Beyond Belief: Logic in multiple attitudes

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Abstract

Choice-theoretic and philosophical accounts of rationality and reasoning address a multi-attitude psychology, including beliefs, desires, intentions, etc. By contrast, logicians traditionally focus on beliefs only. Yet there is logic in multiple attitudes. We propose a generalization of the three standard logical requirements on beliefs – consistency, completeness, and deductive closedness – towards multiple attitudes. How do these three logical requirements relate to *rational* requirements, e.g., of transitive preferences or non-akratic intentions? We establish a systematic correspondence: each logical requirement (consistency, completeness, or closedness) is equivalent to a class of rational requirements. Loosely speaking, this correspondence connects the logical and rational approaches to psychology. Addressing John Broome's central question, we characterize the extent to which reasoning can help achieve consistent, complete, or closed attitudes, respectively.

1 Introduction

Theories of rationality and choice rarely meet logic. An obstacle is that logic is traditionally used to study beliefs and their change through reasoning, whereas theories of rationality address a rich multi-attitude psychology, usually without a formal model of reasoning. Yet, in an important sense, logical relations and reasoning exist not only (as we shall say) 'in beliefs', but also 'in preferences', 'in intentions', etc. For instance, preferences can be inconsistent with other preferences, or be formed through reasoning (e.g., Broome 2006). In fact, logical relations and reasoning even go across attitude types: intentions can be inconsistent with beliefs, or be formed through reasoning from preferences and beliefs, etc. We refer to this as logic and reasoning 'in (multi-)attitudes', a topic at the heart of current philosophical work on rationality and reasoning. Broome's (2013) influential account of rationality and reasoning will be our benchmark, but our paper should also resonate with other important accounts (e.g., Kolodny 2005, Boghossian 2014). Reasoning in attitudes is what Broome calls reasoning 'with' attitudes.

We prefer saying 'in attitudes' to mark the distinction to reasoning *about* attitudes, something different, about which formal logic has much to say. One may reason about the attitudes (the beliefs, intentions, etc.) of someone else, or perhaps even of oneself; various logics to study such reasoning have been developed.[1] Reasoning about attitudes is reasoning in beliefs: in beliefs about attitudes, normally of someone else. It lets you discover these attitudes, not form them.

To relate rationality to logic, we shall import some 'logic' into rationality theory, by extending the three classical logical conditions on beliefs – consistency, completeness, and deductive closedness – towards multiple attitudes (Sections 2-3). This will naturally lead to a question: how do these logical requirements relate to *rational* requirements, such as preference transitivity? We establish a mathematical correspondence between both types of requirement (Section 4). This correspondence will allow us to identify the extent to which reasoning in multi-attitude can make one's attitudes consistent, complete, or deductively closed, respectively (Section 5). Our analysis of logical requirements on, and reasoning in, multiple attitudes is not 'logical' in the proper sense of employing syntax and semantics. We therefore finally compare our analysis to a properly logical approach (Section 6).

2 Rational attitudes formalised

This section introduces the central Broomean concepts we need, in a formalisation that follows Dietrich et al. (2019).

Mental states. The agent – 'you' – holds mental states: beliefs, desires, preferences, intentions, etc. Let M be the non-empty set of all possible *mental states* or *attitudes*. M might contain: believing that it rains, believing that it is sunny, desiring to stay dry, intending to dress warm, preferring sunshine to rain, etc. Think of attitudes in M as pairs of an attitude content (an object) and an attitude type. For most philosophers, contents are propositional: they are *single* propositions for monadic attitudes such as belief or desire, *pairs* of propositions for dyadic attitudes such as preference, etc. We shall say 'attitude' not only for mental states in M (such as: intention to swim), but also for attitude types (such as: intention).

Constitutions. Those mental states in M you possess form your 'constitution'. Formally, a **(mental) constitution** is thus any set $C \subseteq M$ of mental states, 'your' states.

Rationality. Certain constitutions are 'rational', the others are 'irrational'. We identify a notion or theory of rationality with the set of constitutions it deemed

[1] Examples are logics for reasoning about preferences (e.g., Liu 2011), about beliefs (e.g., Halpern 2017), or about beliefs, desires and intentions (so-called 'BDI logics').

rational. So, formally, a **notion or theory of rationality** is simply a set T of constitutions, the 'rational' constitutions.

An illustration. In practice, theories of rationality are defined by specifying requirements. Rational constitutions are then constitutions satisfying those requirements. To state typical requirements, let us first formalise the structure of states. Let L be a set of *propositions*. Let A be a set of *attitude types*, each endowed with an *arity* $n \in \{1, 2, ...\}$, which is usually 1 (for unary or monadic attitudes) or 2 (for binary or dyadic attitudes). A could contain (monadic) attitudes of belief bel, desire des, and intention int, and (dyadic) attitudes of preference $\succ$ and indifference $\sim$. Let the states in M be all tuples $m = (p_1, ..., p_n, a)$ in which a is an attitude type in A, n is its arity, and $p_1, ..., p_n$ are propositions in L. So, (p, bel) is belief of p, (p, int) intention of p, $(p, q, \succ)$ preference of p to q, etc. Here are some typical requirements on C, more precisely requirement schemas parameterized by propositions:

R1: *Modus Ponens*: Believing p and $if\ p\ then\ q$ implies believing q, formally, $(p, bel), (if\ p\ then\ q, bel) \in C \Rightarrow (q, bel) \in C$. Parameters: $p, q \in L$.

R2: *Non-Contradictory Desires*: Desiring p excludes desiring *not* p, formally, $(p, des) \in C \Rightarrow (not\ p, des) \notin C$. Parameter: $p \in L$.

R3: *Enkrasia (Non-Akrasia)*: Believing that *obligatorily* p implies intending p, formally, $(obligatorily\ p, bel) \in C \Rightarrow (p, int) \in C$. Parameter: $p \in L$.

R4: *Instrumental Rationality*: intending p and believing $q\ is\ a\ means\ implied\ by\ p$ implies intending q, formally $(p, int), (q\ is\ a\ means\ implied\ by\ p, bel) \in C \Rightarrow (q, int) \in C$. Parameters: $p, q \in L$.

R5: *Preference Transitivity*: preferring p to q and q to r implies preferring p to r, formally, $(p, q, \succ), (q, r, \succ) \in C \Rightarrow (p, q, \succ) \in C$. Parameters: $p, q, r \in L$.

R6: *Preference Acyclicity*: you do not simultaneously prefer p_1 to p_2, p_2 to p_3, ..., p_{k-1} to p_k, and p_k to p_1, formally, $(p_1, p_2, \succ), (p_2, p_3, \succ), ..., (p_{k-1}, p_k, \succ) \in C \Rightarrow (p_k, p_1, \succ) \notin C$. Parameters: any number $k \geq 1$ and any $p_1, ..., p_k \in L$.

R7: *Preference Completeness*: you have some preference or indifference between p and q, formally $(p, q, \succ) \in C$ or $(q, p, \succ) \in C$ or $(p, q, \sim) \in C$. Parameters: $p, q \in L$.

Are these requirements plausible? Should we refine their formulation? Which else should be required? These important questions are not our topic. For us, the lesson is that any given list of requirements defines a theory of rationality: the theory whereby constitutions are rational just when they satisfy these requirements, the theory's 'axioms'.

In stating R1–R7, we have implicitly assumed that certain composite propositions can be formed within L. Specifically, whenever L contains propositions p and q, L contains specific propositions *not* p, $if\ p\ then\ q$, *obligatorily* p, and $q\ is\ a\ means\ implied\ by\ p$.[2] Some readers might want to model propositions syn-

[2]Technically, the assignments $p \mapsto not\ p$ and $p \mapsto obligatorily\ p$ define two unary operators

tactically (intensionally): L contains the well-formed sentences of a suitable formal language. So the mentioned composite propositions are composite sentences: *not* p stands for $\neg p$, *obligatorily* p stands for $O(p)$ where O is a sentential 'obligation' operator, etc. Other readers might want to model propositions semantically (extensionally), as subsets of a given set of possible worlds Ω. So the mentioned composite propositions are constructed semantically: *not* p is the complementary proposition $\Omega\backslash p$, *obliganroily* p is $O(p)$, where O is a semantic 'obligation' operator mapping Ω-subsets to Ω-subsets, etc.[3]

3 Three logical requirements on attitudes

Having a rational constitution is an ideal you rarely meet. We now introduce three weaker requirements. We call them 'logical' requirements because they are counterparts for multiple attitudes of the three standard logical requirements on *beliefs*. The logical requirements on beliefs are:

(a) *Consistency*: do not believe mutually inconsistent propositions, i.e., propositions which cannot be simultaneously true.

(b) *Completeness*. 'Local' completeness says: believe a member of each proposition-negation pair $\{p$, *not* $p\}$. General or 'global' completeness (our focus) says something stronger: believe a member of each set of mutually exhaustive propositions, i.e., propositions that cannot be simultaneously false. There are many such sets: proposition-negation pairs $\{p,$ *not* $p\}$, sets of type $\{p, q,$ [*not* p] *or* [*not* q]$\}$, etc.

(c) *Closedness*: believe all consequences of your beliefs, i.e., all beliefs that are true whenever your existing beliefs are true.

How can we generalize these logical requirements towards your full multi-attitude psychology? That is, when should we count your constitution as consistent? As complete? As closed? The (informal) definitions (a)-(c) cannot be directly trans-

$L \to L$, and the assignments $(p, q) \mapsto$ *if* p *then* q and $(p, q) \mapsto q$ *is a means implied by* p define two dyadic operators $L \times L \to L$.

[3]The syntactic model of propositions is appealing not only if one thinks propositions are sentences (an implausible metaphysical view), but, more interestingly, if one thinks they are the *meaning* (intension, Sinn) of sentences and can be formally represented by these sentences. The latter view reflects an intensional notion of proposition. By contrast, the semantic, i.e., set-theoretic, model of propositions cannot distinguish between logically equivalent propositions: *it neither snows nor rains* and *It is not the case that it snows or rains* are represented by the same set of worlds, hence the same proposition. This can be problematic because attitudes can distinguish between equivalent propositions: we often believe or intend something without believing or intending something equivalent, often out of unawareness of the equivalence. The common label 'semantical' for the extensional, set-theoretic model of propositions assumes that 'semantics' is about the reference (extension, denoted thing, Bedeutung) of sentences. This assumption is questionable, to say the least. One could alternatively take 'semantics' to be about the meaning (intension, Sinn) of sentences, on grounds of etymology and natural usage.

lated from beliefs to multiple attitudes, because notions such as 'truth' and 'possible world' are inappropriate in the realm of desires and other non-representational attitudes. We cannot use truth-based definitions of multi-attitude consistency, (global) completeness, and closedness. But rationality-based definitions are possible. Loosely speaking, rationality can substitute truth, when going beyond belief.

To see how, we first consider beliefs, and re-express the definitions (a)-(c) in terms of rationality rather than truth (we do this informally; for details see Appendix B). A belief set is 'rational' in the fully classical sense if it is consistent, complete and closed ('complete' could be defined locally or globally, and 'closed' could be dropped as it follows from 'consistent' and 'complete'). This characterizes rationality in terms of the three logical requirements. But one can do the opposite: characterize the logical requirements in terms of rationality: as shown in Appendix B, a belief set is

- consistent if and only if it can be made rational by adding (zero or more) suitable beliefs;
- complete in the global sense if and only if it can be made rational by removing (zero or more) suitable beliefs;
- closed if and only if it contains each belief b which it entails, where 'entailment' has a rationality-based characterization, since a belief set entails b just when all its rational extensions contain b.

These rationality-based characterizations of the logical requirements are precisely what we need, since they can be extended to multiple attitudes:

Definition 1 *Given a theory of rationality, a constitution C is*

- ***consistent*** *if there is a rational constitution $C' \supseteq C$,*
- ***complete*** *if there is a rational constitution $C' \subseteq C$,*
- ***closed*** *if C contains each attitude which it entails, where being* **entailed** *by C means being contained in all rational constitutions $C' \supseteq C$.*

These definitions make intuitive sense. They treat a constitution as

- *consistent* if one can rationally hold at least the attitudes in it,
- *complete* if one can rationally hold at most the attitudes in it,
- *closed* if no other attitudes rationally follow.

Also formally speaking, our logical requirements for multi-attitudes generalize their classic counterparts for beliefs: the classic requirements are special cases, obtained if M contains only beliefs (no intentions, etc.). Why? In this belief-only case, constitutions are equivalent to belief sets: to each constitution (a set of belief states) corresponds a belief set (the set of contents of these beliefs), and our logical requirements are equivalent to the classic requirements applied to the corresponding belief set:

Proposition 1 *(stated informally) In the belief-only case, a constitution C, with corresponding belief set B, is*

- *consistent if and only if B is classically consistent,*
- *complete if and only if B is classically complete (understood globally),*
- *closed if and only if B is classically closed.*

This result is re-stated formally and proved in Appendix B.

4 Logical versus rational requirements

We have generalized the three logical requirements – consistency, completeness, closedness – from beliefs to multi-attitudes. This allows logical talk about your multi-attitude psychology, as opposed to standard rationality talk. But how do the three logical requirements relate to typical rational requirements such as those in R1–R7? The difference between typical rational requirements and logical requirements is fundamental. For one, generic rational requirements are concrete and refer to specific attitudes (such as preferences in R5–R7, or intentions and beliefs in R3–R4), whereas the three logical requirements are purely structural, without reference to specific attitude types. For another, generic rational requirements have a status of 'axioms' used to *define* theories of rationality, whereas logical requirements have a *derived* status, as they follow from (the axioms of) a theory. For instance, transitivity of preferences, a typical rational requirement (or schema thereof), counts as an axiom in choice theory.

Despite their fundamental differences, the two kinds of requirement stand in a tight formal relationship: each logical requirement is equivalent to a particular class of rational requirements. Before stating this theorem, we note that most rational requirements found in philosophy or choice theory fit into the following three-kind typology, which is implicit in various works and spelt out formally in Dietrich et al. (2019):

Definition 2 *A* ***requirement*** *is a condition on constitutions, formally a set R of constitutions (those* ***satisfying*** *the requirement). It is a*

- ***consistency requirement*** *if it forbids having all of certain mental states, i.e., $R = \{C : \text{not } F \subseteq C\}$ for some set $F \neq \varnothing$ of states (the 'forbidden set'),*
- ***completeness requirement*** *if it forbids having none of certain attitudes, i.e., $R = \{C : \text{not } C \cap U = \varnothing\}$ for some set $U \neq \varnothing$ of attitudes (the 'unavoidable set'),*
- ***closedness requirement*** *is if it demands that having certain attitudes implies having a certain attitude, i.e., $R = \{C : P \subseteq C \Rightarrow c \in C\}$ for some set of ('premise-')attitudes P and some ('conclusion-')attitude c.*

Standard requirements, like those in R1–R7, fall into this typology:

- Non-Contradictory Desires (R2) and Preference Acyclicity (R6) are schemas of consistency requirements, with forbidden set $\{(p, des), (not\ p, des)\}$ or $\{(p_1, p_2, \succ), (p_2, p_3, \succ), ..., (p_{k-1}, p_k, \succ), (p_k, p_1, \succ)\}$, respectively.
- Preference Completeness (R7) is a schema of completeness requirements, with unavoidable set $\{(p, q, \succ), (q, p, \succ), (p, q, \sim)\}$.
- Modus Ponens (R1), Enkrasia (R3), Instrumental Rationality (R4), and Preference Transitivity (R5) are schemas of closedness requirements, in R1 with premise set $\{(p, bel), (if\ p\ then\ q, bel)\}$ and conclusion state (q, bel).

A requirement is a *rational* requirement if it follows from the given theory of rationality:

Definition 3 *The* ***requirements of a theory of rationality*** T *– or* ***rational requirements*** *– are those requirements* R *which follow from* T*, i.e., for which* $T \subseteq R$*.*

Typical theories of rationality make several consistency, completeness, or closedness requirements, such as those in R1–R7. More is true: the axioms or principles by which theories of rationality are defined in practice – such as transitivity of preferences – are usually (schemas of) consistency, completeness, or closedness requirements.

Logical requirements are special rational requirements, though non-generic ones as they make no reference to attitude types (such as preferences) and have a flavour of logic rather than rationality. Formally:

Remark 1 *Given a theory of rationality* T*, the three logical requirements*[4] *are requirements of* T*, i.e., special rational requirements.*

So, the right contrast to draw is not one between logical and rational requirements, but one between logical requirements and *generic* rational requirements – the sort of requirements encountered in rationality theory, such as those in R1–R7. Unlike logical requirements, generic rational requirements normally belong to our three-kind typology.

A question has become pressing: how does the logical approach to multi-attitude psychology, with its three logical requirements, relate to the rationality-based approach, with its three classic types of rational requirement? There is an exact correspondence:

Theorem 1 *Given any theory of rationality* $T \neq \varnothing$*, a constitution* C *is*

(a) consistent if and only if it satisfies all consistency requirements of T*,*

(b) complete if and only if it satisfies all completeness requirements of T*,*

(c) closed if and only if it satisfies all closedness requirements of T*,*

[4] I.e., consistency, completeness and closedness, formally reprsented as the sets of constitutions $R = \{C : C \text{ is consistent}\}$, $R = \{C : C \text{ is complete}\}$ and $R = \{C : C \text{ is closed}\}$, respectively.

(d) fully rational if and only if it satisfies all requirements of T.

Parts (a)–(c) of this result connect the logical world of structural (global) requirements to the choice-theoretic or philosophical world of concrete (local) requirements of three types. Part (d) is an addendum, of interest in its own right.

This result is purely static, by concerning coherence at a given time. But it has important implications for the dynamic process of reasoning. Which implications?

5 Are logical requirements achievable through reasoning?

Your constitution is usually inconsistent, incomplete, and unclosed. Can reasoning help you satisfy these logical requirements? This question is a cousin of Broome's central question: can reasoning help achieve *rational* requirements, such as (instances of) Preference Transitivity or Enkrasia? The tight connection between rational and logical requirements (Theorem 1) suggests an equally tight connection between Broome's and our question, i.e., between whether reasoning makes 'more rational' and whether it makes 'more logical'. We will confirm this conjecture.

Unlike Broome, we set aside whether reasoning is *correct* in some objective sense. Our conclusions about becoming 'more logical' through reasoning will be largely negative, and would get further reinforced by excluding incorrect reasoning.

5.1 Reasoning in attitudes

To pave the way, we now briefly discuss the Broomean notion of reasoning adopted here, and formalise it following Dietrich et al. (2019). For Broome (2013) and others, reasoning is a process of forming attitudes from existing attitudes: forming intentions from intentions and beliefs, or preferences from preferences, etc. The process is causal. Unlike other causal processes, it is conscious and constitutes a mental act. You bring the premise-attitudes to mind by 'saying' their contents to yourself, usually through internal speech. This causes you to 'construct' and thereby acquire some conclusion-attitude, again using (usually internal) speech. Here is a stylised instance of reasoning with a single premise. You say this to yourself:

Doctors recommend resting. So, I shall rest.

This is reasoning from a belief to an intention. The 'So' is not part of the conclusion, but expresses the act of drawing the conclusion. Note the use of 'shall': it is a linguistic marker indicating that the conclusion forms an intention. Had you concluded in a belief with same content, you would have said '*I will rest*'. In general, you say to yourself, not contents of attitudes simpliciter, but *marked* *contents*, i.e., contents with a marker indicating *how* you entertain the content: as a belief, or an intention, etc. The English language provides markers for various attitudes. Beliefs are special: they need no explicit marker (in English), as the same sentence expresses the content and the marked content.

Reasoning is rule-governed: you draw the conclusion by following a *rule*. Rules can be individuated more or less broadly. In the example, the rule could be:

- specific: from believing that doctors recommend resting, towards intending to rest.
- broader: From believing that doctors recommend ϕ-ing towards intending to ϕ. Parameter: any act ϕ.
- even broader: From believing that experts E recommend ϕ-ing towards intending to ϕ. Parameters: any experts E and act ϕ.

We shall work with specific rules, to avoid dealing with schemas or parameters. Nothing hinges on this technical choice: our results could be re-stated (more clumsily) using a broader notion of rules. Given our choice, we can identify a rule with a specific premises/conclusion combination. Technically, a **reasoning rule** is a pair (P, c) of a set of ('premise-')attitudes $P \subseteq M$ and a ('conclusion-')attitude $c \in M$, representing the formation of c from P. In the rule of the example above, P contains just the belief that doctors recommend resting, and c is the intention to rest. You follow certain rules – 'your' rules. The totality of your rules is your 'reasoning system', representing your reasoning policy. Technically, a **reasoning system** is a set S of reasoning rules. Starting from your initial constitution, you can reason which each of your rules: whenever you already have a rule's premise-attitudes, you form the conclusion-attitude, which gets added to your constitution. You can do this until your constitution is stable. A constitution C is **stable under** S ('**under reasoning**') if reasoning makes no change, i.e., C already contains the conclusion-attitude of each rule in S whose premise-attitudes it contains. The stable constitution reached by reasoning from your initial constitution C using your reasoning system S is denoted $C|S$ and called the **revision of** C **through** S ('**through reasoning**'). Technically, $C|S$ is defined as the minimal extension of C stable under S.[5] Provided your reasoning system S is finite, you can reach $C|S$ is in finitely many reasoning steps. You first apply a rule (P, c) in S that is effective (difference-making) on C, i.e., for which $P \subseteq C$ but $c \notin C$; your constitution becomes $C \cup \{c\}$. You then apply another rule (P', c') in S that is effective on $C \cup \{c\}$; your constitution becomes $C \cup \{c, c'\}$. You continue until all your rules are ineffective. The order in which you reason, i.e., apply rules, is irrelevant: you inevitably converge to the same stable constitution $C|S$. All this can be stated formally.[6]

[5] This (with respect to set-inclusion) minimal stable extension exists and is unique. It is the intersection of all stable extensions $C' \supseteq C$.

[6] Write $C|r_1|r_2|\cdots|r_n$ for the result of revising C through rule r_1, then through rule r_2, etc. until r_n. For finite S, $C|S$ can be shown to equal $C|r_1|r_2|\cdots|r_n$ for any sequence $(r_1, ..., r_n)$

5.2 Which logical requirements are achievable?

What would it mean to achieve a logical requirement (or even full rationality) through reasoning? Given a theory of rationality, a reasoning system S **achieves** consistency, completeness, closedness, or (full) rationality if for each constitution C the revision $C|S$ is, respectively, consistent, complete, closed, or rational. There would be little point in achieving completeness or closedness if one thereby sacrifices consistency, the arguably most basic and 'least sacrificable' of the three logical requirements. We therefore want reasoning to preserve consistency. Formally, a reasoning system S **preserves consistency** if for each consistent constitution C its revision $C|S$ is still consistent.

By Theorem 1, achieving consistency, completeness, or closedness is respectively equivalent to achieving special rational requirements. But whether these special requirements are achievable is known; it is informally contained in Broome's work, and formally worked out in Dietrich et al. (2019). Details aside, reasoning can successfully achieve closedness requirements, but not consistency or completeness requirements. Using this fact (including details that we have skipped), Theorem 1 has another theorem as a corollary. Informally,

- reasoning can achieve closedness while preserving consistency,
- reasoning cannot achieve consistency,
- reasoning can achieve completeness, but only while sacrificing consistency.

Formally:

Theorem 2 *Given any theory of rationality,*

(a) some reasoning system achieves closedness while preserving consistency,

(b) no reasoning system achieves consistency (unless consistency is trivial[7]),

(c) no reasoning system achieves completeness while preserving consistency (unless completeness is essentially trivial[8]),

(d) no reasoning system achieves full rationality (unless consistency is trivial).

In (b)–(d), 'unless' can be read not only in its weak sense ('if it is not the case that'), but even in its strong sense ('if *and only if* it is not the case that'). So Theorem 2 provides necessary and sufficient conditions for the possibility of successful reasoning, in the four senses of becoming consistent, complete, closed, or fully rational.[9]

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of S-rules that is maximal subject to each rule r_i being effective on the previously reached constitution $C|r_1|r_2|\cdots|r_{i-1}$. In this representation of $C|S$ through consecutive reasoning, the sequence $(r_1, ..., r_n)$ (the way to reason) is only to a limited extent unique: all such sequences $(r_1, ..., r_n)$ have the same length (number of reasoning steps) n and the same set of conclusion-attitudes $\{c : \text{some of } r_1, ..., r_n \text{ concludes in } c\}$.

[7] i.e., unless the theory deems all constitutions consistent (or equivalently, deems the all-attitudes constitution $C = M$ rational).

[8] i.e., unless the theory deems essentially every constitution complete, in a sense defined below.

[9] In part (c), the stronger reading of 'unless' however requires a *compactness* assumption: each

The message of Theorem 2 is gloomy, though 'Broomean': you cannot reason towards two of three logical requirements, just as (following Broome) you cannot reason towards many rational requirements. This result is independent of the attitude type: it even holds for ordinary reasoning in beliefs. A more nuanced picture emerges after cashing in that other mental processes than reasoning could jump in to make your attitudes inch closer to completeness (by creating attitudes) or consistency (by removing attitudes). For instance, some beliefs or intentions might crowd out other ones that are inconsistent with them, making you 'more consistent'. We *can* become 'more logical', but not through reasoning alone.

We now discuss each part in turn.

Part (a): the achievability of closedness. By part (a), you can develop deductively closed attitudes through reasoning, without losing consistency. Why? By Theorem 1, closedness is achieved once all the theory's closedness *requirements* are achieved. A closedness requirement says: having a certain set of attitudes P implies having a certain attitude c. You achieve this requirement if you have the rule $r = (P, c)$. You achieve *all* of the theory's closedness requirements if you have *all* corresponding rules. If these are your only rules, reasoning provably preserves consistency. Although this reasoning system does the job, it is peculiar: it is so rich in rules that you can reason towards each closedness requirement of the theory in a single step. In practice, much slimmer (and cognitively more plausible) reasoning systems do the same job of achieving closedness while preserving consistency. You only need rules corresponding to *certain* closedness requirements of the theory. Suppose rationality requires that believing p and $if\ p\ then\ q$ implies believing q, and that believing q implies intending r. Then rationality also requires that believing p and $if\ p\ then\ q$ implies intending r. If you have the rules corresponding to the first two closedness requirements,

$$r = (\{(p, bel), (if\ p\ then\ q, bel)\}, (q, bel)) \text{ and } r' = (\{(q, bel)\}, (r, int)),$$

then you need not have the rule corresponding to the third requirement, $r'' = (\{(p, bel), (if\ p\ then\ q, bel)\}, (r, int))$, because the third requirement is achievable through applying first r and then r'. Real people presumably reason with few and simple rules.

Part (b): the unachievability of consistency. Part (b) is mathematically trivial, but philosophically disturbing. It is trivial (without even consulting Theorem 1) because Broomean reasoning never removes attitudes, hence never makes inconsistent constitutions consistent. Broome acknowledges that inconsistencies often disappear, but insists that they disappear, not through reasoning, but through automatic psychological processes, such as when you find yourself

inconsistent set of states $C \subseteq M$ has a finite inconsistent subset. Compactness holds trivially if M is finite. Compactness is the multi-attitude counterpart of ordinary logical compactness.

losing a belief after realizing a conflict with other beliefs. The impossibility to *reason* yourself out of inconsistency is disturbing because consistency is a more basic normative desideratum than completeness and closedness. One would have hoped that reasoning can *at least* make consistent; instead it can make closed, but not consistent. The problem is only avoided for trivial theories of rationality that deem all constitutions consistent.

Part (c): the unachievability of completeness. Why does part (c) hold? Given the theory of rationality, we call a set of attitudes **avoidable** if some rational constitution contains none of its states, and **unavoidable** otherwise. Typical unavoidable sets are $\{(p, bel), (not\ p, bel)\}$, $\{(p, int), (q, int), (r, int)\}$, and $\{(p, q, \succ), (q, p, \succ), (p, q, \sim)\}$, for propositions p and q. The theory's unavoidable sets stand in one-to-one correspondence with the theory's completeness requirements: a set $U \subseteq M$ is unavoidable if and only if the theory makes the completeness requirement of having some attitude from U. Now by Theorem 1, completeness is achieved once you satisfy the theory's completeness *requirements*, or equivalently, once you have acquired some attitude from each unavoidable set. There is a trivial (but problematic) way to acquire such attitudes: for each unavoidable set U, you simply have a rule that always generates a given attitude in U (formally, a rule $r = (\varnothing, m)$ which has no premise-attitudes and some conclusion-attitude m in U).

This trivial way to reason towards completeness is unconvincing. It seems ad hoc, if not stubborn and blind, to always acquire the same attitude from a given unavoidable set U, regardless of the web of existing attitudes. What matters is not that *that* you form an intention (from an unavoidable set of intentions U), but also which *intention* you form. Otherwise you can become inconsistent with your beliefs, preferences, or other existing attitudes. Formally, the trivial reasoning system achieves completeness by sacrificing consistency. Unfortunately, also all other reasoning systems that achieve completeness fail to preserve consistency.

This argument presupposes that completeness is not essentially trivial, as shown in the appendix. Completeness is **trivial** if the theory deems all constitutions complete; or equivalently, the empty constitution is rational. Here there are no unavoidable sets U. Slightly more generally, completeness is **essentially trivial** if all constitutions *containing at least the unfalsifiable attitudes (if any)* are complete; or equivalently, some constitution containing at most unfalsifiable attitudes is rational. An attitude m is **unfalsifiable** if it never conflicts with other attitudes, i.e., if $\{m\} \cup C$ is consistent whenever C is consistent. Standard theories of rationality deem no attitudes unfalsifiable: desiring p is falsifiable by conflicting with desiring *not* p; preferring p to q is falsifiable by conflicting with being indifferent between p and q, or with preferring q to p, or with preferring q to r and also r to p; etc. So, for standard theories of rationality, essentially trivial completeness just means trivial completeness.

Part (d): the unachievability of full rationality. Since consistency is unachievable by part (b), so is full rationality. This again presupposes that not all constitutions count as consistent – otherwise you could trivially become rational by having *all* reasoning rules, making you form all attitudes.

6 The different way formal logic goes beyond belief

Our analysis of multi-attitude psychology has been 'logical' in a light sense: we have analysed logical requirements and reasoning without ever employing formal syntax or semantics. But also formal logic has much to say about attitudes, using logics of beliefs, preferences, or other attitudes. How does our light-logical analysis relate to a formal-logical analysis? As we shall see, both go beyond belief in different senses, and pursue different objectives. Formal logic provides a third-personal description of attitudes, suitable for reasoning about someone's attitudes, but not for addressing multi-attitude psychology in an internal, first-personal sense. We shall distinguish between the statics (Section 6.1) and the dynamics (Section 6.2) of multiple attitudes, arguing that the difference between both approaches emerges in the dynamics, i.e., in reasoning.

6.1 The statics of multiple attitudes

The statics of multi-attitudes concern your attitudes *at a given time*. The three logical requirements – consistency, completeness, closedness – are purely static requirements. We were able to formalise them without properly 'logical' tools. An alternative approach would use some formal logic of attitudes. Logics of attitudes exist in abundance. Mono-modal logics involve just one attitude, for instance belief in 'doxastic logics' (e.g., Halpern 2005), or preferences in 'preference logics' (e.g., Liu 2011). Multi-modal logics involve two or more attitudes, for instance beliefs, desires and intentions in 'BDI logics' (e.g., Van der Hoek and Wooldridge 2003). Attitudes are represented by modal operators, and rationality by axioms. No doubt, this machinery is well-suited for studying the statics of multiple attitudes, including nested attitudes (meta-attitudes) such as intentions to desire to believe something. Like our light-logical machinery, such modal logics could be used to define notions of attitudinal consistency, completeness, and closedness, though one would be limited to the (often few) attitudes present in the logic in question.

6.2 The dynamics of multiple attitudes

The dynamics of multi-attitudes concern attitude change. Modal logics of the sort just discussed can address reasoning *about* rather than *in* attitudes – for reasons independent of the attitude type, hence valid even for ordinary reasoning in beliefs. Establishing that this difference is real and could not be overcome through some formal reduction requires a careful analysis, which we undertake in Dietrich et al. (2020). Here, a few remarks should suffice. If someone reasons about your attitudes, then what changes are not your attitudes, but the reasoner's beliefs about them. Even if it is you yourself who reasons about some of your attitudes, then not those attitudes change, but your (meta-)beliefs about them.[10] In our earlier example, you reason *in* your attitudes by saying:

Doctors recommend resting. So, I shall rest.

You thereby form an intention from a belief. An observer (possibly you) might reason *about* your attitudes by saying:

You believe doctors recommend resting. So, you intend to rest.

This and other reasoning about attitudes can be represented modal-logically, as following entailments between atomic attitude-sentences of type 'you hold attitude such-and-such towards such-and-such', formally $O(\phi)$ with an operator O representing the attitude type and a sentence ϕ representing the attitude content. Thanks to appropriate axioms, the right entailments hold between such 'atomic' attitude-sentences. The logic also provides entailments between plenty of 'non-atomic' attitude-sentences, such as: 'you do *not* desire this', 'you *either* believe this *or* intend that', etc. Reasoning about attitudes can thus start from, or conclude in absences of attitudes, disjunctions of attitudes, etc. Reasoning cannot, as Broome insists. This marks another fundamental difference between internal reasoning and logical entailment, hence between practical reasoning and theoretical reasoning about attitudes.

A Proof of Theorem 1

In this appendix, we fix a theory of rationality T and a constitution C. Let $T \neq \varnothing$, an assumption needed only in parts (a) and (b). We now prove each part.

Part (a). We prove both directions of implication. We may assume $C \neq \varnothing$, since otherwise C is trivially consistent (as $T \neq \varnothing$) and satisfies all consistency requirements.

- First let C satisfy T's consistency requirements. We show that C is consistent. Consider the consistency requirement R^* of not holding all states in C: $R^* = \{C' : C \nsubseteq C'\}$. Since C violates R^* while satisfying T's consistency requirements, R^* cannot be a requirement of T. So some rational constitution $C' \in T$ violates R^*, i.e., $C \subseteq C'$. So C is consistent.

[10] Your introspective reasoning may spark some causal process that changes your attitudes (in some direction), but this is another issue.

- Conversely, assume C is consistent. Consider any consistency requirement R of T; we must prove that C satisfies it. R takes the form $R = \{C' : F \nsubseteq C'\}$ for some 'forbidden set' F. Being consistent, C has a rational extension C^+. As C^+ is rational, it satisfies T's requirements, so satisfies R, i.e., $F \nsubseteq C^+$. As $C \subseteq C^+$, it follows that $F \nsubseteq C$. So C satisfies R.

Part (b). The proof is the 'dual' of that for part (a). We may suppose $C \neq M$, because otherwise C is trivially complete (as $T \neq \varnothing$) and satisfies all completeness requirements.

- First let C satisfy T's completeness requirements. We show that C is complete. Note that C violates the (completeness) requirement of containing a state outside C, $R^* = \{C' : (M \backslash C) \cap C' \neq \varnothing\}$. So, as C satisfies T's completeness requirements, R^* is not a requirement of T. So some rational constitution $C' \in T$ violates R^*; hence $(M \backslash C) \cap C' = \varnothing$, i.e., $C' \subseteq C$. So C is complete.
- Conversely, let C be complete. Let R be any completeness requirement of T; we show that C satisfies it. R requires having at least one states from an (unavoidable) set U: $R = \{C' : C' \cap U \neq \varnothing\}$. As C is complete, it has a rational subset C^-. Being rational, C^- satisfies T's requirements, hence satisfies R, i.e., $C^- \cap U \neq \varnothing$. So, as $C^- \subseteq C$, $C \cap U \neq \varnothing$. Hence, C satisfies R.

Part (c). Again, both directions of implication are to be shown.

- First, let C satisfy T's closedness requirements. To show that C is closed, consider a state m entailed by C; we must show that $m \in C$. Consider the closedness requirement R^* with set of premise states C and conclusion state m: $R^* = \{C' : C \subseteq C' \Rightarrow m \in C'\}$. As C entails m, R^* is a requirement of T. So, as C satisfies T's closedness requirements, it satisfies R^*. Hence, as $C \subseteq C$, we have $m \in C$.
- Conversely, assume C is closed. Consider a closedness requirement R of the theory, say $R = \{C' : P \subseteq C' \Rightarrow c \in C'\}$ for some (premise) set $P \subseteq M$ and some (conclusion) state $c \in M$. To show that C satisfies R, assume $P \subseteq C$; we must prove $c \in C$. Since R is a requirement of T, all rational constitutions which include P contain c, which in turn means that P entails c (by definition of entailment). So also the larger set $C \supseteq P$ entails c (again by definition of entailment). Hence $c \in C$, as C is closed.

Part (d). Trivially, rationality is equivalent to satisfaction of the theory's strongest requirement $R = T$, which is equivalent to satisfaction of all the theory's requirements $R \supseteq T$. ■

B Proposition 1 stated formally and proved

Our claim to have generalized consistency, completeness and closedness from beliefs towards multiple attitudes rests on Proposition 1, which we now re-state formally and prove. This appendix follows the formalism in the 'illustration' in Section 2, and assumes the belief-only case $A = \{bel\}$. So M contains only belief-attitudes: $M = \{(p, bel) : p \in L\}$. The set L of propositions is modelled either syntactically or semantically; cf. Section 2.[11]

A *belief set* is any set of ('believed') propositions $B \subseteq L$. As belief is the only attitude, constitutions are notational variants of belief sets: to a constitution $C \subseteq M$ corresponds a belief set $B = \{p : (p, bel) \in C\}$, and to a belief set B corresponds a constitution $C = \{(p, bel) : p \in B\}$.

Classically, a belief set $B \subseteq L$ is

- *consistent* if it is members can be jointly true. Given the semantic model, this means that $\cap_{b \in B} b \neq \varnothing$. Given the syntactic model, it means that B entails no contradiction.
- *(deductively) closed* if it contains all $p \in L$ which it entails. In the semantic model, this means that $\cap_{b \in B} b \subseteq p$.
- *locally complete* if it contains a member of each proposition-negation pair, i.e., each pair $\{p, \Omega \backslash p\} \subseteq L$ (given the semantic model) or each pair $\{p, \neg p\} \subseteq L$ (given the syntactic model).
- *globally complete* if it contains a member of each exhaustive set $Y \subseteq L$. A set $Y \subseteq L$ is *exhaustive* if necessarily at least one member is true. i.e., if $\cup_{p \in Y} p = \Omega$ (given the semantic model) or if the set $\{\neg p : p \in Y\}$ is inconsistent (given the syntactic model). The simplest exhaustive sets are the proposition-negation pairs. Global completeness implies local completeness, by quantifying over *all* exhaustive sets, not just the proposition-negation pairs. An equivalent definition of 'globally complete' is given in Lemma 1(b).

The four conditions on belief sets are far from independent: any consistent and locally complete belief set is automatically deductively closed and globally complete. The gold standard of rational beliefs in logic is to satisfy all these conditions. We can talk of 'classical rationality' if that gold standard is met. Formally, in our belief-only case the *classical* theory of rationality deems a constitution $C \subseteq M$ rational just when the corresponding belief set is consistent and complete (and hence closed). Formally:

[11] In the syntactic case we assume that the logic is a standard propositional logic, or more generally any well-behaved logic such as a standard propositional, predicate, modal, or conditional logic. Formally, the logic must obey a few classic conditions (namely L1–L4 in Dietrich 2007) which guarantee 'regular' notions of logical consistency and logical entailment. The notable condition is monotonicity, whereby entailments are preserved under adding premises, and so consistency of a set is preserved under removing elements.

Definition 4 *In the belief-only case* $A = \{bel\}$ *(with the semantic or syntactic model of* L*), the* ***classical*** *theory or notion of rationality is*

$$T = \{C : \text{the belief set } \{p : (p, bel) \in C\} \text{ is consistent \& complete}\}.$$

We are ready to re-state Proposition 1 formally:

Proposition 1 *Under the belief-only case* $A = \{bel\}$ *(with the semantic or syntactic model of* L*) and the classical theory of rationality, a constitution is*
- *consistent if and only if the corresponding belief set is consistent,*
- *complete if and only if the corresponding belief set is globally complete,*
- *closed if and only if the corresponding belief set is closed.*[12]

Since complete constitutions correspond not to complete, but to strongly complete belief sets, one might ask what type of constitutions correspond to locally complete belief sets. The answer is obvious: those constitutions C such that each proposition-negation pair in L has a member q such that $(q, bel) \in C$.

To prove the result, we first show that the notions of consistency, strong completeness and closedness for belief sets can be re-described in a way that corresponds precisely to our definitions of consistency, completeness and closedness for constitutions. The result should be partly familiar to logicians:

Lemma 1 *Given the semantic or syntactic model of* L*, a belief set* $B \subseteq L$ *is*
(a) consistent if and only if $B \subseteq B'$ *for some complete and consistent belief set* $B' \subseteq L$*,*
(b) complete (understood globally) if and only if $B \supseteq B'$ *for some complete and consistent belief set* B'*,*
(c) closed if and only if B *contains each proposition contained in all complete and consistent extensions* $B' \supseteq B$ *(equivalently,* B *is the intersection of these extensions).*[13]

Proof. Suppose the lemma's assumptions. Let $B \subseteq L$ be a belief set, and $\mathbf{B}$ the set of complete and consistent belief sets.

(a) We distinguish between the semantic and syntactic model of L. In the semantic case the equivalence holds trivially (if B is consistent, we can pick a $w \in \cap_{p \in B} p$ and define B' as $\{p \in L : w \in p\}$). In the syntactic case the equivalence follows from a basic property in logic, often referred to as 'Lindenbaum's lemma', which states that any consistent set of sentences in a logic is extendable to a complete and still consistent set. This property holds in well-behaved logics of the sort assumed here (see footnote 11).

(b) First let B have a subset $B' \in \mathbf{B}$. To show that B is strongly complete, consider any exhaustive set $Y \subseteq L$. We must prove that $B \cap Y \neq \varnothing$. As $B' \subseteq B$

[12] In the syntactic case we assume the logic is well-behaved as defined in footnote 11.

[13] In case of the syntactic model we assume the logic is well-behaved as defined in footnote 11.

it suffices to show that $Y \cap B' \neq \varnothing$, which holds by the following argument to be spell out for the syntactic and the semantic case:

- *In the syntactic case*, note that the (inconsistent) set $\{\neg p : p \in Y\}$ cannot be a subset of the (consistent) set B'. So there is a $p \in Y$ such that $\neg p \notin B'$, and thus $p \in B'$ as B' is complete. So $Y \cap B' \neq \varnothing$.
- *In the semantic case*, since $\{\Omega \backslash p : p \in Y\}$ has empty intersection (as Y has union Ω) while B' has non-empty intersection (as B' is consistent), the set $\{\Omega \backslash p : p \in Y\}$ cannot be a subset of B'. So there is a $p \in Y$ such that $\Omega \backslash p \notin B'$, and hence $p \in B'$ as B' is complete. So $Y \cap B' \neq \varnothing$.

Conversely, assume that B does *not* include any $B' \in \mathbf{B}$, and let us show that B is not strongly complete. By assumption, for each $B' \in \mathbf{B}$ we may pick a $p_{B'} \in B' \backslash B$. Let $Y := \{p_{B'} : B' \in \mathbf{B}\}$. This set Y is exhaustive, both in the semantic case (here each world $\omega \in \Omega$ belongs to some member of Y, namely to $p_{B'}$ with $B' := \{p \in L : \omega \in p\}$) and also in the syntactic case (here $\{\neg p : p \in Y\}$ is not included in any $B' \in \mathbf{B}$ and so is inconsistent by (a)). Yet $Y \cap B = \varnothing$ by construction of Y. So B is not strongly complete.

(c) We must show that B is closed if and only if $B = \cap_{B' \in \mathbf{B}: B' \supseteq B} B'$. In the syntactic case, this is a familiar fact, valid in in well-behaved logics of the sort considered here (see footnote 11). Now consider the semantic case. Note that $\cap_{B' \in \mathbf{B}: B' \supseteq B} B'$ is closed (in fact, not just in the semantic case). So if $B = \cap_{B' \in \mathbf{B}: B' \supseteq B} B'$ then B is automatically closed. Conversely, if B is closed, then $B = \{p \in L : p \supseteq \cap_{q \in B} q\}$, from which it easily follows that $B = \cap_{B' \in \mathbf{B}: B' \supseteq B} B'$. ■

Proof of Proposition 1. Suppose the proposition's assumptions. Let C be a constitution. We denote the content of a (belief) state m by $\widehat{m}$ and the belief set corresponding to a constitution $C \subseteq M$ by $\widehat{C} = \{\widehat{m} : m \in C\}$.

First,

$$\begin{aligned} C \text{ is consistent} \quad &\Leftrightarrow \quad C \subseteq C' \text{ for some } C' \in T \\ &\Leftrightarrow \quad \widehat{C} \subseteq \widehat{C'} \text{ for some } C' \in T \\ &\Leftrightarrow \quad \widehat{C} \subseteq B \text{ for some consistent and complete } B \subseteq L \\ &\Leftrightarrow \quad \widehat{C} \text{ is consistent, by Lemma 1(a).} \end{aligned}$$

Second,

$$\begin{aligned} C \text{ is complete} \quad &\Leftrightarrow \quad C \supseteq C' \text{ for some } C' \in T \\ &\Leftrightarrow \quad \widehat{C} \supseteq \widehat{C'} \text{ for some } C' \in T \\ &\Leftrightarrow \quad \widehat{C} \supseteq B \text{ for some consistent and complete } B \subseteq L \\ &\Leftrightarrow \quad \widehat{C} \text{ is strongly complete, by Lemma 1(b).} \end{aligned}$$

Third, writing $\widehat{T} := \{\widehat{C} : C \in T\} = \{B \subseteq L : B \text{ is complete and consistent}\}$,

$$\begin{aligned} C \text{ is closed} \quad &\Leftrightarrow \quad C \ni m \text{ for all } m \text{ entailed by } C\text{, i.e., all } m \in \cap_{C' \in T: C' \supseteq C} C' \\ &\Leftrightarrow \quad \widehat{C} \ni \widehat{m} \text{ for all } m \text{ entailed by } C\text{, i.e., all } m \in \cap_{C' \in T: C' \supseteq C} C' \\ &\Leftrightarrow \quad \widehat{C} \ni b \text{ for all } b \text{ entailed by } \widehat{C}\text{, i.e., all } b \in \cap_{B \in \widehat{T}: B \supseteq \widehat{C}} B \\ &\Leftrightarrow \quad \widehat{C} \text{ is closed, by Lemma 1(c).} \ ■ \end{aligned}$$

C Proof of Theorem 2

Throughout the proof, let T be any theory of rationality. A reasoning system S **achieves** a requirement R if $C|S$ satisfies R for all constitutions C. Note that for each of parts (b), (c) and (d) we have to prove two directions of implication, as we read 'unless' as 'if *and only if* it is not the case that'.

Given the contradictory theory $T = \varnothing$, all four parts hold trivially. Part (a) holds because the maximal reasoning system S, which contains all rules, does the job: it achieves closedness by transforming each constitution into M (the only closed constitution), and it vacuously preserves consistency by the absence of consistent constitutions. Parts (b), (c) and (d) hold because consistency, completeness and rationality are all trivially unachievable by the absence of any consistent, complete or rational constitutions (regarding (c), note also the absence of avoidable sets).

Henceforth let $T \neq \varnothing$. We prove the four parts in turn.

Part (a). By Theorem 1(c), achieving closedness is equivalent to achieving all closedness requirements of T. Meanwhile, by Theorem 1 in Dietrich et al. (2019) there exists a reasoning schema S which achieves all closedness requirements and preserves consistency. So S achieves closedness while preserving consistency.

Part (b). First, if consistency is trivial (i.e., $C = M$ is rational), then consistency is achieved by any reasoning system. Conversely, assume consistency is non-trivial. Let S be any reasoning system. It fails to achieve consistency, because by non-triviality there is an inconsistent constitution C (e.g., $C = M$), and as $C|S \supseteq C$ also $C|S$ is inconsistent.

Part (c). First, assume completeness is trivial (along with the background assumption of compactness, whereby each inconsistent set of states has a finite inconsistent subset). For each unavoidable set U we can pick an unfalsifiable state $m_U \in U$. The reasoning system $S = \{(\varnothing, m_U) : U \text{ is unavoidable}\}$ achieves each completeness requirement of theory T, because for each completeness requirement of T a state from its unavoidable set is formed. So S achieves completeness simpliciter, by Theorem 1. We now show that S preserves consistency. For a contradiction, consider a consistent constitution C such that $C|S$ is inconsistent. By compactness, $C|S$ has a finite inconsistent subset C'. By definition of S, $C|S = C \cup \{m_U : U \text{ is an unavoidable set}\}$. So we may pick finitely many unavoidable sets $U_1, ..., U_k$ such that $C' \subseteq C \cup \{m_{U_1}, m_{U_2}, ..., m_{U_k}\}$. Since C is consistent, so is $C \cup \{m_{U_1}\}$, as m_{U_1} is non-falsifiable; hence so is $C \cup \{m_{U_1}, m_{U_2}\}$, as m_{U_2} is non-falsifiable. Repeating this argument k times, it follows that $C \cup \{m_{U_1}, m_{U_2}, ..., m_{U_k}\}$ is consistent. Hence its subset C' is consistent.

Conversely, suppose some set of falsifiable states is unavoidable. Let R be the corresponding completeness requirement. It suffices to show that no reasoning system achieves R, because by Theorem 1 achieving completeness is equivalent to achieving all completeness requirements of the theory. By Theorem 3 in Dietrich et al. (2019), no reasoning system achieves any completeness requirement of the theory whose unavoidable set consists of falsifiable states. So no reasoning system S achieves R.

Part (d). First, for (degenerate) theories that deem $C = M$ rational, rationality is trivially achieved by the reasoning system S containing *all* rules, for which $C|S = M$ for all initial constitutions C. Conversely, if $C = M$ is irrational, the unachievability of rationality follows from that of the weaker demand of consistency (see part (b)). ■

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