Understanding expectational coordination as a major intellectual challenge : the “eductive” guide line

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WORKING PAPER N° 2020 – 59

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JEL Codes:
Keywords:

Funded by a French government subsidy managed by the ANR under the framework of the Investissements d'avenir programme reference ANR-17-EURE-001

Understanding expectational coordination as a major intellectual challenge : the "eductive" guide line.

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Abstract

The paper puts emphasis on the so-called "eductive" approach for a critical assessment of the Rational Expectations hypothesis. Section 2 makes an intuitive unformal presentation, aimed at comparing the approach and the results of "eductive" learning and "real time" learning in two polar models, (a two period partial equilibrium model and a simple Real Business Cycle mode). A segment of theoretical literature, taking an eductive view of stability in the fields of finance, trade, general equilibrium, short term or long term macroeconomics….is reviewed in Section 3.

Résumé

Le texte met en exergue ce que l'on peut appeler l'approche « divinatoire » pour un examen critique de l'Hypothèse d'Anticipations Rationnelles. Le paragraphe 2 compare de façon informelle et intuitive les approches et résultats de l'apprentissage « divinatoire » et de l'apprentissage « en temps réel » dans deux cas polaires (un modèle à deux périodes d'équilibre partiel et un modèle simple de cycles réels). Le paragraphe 3 passe en revue partie de la littérature théorique analysant la stabilité « divinatoire » dans les champs de la finance, du commerce international, de l'équilibre général, de la macroéconomie de court ou plus long terme

[1] I thank two anonymous referees for helpful comments.

1-Introduction.

Homo-oeconomicus has been a central reference in the construction of modern economic knowledge. Homo-oeconomicus is essentially rational: his actions are being driven by well-defined objectives and in formal models described as the outcome of utility maximization.

Homo-oeconomicus has also often been given the aptitude of making forecasts (on the collective future) which are essentially correct on average; it is the Rational Expectations Hypothesis, from now the REH, which has played an increasing explicit role in modern theoretical modelling.

The vocabulary suggests that the hypothesis is an extension to expectations of the rationality assumption. Such a terminological proximity, which may partly explain the success of the concept, is conceptually misleading. The REH is not the extension of the rationality hypothesis to expectations : it is rational to have rational expectations only if others have rational expectations. Indeed, the difference of nature in the assessment of objectives and the explanation of expectations was already underlined in the letter that Poincaré wrote to Walras in 1905 :

« Vous regardez les hommes comme infiniment égoistes (infinitely selfish) ce qui peut être une première approximation, (which may be a first approximation) mais aussi infiniment clairvoyants, (infinitely clairvoyant), ce qui est plus douteux (which is more debatable)".

The clairvoyance assumption, that Poincaré clearly stressed in Walras' construct, has been, for long, in the shadow of the economic debate. More recently, following an article of Muth (1961), it has become an object of central concern in the profession. Indeed, this article has triggered the explicit adoption in most fields of formalized economic theory, of the REH, and given an often hegemonic position to the form of clairvoyance that goes with it. At the end of the twentieth century, formal models, not only in the field of general equilibrium, but also in subfields like industrial organization, trade, finance, usually assumed that economic agents' forecasts reflected a correct image of the future..

The objective of the present paper is to present an ongoing line of research, aiming at a *theoretical assessment of the validity of the REH*[2]*.*

The plausibility of the REH has been assessed initially, let us say from the eighties - and associated for example with the names of Marcet-Sargent (1989), Evans- Honkapohja (2001) - on the examination *of real time learning* where boundedly rational agents try to guess the future from repeated examination of past data.

I have been concerned from the nineties not with the just evoked adaptive learning viewpoint but with "eductive learning". On the one-hand, *eductive learning*, in its high tech version, has *a strong game theoretical flavor* and refers to mental processes aiming at understanding the implications of

[2] The assessment of the REH is conceptually connected with the reflection of game theory around the concept of Nash Equilibrium. Next footnote emphasizes the game-theoretical origin of the "eductive" ideas.

"Common Knowledge"[3]. On the other hand, the low tech interpretation of the concepts under scrutiny puts emphasis on the elasticity of realizations to expectations in a very *intuitive* way.

- The first objective of the paper is then to provide *an introductory presentation* (Section 2) of the just introduced lines of investigation. The presentation is organized around *two polar examples*. The analysis emphasizes the "eductive" viewpoint, but puts in a similar perspective the argument and results of adaptive versus "eductive" learning.

- The second objective of the paper is to illustrate the potential intellectual impact of the "eductive" line of research (Section 3). The associated critical assessment of the REH is *already changing our views on some chapters of economic knowledge* but is also likely to drastically modify our understanding of other chapters. Indeed, the paper will show how the eductive perspective brings a number of original insights on expectational coordination. For example, the effect of opening new markets, in a finance context, in an industrial organization or in a trade context, disturbs expectational coordination, along lines on which the standard analysis is in general silent. And such new insights are particularly interesting in macroeconomic contexts.

Economics is traditionally concerned with the assessment of the allocational effects and distributional effects of policies. This paper will argue that the twentieth one century opens the road to a completely new exploration of what many think to be a blind point of theory, the coordination issue.

2- Assessing the REH, adaptive versus eductive learning, an introductory presentation.

We will make this presentation in two polar models, the textbook partial equilibrium model in which Muth's analysis takes place, and the infinite horizon general equilibrium RBC model.

We first come back, following Guesnerie (1992), on the "eductive" story developed in the elementary model of the market.

2A- Back on the text-book partial equilibrium model.

A large number of small farmers, formally a continuum of infinitesimal agents, has to decide at period zero on the size of their crop. The product will be sold to-morrow, at period 1, on a market characterized by a demand curve D(p), (D decreases when p increases) and such that the price that will take place if the quantity supplied is Q will be p/D(p) = Q, or p = D^{-1} (Q). If p were announced

[3] Indeed, the "eductive" viewpoint originates from game theory discussions. In game-theoretical terms, the approach presented here is associated with the concept of "rationalizable" solutions, as introduced by Bernheim (1984) and Pearce (1984) (a small sample of later articles on the subject includes Basu-Weibull (1991), Matsui-Oyama (2006), Weinstein-Yildiz (2007)) . Note that game theory has also put attention on "real time learning" of Nash equilibria, of which the economic counterpart is "adaptive" learning, For early connections between the two viewpoints, see Moulin (1984).

for sure to-day, total supply, the sum of individual supplies would be S(p), which would be increasing in p.

The market clearing price could be determined at date 0, through a very simple tâtonnement process, where the Walrasian auctioneer, communicating with farmers and with to-morrow buyers would take into account to-morrow demand : the market clearing price would be p*/S(p*) = D(p*). This is also the price that will emerge to-morrow, if farmers have perfect foresight, i.e if the price, that they all expect to-day, occurs to-morrow. This is Muth's relevant economic theory[4]

However, the relevance of the theory for the farmers has to be explained : why and how do they adopt the theory? Either the correct price forecast comes from a collective thought process or it comes from real time learning. The first option is associated with "eductive learning", the second one with "adaptive learning".

a) Eductive learning

Eductive learning assumes that the farmers know the world in which they live: they know the demand curve to-morrow, and they know, beyond their own supply curve the aggregate supply curve. But each farmer not only knows aggregate demand and supply, he knows that the others know, know that the others know that the others know..etc.. The information is not only known, it is Common Knowledge (from now CK).

CK is the starting point of a collective thought process. Let us describe it when the demand curve is D(p) =A- Bp and aggregate supply is S(p) = Cp

Let p(0) such that A - Bp(0) = 0. The price to-morrow cannot be greater than p(0). As everybody knows that, total supply cannot be greater than Cp(0) and everybody knows that, so that everybody knows that the price cannot be smaller than p(1)/ A-Bp(1) = Cp(0). Hence everybody knows that supply will be higher than Cp(1), hence everybody knows that the price will be smaller than p(2)/ A - Bp(2) = Cp(1) and the argument goes on starting from p(2). …As it is easily seen when C<B, p(1)>0 and p(2)<p(0), ..and the sequence p(t) oscillates above and below the equilibrium price p*, and converges to it, at a speed that increases as B increases.

When C<B , CK of the model implies CK of the market clearing price : the associated economic theory is relevant because it reflects a collective converging thought process. The equilibrium is said "eductively stable" or Strongly Rational. Two points :

-Obviously the argument applies when demand and supply are non-linear, although non-linearities may affect the conditions of convergence. When global convergence of the collective process does not hold, the criterion has a local counterpart ; can CK that the system will be in a neighborhood of the equilibrium imply CK of the equilibrium? If it is so, the equilibrium is said *"locally eductively stable"*. In the market model, local eductive stability obtains whenever S'(p*)/ D'(p*)<1

[4] We may note that Muth's argument according to which agents refer to the "relevant economic theory" is self-referential since an economic theory can be viewed as fully "relevant" only if it explains agents' expectations.

-The result found here has an intuitive economic flavor : a system is more predictable when the price elasticity of supply is not too large ; it is easier to predict what the others predict and do, when their actions are not too sensitive to predictions.

Note that the local version of the "eductive" process refers strictly speaking to the iterated elimination of dominated strategies based on CK argument. However, the first step of iteration triggers a local condition : there is a neighbourhood of the equilibrium s.t such that the state of the economy will be in this neighbourhood if everybody believes that it will be. And this condition only refers to the rationality of choices under local beliefs, and states, very roughly speaking, *that the elasticity of realizations to expectations is smaller than one.* And this condition does not, strictly speaking imply "local eductive stability" it does it often, (generically in one-dimensional settings, when the agents individual response has the same sign). We will refer to this condition as the "weak local E-stability" condition. And it obviously obtains in the market model whenever S'(p*)/ D'(p*)<1.

b) An alternative explanation of the emergence of the market clearing price is associated with *real time learning.*

Farmers at time t, have a price expectation for to-morrow, p (e, t, t+1) an expectation that differs from the yesterday's expectation p(e,t-1,t) when to-day observed equilibrium price p(t) was not correctly predicted. The change in expectations p(e,t,t+1) - p(e,t-1,t) = a [p(t) - p(e,t-1,t)] reacts, with a<1, to the expectational mistake made at time t. Equivalently, p(e,t,t+1)= a p(t) + (1-a) p(e,t-1,t).

Learning is successful when this real time process generates a sequence of prices p(t) converging towards the equilibrium price p*. In the present case of linear supply and demand, success of the learning process depends on the characteristics of supply and demand, i.e of B and C and of the coefficient that determines the adjustment a, associated in some sense with the speed of adjustment. The results are summarized as follows : When C<B, (C/B<1) the learning algorithm converges whatever a belonging to (0-1); when C/B increases beyond 1, the set of adjustment coefficients shrinks to the set (0, A*(C/B)) where A* is decreasing in C/B and tends to zero when C/B becomes large.

In a sense, the results associated with our two different stories, one associated with sophisticated collective and instantaneous thought process, the other one with a possibly long time movement, have a similar flavor : a lower value of C/B favors expectational coordination, Either because it triggers more rapid convergence of the mental process, when C/B is smaller than 1, a case where all real time learning processes converge. Or C/B is greater than 1, and real time learning converges with smaller and smaller a, when C/B increases, i.e for longer time of adjustment.

2-B Insights into a Real Business Cycle (RBC) model.

We go here to another extreme, passing from a partial equilibrium, two-period model to a general equilibrium infinite-horizon model. More precisely, following Evans-Guesnerie- Mc Gough (2018), we explore expectational coordination within a stylized RBC model, without uncertainty,. Indeed, the world consists of a continuum of infinitely lived identical agents, whose preferences are represented by a discounted sum of additively separable iso-elastic utilities. The economy produces one good per period which is either consumed or invested, and there is a time-independent production function, F(K,L), the inputs of which are capital (which depreciates with time) and labor

(in inelastic supply at each period). The perfect foresight equilibrium is associated with an infinite sequence of interest rates, r(t) and wages q(t) that generate equilibrium on the labor, capital and product markets at every period. For the sake of simplicity, we focus attention on a steady-state equilibrium where the capital stock is constant through time, so that all periods equilibria are identical.

-Can such a simple equilibrium be "eductively" stable, in the sense that CK of the model would generate a converging collective thought process ?

If we think of global "eductive stability, the answer is rather straightforwardly no. Can it be "locally eductively" stable ? Or more prosaically, forgetting about the sophistication of CK, is there a neighborhood of the intertemporal equilibrium, such that the fact that every agent believes that the economy will remain in this neighborhood, is, in a sense sustainable?

Consider the case where the neighborhood of the equilibrium K*(t) is, whatever t, of the form [K*+e, K*- e].

Ask first whether beliefs in this cylinder generate first period actions inside the time 1 interval of the cylinder. For beliefs fixed at K*+ e, for ever, (which induce a constant belief in the interest rate decrease), individual savings at the beginning of time are decreasing triggering a decrease of capital available in period 1. Will the capital decrease leave it above K*- e ? The answer is not always positive, but is so when the value of an index- denoted ϵ - that measures expectational sensitivity, is small enough. Such an index depends on the product of the second derivative of the production function, which governs the sensitivity of interest rates to the stock of capital, and of the inverse of the intertemporal elasticity of substitution of the individual utility function, which triggers the response of individual savings to the change of expected interest rates.

Naturally, we require more, i.e that beliefs in the cylinder not only generate first period aggregate plans in the cylinder but also generate aggregate plans remaining in the cylinder at any period.

In fact this is impossible: the reason is *that plans are too much sensitive to beliefs* as the following examples suggest. Suppose that beliefs are K* for any future period, but for period T, where K*+e is expected – i.e the expected interest rate decreases only at period T : the best response to such a belief is to plan a decrease in savings at the beginning, implying a constant increase of consumption during the T-1 first periods, followed by a constant decrease in consumption starting in period T+1. But such plans would generate a decrease in planned capital accumulation after T. Suppose now a T-period deviation, so that beliefs are K*+e, during the first T periods, and then come back to K*so that agents believe that, after a constant T-period decrease, the interest rate will come back to its steady state value : this will induce agents to shift consumption from the future to the present and lowers savings and capital accumulation, as in the first example. But here if the beliefs are maintained long enough to K*+ e, the planned capital will fall below K*- e. Hence beliefs close to the steady state beliefs generate plans away from the initial neighbourhood of beliefs.

It follows that the equilibrium cannot be, in the previous terminology, locally weakly E-stable with respect to the considered cylinder beliefs.

Can we find another neighborhood of the steady state equilibrium for which E-stability would obtain ? To answer the question, one can construct the (infinitely dimensional) *matrix which associates changes in planned capital at each period to* the (infinitely dimensional) *change of beliefs on the aggregate capital stock*. Such a tool provides an answer to all questions concerning the effect of changes of expectation on changes of aggregate plans. It allows to understand the many ways in which beliefs determine individual and aggregate plans and why the different dimensions of sensitivity prevent the existence of a neighborhood of trajectories, which would support beliefs generating trajectories necessarily in this neighborhood. Hence, the infinite-time equilibrium trajectory of the RBC model is never "locally weakly E-stable", a fortiori never "locally eductively stable", whatever the local restriction under consideration.

c) let us come to the real time learning viewpoint[5].

Agents at time t observe the stock of capital, and make plans associated with an aggregate index of future capital. Such an index is revised , in an adaptive way that puts a weight α to present observed capital. Does learning leads to the convergence to the steady state equilibrium? In fact, it makes sense to require that the learning process not only converges asymptotically, - asymptotic stability- but also does not go away too far from the equilibrium, (for convergence not being too long). Hence, one can look for processes remaining in the cylinder they start from. Under such a requirement, another type of impossibility result holds : whatever the parameters of the economy, real-time learning schedules cannot converge whatever the correction coefficient α belonging to [0,1]. However there are cases where convergence to the steady state, with a trajectory in the cylinder obtains : it is the B-stability case, a case which obtains when the sensitivity coefficient, denoted previously ϵ, associated with the elasticity of first-period aggregate savings to long term beliefs of the form K*+ e, is in absolute value smaller than one, and for a subset of correction coefficients, α belonging to [G(ϵ), 1]. Note also that asymptotic stability always obtain for a correction coefficient close enough to zero, (and for any correction coefficient, when $\epsilon < 1$).

B-stability, which avoids long disturbances in the learning process, is then more satisfactory but also much more demanding than asymptotic stability. It leads to results for adaptive learning which, although not uniformly negative, (B-stability obtains for a subset of cases, depending on the sensitivity coefficient and on appropriate correction coefficients) have the same flavor as the results of "eductive" stability.

The just sketched reminders of an expectational analysis of two different extreme economic models will hopefully convince the reader of the relevance of the question. Similar insigths can be obtained from the examination of a number of issues.

3- The "eductive" approach : a random walk in different chapters of economic knowledge.

[5] The text reports the real-time learning approach of Evans-Guesnerie-Mc Gough (2018), which is connected with earlier work, for example Mitra-Evans-Honkapohja (2013)

Analysing coordination from a general viewpoint is, in the present state of knowledge, unrealistic. But going into a variety of models, will enrich our understanding of the conditions of success or of failures of expectational coordination.

We will successively consider the subfields of partial and general equilibrium, trade, intertemporal macroeconomics; finance and Industrial organization.

3-A Generalities on partial Equilibrium

1- Remaining in a partial equilibrium context, of the Muth model, the fact that farmers decide successively on the size of their crop, and observe previous decisions, (as happens for winter wheat and spring wheat), makes "eductive" coordination easier. The condition C<B becomes C<TB, where T is the number of decision periods : as it is intuitively plausible and desirable, *partial observation* makes guessing easier, and improves expectational coordination. See Guesnerie (1992), (2002)

2 - The next question concerns the difficulty of coordinating expectations *when the sensitivity of the market outcome to agents' forecasts is not common knowledge.*

For example, in the Muth model, suppose that the sensitivity coefficient relating actions to expectations, C/B, depends on a state of nature w=1,2, being smaller than one in state 1, greater than one in state 2. If all farmers are perfectly informed, the equilibrium is stable in state 1, unstable in state 2. If all agents are uniformed of the occurrence of the state of nature, and if the expected sensitivity coefficient is smaller than one, then the uninformed equilibrium, with here a price being the expectation of the informed prices, is stable. Now suppose that information in state 2, is limited to a small number of agents; a continuity argument suggests that information modifies the equilibrium prices, without affecting stability. But if many agents are informed in state 2, the state 2 equilibrium fails to be predictable, and by contagion the eductive process also fails in state 1 - since the volatile beliefs of uniformed agents in state 2 affect the state 1 process. Hence in the considered situation, the arrival of information which is not CK, affects the plausibility of expectational stability . As stressed in the conclusion of Desgranges-Gauthier (2013) where the sketched argument is developed and discussed, "A government agency or a central bank revealing that the underlying sensitivity is low may destabilize the equilibrium if it cannot convince all the agents to believe its announcement".

3 -The Muth model describes a world in which agents' decisions, size of the crops, are strategic substitutes. It follows that agents react negatively to a higher expectation of the size of the crop. Many economic problems involve, *strategic complementarities*[6], it is the case, for example of bank runs, currency attacks, in which the probability of success of an attack increases with the size of the attacking group. It makes sense to contrast the "eductive" stability conditions in the two polar cases of strategic complementarities and strategic substitutabilities[7] In simple one-dimensional models with strategic complementarities, "eductive" stability of the equilibrium often obtains when the equilibrium is unique. When there are several equilibria, the set of rationalizable equilibria is a convex set containing the two extreme equilibria. In the same simple one-dimensional model with

[6] There is a large literature on "super-modular" games with strategic complementarities. Global games, presented just below display strategic complementarities.

[7] The reader may refer to Guesnerie-Jara-Moroni (2011), Harrison-Jara-Moroni (2015).

strategic substitutabilities, uniqueness does not imply local stability (as is known from the Muth model), but global stability will obtain in the absence of cycles of order 2 for the time repeated version of the model.

4-Let us now introduce the expectational questions associated with the *global games* literature, starting from Carlsonn- van Damme (1993)[8]. For that, let us consider the expectational aspects of a model of simultaneous attacks, and assume that the attack is individually costly but succeeds when the size of the attacking group is greater than a parameter b included in [0-1]. If b is public information, there are two equilibria: 1- nobody attacks, or 2- everybody (a group of size 1) attacks. Consider the case where b is not public, but where public information is transmitted by a one-dimensional signal y. For a low (resp. high) value of the signal, smaller than y*, (resp. greater than y**), the probability of a success (resp. failure) of an attack is high enough to overcome the expected cost of failure (resp. the expected benefit of success). Then coordination obtains for low ($\leq$ y*) and high values ($\geq$y**) of the signal, but multiple equilibria still occur in the interval (y*,y**)

Assume now that agents know the stochastic process which governs the value of b, and that in addition, each one, receives a private noisy signal of b, b(i) . Note that attack is a dominant strategy for agents with a low enough private signal, x $\leq$ x(1) –since their a posteriori probability that b is negative, is large enough. But those who receive x slightly above x(1), think that attack is beneficial, since the attacking group has a positive size. When x(2) increases, the benefits of attack decrease. Then there exists x(2) such that when receiving x(2), I attack, knowing that all those who have received less than x(1) attack. But the mental process goes on: the agents with signal less than x(3) attack, as they know that those who had x $\leq$ x(2) attack. And so, on… one generates an increasing sequence x(n) which converges to x*. Hence, relying on the "eductive-like" process, there exists x* such that the agents' strategy : attack iif x is smaller than x*, is an equilibrium strategy. In such a case, the size of the attacking group increases when b decreases. Then, in such an equilibrium, there exists b* s.t the attack succeeds when b $\leq$ b*, fails when b>b*. But is it unique ?

Indeed, one can prove that such an equilibrium is unique whenever the precision of the private information is high enough, compared to the variance of the process determining b, see Morris-Shin (2003). In this case, the equilibrium is globally "eductively" stable. Although perfect information goes with multiplicity of equilibria, dispersed but enough precise private information favors expectational coordination, although the equilibrium success of attack is random.

3-B From Equilibrium to general Equilibrium :

1-What about *market integration*? Although remaining in a partial equilibrium context, Calvo-Pardo, (2009) wants to shed light on the expectational effects of trade. The paper considers two regions (countries) H (home) and F (foreign), and a good market in each region. It is rather straightforward that opening the home market to foreign competitors has a positive welfare effect but is expectationally destabilizing, but conversely opening new markets to the home producers has a stabilizing effect. What happens in case of integration of markets H and F? If they are similar, same supply and demand functions, regional integration affect neither the allocation of goods nor expectational stability. It is also the case that if the demand is linear with the same price inducing

[8] This global games literature starts from Carlsson-Van Damme (1993) and includes Morris-Shin (1998),(2003)

zero demand (the same A/B in the notation of the Muth model), the integration of the autarkic expectationally stable regions is expectationally stable. But, outside this special case, it will happen that the integration of autarkic expectationally stable markets, involving gains from trade, generates expectational instability, either from a local or global viewpoint..

2- Let us consider now a *2-period exchange economy* with n goods at each period, and associated spot markets, and one asset market in period 1. The computation of an equilibrium involves no tâtonnement process since agents are supposed to transmit to an auctioneer their first period and then second period excess demand functions, from which price equilibria are obtained, but naturally expectations matter. Let us assume that, at the margin of the sequential price equilibrium, agents can assess the effect of a change of the second period price expectation, dp(2, n); assumed here to be common expectation, on the first period asset and goods equilibrium prices, dq(n), d (p,1,n) and then deduce the second period equilibrium, d(p,2,n+1) triggered by the first period change…. The process partially mimicks the "eductive" thought process, associated with the CK existence of a neighborhood of the second period equilibrium. With such an eductive viewpoint, the main sources of instability are (i) the effect of a change in asset demand on second period spot market prices and (ii) the effect on asset demand of a small change in second period prices. When these effects are weak the perfect foresight equilibrium is eductively stable. And conversely, if a perfect foresight equilibrium is eductively stable, it is often the case that these effects, in the vicinity of equilibrium, must also be weak. (see Ghosal (1994) Ghosal (2006))[9].

3-Let us examine now the *simple 2- periods production economy*, considered in Guesnerie (2001), with three goods, one final good, labor and money, the prices of which are p,w, and 1. In the first period, firms hire labor, at a market wage w. Workers in the first period, let us say with an inelastic labor supply, are consumers in the second period, and receive profits of the firms with labor income. They buy the produced good at price p and save money. A walrassian (perfect foresight) equilibrium obtains with p,w clearing all markets, w being obtained on the first period labor market, where the expectation of p is self-fulfilling in the second period, when the good market and the money markets clear.

Note that the firms' decisions depend on w, which they observe and on expected p, which depend on the volume of the first period production. This is reminiscent of the farmers' problem in the Muth model. But here an expected higher production does not face the Muth's fixed demand, but entails, because of higher distributed income, a higher associated demand, and hence a smaller decline of prices. Hence the sensitivity coefficient is smaller than the ratio of supply price-elasticity to demand-elasticity, the number S'/D' in the Muth model. Here, the sensitivity coefficient obtains as the product of the S'/D' like-ratio and of a number smaller than one. This number decreases when the multiplier effect, (which relates the income spent increase to the production increase, in other words which reflect the marginal propensity to consume), increases.

Indeed, the argument can be transposed to the case where the wage is fixed in the first period so that a Keynesian equilibrium, with excess supply of labor in the first period, and flexible price and market clearing in the second period, obtains. The sensitivity coefficient is then simply the product of

[9] See also Chapter 6 of Guesnerie (2005), (joint with H. Calvo Pardo)

S'/D' and of *the inverse of the Keynesian multiplier* (which equals 1-c, c being the marginal propensity to consume).

The success of "eductive" expectational coordination depends on a very simple inequality, either in a Keynesian or Walrassian context. Indeed, successful wage adjustment in the first period leads to a weaker stability condition - the ratio of supply and demand elasticities has to be smaller than something higher than the Keynesian multiplier[10]. So there is a sense in which wage flexibility, if successful, favors expectational coordination.

3-Let us add one word on the multi-goods version of the two-period Walrasian equilibrium[11], with n-goods, and a large number of small firms, with labor as the only input, in each sector. The stability condition generalizes the one-good condition, S'/D' becoming $(\partial S)(\partial D^{-1})$, the product of the Jacobian matrix of supply and of the inverse of the Jacobian derivative of the demand vector, (evaluated at equilibrium), and this product should be smaller than a matrix involving the income derivatives of demand. Although the formula generalizes the one-good formula, the inequality has a less straigthforward interpretation in terms of income effects.

Although, too short, the present summary puts the emphasis on a number of intuitive insights for "eductive" stability, which go beyond the 2-period setting under consideration. A high elasticity of supply is an obstacle to convergence of the mental process, the same is true for a low elasticity of demand. The income effect due to the change of income induced by the firms' decisions is good for expectational coordination, although new information has an ambiguous effect.

3-C : Back to Partial equilibrium, Finance and Industrial Organisation.

Modern finance has often been associated with an optimistic view of financial markets. For example, the so-called "efficient market hypothesis" has been assessed a number of contributions focusing on the quality of the information transmitted by financial markets. Also, a number of models stress the merits of new markets, like options markets for example, as insurance-improving devices. Most of these models, however refer to the REH, and ignore the expectational stability dimension of the problems. From this viewpoint, we will consider successively the question of information transmission and of the merits of the market completion.

a) Do markets convey reasonably well[12] the *information* available to the actors? In the litterature on information transmission under the REH, let us refer to two models.

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[10] An example of an "eductively" unstable walrasian equilibrium is provided in Guesnerie (2002). It coexists with another rationalizable equilibrium which has the following characteristics: part of the producing firms F1, the most productive, have walrasian beliefs on the equilibrium price, the other active firms F3, believe in a higher price, the one which will obtain. Such firms F3 would not produce with walrasian beliefs: indeed, they replace slightly more productive firms F2 which due to walrasian beliefs do not produce at the going wage, higher than the walrasian wage. Indeed, at this going wage the replacing firms F3 produce because the (actual) rationalizable price, is higher than the walrasian price, a fact that occurs because these replacing firms F3 are less productive than the replaced firms F2.

[11] Discussed in detail in chapter 5 of Guesnerie (2005).

[12] An extreme version of this assertion is that markets transmit all the information available to decentralized agents, which is sometimes wrongly presented as equivalent to the assertion that you cannot beat the market (see later for discussion)

The first one is presented in Grossman-Stiglitz (1980): a continuum of small agents receive information on the value of a financial asset and transmit a demand curve – demand depending on the price that will occur - to an auctioneer: such a demand curve reflects both individual information and the information transmitted by the occurrence of the price. It turns out that under Rationall Expectations, agents understand the information associated with the occurrence of any equilibrium price. In such an equilibrium a significant amount- although not all - of existing decentralized information will be reflected in prices and the market is efficient in some specific although limited sense.

The study of "eductive stability" of such an outcome, i.e of the plausibility of the REH in this context, brings interesting insigths[13]. In the CARA-Gaussian model of Desgranges (2000), where all small agents are similar but receive independent signals, *the equilibrium is "locally eductively stable" only if it does not transmit too much information*. Let us give some intuition on such a formal result. When the equilibrium transmit too much information, the mental process of elimination is basically perturbed : since the information transmitted by the market is high, in the "eductive" mental process agents tend to trust less their own information, and then to transmit it less to the market. But as the information transmitted to the market is an aggregate of individual information, there is a contradiction: in out of equilibrium reasoning, too much trust in the market information lead to reducing the information sent to this market. To say it in another way : more "informational efficiency" induces the agents to make their potential demands excessively reactive to the information that is expected to be contained in the price and to discard partly their personal information. It is why intuitively, more informational efficiency of the equilibrium makes it less plausible, plausibility being here associated with "eductive stability".

The model considered by Desgranges-Geoffard-Guesnerie (2003), is concerned with the same question, as conveyed by its title: do prices transmit rationally expected information ? The mechanism just stressed is not at work in this model. The reason is that the conflict in the relative trust in the market and in personal information can play no role: informed agents are fully informed of a two-states (B or G) true value of a financial asset, and hence transmit a demand curve that fully reflect their information (they cannot learn anything from the market) and non- informed agents are fully non-informed (they cannot transmit anything to the market).

In this model, the market excess demand is the sum of noise trading, of the linear excess demand of informed agents, and of the excess demand of non-informed agents, which in equilibrium extracts the information imperfectly, because of the random noise conveyed by prices. Under the REH hypothesis, equilibrium market excess demand is necessarily decreasing in prices.

Here, three factors favor "eductive stability". The first one is an increase of the noise variance, which affects negatively the information conveyed by the price, the second one is a diminution of the impact of information on individual demand: information transmitted should not be too large or too important, an intuition that a flavor reminiscent of the one previously emphasized. The third effect concerns the role of the number of informed agents; it favors coordination to have either a small number – hence they do not have much impact on excess demand – or a large number – information is almost public.

[13] See Desgranges (2000), Desgranges-Heinemman (2005), Desgranges (2014), and chapter 8, Guesnerie (2005)

Here are two models in which the analysis of expectational coordination provides a less optimistic view of the transmission of information through prices than when the REH is axiomatically adopted.

b) Let us stay in the field of finance and let us put attention in *the merits of the opening of new markets.*

Consider a world in which a crop falls at each period and in which storage allows to transfer wheat from a favorable time to the next one. Following Guesnerie-Rochet (1993), let us take a two-period world. Here, in period 1, supply equals the actual volume of the crop minus the stored quantity and faces an exogenous demand curve. In period 2, supply equals the random crop production plus the quantity stored and faces an exogenous demand curve. The economic actors are first primary traders who can costly store inventories, and secondary traders who cannot, but who can participate in a market for futures, if such a market exists.

Let us consider the storage problem in the absence of a futures market. Primary traders are risk averse and decide on the basis of the total storage they expect – which determines the expected difference between the random to-morrow price and the to-day price. The higher the total expected crop, the lower the selling price, and the lower the desired stored quantity.[14] The RE solution obtains when the realized inventories equals the expected total inventories. The solution has intuitive features: the lower the cost of inventories, the more inventories, the higher the risk aversion of primary traders, the lower the equilibrium level of inventories…

What about expectational coordination? "Eductive" stability intuitively obtains when the slope of the response of the level of proposed inventories to expected inventories is not too large. A more specific assessment shows, that in the context under consideration, the factors triggering a high inventories equilibrium, affect negatively its "eductive" stability

Introduce now a market for futures, taking place at the first period, in which all traders can submit a demand schedule conditional on the futures' price, when primary traders decide on the level of inventories after having observed the futures equilibrium price. In the new setting, primary traders decide on the level of inventories as if they were sold on the futures market, and in addition, as the secondary traders do, react to the random expected difference between the second period price and the futures market price.

The new equilibrium illustrate the merits of this new market opening : the variance of the wheat equilibrium price decreases. But expectational stability is more demanding. Hence, in a number of cases, the new equilibrium would be better but becomes expectationally fragile. *Speculation turns out to be destabilizing*[15].

3-Let us come back to the single market model, but in which the supply is not the competitive supply, but is associated with *Cournot competition* : what can be said on expectational stability of a Cournot like equilibrium ? Indeed, this is one of the first subjects, at the intersection of game-theory and economics, where the ideas of rationalizability and of iterated dominance solvability, have been considered (see Basu (1992)).

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[14] We are, like in the Muthian case, in the context of strategic substitutabilities.

[15] A similar conclusion obtains with real time learning, in the model of Brock-Hommes-Wagener (2009)

Following Gaballo (2013) there is a significant recent literature. Let me summarize here the paper by Desgranges-Gauthier (2016). It considers a world in which a number of small units of production has to be allocated to firms, which are competing à la Cournot. From an efficiency viewpoint, the optimal arrangement is to disseminate the units in a large number of firms, a situation which triggers a competitive outcome. The equilibrium production is smaller if the units are managed within a single firm, in situation of monopoly. In this setting, in a situation of oligopoly with n identical firms, total equilibrium production increases with the number of oligopolistic firms[16]. Increased competition increases production and welfare.

What about expectational coordination ? The monopoly outcome is the unique rationalizable outcome, hence globally "eductively stable. In an oligopoly of size m, conditions for the "eductive " stability of the Cournot equilibrium, become stronger with m, and converge to the conditions stressed in the Muth model analysed in Section 1-A. Assume that the regulator's objective is to maximizes production under the "eductive stability" constraint. When the conditions of pure competition stability are not satisfied, the solution is to have an oligopoly with m identical firms, m being the maximal size compatible with "eductive stability".

This type of argument opens a new dimension of reflection for the discussion of competition policies.

3-D Long horizon and from short term to long term Macroeconomics.

At this stage, the emphasis has been put on the short horizon dimension of problems, which can be associated with 2-period modelling. But, modern macroeconomics has often considered long horizon models, and the analysis of the infinite-horizon RBC model has stressed the difficulties of successful expectational coordination. But, many macroeconomic models refer to infinite horizon models, in which successive generations overlap, the so-called OLG models. Indeed, much more reflection has been put on expectational coordination in this setting than in any other modelling field.

1-In order to illustrate this fact, let us consider the simplistic model, which is *one-dimensional, one-step forward looking*. The state of the economy at time t equals α times a weigthed sum, over a large number of small agents, of the individual expectations of the state of the economy at time t+1. The model is one-dimensional, one step forward-looking, with no memory. The sequence y(t)=0, whatever t is a (reference) perfect foresight equilibrium. But there a continuum of perfect foresight equilibria indexed by y(0)=b, of the form $y(t)= (1/\alpha)y(t-1)$. Is one of them more plausible? The literature has provided here a number of criteria for assessing such plausibility from the viewpoint of expectational coordination.

The first one is called "*determinacy*" : an equilibrium trajectory is determinate, if there exists no other trajectory which is "close". For example, when $\alpha < 1$, the reference trajectory y(t)=0,

[16] And total production is higher when the n firms have an identical number of units

whatever t, is determinate, since other trajectories with b for example greater than zero, go increasingly away from 0. In the case $\alpha > 1$, the reference trajectory is indeterminate.

The second criterion is the absence of *stationary sunspot equilibria* in the neighborhood of the reference solution. Sunspots are stochastic exogenous events, which can be interpreted as self-fulfilling collective states of minds, which trigger equilibrium[17].

Third criterion, Iterative *E-Stability* : there is a neighborhood of the reference solution such that if beliefs are in this neighborhood, the economy remains in this neighborhood. This criterion is the closest one to the "eductive" stability test, under consideration in this paper : indeed *"eductive stability",* which requires that CK of a neighborhood implies CK of the reference trajectory, implies IE stability but *is in general more demanding*[18].

Fourth criterion, appropriately chosen[19] *evolutive learning rules*, based on the adaptive response of expectations to the observation of the state, converge to the reference trajectory.

In the simple model sketched here, the 4 criteria are equivalent in the one-dimensional, one-step forward linear model just sketched, where they select the reference trajectory, when $\alpha < 1$. The just sketched argument extends to a number of situations.

First, it remains true in non-linear versions of the just introduced one-dimensional one-step forward looking model, when we restrict attention, when needed, to the local version of the criteria under consideration.[20]

Second, a similar equivalence property obtains, when one introduces memory one in the one-step forward looking one-dimensional model, a model in which a reference solution, is, when it exists the saddle-path solution. The intuition for the extension is that such a model generates a somewhat equivalent model without memory, when attention is put on the growth rates[21].

Third, the argument generalizes in the case of the n-dimensional version of the previous one-step forward looking, memory one setting, although it leads to a less strict equivalence connection of the criteria under consideration[22].

2-Let us illustrate this dimension *of long horizon expectational stability with some insights into monetary theory, with emphasis on the Taylor rule.*

The simplest formal modelling, that serves to guide the discussion is an OLG model, in which a continuum of two-period lived infinitesimal agents is endowed with one unit of the single good at each period. The good has a money price, P(t), which determines a path of inflation rates, I(t) and the central bank implements a monetary interest rate i(m,t). Here, sequential perfect foresight equilibria

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[17] See for example Azariadis-Guesnerie (1986), and Woodford-Guesnerie (1992) and Chiapppori-Guesnerie (1996) for surveys.

[18] See Evans-Guesnerie (1993).

[19] Cf the adaptive learning rules detecting cycles of order 2, in Guesnerie-Woodford (1991)

[20] For example, the second condition refers to the absence of neighbor local sunspot equilibria.

[21] For determinacy in this setting, see Gauthier (2002), (2004). For the focus on the equivalence result see Evans-Guesnerie (2003)

[22] See Evans-Guesnerie (2005). For an overview on the whole subject, see Gauthier-Guesnerie (2005).

obtains with a constant real interest rate supporting no-trade, but possibly associated with varying inflation and monetary interest rates. The role of the central bank is to choose the rule of the game on the choice of the monetary interest rate. For that, its announcement aims at implementing a "reference" equilibrium, associated with a "target" constant inflation rate and a constant monetary rate. Such a target announcement, if believed by the agents is self-fulfilling and triggers the constant equilibrium real interest rate.

But, as the central bank cannot impose the target inflation path, the rule of the game has also to make explicit the central bank commitment on how its choice of the monetary rate will react if the rate of inflation differs from the reference rate. And such a credible announcement will trigger, besides the target equilibrium, other equilibria with varying perfectly foreseen inflation: indeed, such equilibria are governed by a relationship between present inflation and (perfectly foreseen) tomorrow inflation. If these neighbor equilibria go away from the reference target equilibrium, this one will be what we have called "determinate", which is one of the 4 conditions stressed above for expectational stability, conditions which are equivalent in this setting. Indeed, local determinacy here obtains when the derivative of the function that link the monetary rate to the observed inflation, is greater than 1 plus the equilibrium perfect foresight real rate. This condition defines the Taylor rule, a rule which is aimed at making to the central announcement of the Bank expectationally plausible.

Naturally, the sketched argument relies on the fact that the agents are short-lived. Consider the same economy with a continuum of identical infinitely lived agents, facing a constant small deviation of expected inflation and hence, given the announced rule, a constant associated increase of the future monetary and real interest rates. Given this belief, agents at period 1, would consider changing their consumption not only at period 2, but at any future period, and then would decrease, ceteris paribus, their first period consumption more than when facing the same problem with a two-period life. Hence, it is intuitively plausible that IE-stability[23], which is needed for "eductive" stability, will be more demanding in the case of long-lived agents. Indeed, it can be shown, in this context, that "eductive" stability requires that the derivative of monetary interest rate with respect to excess target inflation, has to be greater than 1+ the real interest rate -this is the Taylor rule-but smaller than a coefficient close to 1+twice the real interest rate[24] (see Guesnerie 2008)[25].

Let us come back to short-term macroeconomics. In the fixed wage two period model considered above, strategic substituabilities dominate strategic complementarities, but the latter, associated with the keynesian multiplier favor expectational stability. The global game setting, presented above, puts the emphasis on the informational aspects of coordination. It views macroeconomic equilibrium as a game between economic agents, firms, the decisions of which are not strategic substitutes but strategic complements.

For example, Morris and Yildiz (2019) model the firms' choices as a 0-1 decision reflecting the comparison of a unit cost and of a benefit. This benefit depends on the proportion of producing

[23] In the previous one-step forward-looking setting, E-stability is equivalent to determinacy. In the present setting, such an equivalence fails and E-stability is less demanding than "eductive" stability.
[24] Additional reflection is required to provide an appropriate analysis of « eductive" stability in the just sketched model. Note however that the impossibility result of "eductive" stability in the RBC model, stressed in the first section of the paper, does not hold here (because savings for production purpose are not introduced).
[25] For recent literature on this subject, let us mention Christiano-Takahashi (2020)

firms, (which brings strategic complementarities into the picture) plus an aggregate parameter plus a linear function of the signal z(i) received by the firm, a signal adding up an idiosyncratic noise and a common noise, When receiving z, an agent can derive from the knowledge of the laws of probability of the noise, the proportions of agents who have received a signal smaller than the one he received - this proportion is a simple function of z when, for example, the common noise has fat tails. It is easy to derive conditions, that will generate for example Nash-Bayesian equilibria associating invest with a signal higher than some equilibrium level. The question is when is it the case that the equilibrium strategy is the unique rationalizable one? in the present terminology, when is the solution globally eductively stable ? The answer is positive under appropriate conditions: crisis and non-crisis areas are identified. This analysis stresses the informational aspects of macroeconomic crisis. It can be reinterpreted in a dynamic context in order to assess the intertemporal dimension of the crisis , and to suggest interpretations of recent events..

4- Conclusion.

The analysis of economic problems has been associated with a number of different lines of approach. A first one puts emphasis on the allocational dimension, how efficient is the allocation of resources and the arrangement of production? A second one concerns the distributional dimension, how is income distribution determined and possibly improved? A third one has been stressed here, the expectational dimension.

The allocation dimension has been a constant subject of attention of economists. The distributional issue has been explicitely or implicitly present in the economic debate, and has been in the front line of the 20th century reflection after 1970, when the second best approach was introduced. The emergence of the expectational dimension, as a specific and large territory of investigation, is recent; but the issue is likely to remain actively explored for some time.

The plausibility of robust expectational coordination can be explored with the theoretical glasses of rationalizability and "eductive" learning. And the survey presented here focus attention on a subset of the literature on the subject which is connected to my own research in this direction[26]. Hopefully, the reader will get convinced that the line of research opens a renovated reflection in different fields, like macroeconomics[27], finance or industrial organization.

Let us briefly evoke other theoretical work going in connected directions.

[26] Recent references include Angelotos-Sastry (2020).

[27] See Kirman (2011) for a critical view and note that part of macroeconomic modelling is giving up the REH, and develops the so-called agents-based models (see Tesfation (2017). Indeed, the present effort of re-assessing macroeconomic theory, see for example Guzman-Stiglitz (2020), should benefit from progresses on expectational coordination.

1- Ongoing research on real time learning provide complementary analysis of the coordination issue, which has been stressed in Section 2, but only been briefly evoked in Section 3 and which call for more systematic comparison[28]
2- Other theoretical glasses include the reflection on herd behavior (see Banerjee (1992), Chamley (2002)) or on the role of imperfect treatment of information, as analysed in Kurz-Motolese (2001), (2011), or along the line of Frydman-Goldberg (2011). Also, the recent rise of the so-called field of "behavioral economics" challenges some dimensions of the rationality hypothesis which have consequences for the justification of the REH[29].
3- And a significantly large literature has developed on "Expectations in Experiments",[30] at the frontier between economics and game theory. Nagel (1995) introduced the "eductive" ideas in experiments. Hommes-Wagener (2011) and for example Bao-Duffy (2016) provide an experimental comparison of the eductive and evolutive viewpoints[31]

Paralleling the theoretical lines of reflection presented here, or just evoked, a lot of empirical work aims at assessing the discrepancy between expectations and realizations in different contexts and circumstances (in the abundant literature along this line, let me quote the book of Gennaioli-Shleifer (2018)). The theoretical reflection should be confronted with such empirical work more than what has been done until now. In particular, the successful attempts of linking facts with "eductive" argument, in the case of industrial organisation[32], will trigger systematic research along this line….

What will be the outcome of the development of the field ? How will expectational coordination be assessed in 30 years from now ? The answer is unclear and it may still be a long way to go.. Hopefully, the present advance provides indications on the path to follow.

[28] To the small subset of the evolutive learning literature evoked in Section 3, let me add Brock-Hommes (1997), Hommes (2011).
[29] Cf the argument associated with rational inattention, see Sims (2011)
[30] to take the title of Wagener's review (2014)
[31] The survey by Mauersberger-Nagel (2019) on the experiment literature concerned with expectational coordination puts emphasis on the first iterations of "eductive" reasoning (k-level thinking).

[32] For example, the effect of competition between airlines on the volatility of traffic tends to support the theoretical conclusions of Desgranges-Gauthier(2016). Empirical research in this direction is an active domain see Belova-Gagnepain-Gauthier- (2018)

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