Market share transparency, signaling and welfare: Cournot and Bertrand

David Spector

WORKING PAPER N° 2020 – 56

Market share transparency, signaling and welfare: Cournot and Bertrand

David Spector

JEL Codes:
Keywords:

Funded by a French government subsidy managed by the ANR under the framework of the Investissements d'avenir programme reference ANR-17-EURE-001

Market share transparency, signaling and welfare: Cournot and Bertrand

David Spector*

June 2020

Abstract

When demand is noisy and firms' costs are uncertain, the availability of market share data increases the accuracy of each firm's information, and it creates incentives for signaling. Taking both effects into account, we find that under quantity competition with a homogeneous good, the availability of market share data has a positive impact on total surplus and an ambiguous one on consumer surplus. Under price competition with differentiated substitutes, it has a negative impact on consumer surplus and an ambiguous one on total surplus. If the cost difference is small, the effect of first-period signaling dominates the effect of second-period full information. Accordingly, in this case, the availability of market share data causes total and consumer surplus to increase in the case of quantity competition and to decrease in the case of price competition.

1 Introduction

How are market outcomes impacted when firms have access to more accurate information on market shares? This question is of paramount importance for antitrust policy and it is the focus of a lively debate on the proper handling of information exchanges between competitors.[1] For the most part, economists address this issue through the lens of the theory of collusion: many papers attempt to identify under which circumstances the availability of more precise market share data facilitates mutual monitoring and thus ensures compliance

*Paris School of Economics and CNRS. Email: spector@pse.ens.fr

[1] See, e.g., OECD (2010); European Commission (2011).

with a tacit or explicit collusive agreement, and whether firms have incentives to truthfully report their sales.[2]

This paper addresses another possible channel. We ask how better information on market shares may affect non-collusive outcomes in a setting where firms interact repeatedly, but with a finite horizon - in practice, the model considers only two periods. We consider competitors whose costs are private information. If demand is noisy enough, a firm cannot infer much about its competitor's actions by simply observing the market price (in a Cournot setting) or its own sales (in a Bertrand setting). However, if market share data are available, each firm can make more precise inferences on its competitors' actions, and indirectly on their costs. In such a setting, more precise information on market shares implies more precise information on rivals' costs. But this mechanism in turn gives rise to another effect, because it creates an incentive for signaling behavior. For instance, if firms producing a homogeneous good compete in quantities, each has an incentive to produce more so as to convince its competitors that its cost is low and induce them to produce less in the future. Likewise, in the case of price competition with differentiated substitutes, each firm has an incentive to raise prices in the first period, in order to signal a high cost and induce its rivals to raise their prices.

Each of these two effects is well-known and has been extensively covered in the economic literature. The contribution of this paper is to construct a simple and tractable model that allows us to assess their overall combined impact on consumer surplus and total surplus, in order to cast light on the desirability of transparency on market share data.

We first consider the case of quantity competition with homogeneous goods. As has been established long ago, with linear demand and linear costs, transparency on costs causes total surplus to increase and consumer surplus to decrease. Also, earlier papers have shown that signalling in the first period leads to an increase in expected output. If market share data are public, then signalling occurs in the first period, and second-period competition takes place under full information on cost. We find the following results about the overall effect of these two changes: if the cost differential is so high that distortionary signaling is too costly to occur in equilibrium, then the availability of market share information only changes second-period outcomes: market share data reveal costs, which has a positive impact on total surplus and a negative one on

[2]See, e.g., Awaya and Krishna (2020) and the references therein.

consumer surplus. If on the contrary signaling occurs in equilibrium, then the overall impact (combining both periods) of the availability of market share data on total surplus is positive, but the effect on consumer surplus is ambiguous. However, if the cost gap (and hence the magnitude of the information asymmetry and of the signaling distortion) is small, then the impact of the availability of market share data on consumer surplus is positive. This is because, in this case, the increase in consumer surplus caused by the signaling-driven increase in output in the first period is much larger than the second-period loss caused the availability of cost information.

We then consider the case of Bertrand competition with differentiated substitute products. It is already known that, in the case of linear demand at least, the availability of cost information causes both social and consumer surplus to decrease. In addition, signaling takes the form of price increases. These two effects are unambiguously detrimental to consumer surplus. However, the availability of market share data has an ambiguous effect on total surplus: for some parameter values, the signaling-driven distortion brings about a reallocation of production towards the most efficient firm that causes total surplus to increase in spite of higher prices, to an extent that more than offsets the second-period decrease caused by the shift to public information on costs. An unambiguous result can however be stated if the cost gap is small: in that case, signaling causes total surplus to fall in the first period, and the overall effect of the availability of market share data on consumer and social welfare is negative.

This paper is related to several branches of the industrial organization literature. Several papers have assessed how market outcomes are modified when cost information is public rather than private - which underlies our analysis of the second period of competitive interaction when market share data are available. In fact, our assessment of the second period is identical to that of Shapiro (1986) in the case of Cournot competition with homogeneous products, and to that of Sakai and Yamato (1990) in the case of Bertrand competition with differentiated products. These two papers are part of a broader literature that examines the incentives of oligopolists to share cost information.[3]

This paper is also related to the literature on simultaneous signaling. The contribution closest to ours is Mailath's (1989), which studies cost signaling

[3] See e.g., Fried, Li, Gal-Or (1986), Raith (1996), Myatt and Wallace (2015). Several papers also analyze the case where the uncertainty is about demand rather than costs (e.g., Vives, 1984; Gal-Or, 1985).

in a model of price competition with differentiated products.[4] The differences between his approach and ours are his exclusive focus on Bertrand competition (whereas we study both price and quantity competition) and the choice of benchmark: whereas we compare the signaling equilibrium to one where, absent market share data, firms neither signal (in the first period) nor know their competitors' costs (in the second period), his benchmark is one where, 'for some reason', costs become common knowledge at the end of the first period. Other models of signaling in oligopoly consider the case where uncertainty is about the parameters of demand rather than costs (see, e.g., Jin, 1994). Another strand of the literature on signaling (Bonatti et al., 2017; originating with Mester, 1992) study signaling in long-horizon models in which the underlying uncertain parameters may vary, or they extend earlier approaches to supply-function equilibria (Vives, 2011; Bernhardt and Taub, 2015). Whereas these sophisticated models probably reflect actual markets better than our simple two-period model, they do not easily lend themselves to the kind of comparative statics exercise that is the object of this paper.

2 The case of a homogeneous product Cournot duopoly

2.1 The model

2.1.1 Demand and costs

Two firms compete in two periods. Each firm' cost is drawn from the same probability distribution: it is either low (c_L), with probability π, or high (c_H, with $c_H > c_L$) with probability $1 - \pi$. The two draws are independent. Price in period i ($i = 1$ or $i = 2$) is given by $P_i = Max\,(0; (A - Q_i)\,(1 + \varepsilon))$, with A, Q_i and ε_i denoting respectively the demand intercept, total output in period i, and a demand shock with zero mean and support above -1 (it would make no difference to assume the demand shock to be period-specific). Firms maximize the expected sum of their profits in periods 1 and 2.

The game is as follows: in both periods, firms simultaneously set their quantities. At the end of period 1, firms observe the prevailing price, and, in one of the two variants we will compare, total output as well (or, equivalently, market shares).

[4] A closely related paper is Caminal (1990).

The cost and demand intercept parameters are such that in the static Cournot duopoly equilibrium under full information, a high-cost firm produces even if the other firm has a low cost: $c_H - c_L < A - c_H$.

A key assumption is that the distribution of the demand shock is such that a firm can infer almost nothing about total output from observing the price. This admittedly extreme assumption implies that there is no scope for cost signaling in the variant in which firms observe only the prevailing price, whereas there is in the variant in which firms also observe total output (i.e., they can infer their competitor's output by subtracting their own ouput from the total).

Notations: c denotes the expected cost ($c = \pi c_L + (1-\pi)c_H$), δ denotes the cost difference $c_H - c_L$, and $\sigma_c^2 = \pi(1-\pi)\delta^2$ denotes the variance of the cost distribution.

2.1.2 Consumer and total surplus as a function of output and costs

Let Q, q^i, and c^i denote respectively total output, firm i's output and firm i's cost in a given period. Consumer surplus and total surplus are given by the following equalities:

$$CS = \frac{Q^2}{2}$$

$$TS = AQ - \frac{Q^2}{2} - q^1c^1 - q^2c^2$$

implying the following equalities for expected consumer and total surplus (with $\overline{Q}$ denoting expected total output):

$$ECS = \frac{\overline{Q}^2}{2} + \frac{\sigma_Q^2}{2} \tag{1}$$

$$ETS = (A - c)\,\overline{Q} - \frac{\overline{Q}^2}{2} - \frac{\sigma_Q^2}{2} - 2Cov(q^i, c^i) \tag{2}$$

2.2 Surplus under no information and under full information

If a firm cannot infer anything about its competitor's first period output, then, with c denoting the expected cost ($c = \pi c_L + (1-\pi)c_H$), the equilibrium output in each period of a firm with cost c_i ($i = H$ or $i = L$) is given by

$$q_{ns}^i = \frac{A-c}{3} + \frac{c-c_i}{2}.$$

This leads to the following expected values for output, consumer and total surplus:

$$\overline{Q}_{ns} = \frac{2(A-c)}{3}$$

$$ECS_{ns} = \frac{\overline{Q}_{ns}^2}{2} + \frac{\sigma_c^2}{4} = \frac{2\left(A-c\right)^2}{9} + \frac{\sigma_c^2}{4}$$

$$ETS_{ns} = \frac{4\left(A-c\right)^2}{9} + \frac{3\sigma_c^2}{4}$$

Under full information, with the notation $\{i, j\} = \{1, 2\}$, firm i's equilibrium output is given by

$$q_{full}^i = \frac{A-c}{3} + \frac{2(c-c^i)-(c-c^j)}{3},$$

leading to

$$\overline{Q}_{full} = \overline{Q}_{ns} = \frac{2(A-c)}{3}$$

$$ECS_{full} = \frac{\overline{Q}_{full}^2}{2} + \frac{\sigma_c^2}{9} = \frac{2\left(A-c\right)^2}{9} + \frac{\sigma_c^2}{9}$$

$$ETS_{full} = \frac{4\left(A-c\right)^2}{9} + \frac{11\sigma_c^2}{9}.$$

We find the well-known result that in the case of homogeneous Cournot competition, full information on costs increases consumer surplus and decreases total surplus.[5]

2.3 Signaling

We assume now that at the end of the first period, firms observe their market shares, which is equivalent to assuming each firm observes its competitor's output. Each firm now has an incentive to raise its output to signal lower costs, in order to induce its competitor to produce less in the next period. We describe

[5] Shapiro (1986).

hereafter the corresponding equilibrium output levels. The equilibrium concept we use is the subgame perfect equilibrium, with an additional selection criterion, namely, Cho and Kreps' 'intuitive criterion' that rules out all implausible out-of-equilibrium beliefs.[6]

2.3.1 Signaling equilibria

As we prove in the appendix, there exists a unique such equilibrium, which is fully separating.

We start by calculating the gain that a high-cost firm obtains in the second period if it causes its competitor to wrongly believe it has a low cost. A high-cost firm mimicking a low-cost firm in the first period expects its competitor to produce $\frac{A-c_L}{3}$ with probability π and $\frac{A-2c_H+c_L}{3}$ with probability $1-\pi$, leading to an expected profit for the mimicking firm (which maximizes its second period profit) of $\frac{\pi\left(A-\frac{A-c_L}{3}-c_H\right)^2+(1-\pi)\left(A-\frac{A-2c_H+c_L}{3}-c_H\right)^2}{4}$. If a high-cost firm reveals its high-cost, then its second-period expected profit is $\frac{\pi(A+c_L-2c_H)^2+(1-\pi)(A-c_H)^2}{9}$. Let Δ denote the difference between the former and the latter expression, that is, the expected gain that a high-cost firm obtains in the second period if is causes its competitor to wrongly believe it has a low cost. A few calculations show that

$$\Delta = \frac{1}{9}\delta\left(A - c_H + \delta\left(\frac{1}{4} - \pi\right)\right).$$

We first investigate under which conditions the output levels of the no-signaling equilibrium described above can be equilibrium levels in the first period of the game in which firms observe their market shares. Intuitively, this must be the case if a high-cost firm finds that the first period loss induced by producing q_{ns}^H rather than q_{ns}^L (assuming its competitor's expected output is $\frac{A-c}{3}$ as per the no-signaling equilibrium) is greater than or equal to Δ. A few calculations show that it is the case if $\delta \geq \frac{4}{8+4\pi}(A - c_H)$. This yields the first result.

Proposition 1. *If* $\frac{\delta}{A-c_H} \geq \frac{4}{8+4\pi}$, *then the game in which firms observe their market shares has only one subgame-perfect equilibrium satisfying the intuitive criterion. This equilibrium is such that firms set the no-signaling output levels* q_{ns}^i *in the first period, and the full information output levels* q_{full}^i *in the second*

[6] Cho and Kreps (1987). In our model, just like for others, the intuitive criterion eliminates all equilibria but a separating equilibrium such that one firm (the 'non-signaling one') chooses an action (an output level - or, in the case of price competition, a price) that maximizes its short-run profit, whereas the other one chooses an action that makes the non-signaling firm indifferent between the two actions.

period. In this equilibrium, output and total surplus are greater than in the equilibrium of the game in which firms do not observe their market shares, whereas consumer surplus is lower.

The condition in the statement of Proposition 1 can be interpreted as follows. Since $A - c_H$ is three times firms' gross margins if both firms have high costs, it means that the cost difference between efficient and less efficient firms is greater than $\frac{12}{8+4\pi}$ times high-cost firms' gross margins (when both firms have high costs), that is, greater than these margins since $\pi < 1$.

If this condition does not hold, then one can easily show that, likewise, there exists a unique equilibrium satisfying the intuitive criterion, and this equilibrium is separating (see the appendix for the detailed reasoning). In the first period, according to this equilibrium, a high-cost firm sets an output level $\widetilde{q_H}$ that maximizes its first-period expected profit, while a low-cost firm sets an output level $\widetilde{q_L}$ such that a high-cost firm is indifferent between producing $\widetilde{q_H}$, thus revealing its cost is high and inducing its competitor to produce more in period 2, and producing $\widetilde{q_L}$, which reduces its first-period profit but causes its competitor to believe it has a low cost, and to produce less. According to such an equilibrium, after costs have been revealed in the first period, firms in the second period set full information output levels.

Let $\widetilde{q} = \pi\widetilde{q_L} + (1-\pi)\widetilde{q_H}$ denote the competitor's expected output in the first period of such an equilibrium. The output level maximizing a high-cost firm's expected first period profit is $\widetilde{q_H} = \frac{A-\widetilde{q}-c_H}{2}$. The abovementioned indifference condition is equivalent to

$$(\widetilde{q_L} - \widetilde{q_H})^2 = \Delta,$$

leading to

$$\widetilde{q_L} = \frac{A - c_H - \pi\sqrt{\Delta}}{3}$$

$$\widetilde{q_H} = \frac{A - c_H + (3-\pi)\sqrt{\Delta}}{3}$$

$$\widetilde{Q} = \frac{2(A - c_H) + 2\pi\sqrt{\Delta}}{3},$$

with $\widetilde{Q}$ denoting expected first period output in the signaling equilibrium. It can be checked that if the condition stated in Proposition 1 does not hold, then $\widetilde{Q} > \overline{Q}_{ns}$ (see the appendix). In line with intuition, expected equilibrium output is greater when the information structure (i.e., the observability of market shares) creates an incentive for cost signaling. We recapitulate this finding as follows.

Proposition 2. *If* $\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$, *then the game in which firms observe their market shares has only one subgame-perfect equilibrium satisfying the intuitive criterion. This equilibrium is such that in the first period, a firm with a low (resp. high) cost sets an output level equal to* $\widetilde{q_L}$ *(resp.* $\widetilde{q_H}$*) defined by the above formulas, and the full information output levels in the second period.*

2.4 Comparative statics

The above characterization allows us to assess the impact of the availability of market share data. If the condition stated in Proposition 1 holds, then the cost gap is so large that no signal occurs in equilibrium and the availability of market share data only has one effect, namely, it causes each firm to know its competitor's cost ahead of period 2. The effect is thus the same as that of a move from no information to full information on costs: consumer surplus falls and total surplus increases.

In the other case (the one covered in Proposition 2), the availability of market share information creates incentives for signaling. As a result, low-cost firms produce more than they would in the no-signaling case, implying that high-cost firm (which set a myopic best-response output level to the expected competitor's output level) produce less, and total expected output increases. Intuitively, this should cause consumer and total surplus to increase. A short calculation shows that this is indeed the case (see the appendix).

Proposition 3. *If the parameters are such as signaling occurs in the first period in equilibrium if market share information is available* ($\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$) *then, in the first period, total expected output, expected consumer surplus and expected total surplus are greater if market share information is available than if it is not. As a result, overall expected total surplus (considering both periods) is greater if market share information is available.*

A corollary of this result is that making market share information available unambiguously increases expected total surplus, whatever the model parameters: if the cost gap is so large that there is no signaling in equilibrium, then total expected surplus rises only because of the move to full information in the second period; otherwise, it also rises because of the direct impact of first-period signaling.

In contrast, there is no general result on the impact of the availability of market-share information on consumer surplus. By continuity, the result stated in Proposition 1 implies that if the condition stated in Proposition 2 barely holds (in the sense that the cost gap is large), then the incentive for a high-cost firm to mimick a low-cost firm is weak and the equilibrium output levels in period 1 are barely different from those in the no-signaling case: the low-cost firm's output level and total expected output are only slightly above their values in the no-signaling case. If that is the case, then consumers' second-period loss (caused by the shift to public information on costs) dominates their small first-period gain and, overall, expected consumer surplus falls.

However, an unambiguous result holds if the cost gap is small: if that is the case, then consumer surplus unambiguously increases.

Proposition 4. *If the cost gap is small relative to high-cost firms' margins ($\frac{\delta}{A-c_H} << 1$), then, taking both periods into account, expected consumer surplus is greater if market share information is available than if it is not.*

The mechanism behind the proof (presented in the appendix) is quite intuitive and general: the second-period expected gain induced by having the competitor believe one has a low cost has the same order of magnitude as the cost difference (times the margin). But a high-cost firm's first period loss from distorting its price to mimick a low-cost firm has the order of magnitude of the square of the price increase. Therefore, the indifference condition stated above implies that the first-period price increase of low-cost firms in a signaling equilibrium has the same order of magnitude as the square root of the cost difference (times the square root of the margin). If the cost gap is small relative to the margin, this implies that, in absolute value, the (positive) first-period effect of signaling on consumer surplus is much greater than the (negative) second-period effect.

3 The case of a differentiated Bertrand duopoly

We consider hereafter the case of price competition, relying on a a symmetric Bertrand duopoly model with linear demand and differentiated substitute products.

3.1 The model

3.1.1 Demand and costs

The assumptions on the timing of the game and on the distribution of firms' costs, as well as the corresponding notations, are the same as in the previous section.

In each period, demand for product i is given by the following function (with $\{i, j\} = \{1, 2\}$): $q_i = Max\,(0; (1 - p_i + \beta p_j)\,(1 + \varepsilon))$, with p_k and ε denoting respectively firm k's price ($k = 1$ or $k = 2$) and a demand shock with zero mean and support above -1 (which can be assumed identical in both periods without loss of generality). The parameter β captures the degree of substitutability between both goods. Firms maximize the expected sum of their profits in periods 1 and 2.

The game is as follows: in both periods, firms simultaneously set their prices. At the end of period 1, each firm observes its sales and, in one of the two variants we will compare, total output as well (or, equivalently, the other firm's sales or its price). Firms then compete in prices again in period 2.

The cost and demand intercept parameters are such that in the static Bertrand duopoly equilibrium under full information, consumers purchase both goods in equilibrium: $c_H < \frac{1}{1-\beta}$.

Just like in the case of Cournot competition, we asssume that the distribution of the demand shock is such that a firm can infer almost nothing about its competitor's price by observing its own sales, so that there is (almost) no scope for cost signaling in the variant in which a firm observes only its own sales, whereas information on total output (in additional to knowing its own price and sales) allows a firm to infer its competitor's price.

Just like in the previous section, c, δ and σ_c^2 denote respectively the expected cost ($\pi c_L + (1 - \pi)c_H$), the cost difference ($c_H - c_L$), and the variance of the cost distribution ($\pi(1 - \pi)\delta^2$).

3.1.2 Consumer and total surplus as a function of output and costs

The above demand function corresponds to the following formulas for prices, the utility function, consumer surplus and total surplus:

$$p_i = -\frac{q_i}{1-\beta^2} - \frac{\beta q_j}{1-\beta^2} + \frac{1}{1-\beta}$$

$$(1-\beta)U = q_1 + q_2 - \frac{q_1^2 + q_2^2}{2(1+\beta)} - \frac{\beta q_1 q_2}{1+\beta}$$

$$CS = \frac{1}{1-\beta} - (p_1 + p_2) + \frac{p_1^2 + p_2^2}{2} - \beta p_1 p_2$$

$$TS = \frac{1}{1-\beta} - (c_1 + c_2) + (c_1 - \beta c_2)\, p_1 + (c_2 - \beta c_1)\, p_2 - \frac{p_1^2 + p_2^2}{2} + \beta p_1 p_2$$

implying the following equalities for expected consumer and total surplus (with $\overline{p}$ denoting each firm's expected price):

$$ECS = \frac{1}{1-\beta} - 2\overline{p} + (1-\beta)\overline{p}^2 + \sigma_p^2 - \beta Cov\,(p_1, p_2)$$

$$ETS = \frac{1}{1-\beta} - 2c + 2(1-\beta)c\overline{p} + 2Cov(p_1, c_1) - 2\beta Cov(p_1, c_2) - (1-\beta)\overline{p}^2 - \sigma_p^2 + \beta Cov\,(p_1, p_2)$$

3.2 Surplus under no information and under full information

If firms cannot infer anything about their competitor's first period output, then, with c denoting the expected cost ($c = \pi c_L + (1-\pi)c_H$), the equilibrium price in each period of a firm with cost c_i ($i = H$ or $i = L$) is given by

$$p_{ns}^i = p^* + \frac{c_i - c}{2},$$

(with the notation $p^* = \frac{1+c}{2-\beta}$), implying $\sigma_{p,ns}^2 = \frac{\sigma_c^2}{4}$, $Cov_{ns}(p_1, c_1) = 1/2$ and $Cov_{ns}(p_1, c_2) = Cov_{ns}\,(p_1, p_2) = 0$.

This leads to the following expected values for output, consumer and total surplus:

$$\overline{p}_{ns} = p^* = \frac{1+c}{2-\beta}$$

$$ECS_{ns} = CS_0 + \frac{\sigma_c^2}{4}$$

$$ETS_{ns} = TS_0 + \frac{3\sigma_c^2}{4},$$

where CS^* and TS^* denote respectively consumer surplus and total surplus when both firms have cost c ($CS_0 = \frac{1}{1-\beta} - \frac{2(1+c)}{(2-\beta)} + \frac{(1-\beta)(1+c)^2}{(2-\beta)^2}$ and $TS_0 = \frac{1}{1-\beta} - 2c + \frac{2(1-\beta)c(1+c)}{(2-\beta)} - \frac{(1-\beta)(1+c)^2}{(2-\beta)^2}$).

Under full information, with the notation $\{i,j\} = \{1,2\}$, firm i's equilibrium output is given by

$$p_{full}(c_i, c_j) = p^* + \frac{2}{4-\beta^2}(c_i - c) + \frac{\beta}{4-\beta^2}(c_j - c),$$

leading to

$$\sigma^2_{p,full} = \frac{\left(4+\beta^2\right)}{\left(4-\beta^2\right)^2}\sigma_c^2$$

$$Cov_{full}(p_1, c_1) = \frac{2}{\left(4-\beta^2\right)}\sigma_c^2$$

$$Cov_{full}(p_1, c_2) = \frac{\beta}{\left(4-\beta^2\right)}\sigma_c^2$$

$$Cov_{full}(p_1, p_2) = \frac{4\beta}{\left(4-\beta^2\right)^2}\sigma_c^2$$

$$\overline{p}_{full} = \overline{p}_{ns} = \frac{1+c}{2-\beta}$$

$$ECS_{full} = CS^* + \frac{\left(4-3\beta^2\right)\sigma_c^2}{\left(4-\beta^2\right)^2}$$

$$ETS_{full} = TS^* + \frac{\left(2\beta^4 - 9\beta^2 + 12\right)\sigma_c^2}{\left(4-\beta^2\right)^2}$$

It turns out that under full information, both consumer surplus and total surplus are lower than under no information:

$$ECS_{full} - ECS_{ns} = -\frac{\beta^2\left(\beta^2+4\right)}{4\left(4-\beta^2\right)^2}\sigma_c^2$$

$$ETS_{full} - ETS_{ns} = -\frac{\beta^2\left(12-5\beta^2\right)}{4\left(4-\beta^2\right)^2}\sigma_c^2,$$

which are both strictly negative since $0 < \beta < 1$.[7]

[7]These findings are identical to those in Sakai and Yamato (1990).

3.3 Signaling

Just like in the previous section, we assume now that the end of the first period, firms observe their market shares, which is equivalent to assuming each firm observes its competitor's price. Each firm now has an incentive to raise its price to signal high costs, in order to induce its competitor to set a high price in the next period. We describe hereafter the corresponding equilibrium price levels, with the same equilibrium concept (subgame perfection with the intuitive criterion to eliminate certain counterintuitive equilibria).

The results follow the same logic as in the previous section: if the static (no signalling) equilibrium is such that a low-cost firm would have no interest in mimicking the high-cost firm in order to cause its competitor to believe it has a high cost, the only equilibrium satisfying the intuitive criterion is the no signaling equilibrium. Otherwise, in the signaling equilibrium, the prices set by both types of firms are such that (i) the low-cost firm maximizes its short-run profit given its correct expectation of the other firm's price, and (ii) the low-cost firm is indifferent between maximizing its short-run profit and facing a second-period competitor that knows its low cost, or mimicking a high-cost firm and manipulating its competitor's belief;

3.3.1 Signaling equilibria

A reasoning very similar to the one in the quantity competition case (and hence left to the reader) shows that only one equilibrium satisfies the intuitive criterion, and that this equilibrium is separating, with the low-cost firm maximizing its short-term profit and the high-cost firm's price making the low-cost firm indifferent between its own equilibrium price and the high cost firm's.

We now characterize this unique separating equilibrium. Let π^* denote the profit that each firm would earn if costs were both equal to c: $\pi^* = \left(\frac{1-(1-\beta)c}{2-\beta}\right)^2$. If firm 1 sets its price to maximize its profit (as it does in equilibrium, in the second period), then its profit depends on its own cost and its competitor's price as follows: $\pi(c_1, p_2) = \left(\sqrt{\pi^*} - \frac{c_1-c}{2} + \frac{\beta(p_2-p^*)}{2}\right)^2$. Let Γ denote the expected second-period gain that a low-cost firm (say, firm 1) earns from having its competitor (firm 2) believe it is high cost. It is equal to

$$\Gamma = E\pi\left(c_L, p_{full}\left(c_2, c_H\right)\right) - E\pi\left(c_L, p_{full}\left(c_2, c_L\right)\right)$$
$$= E\left(\sqrt{\pi^*} - \frac{c_L - c}{2} + \frac{\beta}{2}\left(\frac{2(c_2 - c)}{4-\beta^2} + \frac{\beta(c_H - c)}{4-\beta^2}\right)\right)^2 - E\left(\sqrt{\pi^*} - \frac{c_L - c}{2} + \frac{\beta}{2}\left(\frac{2(c_2 - c)}{4-\beta^2} + \frac{\beta(c_L - c)}{4-\beta^2}\right)\right)$$
$$= \frac{\beta^2\delta}{2(4-\beta^2)}\left(2\sqrt{\pi^*} - (c_L - c) + \frac{\beta^2(c_H + c_L - 2c)}{2(4-\beta^2)}\right) = \frac{\beta^2\delta}{4-\beta^2}\left(\sqrt{\pi^*} + \frac{8(1-\pi) + (4\pi - 3)\beta^2)}{4(4-\beta^2)}\delta\right)$$

We first investigate under which conditions the prices in the static no-information equilibrium can be first-period equilibrium prices in the first period of the game in which firms observe their market shares. This is the case if a low-cost firm finds that the first-period loss induced by setting p_{ns}^H rather than p_{ns}^L (assuming its competitor's expected price is p^*) is greater than or equal to the above expression. Since this first-period loss is $\left(p_{ns}^H - p_{ns}^L\right)^2 = \frac{\delta^2}{4}$, no signaling is an equilibrium if and only if $4\Gamma \leq \delta^2$, or equivalently $\delta \geq \frac{\beta^2(2+\beta)(1-(1-\beta)c)}{16(4-\beta^2)-\beta^2(24-8\pi-\beta^2(7-4\pi))}$.

Proposition 5. *If $4\Gamma \leq \delta^2$, then the game in which firms observe their market shares has only one subgame-perfect equilibrium satisfying the intuitive criterion. This equilibrium is such that firms set the no-signaling price levels in the first period, and the full information prices in the second period. In this equilibrium, consumer surplus and total surplus are smaller than in the equilibrium of the game in which firms do not observe their market shares.*

If this condition does not hold (if $4\Gamma > \delta^2$), then there exists a unique equilibrium satisfying the intuitive criterion, and this equilibrium is separating. In the first period, according to this equilibrium, a low-cost firm sets price $\widetilde{p_L}$ that maximizes its first-period expected profit, whereas a high-cost firm sets an output level $\widetilde{p_H}$ such that a low-cost firm is indifferent between setting price $\widetilde{p_L}$, thus revealing its cost is low and inducing its competitor to set a lower price in period 2, or setting price $\widetilde{p_H}$, which reduces its first-period profit but causes its competitor to set a higher price in period 2. In such an equilibrium, after costs have been revealed in the first period, firms in the second period set the full information equilibrium prices displayed above.

Let $\widetilde{p} = \pi\widetilde{p_L} + (1-\pi)\widetilde{p_H}$ denote the competitor's expected price in the first period of such an equilibrium. The price maximizing a low-cost firm's expected first period profit is $\widetilde{p_L} = \frac{1+\beta\widetilde{p}+c_L}{2}$. The abovementioned indifference condition is equivalent to

$$(\widetilde{p_L} - \widetilde{p_H})^2 = \Gamma,$$

leading to

$$\widetilde{p_L} = \widetilde{p} - (1-\pi)\sqrt{\Gamma}$$

$$\widetilde{p_H} = \widetilde{p} + \pi\sqrt{\Gamma},$$

with the notation

$$\widetilde{p} = \frac{1+c}{2-\beta} + \frac{(1-\pi)\left(2\sqrt{\Gamma}-\delta\right)}{2-\beta}.$$

Proposition 6. *If* $4\Gamma > \delta^2$ (or, equivalently, $\delta < \frac{\beta^2(2+\beta)(1-(1-\beta)c)}{16(4-\beta^2)-\beta^2(24-8\pi-\beta^2(7-4\pi))}$), *then the game in which firms observe their market shares has only one subgame-perfect equilibrium satisfying the intuitive criterion. This equilibrium is such that in the first period, a firm with a low (resp. high) cost sets an price equal to* $\widetilde{p_L}$ *(resp.* $\widetilde{p_H}$*) defined by the above formulas, and the full information price levels in the second period.*

3.4 Comparative statics

Assessing the impact of the availability of market share data is more straightforward than in the case of Cournot competition with homogeneous products. As seen above, it causes a shift from the no-information to the full information equilibrium in the second period, which is detrimental to both consumer and total surplus. If the condition of Proposition 5 holds ($4\Gamma \leq \delta^2$), this is the only effect since no signaling takes place in the first period.

If on the contrary the condition of Proposition 6 holds ($4\Gamma > \delta^2$), then in addition there is an effect in the first period, as signaling takes place. Proposition 6 implies that both the high-cost firm's and the low-cost firm's prices are greater in the first period, under signaling, than in the no-signaling, no-information equilibrium: $\widetilde{p_L} > p_{ns}^L$ and $\widetilde{p_H} > p_{ns}^H$. To see this, notice that the expected price is greater since $\widetilde{p} - p^* = \frac{(1-\pi)\left(2\sqrt{\Gamma}-\delta\right)}{2-\delta} > 0$. The low-cost firm's price is in both cases a best response to this expected price, which implies it is greater (since prices are strategic complements): $\widetilde{p_L} > p_{ns}^L$. Finally, $\widetilde{p_H} = \widetilde{p_L} + \sqrt{\Gamma} > p_{ns}^L + \sqrt{\Gamma} > p_{ns}^L + \frac{\delta}{2} = p_{ns}^H$. Consumer surplus is therefore less in the first period if market share data are available (so that firms engage in signaling) than if they are not.

In contrast, there is no general result regarding total surplus. Even though the availability ot market share data causes total surplus to fall in the second period, and prices to rise in the first period, it can still cause total surplus to rise taking both periods into account. This is because signaling has two opposing effects. On the one hand, it causes the expected price to rise. On the other hand, it increases the difference between the high-cost firm's and the low-cost firm's prices, which reallocates production inefficient to efficient firms. One can show that for some parameter values, this latter effect may be strong enough to offset both the adverse effect of the average price increase in the first period and the adverse effect of more information in the second period.[8] However, a general result can be stated when the cost difference is very small: in that case, the availability of market share data unambiguously causes total surplus to fall in both periods.

Proposition 7. *If market share information is available, then first-period prices increase, expected consumer decreases in both periods and expected second-period total surplus decreases. There is no general result on the impact on expected first-period total surplus or expected overall total surplus (combining both periods). However, if the cost difference δ is small enough, then the availability of market share information causes expected total surplus to decrease in both periods.*

Appendix

Proof of Propositions 1 and 2. No pooling equilibrium satisfies the intuitive criterion. If there were one, with output level q_{pool} for both types of firms, then the corresponding output would have to be a best-response to itself for the high-cost firm. Consider the function $f_{ij}(q, q')$ defined as the combined (two-period) expected profit of a type i firm ($i = H$ or $i = L$) setting output q in period 1 while its competitor's expected period 1 output is q' and inducing its competitor to believe it has cost j (thus setting a second-period price equal in expectation to $p_{full}(c, c_j)$). These are continuous, concave functions of their first variable, such that for any (q, q'), $f_{iL}(q, q') > f_{iH}(q, q')$, $\frac{\partial(f_{LL}-f_{LH})}{\partial q}(q, q') > \frac{\partial(f_{HL}-f_{HH})}{\partial q}(q, q')$, $\frac{\partial f_{Li}}{\partial q}(q, q') > \frac{\partial f_{Hi}}{\partial q}(q, q')$, $\frac{\partial f_{ij}(q,q')}{\partial q \partial q'} < 0$

[8] In a somewhat different setting, Mailath (1989) proves that total first-period surplus can be greater in the signaling equilibrium in spite of higher prices. Our result shows that the corresponding increase can be large enough to offset the second-period adverse impact of information disclosure on total surplus.

and $\underset{q\to\infty}{Lim} f_i(q,q') = -\infty$. Therefore, there exists q_{dev} such that

$$f_{HL}(q_{dev}, q_{pool}) - (\pi f_{HL} + (1-\pi) f_{HH})(q_{pool}, q_{pool}) < 0$$
$$< f_{LL}(q_{dev}, q_{pool}) - (\pi f_{LL} + (1-\pi) f_{LH})(q_{pool}, q_{pool}).$$

A low-cost firm setting q_{dev} is a deviation that makes the postulated pooling equilibrium violate the intuitive criterion. Notice that this proof applies irrespective of the sign of $\frac{\delta}{A-c_H} - \frac{4}{8+4\pi}$.

Assume now that the condition underlying Proposition 1 is satisfied: $\frac{\delta}{A-c_H} \geq \frac{4}{8+4\pi}$ holds so that a high-cost firm would lose more by setting the one-shot game's low-cost firm's output q^L_{ns} than it would gain by causing its competitor to believe it has a low cost and price accordingly in the second period. This assumption implies that quantities of the equilibrium of the one-period game are still equilibrium quantities of the two-period game. We show now that there exists no other separating equilibrium satisfying the intuitive criterion. Assume such an alternative equilibrium exists, with first-period quantities quantities q^H_{alt}, q^L_{alt}. Let $q_{alt} = \pi q^L_{alt} + (1-\pi) q^H_{alt}$ denote the corresponding level of expected output per firm. Assume first that $q_{alt} > q^*$. Since in any equilibrium the high-cost firm maximizes its short-run profit, this implies that $q^H_{alt} < q^H_{ns}$, which combined with $q_{alt} > q^*$ implies $q^L_{alt} > q^L_{ns}$. The following deviation by the low-cost firm proves that the postulated equilibrium does not satisfy the intuitive criterion: in this deviation, the low-cost firm produces q^L_{ns}. We just need to prove that $f_{HL}(q^L_{ns}, q_{alt}) - f_{HH}(q^H_{alt}, q_{alt}) < 0 < f_{LL}(q^L_{ns}, q_{alt}) - f_{LL}(q^L_{alt}, q_{alt})$. Since $q_{alt} > q^*$, the output maximizing a low-cost firm's short-run profit is less than q^L_{ns}, and $q^L_{alt} > q^L_{ns}$ implies $f_{LL}(q^L_{ns}, q_{alt}) > f_{LL}(q^L_{alt}, q_{alt})$. Also, $f_{HL}(q^L_{ns}, q_{alt}) - f_{HH}(q^H_{alt}, q_{alt}) < f_{HL}(q^L_{ns}, q_{ns}) - f_{HH}(q^H_{alt}, q_{ns}) < f_{HL}(q^L_{ns}, q_{ns}) - f_{HH}(q^H_{ns}, q_{ns}) < 0$. Likewise, assume that $q_{alt} < q^*$. This implies $q^H_{alt} > q^H_{ns}$ and $q^L_{alt} < q^L_{ns}$. Profit maximization by the low-cost firm implies that that $f_{HL}(q^H_{alt}, q_{alt}) < f_{HH}(q^L_{alt}, q_{alt})$, which implies $f_{HL}(q^H_{ns}, q_{alt}) < f_{HH}(q^L_{alt}, q_{alt})$. Also, $q_{alt} < q^*$ implies that the short-run profit-maximizing output for a low-cost firm is greater than q^L_{ns}. Therefore, increasing its output from q_{alt} to q^L_{ns} would increase the low-cost firm's period 1's profit, whereas it would decrease the high-cost firm's profit even if it caused the competitor to wrongly infer it has a low cost (by the previous inequality). Such a deviation thus contradicts the intuitive criterion. Finally, if $q_{alt} = q^*$, which implies $q^H_{alt} = q^H_{ns}$, then a deviation by the low-cost firm from q^L_{alt} to q^L_{ns} would contradict the intuitive criterion.

The above reasoning carries over to the case where the condition underlying Proposition 2 applies ($\frac{\delta}{A-c_H} < \frac{4}{8+4\pi}$), simply replacing $\left(q_{ns}^H, q_{ns}^L\right)$ with $(\widetilde{q_H}, \widetilde{q_L})$.

Proof of Proposition 3. We assume that the condition stated in Propositions 2 and 3 holds:

$$\delta < \frac{4}{8+4\pi}\left(A - c_H\right) \tag{3}$$

We start by introducing the notation $L = A - c_H + \delta\left(\frac{1}{4} - \pi\right)$, which implies that $\Delta = \frac{\delta L}{9}$. Notice that (3) is equivalent to

$$\frac{L}{\delta} > \frac{9}{4}. \tag{4}$$

Substituting $A - c_H + \delta\pi$ for $A - c$, and with B denoting $A - c_H$, we can restate the formulas stated in Sections 2.2. and 2.3 as follows.

$$q_{ns}^H = \frac{B+\pi\delta}{3} - \frac{\pi\delta}{2}$$

$$q_{ns}^L = \frac{B+\pi\delta}{3} + \frac{(1-\pi)\delta}{2}$$

$$\overline{Q}_{ns} = \frac{2(B+\pi\delta)}{3}$$

$$\widetilde{q_H} = \frac{B - \pi/3\sqrt{\delta L}}{3} = \left(\frac{B}{3} + \frac{2\pi\sqrt{\delta L}}{9}\right) - \frac{\pi\sqrt{\delta L}}{3}$$

$$\widetilde{q_L} = \frac{B + (1-\pi/3)\sqrt{\delta L}}{3} = \left(\frac{B}{3} + \frac{2\pi\sqrt{\delta L}}{9}\right) + \frac{(1-\pi)\sqrt{\delta L}}{3}$$

$$\widetilde{Q} = \frac{2B}{3} + \frac{4\pi\sqrt{\delta L}}{9}$$

We prove first that total expected output is greater under signaling than under no signaling.

$$\widetilde{Q} - \overline{Q}_{ns} = \frac{4\pi\sqrt{\delta L}}{9} - \frac{2\pi\delta}{3} = \frac{2\pi\delta}{3}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right) > 0$$

as a result of (4).

(2) implies

$$ETS_{ns} = (B + \delta\pi)\,\overline{Q}_{ns} - \frac{\overline{Q}_{ns}^2}{2} - \sigma_{q_{ns}}^2 - 2Cov(q_{ns}, c)$$

$$\begin{aligned} ETS_{signaling} &= (B + \delta\pi)\,\widetilde{Q} - \frac{\widetilde{Q}^2}{2} - \sigma_{\widetilde{q}}^2 - 2Cov(\widetilde{q}, c^i) \\ &= (B + \delta\pi)\,\widetilde{Q} - \frac{\widetilde{Q}^2}{2} - \frac{\delta L}{9} + \frac{2\pi(1-\pi)\delta^{3/2}L^{1/2}}{3}. \end{aligned}$$

We calculate the difference $ETS_{signaling} - ETS_{ns}$ as the sum of the four following terms:

$$(B + \delta\pi)\left(\widetilde{Q} - \overline{Q}_{ns}\right) = \frac{2\pi\delta}{3}\,(B + \delta\pi)\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right)$$

$$-\frac{\widetilde{Q}^2}{2} + \frac{\overline{Q}_{ns}^2}{2} = -\frac{\pi\delta}{3}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right)\left(\frac{4B}{3} + \frac{4\pi\sqrt{\delta L}}{9} + \frac{2\pi\delta}{3}\right)$$

$$-\sigma_{\widetilde{q}}^2 + \sigma_{q_{ns}}^2 = -\frac{\pi(1-\pi)\delta L}{9} + \frac{\pi(1-\pi)\delta^2}{4} = -\frac{\pi(1-\pi)\delta^2}{4}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right)\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} + 1\right)$$

$$-2Cov(\widetilde{q}, c^i) + 2Cov(q_{ns}, c) = \frac{2\pi(1-\pi)\delta^{3/2}L^{1/2}}{3} - \pi(1-\pi)\delta^2 = \pi(1-\pi)\delta^2\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right),$$

yielding

$$\begin{aligned} \frac{ETS_{signaling} - ETS_{ns}}{\left(\frac{2}{3}\sqrt{L\delta} - \delta\right)\pi} &= \frac{2}{3}\,(B + \delta\pi) - \frac{1}{3}\left(\frac{4B}{3} + \frac{4\pi\sqrt{\delta L}}{9} + \frac{2\pi\delta}{3}\right) - \frac{(1-\pi)\delta}{4}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} + 1\right) + (1-\pi)\delta \\ &= \frac{2B}{9} + \frac{27 - 11\pi}{36}\delta - \sqrt{\delta L}\frac{9 - \pi}{54}. \end{aligned}$$

Replacing L with $\delta\left(\frac{1}{4} - \pi\right)$ and writing x for δ/B, one can check that this expression is positive if and only if

$$\left[\left(\frac{27 - 11\pi}{36}\right)^2 - \left(\frac{1}{4} - \pi\right)\left(\frac{9 - \pi}{54}\right)^2\right]x^2 + \left[\frac{27 - 11\pi}{81} - \left(\frac{9 - \pi}{54}\right)^2\right]x + \frac{4}{81} > 0.$$

This is necessarily the case because each of the three coefficients in the quadratic polynomial above is positive (as a consequence of $0 < \pi < 1$).

As regards consumer surplus, the identity $ECS = \frac{\overline{Q}^2}{2} + \frac{\sigma_Q^2}{2}$ and the above finding that under signaling, both expected output and the variance of expected output are greater than under no signaling imply that consumer surplus is greater under signaling.

Proof of Proposition 4. The identity $ECS = \frac{\overline{Q}^2}{2} + \frac{\sigma_Q^2}{2}$ and the above formulas imply that the first-period difference $ECS_{signaling} - ECS_{ns}$ is equal to

$$ECS_{signaling} - ECS_{ns} = \frac{\pi\delta}{3}\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right)\left[\left(\frac{4B}{3} + \frac{4\pi\sqrt{\delta L}}{9} + \frac{2\pi\delta}{3}\right) + \frac{(1-\pi)\delta}{4}\left(2\sqrt{\frac{L}{\delta}} + 3\right)\right]$$
$$> 4B\pi\delta\left(\frac{2}{3}\sqrt{\frac{L}{\delta}} - 1\right) = 4B\pi\delta\left(\frac{2}{3}\sqrt{\frac{B}{\delta} + \left(\frac{1}{4} - \pi\right)} - 1\right)$$

whereas the second-period difference is equal to

$$ECS_{full} - ECS_{ns} = -\frac{5\pi(1-\pi)\delta^2}{6}.$$

The ratio of the absolute value of the first-period difference over that of the second-period difference is

$$\frac{|ECS_{signaling} - ECS_{ns}|}{|ECS_{full} - ECS_{ns}|} > \frac{24\frac{B}{\delta}\left(\frac{2}{3}\sqrt{\frac{B}{\delta} + \left(\frac{1}{4} - \pi\right)} - 1\right)}{5(1-\pi)},$$

which is greater than 1 if $\frac{\delta}{B}(= \frac{\delta}{A - c_H})$ is close enough to zero.

Proof of Proposition 7. We show here an example of parameter values such that (i) the condition of Proposition 6 holds ($4\Gamma > \delta^2$) and (ii) equilibrium expected total surplus (considering both periods) is greater in the case where market share data are available than in the case when they are not. We define $x = \frac{\sqrt{\Gamma}}{\delta}$. The difference Δ_1 in expected total surplus in the first period is equal to

$$\Delta_1 = 2(1-\beta)c\left(\widetilde{p} - p^*\right) - (1-\beta)\left(\widetilde{p}^2 - p^{*2}\right)$$
$$+ \Delta_{ns\to signaling}\left(2Cov(p_1, c_1) - 2\beta Cov(p_1, c_2) - \sigma_p^2 + \beta Cov\left(p_1, p_2\right)\right)$$

(with obvious notations). The sum of the first two terms is equal to

$$2(1-\beta)c\left(\widetilde{p}-p^{*}\right)-(1-\beta)\left(\widetilde{p}^{2}-p^{*2}\right)=-\frac{(1-\beta)(1-\pi)\delta^{2}\left(2x-1\right)\left(2(1+c)+(1-\pi)\left(2x-1\right)\right)}{\left(2-\delta\right)^{2}}.$$

To calculate the other terms, note that (with obvious notations):

$$\sigma_{p_{ns}}^{2}=\frac{\pi(1-\pi)\delta^{2}}{4}$$

$$\sigma_{\widetilde{p}}^{2}=\pi(1-\pi)\Gamma$$

$$Cov\left(p_{1,ns},p_{2,ns}\right)=Cov\left(p_{1,ns},c_{2,ns}\right)=Cov\left(\widetilde{p_{1}},\widetilde{p_{2}}\right)=Cov\left(\widetilde{p_{1}},\widetilde{c_{2}}\right)=0$$

$$Cov\left(p_{1,ns},c_{1}\right)=\frac{\delta^{2}}{2}$$

$$Cov\left(\widetilde{p_{1}},c_{1}\right)=\pi(1-\pi)\sqrt{\Gamma}\delta.$$

Therefore, with obvious notations again,

$$\varDelta_{ns\rightarrow signaling}\left(2Cov(p_{1},c_{1})-2\beta Cov(p_{1},c_{2})-\sigma_{p}^{2}+\beta Cov\left(p_{1},p_{2}\right)\right)=\delta^{2}\pi(1-\pi)\left(x-\frac{1}{2}\right)\left(\frac{3}{2}-x\right).$$

Assume that β is very close to 1. Then $\varDelta_{1}$ is almost equal to $\delta^{2}\pi(1-\pi)\left(x-\frac{1}{2}\right)\left(\frac{3}{2}-x\right)$. The change in second period surplus, as mentioned in Section 3.2, is $-\frac{\beta^{2}\left(12-5\beta^{2}\right)}{4(4-\beta^{2})^{2}}\delta^{2}\pi(1-\pi)$, that is, arbitrarily close to $-\frac{7}{36}\delta^{2}\pi(1-\pi)$. If $x=1$ (which implies that the condition of Proposition 6, namely $x>\frac{1}{2}$, is satisfied), then the change in total surplus (considering both periods) is arbitrarily close to $\delta^{2}\pi(1-\pi)(\frac{1}{4}-\frac{7}{36})=\frac{\delta^{2}\pi(1-\pi)}{18}>0$. The last step of the proof is to show that one can find parameter values such that $x=1$ and $c_{H}<\frac{1}{1-\beta}$. If β is close to 1, then $x=\frac{\sqrt{\Gamma}}{\delta}\approx\frac{1}{2}\sqrt{\frac{1}{2\delta}+\frac{5-4\pi}{12}}$. We choose arbitrarily some π strictly between 0 and 1 and we set $\delta=\frac{6}{43+4\pi}$, which implies that $x=1$. Also, one can choose $c_{L}<\frac{1}{2}$ so that $c_{H}=c_{L}+\delta<1<\frac{1}{1-\beta}$.

We prove hereafter that if δ is small enough, then expected total surplus is lower in the first period when market share information is available. To see this, set β and π, and some $c<\frac{1}{1-\beta}$ so that if δ is small enough, $c_{H}(=c+\pi\delta)$ is smaller than $\frac{1}{1-\beta}$, as required by the model. The formula for Γ implies that

$$x = \frac{\sqrt{\Gamma}}{\delta} = \frac{\beta}{\sqrt{4-\beta^2}}\sqrt{\frac{1-(1-\beta)c}{(2-\beta)\delta} + \frac{8(1-\pi)+(4\pi-3)\beta^2)}{4(4-\beta^2)}}.$$

This expression tends to infinity as δ tends to zero, and is thus greater than $\frac{3}{2}$ if δ is small enough. But $x > \frac{3}{2}$ implies that $\Delta_1 < 0$ because, as the above calculations show, Δ_1 is the sum of $2(1-\beta)c\,(\widetilde{p} - p^*) - (1-\beta)\left(\widetilde{p}^2 - p^{*2}\right)$, which is negative, and of $\Delta_{ns\to signaling}\left(2Cov(p_1, c_1) - 2\beta Cov(p_1, c_2) - \sigma_p^2 + \beta Cov\,(p_1, p_2)\right)$, which has the same sign as $\frac{3}{2} - x$.

References

[1] AWAYA, Y. AND V. KRISHNA. (2020). Information Exchange in Cartels, *Rand Journal of Economics*, 51(2), 421-446.

[2] BERNARDT, D. AND TAUB, B. (2015). Learning about Common and Private Values in Oligopoly. *Rand Journal of Economics*, 46(1), 66–85.

[3] CAMINAL, R. (1990). A Dynamic Duopoly Model with Asymmetric Information, *Journal of Industrial Economics*, 38(3), 315-333.

[4] CHO, I-K. AND KREPS, D. M. (1987). Signaling games and stable equilibria. *Quarterly Journal of Economics,* 102,179-221.

[5] EUROPEAN COMMISSION. (2011). Communication from the Commission — Guidelines on the applicability of Article 101 of the Treaty on the Functioning of the European Union to horizontal co-operation agreements, available at https://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:52011XC0114(04)

[6] FRIED D. (1984). Incentives for information production and disclosure in a duopolistic environment. *Quarterly Journal of Economics,* 99(2), 367–381.

[7] GAL-OR, E. (1985). Information sharing in oligopoly. *Econometrica,* 53(2), 329–343.

[8] GAL-OR, E. (1986). Information transmission - Cournot and Bertrand equilibria. *Review of Economic Studies*, 53(1), 85–92.

[9] JIN, J. (1994). Information Sharing Through Sales Report. *Journal of Industrial Economics*, 42(3), 323-331.

[10] LI, L. (1985). Cournot oligopoly with information sharing. *Rand Journal of Economics*, 16(4) 521–536.

[11] MAILATH, G. J. (1989). Simultaneous Signaling in an Oligopoly Model. *Quarterly Journal of Economics*, 104(2), 417–427.

[12] MESTER, L. (1992). Perpetual Signalling with Imperfectly Correlated Costs. *Rand Journal of Economics*, 23(4), 548–563.

[13] MYATT, D. P. AND WALLACE, C. (2015). Cournot competition and the social value of information, *Journal of Economic Theory*, 158(B), 466–506.

[14] OECD. (2010). Information Exchanges Between Competitors, availaible at http://www.oecd.org/competition/cartels/48379006.pdf

[15] RAITH, M. (1996). A general model of information sharing in oligopoly. *Journal of Economic Theory,* 71(1), 260–288.

[16] SAKAI, Y. AND YAMATO, T. (1990). On the Exchange of Cost Information in a Bertrand-type Duopoly Model. *Economic Studies Quarterly*, 30, 48-64

[17] SHAPIRO, C. (1986). Shapiro, Exchange in of cost information oligopoly, *Review of Economic Studies,* 53(3), 433–446.

[18] VIVES, X. (1984). Duopoly information equilibrium: Cournot and Bertrand, *Journal of Economic Theory*, 34(1), 71–94.

[19] VIVES, X. (2011). Strategic Supply Function Competition with Private Information. *Econometrica*, 79(6), 1919–1966.