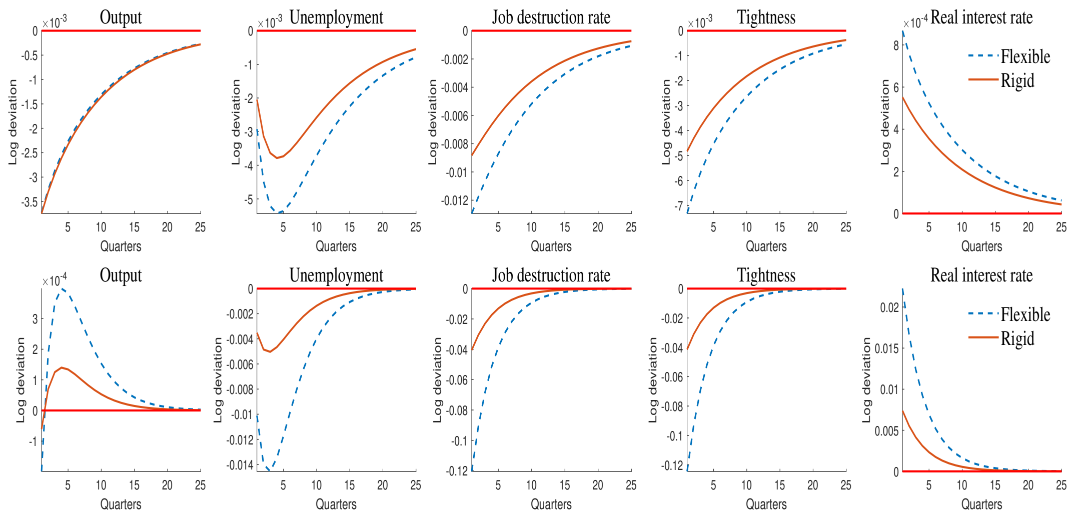


Figure 3: Impulse responses to negative technology shocks (first row) and to positive markup shocks (second row), conditional on $\gamma_w = 0$

Figure 3 displays the case without wage inertia. The dashed blue line illustrates the dynamics without price rigidity, while the solid line displays the dynamics when prices are rigid. Because wages are flexible enough, unemployment declines in response to the allegedly contractionary shocks, regardless of demand feedback. Firms curtail vacancies because shocks reduce the surplus from job creation. However, firms also decrease layoffs because of a sufficient decline in the shadow cost of labor. The dynamics indicate that wage rigidity is slightly lower than $\gamma_\theta$ under rigid prices, which explains why the job destruction rate is much more responsive to shocks than the job finding probability. Although job creation declines, unemployment (and unemployment risk) falls due to a sizable decrease in layoffs. An increase in employment even causes the economy to expand in response to supposedly contractionary markup shocks. The unemployment dynamics suggest that demand feedback dampens the impact of business cycle shocks on

[14]The purpose of the current study is to show how rigid labor cost adjustments affect the impact of demand feedback. The impact of demand feedback on the real economy can be captured by comparing cases with and without price rigidity. Because the real economy is invariable to demand shocks when prices are flexible, demand shocks such as monetary policy shocks cannot clearly illustrate the impact of demand feedback.

(un)employment fluctuations [15]. As demand feedback endogenously curbs an increase in labor demand, output increases by a lesser extent (markup shocks) or decreases more (productivity shocks) than the counterfactual economy with flexible prices.

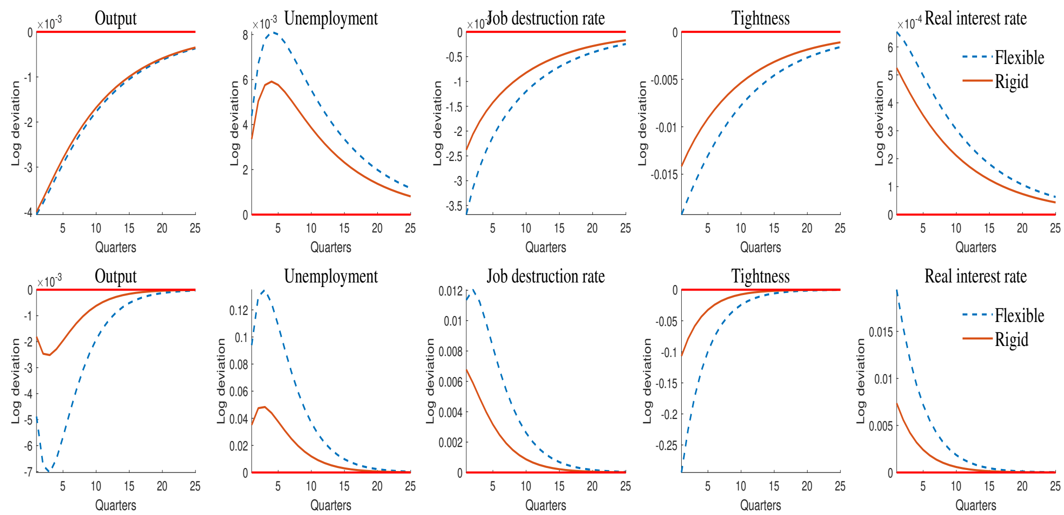


Figure 4: Impulse responses to negative technology shocks (first row) and to positive markup shocks (second row), conditional on $\gamma_w = 0.43$

Figure 4 illustrates the dynamics under the calibrated wage rigidity. Wages are sufficiently rigid to raise unemployment after the shocks. The job finding probability fluctuates procyclically, and a relatively large drop in hiring drives an increase in unemployment in response to recessionary shocks. The dynamics indicate that real wage rigidity is close to $\gamma_{\bar{a}}$ under rigid prices, which implies a relatively more volatile labor market tightness than the separation threshold. Thus, the procyclical job finding probability is more influential than the nearly stable job destruction rate in accounting for unemployment fluctuations. The result is consistent with Shimer (2012). The dampened responses in the economy under rigid prices suggest stabilizing demand feedback. Likewise, the consumption smoothing motive dominates the countercyclical precautionary saving demand.

[15] As the real interest rate responses indicate, saving demand decreases in response to the shocks. After the shocks, flexible wage arrangements lower workers' after-tax income. In addition, the expected decline in unemployment risk dampens precautionary saving demand. Consequently, $\mu_t$ increases in equilibrium as saving demand declines. Unlike the discussion in the previous section, an increase in $\mu_t$ has two effects working in opposite directions under Nash bargaining. On the one hand, it pushes up labor demand as the match surplus increases. On the other hand, it raises the shadow cost of labor and dampens an increase in labor demand due to flexible wage arrangements. Because of the latter effect, we can observe that demand feedback moderates a decline in unemployment, layoffs, and labor market tightness.

Therefore, saving demand fluctuates procyclically, which generates expansionary demand feedback that offsets the contractionary impact of business cycle shocks on employment. As expansionary demand feedback moderates the decreases in employment, output stabilization follows. I find that demand feedback decreases unemployment volatility by 64 percent and output volatility by 41 percent compared to the economy without demand feedback (flexible prices). The model predicts procyclical saving demand under the calibrated wage inertia, and procyclical saving demand results in countercyclical fluctuations in real interest rates to clear the bond market. These model dynamics seem consistent with the cyclicality of real interest rates that are typically countercyclical in data.

The dynamics under the highest level of wage inertia are presented in Figure 5. The shadow cost of labor is almost constant and generates considerable procyclical fluctuations in labor demand. The job destruction rate is strongly countercyclical and as volatile as the job finding probability. Although unemployment risks rise during recessions, demand feedback stabilizes in response to markup shocks as saving demand declines due to the consumption smoothing motive. In contrast, we can observe amplified fluctuations in output and unemployment under rigid prices at the outset of adverse technology shocks. This suggests that destabilizing demand feedback occurs at the beginning of recessions caused by technology shocks. One should note that destabilizing demand feedback co-occurs with an increase in the real interest rate in equilibrium. The contractionary effect of demand feedback at the outset of the adverse technology shocks amplifies a drop in each worker's after-tax income because the tax per worker rises as unemployment increases, while wages decline further as the firm's surplus diminishes. Therefore, contractionary demand feedback eventually leads to a decline in saving demand in general equilibrium due to the consumption smoothing motive. As a result, the real interest rate rises in equilibrium despite destabilizing demand feedback. Instead, destabilizing feedback manifests itself in general equilibrium by the hump-shaped responses in inflation[16]. Although destabilizing demand feedback occurs at the outset of adverse technology shocks, demand feedback turns out to be stabilizing after a few quarters as the consumption smoothing motive

[16]The Phillips curve equation implies that the expected inflation can lower the marginal revenue from production today ($\mu_t$.)

starts dominating the precautionary saving demand. Despite temporal destabilizing effects, demand feedback overall serves as an endogenous stabilizer.

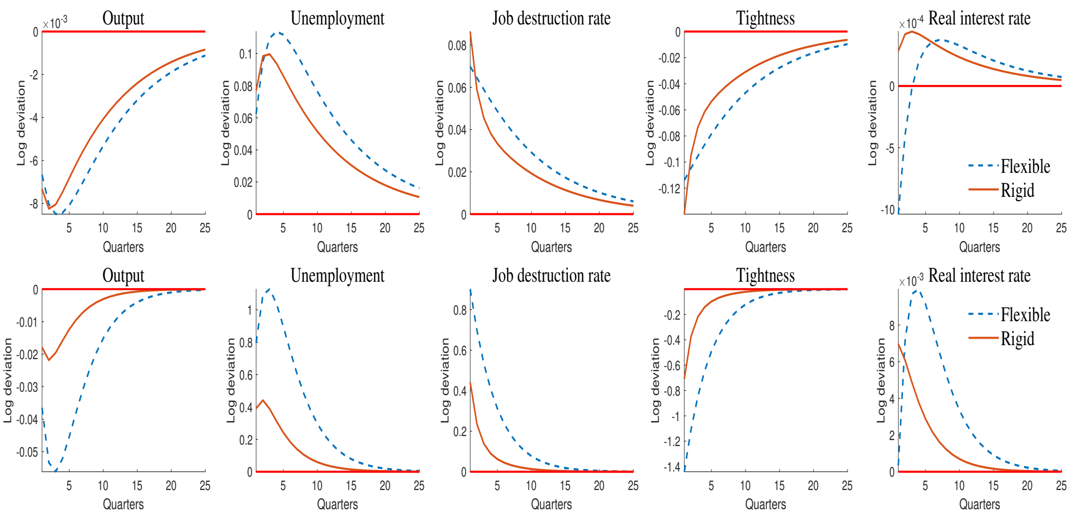


Figure 5: Impulse responses to negative technology shocks (first row) and to positive markup shocks (second row), conditional on $\gamma_w = 0.88$

## 5.3 Sensitivity analysis

### Risk-aversion

This section investigates the robustness of the main findings under alternative calibrations. I begin by examining the case of higher risk aversion. Higher risk aversion tends to stimulate countercyclical precautionary saving demand against unemployment risks, but it also reinforces the incentive to smooth consumption as the intertemporal elasticity of substitution declines. Given other baseline parameter values, I find that demand feedback is still stabilizing under higher risk aversion ($\sigma = 5$), regardless of the level of wage inertia. For instance, conditional on the baseline calibration, including wage inertia, demand feedback under $\sigma = 5$ decreases unemployment volatility by 82 percent and output volatility by 72 percent compared to its counterpart with flexible prices.

The baseline wage inertia, together with higher risk aversion, cannot satisfactorily account for the relative volatility and cyclicality of the variables of interest in the U.S. labor market. Under $\sigma = 5$, wages should be more flexible to match the relative volatility and

Table 4: The model statistics under $\sigma = 5$ and $\gamma_w = 0.18$

| Business cycle statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.12 | 0.12 | 0.21 | 0.08 | 0.03 | 0.02 | 0.01 |
| Correlation | $u$ | 1 | -0.55 | -0.89 | -0.89 | 0.88 | -0.85 | -0.50 |
| | $Y$ | -0.50 | 0.41 | 0.52 | 0.52 | -0.15 | 0.63 | 1 |

| Labor market volatility | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Flexible prices | 0.69 | 0.66 | 1.19 | 0.46 | 0.14 | 0.10 | 0.04 |
| Rigid prices | 0.12 | 0.12 | 0.21 | 0.08 | 0.03 | 0.02 | 0.01 |

correlation across variables. For instance, I find that $\gamma_w = 0.18$ yields better matches to the target moments (See Table 4) [17]. In addition, Table 4 compares the standard deviation of the variables between the economies with flexible and rigid prices. The results suggest stabilizing effects of demand feedback under the new calibration associated with higher risk aversion.

**Consumption upon unemployment**

The baseline calibration supposes that unemployment reduces consumption by 11 percent compared to employees' consumption. Alternatively, I suppose that unemployment decreases consumption by 55 percent, i.e., $\tilde{r} = 0.45$. This target corresponds to the unemployment insurance replacement rate in the United States. A large drop in consumption upon unemployment can induce the precautionary saving demand to be more sensitive to unemployment risk fluctuations of the same size. Therefore, although procyclical labor demand fluctuations might not be sufficiently extensive, precautionary saving demand against unemployment risk can be potent in this case.

Despite a more significant consumption drop upon unemployment, demand feedback is still stabilizing under the range of wage inertia consistent with the empirical labor market

[17] The shock sizes should be adjusted to match the absolute volatility of the selected variables under the new calibration of risk aversion and wage inertia. I find that the model with higher risk aversion and lower wage inertia can match the targeted business cycle moments reasonably well when both $\sigma_A$ and $\sigma_\mu$ are increased proportionally by 75 percent from its baseline levels.

Table 5: The model statistics under $\tilde{r} = 0.45$ and $\gamma_w = 0.41$

| Business cycle statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.11 | 0.14 | 0.23 | 0.09 | 0.01 | 0.02 | 0.01 |
| Correlation | $u$ | 1 | -0.65 | -0.89 | -0.89 | 0.80 | -0.88 | -0.65 |
| | $Y$ | -0.65 | 0.48 | 0.62 | 0.62 | -0.14 | 0.66 | 1 |

| Labor market volatility | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Flexible prices | 0.30 | 0.35 | 0.60 | 0.23 | 0.03 | 0.06 | 0.02 |
| Rigid prices | 0.11 | 0.14 | 0.23 | 0.09 | 0.01 | 0.02 | 0.01 |

dynamics. For instance, given other baseline parameters but the lower UI replacement rate ($\tilde{r} = 0.45$), the economy with $\gamma_w = 0.41$ is consistent with the data (See Table 5). The table shows that the economy is less volatile when demand feedback is relevant.

**The extent of fiscal redistribution**

The effects of fiscal redistribution from workers to unemployed households are ambiguous. First, the tax per worker fluctuates countercyclically due to countercyclical total unemployment insurance spending, which amplifies the procyclical fluctuations in workers' after-tax income. Therefore, fiscal redistribution from workers to the unemployed may intensify the procyclical consumption smoothing motive. Second, countercyclical fiscal redistribution hampers procyclical wage adjustments under bilateral wage bargaining. Intuitively, when workers are liable for higher taxes during recessions, their welfare surplus from employment declines, and it induces them to insist on higher wages. Specifically, countercyclical fiscal redistribution from workers to the unemployed increases real wage rigidity. It intensifies precautionary saving demand against unemployment risk as countercyclical fluctuations in unemployment risk are amplified. Finally, precautionary savers (workers) have a lower marginal propensity to consume than unemployed households in my model environment. Thus, fiscal redistribution from workers to unemployed households has a distributional effect, which is supposed to stabilize an economy in the absence of the contractionary supply-side externality via the wage cost channel mentioned above.

Because the net effect of fiscal redistribution in general equilibrium is ambiguous, I investigate alternative economies such that $\alpha = 0$ and $\alpha = 0.9$, respectively. I find that the consumption smoothing motive still outweighs the precautionary saving demand for both cases under the wage inertia calibrated to match the second-order moments in the U.S. labor market. For instance, Table 6 reports the model statistics under $\alpha = 0$ and $\gamma_w = 0.45$. We can observe that the economy is less volatile when we allow demand feedback. The case of more extensive fiscal redistribution ($\alpha = 0.9$) is similar.

Table 6: The model statistics under $\alpha = 0$ and $\gamma_w = 0.45$

| Business cycle statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.11 | 0.13 | 0.22 | 0.08 | 0.01 | 0.02 | 0.01 |
| Correlation | $u$ | 1 | -0.65 | -0.89 | -0.89 | 0.74 | -0.89 | -0.64 |
| | $Y$ | -0.64 | 0.49 | 0.61 | 0.61 | -0.06 | 0.67 | 1 |

| Labor market volatility | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Flexible prices | 0.31 | 0.36 | 0.61 | 0.23 | 0.03 | 0.06 | 0.02 |
| Rigid prices | 0.11 | 0.13 | 0.22 | 0.08 | 0.01 | 0.02 | 0.01 |

**Directly calibrated wage stickiness**

The baseline calibration indirectly adjusts the degree of wage inertia to reconstruct the (relative) volatility and cyclicality of selected variables. Alternatively, this section directly calibrates the wage inertia to match a cyclical property of real wages in the data. In particular, Hagedorn and Manovskii (2008) estimate the cyclicality of real wages and find that a one percentage point increase in labor productivity is associated with a 0.449 percentage point increase in real wages[18].

[18] They measure labor productivity using seasonally adjusted quarterly real average output per person in the nonfarm business sector constructed by the Bureau of Labor Statistics. The data covers from Q1 1951 to Q4 2004. For estimation, Both time series of real wages and labor productivity are reported in logs and are detrended using an HP filter with a smoothing parameter of 1,600.

The quarterly real average output per worker ($p_t$, henceforth) in my model economy can be written as $p_t \equiv A_t \frac{\int_{\bar{a}_t} a dF(a)}{1-F_t}$. Thus, I find the level of wage inertia that yields

$$d\log(\frac{w_t}{w})/d\log(\frac{p_t}{p}) \approx 0.45$$

where $w$ and $p$ represent the steady-state real wages and labor productivity, respectively[19]. I observe that the elasticity of real wages to labor productivity equals 0.45 when $\gamma_w = 0.657$ [20]. Table 7 reports the model statistics under the directly calibrated wage inertia[21]. Although the model associated with the directly calibrated wage inertia can roughly capture the cyclical behavior of the selected variables, it shows limited accuracy for some target moments. For instance, the job destruction rate shows quite strong countercyclicality, while the vacancy shows too low countercyclicality. In addition, the job finding probability is just 1.2 times more volatile than the job destruction rate. Table 7 also suggests that the directly calibrated wage inertia is still not rigid enough to cause the destabilizing process. Similarly, the economy with rigid prices is less volatile than its counterpart with flexible prices, indicating stabilizing effects of demand feedback.

## 5.4 A caveat when assuming exogenous job destruction

It is currently common in the macroeconomic modeling of a labor market to rule out endogenous job destruction while calibrating outflow hazard rates to reproduce unemployment fluctuations observed in data. The literature that addresses the destabilizing mechanism takes a similar modeling shortcut[22]. This section discusses a caveat to this modeling approach. To this end, I suppose an economy in which job destruction occurs exogenously

[19] The wage inertia is computed based on the simulated time series over a 10,000 quarter horizon. The initial 1,000 periods are discarded.

[20] The baseline calibrated wage inertia, $\gamma_w = 0.43$, indicates $d\log(\frac{w_t}{w})/d\log(\frac{p_t}{p}) \approx 0.798$, which is 1.8 times higher than the estimate of Hagedorn and Manovskii.

[21] Under the baseline calibration of the shock sizes, the economy under the directly calibrated wage inertia is too volatile. Thus, I descale the economy proportionally by reducing the size of both shocks by 45 percent. Again, the extent of wage inertia controls the relative volatility and cyclicality of variables, while the size of shocks controls the absolute volatility of variables.

[22] Ravn and Sterk (2017) suppose constant real wages and exogenous job destruction dynamics for their baseline case. Challe (2019) assumes a constant job destruction rate while imposing a wage inertia close to 1 (0.95). Den Haan et al. (2018) also suppose sufficiently rigid nominal wages while ruling out endogenous job destruction.

Table 7: The model statistics under the directly calibrated wage inertia

| Business cycle statistics: Data | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $Y$ | |
| Standard deviation | | 0.11 | 0.12 | 0.23 | 0.09 | 0.05 | 0.01 | |
| Correlation | $u$ | 1 | -0.91 | -0.96 | -0.95 | -0.03 | -0.91 | |
| | $Y$ | -0.91 | 0.88 | 0.89 | 0.86 | -0.11 | 1 | |

| Business cycle statistics: Directly calibrated wage inertia | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Standard deviation | | 0.15 | 0.11 | 0.22 | 0.09 | 0.07 | 0.01 | 0.01 |
| Correlation | $u$ | 1 | -0.36 | -0.89 | -0.88 | 0.89 | -0.87 | -0.88 |
| | $Y$ | -0.88 | 0.33 | 0.79 | 0.79 | -0.77 | 0.81 | 1 |

| Labor market volatility | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $u$ | $v$ | $v/u$ | $f$ | $F$ | $\lambda$ | $Y$ |
| Flexible prices | 0.43 | 0.28 | 0.59 | 0.23 | 0.17 | 0.02 | 0.02 |
| Rigid prices | 0.15 | 0.11 | 0.22 | 0.09 | 0.07 | 0.01 | 0.01 |

at a constant arrival rate. The model details are presented in Appendix A.3.4.

Conditional on sufficiently high wage inertia ($\gamma_w = 0.98$) and adequate calibration[23], the model generates destabilizing demand feedback that prevails across the entire business cycle. Indeed, one should note that these modeling and calibration approaches are in line with the approaches in the existing studies. Figure 6 shows amplified fluctuations in all variables under rigid prices relative to its flexible price counterpart. I find that demand feedback increases unemployment volatility by 11 percent and output volatility by 8 percent compared to the economy with flexible prices. Moreover, Table 8 illustrates that the model can be calibrated to capture plausible labor market dynamics despite considerable real wage rigidity.

[23] I eliminate the markup shock for simplicity and adjust the size of the technology shock, $\sigma_A = 0.0006$, to match the empirical labor market dynamics conditional on $\gamma_w = 0.98$. Additionally, I limit attention to the case without fiscal redistribution, i.e., $\alpha = 0$. Under the baseline calibration of fiscal redistribution, $\alpha = 0.4$, demand feedback is stabilizing even if real wages are constant. Since the constant wage causes sizable procyclical employment fluctuations, the tax per worker shows considerable countercyclical fluctuations. The after-tax income fluctuates procyclically due to the countercyclical tax, which causes the procyclical consumption smoothing motive to be the main driver of saving demand over the business cycle.

Table 8: Simulated business cycle statistics conditional on exogenous job destruction

| | | $u$ | $v$ | $v/u$ | $f$ | $Y$ |
|---|---|---|---|---|---|---|
| Standard deviation | | 0.12 | 0.12 | 0.23 | 0.09 | 0.01 |
| Correlation | $u$ | 1 | -0.80 | -0.95 | -0.95 | -1.00 |
| | $Y$ | -1.00 | 0.83 | 0.97 | 0.97 | 1 |

*Note*: The variables are reported in logs. The simulated business cycle statistics are based on 1,000 simulations over a 100-quarter horizon.

Regardless of how we treat job destruction dynamics, labor costs should be very rigid to set off the destabilizing mechanism over business cycles. However, when we rule out endogenous job destruction dynamics, it is generally possible to calibrate the model to be consistent with the empirical labor market dynamics, even under the considerable wage inertia necessary to cause the destabilizing process. This is because the exogenous job destruction assumption rules out potentially sizeable countercyclical fluctuations in layoffs under the high level of wage rigidity. The range of wage rigidity consistent with empirical evidence can be dramatically different depending on the way of modeling job destruction, and this yields a fundamentally different implication on the magnitude of countercyclical precautionary saving demand against unemployment risk.

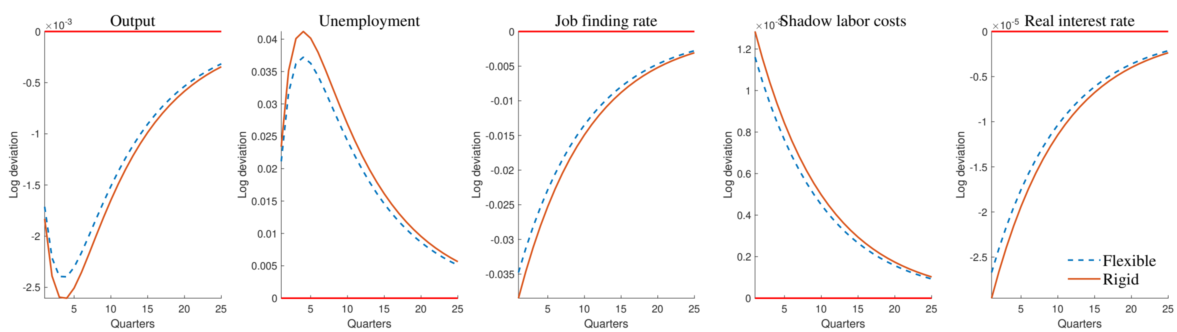


Figure 6: Impulse responses to negative technology shocks under exogenous job destruction

The debatable finding that stable and almost acyclical job destruction rates contribute little to unemployment fluctuations is a general equilibrium outcome. Therefore, this empirical feature cannot be a sufficient rationale when researchers presume exogenous job destruction as innocuous. Instead, empirical evidence may imply that real wages should be neither too flexible nor too rigid. Researchers who assume exogenous job destruction while imposing sufficient real wage rigidity to improve the empirical relevance of the model economy should be aware of this possibility.

# 6 Concluding remarks

This paper investigates the theoretical link between real wage rigidity and the destabilizing mechanism driven by precautionary saving demand against unemployment risk. I show that destabilizing supply-demand feedback is a general equilibrium outcome of rigid labor costs. In addition, I also illustrate that, conditional on the calibrated real wage rigidity, procyclical fluctuations in labor demand are not extensive enough to generate sufficiently potent countercyclical precautionary saving responses. The countercyclical precautionary saving demand against unemployment risk is dominated by the procyclical consumption smoothing saving motive, which renders demand feedback stabilizing.

Empirical evidence may indicate that wages are moderately rigid. If this is the case, then precautionary saving behavior against uninsured unemployment risks alone may not account for the powerful destabilizing spirals in general equilibrium. Instead, destabilizing supply-demand feedback might come from the interactions of additional factors, such as rich household heterogeneity, additional labor market frictions, or endogenous policy reactions with incomplete asset and labor markets. First, as Ravn and Sterk (2017) point out, workers may be heterogeneous in their job search efficiency. Although countercyclical unemployment risk fluctuates moderately, destabilizing demand feedback could be relevant if a number of dismissed workers expect a prolonged unemployment duration. Allowing wealth heterogeneity may also lead to a nontrivial difference in the magnitude of the destabilizing impact of demand feedback under moderate wage rigidity. Second, according to insight in Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2015), the job creation margin might be associated with frictions that make the surplus from job creation much smaller than the surplus from incumbent matches. One possible explanation could be asymmetric worker bargaining power between new employees and incumbent workers. If we impose relevant frictions in the job creation margin, then a model may yield sizable procyclical fluctuations in the job finding probability without generating too much volatility in the job destruction rate. Third, Hagedorn et al. (2013) suggest that the sizable extension in unemployment insurance in the United States could be a reason for a sharp drop in labor market tightness during the Great Recession. Although countercyclical unemployment insurance extension, to some extent, may depress households' incentive for precautionary saving against unemployment risk, it can also amplify procyclical fluctuations in labor demand as long as wages are determined by bilateral bargaining. If the latter effect dominates, cyclical unemployment insurance policy reactions could reinforce the destabilizing impact of demand feedback under moderate wage rigidity. Finally, this paper supposes that cyclical precautionary saving demand is driven exclusively by unemployment risk fluctuations. As the endogenous wedge indicates, the consumption ratio between workers and unemployed households also affects precautionary saving demand. It implies that the countercyclical precautionary saving incentive would be intensified if an economy is associated with countercyclical inequality, i.e., unemployed households become relatively more impoverished during recessions than working households. Identifying the relevant factors that encourage households to engage in countercyclical precautionary saving despite moderate countercyclical fluctuations in unemployment risks may be necessary to strengthen the theory of destabilizing supply-demand feedback. I plan to pursue these issues in future works.

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# A Appendix

## A.1 Data and measurement

### A.1.1 Labor market dynamics in the United States

I estimate the time series of job finding probability and job destruction rates by using Current Population Survey (CPS) gross flows and unemployment duration data. The data are collected monthly and are seasonally adjusted. Because the survey instrument was redesigned in 1994 and it made a nonnegligible discontinuity in the short-term unemployment series (Abraham and Shimer 2001), I restrict the sample period from the first quarter of 1995 to the third quarter of 2019. The detailed estimation strategy is discussed later.

The empirical moments of the vacancy-unemployment ratio are computed by using the Job Openings and Labor Turnover Survey (JOLTS). The data describe the number of job openings in the entire non-farm business sector, which are collected monthly and are seasonally adjusted. Since data are only available after December 2000, vacancy and the vacancy-unemployment ratio are computed over the first quarter of 2001 to the third quarter of 2019.

One of the primary calibration concerns is to precisely capture the cyclicality of the variables. I use the quarterly data for Real GDP per capita, provided by the Bureau of Economic Analysis, to estimate the volatility of output and the cyclicality of the relevant variables. In common with the labor force statistics, the sample covers from the first quarter of 1995 to the third quarter of 2019.

The log-normal distribution of idiosyncratic productivity approximates the empirical distribution of net compensation to employees by using Social Security Administration (SSA) data. The estimation detail is presented below. The data report the net compensation subject to federal income taxes, as reported by employers. The SSA provides data on actual payments to workers by employers, whereas the census data based on the CPS estimates household income. Therefore, the distribution of the net compensation by employers would be a more suitable proxy for the idiosyncratic productivity distribution than the income distribution of households. I approximate the empirical cumulative distribution function of net compensation to employees in the United States for 2012. The reference year is chosen arbitrarily, but the estimation results are generally robust for the year choice.

I detrend the quarterly data by using a Hodrick–Prescott filter with a smoothing parameter of 1600. All monthly series are averaged quarterly.

## A.2 The estimation details

### A.2.1 The estimation of the transition rates

The short-term unemployment of the CPS gross flows and unemployment duration data count the number of workers unemployed less than five weeks in a survey month. Given the data of the employed $(N_t)$, unemployed $(U_t)$, and short-term unemployed $(U_t^s)$, the gross flow of unemployment can be described according to the model in this paper as follows:

$$U_t = (1 - f_t)U_{t-1} + U_t^s$$
$$U_t^s = (1 - f_t)F_t N_{t-1}$$

The short-term unemployed are workers dismissed less than a month ago at a certain date of the survey and are still classified as unemployed. This means that they are participating in job finding activities. Thus, I suppose that the short-term unemployed corresponds to flows specified as $(1 - f_t)F_t N_{t-1}$ rather than $F_t N_{t-1}$. An implicit assumption regarding the structural equations is that workers are fired only once between survey months. However, workers may find a job and be dismissed several times during the same survey month. I treat this case as if firing occurs just once for those workers. Since the model does not allow entry or exit from the labor market and normalizes the entire population to one, the measurement equations are corrected as follows:

$$u_t = g_L(1-f_t)u_{t-1} + u_t^s$$
$$u_t^s = g_L(1-f_t)F_t n_{t-1}$$

where $u_t = \frac{U_t}{L_t}$, $n_t = \frac{N_t}{L_t}$, $u_t^s = \frac{U_t^s}{L_t}$, $g_L = \frac{L_{t-1}}{L_t}$ and $L_t = U_t + N_t$. By using the stream of data for $n_t$, $u_t$ and $u_t^s$, we can recover the job-finding probability from the first equation. Given $u_t^s$, $n_{t-1}$ and $f_t$, we can reconstruct the job destruction rate from the second equation.

### A.2.2 The estimation of the idiosyncratic productivity distribution

The SSA wage data report the number of employers by the level of net compensation, which is classified by 59 class intervals. First, I log-transform the average net compensations in each bin. Next, the empirical distribution of the log wage is standardized to have a unit mean. The distribution parameters, $\mu_a$ and $\sigma_a$, are estimated to produce a minimum distance of the estimated c.d.f from the empirical counterpart.

## A.3 Mathematical appendix

### A.3.1 The proof of proposition 2.1

Given the nominal interest rate $R_t$ and with the condition that $c_{u,t} = \tilde{r}c_{w,t}$, the Euler equations of the employed and unemployed households are written as follows:

$$R_t\mu_{w,t} = c_{w,t}^{-\sigma}\Big[1 - \beta E_t\Big(\frac{1}{\Pi_{t+1}}(P_{w,t+1} + (1-P_{w,t+1})\tilde{r}^{-\sigma})\Big(\frac{c_{w,t+1}}{c_{w,t}}\Big)^{-\sigma}\Big)\Big]$$
$$R_t\mu_{u,t} = b_t^{-\sigma}\Big[1 - \beta E_t\Big(\frac{1}{\Pi_{t+1}}\frac{((1-P_{u,t+1}) + P_{u,t+1}\tilde{r}^{-\sigma})}{\tilde{r}^{-\sigma}}\Big(\frac{c_{w,t+1}}{c_{w,t}}\Big)^{-\sigma}\Big)\Big]$$

From this, we have:

$$\frac{\mu_{w,t}}{\mu_{u,t}} = \tilde{r}^{\sigma}\frac{\Big(1-\beta E_t\Big[\frac{1}{\Pi_{t+1}}(P_{w,t+1} + (1-P_{w,t+1})\tilde{r}^{-\sigma})(\frac{c_{w,t+1}}{c_{w,t}})^{-\sigma}\Big]\Big)}{\Big(1-\beta E_t\Big[\frac{1}{\Pi_{t+1}}(\tilde{r}^{\sigma}(1-P_{u,t+1}) + P_{u,t+1})(\frac{c_{w,t+1}}{c_{w,t}})^{-\sigma}\Big]\Big)}$$

Since $\tilde{r} < 1 < \sigma$, it is clear that

$$\tilde{r}^{\sigma}(1 - P_{u,t+1}) + P_{u,t+1} \leq P_{w,t+1} + (1 - P_{w,t+1})\tilde{r}^{-\sigma}$$

$P_{u,t+1} + \tilde{r}^{\sigma}(1 - P_{u,t+1}) = 1$ if and only if $P_{u,t+1} = 1$. Similarly, $P_{w,t+1} + (1 - P_{w,t+1})\tilde{r}^{-\sigma} = 1$ if and only if $P_{w,t+1} = 1$. Hence, we obtain:

$$\frac{(1 - \beta E_t[\frac{1}{\Pi_{t+1}}(P_{w,t+1} + (1 - P_{w,t+1})\tilde{r}^{-\sigma})(\frac{c_{w,t+1}}{c_{w,t}})^{-\sigma}])}{(1 - \beta E_t[\frac{1}{\Pi_{t+1}}(\tilde{r}^{\sigma}(1 - P_{u,t+1}) + P_{u,t+1})(\frac{c_{w,t+1}}{c_{w,t}})^{-\sigma}])} \leq 1$$

Since $\tilde{r}^{\sigma} << 1$, $\mu_{w,t} < \mu_{u,t}$ follows. □

The equilibrium with any $\mu_{w,t}$ that satisfies $0 \leq \mu_{w,t} < \mu_{u,t}$ can constitute an equilibrium such that $d_t = 0$ for all households. Among the infinite number of feasible equilibrium allocations, the equilibrium such that $\mu_{w,t} = 0$ for all *t* deserves a special note. Suppose that $\mu_{w,t} = \bar{\mu} > 0$, and denote $c_{w,t,1}$, which is the consumption of workers that corresponds to the Lagrange multiplier $\bar{\mu} > 0$. The Euler equation can be written as

$$c_{w,t,1}^{-\sigma} = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} (P_{w,t+1} c_{w,t+1}^{-\sigma} + (1 - P_{w,t+1}) b_{t+1}^{-\sigma}) \Big| \mathbf{S_{t+1}} \right] + R_t \bar{\mu}$$

By keeping the allocation from *t+1* and $R_t$ invariable, suppose an alternative allocation in period *t* such that the Lagrange multiplier takes $\widehat{\mu}$ to satisfy $0 < \widehat{\mu} < \bar{\mu}$, and the new equilibrium consumption, $c_{w,t,2}$, is given as

$$c_{w,t,2}^{-\sigma} = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} (P_{w,t+1} c_{w,t+1}^{-\sigma} + (1 - P_{w,t+1}) b_{t+1}^{-\sigma}) \Big| \mathbf{S_{t+1}} \right] + R_t \widehat{\mu}$$

The new allocation with $\widehat{\mu}$ and $c_{w,t,2}$ is eligible for an equilibrium. The allocation in other periods is unaffected; thus, households can raise the lifetime welfare unambiguously through this marginal adjustment. More generally, as long as $\mu_{w,t}$ is strictly positive, a worker can increase the lifetime utility by reducing $\mu_{w,t}$ while raising $c_{w,t}$ without violating the equilibrium condition. Therefore, among infinite feasible equilibrium allocations, the constrained Pareto optimum that maximizes workers' lifetime utility should satisfy $\mu_{w,t} = 0$, and $c_{w,t}$ is determined by the Euler equation accordingly. That is,

$$c_{w,t}^{-\sigma} = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} (P_{w,t+1} c_{w,t+1}^{-\sigma} + (1 - P_{w,t+1}) b_{t+1}^{-\sigma}) \middle| \mathbf{S_{t+1}} \right]$$

The equilibrium nominal interest rate $R_t^N$ is determined by

$$\frac{1}{R_t^N} = \beta E_t \left[ \frac{1}{\Pi_{t+1}} (\frac{c_{w,t+1}}{c_{w,t}})^{-\sigma} (P_{w,t+1} + (1 - P_{w,t+1}) \tilde{r}^{-\sigma}) \middle| \mathbf{S_{t+1}} \right]$$

Workers who price the bond in equilibrium are indifferent at zero bonds holding under the bond price $\frac{1}{R_t^N}$, which is a global upper bound on the equilibrium bond price.

### A.3.2 The entrepreneur's optimal decision

The risk-neutral entrepreneur's problem can be solved recursively:

$$\begin{aligned}
&\pi_t(n_{t-1}(i), p_{t-1}(i)|\mathbf{S_t}) \\
&= \max_{y_t(i), n_t(i), v_t(i), \bar{a}_t(i), \widehat{a}_t(i), p_t(i)} \frac{p_t(i)}{P_t} y_t(i) - \frac{\Psi}{2}(\frac{p_t(i)}{p_{t-1}(i)} - 1)^2 Y_t - \kappa v_t(i) - w_t n_t(i) \\
&- \Omega_I F(\bar{a}_t(i)) n_{t-1}(i) - \Omega_N F(\widehat{a}_t(i)) \tilde{q}_t v_t(i) + \beta E_t[\pi_{t+1}(n_t(i), p_t(i))|\mathbf{S_t}] - (1 - \alpha) L_t \\
&+ \zeta_t (y_t(i) - (\frac{p_t(i)}{P_t})^{-\eta_t} Y_t) + \mu_t (A_t n_{t-1}(i) \int_{\bar{a}_t(i)} a dF(a) + A_t \tilde{q}_t v_t(i) \int_{\widehat{a}_t(i)} a dF(a) - y_t(i)) \\
&+ \lambda_t (n_t(i) - (1 - F(\bar{a}_t(i))) n_{t-1}(i) - (1 - F(\widehat{a}_t(i))) \tilde{q}_t v_t(i))
\end{aligned}$$

where $\mathbf{S_t}$ denotes the vector of the endogenous and exogenous state variables. Due to the *i.i.d* idiosyncratic productivity shock, firms are symmetric in equilibrium. I do not consider the corner solution cases for $v_t$ or the separation thresholds $\bar{a}_t$ and $\widehat{a}_t$ because they do not bind under a reasonable calibration and in the plausible approximation space.

The first-order conditions are as follows:

$$
\begin{aligned}
y_t &: \mu_t = 1 + \zeta_t \\
v_t &: \kappa = \tilde{q}_t[\mu_t A_t \int_{\widehat{a}_t} adF(a) - \lambda_t(1 - \widehat{F}_t) - \widehat{F}_t \Omega_N] \\
n_t &: -w_t + \beta \frac{\partial E_t[\pi_{t+1}]}{\partial n_t} + \lambda_t = 0 \\
n_{t-1} &: \frac{\partial \pi_t}{\partial n_{t-1}} = -\Omega_I F_t + \mu_t A_t \int_{\bar{a}_t} adF(a) - (1 - F_t)\lambda_t \\
p_{t-1} &: \frac{\partial \pi_t}{\partial p_{t-1}} = \Psi(\Pi_t - 1)\Pi_t Y_t \frac{1}{P_{t-1}} \\
p_t &: Y_t - \Psi(\Pi_t - 1)\Pi_t Y_t + \eta_t \zeta_t Y_t + \beta \frac{\partial E_t[\pi_{t+1}]}{\partial p_t} P_t = 0 \\
\bar{a}_t &: -\Omega_I - \mu_t A_t \bar{a}_t + \lambda_t = 0 \\
\widehat{a}_t &: -\Omega_N - \mu_t A_t \widehat{a}_t + \lambda_t = 0
\end{aligned}
$$

Since entrepreneurs are risk-neutral, $\frac{\partial E_t[\pi_{t+1}]}{\partial p_t} = E_t[\frac{\partial \pi_{t+1}}{\partial p_t}]$ holds. Since $\widehat{a}_t = \bar{a}_t + \frac{\Omega_I - \Omega_N}{\mu_t A_t}$. If $\Omega_I > \Omega_N$, then $\widehat{a}_t$ is strictly greater than $\bar{a}_t$, and $\bar{a}_t = \widehat{a}_t$ if and only if $\Omega_I = \Omega_N$. After arranging the equations, we can obtain the equilibrium conditions in the main text. □

### A.3.3 The proof of proposition 2.2: Nash wage bargaining

#### The firm's match surplus

Since wages are homogeneous regardless of idiosyncratic shocks, a firm takes into account the average match surplus, $J_t$, when bargaining the wage. When a match is over, a firm spends the separation cost $\Omega$, but it obtains the surplus from a vacancy $VC_t$ if it exists. We can write the firm's total surplus from a match $S_t^f$ as follows:

$$
S_t^f \equiv J_t - (\max\{0, VC_t\} - \Omega)
$$

To define a firm's average match surplus, we start from the job destruction condition:

$$
\mu_t A_t \bar{a}_t - w_t + \beta E_t \Big[ -\Omega F_{t+1} + \mu_{t+1} A_{t+1} \int_{\bar{a}_{t+1}} adF(a) - (1 - F_{t+1})\lambda_{t+1} \Big] = -\Omega
$$

$\bar{a}_t$ is the marginal productivity that yields the zero net match surplus. The left-hand side indicates the gross surplus that a firm obtains from the marginal match. If the surplus is smaller than $-\Omega$, then a firm declares separation. The equation suggests that the surplus from a match associated with the idiosyncratic productivity $a$ can be written as follows:

$$J(a) = \mu_t A_t a - w_t + \beta E_t\Big[ - \Omega F_{t+1} + \mu_{t+1} A_{t+1} \int_{\bar{a}_{t+1}} adF(a) - (1 - F_{t+1})\lambda_{t+1}\Big]$$

Since the idiosyncratic productivity is well distributed in each firm, there are $F'(a)n_t$ number of matches associated with the idiosyncratic productivity $a$, which produces the surplus $J(a)$. Thus, the average match surplus, $J_t$, can be computed as

$$J_t = \frac{\int_{\bar{a}_t} J(a)\frac{n_t F'(a)da}{1-F(\bar{a}_t)}}{n_t} = \mu_t A_t \frac{\int_{\bar{a}_t} adF(a)}{1 - F_t} - w_t + \beta E_t\Big[ - \Omega F_{t+1} + \mu_{t+1} A_{t+1} \int_{\bar{a}_{t+1}} adF(a) - (1 - F_{t+1})\lambda_{t+1}\Big]$$

The value of a vacancy is the net gain from posting a vacancy. The job creation condition gives

$$VC_t \equiv \tilde{q}_t\Big[\mu_t A_t \int_{\bar{a}_t} adF(a) - \lambda_t(1 - F_t) - F_t\Omega\Big] - \kappa$$

As long as $v > 0$, $VC_t$ should be zero in equilibrium. The firm's total average match surplus is given by

$$S_t^f = \mu_t A_t \frac{\int_{\bar{a}_t} adF(a)}{1 - F_t} - w_t + \Omega + \beta E_t\Big[ - \Omega F_{t+1} + \mu_{t+1} A_{t+1} \int_{\bar{a}_{t+1}} adF(a) - (1 - F_{t+1})\lambda_{t+1}\Big]$$

From an entrepreneur's optimal condition, $S_t^f = \mu_t \frac{Y_t}{n_t} - \lambda_t + \Omega$ in equilibrium.

**The worker's match surplus**

The worker's surplus from being employed is just the difference between the value functions that correspond to each employment status. Conditional on $\gamma_w = 0$, the surplus of a worker from a match, $S_t^w$, is

$$S_t^w = \Delta V_t = \frac{c_{w,t}^{1-\sigma} - c_{u,t}^{1-\sigma}}{1-\sigma} + \beta E_t\Big[(1-F_{t+1})(1-f_{t+1})S_{t+1}^w\Big]$$

where $c_{w,t} \equiv w_t - \alpha\frac{T_t}{n_t}$. Workers take as a given the lump-sum tax when bargaining the wage.

#### The equilibrium wage

A solution to the Nash wage bargaining problem maximizes the weighted surplus of a match. Formally,

$$w_{n,t} = \underset{w_t}{\operatorname{argmax}}(S_t^w)^\zeta(S_t^f)^{1-\zeta}$$

where $\zeta$ is the worker's bargaining power parameter. The first-order condition is

$$(1-\zeta)S_t^w = \zeta S_t^f c_{w,t}^{-\sigma}$$

By using the equilibrium conditions $S_t^f = \mu_t\frac{Y_t}{n_t} - \lambda_t + \Omega$ and $c_{w,t} = w_t - \alpha\frac{T_t}{n_t}$, we can find the wage equation (22) in the main text. □

### A.3.4 A counterfactual economy: Exogenous job destruction

Job destruction occurs exogenously at a constant arrival rate, but I suppose that dismissals still incur the separation cost. The idiosyncratic productivity shocks are irrelevant. That is, I rule out the productivity dispersion across matches. The entrepreneur's problem is similar:

$$
\begin{aligned}
&\pi_t(n_{t-1}(i), p_{t-1}(i)|\mathbf{S_t}) \\
&= \max_{y_t(i), n_t(i), v_t(i), p_t(i)} \frac{p_t(i)}{P_t} y_t(i) - \frac{\Psi}{2}(\frac{p_t(i)}{p_{t-1}(i)} - 1)^2 Y_t - \kappa v_t(i) - w_t n_t(i) \\
&- \Omega F(n_{t-1}(i) + \tilde{q}_t v_t(i)) + \beta E_t[\pi_{t+1}(n_t(i), p_t(i))|\mathbf{S_t}] - (1-\alpha) L_t \\
&+ \zeta_t(y_t(i) - (\frac{p_t(i)}{P_t})^{-\eta} Y_t) + \mu_t(A_t n_t(i) - y_t(i)) + \tilde{\lambda}_t(n_t(i) - (1-F)(n_{t-1}(i) + \tilde{q}_t v_t(i)))
\end{aligned}
$$

We can find the first-order conditions written as follows:

$$
\begin{aligned}
&\lambda_t = w_t - \frac{\kappa\beta}{\chi} E_t[\theta_{t+1}^{\gamma}] \\
&\kappa = \tilde{q}((1-F)(\mu_t A_t - \lambda_t) - \Omega F) \\
&\Psi(\Pi_t - 1)\Pi_t = 1 - \eta + \eta\mu_t + \beta E_t\Big[\Psi(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\Big] \\
&Y_t = A_t n_t \\
&n_t = (1-F)(n_{t-1} + M_t)
\end{aligned}
$$

where $\lambda_t \equiv \mu_t A_t + \tilde{\lambda}_t$. The other equilibrium conditions are invariable to the baseline case, but we have to replace $F_t$ with the constant job destruction arrival rate $F$. I set $F = 0.032$ that equals the steady-state target in the baseline calibration. The separation cost, $\Omega$, remains the same as in the baseline case. The results are robust for the value of $\Omega$, nonetheless. In particular, the result is robust even for the zero separation cost, which corresponds to the standard matching model with exogenous job destruction.

### A.3.5 The proofs of the propositions and corollaries in Section 3

#### The proof of proposition 3.1

The equations necessary to address the case without demand feedback ($\Psi = 0$) are as follows:

$$\lambda_t = w_t - \beta E_t[\theta^{\gamma}_{t+1}]$$
$$\theta^{\gamma}_t = \mu_t A_t(\int_{\bar{a}_t} adF - (1 - F_t)\bar{a}_t) - \Omega$$
$$w_t = \bar{w}(A_t)^{\gamma_w}$$
$$\lambda_t = \mu_t A_t \bar{a}_t + \Omega$$

The fist-order approximation gives

$$\lambda\hat{\lambda}_t \approx w\hat{w}_t - \beta\gamma E[\theta_{t+1}]$$
$$\lambda\hat{\lambda}_t \approx \mu\bar{a}(\hat{\mu}_t + \hat{\bar{a}}_t + \hat{A}_t)$$
$$\gamma\hat{\theta}_t \approx (1 + \Omega)(\hat{\mu}_t + \hat{A}_t) - \mu(1 - F)\bar{a}\hat{\bar{a}}_t$$
$$\hat{w}_t \approx \gamma_w \hat{A}_t$$

$\hat{x}_t \equiv \frac{x_t - x}{x}$ for the variables $x_t \in \{\bar{a}_t, \lambda_t, \mu_t, \theta_t, w_t, A_t\}$ where $x$ denotes the deterministic steady-state counterpart of each variable. The method of undetermined coefficients gives

$$\lambda\Phi_\lambda = w\gamma_w - \beta\gamma\rho_A\Phi_\theta$$
$$\lambda\Phi_\lambda = \mu\bar{a}(\Phi_\mu + \Phi_{\bar{a}} + 1)$$
$$\gamma\Phi_\theta = (1 + \Omega)(1 + \Phi_\mu) - \mu(1 - F)\bar{a}\Phi_{\bar{a}}$$

where $\hat{x}_t = \Phi_x \hat{A}_t$, and $\rho_A$ denotes the persistence of the technology shock. After some calculation, we can find that

$$\Phi_{\bar{a}} = \frac{w\gamma_w - (1+\Phi_\mu)(\mu\bar{a} + \beta\rho_A(1+\Omega))}{\mu\bar{a}(1-\beta\rho_A(1-F))} \tag{A.1}$$

$$\Phi_\lambda = \frac{w\gamma_w - (1+\Phi_\mu)\beta\rho_A\mu\int_{\bar{a}} adF}{\lambda(1-\beta\rho_A(1-F))} \tag{A.2}$$

$$\Phi_\theta = \frac{-(1-F)w\gamma_w + (1+\Phi_\mu)\mu\int_{\bar{a}} adF}{\gamma(1-\beta\rho_A(1-F))} \tag{A.3}$$

If prices are flexible, then $\Phi_\mu = 0$. The threshold wage rigidity that induces each variable to be invariable to aggregate technology shocks is determined by $\gamma_w$, which results in the policy rule being equal to zero. The threshold wage rigidity of each variable is written as follows:

$$\gamma_\theta = \frac{\mu\int_{\bar{a}} adF}{(1-F)w}$$

$$\gamma_{\bar{a}} = \frac{\mu\bar{a} + \beta\rho_A(1+\Omega)}{w}$$

$$\gamma_\lambda = \frac{\beta\rho_A\mu\int_{\bar{a}} adF}{w}$$

Since $1+\Omega = \mu(\int_{\bar{a}} adF - (1-F)\bar{a})$ and $\beta\rho_A(1-F) < 1$, $\gamma_{\bar{a}} > \gamma_\lambda$ is obvious. Similarly, we can show $\gamma_\theta > \gamma_{\bar{a}}$. From the policy rules, it is obvious that $\left.\frac{\Phi_{\bar{a}}}{\gamma_w}\right|_{\Phi_\mu=0} > 0$, $\left.\frac{\Phi_\lambda}{\gamma_w}\right|_{\Phi_\mu=0} > 0$, and $\left.\frac{\Phi_\theta}{\gamma_w}\right|_{\Phi_\mu=0} < 0$. $\square$

**The proof of corollary 3.1.1**

$u_t = u_t(\bar{a}_t, \theta_t, n_{t-1}) = (1-f_t)(u_{t-1} + F_t n_{t-1})$. By linearizing $u_t$ conditional on $n_{t-1} = n$, $\theta_t = \theta$, $\bar{a}_t = \bar{a}$, the linearized equation gives

$$\hat{u}_t \approx F'(\bar{a})\bar{a}\frac{(n(1-f) + u + Fn)}{u}\hat{\bar{a}}_t - \frac{(1-F)(1-\gamma)}{1-f}\hat{\theta}_t - \frac{(1-F)n}{u+Fn}\hat{n}_{t-1} \tag{A.4}$$

Since $\bar{a}_t$ and $\theta_t$ are affected by $A_t$ but independent of $n_{t-1}$, we can simplify the percentage deviation of $u_t$ from its steady-state $u$ as follows:

$$\hat{u}_t = \Phi_u^A(\gamma_w)\hat{A}_t + \Phi_u^n \hat{n}_{t-1} \tag{A.5}$$

Since $\frac{\partial \Phi_{\bar{a}}}{\partial \gamma_w} > 0$ and $\frac{\partial \Phi_\theta}{\partial \gamma_w} < 0$, equation (A.4) implies that

$$\gamma_w < \gamma_{\bar{a}} \Longrightarrow \Phi_u^A(\gamma_w) < 0$$
$$\gamma_w > \gamma_\theta \Longrightarrow \Phi_u^A(\gamma_w) > 0$$

Since $\Phi_{\bar{a}}$ and $\Phi_\theta$ are continuous in $\gamma_w$, $\Phi_u^A(\gamma_w)$ is also continuous in $\gamma_w$. Since $\Phi_u^A(\gamma_w)$ is a linear function of $\gamma_w$, there exists $\gamma_u$ in the interval $[\gamma_{\bar{a}}, \gamma_\theta]$, which satisfies corollary 3.1.1. □

**The proof of proposition 3.2**

We can find the policy rule of $\mu_t$ by solving the following linearized equations:

$$\hat{R}_t \approx \gamma_R \hat{\Pi}_t \tag{A.6}$$

$$\hat{\Pi}_t \approx \frac{\eta\mu}{\Psi}\hat{\mu}_t + \beta E[\hat{\Pi}_{t+1}] \tag{A.7}$$

$$-\sigma\gamma_w(1-\rho_A)\hat{A}_t \approx \gamma_\Pi \hat{\Pi}_t - E[\hat{\Pi}_{t+1}] + \hat{Z}_{t+1} \tag{A.8}$$

where $Z_t = P_{w,t} + (1-P_{w,t})\tilde{r}^{-\sigma}$. Since $P_{w,t} = 1 - F_t + F_t(1-F_t)$, we can find that

$$\Phi_z = \frac{1-\tilde{r}^{-\sigma}}{Z}(F(1-F)(1-\gamma)\Phi_\theta - 2F'(\bar{a})F\bar{a}\Phi_{\bar{a}}) \tag{A.9}$$

We can solve equations (A.6) to (A.8) by using the method of undetermined coefficients:

$$\begin{aligned}\Phi_\mu &= \frac{-\Psi(1-\beta\rho_A)}{\eta\mu(\gamma_R-\rho_A)}\big(\sigma(1-\rho_A)\gamma_w + \rho_A\Phi_z\big) \\ &= -\frac{\Psi(1-\beta\rho_A)}{\eta\mu(\gamma_R-\rho_A)}\big(\sigma(1-\rho_A)\gamma_w - \frac{(\tilde{r}^{-\sigma}-1)\rho_A F}{P_w + (1-P_w)\tilde{r}^{-\sigma}}((1-F)(1-\gamma)\Phi_\theta - 2F'(\bar{a})\bar{a}\Phi_{\bar{a}})\big)\end{aligned} \tag{A.10}$$

We can obtain proposition 3.2 after arranging equation (A.10). □

**The proof of corollary 3.2.1**

I impose the following parameter restrictions, which generally hold, conditional on reasonable parameter values and plausible steady-state targets.

**Assumption 1** *The parameters and the steady-state equilibrium allocation satisfy*

$$\frac{\Psi(1-\beta\rho_A)}{\eta\mu(\gamma_R-\rho_A)}\frac{(\tilde{r}^{-\sigma}-1)\rho_A F}{(P_w+(1-P_w)\tilde{r}^{-\sigma})}\Big(\frac{(1-F)(1-\gamma)\mu\int_{\bar{a}}adF+\gamma 2F'(\bar{a})(\bar{a}+\beta\rho_A(\int_{\bar{a}}adF-(1-F)\bar{a}))}{\gamma(1-\beta\rho_A(1-F))}\Big)<1$$

**Assumption 2** *Given the steady-state equilibrium allocation, the following inequality holds:*

$$\frac{1-n(f-F)}{n}>\frac{\int_{\bar{a}}adF-(1-F)\bar{a}}{(1-F)\int_{\bar{a}}adF}$$

By jointly solving equations (A.1), (A.3), and (A.10), we can obtain

$$\begin{aligned}\Phi_\mu(\gamma_w)&=-a_1(a_2\gamma_w-a_3(a_4\Phi_\theta-a_5\Phi_{\bar{a}}))\\&=-a_1a_2\gamma_w+a_1a_3a_4(C_\theta-\Phi_\theta^{\gamma_w}\gamma_w+\Phi_\theta^{\mu}\Phi_\mu)-a_1a_3a_5(-C_{\bar{a}}+\Phi_{\bar{a}}^{\gamma_w}\gamma_w-\Phi_{\bar{a}}^{\mu}\Phi_\mu)\end{aligned}$$

where $a_1=\frac{\Psi(1-\beta\rho_A)}{\eta\mu(\gamma_R-\rho_A)}$, $a_2=\sigma(1-\rho_A)$, $a_3=\frac{(\tilde{r}^{-\sigma}-1)\rho_A F}{P_w+(1-P_w)\tilde{r}^{-\sigma}}$, $a_4=(1-F)(1-\gamma)$, $a_5=2F'(\bar{a})\bar{a}$, $C_\theta=\frac{\mu\int_{\bar{a}}adF}{\gamma(1-\beta\rho_A(1-F))}$, $\Phi_\theta^{\gamma_w}=\frac{(1-F)w}{\gamma(1-\beta\rho_A(1-F))}$, $\Phi_\theta^{\mu}=\frac{\mu\int_{\bar{a}}adF}{\gamma(1-\beta\rho_A(1-F))}$, $C_{\bar{a}}=\frac{\mu\bar{a}+\beta\rho_A(1+\Omega)}{\mu\bar{a}(1-\beta\rho_A(1-F))}$, $\Phi_{\bar{a}}^{\gamma_w}=\frac{w}{\mu\bar{a}(1-\beta\rho_A(1-F))}$, and $\Phi_{\bar{a}}^{\mu}=\frac{\mu\bar{a}+\beta\rho_A(1+\Omega)}{\mu\bar{a}(1-\beta\rho_A(1-F))}$. All terms are positive. By solving for $\Phi_\mu$, we can find:

$$\begin{aligned}\Phi_\mu(\gamma_w)&=\frac{1}{1-a_1a_3(a_4\Phi_\theta^{\mu}+a_5\Phi_{\bar{a}}^{\mu})}\Big[-a_1a_2\gamma_w+a_1a_3a_4(C_\theta-\Phi_\theta^{\gamma_w}\gamma_w)-a_1a_3a_5(-C_{\bar{a}}+\Phi_{\bar{a}}^{\gamma_w}\gamma_w)\Big]\\&=\frac{1}{1-a_1a_3(a_4\Phi_\theta^{\mu}+a_5\Phi_{\bar{a}}^{\mu})}\Big[a_1a_3(a_4C_\theta+a_5C_{\bar{a}})-(a_1a_2+a_1a_3(a_4\Phi_\theta^{\gamma_w}+a_5\Phi_{\bar{a}}^{\gamma_w}))\gamma_w\Big]\\&=C_\mu-\Phi_\mu^{\gamma_w}\gamma_w\end{aligned}$$

The equation implies that $C_\mu$ and $\Phi_\mu^{\gamma_w}$ are positive if and only if $a_1a_3(a_4\Phi_\theta^{\mu}+a_5\Phi_{\bar{a}}^{\mu})<1$, which holds due to Assumption 1. □

Define $\gamma_\mu \equiv \frac{C_\mu}{\Phi_\mu^{\gamma_w}}$, which satisfies $\Phi_\mu(\gamma_\mu) = 0$. From $\gamma_\theta = \frac{C_\theta}{\Phi_\theta^{\gamma_w}}$, it is clear that

$$\gamma_\theta - \gamma_\mu = \frac{C_\theta}{\Phi_\theta^{\gamma_w}} - \frac{a_3(a_4 C_\theta + a_5 C_{\bar{a}})}{a_2 + a_3(a_4\Phi_\theta^{\gamma_w} + a_5\Phi_{\bar{a}}^{\gamma_w})} > 0$$

**The proof of proposition 3.3**

From the equation $Y_t = A_t n_t \frac{\int_{\bar{a}} adF}{1-F(\bar{a}_t)}$ and by using the equation (A.4), we can approximate $Y_t$ conditional on $n_{t-1} = n$ and $\theta_t = \theta$ and $\bar{a}_t = \bar{a}$, which gives

$$\begin{aligned}\hat{Y}_t =& \hat{A}_t - \frac{u}{n}(F'(\bar{a})\bar{a}\frac{(n(1-f)+u+Fn)}{u}\hat{\bar{a}}_t - \frac{(1-F)(1-\gamma)}{1-f}\hat{\theta}_t - \frac{(1-F)n}{u+Fn}\hat{n}_{t-1}) \\ &+ \frac{F'(\bar{a})\bar{a}[\int_{\bar{a}} adF - (1-F)\bar{a}]}{(1-F)\int_{\bar{a}} adF}\hat{\bar{a}}_t \end{aligned} \tag{A.11}$$

The equation implies that the policy rule conditional on $n_{t-1}$ can be written as

$$\Phi_Y^A(\gamma_w) = C_Y - \Phi_Y^{\gamma_w}\gamma_w + \Phi_Y^\mu \Phi_\mu(\gamma_w)$$

If $\gamma_w$ is sufficiently low, then policy rules (A.1), (A.3), and (A.5) suggest that output falls in response to adverse technology shocks under flexible prices. That is, $C_Y > 0$. Similarly, the output would rise under flexible prices in response to adverse technology shocks if $\gamma_w$ is sufficiently high. This means that the slope coefficient $\gamma_w$ is negative, i.e., $\Phi_Y^{\gamma_w} > 0$. Lastly, Assumption 2 ensures that $\Phi_Y^\mu$ is strictly positive. I define the stabilizing effect of demand feedback as follows.

**Definition 1** *Demand feedback is **destabilizing** if and only if $|\Phi_Y^A(\gamma_w)| > |C_Y - \Phi_Y^{\gamma_w}\gamma_w|$. Otherwise, it is **stabilizing**.*

$C_Y - \Phi_Y^{\gamma_w}\gamma_w$ denotes the output policy rule conditional on $n_{t-1}$ and flexible prices. Define $\gamma_Y \equiv \frac{C_Y}{\Phi_Y^{\gamma_w}}$. We begin with the case such that $\gamma_Y > \gamma_\mu$. Figure 1 illustrates this case. Since $\gamma_Y > \gamma_\mu$, $|\Phi_Y^A(\gamma_w)| > |C_Y - \Phi_Y^{\gamma_w}\gamma_w|$ for any $\gamma_w$ that satisfies $\gamma_w < \gamma_\mu$. This implies that $\underline{\gamma_w} = \gamma_\mu > 0$. However, since $\Phi_Y^A(\gamma_w)$ is a linear function of $\gamma_w$, a level of wage rigidity $\widehat{\gamma_w}$

exists such that

$$
\begin{aligned}
&C_Y - \Phi_Y^{\gamma_w}\widehat{\gamma_w} + \Phi_Y^{\mu}\Phi_\mu(\widehat{\gamma_w}) < 0 \\
&C_Y - \Phi_Y^{\gamma_w}\widehat{\gamma_w} > 0 \\
&|C_Y - \Phi_Y^{\gamma_w}\widehat{\gamma_w} + \Phi_Y^{\mu}\Phi_\mu(\widehat{\gamma_w})| = |C_Y - \Phi_Y^{\gamma_w}\widehat{\gamma_w}|
\end{aligned}
$$

Since both $\Phi_Y^A(\gamma_w)$ and $\Phi_\mu(\gamma_w)$ are decreasing in $\gamma_w$, $\overline{\gamma_w} = \widehat{\gamma_w}$. The case of $\gamma_Y < \gamma_\mu$ is similar, but the difference is that $\overline{\gamma_w}$ equals $\gamma_\mu$, and a positive $\underline{\gamma_w}$ exists such that $0 < \underline{\gamma_w} < \overline{\gamma_w} = \gamma_\mu$, which satisfies

$$
\begin{aligned}
&C_Y - \Phi_Y^{\gamma_w}\underline{\gamma_w} + \Phi_Y^{\mu}\Phi_\mu(\underline{\gamma_w}) > 0 \\
&C_Y - \Phi_Y^{\gamma_w}\underline{\gamma_w} < 0 \\
&|C_Y - \Phi_Y^{\gamma_w}\underline{\gamma_w} + \Phi_Y^{\mu}\Phi_\mu(\underline{\gamma_w})| = |C_Y - \Phi_Y^{\gamma_w}\underline{\gamma_w}| \quad \square
\end{aligned}
$$

## A.4 Figures

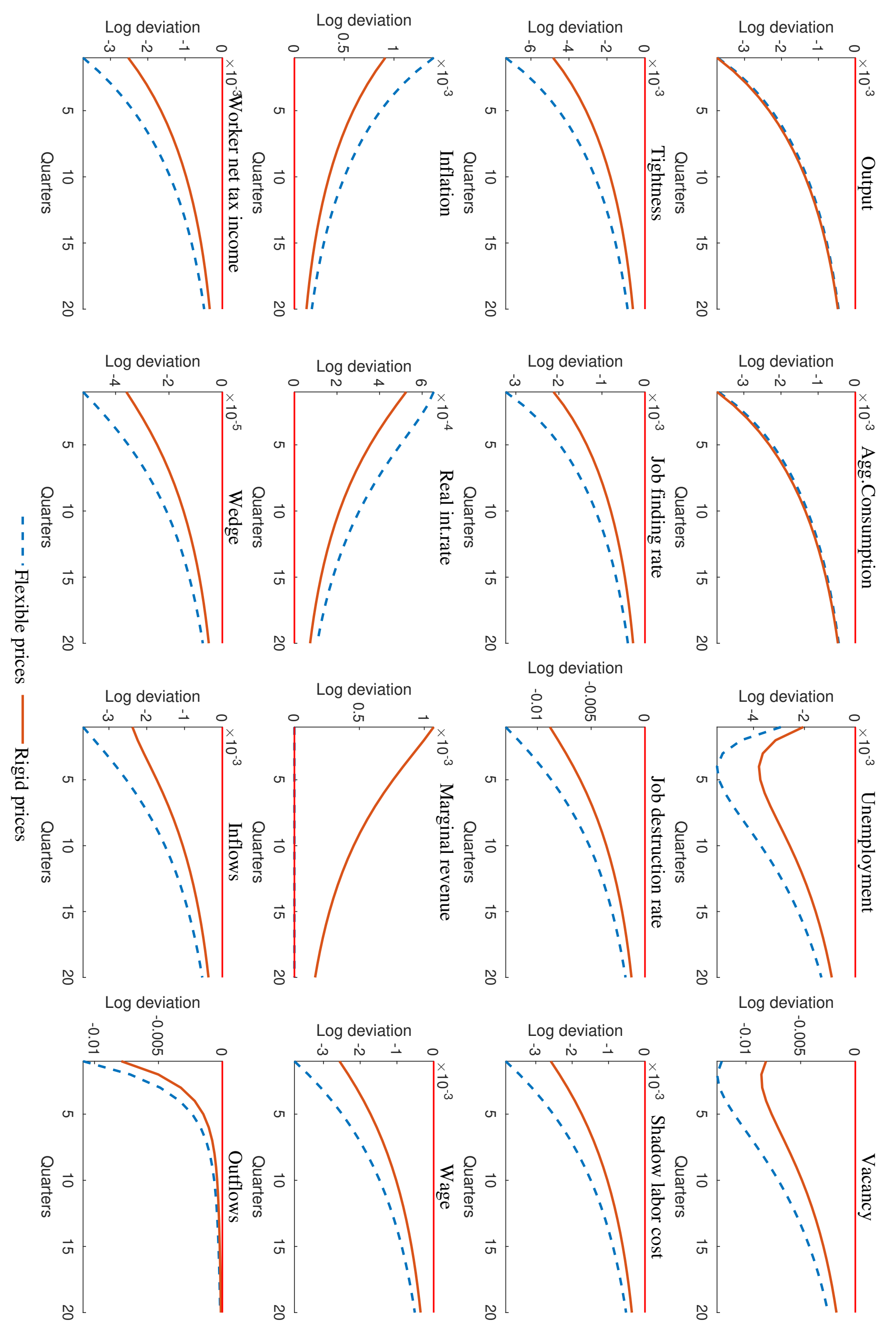


Figure A.1: Impulse-Responses to one standard deviation adverse technology shocks, Without wage inertia

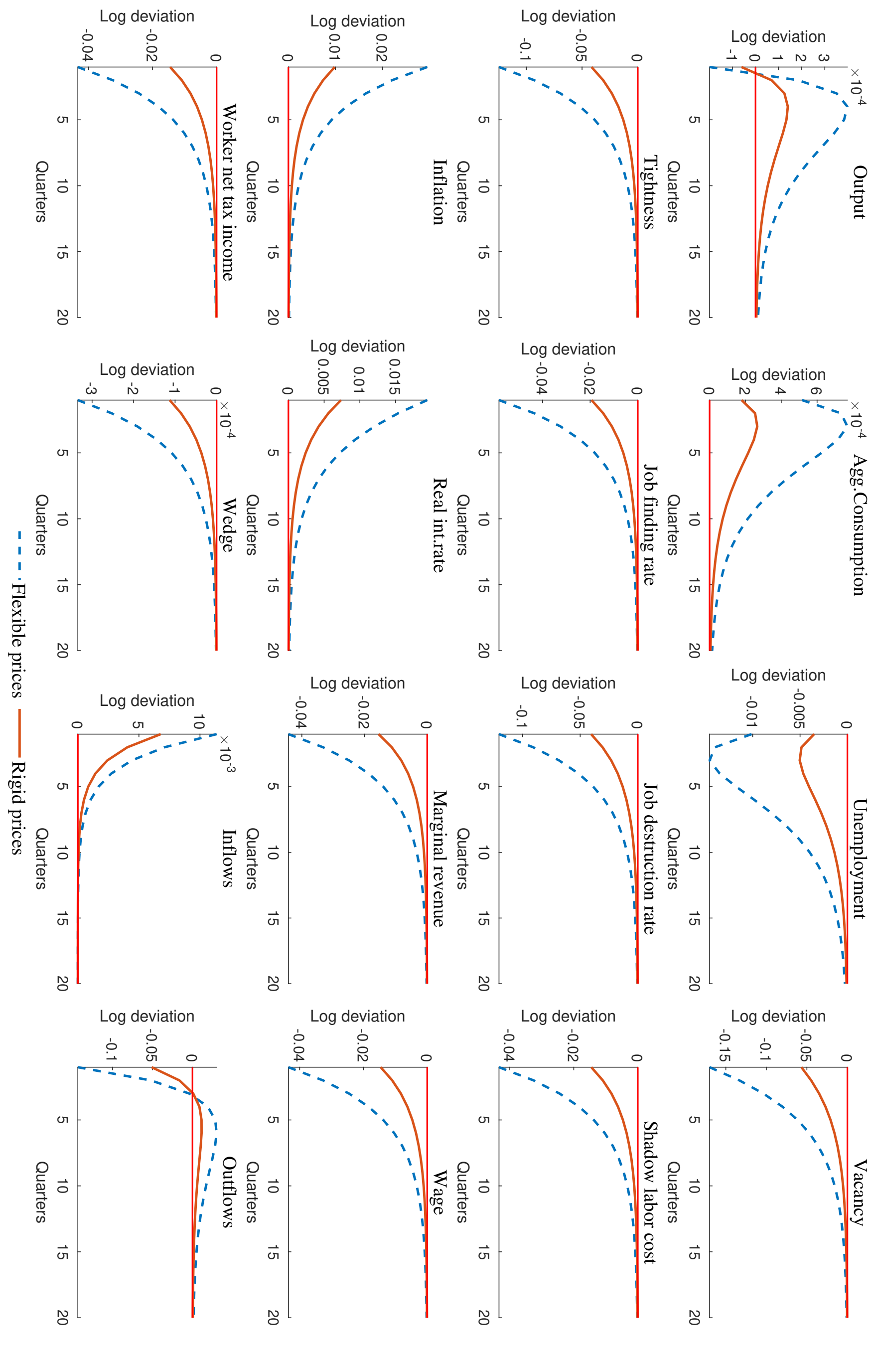


Figure A.2: Impulse-Responses to one standard deviation positive markup shocks, Without wage inertia

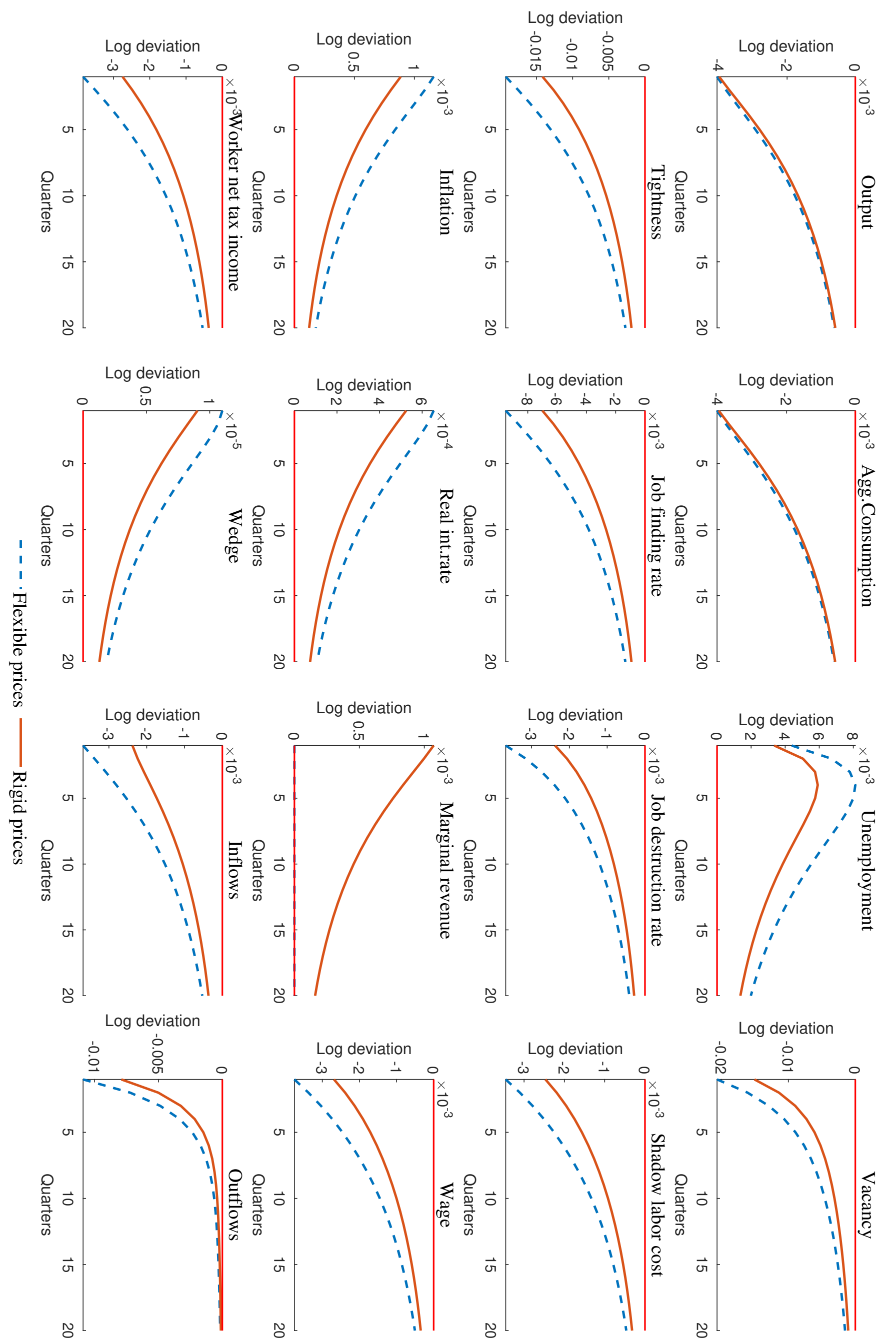


Figure A.3: Impulse-Responses to one standard deviation adverse technology shocks, Calibrated wage inertia

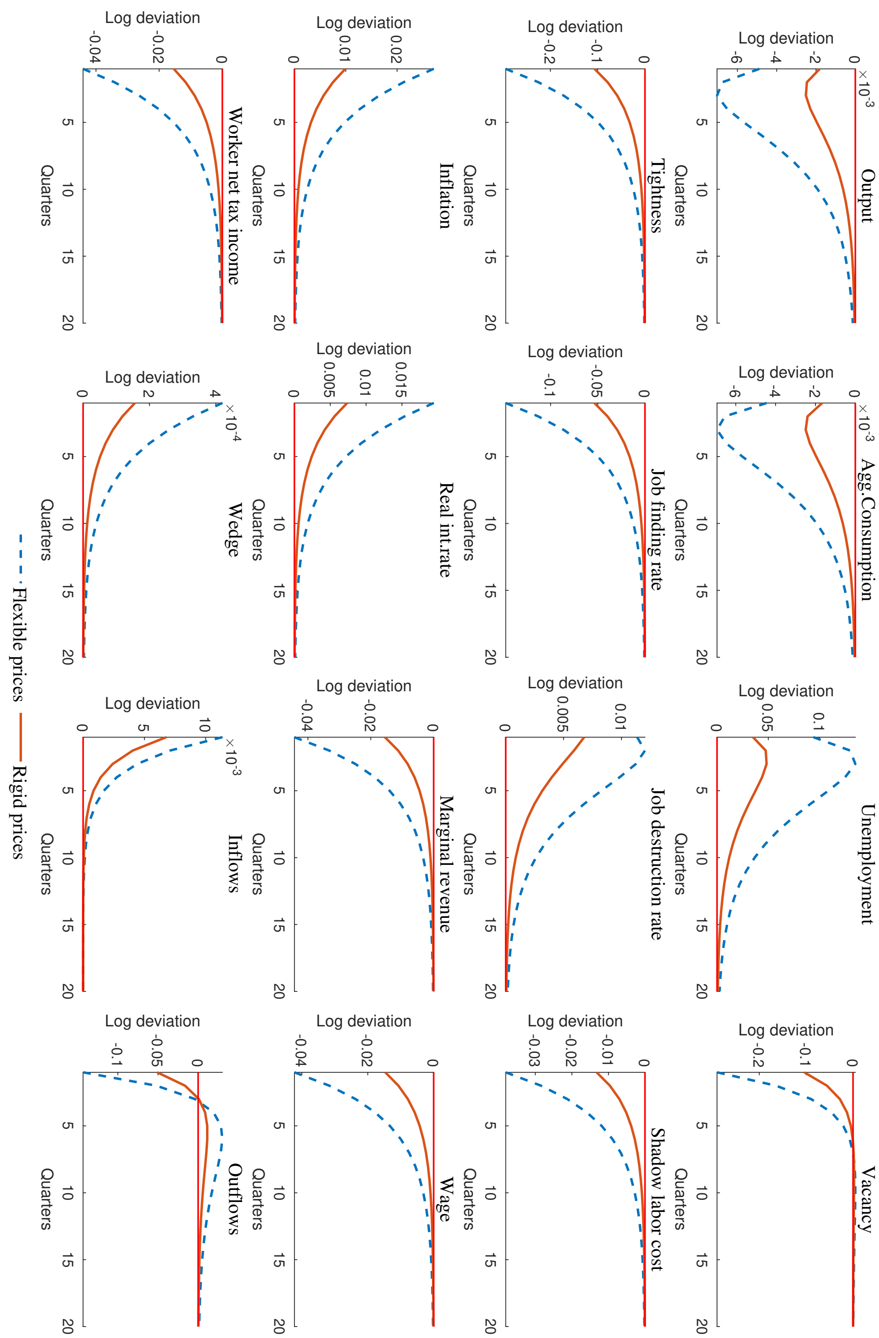


Figure A.4: Impulse-Responses to one standard deviation positive markup shocks, Calibrated wage inertia

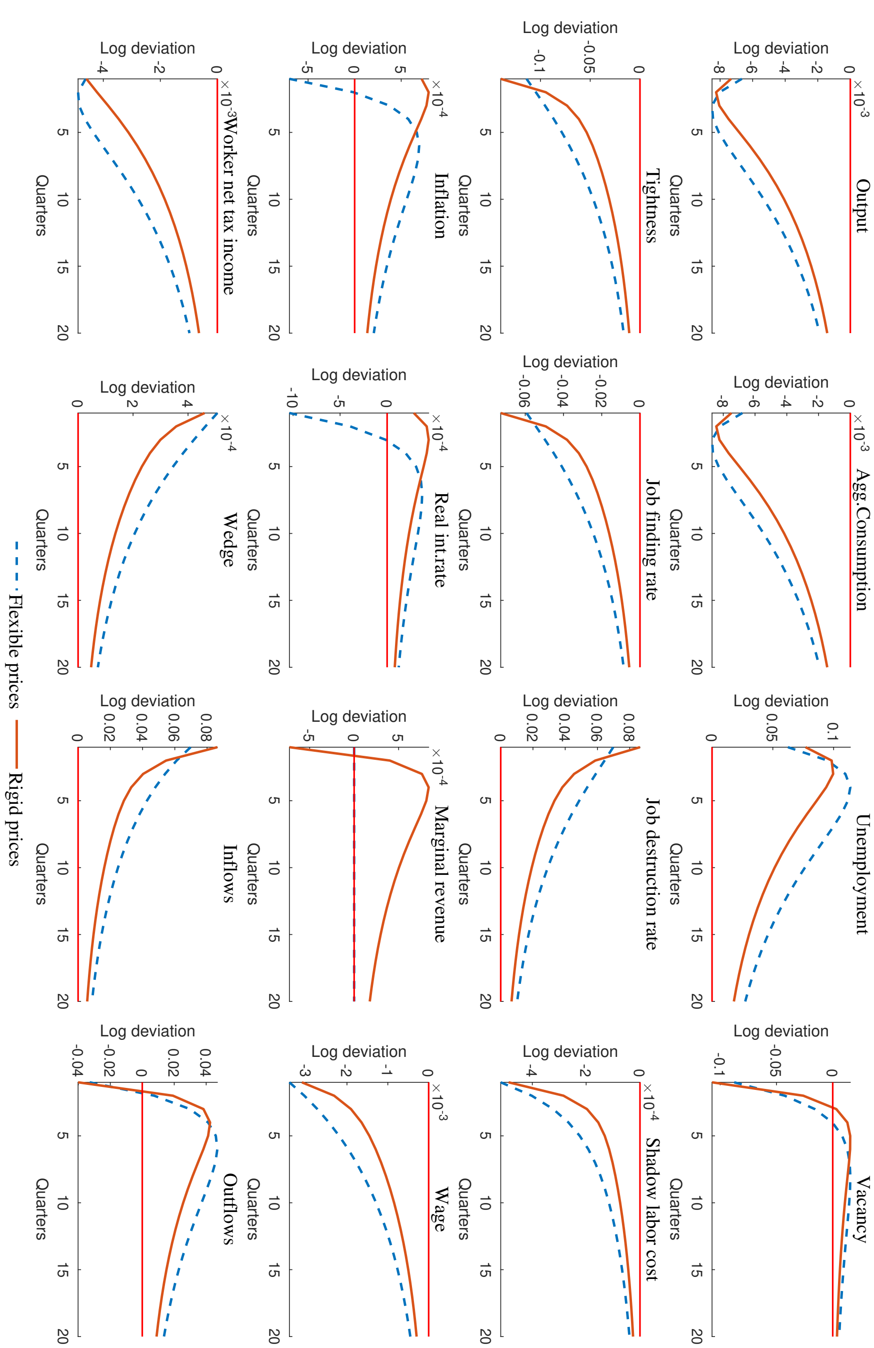


Figure A.5: Impulse-Responses to one standard deviation adverse technology shocks, Highest wage inertia

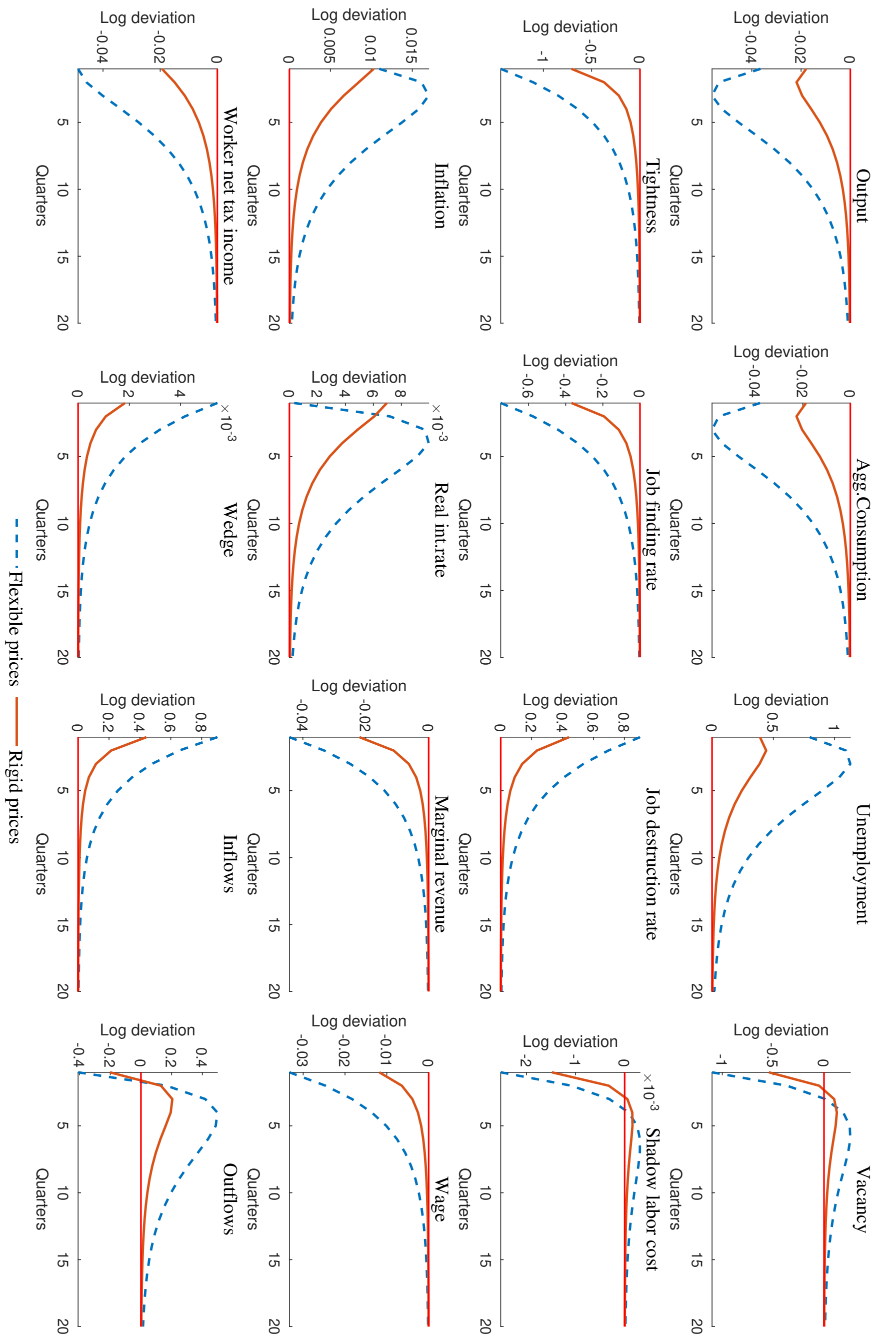


Figure A.6: Impulse-Responses to one standard deviation positive markup shocks, Highest wage inertia