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# Market share transparency, signaling and welfare: Cournot and Bertrand

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**Abstract**

When demand is noisy and firms' costs are uncertain, the availability of market share data increases the accuracy of each firm's information, and it creates incentives for signaling. Taking both effects into account, we find that under quantity competition with a homogeneous good, the availability of market share data has a positive impact on total surplus and an ambiguous one on consumer surplus. Under price competition with differentiated substitutes, it has a negative impact on consumer surplus and an ambiguous one on total surplus. If the cost difference is small, the effect of first-period signaling dominates the effect of second-period full information. Accordingly, in this case, the availability of market share data causes total and consumer surplus to increase in the case of quantity competition and to decrease in the case of price competition.

## 1 Introduction

How does firms' access to information on market shares affect market outcomes? This question is of paramount importance for antitrust policy and it is the focus of a lively debate on the proper handling of information exchanges between competitors.[1] For the most part, economists address this issue through the lens of the theory of collusion: many papers attempt to identify under which circumstances the availability of more precise market share data facilitates mutual monitoring and thus ensures compliance with a tacit or explicit collusive agreement, and whether firms have incentives to truthfully report their sales.[2]

[1] See, e.g., OECD (2010); European Commission (2011).
[2] See, e.g., Awaya and Krishna (2016, 2020) and Harrington and Skrzypacz (2011).

This paper addresses another possible channel. We ask how better information on market shares may affect non-collusive outcomes in a setting where firms interact repeatedly, but with a finite horizon. We consider competitors whose costs are private information and who compete during two periods. If demand is noisy enough, a firm cannot infer much about its competitor's actions (its choice of production or of price, depending on the mode of competition) by simply observing the market price (in a Cournot setting) or its own sales (in a Bertrand setting); and consequently it cannot infer much about its competitor's costs. However, if market share data are available, each firm can make more precise inferences on its competitors' actions, and indirectly on their costs. In such a setting, more precise information on market shares implies more precise information on rivals' costs. But this mechanism in turn gives rise to another effect, because it creates an incentive for signaling behavior. For instance, if firms producing a homogeneous good compete in quantities, each has an incentive to produce more so as to convince its competitors that its cost is low and induce them to produce less in the future. Likewise, in the case of price competition with differentiated substitutes, each firm has an incentive to raise prices in the first period, in order to signal a high cost and induce its rivals to raise their prices.

Each of these two effects is well-known (see 'Relation to the literature' hereafter). The contribution of this paper is to construct a simple and tractable model that allows us to assess their combined effect on consumer surplus and total surplus, in order to cast light on the desirability of transparency on market share data.

We first consider the case of quantity competition with homogeneous goods (Section 2). As has been established long ago, with linear demand and linear costs, transparency on costs causes total surplus to increase and consumer surplus to decrease. Also, earlier papers have shown that signalling in the first period leads to an increase in expected output. If market share data are public, then signalling occurs in the first period, and second-period competition takes place under full information on cost. We find the following results about the overall effect of these two changes: if the cost differential is so high that distortionary signaling is too costly to occur in equilibrium, then the availability of market share information only changes second-period outcomes: market share data reveal costs, which has a positive impact on total surplus and a negative one on consumer surplus. If on the contrary signaling occurs in equilibrium, then the overall effect (combining both periods) of the availability of market share data on total surplus is positive, but the effect on consumer surplus is ambiguous. However, if the cost gap (and hence the magnitude of the information asymmetry and of the signaling distortion) is very small, then the impact of the availability of market share data on consumer surplus is positive. This is because, in this case, the increase in consumer surplus caused by the signaling-driven increase in output in the first period is much larger than the second-period loss caused the availability of cost information.

We then consider the case of Bertrand competition with differentiated substitute products (Section 3). It is already known that, in the case of linear demand at least, the availability of cost information causes both social and consumer surplus to decrease. In addition, signaling takes the form of price increases. These two effects are unambiguously detrimental to consumer surplus. However, the availability of market share data has an ambiguous effect on total surplus: for some parameter values, the signaling-driven distortion brings about a reallocation of production towards the most efficient firm that causes total surplus to increase in spite of higher prices, to an extent that more than offsets the second-period decrease caused by the shift to public information on costs. An unambiguous result can however be stated if the cost gap is very small: in that case, signaling causes total surplus to fall in the first period, and the overall effect of the availability of market share data on consumer and social welfare is negative.

**Relation to the literature.** This paper is related to several branches of the industrial organization literature. Several papers assess how market outcomes are modified when cost information is public rather than private - which underlies our analysis of the second period of competitive interaction when market share data are available. In fact, our assessment of the second period is identical to that of Shapiro (1986) in the case of Cournot competition with homogeneous products, and to that of Sakai and Yamato (1990) in the case of Bertrand competition with differentiated products.[3] These two papers are part of a broader literature that examines oligopolists' incentives to share cost information.[4]

[3]Quantity competition with homogeneous products and price competition with differentiated products are the two simplest frameworks allowing one to analyze signaling and the effect of information in oligopoly. This is why most of the literature focuses on these two cases, as we do in this paper (Gal-Or 1986, however, compares Cournot and Bertrand competition within a single framework, with differentiated products). There is no meaningful way to address price competition in the 'limit case' where product differentiation is very small, because in order to avoid corner solutions (with only one firm producing), one would have to assume the cost difference to be small, and the results would depend on comparing two small variables (the cost difference and the degree of product differentiation), yielding few economic insights.

[4]See e.g., Fried (1984), Li (1985), Gal-Or (1986), Raith (1996), Myatt and Wallace (2015).

This paper is also related to the literature on simultaneous signaling. The contribution closest to ours is Mailath's (1989), which studies cost signaling in a model of price competition with differentiated products.[5] The differences between his approach and ours are his exclusive focus on Bertrand competition (whereas we study both price and quantity competition) and the choice of benchmark: whereas we compare the signaling equilibrium to one where, absent market share data, firms neither signal (in the first period) nor know their competitors' costs (in the second period), his benchmark is one where, 'for some reason', costs become common knowledge at the end of the first period. Other models of signaling in oligopoly consider the case where uncertainty is about the parameters of demand rather than costs (see, e.g., Jin, 1994). Another strand of the literature on signaling (Bonatti et al., 2017; originating with Mester, 1992) studies signaling in long-horizon models in which the underlying uncertain parameters may vary, or they extend earlier approaches to supply-function equilibria (Vives, 2011; Bernhardt and Taub, 2015). Whereas these sophisticated models probably reflect actual markets better than our simple two-period model, they do not easily lend themselves to the kind of comparative statics exercise that is the object of this paper.

# 2 The case of a homogeneous product Cournot duopoly

## 2.1 The model

### 2.1.1 Demand and costs

Two firms compete in two periods. Each firm has linear costs. Both firms' marginal costs are drawn independently from the same probability distribution: a firm's cost is either low ($c_L$), with probability $\pi$, or high ($c_H$, with $c_H > c_L$) with probability $1 - \pi$. The market-clearing price in period $t$ ($t = 1$ or $t = 2$) is given by the equation $P_t = Max\,(0; (A - Q_t)\,(1 + \varepsilon_t))$, with $A, Q_t$ and $\varepsilon_t$ denoting respectively the demand intercept, total output in period $t$, and a demand shock. The demand shock $\varepsilon_t$ has zero mean and support above -1, and

Several papers also analyze the case where the uncertainty is about demand rather than costs (e.g., Vives, 1984; Gal-Or, 1985).

[5] A closely related paper is Caminal (1990). Unlike Mailath (1989) and the present paper, it considers firm-specific demand shocks, and the main results only cover "small" amounts of uncertainty.

it is independent across periods.[6] For simplicity, the discount rate is assumed to be 1: firms maximize the expected sum of their profits in periods 1 and 2.

The game is as follows: in each period, before observing the demand shock, firms simultaneously set their quantities. At the end of period 1, firms observe the prevailing price. In Section 2.3, we also assume that firms observe total output (or, equivalently, market shares).

The cost and demand intercept parameters are assumed to be such that in the static Cournot duopoly equilibrium under full information on costs, a high-cost firm produces even if the other firm has a low cost: $c_H - c_L < A - c_H$.

The key assumption of this paper regards the distribution of the demand shock. If there were no demand shock, observing the price would allow each firm to infer its competitor's output. The more noisy demand is, the less precise this inference can be, since, for instance, a high price could be explained by different combinations of the unobserved total output level and the unobserved demand shock. We take this logic to the extreme, in order to simplify the analysis: we assume that a firm can infer nothing about total output from observing the price. This admittedly extreme assumption implies that there is no scope for cost signaling in the case where firms observe only the prevailing price, whereas there is if firms also observe total output (i.e., they can infer their competitor's output by subtracting their own ouput from the total).

Notations: $c$ denotes the expected cost ($c = \pi c_L + (1-\pi) c_H$), $\delta$ denotes the cost difference $c_H - c_L$, and $\sigma_c^2 = \pi(1-\pi)\delta^2$ denotes the variance of the cost distribution.

#### 2.1.2 Consumer and total surplus as a function of output and costs

Let $Q_t$, $q_t^i$, and $c^i$ denote respectively total output in period $t$, firm $i$'s output in period $t$ and Firm $i$'s cost. Consumer surplus and total surplus in period $t$ are given by the following formulas:

$$CS_t = (1+\varepsilon_t)\frac{Q^2}{2}$$

$$TS_t = (1+\varepsilon_t)\left(AQ_t - \frac{Q_t^2}{2}\right) - q_t^1 c^1 - q_t^2 c^2$$

[6] In fact, only the first-period shock matters for the model: it prevents each firm from inferring its competitor's behavior from the price. The model would work in exactly the same way if there were no second-period shock.

implying (using basic statistics) the following formulas for expected consumer and total surplus before cost and demand uncertainty is resolved (with $\overline{Q}$ denoting expected total output and $\sigma_Q^2$ the variance of total output, and leaving aside the period index, since the expected values are before the resolution of demand uncertainty):

$$ECS = \frac{\overline{Q}^2}{2} + \frac{\sigma_Q^2}{2} \quad (1)$$

$$ETS = (A-c)\,\overline{Q} - \frac{\overline{Q}^2}{2} - \frac{\sigma_Q^2}{2} - 2Cov(q^i, c^i) \quad (2)$$

The way in which expected consumer and total surplus vary with the variance of total output and the covariance of a firm's output level and its cost is in accordance with simple economic intuitions. In this model, output uncertainty (and equivalently, price uncertainty) results from cost uncertainty alone. Since consumers' indirect utility is a convex function of prices, greater price and output uncertainty increases expected surplus. Conversely, since total surplus is a concave function of output (an immediate consequence of the convexity of preferences), in expectation it is a decreasing function of the variance of output. Finally, since it is efficient for a firm with high costs to produce less, the covariance of a firm's output and its costs enters negatively in the formula for total expected surplus.

## 2.2 Surplus under no information and under full information

If a firm cannot infer anything about its competitor's first period output, then, with $c$ denoting the expected cost ($c = \pi c_L + (1-\pi)c_H$) and $q^*$ denoting the quantity produced in a hypothetical Cournot duopoly by firms facing the linear demand function mentioned above and both having a marginal cost of $c$ ($q^* = \frac{A-c}{3}$), the equilibrium output in each period of a firm with cost $c_i$ ($i = H$ or $i = L$) is given by

$$q_{ns}^i = q^* + \frac{c - c^i}{2}.$$

This leads to the following expected values for output, consumer and total surplus:

$$\overline{Q}_{ns} = 2q^*$$

$$ECS_{ns} = \frac{\overline{Q}_{ns}^2}{2} + \frac{\sigma_c^2}{4} = 2q^{*2} + \frac{\sigma_c^2}{4}$$

$$ETS_{ns} = 4q^{*2} + \frac{3\sigma_c^2}{4}$$

Under full information, with the notation $\{i, j\} = \{1, 2\}$, firm $i$'s equilibrium output is given by

$$q^i_{full} = q^* + \frac{2(c - c^i) - (c - c^j)}{3},$$

leading to

$$\overline{Q}_{full} = \overline{Q}_{ns} = 2q^*$$

$$ECS_{full} = \frac{\overline{Q}^2_{full}}{2} + \frac{\sigma_c^2}{9} = 2q^{*2} + \frac{\sigma_c^2}{9}$$

$$ETS_{full} = 4q^{*2} + \frac{11\sigma_c^2}{9}.$$

We find the well-known result that in the case of homogeneous Cournot competition, full information on costs decreases consumer surplus and increases total surplus.[7] This is because quantities are strategic substitutes: under full information, a firm's reaction to the other firm's cost shock mitigates the effect of this shock on total output, reducing the variance of total output, which in expectation causes consumer surplus to decrease and total surplus to increase, as per the formulas at the end of section 2.1.2. In addition, full information on costs reinforces the negative relationship between a firm's cost and its output: reasoning in terms of myopic iterative best responses, if Firm 2 knows that Firm 1's cost is high and that it will cut its output accordingly, Firm 2 increases its output, which causes Firm 1 to decrease its output further, etc. The resulting smaller value of $Cov(q^i, c^i)$ (greater in absolute value since this covariance is negative) further increases total surplus.

## 2.3 Availability of market share information and signaling

We assume now that at the end of the first period, firms observe their market shares, which is equivalent to assuming that each firm observes its competitor's output. Each firm now has an incentive to raise its output to signal lower costs, in order to induce its competitor to produce less in the next period.

We describe hereafter the corresponding equilibrium output levels. The equilibrium concept we use is the subgame perfect equilibrium, with an additional

[7] Shapiro (1986).

selection criterion, namely, Cho and Kreps' 'intuitive criterion' that rules out all implausible out-of-equilibrium beliefs to select stable equilibria.[8]

We start with the following lemma, which is proved in the appendix.

**Lemma 1.** *There exists a unique equilibrium satisfying Cho and Kreps' intuitive criterion. This equilibrium is separating in the first period in the sense that a high-cost firm and a low-cost firm produce different amounts.*

The lemma implies that in equilibrium, each firm knows its competitor's cost at the start of period 2, so that the full information outcome described above prevails in period 2.

We now characterize firms' equilibrium decisions in period 1. The logic of the reasoning is as follows. If the static (no signalling) equilibrium is such that a high-cost firm would have no interest in mimicking the low-cost firm in order to cause its competitor to believe it has a low cost, then the only equilibrium satisfying the intuitive criterion is the no-signaling equilibrium characterized in the previous section.[9] Otherwise, the output levels set by both types of firms in period 1 are such that (i) the high-cost firm's output level maximizes its short-run profit (given its correct knowledge of the equilibrium probability distribution over the possible values of the other firm's output level), and (ii) the low-cost firm's output level is such that the high-cost firm is indifferent between, on the one hand, maximizing its short-run profit and facing a second-period competitor that knows its high cost, and on the other hand mimicking a low-cost firm.

In order to characterize the equilibrium, we start by calculating the gain that a high-cost firm obtains in the second period if it causes its competitor to wrongly believe it has a low cost.

A high-cost firm (say, Firm 1) mimicking a low-cost firm in the first period expects its competitor (say, Firm 2) to produce the output that corresponds to the Cournot equilibrium quantity it would produce if Firm 1 indeed had a low cost, that is $\frac{A-c_L}{3}$ with probability $\pi$ (if Firm 2's cost is low) and $\frac{A-2c_H+c_L}{3}$ with probability $1-\pi$ (if Firm 2's cost is high). Since the maximal profit that a firm with cost $c_H$ can earn, facing a competitor producing $q$, is $\frac{(A-q-c_H)^2}{4}$, a

[8]Cho and Kreps (1987).

[9]The corresponding equilibrium is *separating* because firms with different costs produce different quantities: each firm's output reveals its type. However, we can say that there is no *signaling* in the sense that the quantities set by each firm in this case are the same are those they would set if it were impossible to reveal one's type (for instance if market share information were not available). In this paper, we use the word 'signaling' to refer to agents changing their behavior in order to have an impact on other agents' beliefs, in accordance with the literature.

high-cost firm mimicking a low-cost firm earns in period 2 an expected profit equal to

$$\frac{\pi\left(A-\frac{A-c_L}{3}-c_H\right)^2+(1-\pi)\left(A-\frac{A-2c_H+c_L}{3}-c_H\right)^2}{4}$$
$$=\frac{(A-c_H)^2}{9}\left[\pi\left(1-\frac{\delta}{2(A-c_H)}\right)^2+(1-\pi)\left(1+\frac{\delta}{2(A-c_H)}\right)^2\right].$$

Conversely, if a high-cost firm reveals its high cost, then its second-period profit is simply given by the standard formula for profit in a linear Cournot model. Since in general Firm 1's profit is equal to $\frac{\left(A-2c^1+c^2\right)^2}{9}$, the corresponding expected second-period profit is

$$\frac{\pi\left(A-2c_H+c_L\right)^2+(1-\pi)\left(A-c_H\right)^2}{9}=\frac{(A-c_H)^2}{9}\left[\pi\left(1-\frac{\delta}{(A-c_H)}\right)^2+(1-\pi)\right].$$

Let $\Delta$ denote the difference between the former and the latter expression, that is, the expected gain that a high-cost firm obtains in the second period if it causes its competitor to wrongly believe it has a low cost. A simple calculation shows that

$$\Delta=\frac{1}{9}\delta\left(A-c_H+\delta\left(\frac{1}{4}-\pi\right)\right).$$

We show in the appendix that if $\delta\geq\frac{4}{8+4\pi}(A-c_H)$, then a high-cost firm finds that the first period loss induced by producing $q_{ns}^L$ rather than $q_{ns}^H$ is greater than or equal to $\Delta$: in other words, starting from the "naive" equilibrium, i.e. the one with output levels corresponding to the case of firms maximizing their one-period profit, it would not be profitable for a high-cost firm to mimic a low-cost firm in order to manipulate its competitor's belief and earn a greater second-period profit. If this condition holds, then the only subgame-perfect equilibrium satisfying the intuitive criterion is the one such that in period 1, firms produce the output levels $q_{ns}^i$ of the no-signaling equilibrium in the first period, and the full information output levels $q_{full}^i$ in the second period.

If this condition does not hold, then in the only subgame-perfect equilibrium satisfying the intuitive criterion (which is separating, as per the above lemma), a low-cost firm engages in signaling in the first period and produces an output level greater than the one that would maximize its short-run profit: in the first period, a high-cost firm sets an output level $\widetilde{q_H}$ that maximizes its first-period expected profit, while a low-cost firm sets an output level $\widetilde{q_L}$ such that a high-cost firm is indifferent between producing $\widetilde{q_H}$ (thus revealing its cost is high and inducing its competitor to produce more in period 2) and producing $\widetilde{q_L}$, which reduces its first-period profit but causes its competitor to believe it has a low cost and to produce less in the second period. In the second period, each firm sets an output level corresponding to the full-information equilibrium.

Let $\widetilde{q} = \pi\widetilde{q_L} + (1-\pi)\widetilde{q_H}$ denote the competitor's expected output in the first period of such an equilibrium. The output level maximizing a high-cost firm's expected first period profit is $\widetilde{q_H} = \frac{A-\widetilde{q}-c_H}{2}$. The abovementioned indifference condition is equivalent to

$$(\widetilde{q_L} - \widetilde{q_H})^2 = \Delta,$$

leading to

$$\widetilde{q_L} = \frac{A - c_H - \pi\sqrt{\Delta}}{3}$$

$$\widetilde{q_H} = \frac{A - c_H + (3-\pi)\sqrt{\Delta}}{3}$$

$$\widetilde{Q} = \frac{2(A - c_H) + 2\pi\sqrt{\Delta}}{3},$$

with $\widetilde{Q}$ denoting expected first period output. These results are summarized below (see the proof in the appendix).

**Proposition 1.** *If $\frac{\delta}{A-c_H} \geq \frac{4}{8+4\pi}$, then the game in which firms observe their market shares has only one subgame-perfect equilibrium satisfying the intuitive criterion. This equilibrium is such that firms set the no-signaling output levels $q^i_{ns}$ in the first period, and the full information output levels $q^i_{full}$ in the second period. If $\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$, then the game in which firms observe their market shares has only one subgame-perfect equilibrium satisfying the intuitive criterion. This equilibrium is such that in the first period, a firm with a low (resp. high) cost sets an output level equal to $\widetilde{q_L}$ (resp. $\widetilde{q_H}$) defined by the above formulas, and the full information output levels in the second period.*

## 2.4 The effect on information on welfare

The above characterization allows us to assess the impact of the availability of market share data. If $\frac{\delta}{A-c_H} \geq \frac{4}{8+4\pi}$, then the cost gap is so large that in equilibrium, a firm has no incentive to move away from the equilibrium of the one-shot game in order to manipulate its competitor's beliefs. The availability of market share data thus only modifies the second-period outcome, namely, it causes each firm to know its competitor's cost ahead of period 2. The effect is thus the same as that of a move from no information to full information on costs: consumer surplus falls and total surplus increases.

In the other case ($\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$), the availability of market share data changes the first-period equilibrium. In the first period, the possibility of signaling its cost would induce the high-cost firm to try and mimic the low-cost firm if the expected output levels were the same as in the no-signaling equilibrium. In equilibrium, such mimicking does not occur, but in order for it not to occur, it must be the case that the first-period equilibrium output of a low-cost firm is so high that mimicking would be too costly for a low-cost firm. Intuitively, this suggests that in the first period, total output, and therefore consumer and total surplus, should be greater than in the no-signaling equilibrium. Proposition 2 (proved in the appendix) confirms this.

**Proposition 2.** *If the parameters are such the availability of market share data changes the first-period equilibrium ($\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$) then, in the first period, total expected output, expected consumer surplus and expected total surplus are greater if market share information is available than if it is not. As a result, overall expected total surplus (considering both periods combined) is greater if market share information is available.*

A corollary of this result is that making market share information available unambiguously increases expected total surplus, whatever the model parameters: the second-period effect that is present irrespective of parameter values (the shift to full information) and the first-period effect that is present only if the cost difference is not too large both tend to increase total surplus.

In contrast, there is no general result on the impact of the availability of market-share information on consumer surplus. By continuity, the result stated in Proposition 1 implies that if the condition $\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$ barely holds (in the sense that the difference between the two sides of the inequality is small), then the incentive for a high-cost firm to mimick a low-cost firm is weak and the equilibrium output levels in period 1 are barely different from those in the no-signaling case. If that is the case, then consumers' second-period loss (caused by the shift to public information on costs) dominates their small first-period gain and, overall, expected consumer surplus falls.

However, an unambiguous result holds if the cost gap is small enough: if that is the case, then considering both periods combined, consumer surplus increases.

**Proposition 3.** *If the cost gap is small relative to high-cost firms' margins* ($\frac{\delta}{A-c_H} << 1$)*, then, taking both periods into account, expected consumer surplus is greater if market share information is available than if it is not.*

The mechanism behind the proof (presented in the appendix) is intuitive: the second-period expected gain induced by having the competitor believe one has a low cost has the same order of magnitude as the cost difference (times the margin). But a high-cost firm's first-period loss from distorting its output to mimick a low-cost firm has the order of magnitude of the square of the output increase, because it is in the vicinity of the profit-maximizing output level. Therefore, the indifference condition stated above implies that the first-period output increase of low-cost firms has the same order of magnitude as the square root of the cost difference (times the square root of the margin). If the cost gap is small relative to the margin, this implies that, in absolute value, the (positive) first-period effect of signaling on consumer surplus is much greater than the (negative) second-period effect.

# 3 The case of a differentiated Bertrand duopoly

We consider hereafter the case of price competition, relying on a a symmetric Bertrand duopoly model with linear demand and differentiated substitute products.

## 3.1 The model

### 3.1.1 Demand and costs

The assumptions on the timing of the game and on the distribution of firms' costs, as well as the corresponding notations, are the same as in the previous section.

In each period, demand for product $i$ is given by the following function (with $\{i,j\} = \{1,2\}$): $q_i = Max\,(0; (1 - p_i + \beta p_j)\,(1+\varepsilon))$, with $p_k$ and $\varepsilon$ denoting respectively firm $k$'s price ($k = 1$ or $k = 2$) and a demand shock with zero mean and support above -1 (which can be assumed to be identical in both periods without loss of generality). The parameter $\beta$ captures the degree of substitutability between both goods. Firms maximize the expected sum of their profits in periods 1 and 2.

Just like in the previous section, $c$, $\delta$ and $\sigma_c^2$ denote respectively the expected cost $(\pi c_L + (1-\pi)c_H)$, the cost difference $(c_H - c_L)$, and the variance of the cost distribution $(\pi(1-\pi)\delta^2)$.

The game is as follows: in both periods, firms simultaneously set their prices. At the end of period 1, each firm observes its sales and, in one of the two models we will compare, total output as well (or, equivalently, the other firm's sales or its price). Firms then compete in prices again in period 2.

The cost and demand intercept parameters are assumed to be such that in the static Bertrand duopoly equilibrium under full information, consumers purchase both goods in equilibrium irrespective of firms' costs.[10]

Just like in the case of Cournot competition, we asssume that the distribution of the demand shock is such that a firm can infer almost nothing about its competitor's price by observing its own sales, so that there is (almost) no scope for cost signaling if a firm observes only its own sales, whereas information on total output (in additional to knowing its own price and sales) allows a firm to infer its competitor's price.[11]

### 3.1.2 Consumer and total surplus as a function of output and costs

The above demand function corresponds to the following formulas for prices, the utility function, consumer surplus and total surplus:

$$p_i(1+\varepsilon) = -\frac{q_i}{1-\beta^2} - \frac{\beta q_j}{1-\beta^2} + \frac{1}{1-\beta}$$

$$(1-\beta)(1+\varepsilon)U = q_1 + q_2 - \frac{q_1^2+q_2^2}{2(1+\beta)} - \frac{\beta q_1 q_2}{1+\beta}$$

$$(1+\varepsilon)CS = \frac{1}{1-\beta} - (p_1+p_2) + \frac{p_1^2+p_2^2}{2} - \beta p_1 p_2$$

[10]One can check that this is equivalent to $c < \frac{1}{1-\beta} - \frac{2\beta(1+\pi)+(4-2\beta^2)\pi}{(1-\beta)(2+\beta)}\delta$.

[11]The assumption that a firm cannot observe its competitor's prices nor infer them from its own sales volume is in accordance with a vast theoretical literature on 'secret price-cutting' in collusion, starting with Stigler (1964), who notes that while consumers may have an incentive to disclose the low prices offered by a firm to another firm, they may in some markets have trouble doing so credibly. The discussion of this assumption in Kandori and Matsushima (1998, p. 628) applies perfectly to this paper, once one replaces the words "secret price cutting" (that refer to some hypothetical collusive price) with "low prices": *"Firms cannot directly observe others' effective prices. However, each firm can observe its own sales level, which serves as an imperfect signal about other firms' pricing behavior. If sales are low, for example, it may be an indication of other firms' secret price cutting. Or, it may just be the case that market demand is low."*

$$(1+\varepsilon)TS = \frac{1}{1-\beta} - (c_1+c_2) + (c_1-\beta c_2)\, p_1 + (c_2-\beta c_1)\, p_2 - \frac{p_1^2+p_2^2}{2} + \beta p_1 p_2,$$

implying the following equalities for expected consumer and total surplus (with $\overline{p}$ denoting each firm's expected price - the two expected prices are identical because of the symmetry of the model):

$$ECS = \frac{1}{1-\beta} - 2\overline{p} + (1-\beta)\overline{p}^2 + \sigma_p^2 - \beta Cov\,(p_1, p_2)$$

$$ETS = \frac{1}{1-\beta} - 2c + 2(1-\beta)c\overline{p} + 2Cov(p_1, c_1) - 2\beta Cov(p_1, c_2) - (1-\beta)\overline{p}^2 - \sigma_p^2 + \beta Cov\,(p_1, p_2)$$

## 3.2 Surplus under no information and under full information

If firms cannot infer anything about their competitor's first-period price, then, with $c$ denoting the expected cost ($c = \pi c_L + (1-\pi)c_H$) and $c^i$ denoting firm $i$'s marginal cost, the equilibrium price in each period is given by

$$p_{ns}^i = p^* + \frac{c^i - c}{2},$$

(with the notation $p^* = \frac{1+c}{2-\beta}$ and $\{i,j\} = \{1,2\}$), implying $\sigma_{p,ns}^2 = \frac{\sigma_c^2}{4}$, $Cov_{ns}(p_i, c^i) = 1/2$ and $Cov_{ns}(p_i, c^j) = Cov_{ns}\,(p_1, p_2) = 0$.

This leads to the following expected values for prices, consumer and total surplus:

$$\overline{p}_{ns} = p^* = \frac{1+c}{2-\beta}$$

$$ECS_{ns} = CS^* + \frac{\sigma_c^2}{4}$$

$$ETS_{ns} = TS^* + \frac{3\sigma_c^2}{4},$$

where $CS^*$ and $TS^*$ denote respectively consumer surplus and total surplus when both firms have cost $c$ ($CS^* = \frac{1}{1-\beta} - \frac{2(1+c)}{(2-\beta)} + \frac{(1-\beta)(1+c)^2}{(2-\beta)^2}$ and $TS^* = \frac{1}{1-\beta} - 2c + \frac{2(1-\beta)c(1+c)}{(2-\beta)} - \frac{(1-\beta)(1+c)^2}{(2-\beta)^2}$).

Under full information, with the notation $\{i,j\} = \{1,2\}$, firm $i$'s equilibrium price is given by

$$p_{full}(c^i, c^j) = p^* + \frac{2}{4-\beta^2}\left(c^i - c\right) + \frac{\beta}{4-\beta^2}\left(c^j - c\right),$$

leading to

$$\sigma^2_{p,full} = \frac{\left(4+\beta^2\right)}{\left(4-\beta^2\right)^2}\sigma_c^2$$

$$Cov_{full}(p_1, c^1) = \frac{2}{\left(4-\beta^2\right)}\sigma_c^2$$

$$Cov_{full}(p_1, c^2) = \frac{\beta}{\left(4-\beta^2\right)}\sigma_c^2$$

$$Cov_{full}\left(p_1, p_2\right) = \frac{4\beta}{\left(4-\beta^2\right)^2}\sigma_c^2$$

$$\overline{p}_{full} = \overline{p}_{ns} = \frac{1+c}{2-\beta}$$

$$ECS_{full} = CS^* + \frac{\left(4-3\beta^2\right)\sigma_c^2}{\left(4-\beta^2\right)^2}$$

$$ETS_{full} = TS^* + \frac{\left(2\beta^4 - 9\beta^2 + 12\right)\sigma_c^2}{\left(4-\beta^2\right)^2}$$

It turns out that under full information, both consumer surplus and total surplus are lower than under no information:

$$ECS_{full} - ECS_{ns} = -\frac{\beta^2\left(\beta^2+4\right)}{4\left(4-\beta^2\right)^2}\sigma_c^2$$

$$ETS_{full} - ETS_{ns} = -\frac{\beta^2\left(12-5\beta^2\right)}{4\left(4-\beta^2\right)^2}\sigma_c^2,$$

which are both strictly negative since $0 < \beta < 1$. These findings, which are identical to those in Sakai and Yamato (1990), can be interpreted as follows. With substitute products and price competition, prices are strategic complements. Full information on costs thus increases the variance of prices: prices are higher (resp. lower) when both firms have high (resp. low) costs and know it, than when they do not know each other's costs. Since total surplus is concave function of each price, this increased variance causes expected total surplus to fall relative to the no-information case, just like in the case of Cournot competition, the decrease in the variance of prices and quantities (caused by the fact that quantities are strategic substitutes) causes expected total surplus to rise.

The mechanism regarding the effect on consumer surplus is less intuitive because it involves two opposing effects. On the one hand, the greater variance of each price, by itself, cause consumer surplus to rise. However, the fact that prices are strategic complements also implies that, whereas in the no-information case the covariance of both prices is zero, the presence of information makes it positive. This effect is detrimental to consumer surplus, because consumers lose more from a high price when that high price affects a product in high demand, which is precisely the case if the other product's price is high. The calculation above shows that thie detrimental effect dominates, hence the negative effect on expected consumer surplus.

### 3.3 Availability of market share information and signaling

Just like in the previous section, we assume now that at the end of the first period, firms observe their market shares, which is equivalent to assuming each firm observes its competitor's price. Each firm now has an incentive to raise its price to signal high costs, in order to induce its competitor to set a high price in the next period. We describe hereafter the corresponding equilibrium price levels, with the same equilibrium concept (subgame perfection with the intuitive criterion to eliminate certain counterintuitive equilibria).

The results follow the same logic as in the previous section: if the static (no signalling) equilibrium is such that a low-cost firm would have no interest in mimicking the high-cost firm in order to cause its competitor to believe it has a high cost, the only equilibrium satisfying the intuitive criterion is the no-signaling equilibrium. Otherwise, the prices set by both types of firms are such that (i) the low-cost firm maximizes its short-run profit given its (correct) knowledge of the probability distribution over the possible values of the other firm's price, and (ii) the low-cost firm is indifferent between maximizing its short-run profit and facing a second-period competitor that knows its low cost, or mimicking a high-cost firm and manipulating its competitor's belief.

Unlike in the case of quantity competition, one cannot prove in general that no pooling equilibium satisfies the intuitive criterion. This result holds, but only under a certain condition, spelled out below.[12] This condition is

$$\left(\frac{1-(1-\beta)c}{2-\beta}\right)^2 > \frac{2\delta}{2-\beta} + \frac{2\beta\left(2(1-\pi)\delta\right)^{1/2}}{(2-\beta)\left(4-\beta^2\right)^{1/2}}\left(\frac{2-2(1-\beta)c}{2-\beta} + \frac{4-2\beta^2}{4-\beta^2}(1-\pi)\delta\right)^{1/2} \tag{3}$$

$$+\frac{\beta^2\pi\delta}{2(4-\beta^2)}\left[2\frac{1-(1-\beta)c}{2-\beta} + \delta\left(1-\pi+\frac{\beta^2\pi}{4-\beta^2}\right)\right] - \left(\frac{1-(1-\beta)c}{2-\beta}\right)^2$$

**Lemma 2.** *If inequality (3) holds, then if an equilibrium satisfies Cho and Kreps' intuitive criterion, the corresponding first-period prices set by the high-cost and the low-cost firms are different.*

In the remainder of the paper, we assume that condition (3) holds.

We prove now that there exists exactly one (separating) equilibrium satisfying the intuitive criterion. Its characterization is based on a reasoning that is the mirror image of the one in the case of quantity competition. Let $\rho^*$ denote the profit that each firm would earn if costs were both equal to $c$: $\rho^* = \left(\frac{1-(1-\beta)c}{2-\beta}\right)^2$. If Firm 1 sets its price to maximize its profit (as it does in equilibrium, in the second period), then its profit depends on its own cost and its competitor's price as follows: $Profit(c_1, p_2) = \left(\sqrt{\rho^*} - \frac{c^1-c}{2} + \frac{\beta(p_2-p^*)}{2}\right)^2$. Let $\Gamma$ denote the expected second-period gain that a low-cost firm (say, Firm i) earns from having its competitor (Firm j) believe its cost is high rather than low. It is equal to

$$\begin{aligned}
\Gamma &= EProfit\left(c_L, p_{full}\left(c^j, c_H\right)\right) - EProfit\left(c_L, p_{full}\left(c^j, c_L\right)\right)\\
&= E\left(\sqrt{\rho^*} - \frac{c_L-c}{2} + \frac{\beta}{2}\left(\frac{2(c^j-c)}{4-\beta^2} + \frac{\beta(c_H-c)}{4-\beta^2}\right)\right)^2\\
&\quad - E\left(\sqrt{\rho^*} - \frac{c_L-c}{2} + \frac{\beta}{2}\left(\frac{2(c^j-c)}{4-\beta^2} + \frac{\beta(c_L-c)}{4-\beta^2}\right)\right)^2
\end{aligned}$$

$$= \frac{\beta^2\delta}{2(4-\beta^2)}\left(2\sqrt{\rho^*} - (c_L-c) + \frac{\beta^2(c_H+c_L-2c)}{2(4-\beta^2)}\right) = \frac{\beta^2\delta}{4-\beta^2}\left(\sqrt{\rho^*} + \frac{8(1-\pi)+(4\pi-3)\beta^2)}{4(4-\beta^2)}\delta\right)$$

We first investigate under which conditions the prices in the static no-information equilibrium can be first-period equilibrium prices in the first period of the game in which firms observe their market shares. This is the case

[12]This difference is because in the case of quantity competition, very high ouput levels can lead to arbitrarily large losses, whereas in the case of price competition, very high prices only lead to zero profits. For some 'candidate' pooling equilibria, this difference makes it harder to construct a deviation that 'undoes' it by virtue of the intuitive criterion in the case of price competition.

if a low-cost firm finds that the first-period loss induced by setting $p^H_{ns}$ rather than $p^L_{ns}$ (assuming its competitor's expected price is $p^*$) is greater than or equal to the above expression. Since this first-period loss is $\left(p^H_{ns} - p^L_{ns}\right)^2 = \frac{\delta^2}{4}$, the no-signaling prices correspond to an equilibrium if and only if $4\Gamma \leq \delta^2$, or equivalently $\delta \geq \frac{\beta^2(2+\beta)(1-(1-\beta)c)}{16(4-\beta^2)-\beta^2(24-8\pi-\beta^2(7-4\pi))}$. Proposition 4 (proved in the appendix) shows that if this condition holds, then the first-period price levels of the no-signaling equilibrium also characterize the only separating equilibrium satisfying the intuitive criterion.

**Proposition 4.** *If $4\Gamma \leq \delta^2$, or equivalently $\delta > \frac{\beta^2(2+\beta)(1-(1-\beta)c)}{16(4-\beta^2)-\beta^2(24-8\pi-\beta^2(7-4\pi))}$, then the game in which firms observe their market shares has only one subgame-perfect separating equilibrium satisfying the intuitive criterion. This equilibrium is such that firms set the no-signaling price levels in the first period, and the full information prices in the second period.*

Notice that for some parameter values, both the condition in Proposition 4 and condition (3) hold. In this case, the separating equilibrium mentioned in Proposition 4 is the only equilibrium satisfying the intuitive criterion.

A similar reasoning shows that if the condition stated in Proposition 4 does not hold (if $4\Gamma > \delta^2$), then there also exists a unique separating equilibrium satisfying the intuitive criterion (Proposition 5 below, proved in the appendix). In the first period, according to this equilibrium, a low-cost firm sets price $\widetilde{p_L}$ that maximizes its first-period expected profit, whereas a high-cost firm sets an output level $\widetilde{p_H}$ such that a low-cost firm is indifferent between setting price $\widetilde{p_L}$, thus revealing its cost is low and inducing its competitor to set a lower price in period 2, or setting price $\widetilde{p_H}$, which reduces its first-period profit but causes its competitor to set a higher price in period 2. In such an equilibrium, after costs have been revealed in the first period, firms in the second period set the full information equilibrium prices displayed above.

Let $\widetilde{p} = \pi\widetilde{p_L} + (1-\pi)\widetilde{p_H}$ denote the competitor's expected price in the first period of such an equilibrium. The price maximizing a low-cost firm's expected first period profit is $\widetilde{p_L} = \frac{1+\beta\widetilde{p}+c_L}{2}$. The abovementioned indifference condition is equivalent to

$$(\widetilde{p_L} - \widetilde{p_H})^2 = \Gamma,$$

leading to

$$\widetilde{p_L} = \widetilde{p} - (1-\pi)\sqrt{\Gamma}$$

$$\widetilde{p_H} = \widetilde{p} + \pi\sqrt{\Gamma},$$

with the notation

$$\widetilde{p} = \frac{1+c}{2-\beta} + \frac{(1-\pi)\left(2\sqrt{\Gamma}-\delta\right)}{2-\beta}.$$

**Proposition 5.** *If* $4\Gamma > \delta^2$, or equivalently $\delta < \frac{\beta^2(2+\beta)(1-(1-\beta)c)}{16(4-\beta^2)-\beta^2(24-8\pi-\beta^2(7-4\pi))}$, *then the game in which firms observe their market shares has only one subgame-perfect separating equilibrium satisfying the intuitive criterion. This equilibrium is such that in the first period, a firm with a low (resp. high) cost sets an price equal to* $\widetilde{p_L}$ *(resp.* $\widetilde{p_H}$*) defined by the above formulas, and the full information price levels in the second period.*

Notice that there exist parameter values such that both condition (3) and the condition stated in Proposition 5 hold, implying that the separating equilibrium mentioned in Proposition 5 is the only equilibrium satisfying the intuitive criterion.

### 3.4 The effect of information on welfare

Assessing the impact of the availability of market share data is more straightforward than in the case of Cournot competition with homogeneous products. As seen above, it causes a shift from the no-information to the full information equilibrium in the second period, which is detrimental to both consumer and total surplus. If the condition of Proposition 4 holds ($4\Gamma \leq \delta^2$), this is the only effect since firms have no incentive to deviate from the prices in the no-signaling equilibrium.

If on the contrary the condition of Proposition 5 holds ($4\Gamma > \delta^2$), then in addition there is an effect in the first period, as signaling takes place. Proposition 5 implies that both the high-cost firm's and the low-cost firm's prices are greater in the first period than in the no-signaling, no-information equilibrium: $\widetilde{p_L} > p^L_{ns}$ and $\widetilde{p_H} > p^H_{ns}$.[13] Consumer surplus is therefore less in the first period if market share data are available (so that firms engage in signaling) than if they are not.

In contrast, there is no general result regarding total surplus. Even though the availability ot market share data causes total surplus to fall in the second

[13] The expected price is greater since $\widetilde{p} - p^* = \frac{(1-\pi)(2\sqrt{\Gamma}-\delta)}{2-\delta} > 0$. The low-cost firm's price is in both cases a best response to this expected price, which implies it is greater because prices are strategic complements: $\widetilde{p_L} > p^L_{ns}$. Finally, $\widetilde{p_H} = \widetilde{p_L} + \sqrt{\Gamma} > p^L_{ns} + \sqrt{\Gamma} > p^L_{ns} + \frac{\delta}{2} = p^H_{ns}$.

period, and prices to rise in the first period, it can still cause total surplus to rise taking both periods into account. This is because signaling increases the difference between the high-cost firm's and the low-cost firm's prices, which increases the gap between the efficient and the inefficient firm's output level. One can show that for some parameter values, this latter effect may be strong enough to offset both the adverse effect of the average price increase in the first period and the adverse effect of more information in the second period.[14] However, a general result can be stated when the cost difference is very small: in that case, the availability of market share data unambiguously causes total surplus to fall in both periods.

**Proposition 6.** *If market share information is available, then first-period prices increase, expected consumer decreases in both periods and expected second-period total surplus decreases. There is no general result on the impact on expected first-period total surplus or expected overall total surplus (combining both periods). However, if the cost difference $\delta$ is small enough, then the availability of market share information causes expected total surplus to decrease in both periods.*

## 4 Conclusion

The above findings show that when assessing the possible impact of market share transparency on non-collusive market outcomes, that is, taking into account both the effect of increased transparency about costs that of increased incentives for firms to signal their costs, competition authorities should be lenient in the case of quantity competition and more concerned in the case of price competition. In the former case, the availability of market share information unambiguously increases total surplus, and it raises consumer surplus if the range of possible costs is narrow enough; whereas in the latter, it unambiguously causes consumer surplus to fall, and it also causes total surplus to fall if the range of possible costs is narrow enough.

Another general finding is that, in order to assess the combined effect of cost signaling and of cost transparency, it is sufficient to focus on the former if the range of possible costs is narrow enough, since it tends to dominate the latter.

[14]In a somewhat different setting, Mailath (1989) proves that price signaling may increase total first-period surplus even though it leads to higher prices. Our result shows that the corresponding increase can be large enough to offset the second-period adverse impact of information disclosure on total surplus.

Whether these results carry over to other settings (in particular, to the case of demand uncertainty) should be the focus of future research.

# Appendix

**Proof of Lemma 1.** There exists no pooling equilibrium satisfying the intuitive criterion. If there were one, with output level $q_{pool}$ for both types of firms, then the corresponding output would have to be a best-response to itself for the high-cost firm. Let $g_{ij}$ denote the expected profit, in the equilibrium of the one-period Cournot competition game, of a firm with cost $c_i$ facing a competitor believing it has cost $c_j$, and let $g_{i,pool}$ denote the expected profit, in the equilibrium of the one-period Cournot competition game, of a firm with cost $c_i$ facing a competitor not knowing its cost (thus believing it has low cost with probability $\pi$). The competitor's expected cost is $c$ and its expected output is $\frac{A-2c+c_j}{3}$, so that $g_{ij} = \frac{(A-c)^2}{9}\left(1 - \frac{3(c_i-c)+(c_j-c)}{2(A-c)}\right)^2$ and $g_{i,pool} = \frac{(A-c)^2}{9}\left(1 - \frac{3(c_i-c)}{2(A-c)}\right)^2$.

Consider the function $f_{ij}(q, q')$ defined as the combined (two-period) expected profit of a type $i$ firm ($i = H$ or $i = L$) setting output $q$ in period 1 while its competitor's expected period 1 output is $q'$ and inducing its competitor to believe it has cost $j$: $f_{ij}(q, q') = q\,(Max\,(0, A - q - q') - c_i) + g_{ij}$, and define $f_{i,pool}(q) = q\,(Max\,(0, A - 2q) - c_i) + g_{i,pool}$. Define the function $z_i(q) = f_{iL}(q, q_{pool}) - f_{i,pool}(q_{pool})$. $z_i(q)$ is the net gain a firm with cost $c_i$ would earn from inducing its competitor to believe it has a low cost while producing $q$ in period 1, relative to the pooling equilibrium output level and induced belief. Its definition implies that $z_i(q_{pool}) = g_{iL} - g_{i,pool} > 0$ and $\underset{q\to\infty}{Lim} z_i(q) = -\infty$. Also, the above formulas imply that $(z_L - z_H)(q_{pool}) = (g_{LL} - g_{L,pool}) - (g_{HL} - g_{H,pool}) = \frac{(c_H-c_L)(c-c_L)}{6} > 0$. Finally, notice that $\frac{\partial(z_L-z_H)}{\partial q} = c_H - c_L > 0$. These observations imply that there exists some $q_{dev}$ such that $z_H(q_{dev}) < 0 < z_L(q_{dev})$.

A low-cost firm setting $q_{dev}$ is a deviation that makes the postulated pooling equilibrium violate the intuitive criterion because, assuming the other firm believes that a firm producing $q_{dev}$ has a low cost, then only a low-cost firm has an interest in producing $q_{dev}$ and cause its competitor to believe it has a low cost with probability 1, rather than producing the equilibrium quantity $q_{pool}$ and having its competitor believe it has a low cost with probability $\pi$.

**Proof of Proposition 1.** We start by identifying under which conditions there exists a separating equilibrium such that in the first period firms produce the same output as in the no-signaling equilibrium of the one-shot game, and any out-of-equilibrium output level (any output level other than the two corresponding to the high- and low-cost firm) lead to the belief that the firm producing it has a high cost.

In such a hypothetical equilibrium, firms reveal their cost and the second period output levels are those of the static full information equilibrium. The only condition for this to be an equilibrium of the two-period game is that a high-cost firm would not gain by mimicking a low-cost firm: the decrease in first period profits should exceed the expected gain $\Delta$ from manipulating the competitor's belief.

A high-cost firm's loss from setting an output level $q$ different from the profit-maximizing one $q_{ns}^H$ is $\left(q_{ns}^H - q\right)^2$. Therefore the condition described above is $\left(q_{ns}^H - q_{ns}^L\right)^2 \geq \Delta$, or $\frac{\delta^2}{4} \geq \Delta$. Since $\Delta = \frac{1}{9}\delta\left(A - c_H + \delta\left(\frac{1}{4} - \pi\right)\right)$, this is equivalent (after some simplification) to the inequality $\delta \geq \frac{4}{8+4\pi}\left(A - c_H\right)$. If this inequality holds, then the two-period game has a separating equilibrium such that the first-period output levels corresponding to both types are $q_{ns}^H$ and $q_{ns}^L$. This equilibrium satisfies the intuitive criterion because there is no conceivable deviation that would defeat it: such a deviation should be one by the low-cost firm, that only the low-cost firm would find profitable relative to its equilibrium behaviour, if it caused the other firm to believe it is low cost. But no such deviation can exist since in this equilibrium, the low-cost firm by producing $q_{ns}^L$ simultaneously maximizes its period 1 profit while signaling its low cost: nothing better can be achieved by any deviation.

We show now that there exists no other separating equilibrium satisfying the intuitive criterion. Assume such an alternative equilibrium exists, with first-period quantities quantities $q_{alt}^H, q_{alt}^L$. Let $q_{alt} = \pi q_{alt}^L + (1-\pi)q_{alt}^H$ denote the corresponding level of expected output per firm. Assume first that $q_{alt} > q^*$. Since in any equilibrium the high-cost firm maximizes its short-run profit, this implies that $q_{alt}^H < q_{ns}^H$, which combined with $q_{alt} > q^*$ implies $q_{alt}^L > q_{ns}^L$. The following deviation by the low-cost firm proves that the postulated equilibrium does not satisfy the intuitive criterion: in this deviation, the low-cost firm produces $q_{ns}^L$. We just need to prove that $f_{HL}(q_{ns}^L, q_{alt}) - f_{HH}(q_{alt}^H, q_{alt}) < 0 < f_{LL}(q_{ns}^L, q_{alt}) - f_{LL}(q_{alt}^L, q_{alt})$. Since $q_{alt} > q^*$, the output maximizing a low-cost firm's short-run profit is less than $q_{ns}^L$, and $q_{alt}^L > q_{ns}^L$ implies $f_{LL}(q_{ns}^L, q_{alt}) > f_{LL}(q_{alt}^L, q_{alt})$. Also, $f_{HL}(q_{ns}^L, q_{alt}) - f_{HH}(q_{alt}^H, q_{alt}) <$

$f_{HL}(q^L_{ns}, q_{ns}) - f_{HH}(q^H_{alt}, q_{ns}) < f_{HL}(q^L_{ns}, q_{ns}) - f_{HH}(q^H_{ns}, q_{ns}) < 0$. Likewise, assume that $q_{alt} < q^*$. This implies $q^H_{alt} > q^H_{ns}$ and $q^L_{alt} < q^L_{ns}$. Profit maximization by the low-cost firm implies that that $f_{HL}(q^H_{alt}, q_{alt}) < f_{HH}(q^L_{alt}, q_{alt})$, which implies $f_{HL}(q^H_{ns}, q_{alt}) < f_{HH}(q^L_{alt}, q_{alt})$. Also, $q_{alt} < q^*$ implies that the short-run profit-maximizing output for a low-cost firm is greater than $q^L_{ns}$. Therefore, increasing its output from $q_{alt}$ to $q^L_{ns}$ would increase the low-cost firm's period 1's profit, whereas it would decrease the high-cost firm's profit even if it caused the competitor to wrongly infer it has a low cost (by the previous inequality). Such a deviation thus contradicts the intuitive criterion. Finally, if $q_{alt} = q^*$, which implies $q^H_{alt} = q^H_{ns}$, then a deviation by the low-cost firm from $q^L_{alt}$ to $q^L_{ns}$ would contradict the intuitive criterion.

The above reasoning carries over to the case where the opposite inequality holds ($\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$), simply replacing $\left(q^H_{ns}, q^L_{ns}\right)$ with $(\widetilde{q_H}, \widetilde{q_L})$. We show that the separating equilibrium with quantities $\widetilde{q_H}, \widetilde{q_L}$ satisfies the intuitive criterion. The equality defining $\widetilde{q_L}$ implies that if it were possible to signal a low cost by setting an output level below $\widetilde{q_L}$, the high-cost firm would find it profitable to do so, making such a deviation not credible in the sense of the intuitive criterion. There is also no credible deviation for the high-cost firm with an output level above $\widetilde{q_L}$, because the indifference condition that defines $\widetilde{q_L}$ implies that a high-cost firm would be made worse off by such an output level, even taking into account the manipulation of its competitor's belief. Finally, a low-cost firm would not want to deviate to an output level above $\widetilde{q_L}$. Such a deviation would have no impact on beliefs, but it would reduce a low-cost firm' profit because $\widetilde{q_L}$ is greater than a low-cost firm's best-response to $\widetilde{q}$. This is because the condition $\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$ implies $\left(q^H_{ns} - q^L_{ns}\right)^2 \leq \Delta$ so that $\Delta = \widetilde{q_L} - \widetilde{q_H} > q^L_{ns} - q^H_{ns} = \frac{\delta}{2}$. Since by construction $\widetilde{q_H}$ is a high-cost firm's best response to $\widetilde{q}$, a low-cost firm's best response is $\widetilde{q_H} + \frac{\delta}{2} < \widetilde{q_L}$.

**Proof of Proposition 2.** We assume that the inequality

$$\delta < \frac{4}{8+4\pi}(A - c_H) \tag{4}$$

holds. We start by introducing the notation $L = A - c_H + \delta\left(\frac{1}{4} - \pi\right)$, which implies that $\Delta = \frac{\delta L}{9}$. Notice that (3) is equivalent to

$$\frac{L}{\delta} > \frac{9}{4}. \tag{5}$$

Substituting $A - c_H + \delta\pi$ for $A - c$ , and with $B$ denoting $A - c_H$, we can restate the formulas stated in Sections 2.2. and 2.3 as follows.

$$q^H_{ns} = \frac{B + \pi\delta}{3} - \frac{\pi\delta}{2}$$

$$q^L_{ns} = \frac{B + \pi\delta}{3} + \frac{(1-\pi)\delta}{2}$$

$$\overline{Q}_{ns} = \frac{2(B + \pi\delta)}{3}$$

$$\widetilde{q_H} = \frac{B - \pi/3\sqrt{\delta L}}{3} = \left(\frac{B}{3} + \frac{2\pi\sqrt{\delta L}}{9}\right) - \frac{\pi\sqrt{\delta L}}{3}$$

$$\widetilde{q_L} = \frac{B + (1 - \pi/3)\sqrt{\delta L}}{3} = \left(\frac{B}{3} + \frac{2\pi\sqrt{\delta L}}{9}\right) + \frac{(1-\pi)\sqrt{\delta L}}{3}$$

$$\widetilde{Q} = \frac{2B}{3} + \frac{4\pi\sqrt{\delta L}}{9}$$

We prove first that total expected output is greater under signaling than under no signaling.

$$\widetilde{Q} - \overline{Q}_{ns} = \frac{4\pi\sqrt{\delta L}}{9} - \frac{2\pi\delta}{3} = \frac{2\pi\delta}{3}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right) > 0$$

as a result of (4).

(2) implies, with $\sigma^2_{q_{ns}}$ denoting the variance of $q_{ns}$:

$$ETS_{ns} = (B + \delta\pi)\,\overline{Q}_{ns} - \frac{\overline{Q}^2_{ns}}{2} - \sigma^2_{q_{ns}} - 2Cov(q_{ns}, c)$$

$$\begin{aligned} ETS_{signaling} &= (B + \delta\pi)\,\widetilde{Q} - \frac{\widetilde{Q}^2}{2} - \sigma^2_{\widetilde{q}} - 2Cov(\widetilde{q}, c^i) \\ &= (B + \delta\pi)\,\widetilde{Q} - \frac{\widetilde{Q}^2}{2} - \frac{\delta L}{9} + \frac{2\pi(1-\pi)\delta^{3/2}L^{1/2}}{3}. \end{aligned}$$

We calculate the difference $ETS_{signaling} - ETS_{ns}$ as the sum of the four following terms:

$$(B+\delta\pi)\left(\widetilde{Q}-\overline{Q}_{ns}\right)=\frac{2\pi\delta}{3}(B+\delta\pi)\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}-1\right)$$

$$-\frac{\widetilde{Q}^2}{2}+\frac{\overline{Q}_{ns}^2}{2}=-\frac{\pi\delta}{3}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}-1\right)\left(\frac{4B}{3}+\frac{4\pi\sqrt{\delta L}}{9}+\frac{2\pi\delta}{3}\right)$$

$$-\sigma_{\widetilde{q}}^2+\sigma_{q_{ns}}^2=-\frac{\pi(1-\pi)\delta L}{9}+\frac{\pi(1-\pi)\delta^2}{4}=-\frac{\pi(1-\pi)\delta^2}{4}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}-1\right)\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}+1\right)$$

$$-2Cov(\widetilde{q},c^i)+2Cov(q_{ns},c)=\frac{2\pi(1-\pi)\delta^{3/2}L^{1/2}}{3}-\pi(1-\pi)\delta^2=\pi(1-\pi)\delta^2\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}-1\right),$$

yielding

$$\frac{ETS_{signaling}-ETS_{ns}}{\left(\frac{2}{3}\sqrt{L\delta}-\delta\right)\pi}=\frac{2}{3}(B+\delta\pi)-\frac{1}{3}\left(\frac{4B}{3}+\frac{4\pi\sqrt{\delta L}}{9}+\frac{2\pi\delta}{3}\right)-\frac{(1-\pi)\delta}{4}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}+1\right)+(1-\pi)\delta$$
$$=\frac{2B}{9}+\frac{27-11\pi}{36}\delta-\sqrt{\delta L}\frac{9-\pi}{54}.$$

Replacing $L$ with $\delta\left(\frac{1}{4}-\pi\right)$ and writing $x$ for $\delta/B$, one can check that this expression is positive if and only if

$$\left[\left(\frac{27-11\pi}{36}\right)^2-\left(\frac{1}{4}-\pi\right)\left(\frac{9-\pi}{54}\right)^2\right]x^2+\left[\frac{27-11\pi}{81}-\left(\frac{9-\pi}{54}\right)^2\right]x+\frac{4}{81}>0.$$

This is necessarily the case because each of the three coefficients in the quadratic polynomial above is positive (as a consequence of $0<\pi<1$).

As regards consumer surplus, the identity $ECS=\frac{\overline{Q}^2}{2}+\frac{\sigma_Q^2}{2}$ and the above formulas imply that the first-period difference $ECS_{signaling}-ECS_{ns}$ is equal to

$$ECS_{signaling}-ECS_{ns}=\frac{\pi\delta}{3}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}-1\right)\left[\left(\frac{4B}{3}+\frac{4\pi\sqrt{\delta L}}{9}+\frac{2\pi\delta}{3}\right)+\frac{(1-\pi)\delta}{4}\left(2\sqrt{\frac{L}{\delta}}+3\right)\right]$$
$$>4B\pi\delta\left(\frac{2}{3}\sqrt{\frac{L}{\delta}}-1\right)>0,$$

the last inequality being a consequence of (4). The above findings imply that if market share data are available and (3) holds, then in the first period, expected output, the variance of expected output, expected total surplus and expected consumer surplus are greater than if these data are not available.

**Proof of Proposition 3.** The inequality at the end of the proof of Proposition 2 implies

$$ECS_{signaling} - ECS_{ns} > 4B\pi\delta \left( \frac{2}{3}\sqrt{\frac{L}{\delta}} - 1 \right) = 4B\pi\delta \left( \frac{2}{3}\sqrt{\frac{B}{\delta} + \left(\frac{1}{4} - \pi\right)} - 1 \right)$$

whereas the identities in Section 2.2. imply

$$ECS_{full} - ECS_{ns} = -\frac{5\pi(1-\pi)\delta^2}{6}.$$

The ratio of the absolute value of the first-period difference over that of the second-period difference is

$$\frac{|ECS_{signaling} - ECS_{ns}|}{|ECS_{full} - ECS_{ns}|} > \frac{24\frac{B}{\delta}\left(\frac{2}{3}\sqrt{\frac{B}{\delta} + \left(\frac{1}{4} - \pi\right)} - 1\right)}{5(1-\pi)},$$

which is greater than 1 if $\frac{\delta}{B} (= \frac{\delta}{A - c_H})$ is close enough to zero.

**Proof of Lemma 2.** We assume (3) holds and we consider a hypothetical equilibrium such that both types of firms set price $p_{pool}$ in the first period, irrespective of their cost. Let $h_{ij}$ denote the expected profit, in the equilibrium of the one-period price competition game, of a firm with cost $c_i$ facing a competitor believing it has cost $c_j$ ($i$ and $j$ can each be $H$ or $L$), and let $h_{i,pool}$ denote the expected profit, in the equilibrium of the one-period price competition game, of a firm with cost $c_i$ facing a competitor not knowing its cost (thus believing it has low cost with probability $\pi$). The competitor's expected cost is $c$ and its expected price is $p_{full}(c, c_j) = p^* + \frac{\beta}{4-\beta^2}(c_j - c)$ so that $h_{ij} = (p^* - c)^2 \left(1 - \frac{c_i - c}{2(p^*-c)} + \frac{\beta^2}{4-\beta^2}\frac{c_j - c}{2(p^*-c)}\right)^2$ and $h_{i,pool} = (p^* - c)^2 \left(1 - \frac{c_i - c}{2(p^*-c)}\right)^2$.

The price maximizing the short-run profit of a firm with cost $c_i$ if the other firm sets $p_{pool}$ is $\frac{1+\beta p_{pool}+c_i}{2}$, and a few lines of calculation show that the difference between the corresponding profit, and the profit resulting from setting price $p_{pool}$, is equal to $\frac{(2-\beta)^2}{4}\left(p_{pool} - p^* + \frac{c-c_i}{2-\beta}\right)^2$. If a firm with cost $c_i$ finds it optimal to set $p_{pool}$ in the first period rather than the price maximizing its first-period profit, the corresponding first-period foregone profit must be less than the loss induced by causing the other firm to believe that it has a low cost:

$$\frac{(2-\beta)^2}{4}\left(p_{pool}-p^*+\frac{c-c_i}{2-\beta}\right)^2 \leq h_{i,pool}-h_{i,L}=\frac{\beta^2(c-c_L)}{2(4-\beta^2)}\left[2(p^*-c)-(c_i-c)-\frac{\beta^2}{4-\beta^2}(c-c_L)\right], \tag{6}$$

for $i=H$ or $i=L$. Consider the function $\varphi_{ij}(p,p')$ defined as the combined (two-period) expected profit of a type $i$ firm ($i=H$ or $i=L$) setting price $p$ in period 1 while its competitor's expected period 1 price is $p'$ and inducing its competitor to believe it has cost $j$: $\varphi_{ij}(p,p')=Max\,(0,1-p+\beta p')\,(p-c_i)+h_{ij}$ and define $\varphi_{i,pool}(p)=Max\,(0,1-p+\beta p)\,(p-c_i)+h_{i,pool}$. Define the function $\zeta_i(p)=\varphi_{iH}(p,p_{pool})-\varphi_{i,pool}(p_{pool})$. $z_i(q)$ is the net gain a firm with cost $c_i$ would earn from inducing its competitor to believe it has a high cost while setting price $p$ in period 1, relative to the pooling equilibrium price level and the induced belief. By definition, $\zeta_i(p_{pool})=h_{iH}-h_{i,pool}>0$ .

We prove now that there exists $p$ such that $\zeta_L(p)<0$ . Consider the price $1+\beta p_{pool}$. Such a price leads to zero sales, so that, with the notation $\rho_L(p)=Max\,(0,1-(1-\beta)p)\,(p-c_L)$ .

$$\begin{aligned}\zeta_L(1+\beta p_{pool}) &= -\rho_L(p_{pool})+h_{LH}-h_{L,pool}\\ &= -\rho_L(p_{pool})+\frac{\beta^2(c_H-c)}{2(4-\beta^2)}\left[2(p^*-c)-(c_L-c)+\frac{\beta^2}{4-\beta^2}(c_H-c)\right]\\ &= -\rho_L(p_{pool})+\frac{\beta^2\pi\delta}{2(4-\beta^2)}\left[2\frac{1-(1-\beta)c}{2-\beta}+\delta\left(1-\pi+\frac{\beta^2\pi}{4-\beta^2}\right)\right]\end{aligned}$$

One can check that $|\rho_L'|<2$, which, combined with (6), implies

$$\begin{aligned}\rho_L(p_{pool}) &> \rho_L(p^*)-2\,|p_{pool}-p^*|>\rho_L(p^*)-\frac{2\delta}{2-\beta}-2\left(\frac{4Max\,(h_{H,pool}-h_{HL},h_{L,pool}-h_{LL})}{(2-\beta)^2}\right)^{1/2}\\ &>\rho_L(p^*)-\frac{2\delta}{2-\beta}-2\left(\frac{4Max\,(h_{H,pool}-h_{HL},h_{L,pool}-h_{LL})}{(2-\beta)^2}\right)^{1/2}\\ &>\rho^*-\frac{2\delta}{2-\beta}-\frac{2\beta\,(2(1-\pi)\delta)^{1/2}}{(2-\beta)\,(4-\beta^2)\,{}^{1/2}}\left(\frac{2-2(1-\beta)c}{2-\beta}+\frac{4-2\beta^2}{4-\beta^2}(1-\pi)\delta\right)^{1/2}\\ &=\left(\frac{1-(1-\beta)c}{2-\beta}\right)^2-\frac{2\delta}{2-\beta}-\frac{2\beta\,(2(1-\pi)\delta)^{1/2}}{(2-\beta)\,(4-\beta^2)\,{}^{1/2}}\left(\frac{2-2(1-\beta)c}{2-\beta}+\frac{4-2\beta^2}{4-\beta^2}(1-\pi)\delta\right)^{1/2},\end{aligned}$$

implying

$$\zeta_L(1+\beta p_{pool}) < \frac{2\delta}{2-\beta} + \frac{2\beta\,(2(1-\pi)\delta)^{1/2}}{(2-\beta)\,(4-\beta^2)^{1/2}} \left(\frac{2-2(1-\beta)c}{2-\beta} + \frac{4-2\beta^2}{4-\beta^2}(1-\pi)\delta\right)^{1/2}$$
$$+ \frac{\beta^2\pi\delta}{2(4-\beta^2)}\left[2\frac{1-(1-\beta)c}{2-\beta} + \delta\left(1-\pi+\frac{\beta^2\pi}{4-\beta^2}\right)\right] - \left(\frac{1-(1-\beta)c}{2-\beta}\right)^2,$$

which is strictly negative by (3). Therefore $\zeta_L(1+\beta p_{pool}) < 0$. Since $\zeta_L(p_{pool}) > 0$, there exists some $p > p_{pool}$ such that $\zeta_L(p) = 0$, and a few lines of calculations show that $\zeta_H(p) > 0$. By continuity, there exists some $p_{dev}$ such that $\zeta_L(q_{dev}) < 0 < \zeta_H(q_{dev})$.

A high-cost firm setting $p_{dev}$ is a deviation that makes the postulated pooling equilibrium violate the intuitive criterion because, assuming the other firm believes that a firm setting $p_{dev}$ has a high cost, then only a high-cost firm has an interest in setting $p_{dev}$ and cause its competitor to believe it has a high cost with probability 1, rather than setting the equilibrium price $p_{pool}$ and having its competitor believe it has a low cost with probability $\pi$.

**Proof of Proposition 4.** We assume that the condition stated in Proposition 4 holds and we consider the following candidate equilibrium: in the first period, a low-cost firm sets price $p^L_{ns}$ and a high-cost firm sets price $p^H_{ns}$ , and a firm observing that its competitor set a price other than $p^H_{ns}$ believes the competitor has a low cost; and in the second period, firms set the prices corresponding to the full-information equilibrium. We show hereafter that this candidate equilibrium is indeed an equilibrium, that it sastifies the intuitive criterion, and that no other separating equilibrium satisfies it.

Assuming the other firm follows this strategy, consider the choice facing a low-cost firm: it can either maximize its short-run profit while revealing it has a low-cost, by setting price $p^L_{ns}$ , or set price $p^H_{ns}$ and cause its competitor to believe it has a high cost (all the other prices are obviously dominated by one of these two possibilities). The condition stated in Proposition 4 implies that the expected gain from manipulating the competitor's belief, $\Gamma$, is less than the corresponding decrease in the first-period profit, $\frac{\delta^2}{4}$, so that setting price $p^L_{ns}$ is a best response. As for a high-cost firm, the strategy prescribed by the candidate equilibrium maximizes its first-period profit and reveals the high cost to the competitor, which induces it to set a high price in the second period. It therefore dominates any other strategy. To prove that this equilibrium satisfies the intuitive criterion, it suffices to show that there is no other price that the high-cost firm could set and that would be a profitable deviation for the high-cost firm only, if it triggered the belief that the deviator has a high cost. The reason is that in the candidate equilibrium, the high-cost firm maximizes its first-period profit and reveals its high cost, so that no deviation of any kind, whatever the induced beliefs, would dominante the candidate equilibrium strategy.

Finally, we show that there exists no other separating equilibrium satisfying the intuitive criterion. Assume such an alternative equilibrium exists, with first-period prices $p^L_{alt}, p^H_{alt}$. In a separating equilibrium, the low-cost firm necessarily maximizes its expected first-period profit: since, by the definition of a separating equilibrium, it reveals its low cost (which is detrimental to its second-period profit), there is nothing to gain by setting a price that does not maximize profit in the first period. Let $p_{alt} = \pi p^L_{alt} + (1-\pi)p^H_{alt}$ denote the corresponding expected price. Define $p^H_{alt,dev} = \frac{1+\alpha p_{alt}+c_H}{2}$. By construction, $p^H_{alt,dev}$ maximizes the profit of a high-cost firm facing a competitor whose expected price is $p_{alt}$. It must be the case that $p^H_{alt,dev} \neq p^H_{alt}$ because otherwise, $p^L_{alt}, p^H_{alt}$ would be, respectively, the profit-maximizing price set by a low-cost and a high-cost firm in response to an expected price $p_{alt} = \pi p^L_{alt} + (1-\pi)p^H_{alt}$, implying they would coincide with $p^L_{ns}, p^H_{ns}$. By construction, $p^H_{alt,dev} - p^L_{alt} = \frac{\delta}{2}$. Since the condition stated in Proposition 4 implies $\frac{\delta^2}{4} > \Gamma$, a low-cost firm would have no interest setting price $p^H_{alt,dev}$ even if this caused its competitor to believe it has a high cost. This implies that the candidate alternative equilibrium cannot satisfy the intuitive criterion. A high-cost firm could indeed set its profit-maximizing price $p^H_{alt,dev}$ and credibly claim it has a high cost.

**Proof of Proposition 5.** We assume that the condition of Proposition 5 holds so that $\delta^2 < 4\Gamma$. Let $\Gamma'$ denote the expected second-period gain that a high-cost firm earns from having its competitor believe it is high cost rather than low cost:

$$\Gamma' = EProfit\left(c_H, p_{full}\left(c^j, c_H\right)\right) - EProfit\left(c_H, p_{full}\left(c^j, c_L\right)\right)$$

$$= \frac{\beta^2\delta}{2(4-\beta^2)}\left(2\sqrt{\rho^*} - (c_H - c) + \frac{\beta^2(c_H + c_L - 2c)}{2(4-\beta^2)}\right) = \Gamma - \frac{\beta^2\delta^2}{2(4-\beta^2)}$$

We consider now the prices $\widetilde{p_L}$ and $\widetilde{p_H}$ defined in section 3.3, such that $\widetilde{p_L}$ is a low-cost firm's profit-maximizing price in response to a competitor setting an expected price $\widetilde{p} = \pi\widetilde{p_L} + (1-\pi)\widetilde{p_H}$, and $\widetilde{p_H} = \widetilde{p_L} + \sqrt{\Gamma}$. We show that these prices, together with induced beliefs such that if a firm sets a price that is different from these two, its competitor infers it has a low cost, characterize a separating equilibrium satisfying the intuitive criterion. The low-cost firm's choice is between setting $\widetilde{p_L}$, thereby maximizing its short-run profit but revealing it has a low cost, or setting $\widetilde{p_H}$ and inducing the belief it has a high cost. The low-cost firm's first-period gain from setting $\widetilde{p_L}$ rather than $\widetilde{p_H}$ is $(\widetilde{p_H} - \widetilde{p_L})^2 = \Gamma$, which is also equal to the second-period loss from revealing its low cost. Therefore, setting $\widetilde{p_L}$ is a best response. The price that would maximize the high-cost firm's profit in the first period is $\widetilde{p_L} + \frac{\delta}{2}$, so that the first-period loss from setting price $\widetilde{p_H}$ instead is $\left(\widetilde{p_H} - \left(\widetilde{p_L} + \frac{\delta}{2}\right)\right)^2 = \left(\sqrt{\Gamma} - \frac{\delta}{2}\right)^2$. We prove now that this first-period loss is less than the second-period gain induced by revealing its high cost, namely $\Gamma'$. The difference between the latter and the former is

$$
\begin{aligned}
\Gamma' - \left(\sqrt{\Gamma} - \frac{\delta}{2}\right)^2 &= \Gamma - \frac{\beta^2\delta^2}{2(4-\beta^2)} - \left(\sqrt{\Gamma} - \frac{\delta}{2}\right)^2 \\
&= \delta\sqrt{\Gamma} - \left(\frac{\beta^2}{2(4-\beta^2)} + \frac{1}{4}\right)\delta^2 \\
&= \delta\left(\sqrt{\Gamma} - \delta\left(\frac{\beta^2}{2(4-\beta^2)} + \frac{1}{4}\right)\right) \\
&> \delta\left(\sqrt{\Gamma} - \delta\left(\frac{1}{6} + \frac{1}{4}\right)\right) = \delta\left(\sqrt{\Gamma} - \frac{5\delta}{12}\right) > \delta\left(\sqrt{\Gamma} - \frac{\delta}{2}\right) > 0,
\end{aligned}
$$

the last inequality being the condition of Proposition 5. Therefore, a high-cost firm finds it better to set $\widetilde{p_H}$ and reveal its high cost rather than setting another price and inducing its competitor to believe it has a low cost.

We show finally that this equilibrium satisifies the intuitive criterion. We need to show that there is no other price such that, if setting that price could signal a high cost, a high-cost firm would find that setting that price, rather than the price prescribed by the candidate equilibrium would strictly increase its profit, but the low-cost firm would not. The construction of $\widetilde{p_L}$ and $\widetilde{p_H}$ implies that any price smaller than $\widetilde{p_H}$ and signaling a high price would be optimal for the low-cost firm (and hence could not support a deviation violating the intuitive criterion). As for prices greater than $\widetilde{p_H}$, they would lead to a smaller first-period profit for a high-cost firm, because $\widetilde{p_H}$ is greater than the price $\widetilde{p_L} + \frac{\delta}{2}$ that would maximize its first-period profit; therefore, a high-cost firm would have no incentive to undertake such a deviation. Finally, there exists no other separating equilibrium satisfying the intuitive criterion: in such an equilibrium, the low-cost firm's price must maximize its first-period profit, the high-cost firm's price must be greater than or equal to $\widetilde{p_L} + \sqrt{\Gamma}$ (otherwise the low-cost firm would have an incentive to mimic a high-cost firm) and it cannot be strictly greater (otherwise a deviation to a slightly lower price, still greater than $\widetilde{p_L} + \sqrt{\Gamma}$ would violate the intuitive criterion because only a high-cost firm would have an incentive to set such a price if such a price triggers the belief it has a high price).

**Proof of Proposition 6.** We show here an example of parameter values such that (i) the condition of Proposition 6 holds ($4\Gamma > \delta^2$) and (ii) equilibrium expected total surplus (considering both periods) is greater in the case where market share data are available than in the case when they are not. We define $x = \frac{\sqrt{\Gamma}}{\delta}$. The difference $\Delta_1$ in expected total surplus in the first period is equal to

$$\begin{aligned}&\Delta_1 = 2(1-\beta)c\left(\widetilde{p} - p^*\right) - (1-\beta)\left(\widetilde{p}^2 - p^{*2}\right)\\&+ Change_{ns\to signaling}\left(2Cov(p_i, c^i) - 2\beta Cov(p_1, c^j) - \sigma_p^2 + \beta Cov\left(p_1, p_2\right)\right),\end{aligned}$$

with $\{i,j\} = \{1,2\}$ and $Change_{ns\to signaling}(X)$ denoting the value of $X$ in the equilibrium such that firms engage in signaling (that is, the equilibrium of the game when market share data are available) minus its value in the no-signaling equilibrium (corresponding to the game when market share data are not available). The sum of the first two terms is equal to

$$2(1-\beta)c\left(\widetilde{p} - p^*\right) - (1-\beta)\left(\widetilde{p}^2 - p^{*2}\right) = -\frac{(1-\beta)(1-\pi)\delta^2\left(2x-1\right)\left(2(1+c)+(1-\pi)\left(2x-1\right)\right)}{\left(2-\delta\right)^2}.$$

To calculate the other terms, note that (with obvious notations for the variance of $p_{ns}$ and $\widetilde{p}$, and $\{i,j\} = \{1,2\}$):

$$\sigma^2_{p_{ns}} = \frac{\pi(1-\pi)\delta^2}{4}$$

$$\sigma^2_{\widetilde{p}} = \pi(1-\pi)\Gamma$$

$$Cov\left(p_{i,ns}, p_{j,ns}\right) = Cov\left(p_{i,ns}, c^j\right) = Cov\left(\widetilde{p_1}, \widetilde{p_2}\right) = Cov\left(\widetilde{p_i}, c^j\right)\right) = 0$$

$$Cov\left(p_{i,ns}, c^j\right) = \frac{\delta^2}{2}$$

$$Cov\left(\widetilde{p_i}, c^i\right) = \pi(1-\pi)\sqrt{\Gamma}\delta.$$

Therefore,

$$Change_{ns\to signaling}\left(2Cov(p_i,c^i) - 2\beta Cov(p_i,c^j) - \sigma_p^2 + \beta Cov\left(p_1,p_2\right)\right) = \delta^2\pi(1-\pi)\left(x-\frac{1}{2}\right)\left(\frac{3}{2}-x\right).$$

Assume that $\beta$ is very close to 1. Then $\Delta_1$ is almost equal to $\delta^2\pi(1-\pi)\left(x-\frac{1}{2}\right)\left(\frac{3}{2}-x\right)$. The change in second period surplus, as mentioned in Section 3.2, is $-\frac{\beta^2\left(12-5\beta^2\right)}{4(4-\beta^2)^2}\delta^2\pi(1-\pi)$, that is, arbitrarily close to $-\frac{7}{36}\delta^2\pi(1-\pi)$. If $x=1$ (which implies that the condition of Proposition 6, namely $x>\frac{1}{2}$, is satisfied), then the change in total surplus (considering both periods) is arbitrarily close to $\delta^2\pi(1-\pi)(\frac{1}{4}-\frac{7}{36}) = \frac{\delta^2\pi(1-\pi)}{18} > 0$. The last step of the proof is to show that one can find parameter values such that $x=1$ and $c_H < \frac{1}{1-\beta}$. If $\beta$ is close to 1, then $x = \frac{\sqrt{\Gamma}}{\delta} \approx \frac{1}{2}\sqrt{\frac{1}{2\delta}+\frac{5-4\pi}{12}}$. We choose arbitrarily some $\pi$ strictly between 0 and 1 and we set $\delta = \frac{6}{43+4\pi}$, which implies that $x=1$. Also, one can choose $c_L < \frac{1}{2}$ so that $c_H = c_L + \delta < 1 < \frac{1}{1-\beta}$.

We prove hereafter that if $\delta$ is small enough, then expected total surplus is lower in the first period when market share information is available. To see this, set $\beta$ and $\pi$, and some $c < \frac{1}{1-\beta}$ so that if $\delta$ is small enough, $c_H (= c+\pi\delta)$ is smaller than $\frac{1}{1-\beta}$, as required by the model. The formula for $\Gamma$ implies that

$$x = \frac{\sqrt{\Gamma}}{\delta} = \frac{\beta}{\sqrt{4-\beta^2}}\sqrt{\frac{1-(1-\beta)c}{(2-\beta)\delta} + \frac{8(1-\pi)+(4\pi-3)\beta^2)}{4(4-\beta^2)}}.$$

This expression tends to infinity as $\delta$ tends to zero, and is thus greater than $\frac{3}{2}$ if $\delta$ is small enough. But $x > \frac{3}{2}$ implies that $\Delta_1 < 0$ because, as the above calculations show, $\Delta_1$ is the sum of $2(1-\beta)c\left(\widetilde{p}-p^*\right)-(1-\beta)\left(\widetilde{p}^2-p^{*2}\right)$, which is negative, and of $Change_{ns\to signaling}\left(2Cov(p_i,c^i)) - 2\beta Cov(p_i,c^j)) - \sigma_p^2 + \beta Cov\left(p_1,p_2\right)\right)$, which has the same sign as $\frac{3}{2}-x$.

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