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Bet against the trend and cash in profits.
An agent-based model of endogenous fluctuations of exchange rates

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Abstract. This paper intends to contribute to the theoretical literature on the determinants of exchange rate fluctuations. We build an agent-based model, based on behavioral assumptions inspired by the literature on behavioral finance and by empirical surveys about the behavior of foreign exchange professionals. In our artificial economy with two countries, traders can speculate on both exchange and interest rates, and allocate their wealth across heterogeneous assets. Fundamentalists use both fundamental and technical analysis, while chartists only employ the latter, and are either trend followers or trend contrarians. In our model, trend contrarians and cash in mechanisms provide the sufficient stability conditions, and allow explaining and replicating most stylized facts of foreign exchange markets, namely (i) the excess volatility of the exchange rate with respect to its fundamentals, (ii) booms, busts and precarious equilibria, (iii) clusters of volatility, (iv) long memory and (v) fat tails.

Keywords: Foreign exchange markets, clusters of volatility, fat tails, heterogeneous beliefs, agent-based models, Stock-flow consistent models.

JEL codes: D40, D84, G11, G12.

1. Introduction

The emergence of the literature now known as the behavioural finance approach to exchange rates is grounded on three strands of literature: the works led in the 1980’s that attempted to explain the US dollar’s pattern, the concomitant general criticism on the rational expectations hypothesis and the literature trying to grasp the workings of asset markets in general. For exchange rate modelling, these strands interconnected when, following Meese and Rogoff’s (1983) major criticism, scholars highlighted a series of exchange rate patterns that puzzled the literature, indicating the stylized facts about exchange rates that previous models could not account for. They raised the need for analytical frameworks that were “not constrained by the assumption of rational expectation” (Frankel and Froot, 1986, p. 24) or not written with “the specific intention of finding a steady state to which an economy or market will finally converge” (Kirman, 1993, p. 154).

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1 The literature on exchange rate puzzles is vast and included: i) the disconnection of exchange rates from macroeconomic aggregates (Obstfeld and Rogoff, 2001) and the idea that they would be better explained by a random walk (Meese and Rogoff, 1983); ii) the weakness of the connection between exchange rates and price levels, in contrast from what would be expected from the purchasing power parity (Rogoff, 1996); iii) or the slow convergence of exchange rates to price levels (Engel and Morley, 2001); iv) the non-observance of the uncovered interest parity (Engel, 1996); and v) the very high volatility of exchange rates as compared to the one of the underlying economic variables (Flood and Rose, 1995).
As a matter of fact, exchange rate models had until then aimed at explaining exchange rates through macroeconomic variables. But as the two have very different time series properties, linear deterministic dynamics cannot be successful in explaining exchange rates. As put by Lux and Marchesi (2000, p. 677), “an elementary requirement for any adequate analytical approach is that it must have the potential for bringing about the required behavior in theoretical time series”. For that end, scholars had not only to develop other models, but also to document the statistical properties that these models should replicate, as well as every detail concerning the workings of FX markets (Guillaume et al, 1997). Following this idea, the list of stylized facts concerning exchange rates (and assets prices in general) grew fast, the most explored being (i) the excess volatility of the exchange rate with respect to its fundamentals; (ii) the existence of sequences of booms, busts and precarious equilibria (Schulmeister, 1987); (iii) the presence of clusters of volatility, with long periods of tranquility following long periods of large fluctuations; (iv) long memory in power transformations or in absolute values of asset price returns; and (v) fat tails in the distribution of asset price returns (Cont, 2007; Chen et al, 2012).

Simultaneously, a broader criticism had been growing on the idea of rational expectations and alternatives to those were being presented. Shiller (1981; 1990, p. 59) argued that fluctuations in stock markets were larger than news about dividends would predict, calling attention to the workings of “fads and fashion” in determining prices, as opposed to the idea of speculative bubbles. In line with this theoretical rethinking, we observed the emergence of theoretical models explaining the US dollar’s path through the interaction of two types of agents: those who “think of the exchange rate according to a model” – the fundamentalists – and those who extrapolate past exchange rate changes – the chartists – (Frankel and Froot, 1986, p. 24). The existence of chartists has been backed empirically (the number of traders using technical analysis increased from 1978 to 1988) and they have been said to be responsible for large exchange rate movements “with little basis in macroeconomic fundamentals” (Frankel and Froot, 1990, p. 184-185). Non-fundamentalists also predict short-term exchange rates more accurately (Allen and Taylor, 1990, based on phone-interviews). Among models featuring different types of agents and behaviours, with at least one that does not follow the “rational” strategy, stand out the rational and noise traders of De Long et al (1990), the smart and dumb managers of Scharfstein and Stein (1990) and the fundamentalist and chartist traders of Frankel and Froot (1986), Day and Huang (1990) and De Grauwe et al. (1995), whereby the chartists provide positive feedback rules pushing the exchange rate away from its fundamental value, while fundamentalists observe the increasing deviation and react accordingly, bringing the rate back to the fundamental value.

Models with two types of behaviour brought interesting perspectives, but also questions. In Frankel and Froot’s (1986) fundamentalists and chartists set up, it is the weight of their forecasts in determining prices that changes. But wouldn’t it be more intuitive to think that traders themselves would change their minds? And if that would be the rule, why would a smart manager, as in Scharfstein and Stein (1990) decide to go dumb? (Kirman, 1993). Two answers have emerged.

For Kirman (1993), the change of forecasts results from a herd behaviour. He suggests a decision of the forecasting strategy based on the behaviour of ants searching for a food or of humans deciding in which restaurant to go to. This herd behaviour results in the majority choosing one of the options but constantly
changing opinion, resulting in no equilibrium, but in each state being constantly revised. In a similar vein, Lux (1995) models a contagion mechanism pushing agents to move from pessimism to optimism and markets from bear to bull. Also based on the idea of herding behaviour, with a simple model featuring fundamentalist and noise traders, Alfarano et al (2005) show that introducing this propensity to switch strategies allows reproducing fat tails and volatility clustering.

A second answer derived from the debate on whether heterogenous strategies could survive in the same market, or whether the market would eventually select the (unique) profit-maximizing strategy and kick out the others, just as nature selects the fittest species and let the others dying. Starting from this Market selection hypothesis (MSH) (Dutta and Radner, 1999), inspired by Friedman (1953), who argued that businessmen not maximizing profits would hardly survive in a competitive market, scholars questioned how chartists would survive if fundamentalists are able to accurately predict the fundamental value and bring the exchange rate to this value. The answer came from the demonstration that chartists, for being more often right than wrong, accumulate profits. Following this discussion on the role of profitability on the existence of strategies, Lux (1998) and Brock and Hommes (1998) modelled a market whereby agents switch strategies over time according to their relative performance. Assessment of past performance became key in a series of studies, and scholars studied the role of agents’ sensitivity to the difference of performances and how fast they change strategies in price fluctuations (Brock and Hommes, 1997; Chiarella and He, 2003; Hommes, 2006; Kirman et al, 2007). The switching hypothesis has become a common feature of the models, and has been constantly complexified to consider, for instance, the costs and risks involved with each strategy (De Grauwe and Grimaldi, 2006) or the consideration that agents only switch strategies if they meet others and learn about their gains (Lux and Marchesi, 2000).

Agents switching strategies is indeed of central importance to this literature. It is the crucial feature for generating time series whose characteristics, like volatility, change drastically from period to period; the pool of strategies available to the agents being the element that varies from one model to the other (Cont, 2007). The underlying idea of these exercises has been clearly put by Giardina and Bouchaud (2003, p. 422, italics added): it is to “explore generic classes of models, with the hope of finding some plausible mechanisms that reproduce at least part of the stylized facts”. We present the reasoning behind the criticism and the mechanisms proposed in what follows.

If in the late 1980s the idea of an increasing weight of chartists’ expectations came from the empirical observation of their long-term increasing weight in FX markets (Frankel and Froot, 1990), models in

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2 The existence of chartists is confirmed by empirical analyses, that also show how their strategies has changed with time (Schulmeister, 2009b; Hsu et al, 2016).

3 Apart from the models of herding behavior and chartists and fundamentalists that switch according to past performance, there are also i) models in which agents switch strategies partly randomly, and partly because they all try, more or less successfully, to convince other traders to follow their own strategy (Gilli and Winker, 2003); ii) models in which the probability of an agent becoming fundamentalist evolves according to market circumstances as the speed of a trend (Giardina and Bouchaud, 2003), price volatility (Cont, 2007), the distance to fundamentals (Westerhoff, 2003); and iii) models in which past performance guides the choice among heuristics-based strategies (Anufriev and Hommes, 2012; Agliari et al, 2016).
the 1990s included the hypothesis of constant alternation of expectation formation rules, because it would be the (bounded) rational option (Brock and Hommes, 1998) in line with psychological insights and behavioural economics (Tversky and Kahneman, 1974). Traders would have adaptive learning capacities (Hommes, 2006). They switch strategies because they are not able to process all “the full (but too complex) information set available in the world”. But “they are not fools (…) and want to find out whether the rule they use is a good one” (De Grauwe and Grimaldi, 2006, p. 14). For comparing the profitability of different strategies, traders would need, however, to consider that the two are based in the same time trend, which is “very far from reality” (Giardina and Bouchaud, 2003, p. 423). Indeed, heterogeneity of time horizons is a documented feature of FX markets, ranging from intraday trading to pension funds’ hedging of long-term positions in other asset markets (Müller et al, 1993; Dacorogna et al, 2001). Heterogeneity of time horizons is such a distinct feature in FX markets that is considered as the element of differentiation among agents in a branch of the literature – and has major consequences on volatility clustering (Müller et al, 1997; LeBaron, 2001).

If profitability of the two strategies are not comparable, traders choose the strategy they think will work according to the time horizon of their activity. Eighty-seven percent (87%) of traders surveyed by Cheung et al (2004) believe that long-run exchange rate movements (after 6 months) reflect changes in fundamental values, while only 3% agree that intraday changes reflect them. If fundamentals can predict long-term changes, that is the strategy to be used in long-term forecasts. This association between strategies and time horizon has been confirmed by empirical surveys (Taylor and Allen, 1992; Cheung et al, 2004; Gehrig & Menkhoff, 2005; Menkhoff and Taylor, 2007). Different strategies could therefore be used as complementary by the same FX professional. Surveys show that agents who are expected to behave as fundamentalists also use other pieces of information than fundamentals when building their forecasts if the time horizon is shorter. This pattern is also intensifying with time: year after year, fundamentalists make more use of other, non-fundamentalist, strategies (Taylor and Allen, 1992; Frankel and Froot, 1987; Gehrig and Menkhoff, 2006; Menkhoff and Taylor, 2007).

Based on this evidence, we aim at contributing to the literature, by proposing a model that does not rely on the assumption of a continuous switching among heterogeneous strategies, but refers instead to procedures and heuristics empirically observed in FX markets in order to explain the stylized facts of these markets. Our model portrays a market with a constant share of fundamentalists and chartists, but in which fundamentalists also resort to technical analysis under some circumstances, as in Hommes (2006), and similarly to the chartist trader of Giardina and Bouchaud (2003), who choose to be a fundamentalist when the price reaches “unreasonable” levels. Still from the recognition of the different time horizons, our fundamentalists update their forecasts less often than traders who follow technical analysis, creating non-uniform reactions, as proposed by Dacorogna et al. (2001). Agents’ memories also vary inside a category. Traders’ heterogeneity in terms of memory is important for explaining bandwagons (Schulmeister, 2009), which are pointed by traders as the second-most important feature determining intraday movements (after over-reaction to news; Cheung et al. 2004). Longer memory for some agents is also a feature in LeBaron (2001), De Grauwe and Kaltwasser (2012). Different timing for updating forecasts and acting is included in Cont (2007). Müller et al. (1997) include the two features.
Our model also includes two other elements observed in FX markets and recognized as important for explaining trend reversals, namely *cash in* and *contrarian trading*. The fact that traders decide to cash in their gains is pointed as a common practice among traders⁴, and it is an important element to explain the end of a trend (Schulmeister, 1988). Moreover, it is in line with theoretical and empirical findings about agents looking for *satisficing* outcomes (Simon, 1955, 1978) in a world of uncertainty (Knight, 1921): if traders were able to predict the future, they would maximize profits by closing positions right before a trend’s reversal—a timing that is however impossible to know. This is also in line with behavioral studies showing that people weight losses more than gains (Kahneman and Tversky, 1991), and therefore prefer to cash in paper (virtual) profits even if they do not expect a reversal of the trend, instead of risking to record a loss if their expectations turn out to be wrong. The second key element of our model is the presence of contrarian traders, who follow a third type of trading strategy, consisting in betting against the current trend by selling (buying) when trend-followers would rather buy (sell). The presence of contrarians is broadly recognized in the literature (Galariotis, 2014; Brock and Hommes, 1997; Chiarella and He, 2002; Sansone and Garofalo, 2007), and their existence is important for explaining trend reversals (Schulmeister, 2009). Furthermore, empirical studies recognize certain classes of (actual) traders as contrarians (Barber and Odean, 2000, Kumar and Lee, 2006, Kaniel et al, 2008) and experiments have shown that some people act as contrarians (Drehmann et al, 2005).

Apart from these elements of expectation formation, we suggest few innovations to understand exchange rate determination as embedded into a broader setting of international portfolio allocation, by combining the stock-flow consistent (SFC) approach with an agent-based model⁵. We do that in two interconnected steps. First, the exchange rate is embedded in a broader (although simplified) macroeconomic framework by explicitly tracking both sectoral and individual balance-sheets and the flows between them. Also, consistently with the SFC modelling tradition, in our model households/traders allocate their wealth across heterogeneous assets, namely currencies and government bonds, according to their expectations on assets’ future profitability. Hence, the demand for assets is a product of traders’ expectations and portfolio allocation decisions. Moreover, as flows and stocks are rigorously accounted for, our agents’ trading is limited by their wealth (a constraint that is said to explain a mean-reverting behaviour in Giardina and Bouchaud, 2003).

Secondly, by modelling treasury bonds markets and the interaction between the latter and the foreign exchange market, we allow fundamentalists to have endogenous fundamental exchange rate expectations. In the behavioural literature, fundamentalists’ expectations are always modelled as

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⁴ This is a common practice in financial markets. See https://www.cmegroup.com/education/courses/things-to-know-before-trading-cme-futures/closing-your-position.html

⁵ Even if written in accordance with the accounting constraints of SFC models, our model does not include all of the features of these models. For instance, our model respects the accounting constraints of stock-flow consistency, namely the principle of *quadruple entry* (Godley and Lavoie, 2007). Nevertheless, because it focuses on very short-term (daily) fluctuations of asset prices, and considers aggregate variables as constant or exogenous for simplicity, the model does not account for endogenous feedbacks between macroeconomic stocks and flows, therefore omitting an important feature of traditional SFC models. Extensions of this model might fill this gap, in line with the agent-based stock-flow consistent tradition (AB-SFC; Caverzasi and Godin, 2015; Caiani et al, 2016).
exogenous, often using a random walk or a stochastic process (De Grauwe and Grimaldi, 2006; Manzan and Westerhoff, 2005). Those expectations are therefore disconnected from the model itself, which can be viewed as an oversimplifying assumption, given that exchange rate movements also influence other variables affecting fundamentals (De Grauwe and Kaltwasser, 2012). We endogenize fundamentalists’ expectation formation by associating it to interest rates. Despite of the lack of consensus about the fundamental variables governing exchange rates, most economists would agree that interest rates are highly ranked in the list of probable determinants of the exchange rates (Harvey, 2001; Kirman et al, 2007) and that traders follow monetary policies announcements closely (Cheung et al 2004; Müller et al, 2017). Our fundamentalists therefore base their exchange rates forecasts on the expected interest rate differential between the two countries, given a certain degree of heterogeneity of interest rate and fundamental exchange rate expectations. This is also consistent with the empirical evidence regarding the recent dramatic increased magnitude of cross-border portfolio assets and liabilities, from 43% of the world’s GDP in 2001 to 76% in 2015 (Camanho et al, 2018). Portfolio allocation choices between heterogeneous assets might also be considered a source of several stylized facts in the foreign exchange market, such as the emergence of booms, busts and precarious equilibria (Schulmeister, 1987), which are statistical equilibria characterized by a non-fundamental centre of gravity. Hence, although our model does not currently include a comprehensive macroeconomic structure, it provides several interconnections between markets and economic sectors that allow to explore, in future research, the micro-to-macro and the-macro-to-micro relationships that explain exchange rates and interest rates fluctuations. To put it differently, with this model we aim at creating the microeconomic structure of a future micro-founded macroeconomic model of exchange rate fluctuations.

To sum up, this article proposes to build an agent-based model of the determination of exchange rates, focused on exploring the role of procedures and heuristics observed in actual FX markets, in order to explain the major stylized facts of these markets. Our model includes a fixed proportion of fundamentalists, chartists and contrarians, who are heterogeneous regarding a series of aspects, such as memory, options regarding when to cash in paper profits, or the weight given to past expectations (anchoring). Our model also comprises a treasury bonds market, whereby the interest rate responds to the traders’ demand for treasury bonds, resulting in endogenous expectations of the exchange rate by fundamentalists. As we will show, our model is able to replicate and explain most of the stylized facts of financial markets that we highlighted previously, thereby providing an original contribution to the literature on exchange rates focused on foreign exchange (FX) markets participants’ heterogeneous expectations.

The remainder of the paper is structured as follows. In section 2, we present the main features of our model, namely how traders form their expectations, make their portfolio allocation choices and open or close positions in financial markets. In section 3, we present, discuss and validate the results of the model by showing that it is able to reproduce the vast majority of the main stylized facts regarding FX markets. In section 4, we perform experiments on the key parameters of the model, in order to identify the main determinants of exchange rate fluctuations. Section 5 concludes and discusses possible avenues for future research.
2. The model

2.1. A general overview

Our artificial economy contains two countries – A and B – and three institutional sectors: households, governments and central banks (Table A.2). Each country has its own national currency, which is created by the central bank by purchasing domestic treasury bonds at issuance. Since we abstract from firms’ investments, bank loans and central banks’ open market operations, the central bank buying bonds is the only channel for money creation and thus for the supply of new liquid assets (currencies). The supply of domestic and foreign assets (currencies and treasury bonds) is therefore endogenous.

The demand for domestic and foreign assets is also endogenous and it is determined by the N traders who populate our artificial economies and speculate on the four existing assets, namely currency A, currency B, country A’s government bonds and country B’s government bonds. Consequently, there are three asset prices: the exchange rate between currency A and currency B, the price of country A’s government bonds and the price of country B’s government bonds. Traders have heterogeneous time horizons and, therefore, heterogeneous trading strategies to exploit the fluctuations of the asset prices. There are three different trading rules. Trend-follower chartists follow the very simple and adaptive rule “the trend is your friend”: if the price is increasing, they believe that it will keep increasing, and if the price is falling, they believe that it will keep falling. Trend-contrarian chartists seek to exploit fluctuations around the trend when the market is volatile, and follow the trend when the market is relatively quiet. Hence, in periods of high volatility, they bet against the trend, selling if a rapid price increase is seen and buying in the case of a rapid price fall. In periods of tranquility, however, with only limited fluctuations around the trend and less space for speculation, they follow the “trend is your friend” rule. Fundamentalists seek to speculate out of fluctuations of the market price around what they believe is the true, or fundamental price. For believing that prices cannot go too far from fundamentals for too long, they bet for convergence when the asset’s price is relatively far from its supposed fundamental value, but when the asset’s price is relatively close to what they believe is the fundamental value, they follow technical trading’s buy and sell signals. As we stressed in the introduction, these assumptions are inspired by empirical surveys showing that traders defining themselves as “chartists” pay little, if any, attention to fundamental analysis, while traders defining themselves as “fundamentalists” pay attention.

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6 If the central bank purchased all the newly issued bonds, there would be no market for government bonds. On the other hand, if the central bank did not accommodate government’s expenditures at all, there would be a fixed supply of currency with a non-fixed supply of assets, pushing the interest rate to infinity. This assumption is thus necessary to introduce money in our artificial economy. We assume for simplicity that the fraction of newly issued bonds purchased by the central bank is exogenous and fixed, and that the remaining is supplied to the market.
to technical analysis but only in the short run, since in the medium- to long-run they firmly believe in fundamentals (See particularly Gehrig & Menkhoff, 2005).

As we will show, the interaction among these heterogeneous speculators who seek to make profits out of the fluctuations of prices, and also between these speculators and governments, who seek to issue bonds to finance public spending, is able to generate endogenous and realistic fluctuations of the exchange rate and to replicate most of the stylized facts of FX markets.

2.2. The sequence of the model

Every period of the model corresponds to one day, and is composed of five successive steps:

1. Governments spend their resources, as decided in the previous period, and decide how many bonds to issue in order to finance next period’s desired spending. Current government’s expenditure finances social transfers to households. For the sake of simplicity, we assume that all households are traders and that they receive an equal amount of government benefits in each period.

2. Traders form their expectations about the evolution of asset prices, according to their own strategy, and decide how to allocate their wealth across the four different assets. After setting their portfolio choices, they decide whether to open, keep open or close their positions in the markets, given the difference between the desired and the current amount they have of each asset.

3. The market for government bonds opens. Governments seek to sell new bonds at the current interest rate and traders seek to buy or sell bonds according to their portfolio choices.

4. The currency market opens. Traders exchange currencies according to the portfolio choices of step 2.

5. According and proportionately to the gap between demand and supply, the prices of assets increase (decrease) if the gap is positive (negative).

2.3. The Governments and the Central Banks

The government of country $j$ issues treasury bonds that pay a face value set to 1 at 10 years of maturity; their price $P(B^j)_t$ being an inverse function of the interest rate $(i^j_t)$:

$$P(B^j)_t = 1/(1 + i^j_t)^{10}$$

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7 In order to abstract from assets’ heterogeneity in terms of maturity and interest payments, which would add further complexity in agent’s portfolio choices, we assume that there is a unique type of treasury bill, with 10 years of maturity and a homogeneous face value, and that there are no interest payments before maturity.
Treasury bonds issued at any period \( t \) in country \( j \) (\( \Delta B^j_{t} \)) are primarily used to finance desired public expenditure (\( \bar{G}^j \)), which is held constant for simplicity. From here on, the tilde above a variable will denote the desired level of a variable. Moreover, in order to avoid a lack of liquidity due to the inability to sell the desired amount of bonds at the current interest rate, the government owns deposits at the central bank (\( D^j_{g} \)) and targets a fixed ratio of deposits to desired expenditure (\( \bar{d}^j \)):

\[
\bar{d}^j = (\bar{G}^j - D^j_{g}) 
\]  

(2)

\( \bar{D}^j_{g} \) is the desired amount of liquidity that the government deposits at the central bank. Hence, bonds issued by national governments finance both desired expenditure and liquidity targets:

\[
\Delta B^j_{t} = [\bar{G}^j + (\bar{D}^j_{g} - D^j_{g})] * (1 + i^j_{t})^{10} 
\]  

(3)

The central bank’s desired demand for bonds is a fixed share \( c^j \) of government’s desired expenditure:

\[
\Delta B^j_{CB,t} = (\bar{G}^j - c^j B^j) * (1 + i^j_{t})^{10} 
\]  

(4)

The desired amount of bonds that the government seeks to sell to the market is equal to the residual of the total desired amount of new bonds minus the desired share of bonds purchased by the central bank.

\[
\Delta B^j_{H,t} = [\bar{G}^j * (1 - c^j B^j) + (\bar{D}^j_{g} - D^j_{g})] * (1 + i^j_{t})^{10} 
\]  

(5)

If the market is not willing to purchase the total amount of bonds at the current interest rate, the effective amount will be different from the desired amount (\( \Delta B^j_{H,t} \neq \Delta B^j_{H,t} \)). If this difference is such that the government is unable to finance public expenditure with new liquidity, it will use its own liquidity deposited at the central bank to finance the residual. Nevertheless, if the government runs out of liquidity and is not able to finance current expenditure, the central bank steps in and buys the necessary amount of bonds to cover current expenditure. Hence, current expenditure is always equal to desired expenditure (\( G^j_{t} = \bar{G}^j \)) and the variation of government deposits will be equal to the effective amount of newly issued debt to the central bank and to households, minus government’s current expenditure:

\[
\Delta D^j_{g,t} = \frac{\Delta B^j_{CB,t} (1 + i^j_{t})^{10}}{(1 + i^j_{t})^{10}} + \frac{\Delta B^j_{H,t} (1 + i^j_{t})^{10}}{(1 + i^j_{t})^{10}} - G^j 
\]  

(6)

With \( \Delta B^j_{CB,t} = (\Delta B^j_{CB,t} + R^j_{t}) \) and \( R^j_{t} \) the amount of undesired bonds purchased by the central bank, which is equal to 0 in normal times, and can take a positive value in the above-mentioned extreme case where the government runs out of liquidity and is unable to finance current expenditures.

2.4. The traders

Each of the N traders who populate countries A and B of our artificial economy owns a financial portfolio composed of all the four assets traded in the markets, namely currency A (\( M^A \)), currency B (\( M^B \)), country A’s treasury bonds (\( B^A \)) and country B’s treasury bonds (\( B^B \)). We define the exchange
rate \((E_{r_t})\) as the amount of currency A necessary to buy one unit of currency B, (an increase of the exchange rate reflects an appreciation of currency B):

\[ E_{r_t} = \frac{S^A}{S^B} \]  \tag{7}

Hence, the wealth of traders is given by:

\[ W_{i,j,t} = M^A_{i,j,t} + (B^A_{i,j,t} \times P(B^A_t)) + (M^B_{i,j,t} \times E_{r_t}) + (B^B_{i,j,t} \times P(B^B_t) \times E_{r_t}) \]  \tag{8}

\[ W_{i,j,t} = \frac{M^A_{i,j,t}}{E_{r_t}} + \frac{B^A_{i,j,t} \times P(B^A_t)}{E_{r_t}} + M^B_{i,j,t} + \frac{B^B_{i,j,t} \times P(B^B_t)}{E_{r_t}} \]  \tag{8'}

\(W_{i,j,t}\) is the wealth of trader \(i\) living in country \(j\) at time \(t\). Hereafter, the subscripts refer to traders while the superscripts refer to assets. Hence, \(M^A_{i,j,t}\) denotes the amount of currency A held by trader \(i\), who lives in country B, at time \(t\). Because traders define their own portfolio in terms of their national currency, in what follows we will specify behavioral equations twice, for traders in country A and for traders in country B. For traders living in country \(j\), the share of each asset in their own portfolio, represented by the vector of parameters \((\alpha_{i,j,t}; \beta_{i,j,t}; \gamma_{i,j,t}; \Delta_{i,j,t})\), is:

\[ \begin{align*}
\text{For } j = A & \quad \begin{cases}
\frac{M^A_{i,j,t+1}}{W_{i,j,t}} = \alpha_{i,j,t} \\
\frac{(B^A_{i,j,t} \times P(B^A_t))}{W_{i,j,t}} = \beta_{i,j,t} \\
\frac{(M^B_{i,j,t} \times E_{r_t})}{W_{i,j,t}} = \gamma_{i,j,t} \\
\frac{(B^B_{i,j,t} \times P(B^B_t) \times E_{r_t})}{W_{i,j,t}} = \Delta_{i,j,t}
\end{cases} \\
\text{For } j = B & \quad \begin{cases}
\frac{M^A_{i,j,t+1}}{E_{r_t} \times W_{i,j,t}} = \frac{(B^A_{i,j,t} \times P(B^A_t))}{W_{i,j,t}} = \beta_{i,j,t} \\
\frac{M^B_{i,j,t}}{W_{i,j,t}} = \gamma_{i,j,t} \\
\frac{(B^B_{i,j,t} \times P(B^B_t))}{W_{i,j,t}} = \Delta_{i,j,t}
\end{cases}
\]  \tag{9}

2.4.1. Open or expand a position

The vector of parameters \((\alpha_{i,j,t}; \beta_{i,j,t}; \gamma_{i,j,t}; \Delta_{i,j,t})\) is an endogenous variable determined by traders’ decisions to open or close positions in the foreign exchange and in the treasury bonds markets, which are in turn determined by traders’ expectations about asset prices’ fluctuations in these markets. Namely, traders wish to increase the share of those assets that they expect to appreciate, and wish to reduce the share of those assets that they expect to depreciate. Hence, they can either open a long position (buy an asset in order to sell it back at a hopefully higher price) or open a short position (sell an asset in order to buy it back at a hopefully lower price), if they expect, respectively, an increase or a fall in an asset price. For example, if they expect a positive variation of the exchange rate \((\Delta E_{r_t} > 0)\) implying an appreciation of B (hence, a depreciation of A), they can either go long (buying) by increasing the share of currency B or go short (selling) by decreasing the share of currency A. In our model, we treat traders’ demand for national currency as an exogenous and stochastic variable. This allows us to capture and simulate the effect of changes to factors affecting traders’ liquidity preference beyond speculative
motives. Together with fundamentalists’ expectations about fundamental prices (see section 2.4.3.3), changes in liquidity preference are the only stochastic triggers in our model. Hence, in the foreign exchange market, traders open a long position (buy) if they expect an appreciation of the foreign currency and a short position (sell) if they expect a depreciation of the foreign currency:

\[
\begin{align*}
\text{For } j = A, B, \quad &
\begin{cases}
\tilde{y}_{i,j,t+1} = \tilde{y}_{i,j,t} + (1 + \omega^{er} E_{i,t}(|\Delta E_{r,t+1}|)) & \text{if } E_{i,t}(\Delta E_{r,t+1}) > 0 \\
\tilde{\alpha}_{i,j,t+1} = \tilde{\alpha}_{i,j,t} + \tilde{\alpha} + \epsilon_{i,j,t} & \text{with } \epsilon_{i,j,t} - U(0, \epsilon^*) \\
\tilde{y}_{i,j,t+1} = \tilde{\alpha}_{i,j,t}^* (1 + \omega^{er} E_{i,t}(|\Delta E_{r,t+1}|)) & \text{if } E_{i,t}(\Delta E_{r,t+1}) < 0 \\
\tilde{\beta}_{i,j,t+1} = \tilde{\beta}_{i,j,t}^* (1 + \omega^{ir} E_{i,t}(|\Delta i^{A}_{t+1}|)) & \text{if } E_{i,t}(\Delta i^{A}_{t+1}) < 0 \\
\tilde{\Delta}_{i,j,t+1} = \tilde{\Delta}_{i,j,t}^* (1 + \omega^{ir} E_{i,t}(|\Delta i^{B}_{t+1}|)) & \text{if } E_{i,t}(\Delta i^{B}_{t+1}) < 0 \\
\end{cases}
\end{align*}
\]

\(E_{i,t}(\Delta E_{r,t+1})\) is the expectation at time \(t\) of the variation of the exchange rate from \(t\) to \(t+1\), \(\omega^{er}\) is a parameter defining the sensitivity of speculative demand for foreign currency to expected variations of the exchange rate, and \(\epsilon_{i,j,t}\) is a uniformly distributed shock reflecting cyclical changes to the demand for national currency, our exogenous variable.

The same mechanism applies to treasury bonds. If traders expect an appreciation (depreciation) of treasury bonds of country \(j\), they can go long (short) on the treasury bonds market by increasing (reducing) the share of treasury bonds in their portfolio. Because price is inversely related to the interest rate (equation (1)), traders in country \(j\) will open or keep expanding their positions on the treasury bonds market according to the expected variations of the interest rate:

\[
\begin{align*}
\text{For } j = A, B, \quad &
\begin{cases}
\tilde{\beta}_{i,j,t} = \tilde{\beta}_{i,j,t-1} + (1 + \omega^{ir} E_{i,t}(|\Delta i^{A}_{t+1}|)) & \text{if } E_{i,t}(\Delta i^{A}_{t+1}) > 0 \\
\tilde{\alpha}_{i,j,t} = \tilde{\alpha}_{i,j,t-1} + \tilde{\alpha} & \text{with } \epsilon_{i,j,t} - U(0, \epsilon^*) \\
\tilde{\Delta}_{i,j,t} = \tilde{\Delta}_{i,j,t-1} + \tilde{\Delta} & \text{if } E_{i,t}(\Delta i^{B}_{t+1}) > 0 \\
\end{cases}
\end{align*}
\]

\(\omega^{ir}\) represents the sensitivity of speculative demand for treasury bonds to expected variations of the interest rate. By normalizing parameters \((\tilde{\alpha}_{i,j,t}; \tilde{\beta}_{i,j,t}; \tilde{\gamma}_{i,j,t}; \tilde{\Delta}_{i,j,t})\) such that they sum to 1, we can compute, for each trader, the desired share of each asset in her or his own portfolio, and derive his or her desired amount \((\tilde{M}_{i,j}^{A}, \tilde{M}_{i,j}^{B})\) by substituting the normalized desired shares within equation (9), given the assets’ prices and the exchange rate. This ultimately allows computing, for each trader, the demand (supply) of each asset, which is equal to the difference between desired and current amount. Namely, in the treasury bonds market, the demand or supply of treasury bonds is equal to:

\[
\begin{align*}
B_{i,j,t}^{AD} &= \tilde{B}_{i,j,t+1}^{A} - B_{i,j,t}^{A} \\
B_{i,j,t}^{BD} &= \tilde{B}_{i,j,t+1}^{B} - B_{i,j,t}^{B}
\end{align*}
\]

\(11\)
Because traders cannot *barter* national (foreign) bonds against foreign (national) bonds, nor they can *barter* national (foreign) bonds against foreign (national) currency, the desired demand of bonds is limited by their liquidity. Moreover, transactions can take place at a non-market clearing price, therefore the actual shares of bonds can be different from the desired shares of bonds *ex post*. In this case, because the currency market comes after the treasury bonds market (see section 2.2), traders can take account of the unsatisfied supply or demand of treasury bonds when trading in the FX market, such that:

\[
\begin{align*}
(M_{i,j,t}^{B,D})_t &= (\tilde{M}_{i,j,t+1}^B - M_{i,j,t}^B) + [(B_{i,j,t+1}^B - B_{i,j,t}^B) \ast P(B_{t}^B)] \\
(M_{i,j,t}^{A,D})_t &= -(M_{i,j,t}^{B,D})_t \ast E_{r_t} \\
\end{align*}
\]

Equation (12’) tells that traders demand the exact amount of currency that is necessary to buy the desired bonds and simultaneously hold the desired share of currency in the portfolio. Because traders cannot sell assets that they do not hold, and because they are liquidity constrained, it follows that the desired demand and supply of currency can be different from the actual demand or supply *ex post*.

2.4.2. Wait or close a position

Traders open or keep expanding a long (short) position, if they expect that the price of an asset is going to increase (decrease). After opening a position in a market, they wait without doing anything if they are not confident about their expectations of market price evolution, or they close the position if they are sufficiently confident that the trend is going to reverse\(^8\). Traders can also decide to close a position before the reversal of the trend, because they enjoy realizing “what have been up to now only paper profits” (Harvey, 2009, p.53, italics added). In practice, when opening a position, traders usually set an *ex-ante* price target such that if the market price reaches this target, they automatically send an exit order. As already stressed in the introduction, this behavior is consistent with both theoretical and behavioral findings and reflects common practices in financial markets. We thus assume that traders fix a target price \(\bar{p}(Z^t)\) of asset \(Z^t\) when they open a position, by setting an additive parameter \(\psi_{i,j}\) to the current price, and if the market price reaches the target price, they automatically close the position:

\[
\bar{p}(Z^t)_{i,j} = p(Z^t)_t + \psi_{i,j}
\]

\(\psi_{i,j}\) is constant and heterogeneous across traders and follows the uniform distribution, \(\psi \sim U(0, \psi^+).\)

To close a position, traders set the *desired* amount of the asset that they still wish to keep in the portfolio after closing the position, which depends on the asset price at the time of closing. The rationale is that for traders in country A, the higher the value of currency B (the higher \(E_{r_t}\)), the higher the incentive of selling currency B against currency A. Conversely, for traders belonging to country B, a lower exchange rate implies a higher value of currency A, hence a higher incentive of selling currency A against currency.

---

\(^8\) We explain in section 2.4.3 what do we mean by *confidence* and how we model confidence in expectations.
B. The same applies for the interest rate: the lower the interest rate, the higher the price of bonds and thus the higher the incentive of selling bonds against currency. Hence, for each trader in country $j$, the desired share of each asset after closing a position is equal to:

$$
\begin{align*}
&\text{For } j = A, \begin{cases} 
\tilde{\beta}_{i,j,t} = \lambda \ast i_{A,t} \\
\tilde{\gamma}_{i,j,t} = \frac{\mu}{E_{r_t}} \\
\tilde{\Delta}_{i,j,t} = \lambda \ast i_{B,t}
\end{cases} \\
&\text{For } j = B, \begin{cases} 
\tilde{\alpha}_{i,j,t} = \mu \ast E_{r_t} \\
\tilde{\beta}_{i,j,t} = \lambda \ast i_{A,t} \\
\tilde{\Delta}_{i,j,t} = \lambda \ast i_{B,t}
\end{cases}
\end{align*}
$$

(14)

$\lambda$ and $\mu$ are positive constants, equal for all traders for simplicity. To finally close the position on a given asset, traders simply follow equations (12) and (12'), and sell (or buy) the amount that is necessary to reach the desired share of that asset in the portfolio. For the sake of clarity, Figure 1 summarizes traders’ sequential choices on assets $Z^j$, with $Z^j = (M^A, M^B, B^A, B^B)$, traded at their price $p(Z^j)_t$.

**Figure 1: Traders’ sequential open and close decisions**

Figure 1 summarizes traders’ sequential choices. At time $t=0$, traders open a long position (buy) if they expect the price to increase, open a short position (sell) if they expect the price to fall and just wait if they expect that the price will remain fairly constant. At time $t=1$, they keep expanding the long (short) position if positive (negative) expectations are confirmed, close the long (short) position either if they expect a fall (increase) in future price or if the market price hits their target price, or just wait if they expect that the price will remain fairly constant. As of time $t=2$, traders repeat these same sequential choices until the end of the simulation.
2.4.3. Trading rules and expectations

We consider three different trading rules, hence three different type of traders: chartist trend-followers, chartists trend contrarians and fundamentalists. Each type of trader represents a fixed share of the whole population of traders. Namely, the share of fundamentalists on the whole population is equal to the fixed parameter \( f \), the share of trend contrarians is equal to the fixed parameter \( (1 - f)c \) and the share of trend followers is equal to the fixed parameter \( (1 - f)(1 - c) \).

2.4.3.1. Trend-follower chartists

We model chartists, or technical traders, as speculators who seek to exploit short to medium run fluctuations. Trend follower chartists do not react to short-run changes in the trend, unless they observe that these changes have been persistent enough to represent a reversal. Their strategy consists of buying when the price is relatively low but already increasing, and selling when the price is high but already decreasing. These traders have bandwagon expectations, as Frankel and Froot (1990) would put it. Hence, they open or close positions when asset prices have a well-defined positive or negative trend and bet that this trend is going to persist, whereby a well-defined trend is an average change in the market price in the last \( n \) days that is larger than a given threshold \( s^Z_t \) (Cont, 2007). Hence, the parameter \( s^Z_t \) captures a measure of confidence in their own expectations or risk aversion: traders with a lower \( s^Z_t \) enter and exit relatively faster, because they are relatively confident that the expected positive or negative trend is going to persist. Traders with a larger \( s^Z_t \), on the other hand, will wait relatively longer before entering or quitting a market because they are relatively less confident about their own expectations about the trend. The presence of \( s^Z_t \) therefore creates a band within which the exchange rate can fluctuate without generating buy or sell signals, preventing whiplash signals (Brock et al, 1992). Their trading strategy, which is directly inspired by technical trading’s momentum models (Schulmeister, 2008), is thus the following:

\[
\begin{align*}
E_t(\Delta P(Z)_{t+1}) = \frac{(P(Z)_{t+n_i} - P(Z)_{t-n_i})}{n_i} & \quad \text{if} \quad \left| \frac{(P(Z)_{t+n_i} - P(Z)_{t-n_i})}{n_i} \right| \geq s^Z_t \\
E_t(\Delta P(Z)_{t+1}) = 0 & \quad \text{if} \quad \left| \frac{(P(Z)_{t+n_i} - P(Z)_{t-n_i})}{n_i} \right| < s^Z_t
\end{align*}
\]  

(15)

\( E_t(\Delta P(Z)_{t+1}) \) is the expected future price variation of asset \( Z \), \( P(Z)_{t} \) is the current price of asset \( z \) and \( P(Z)_{t-n_i} \) is the price of asset \( Z \) at a certain past date \( t-n_i \). Trend-followers are heterogeneous in the length of the trend \( (n_i) \) that they consider and in their switching threshold \( (s^Z_t) \). In particular, we assume uniform distributions such that \( n_i ~ U(n^-, n^+) \) and \( s^Z_t ~ U(0, s^Z+) \).

2.4.3.2. Trend-contrarian chartists

Trend-contrarian traders, also called contrarians, are chartists going against the trend (Brock and Hommes, 1998; Chiarella and He, 2002; Sansone and Garofalo, 2007; Chen et al, 2012). We model them
as speculators who bet that the current trend is going to persist, following the “trend is your friend” rule, if this trend is relatively stable, but they also try to exploit short run fluctuations around the medium run trend if these fluctuations are sufficiently large to create opportunities for speculation. In particular, closely inspired by the moving average models (Brock et al, 1992; Schulmeister, 2008), we assume that they observe the difference between the slope of price changes in the latest m days and the slope of price changes in the latest n days, with m+n, whereby a positive difference between the two (V>0) reflects an upward diverging trend and a negative difference (V<0) denotes a downward diverging trend. Such as trend followers, they set a band of inaction following their own risk aversion: if they observe an upward (downward) divergence in the short run trend larger than the reference threshold \( s_t^{tc} \), they bet that the price is going to revert to the medium run trend and sell (buy) the asset in order to buy (sell) it back at a hopefully lower (higher) price (hence, they bet against the trend); otherwise they behave like trend followers and bet that the medium run trend will persist. Their trading strategy is the following:

\[
V = \left[ \frac{P(Z)_{t-m_i} - P(Z)_{t-n_i}}{m_i} \right] - \frac{P(Z)_{t-n_i} - P(Z)_{t-n_i}}{n_i} \\
\begin{align*}
\text{For } V < s_t^{tc} \\
E_{t,t}(\Delta P(Z)_{t+1}) &= \frac{P(Z)_{t-m_i} - P(Z)_{t-n_i}}{n_i} \\
E_{t,t}(\Delta P(Z)_{t+1}) &= 0
\end{align*}
\text{if } \left| \frac{P(Z)_{t-m_i} - P(Z)_{t-n_i}}{n_i} \right| \geq s_t^{Z} \\
\text{if } \left| \frac{P(Z)_{t-n_i} - P(Z)_{t-n_i}}{n_i} \right| < s_t^{Z}
\]  

As for trend followers, we introduce heterogeneity among trend contrarians in the form of a uniform distribution for both \( n_i \) and \( m_i \), with \( n_i \sim U(n^-, n^+) \), \( m_i \sim U(m^-, m^+) \) and \( m^+ < n^- \). We also assume a uniform distribution for \( s_t^{Z} \sim U(0, s_{Z,+}^{Z}) \) and for \( s_t^{tc} \sim U(0, s_{tc,+}^{zc}) \).

2.4.3.3. Fundamentalists

We model fundamentalists as traders that seek to exploit long run fluctuations. As already stressed in the introduction, fundamentalists believe that the price cannot fluctuate too far and for too long from its fundamental value, but are also aware that chartists exist and refer to past movements, and that this has a short-term influence on the exchange rates. Thus, fundamentalists do not refrain from profiting from these short-run fluctuations. For this reason, we assume that fundamentalists refer to buy and sell signals produced by technical trading models in the short run, but stop doing so if they think that the exchange rate is too far away from its fundamental value (similarly to Westerhoff, 2003). More precisely, before opening or closing positions, they estimate the gap between the current market price and the fundamental price (FG), and if this gap is lower than their reference threshold \( s_t^{f} \), they act as trend-followers.

\[^9\text{In moving average models, traders take the moving average of the observed price. By contrast, our contrarians take the moving average of observed price variation. This allows us to reconcile trend followers and contrarians, as they both look at the same trend, but while the former bet that it will continue, the latter bet that it will revert.}\]
Conversely, if the gap is larger than the reference threshold $s_i^r$, they expect a convergence to the fundamental price and bet for it. Their trading strategy is synthetized by the following equations: 

$$
FG = \left| P^f(Z)_{t+1} - P(Z)_t \right|
$$

For $FG < s_i^f$ 

$$
E_{i,t}(\Delta P(Z)_{t+1}) = \frac{P(Z)_{t+1} - P(Z)_{t}}{n_i}
$$

if 

$$\frac{P(Z)_{t+1} - P(Z)_{t}}{n_i} \geq s_i^F$$

For $FG > s_i^f$ 

$$
E_{i,t}(\Delta P(Z)_{t+1}) = \zeta_i(p^f(Z)_{t+1} - P(Z)_t)
$$

$p^f(Z)_{t+1}$ is the expected future fundamental price of asset $Z$, $\zeta_i \sim U(0, \zeta^*)$ is a coefficient that reflects the expected speed of adjustment of the current market price to the fundamental price. The reference thresholds $s_i^f$ and $s_i^Z$ follow a uniform distribution, such that $s_i^f \sim U(0, s^f\uparrow)$ and $s_i^Z \sim U(0, s^Z\uparrow)$.

Fundamentalists’ exchange rate expectations are centered on the treasury bonds’ market, as a change in the interest rate differential of the two countries can trigger financial flows, affecting exchange rates. Interest rate expectations, in turn, depend on common information received by the market. This is modelled as a stationary but persistent stochastic process, which may be thought of as a formalization of announcements related to monetary policies, or to other macroeconomic shocks that can affect interest rates, but that are not modelled explicitly within our framework.

The forecasted value however varies among traders, as they anchor expectations to their prior forecasts with different degrees. Moreover, because fundamentals are relatively stable, we assume that fundamentalists do not update their expectations too often (they do it with a probability $\pi < 1$). In both countries A and B, expectations concerning interest rates are thus given by:

$$
\text{For } j = A, B
\begin{cases}
\epsilon^A_1 = (\tau_i * \frac{1}{n^r} \sum_{i=0}^{n^r} \epsilon^A_{t-i} + (1 - \tau_i) * \phi^A_i * (1 + |\epsilon^A_t|)) & \text{if } \epsilon^A_t > 0 \\
\epsilon^A_1 = (\tau_i * \frac{1}{n^r} \sum_{i=0}^{n^r} \epsilon^A_{t-i} + (1 - \tau_i) * \phi^A_i * (1 + |\epsilon^A_t|)^{-1}) & \text{if } \epsilon^A_t < 0 
\end{cases}
$$

The parameter $\tau$ reflects the degree of anchoring of current information to the average of past expected values over 3 periods ($n^r = 2$), and follows a uniform distribution with $\tau_i \sim U(0, \tau^*)$, while news affecting the fundamental interest rate ($\phi^A_i * (1 + |\epsilon^A_t|)$) follow a stochastic process. In particular, by assuming that the fundamental interest rates of countries A and B are correlated, we model $\epsilon^A_t$ as a stochastic and persistent process centered around 0:

$$
\begin{align*}
\epsilon^A_t &= \theta * \epsilon^A_{t-1} + \omega^A_t \\
\epsilon^B_t &= \epsilon^A_t + \omega^B_t
\end{align*}
$$

With $\omega^A_t - \omega^B_t \sim N(0, 0.05)$.

When building their expectations concerning the fundamental exchange rates ($Er^f$), fundamentalists take information related to the fundamental interest rate differential in the two countries
\((i^f_B/i^f_A - 1)^{10}\). The sensitivity of fundamental exchange rate fluctuations to fundamental interest rates fluctuations is given by \(\rho_i - U(0, \rho^+).\) Current expectations are anchored to a fixed center of gravity, \(\overline{E}_f^r\), according to the parameter \(\tau\), which follows the same distribution as in the treasury bonds market. \(\overline{E}_f^r\) reflects the (exogenous) macroeconomic variables that affect the fundamental exchange rate, and it is constant for simplicity. Therefore, the fundamental exchange rate follows the process:

\[ E_r^f_{t+1} = (\tau_i + \frac{1}{n} \sum_{i=0}^n E_r^f_{t-i} + (1 - \tau_i) * \overline{E}_r^f * (1 + \rho_i (i^f_B_{t+1} - 1)) \]  

(20)

2.4.4. Trading

On the basis of their expectations, traders determine their desired portfolio composition and go to the market to buy or sell assets. They go first to the treasury bonds market (see section 2.2 about the sequential steps of the model), in which governments try to sell newly issued bonds, while traders try to buy or sell newly issued treasury bonds or bonds on the secondary market, according to the current asset price. To trade assets, each trader selects randomly and sequentially all other traders who are willing to exchange domestic (foreign) treasury bonds against domestic (foreign) currency – since they cannot exchange national (foreign) bonds against foreign (national) bonds or against foreign (national) currency, as already explained in section 2.4.1 – until his or her demand or supply of treasury bonds is exhausted, or until there are no traders left. Then, they go to the FX market and try to exchange currency B against currency A (or vice versa) following the same sequential random pairing that they followed in the treasury bonds market. Because we do not have any central auctioneer and transactions can take place at a non-market-clearing price, some traders might not be able to reach their desired portfolio composition if the current price is not the market-clearing price. Therefore, ex-post positive or negative gaps between demand and supply will simply start an adjustment mechanism leading to a higher interest rate (hence, a lower price) if the supply of treasury bonds is higher than its demand, or a lower interest rate (hence, a higher price) if the supply of treasury bonds is lower than demand. The process of updating interest rates and exchange rates entirely depends on the gap between demand and supply (\(DSG_{Bonds,j}\)).

\[
\begin{align*}
DSG_{Bonds,A} &= \frac{1}{N} \left( \sum_{i=1}^{N_A} \frac{P(A^B)_{il} \cdot (A^D_{lA} - A^B_{sA})}{W_{lA}} + \sum_{i=1}^{N_B} \frac{P(A^A)_{il} \cdot (A^B_{lB} - A^D_{sB})}{W_{lB}} \right) \\
DSG_{Bonds,B} &= \frac{1}{N} \left( \sum_{i=1}^{N_A} \frac{P(B^B)_{il} \cdot (B^D_{lA} - B^B_{sA})}{W_{lA}} + \sum_{i=1}^{N_B} \frac{P(B^A)_{il} \cdot (B^B_{lB} - B^D_{sB})}{W_{lB}} \right)
\end{align*}
\]

\[
\begin{align*}
\bar{i}^j_{t+1} &= i^j_t \cdot (1 + \delta^{ir} |DSG^{B,j}|) & \text{ if } DSG^{B,j} < 0 \\
\bar{i}^j_{t+1} &= i^j_t \cdot (1 + \delta^{ir} |DSG^{B,j}|)^{-1} & \text{ if } DSG^{B,j} > 0
\end{align*}
\]

(21)

10 The rationale is the following: if fundamentalists expect that the fundamental interest rate of country B will increase above the fundamental exchange rate of country A, they expect capital flowing from country A to country B to benefit from the higher interest rate. This will lead to a higher demand of currency B, in order to buy treasury bonds of country B, which will lead to an appreciation of currency B, thus an increase in the exchange rate.
The same applies for the foreign exchange market. *Ex-post* positive or negative gaps between demand and supply ($DSG^{Er}$) trigger an adjustment mechanism leading to a higher exchange rate (appreciation of currency B) if the demand for currency B is higher than the supply of currency B, or a lower exchange rate (depreciation of currency B) if the supply of currency B is higher than the demand for currency B.

$$\begin{align*}
DSG^{Er} & = \frac{1}{N} \left( \sum_{l=1}^{N_A} \left( \frac{M^{B,D}_l - M^{B,S}_l}{W_{l,A,t}} \right) \right) + \sum_{i=1}^{N_B} \left( \frac{M^{B,D}_i - M^{B,S}_i}{W_{l,B,t}} \right) \\
Er_{t+1} & = Er_t \times \left( 1 + \delta^{Er} |DSG^{Er}| \right) \quad \text{if } DSG^{Er} > 0 \\
Er_{t+1} & = \frac{Er_t}{\left( 1 + \delta^{Er} |DSG^{Er}| \right)} \quad \text{if } DSG^{Er} < 0
\end{align*}$$

(22)

3. Results of the simulations

3.1. Endogenous cycles and precarious equilibria

We run the model for 4000 periods, which correspond to 4000 artificial trading days, using the baseline parameters (Table A.1), and discard the first 1000 periods in order to eliminate initial fluctuations caused by the initial values, which are set arbitrarily to conform to stock-flow consistent norms (Table A.2)\textsuperscript{11}. The only exogenous stochastic shocks in the model are shocks in the demand for national currency (section 2.4.1) and shocks in the expectations about fundamental interest rates (section 2.4.3.3).

Figure 2: The US-EU exchange rate (DEXUSEU) against the artificial A-B exchange rate (DEXAB).

Figure 2 shows 1000 periods of a representative artificial time series of the exchange rate, obtained by simulating the model with the baseline parameters, as compared to the observed US-EU exchange rate between the fourth quarter of 2005 up to the third quarter of 2009.

Long cycles are characterized by a slowly increasing exchange rate, followed by a sudden fall and a new upwards cycle, and by a multitude of medium- to short-run cycles that are the rationale for the existence of multiple trading strategies. Namely, *trend-contrarians* seek to exploit the short run cycles around the

\textsuperscript{11} The model, the computation files as well as the generated data are available upon request.
medium-run trend. *Trend followers* seek to exploit the medium run cycles without reacting to short run changes in the cycle. *Fundamentalists* seek to exploit reversals in the long run trend by selling what they see as overpriced assets before the peak of the cycle.

Figure 3 shows the different reactions of trend-followers, trend-contrarians and fundamentalists in the different phases of the cycle. The left chart in Figure 3 shows that fundamentalists are the *early-bird* traders, who quit the market first, before the peak, while chartists (trend followers and trend contrarians) keep fueling the trend. Nevertheless, as soon as the fastest chartists also quit the market, the exchange rate collapses. By playing first, fundamentalists seem to play the role of the trigger.

![Figure 3: Simulated FX trading and the artificial exchange rate](image)

Figure 3 shows 1000 periods of a representative artificial time series of the exchange rate, obtained by simulating the model with the baseline parameters, against the corresponding moving average of the share of fundamentalists buying currency B (left chart), the moving average of the share of contrarians buying currency B (right chart) and the moving average of the share of trend followers buying currency B (bottom chart).

When the price is below what they think is the *fundamental* price, they open long positions and generate an upward cycle. Trend-followers detect the upward cycle and keep fueling it, while trend contrarians create some short run noise. When the price keeps rising above the *fundamental* price, fundamentalists start closing their positions and slow down the cycle, which is still fueled by trend followers and trend contrarians, until they realize that the trend is about to revert or has already reverted. As a consequence, the exchange rate and the interest rates of countries A and B, although constantly gravitating around their *fundamental price*, display a much larger volatility than these latter, consistently with one of the main stylized facts of financial markets presented in section 1 (Figure 4).
Figure 4: The artificial exchange rate and interest rates against their fundamentals

Figure 4 shows 1000 periods of representative artificial time series of the interest rate of country A (left chart), the interest rate of country B (right chart) and the exchange rate (bottom chart), against their fundamental values, obtained by simulating the model with the baseline parameters.

The origin of this larger volatility of market prices with respect to their fundamental anchors lies in the heterogeneous portfolio choices of traders and, particularly, in the interactions taking place in assets’ markets. The cycle starts within the treasury bonds markets, when traders translate news into changes of the fundamental interest rates (see Figure 5). Fundamentalists expect an increase in the fundamental interest rates of both countries (hence, a fall in the fundamental price of treasury bonds), and wish to reduce the shares of treasury bonds in their portfolios. They thus start selling treasury bonds, pushing the interest rates upwards. Because the increase in the fundamental interest rate of country B is larger than the increase in the fundamental interest rate of country A, the fundamental exchange rate also increases (see equation (20)). Consequently, fundamentalists wish to increase the share of currency B faster than the share of currency A, and they start selling currency A against currency B. This pushes also the exchange rate upwards. Chartists (trend followers) detect the positive trends and create a bandwagon effect (Figure 5, from 625 to 700). As soon as the exchange rate starts to deviate significantly from its fundamental value, the fastest fundamentalists close their positions. This leads gradually to an increase in the average desired share of currency A and a fall in the average desired share of currency B in traders portfolio, thereby weakening the trend and creating the conditions for a reversal (Figure 5, from 700 to 775). The end of this downward cycle of the exchange rate, however, does not lead to a sudden reversal but to a long period of moderate fluctuations of the exchange rate around a relatively constant trend, although below the fundamental exchange rate (Figure 5, between 775 to 875). This precarious equilibrium (Schulmeister, 1987) is the consequence of two effects: the first one is the
persistence effect created by the slowest chartists, who keep following the downward trend by selling currency B to the fastest fundamentalists, who already bet on a new upward cycle. The second effect depends on conflicting portfolio choices of fundamentalist traders. This last effect can be explained by looking at Figure 5, from period 775 to 875. The fundamental exchange rate is significantly above the current exchange rate. Fundamentalists should thus wish to decrease the share of currency A and increase the share of currency B in their portfolios, by buying currency B against currency A, thus letting the exchange rate appreciate. Nevertheless, because the interest rate of country B fell largely below its fundamental value, fundamentalists also wish to decrease the share of treasury bonds B, thus selling bonds B against currency B. As a consequence, the supply of treasury bonds B finances the demand of currency B, without need to supply currency A in the FX market. The excess of currency A, for instance, finances the (slowly) increasing demand of treasury bonds A, whose interest rate keeps falling and stabilizes around its fundamental value, such that the desired shares of currency A and B are overall stable. This generates a precarious equilibrium in the foreign exchange market, with the exchange rate fluctuating persistently below the fundamental value without any clear tendency to converge towards it. Eventually, as soon as traders stop buying treasury bonds A and close their positions, the interest rate A starts to increase. Chartists detect the positive trend and start selling treasury bonds A against currency A (from 825 to 875). The excess of currency A that follows generates a demand for currency B, which makes the exchange rate appreciate, fueled by the bandwagon effect of chartists (from 850 to 925).

Figure 5: Portfolio choices, booms, busts and precarious equilibria

Figure 5 shows 300 periods of a representative baseline simulation of the average desired portfolio shares of currencies A and B (upper left chart) and treasury bonds A and B (upper right chart). The bottom charts show the corresponding artificial time series of the exchange rate and its fundamental value, against the interest rate and its fundamental value in country A (bottom left chart) and in country B (bottom right chart).
3.2. Heterogeneous strategies vs. market selection hypothesis

According to the market selection hypothesis (MSH hereafter), heterogeneous strategies cannot coexist in the same market, since the most profitable strategy – which is the fundamentalists’ strategy consistent with rational expectations – would eventually kick the less profitable strategies out of the market. Theoretical and empirical studies reject the MSH by proving that, contrarily to conventional wisdom, financial speculators would be able to make consistent profits in real markets by using purely statistical, technical trading strategies (Brock et al, 1992; Dutta and Radner, 1999; Hsu et al, 2016; Bottazzi et al, 2018). This result is consistent with the empirical surveys that we have referred to in the first section of the paper, which show the co-existence of heterogenous trading rules within the same markets.

Our model is in line with these theoretical and empirical findings, as the portfolio composition of traders who follow different rules is relatively stable in time, a result that suggests the absence of systematic accumulation of profits and assets by a leading group of traders. Figure 6 shows the evolution of traders’ portfolios in country A. The shares of assets in traders’ portfolios follow the relative proportion of traders in the population, without any systematic increase or fall, suggesting that there is no trading strategy allowing a systematic accumulation or a systematic drain of wealth.

Figure 6: Relative portfolio composition of traders from country A.

Figure 6 shows 1000 periods of representative, baseline artificial time series of the shares of each asset held by traders of country A, relative to the amount of this asset circulating in country A, which sum to 1 by definition. However, for comparison purposes, we plot the difference between the share of the asset held by each type of traders and their proportion in the population. Thus, if fundamentalists of country A represent 70% of the population, a share of currency A equal to 0 means that they exactly 70% of currency A circulating in country A.
The boxplots of traders’ profits confirm that there are no significant differences in terms of profitability. We compute daily profits as the average rate of change of traders’ wealth for each type of agent (trend-followers, trend-contrarians and fundamentalists). Interestingly, we can observe that all trading strategies have a comparable range of gains and losses, and that none of the three types of agents is able to run, on average, significantly higher profits, a result that is in sharp contrast with the market selection hypothesis. Hence, our model supports the claim that heterogeneous strategies can co-exist and survive in the same market.

Figure 7: Boxplot of traders’ profits by type of trader

Figure 7 shows the boxplot of the average daily profit of each type of trader, in a representative simulation of the model using the baseline parameters.

3.3. Long memory, fat tails and clusters of volatility

To validate our model, we compare the statistical properties of our artificial series with the statistical properties of the corresponding empirical series. Financial markets are characterized by six main statistical properties or stylized facts (Sansone and Garofalo, 2007; Cont, 2007; Chen et al, 2012). Namely, the exchange rate (i) is more volatile than its fundamental value and (ii) follows a cyclical dynamic characterized by booms, crashes and periods of precarious equilibria (Schulmeister, 1987). Exchange rate returns (iii) are uncorrelated, (iv) cluster together, alternating periods of high volatility with periods of low volatility and (v) their distribution displays heavier tails than the Gaussian distribution. Absolute returns, on the other hand, (vi) are highly correlated and their autocorrelation function decays slowly, exhibiting long memory. A graphical inspection convinces that our model is able to reproduce most of these facts.
Figure 8: Clusters of volatility, autocorrelation of returns and absolute returns, and heavy tailed returns

Figure 8 shows the exchange rate’s daily returns (upper left chart), autocorrelation function of absolute daily returns (upper right chart), quantiles of the distribution of daily returns against gaussian quantiles (bottom left chart) and autocorrelation function of daily returns (bottom right chart), in a representative baseline simulation.

The excess volatility of market prices with respect to fundamentals, as well as the sequence of booms, busts and precarious equilibria, can be observed graphically in Figures 4 and 5, which plot the exchange rate and the interest rates against their fundamental values. Moreover, looking at Figure 8, we can see that our model is also able to reproduce most of the other stylized facts. Namely, our simulations show that the exchange rate returns tend to cluster together, alternating sustained periods of high volatility and sustained periods of tranquillity (upper left chart). The autocorrelation function of absolute returns is consistent with an exponentially decaying function, which is symptomatic of long memory (upper right chart)\(^\text{12}\). Moreover, by comparing the quantiles of the distribution of returns in our simulated series with the quantiles of a Gaussian distribution, we can see that the distribution of our artificial returns is significantly leptokurtic (bottom left chart). Nevertheless, our artificial returns show significant autocorrelation that decays too slowly with respect to empirical observation (bottom right chart), failing to account properly for this last statistical fact. To corroborate the graphical analysis, we compute some typical indices that summarize these statistical properties, and compare our artificial indices with the empirical indices in four of the biggest foreign exchange markets: the US Dollar-Euro market, the US...

\(^{12}\) Long memory is typically associated to hyperbolic decay. Nevertheless, an exponentially decaying function with slow decay is hardly distinguishable from a power law with hyperbolic decline (Alfarano et al, 2005).
Dollar-Canadian Dollar market, the US Dollar-Australian Dollar market and the US Dollar-UK Pound market. To capture volatility clustering and long memory, we refer to the parameters of the autocorrelation function of absolute returns by estimating the exponentially decaying function:

\[ \sigma_{t-1-j} = N_0 \cdot e^{-\lambda \cdot j} \]  

(23)

To capture heavy tails, we estimate the index of Kurtosis of exchange rate returns. Indeed, a heavy-tailed distribution shows a value of excess Kurtosis (that is the difference between actual Kurtosis and the value of Kurtosis of a Gaussian distribution) larger than 0\(^1\). By comparing the estimated parameters of the exponentially decaying function and the index of excess Kurtosis of exchange rate returns in our artificial simulation (DEXAB) with the empirical counterparts, we can confirm that our model produces empirically consistent exchange rate series, as shown in table 1.

Table 1: Parameters of the exponentially decaying ACF and excess kurtosis of exchange rate returns

<table>
<thead>
<tr>
<th>ACF</th>
<th>returns</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(N_0)</td>
<td>(\lambda)</td>
</tr>
<tr>
<td>DEXAB</td>
<td>0.223</td>
<td>0.012</td>
</tr>
<tr>
<td>DEXUSDEUR</td>
<td>0.108</td>
<td>0.010</td>
</tr>
<tr>
<td>DEXCADUSD</td>
<td>0.221</td>
<td>0.001</td>
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<tr>
<td>DEXUSDAUD</td>
<td>0.149</td>
<td>0.002</td>
</tr>
<tr>
<td>DEXUSDGBP</td>
<td>0.122</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 1 reports the index of kurtosis of exchange rate returns and the estimated parameters of the exponentially decaying function of absolute exchange rate returns, comparing a representative simulation of the model using baseline parameters (DEXAB), the US Dollar-Euro exchange rate (USDEUR), the Canadian Dollar-US Dollar exchange rate (CADUSD), the Australian Dollar-US Dollar exchange rate (AUDUSD) and the US Dollar-UK Pound exchange rate (USDBGP). Source: Federal Reserve Bank of Saint Louis, Economic Research Division.

4. Experiments and sensitivity analysis

In order to explore the origin of the statistical properties associated to our artificial series and test for the sensitivity of the model to changes in the values of the parameter, we perform a series of experiments by changing the initial distributions of the main parameters of the model (see Table 2).

\(^{13}\) An alternative indicator of heavy tails in the literature is the Hill index (De Grauwe and Grimaldi, 2006). The advantage of the Hill index, with respect to the index of Kurtosis, is the smaller sensitivity to outliers. Nevertheless, because we focus on the qualitative results of our model, i.e. its ability to reproduce heavy tails and clusters of volatility, and we do not calibrate the model to reproduce quantitative empirical results, we only use the Kurtosis index, which is easier to compute and provides a good approximation of the qualitative properties of our model.
We find that entry and exit thresholds for traders, captured by the parameter $s_i^Z$, explain both clusters of volatility and heavy tails, in line with the existing literature (Cont, 2007). Indeed, a larger average value of $s_i^Z$ generates heavier tails and more persistent autocorrelation functions of absolute returns, by amplifying the effect of returns that exceed the threshold, and minimizing the effect of returns below the threshold. The memory parameter of chartists – how far back in time they look when building their expectations and projecting past trends into the future – also plays a relevant role on the overall volatility. Namely, we find that a longer memory parameter for chartists, captured by the interval $(n^-, n^+)$, generates fatter tails, measured by the index of kurtosis. Moreover, a longer memory is also associated with more persistent autocorrelation of absolute returns (captured by $\lambda$ in (23)). This is explained by the fact that a shorter memory implies fast and frequent cycles for the exchange rate and overall large volatility, homogeneous through time; while a longer memory increases the length of each cycle and creates sequences of long periods of tranquility and long periods of volatility (Figure 9). In other words, if chartists have a short memory and are sensitive to short run reversals of the trend, the exchange rate is always volatile, rather than alternating between periods of tranquility and periods of volatility. On the other hand, when chartists have a longer memory and keep fueling the trend despite short run deviations, the exchange rate will be on average less volatile, and long periods of tranquility will alternate with long periods of volatility. This creates both heavier tails and clusters of volatility. A too long memory, however, stabilizes the cycle excessively, thereby reducing the kurtosis and homogenizing volatility.

We might thus conclude that the precondition for heavy tails and clusters of volatility is a sufficiently long memory to create periods of tranquility out of the storm but sufficiently short to create periods of volatility out of the dead calm. The memory parameter of contrarians, captured by the interval $(0, m^*)$, seems less significant to explain clusters of volatility, although it is highly significant to explain heavy tails, as captured by the positively increasing kurtosis. Moreover, this parameter seems to affect the shape of the exchange rate cycle significantly. In particular, as shown in Figure 10, for low values of the memory parameter, the model produces moderate exchange rate cycles, but significant clusters of volatility and excess kurtosis. As the memory parameter increases, exchange rate fluctuations become larger, without significant changes in volatility clustering, but displaying heavier tails. Nevertheless, a further increase in the parameter reduces both the amplitude of the exchange rate cycle and the overall volatility. This result suggests that contrarians play a crucial role in shaping the exchange rate cycle, and that their memory parameter should be large enough not to choke off the cycle, but small enough to affect the market price. Indeed, because contrarians follow the trend in periods of tranquility, while they bet against the trend in periods of volatility, a larger memory parameter implies that they need to observe a larger and more persistent volatility before betting against the trend. Hence, an increase (reduction) in the memory parameter produces an effect that is comparable to a reduction (increase) of the share of contrarians in the chartist population.
Figure 9: Chartists’ long memory parameter and exchange rate fluctuations

Figure 9 shows representative artificial time series of the exchange rate (upper series) and exchange rate returns (lower series), when trend-followers memory ranges between 30 and 60 days (upper left chart), between 90 and 120 days (upper right chart) and between 120 and 150 days (bottom chart), using the same random seed.

Figure 10: Contrarians’ short memory parameter and exchange rate fluctuations

Figure 10 shows representative artificial time series of the exchange rate (upper series) and exchange rate returns (lower series), when trend-contrarians’ memory is equal to 1 day (upper left chart) or ranges between 0 and 5 days (upper right chart) or between 0 and 10 days (bottom chart), using the same random seed.
We confirm this result in the next experiment, by changing the initial proportion of chartists and fundamentalists\textsuperscript{14}. We find that a stable share of fundamentalists and chartists is compatible with persistent autocorrelation functions. In other words, we do not need to introduce a herding process, nor to assume that traders shift from the less profitable to the most profitable strategies in order to generate time series that do not explode and that display a long memory, heavy tails and volatility clustering. Nevertheless, the relative share of fundamentalists and chartists plays a relevant role: the lower the share of fundamentalists (hence the larger the share of chartists), the more persistent the autocorrelation of absolute returns, as chartists will dominate the market and generate persistent periods of volatility. The share of trend-contrarians in the population of chartists also plays a relevant role: if their share increases, the autocorrelation of absolute returns becomes more significant and persistent (see Figure 11).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure11.png}
\caption{Long memory in absolute returns}
\end{figure}

Figure 11 shows the average autocorrelation functions of absolute residuals for different shares of fundamentalists on total population (left chart) and different shares of contrarians on total chartists (right chart), after running 20 simulations with 20 different random-seeds for each set of parameters.

This result shows that the interaction between trend-followers, trend-contrarians and fundamentalists is the major source of instability of the exchange rate, which is a necessary pre-condition to generate persistent periods of high volatility and persistent periods of tranquility. In addition, this instability is stronger when chartists dominate the market. Nevertheless, if this instability gets too large, the probability that the exchange rate explodes, and hence that the simulation eventually aborts, increases proportionally. The left hand-side of figure 12 shows the stability condition of the model given different shares of fundamentalists and contrarians. The model is stable with high shares of fundamentalists and low shares of contrarians, although this stability generates lower overall volatility (see Figure 12 and Table 2). For larger shares of trend-contrarians on the other hand, the model generates significant clusters of volatility and heavier tails of the distribution of returns, although the exchange rate can rapidly follow explosive dynamics, such that simulations eventually abort. The left hand-side of figure 12 shows, however, that the stability of the model increases significantly when the cash in parameter

\textsuperscript{14} In our model, the proportion of chartists and fundamentalists is seen as a structural variable, and is constant during each single simulation, but we have analyzed the results of simulations run with different shares.
ψ_{i,j} (that reflects the gap between current price and desired price, see section 2.4.2) decreases: the quicker traders close their positions, the more the exchange rate gains stability and avoids explosive dynamics. Hence, the strategy of setting a desired profit target before opening a position – which can be seen as a rational response in a world of fundamental uncertainty, with limited confidence in own expectations – provides stability to the model, by simultaneously generating volatility clusters and heavy tailed distributions of returns. This result is extremely interesting, as it shows that in financial markets that are virtually unstable because of technical trading, traders resort to simple heuristics to cope with this instability, and this is sufficient to create stability and avoid explosion. Hence, there is no need to coordinate all agents around an exogenous center of gravity (the fundamental price) through a Walrasian auctioneer, nor to assume that, at some point, chartists have to convert to fundamentalism, in order to produce realistic fluctuations and have a stable model.

![Figure 12: Stability analysis](image)

Figure 12 shows the scatter plot of successful simulations (green dots) against aborted simulations (red dots) as a function of the shares of fundamentalists and contrarians, given a cash in parameter equal to 0.5 (left chart), and the probability function of successful simulations according to the share of fundamentalists, in a logistic model, when the share of contrarians is 0.7 and the cash in parameter is equal to, respectively, 0.1, 0.2, 0.3 and 0.5.

The stability condition provided by low cash in thresholds does not weaken the ability of the model to produce heavy tails and clusters of volatility. Rather, it reinforces the qualitative properties of the model. Indeed, the lower the average value of the cash in parameter – which could be interpreted as an increase in traders’ risk aversion leading to a quicker monetization of paper profits – the more the autocorrelation of absolute returns is significant and persistent (respectively, N₀ increases and λ decreases). Moreover, excess kurtosis increases with ψ_{i,j} following a bell shaped curve: too large values of ψ_{i,j} lead to
explosive exchange rate series and constantly volatile exchange rate returns, instead of alternating periods of volatility with periods of tranquility.

Table 2: Experiments

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<th>Open/close thresholds ($s^{Z,+}$)</th>
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<th>0.0025</th>
<th>0.0035</th>
<th>0.005</th>
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<td>$N_0$</td>
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<td>0.493</td>
<td>0.440</td>
<td>0.514</td>
<td>0.353</td>
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<tr>
<td></td>
<td>(0.020)</td>
<td>(0.134)</td>
<td>/</td>
<td>(0.201)</td>
<td>(0.124)</td>
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<tr>
<td>$\lambda$</td>
<td>0.136*</td>
<td>0.115</td>
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<td>0.052</td>
<td>0.082</td>
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<td>/</td>
<td>(0.101)</td>
<td>(0.472)</td>
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<td>32.004</td>
<td>21.678***</td>
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<td>(0.000)</td>
<td>/</td>
<td>(0.044)</td>
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<table>
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<th>Technical trading long memory ($n^{-}, n^{+}$)</th>
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<th>(30, 60)</th>
<th>(60, 90)</th>
<th>(90, 120)</th>
<th>(120, 150)</th>
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<tr>
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<td>0.518**</td>
<td>0.430</td>
<td>0.440</td>
<td>0.396</td>
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<td></td>
<td>(0.182)</td>
<td>(0.042)</td>
<td>(0.417)</td>
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<td>(0.239)</td>
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<tr>
<td>$\lambda$</td>
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<td>0.139***</td>
<td>0.075</td>
<td>0.073</td>
<td>0.103</td>
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<td>(0.003)</td>
<td>(0.473)</td>
<td>/</td>
<td>(0.192)</td>
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<tr>
<td>Tails</td>
<td>Kurtosis</td>
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<td>11.526***</td>
<td>20.284***</td>
<td>32.004</td>
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<td>(0.007)</td>
<td>/</td>
<td>(0.115)</td>
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<table>
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<tr>
<th>Technical trading short memory ($0, m^{+}$)</th>
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<th>5</th>
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<th>10</th>
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<tbody>
<tr>
<td>$N_0$</td>
<td>0.265***</td>
<td>0.442</td>
<td>0.440</td>
<td>0.345**</td>
<td>0.293***</td>
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<tr>
<td></td>
<td>(0.000)</td>
<td>(0.489)</td>
<td>/</td>
<td>(0.034)</td>
<td>(0.002)</td>
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<tr>
<td>$\lambda$</td>
<td>0.042*</td>
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<td>(0.072)</td>
<td>(0.476)</td>
<td>/</td>
<td>(0.340)</td>
<td>(0.150)</td>
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<td>Tails</td>
<td>Kurtosis</td>
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<td>22.833**</td>
<td>32.004</td>
<td>33.625</td>
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<td>(0.002)</td>
<td>(0.042)</td>
<td>/</td>
<td>(0.401)</td>
<td>(0.031)</td>
</tr>
</tbody>
</table>

Fundamentalists' share

| $\lambda$                                    | 0.244*** | 0.035** | 0.084 | 0.052 | 0.070 |
|                                  | (0.005) | (0.012) | / | (0.105) | (0.318) |
| Tails                                       | Kurtosis | 12.599*** | 22.219** | 32.004 | 31.816 | 26.704 |
|                                  | (0.000) | (0.039) | / | (0.489) | (0.179) |

Contrarians' share

| $\lambda$                                   | 0.054 | 0.072 | 0.060 | 0.073 | 0.056 |
|                                  | (0.175) | (0.481) | (0.325) | / | (0.240) |
| Tails                                      | Kurtosis | 18.291*** | 26.140 | 31.828 | 32.004 | 27.138 |
|                                  | (0.003) | (0.151) | (0.488) | / | (0.179) |

Cash in parameter ($\psi_1^{+}$)

| $\lambda$                                   | 0.061 | 0.080 | 0.084 | 0.118 | 0.078 |
|                                  | (0.214) | (0.442) | / | (0.163) | (0.393) |
| Tails                                      | Kurtosis | 15.241*** | 27.423 | 32.004 | 18.238*** | 12.728*** |
|                                  | (0.004) | (0.241) | / | (0.002) | (0.000) |

Note: all values represent the average across 20 simulations with 20 random seeds. The values in parenthesis represent the p-value of the t-test relatively to the baseline scenario. Hence, a low p-value suggests that the estimated parameter is significantly different from the estimated parameter in the baseline scenario (denoted with /). * p < 0.1, ** p < 0.05, *** p < 0.01.
5. Concluding elements

This paper contributes to the theoretical literature on the determinants of exchange rate fluctuations, by building a model based on behavioral assumptions inspired by the behavioral finance literature and by empirical surveys about how foreign exchange (FX) professionals act on this market.

We have built an agent-based model in which heterogeneous traders interact according to different speculative strategies, based on fundamental and technical analysis. In particular, while fundamentalists use both fundamental and technical analysis, chartists only refer to technical analysis, and they either follow the trend or bet against it. By introducing these interactions in an artificial economy with two countries, in which traders can speculate on both exchange and interest rates, and allocate their wealth across heterogeneous assets, we are able to reproduce and explain observable features of financial markets, such as (i) the excess volatility of the exchange rate with respect to its fundamentals, (ii) booms, busts and precarious equilibria, (iii) clusters of volatility, (iv) long memory and (v) fat tails.

In particular, we find that the introduction of contrarians in standard “two-types” models, and the introduction of cash in mechanisms driving traders’ decisions to open or close their positions, both contribute to stabilize the model and simultaneously produce realistic results, despite the absence of any explicit herding mechanism or any endogenous selection of the most profitable strategy. We also find that the co-existence of heterogenous strategies explains most efficiently the stylized facts of foreign exchange markets, in comparison with artificial markets (almost) fully dominated by fundamentalists or chartists. Another finding is that the introduction of the cash in mechanism, together with this persistent heterogeneity of trading strategies, prevents explosions or implosions of the market towards, respectively, an infinite price or a flat equilibrium. These results suggest that potentially unstable markets do not need an external auctioneer or deus ex machina to be stable. The procedures and heuristics implemented by traders, aimed at exploiting profit opportunities while coping with radical uncertainty and with their own bounded rationality, are able by themselves to generate persistent fluctuations but overall stability. Our model therefore suggests considering the empirically observed increased share of technical traders as a structural and long-run feature of exchange rate markets, leading to increased complexity and instability, rather than introducing high-frequency changes from fundamentalist to technical trading, and vice versa, to explain daily fluctuations and stabilize the model.

Our model also has weaknesses, which are as many avenues for future research. Firstly, we are able to account for the vast majority of the stylized facts regarding exchange rate markets, but nevertheless fail to account for uncorrelated returns, as the autocorrelation function decays too slowly as compared to empirical evidence. The (undesired) autocorrelation of returns might be related to the very simple and
stylized behavior of our traders and, most importantly, to the absence of intraday trading in our model, which implies that statistical patterns that in real markets should fade out after a few trading hours (autocorrelation of returns, for instance) can only fade out after a few trading days in our artificial economy. A possible extension of our work might therefore consist of adapting the agents’ forecasting rules more closely to existing technical trading algorithms and fundamental analysis, and including loops in the interactions among traders to simulate intraday trading. Secondly, since the focus of this paper is not on monetary policy, the central bank is currently much too passive, as it does not react to exchange rate and interest rate fluctuations. Introducing a central bank’s reaction function and discretionary monetary policies could help getting interesting insights regarding the effect of different monetary policies on the exchange rate, a topic that is definitely crucial to economic policymaking and to the understanding of the evolution of the exchange rates. Thirdly, for the sake of thoroughness, our model is also stock-flow consistent. Nevertheless, contrarily to the tradition in SFC modelling, our macroeconomic part might be seen as overly simple. We have made this choice deliberately in order to stay focused on daily exchange rates’ dynamics. In future work, this part could be further developed in order to explore the possible interactions among the dynamics of financial markets and macroeconomic outcomes, in both directions: how the key economic sectors’ behavior can influence the exchange rate via fundamental analysis, and how, in return, the speculative dynamics of the exchange rate market might affect the productive side of an economy and its fundamentals. Our model could therefore be seen as laying the bases for the microeconomic structure of this micro-founded macroeconomic model still to be developed.

Finally, some colleagues might argue that an alternative research agenda might be to simplify our model in order to gain more specific insights on key mechanisms and be able to estimate it empirically. Unquestionably, our model is currently too complex to be estimated empirically. Nevertheless, as stated in the previous sections, we did not aim at producing a model simple enough to reproduce quantitative facts but rather a model complex enough to explain qualitative facts. Our main objective in this paper is to reproduce and provide realistic and convincing explanations of the qualitative emerging economic and statistical properties of exchange rate fluctuations, given the still limited yet increasing knowledge that we have, as economists, about the real mechanisms driving financial markets.
Acknowledgements

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References


### Appendix A

**Table A.1: Baseline parameters and initial values**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{b, ij}$</td>
<td>Share of national bonds purchased by the national central bank</td>
</tr>
<tr>
<td>$\omega_{er}$</td>
<td>Sensitivity of foreign currency’s speculative demand to expected price’s fluctuations</td>
</tr>
<tr>
<td>$\omega_{ir}$</td>
<td>Sensitivity of treasury bonds’ speculative demand to expected price’s fluctuations</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>Minimum desired share of domestic currency for traders from country A</td>
</tr>
<tr>
<td>$e^+$</td>
<td>Upper bound of the uniform distribution of the shock to demand for national currency</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>Minimum desired share of domestic currency for traders from country B</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Desired share of foreign currency when closing a position</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Desired share of domestic and foreign treasury bonds when closing a position</td>
</tr>
<tr>
<td>$\bar{\xi}_{er, +}$</td>
<td>Upper bound of the uniform distribution of the cash in parameter for foreign currency</td>
</tr>
<tr>
<td>$\bar{\xi}_{ir, +}$</td>
<td>Upper bound of the uniform distribution of the cash in parameter for treasury bonds</td>
</tr>
<tr>
<td>$s^{z, +}$</td>
<td>Upper bound of the uniform distribution of the open/close position threshold</td>
</tr>
<tr>
<td>$n^+$</td>
<td>Upper bound of the uniform distribution of long memory in technical trading models</td>
</tr>
<tr>
<td>$n^-$</td>
<td>Lower bound of the uniform distribution of long memory in technical trading models</td>
</tr>
<tr>
<td>$s^{ic, +}$</td>
<td>Upper bound of the uniform distribution of contrarians’ behavioral switching value</td>
</tr>
<tr>
<td>$m^+$</td>
<td>Upper bound of the uniform distribution of short memory in technical trading models</td>
</tr>
<tr>
<td>$m^-$</td>
<td>Lower bound of the uniform distribution of short memory in technical trading models</td>
</tr>
<tr>
<td>$\zeta^+$</td>
<td>Maximum sensitivity of expectations to the gap between fundamental and current price</td>
</tr>
<tr>
<td>$s^{f, +}$</td>
<td>Upper bound of the uniform distribution of fundamentalists’ behavioral switching value</td>
</tr>
<tr>
<td>$n_\pi$</td>
<td>Memory parameter of fundamentalists</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Probability of updating fundamental price expectations</td>
</tr>
<tr>
<td>$\tau^+$</td>
<td>Maximum anchoring of current fundamental expectations to past fundamental expectations</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Persistence parameter of the stochastic process for the determination of the shared convention</td>
</tr>
<tr>
<td>$\phi^j$</td>
<td>Shared convention about the fundamental interest rate</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>Persistence of shocks to shared conventions on fundamental prices</td>
</tr>
<tr>
<td>$\bar{E}_{r, f}$</td>
<td>Shared convention about the fundamental exchange rate</td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>Maximum sensitivity of fundamental exchange rate to the ratio of fundamental interest rates</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of traders</td>
</tr>
<tr>
<td>$N^j$</td>
<td>Number of traders from country $j$</td>
</tr>
<tr>
<td>$f, c$</td>
<td>Share of fundamentalists out of traders and share of contrarians out of chartists</td>
</tr>
<tr>
<td>$\delta_{ir}$</td>
<td>Sensitivity of the interest rate to the gap between demand and supply of treasury bonds</td>
</tr>
<tr>
<td>$\delta_{er}$</td>
<td>Sensitivity of the exchange rate to the gap between demand and supply of currency B</td>
</tr>
<tr>
<td>$i^j$</td>
<td>Initial interest rate on sovereign bonds in countries A and B</td>
</tr>
<tr>
<td>$(\alpha_j; \beta_j; \gamma_j; \Delta_j)$</td>
<td>Portfolio composition of domestic and foreign traders</td>
</tr>
<tr>
<td>$D^{i}_G$</td>
<td>Initial Government deposits</td>
</tr>
<tr>
<td>$\bar{G}^i$</td>
<td>Desired expenditure</td>
</tr>
<tr>
<td>$E_r$</td>
<td>Initial exchange rate</td>
</tr>
<tr>
<td>$M^A_{LA}, M^B_{LB}$</td>
<td>Initial national currency of traders</td>
</tr>
<tr>
<td>$M^B_{LA}, M^A_{LB}$</td>
<td>Initial foreign currency of traders</td>
</tr>
<tr>
<td>$P(B^j)B^j_{LA}$</td>
<td>Initial value of sovereign (both national and foreign) bonds of traders</td>
</tr>
<tr>
<td></td>
<td>Country A</td>
</tr>
<tr>
<td>----------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Households</td>
</tr>
<tr>
<td>Transfers</td>
<td>$+G^A$</td>
</tr>
<tr>
<td></td>
<td>$-\Delta M^A_{GA}$</td>
</tr>
<tr>
<td>Δ Deposits A</td>
<td>$-\Delta M^A_{A} * Er$</td>
</tr>
<tr>
<td>Δ Deposits B</td>
<td>$-\Delta M^B_{A} * Er$</td>
</tr>
<tr>
<td>Δ Bonds A</td>
<td>$-\Delta B^A_{A} * p(B^A)$</td>
</tr>
<tr>
<td>Δ Bonds B</td>
<td>$-\Delta B^B_{A} * p(B^B) * Er$</td>
</tr>
<tr>
<td>Δ Reserves A</td>
<td>$+H^A$</td>
</tr>
<tr>
<td>Δ Reserves B</td>
<td>$+\Delta H^A$</td>
</tr>
<tr>
<td>Σ</td>
<td>0</td>
</tr>
</tbody>
</table>

|                      | Households                         | Government                      | Central Bank | Σ   |
| Deposits A           | $+M^A_{A}$                         | $+M^A_{GA}$                      | $-M^A_{CBA}$ |     |
| Deposits B           | $+M^B_{A} * Er$                    | $-M^B_{A} * Er$                  | $+M^B_{B}$  |     |
| Bonds A              | $+B^A_{A} * p(B^A)$                | $-B^A * p(B^A)$                  | $+B^A_{CBA} * p(B^A)$ | $/(Er)$ |
| Bonds B              | $+B^B_{A} * p(B^B) * Er$           | $-H^A$                           | $-B^B * p(B^B)$ | $/(Er)$ |
| Reserves A           | $-H^A$                             | $+M^B_{CBA} * Er$               |             |     |
| Reserves B           | $-H^A$                             | $+M^B_{CBA} * Er$               |             |     |
| Net worth            | $-N W^A_{A}$                       | $-N W^A_{GA}$                    | $-N W^A_{CBA}$ |     |
| Σ                    | 0                                 | 0                                  | 0           | 0   |


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