The Solow model augmented with the net capital inflows to-GDP-ratio

Abdoul A. Ndiaye, Mamadou A. Konte, Yao T. Kpegli

Economics Bulletin

Volume 37, Issue 3

The Solow model augmented with the net capital inflows to-GDP-ratio

Abdoul A. Ndiaye
LARES, Université Gaston Berger de Saint-Louis

Mamadou A. Konte
LARES, Université Gaston Berger de Saint-Louis

Yao T. Kpegli
LARES, Université Gaston Berger de Saint-Louis

Abstract

In this paper, we study the Solow model in open economy. For this purpose, we increase the traditional model with the net capital inflows to-GDP-ratio. We end up with two main implications. First, the steady state net marginal product of capital more correctly predicts the real interest rate than that obtained in traditional model. Second, the golden rule of the savings rate tells us that a savings rate that is below (or above) the share of capital in GDP doesn't necessarily mean that the savings rate is too low (or too high).

We thank anonymous referees whose remarks and suggestions have allowed us to improve considerably the quality of this paper.
Citation: Abdoul A. Ndiaye and Mamadou A. Konte and Yao T. Kpegli, (2017) "The Solow model augmented with the net capital inflows to-GDP-ratio", *Economics Bulletin*, Volume 37, Issue 3, pages 2020-2029
Contact: Abdoul A. Ndiaye - elhadji-abdoul-aziz.ndiaye@ugb.edu.sn, Mamadou A. Konte - kontedoudou@yahoo.fr, Yao T. Kpegli - kpegliyaothibaut@yahoo.fr.
Submitted: November 19, 2016. **Published:** August 31, 2017.

1 Introduction

The Solow model[14] is based on the assumption of closed economy. For this purpose, it predicted under certain conditions that each economy converges to a unique and stable balanced growth situation. However, this convergence in the solow model is subject to controversies about its nature : conditional or absolute ? Also, one can add the criticism (Lucas[10], King and Rebelo[8]) regarding the predictions of the traditional model on the differences in the rate of return of capital (between rich and poor countries) and the movements of capital. Mankiw and al.[11] changed the structure of production of the traditional model by adding human capital. Even if the augmented model could not provide satisfactory response to criticism of interest rate differentials and capital movements [1], it gave a satisfactory answer to criticism of a too high speed of convergence by adding the human capital in the Solow model. Also, we note that this change in the production function by Mankiw and al.[11], avoids a problem of omitted variable bias insofar as human capital can be correlated both with the explanatory variables (savings rates and the rate of population growth) and the outcome variable (GDP per capita). But in the Solow model, the two important equations are the production function and the dynamic equation of capital accumulation which depends on the structure of demand. In tests of the Solow model, the authors (Mankiw and al.[11], Caselli and al.[3], bond and al.[2]) use the investment rate rather than the savings rate that is present in the model. One of the criticisms that one can therefore make is that these empirical studies do not fundamentally test the traditional theory and this reflected the need to review the formulation of the demand in Solow model by disregarding the assumption of closed economy. Another empirical fact that can surprise today is that some countries as China, Singapore, Malaysia or South Korea have sometimes a savings rate higher than one-third while the share of physical capital in production is generally considered near to this value. In the standard neo-classical growth model, the "golden rule" saving rate—the saving rate that maximizes steady state consumption—is equal to the share of capital in GDP (Blanchard and Giavazzi [1]). It is therefore legitimate to ask the question of a over-investment for these countries, as is the case of China (Blanchard and Giavazzi [1], Lee and al.[9],Jha et al.[7]) and Singapore (Hopf [6]).

The aim of this paper is to study the Solow model in open economy by increasing the traditional model with the net capital inflows to-GDP-ratio.

The remainder of this paper is organized as follows. In section 2, the augmented model with the net capital inflows to-GDP-ratio is presented. In sections 3, 4 and 5, we study the implications of this change on golden rule saving rate, the interest rate differentials and the speed of convergence. Concluding and remarks are presented in the last section.

2 Adding net capital inflows to GDP-ratio to the Solow model

As in the Solow model, we assume that the economy produced just one *homogeneous good*, with the difference that it is exchangeable, that we note Y. The supply of this goods is based on a neoclassical production function linking the production of the goods (Y) to the stock of capital (K) and labor (L) and technical progress incorporated into labor (E) and a factor of efficiency of the productive combination (A)

$$Y = A.F(K, EL) \tag{1}$$

1. The augmented model with the human capital and the traditional model predicts at the steady state the same rate of return of physical capital.

with $E(t) = E(0)e^{\gamma t}$ and $L(t) = L(0)e^{nt}$. Thus, the number of effective units of labor, $E(t)L(t)$, grows at rate $n + \gamma$. Additionally, we assume that the production function exhibits constant returns to scale.

In open economy and under the assumption of absence of State we have

$$I = S + M - X \tag{2}$$

Where I, S, M, and X are respectively the investment, savings, imports and exports. Noting that for a given import rate $m = \frac{M}{Y}$, an external deficit rate (relative difference between imports and exports , $v = \frac{M-X}{M}$) is proportional to the rate of deficit in resources relative to GDP (*i.e* a net capital inflows to-GDP-ratio, $\tau^c = \frac{M-X}{Y}$) in accordance with the following equation [2] :

$$\frac{M-X}{Y} = \frac{M-X}{M} \times \frac{M}{Y} \tag{3}$$

Thus equation (2) becomes :

$$I = S + \tau^c Y. \tag{4}$$

This equation (4) reflects that the investment I is financed by domestic savings S and net capital inflow $\tau^c Y$, which is nothing else than a reflection of an external deficit $M - X > 0$.

We denote respectively by k and y the stock of capital per effective unit of labor, $k = \frac{K}{EL}$, and the level of GDP per effective unit of labor, $y = \frac{Y}{EL}$. Denote by δ the depreciation rate of capital. Assuming τ^c is constant [3] and using the common assumption of the traditional model, the fundamental equation of capital per effective worker is given by

$$\dot{k}(t) = [s + \tau^c]\, y(k(t)) - (n + \delta + \gamma)k(t) \tag{5}$$

This equation shows that the accumulation of capital per effective worker $\dot{k}$ is equal investment [4] $(s + \tau^c)\, y(k)$ minus break-even investment $(n+\delta+\gamma)k$. In the Solow model, the analysis of the dynamic equation of capital accumulation reveals us that, in the short run, the economic growth is based on the accumulation of capital which mainly depend of the savings rate. The implication is that a low savings rate is a sign of a bad structure of capital accumulation leading low economic growth rates in the short run and a low level GDP-per-capita in the long run. But, even with a low savings rate, the accumulation of capital (and so economic growth) can be financed by foreign savings as we see with the dynamic equation (5) of the augmented model. Thus, a country with a low savings rate can have a good economic growth in the short run if it records capital inflows. Equation (5) implies that k converges to a steady-state value k^* defined by

$$(s + \tau^c)y(k^*) = (n + \delta + \gamma)k^* \tag{6}$$

Using the Cobb-Douglas function, $y(k) = Ak^\alpha$ we obtain $k^* = \left(\frac{A(s+\tau^c)}{n+\delta+\gamma}\right)^{\frac{1}{1-\alpha}}$. Thus, the steady-state capital-labor ratio is related positively to the rate of investment— which is defined

2. inspired by the work of Combes j.-L and al [4]

3. Note that this assumption of constancy of the net capital inflow to-GDP-ratio is simplified to the extent that it does not take into account the fact that net capital inflows may be determined by the differences in interest rates. But it avoids an amplification of criticism of a high speed of convergence (see appendix (1) for detail).

4. The investment rate equal $i = s + \tau^c$

as the sum of the savings rate and the net capital inflows to GDP-ratio— and negatively to the rate of population growth. The adding of the constant net capital inflows to GDP-ratio to the Solow model have implications about the golden rule of savings rate, the speed of convergence and the capital movements.

3 The golden-rule of the savings rate

As in the Solow model, the issue of the golden rule of capital accumulation is given by

$$\begin{cases} \max\limits_{s} \quad c = (1-s)y \\ s.t: \\ \dot{k} = 0 \end{cases} \tag{7}$$

where c is the consumption per effective worker. Solving this problem, the savings rate which corresponds to the golden rule is such that

$$MPK = \frac{n+\delta+\gamma}{1+\tau^c} \tag{8}$$

If we consider the Cobb-Douglas function, the steady-state marginal product of capital is given by $\alpha \times \dfrac{n+\delta+\gamma}{s+\tau^c}$ and the equation (8) becomes

$$s^{gold} = \alpha - (1-\alpha)\tau^c \quad or \quad i^{gold} = \alpha(1+\tau^c) \tag{9}$$

with $i^{gold} = s^{gold} + \tau^c$

This golden rule indicates that :

- with a net capital inflows to-GDP ratio zero (external balance), the rate of savings and investment rates are equal to the share of capital in GDP. This is the traditional golden rule.

- more the net capital inflows to-GDP-ratio is high, more the savings rate (respectively investment rate) required to maximize consumption per effective worker at the steady state is low (respectively high) compared to the share of capital in GDP. Conversely, more an economy registers net capital outflows—*i.e* a negative net capital inflows to-GDP-ratio—, more the savings rate (respectively investment rate) required to maximize consumption per effective worker at the steady state will be high (respectively low) compared to the share of capital in GDP.

As in the Solow model, to assess the savings rate to see if it is roughly correct, insufficient or excessive; we should compare the savings rate with its level dictated by the golden rule. Because of the decrease in the productivity of capital per effective worker and the growing relationship between capital per effective worker and the savings rate, the savings rate will be considered :

- low if $MPK > \dfrac{n+\delta+\gamma}{1+\tau^c} \quad or \quad s < \alpha - (1-\alpha)\tau^c \quad or \quad i < \alpha(1+\tau^c)$
- right if $MPK = \dfrac{n+\delta+\gamma}{1+\tau^c} \quad or \quad s = \alpha - (1-\alpha)\tau^c \quad or \quad i = \alpha(1+\tau^c)$
- excessive if $MPK < \dfrac{n+\delta+\gamma}{1+\tau^c} \quad or \quad s > \alpha - (1-\alpha)\tau^c \quad or \quad i > \alpha(1+\tau^c)$

Thus, the equation (9) tells us that a savings rate that is below (or above) the share of capital in GDP doesn't necessarily mean, as in the traditional model, that the savings rate is too low (or too high). A country that has a savings rate higher (respectively less) than the share of capital in GDP may be a net creditor (respectively a net debtor) of capital so as to avoid over-investment (respectively under-investment).

4 Interest rate differentials and capital movements

In this section, we analyse interest rate differentials and the capital movements.

4.1 Interest rate differentials

Several economists, including Lucas[10] and King and Rebelo[8] , argue that the Solow model fails to explain rate-of-return differences or international capital flows. This criticism comes because in economics one considers that capital flows from the countries where the interest rate is lower (generally the rich countries) to the countries where the interest rate is higher (generally the poor countries). In the traditional model and the augmented model with human capital (Mankiw and al.[11]), the steady-state marginal product of physical capital, net of depreciation rate which is equal to interest rate is given by :

$$MPK^* - \delta = \alpha \frac{n+\delta+\gamma}{s} - \delta \tag{10}$$

Mankiw and al.[11] explain that, because the cross-country differences in savings rate and population growth rates are large, the differences in rates of return should also be large. These authors, using $(\gamma, \delta) = (0.02, 0.03)$ find the mean of the steady-state net marginal product is 0.12 in the intermediate sample, and the standard deviation is 0.08. As noted by authors, observed differentials in real interest rates appear smaller than the predicted differences in the net marginal product of capital. The problem with the equation (10) is that one forget that more capital through the net capital inflows can reduce the rate of return under the assumption of diminishing marginal product. In the augmented model with the net capital inflows to-GDP-ratio, the net marginal product is decreasing function of net capital inflows.

$$MPK^* - \delta = \alpha \frac{n+\delta+\gamma}{s+\tau^c} - \delta \tag{11}$$

Since rich countries generally have $\tau^c < 0$, the augmented model predicts an interest rate higher for rich countries compared to the solow model. Also, since poor countries generally have $\tau^c > 0$, the augmented model predicts an interest rate lower for poor countries. Thus, the equation (11) predict the small difference in return of capital between rich and poor countries than equation (10).

To check whether or not the equation (11) correctly predicted the real interest rate, you can proceed in two ways :

i) The first is similar to the approach in Mankiw and al.[11]. We can suppose $\alpha = \frac{1}{3}$ and $(\gamma, \delta) = (0.02, 0.03)$ and compute the steady-state net marginal product using the equation (11). The results are given in the table (1). Using equation (11), the mean of the steady-state net marginal product is 0.0556 (with a standard deviation of 0.016) in the rich countries and 0.1238 (with a standard deviation of 0.037) in the poor countries. The mean of real interest rate is 0.0480 (with a standard deviation 0.018) in rich countries and 0.1226 (with a standard deviation 0.058) in poor countries.

Table 1–Net marginal product of capital and real interest rate

	MPK^*	Real interest rate
poor countries (a)	0.1238 (0.037)	0.1226 (0.058)
rich countries (b)	0.0556 (0.016)	0.0480 (0.018)
Ratio $\left(\frac{a}{b}\right)$	2.23	2.55

(.) standard deviation
source : authors using the World Bank data on 1990 to 2014.

One can remark that the means of net marginal product are close to the means of real interest rate using the equation (11).

ii) Because α is considered near to one-third in more country in the world, another way to make this comparison is to estimate the parameter α using the equation (11) with $MPK = \delta + r$ and $(\gamma, \delta) = (0.02, 0.03)$. The comparison of α statistically with $\frac{1}{3}$ permit to evaluate the consistency of the equation (11) with empiricals facts. Formally, we estimate α using the following equation

$$r_i + 0.03 = \alpha \frac{n_i + 0.05}{s_i + \tau_i^c} + \epsilon_i \quad , \quad \epsilon_i \rightsquigarrow \mathcal{N}(0, \sigma_\epsilon^2) \tag{12}$$

where r_i, n_i , s_i and τ_i^c designate respectively the mean of real interest rate, the mean of the population growth, the mean of the savings rate and the mean of the net capital inflows to-GDP-ratio in country i. Ordinary least squares (OLS) is used to estimate (12). The results of regressions are given in the table (2). For all countries of sample[5], the estimation of this equation gives $\hat{\alpha} = 0.313$ with a $R^2 = 0.86$. The confidence interval indicate that α is not significantly different of $\frac{1}{3}$ at 95 % level. Also, in the sub-sample of the poor countries and the rich countries, $\hat{\alpha}$ is respectively equal to $\hat{\alpha}_p = 0.318$ (with $R^2 = 0.85$) and $\hat{\alpha}_r = 0.293$ (with $R^2 = 0.92$). The confidence interval indicate that α_p and α_r are not significantly different of $\frac{1}{3}$ at 95 % level. These results also lead us to say that equation (11) correctly predicts the real interest rate.

Table 2– Estimation of α

	Dependent variable : $r + 0.03$		
	all	poor	rich
$\frac{n+0.05}{s+\tau^c}$	0.313*** (16.55)	0.318*** (11.35)	0.293*** (14.75)
95% Conf. Interval	[0.2745; 0.3506]	[0.2600; 0.3760]	[0.2517; 0.3346]
N	44	23	21
R^2	0.8644	0.8542	0.9158
$F_{(1,N-1)}$	274.1	128.9	217.5

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4.2 On the capital movements

Even if it isn't with the great magnitude as predicted by the traditional model, we can notice that there are differences in marginal product between rich and poor countries. But this

5. See the list of countries in the sample in appendix (2).

doesn't mean that all capital would flow from rich countries to poor countries : the opposite is possible. The rule is that capital flows from a country (poor or rich) where the interest rate is lower to a country (poor or rich) where the interest rate is higher . Thus, when we take the example of net capital inflows of the United States in recent years, it must come from countries where the real interest rate and the rate of return are lower. But Prasad and al.[13] noting that "the average income, relative to the United States, of capital-exporting countries has fallen well below that of capital-importing countries" conclude that "capital has been flowing from poor to rich countries". These authors also note that the "recent U.S. current account deficits and Chinese current account surpluses are, of course, a big part of the reason why capital is flowing uphill". The problem is that if we consider that the capital must flow from a country where the rate of return is lower to a country where the rate of return is higher, there is no paradox in this flow. Indeed, both the real interest rate and net marginal product of capital are lower in China than in the United States. The mean of the real interest rate over the period of 1990 to 2014 is 0.019 in China and 0.040 in the United States. If we assume $\alpha = \frac{1}{3}$ and $(\gamma, \delta) = (0.02, 0.03)$, the net marginal product of capital according to the augmented model is 0.018 in China and 0.064 in United States.

5 The speed of convergence and nature of convergence

In this section, we examine the nature of convergence on the one hand and the speed of convergence on the other.

5.1 The nature of convergence

Another problem is the nature of convergence. Is it absolute or conditional? Mankiw and al.[11] argue that the Solow model does not predict absolute convergence, but it predicts convergence only after controlling for the determinants of the steady state, a phenomenon that might be called *conditional convergence*. This conditional convergence can be considered as the most likely case, but that does not exclude a possibility of absolute convergence. One can rewrite the expressions [6] of capital per capita and GDP per capita of steady-state :

$$k^* = \left[\frac{A(s+\tau^c)}{n+\delta+\gamma}\right]^{\frac{1}{1-\alpha}} \quad and \quad y^* = A\left[\frac{A(s+\tau^c)}{n+\delta+\gamma}\right]^{\frac{\alpha}{1-\alpha}} \tag{13}$$

For example, a poor country that has identical characteristics (A, n, δ, γ) with a rich country except in savings rate [7] $(s_p < s_r)$ and net capital inflow to-GDP-ratio ($\tau_p^c > \tau_r^c$) , can reach the same steady-state than the rich countries if it has the same investment rate than the rich country under the effect of the net capital inflow. Although our model doesn't explicitly take into account these aspects, it should be noted that there are at least two conditions for capital inflows :

i) the net capital inflows increase the stock of debt and can lead to the problem of sustainability of the debt. To avoid this problem of sustainability and a crisis of confidence among lenders of capital, the poor countries will have to at least stabilize its debt to GDP ratio. Choosing to stabilize the debt ratio to GDP at level $\overline{d}$, the net capital inflows to GDP-ratio $\overline{\tau}^c$ compatible with this stabilisation (given of interest rate r and economic growth rate g) is given

6. under the assumption of Cobb-Douglas production function
7. The subscripts p and r refers respectivelly poor country and rich country.

by [8] : $\overline{\tau}^c = (g - r)\overline{d}$.

ii) also, since the net marginal product of capital $MPK - \delta$ will tend to move towards the international interest rate r^* during the process of accumulation of capital, the country should develop policies to stimulate the domestic savings rate. Indeed, if the net marginal product of capital becomes lower than interest rates r^*, net capital inflows will also stop and domestic savings should be developed in order to maintain the country to a high level of GDP per capita.

5.2 The speed of convergence

With the equation (5), the first-order Taylor-series approximation of $\dot{k}(k)$ around $k = k^*$ conduce at the same [9] expression of the speed of convergence to that obtained in a traditional model

$$\lambda = (1 - \alpha(k^*))(n + \delta + \gamma) \tag{14}$$

However the value of the speed of convergence can be different to that obtained in the model of Solow depending on whether $\alpha_k(k^*)$ dependent or not of k^*. For example, if we take a production function such as $\alpha_k(k^*)$ is independent of k^* (example of a Cobb-Douglas function, $\alpha_k(k^*) = \alpha$), then the value of the speed of convergence is identical to the one obtained in the traditional model. If $\alpha_k(k^*)$ depends on k^* (as in a CES production function [10] with an elasticity of substitution between capital and effective labor 0.5) , then the speed of convergence λ predicted by the augmented model is different to the one obtained in the solow model.

6 Concluding and remarks

The aim of this paper is to study the Solow model in open economy by increasing the traditional model with the net capital inflows to-GDP-ratio. The implications of this introduction are :

First, the golden rule of the savings rate assessment tells us that a savings rate that is below (or above) the share of capital in GDP doesn't necessarily mean that the savings rate is too low (or too high). A country that has a savings rate higher (respectively less) than the share of capital in GDP may be a net creditor (respectively a net debtor) of capital so as to avoid over-investment (respectively under-investment).

Second, the net marginal product of capital of steady-state is a decreasing function of

8. Noting by D the stock of debt and Y the GDP, one can write $\dot{D} = M - X + r.D$ and $\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{Y}}{Y}$ with $d = \frac{D}{Y}$. At a constant debt-GDP ratio $\overline{d}$, the change in debt as a percent of GDP is equal to the debt ratio times the rate of growth of GDP : $\frac{\dot{D}}{Y} = \overline{d}\frac{\dot{Y}}{Y}$ (see Howard[5] , Oliveira and Plihon [12]) . We can draw from this equation the $\overline{\tau}^c$ compatible with $\overline{d}$, given g and r : $\overline{\tau}^c = (g - r)\overline{d}$.

9. We recall that this is because we assume a constant net capital inflow to-GDP-ratio rather than assume it is determined by the interest rate differential. As noted in Appendix appendix (1), this avoids to predict a higher convergence speed than that predicted in the traditional model (with Cobb-Douglas function).

10. A CES production function is written in the form of $Y = A\left(K^{\frac{\sigma-1}{\sigma}} + (EL)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ where $0 < \sigma < +\infty$ and $\sigma \neq 1$ represents the elasticity of substitution between capital and effective labor. Thus $y(k) = A\left(k^{\frac{\sigma-1}{\sigma}} + 1\right)^{\frac{\sigma}{\sigma-1}}$. As in the example if $\sigma = 0.5$ then $Y = A\left(K^{-1} + (EL)^{-1}\right)^{-1}$, $y(k) = \frac{Ak}{k+1}$, $\alpha_k(k^*) = \frac{1}{k^*+1}$ and $k^* = \frac{A(s+\tau^c)}{n+\delta+\gamma} - 1$ is an increasing function of τ^c. If $\tau^c > 0$ (respectively $\tau^c < 0$), the speed of convergence λ is more high (respectively more low) in the augmented model comparatively to the traditional model.

the net capital inflow to GDP-ratio. Because rich countries generally have negative net capital inflow to GDP-ratio and poor countries have positive net capital inflow to GDP-ratio, the rate of return and the interest rate are not supposed to be too different between rich and poor countries.

Third, it is not excluded that a poor country that has a low savings rate to converge towards a GDP per capita of a rich country because it can benefit the net capital inflows. It must however avoid the problem of the sustainability of debt underlying to the net capital inflows and stimulate its savings rate before its rate of return tend to equalize the rate of return of net capital creditor.

If the change in the structure of production by Mankiw and al.[11] brings elements of answers to criticism of a too high convergence speed in the traditional model, their model have the same predictions on international differences in the rate-of-return of capital than the traditional model at the steady state. Conversely, the augmented model with the net capital inflow to-GDP-ratio brings elements of answers to criticism on the international differences in rate-of-return of capital at the steady state without able to find an answer to the criticism of a too high convergence speed. Thus, future work may be to make a synthesis of these two models and see how to remove the hypothesis of exogenous net capital inflow to-GDP-ratio without amplify criticism of a too high convergence speed.

REFERENCES

[1] Blanchard, Olivier and Francesco Giavazzi (2006), "Rebalancing China : A Three Handed Approach," CEPR Discussion Paper No. 5403.

[2] Bond S., Hoeffler A., Temple J.(2001)."GMM Estimation of Empirical Growth Models".CEPR Discussion Paper 3048.

[3] Caselli F., Esquivel G.,Lefort F.(1996)."Reopening the Convergence Debate : A New Look at Cross-Country Growth Empirics". Journal of Economic Growth, Vol. 1, No. 3, pp. 363-389

[4] Combes J.-L., Guillaumont Patrick, Guillaumont Jeanneney S., Motel Combes P.(2000)." Ouverture sur l'extérieur et instabilité des taux de croissance", Revue française d'économie, volume 15, $n°1$, pp. 3-33.

[5] Howard, D.(1989)."Implications of The US Account Deficit", Journal of Economic Perspectives, Vol. 3, $n°$ 4.

[6] Hopf, G.(2009)."Saving and investment : the economic development of Singapore 1965-99". Saarbrücken, Germany : VDM Verlag Dr. Müller, pp. 241-318.

[7] Jha, Shikha ; Prasad, Eswar ; and Terada-Hagiwara, Akiko (2009), "Saving in Asia : Issues for Rebalancing Growth," ADB Economics Working Paper Series no. 162, Asian Development Bank, Manila, Philippines.

[8] King, Robert G., and Sergio T. Rebelo,.(1989)."Transitional Dynamics and Economic Growth in the Neoclassical Model," NBER Working Paper 3185.

[9] Il Houng Lee, Murtaza Syed, and Liu Xueyan.(2012). "Is China Over-Investing and Does it Matter ?".IMF Working Paper.WP/12/277.

[10] Lucas, Robert E.(1988), "On the Mechanics of Economic Development," Journal of Monetary Economics, XXII , 3-42.

[11] Mankiw, N. G., D. Romer and D. N. Weil (1992). "A contribution to the empirics of economic growth." Quarterly Journal of Economics, 107(2), 407-437.

[12] Oliveira M. J, Plihon D.(1990)."L'impact des transferts internationaux d'épargne sur les déséquilibres extérieurs". Economie et statistique, $n°$ 232,. Numéro spécial : Déséquilibres internationaux. pp. 33-48.

[13] Prasad E., Rajan R. , Subramanian A.(2007). "The Paradox of Capital". Finance and Development, IMF.pp. 10-13.

[14] Solow R. (1956)."A Contribution to the Theory of Economic Growth", Quarterly Journal of Economics. vol.70, pp. 65-94.

Appendix 1

- **Steady-state**

Assuming that $\tau^c = \mu(r - r^*)$ with $r = f'(k) - \delta$ and $\mu > 0$, we obtain the following differential equation :

$$\dot{k} = [s + \mu(f'(k) - \delta - r^*)]\, f(k) - (n + \delta + \gamma)k$$

The steady state is characterized by $\dot{k} = 0$. Then

$$[s + \mu(f'(k^*) - \delta - r^*)]\, f(k^*) = (n + \delta + \gamma)k^*$$

- **Speed of convergence**

Under the assumption $\tau^c = \mu(r - r^*)$, we obtain :

$$k(t) - k^* = e^{-((1-\alpha_k(k^*))(n+\delta+\gamma)-\mu f''(k^*)f(k^*))t}(k(0) - k^*)$$

Thus the speed of convergence is given by :

$$\lambda_o = (1 - \alpha_k(k^*))(n + \delta + \gamma) - \mu f''(k^*)f(k^*)$$

Because $f''(k^*) < 0$ we have $\mu f''(k^*)f(k^*) < 0$. Then

$$\lambda_o = (1 - \alpha_k(k^*))(n + \delta + \gamma) - \mu f''(k^*)f(k^*) > (1 - \alpha_k(k^*))(n + \delta + \gamma) = \lambda$$

Using the Cobb-Douglas function, we obtain $\lambda_o = (1-\alpha)(n+\delta+\gamma)+\mu\alpha(1-\alpha)\left(\dfrac{n+\delta+\gamma}{s+\tau^c}\right)^2 > (1 - \alpha)(n + \delta + \gamma) = \lambda$. In the empiricals studies, α is considered near of one-third and $n + \delta + \gamma \approx 0.06$. Then the standard convergence rate λ would equal 0.04. This implies, in the traditional model, that the economy moves halfway to steady state in 17 years. If we suppose $s+\tau^c = 0.25$, then $\lambda_o = 0.04+0.0128\mu$. This implies, in the augmented model, that the economy moves halfway to steady state in 13 years if $\mu = 1$. The majority of empirical studies conclude a speed of convergence between 2 and 3 percent. Caselli and al.[3] are the main authors who find the opposite (convergence rate equal 0.1). But Bond and al.[2] revisit this study and shows that this result of Caselli and al.[3] is biased. Thus the consideration of the relationship between τ^c and $r - r^*$ amplify this empirical contradiction.

- **Golden-rule of the savings rate**

With the assumption of $\tau^c = \mu(r - r^*) = \mu(f'(k) - \delta - r^*)$, the savings rate which corresponds to the golden rule is such that :

$$Pmk = \frac{n + \delta + \gamma - \mu f''(k^*) f(k^*)}{1 + \tau^c}$$

Appendix 2

Sample of poor countries :

Afghanistan ; Burkina Faso ; Burundi ; Benin ; Central African Republic ; Congo, Dem. Rep. ; Comoros ; Gambia ; Guinea ; Guinea Bissau ; Haiti ; Liberia ; Madagascar ; Malawi ; Mali ; Mozambique ; Niger ; Rwanda ; Sierra Leone ; Senegal ; Togo ; Uganda ; Nepal.

Sample of rich countries :

United States ; Germany ; Argentina ; Australia ; Belgium ; Canada ; China ; France ; India ; Indonesia ; Italy ; Japan ; Mexico ; Netherlands ; Poland ; Spain ; R. Korea ; United Kingdom ; Russia ; Switzerland ; Sweden.