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BEQUESTS OR EDUCATION

Julio Dávila[1]

Abstract. Whether parents choose to endow their offspring with bequests, or with human capital —the effectiveness with which they do so surely depending on their own human capital— or with both, markets cannot deliver, under laissez-faire, the egalitarian planner's mix of bequests and education that maximises the representative agent's welfare. Specifically, at the steady state and for a close enough to linear human capital production —out of educational investment and parents human capital— the market wage per efficient unit of labor is too high compared to the marginal productivity of labor resulting from the steady state the planner would choose, so that the market human capital is too low. In other words, the market misses the planner's allocation by leading households to transfer to their offspring more in bequests and less in education than would be advisable. This is so even if parents internalise in their utility the value of their bequests and educational investment for their children. The problem is not, therefore, one of an externality not internalised, but rather the impossibility of replicating in a decentralised way, under laissez-faire, the kind of intergenerational coordination that a planner constrained only by the feasibility of the allocation of resources can achieve. The planner's allocation can, nonetheless, be decentralised through the market by means of subsidising labor income at the expense of a lump-sum tax on saving returns.

1. Introduction

Households choose, among other things, the mix of bequests and human capital they endow their offspring with for altruistic reasons. The latter takes typically the form of education expenditures that determine their offspring human capital. The

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effectiveness of households' education efforts arguably depends also on the parents' own human capital, in such a way that —all other things equal— any given amount invested translates into a higher human capital for the offspring when compounded with a high human capital of the parents. Be as it may, one would expect, at any rate, that when parents take into account the value for their children of their mix of bequests and education effort, as well as internalise the impact that their own human capital entails, they would be able to choose the right one, from a social viewpoint. Interestingly enough, as it is shown below, this is not the case. It turns out that even when parents assess correctly the value for their children of their mix of bequests and education expenditures effort —as well as how their own education compounds with it— and choose it in a decentralised way through the market, the latter cannot deliver (under laissez-faire) the right allocation[1] —specifically, the market steady state wage per efficient unit of labor is too high relative to the marginal productivity of labor at the planner's steady state.[2]

To grasp the extent to which this should be surprising, it should be noted that when I refer to a situation in which "parents assess correctly the value for their children of their education effort and how their own education compounds with it" I mean exactly that: parents add to their utility from consumption *the actual* value function $V(e^t, h^t, b^t; \mathbf{x}_t)$ —weighted by a parameter measuring their degree of altruism— of each of their children's optimisation, as a function of the parents' choices of education e^t and bequests b^t for their children as well as the parents' own human capital h^t.[3] This modelling choice is not only the only one consistent with the parents' rationality —since they know that their children's decision problem is the same as theirs and, knowing their own value function, they therefore know their children's, from which they could only depart in their objective at the price of being irrational— but is also made in order to give all the chances to the decentralised allocation of resources to deliver the best outcome for the representative agent. And still, it falls short of doing so. Why is it so?

From the analysis below it follows that —since the parents internalise correctly the impact of their educational and bequest choices, as well as their own human capital, on their offspring's utility through the value function $V(e^t, h^t, b^t; \mathbf{x}_t)$— it is not a missing externality in their optimisation which lies at the heart of the result.

[1]By the "right" allocation I mean the allocation that a planner would choose in their stead in order to maximise the well being of the representative agent of the economy.

[2]See footnote 13 below for the reasons to focus on steady states.

[3]The value function will depend also —because of the forward-looking nature of households' problem— on the sequences, from t onwards, of all future prices for consumption as well as for production factors, summarised by $\mathbf{x}_t \equiv \{x_\tau\}_{\tau \geq t} \equiv \{p_t, p_{t+1}, w_t, r_{t+1}\}_{\tau \geq t}$.

What drives the result is instead the fact that, while parents can take into account how their choices impact their children's utility, they nonetheless cannot choose for them. This is a constraint from which the planner is, by definition, freed: he or she chooses for everyone, and therefore can improve upon what households can do in a decentralised way.

In some sense, one lesson to be drawn from this result is that internalising all kinds of externalities needs not always be enough: there are limits to what can be done in a decentralised way *under laissez-faire*. Having said so, the planner's allocation can be decentralised indeed, but this requires a policy that steers households choices towards it through the right incentives. Specifically, labor income needs to be subsidised and bequests taxed —funding the subsidy by means of a non-distortionary lump-sum tax on the savings returns— in order to give parents the right incentive to invest more in their children's education and bequeath them less.

Although the literature on parental investment in their offspring human capital is abundant, as well as that on bequests, there is surprisingly little on, specifically, what is the right mix of the two, for altruistic households to pass on their children. The question addressed in this paper is actually reminiscent of the point made in Drazen (1978), namely that —contrarily to what was argued in Barro (1974)— government debt is net wealth[4] even for altruistic households with limited life-spans as soon as the possibility of bequeathing through educational investments in their offspring's human capital is added to the model. The gist of the point made in Drazen (1978) is that —since the implicit return to investments human capital seems, by revealed preferences, to be empirically higher than the return to physical capital, at least up to some threshold— a liability passed on their children allows households to increase the return to their savings for retirement by investing in their children education and making them pay the necessary taxes to repay government debt, instead of investing in physical capital. From making the assumption that the return to investments in children's human capital exceeds that of physical capital up to some threshold, Drazen (1978) concludes that bequests would be, as much as possible, in human capital, the composition depending thus on whether the amount of the bequest exceeds or not the threshold. Nevertheless, the literature in

[4]That is to say, the introduction of government bonds expands the budget set of households by allowing for negative bequests capturing resources from future generations by means of imposing to them the liability of the future taxes needed to pay for the interest and principal of the debt issued. Barro (1974) claimed that as long as households' choice is to make positive bequests —which seems to be the empirically relevant case— this possibility of imposing negative bequests would not effectively change the equilibrium allocation, since even in their absence households can reduce the positive bequests they make but they choose not to.

the wake of Drazen (1978) kept focusing rather on the Ricardian equivalence (or debt neutrality) debate, e.g. Weil (1987), instead of on the right mix of bequests and education. Regarding this issue of the education-bequests mix, this paper therefore goes beyond Drazen (1978) insofar it establishes that, regardless the role of government bonds[5] in expanding the budget set of altruistic households with limited life-spans in the presence of human capital, the mix of bequests and education provided by parents to children is, at a market equilibrium, inefficient. The paper provides too —without having to resort to any assumption on the returns to human or physical capital— an assessment of the direction of the inefficiency —parents provide less education and higher bequests than the planner would— as well as a policy to undo it.

It is worth mentioning too that Caballé (1995) considers a model similar to that of this paper, except that there human capital productivity of a household's educational investment increases with the average investment —and not on parents' human capital, as in this paper— leading to inefficient endogenous growth. As it is known to be the case for similar externalities,[6] households do not internalise —when interacting in a decentralised way through the market— the positive externality of their educational investments in their children's human capital on that of everybody else's children, resulting in a market allocation that delivers an inefficient underinvestment in human capital. Nevertheless, contrarily to what is generally argued, Caballé (1995) points that subsidising education might not increase the rate of growth in an economy of altruistic overlapping generations when young agents cannot borrow to make educational investment in their own human capital —which only their parents can do for them, as in the current paper— in the case in which the "physical bequest motive is not operative", that is to say when at equilibrium the non-negativity constraint on bequests is binding, meaning that households would have actually liked to bequeath liabilities rather than assets to their offspring. This result hinges, nonetheless, on the interplay of the inefficiency resulting from the lack of internalisation of the positive externality from average education on human capital formation, with the usual inefficiency resulting from over-accumulation of physical capital when the latter is the only means of saving, a problem that is not present when households can save in some other asset —like fiat money or rolled-over government debt— or there is some mechanism allowing, equivalently, to implement transfers from young to old —like, for instance, a pay-as-you-go pension scheme. Therefore, since this paper differs fom Caballé (1995)

[5]Government bonds are absent in the framework considered here, for the sake of delivering transparently what really drives the point.
[6]For instance, in Arrow (1962), Romer (1986), and Lucas (1988).

—on top of because of human capital production differing as mentioned above— in that households can save in an alternative asset too, namely fiat money —both for empirical relevance and as a fix to the risk of over-accumulation problem— the analysis in Caballé (1995) does not apply.

The rest of the paper is organised as follows. Section 2 presents the key elements of the economy, namely its demographics and production possibilities. Section 3 characterises the planner's steady state for the economy. Section 4 its market equilibria and, more specifically, steady state. Section 5 compares the planner's and the market steady states and shows how the market steady state misses the planner's. Section 6 presents a policy allowing to decentralise the planner's steady state through the market as a competitive equilibrium. Some concluding remarks are made in the final Section 7.

2. The economy

Consider an economy of identical 2-period lived overlapping generations of households multiplying by a factor $n > 0$ each period. The representative household born at t derives a utility $u(c_0^t, c_1^t)$ from its consumption —c_0^t when young and c_1^t when old[7]— of output produced each period out of the young household's efficient units of labor[8] h^t and the (*per* young) previously unconsumed output[9] k^{t-1}/n through a neoclassical production function[10] F delivering a (*per* young) output $F(\frac{k^{t-1}}{n}, h^t)$ at t. A household also derives utility from the utility of each of its n children households, discounted by an altruism factor γ that, for the allocation of resources problem to be well defined, is bounded above by the reciprocal of the population growth factor n, i.e. $\gamma n < 1$. The efficient units of labor (or human capital) h^t a household born at t is endowed with when young results from both a *per* child educational investment e^{t-1} made at $t-1$ by (or on behalf of)[11] its parent household, and the parent household's own human capital h^{t-1}, through a human

[7]With u being differentiably strictly increasing and differentiably strictly quasi-concave, i.e. $Du(c_0^t, c_1^t) \in \mathbb{R}^2_{++}$ and $D^2u(c_0^t, c_1^t)$ negative definite in the orthogonal space to $Du(c_0^t, c_1^t)$, for all (c_0^t, c_1^t), so that consumption demands in the face of positive prices will be interior.
[8]Only young agents supply labor.
[9]Without loss of generality, capital fully depreciates in one period, for the sake of simplicity.
[10]That is to say, a linearly homogeneous, concave function, satisfying Inada —marginal productivities with respect to any factor increase without bound as the latter converges to zero— and the condition that output from no capital or no labor is nil.
[11]If chosen by a planner.

capital production function H, such that

$$h^t = H(e^{t-1}, h^{t-1}) \quad (1)$$

Later on, concavity of H will be assumed too, in order to guarantee the uniqueness of the utilitarian planner's steady state. Moreover, H will be assumed when needed to be close enough to be linear, at least in a neighbourhood containing the planner's and the market steady states.

In what follows, I will characterise the steady state allocation that an egalitarian planner would choose for such an economy and show that it has to provide positive bequests and educational investments. I then characterise as well the competitive equilibrium allocations when households can save by means of both lending to firms and holding fiat money.[12] For the sake of subsequently addressing the question of whether a planner's steady state can be decentralised by the market I specifically characterise the equilibria delivering positive educational investments too. By means of these characterisations, I establish then that an egalitarian planner's steady state cannot be a market allocation under laissez-faire for altruistic households making bequests and education investments for their children. Finally, in order to address this market inefficiency, I identify a balanced policy of taxes and subsidies that decentralises through the market the egalitarian planner's steady state.

3. The egalitarian planner's steady state

An egalitarian planner would choose a feasible steady state profile of consumptions c_0, c_1 that maximises the representative household's overall steady state utility, which comprises the utility the household derives from its own consumption profile $u(c_0, c_1)$ plus the overall steady state utility of each of its children households —of which it has n and, for each of them, their utility is weighted by the altruism factor γ— which comprises the utility they derive from their own consumption profile $u(c_0, c_1)$, plus the overall steady state utility of each of their own children, which

[12]Since, generically, whenever capital is the only means of saving, it cannot achieve simultaneously the two goals of equalising the marginal return to capital to both the representative agent's intertemporal rate of substitution —necessary for the optimality of the household savings— and the population growth factor —necessary for the maximisation of the output net of investment.

comprises... and so on. That is to say, the planner maximises

$$u(c_0, c_1) + n\gamma\Big(u(c_0, c_1) + n\gamma\big(u(c_0, c_1) + n\gamma(u(c_0, c_1) + \dots)\big)\Big) = \frac{1}{1 - n\gamma}\, u(c_0, c_1) \tag{2}$$

given that $\gamma n < 1$, or, equivalently, the planner maximises just $u(c_0, c_1)$, since the first factor in the right-hand side of the equation (2) above amounts to a mere scaling factor.[13] Moreover, the planner is constrained to satisfy the steady state feasibility conditions, in *per* young terms, imposed by the technologies allowing for the production of output

$$c_0 + \frac{c_1}{n} + k + ne \leq F(\frac{k}{n}, h) \tag{3}$$

and of human capital

$$h \leq H(e, h) \tag{4}$$

A planner's steady state is, therefore, a profile c_0, c_1, k, e, h solution to[14]

$$\begin{aligned} &\max_{0 \leq c_0, c_1, k, e, h} u(c_0, c_1) \\ &c_0 + \frac{c_1}{n} + k + ne \leq F(\frac{k}{n}, h) \\ &h \leq H(e, h) \end{aligned} \tag{5}$$

It is thus worth noting that, although the planner's ability to choose for all households a stationary allocation makes the households' altruism appear to be seemingly

[13]Bernheim (1989) adresses the issue of the difficulty of defining in general an objective for the planner in the case of altruistic agents and how it necessarily differs from a dynastic objective. Indeed, since each altruistic generation takes into account —through its offspring's— the utility of *all* future descendants, then maximising a sum of all generations' utilities —weighted by a positive sequence ρ_t such that $\sum_t \rho_t = 1$— i.e. maximising $\sum_{t=1}^{+\infty} \rho_t \Big[\sum_{t'=t}^{+\infty} (\gamma n)^{t'-t} u(c_0^{t'}, c_1^{t'})\Big]$, leads to a "double"-counting of future consumption utilities that *de facto* weights generations t's utility from consumption $u(c_0^t, c_1^t)$ by a factor $\sum_{t'=1}^{t} \rho_{t'} (\gamma n)^{t-t'}$ and is therefore not equivalent to the dynastic objective of the representative generation weighting $u(c_0^t, c_1^t)$ by $(\gamma n)^{t-1}$. As shown above, focusing on the steady state allocations sidesteps this problem.

[14]For a discussion of the planner's steady state as, equivalently, (the limit of) the steady state solution to the problem of a planner discounting exponentially future generations' utility (as it discounts them less and less) —while disregarding future generation altruism— see the concluding remarks in the last section.

irrelevant in the planner's problem, the problem above *does* capture the households' altruism towards their children. Indeed, while taking into account the households' altruism does not make the planner consider an objective distinct from the one that would correspond to the case of selfish households, as opposed to altruistic ones,[15] even if the planner cares about the representative agent's consumptions only *directly*, it actually realises that the education effort e enters in the determination of the *contemporaneous* —because of the stationarity— efficient units of labor necessary for the production of the steady state level of output and, hence, consumption, which makes the planner provide for h through e, behaving this way altruistically *de facto*, even if such behaviour coincides with the one that results from tending to the selfish interest of the representative agent in his or her own consumption only.

A planner's steady state solution to the problem above is, therefore, a profile c_0, c_1, k, e, h necessarily satisfying[16]

$$\begin{pmatrix} u_0(c_0,c_1) \\ u_1(c_0,c_1) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{1}{n} \\ 1 - F_K(\frac{k}{n},h)\frac{1}{n} \\ n \\ -F_L(e,h) \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 0 \\ -H_e(e,h) \\ 1 - H_h(e,h) \end{pmatrix} \tag{6}$$

—for some positive multipliers λ and μ— along with the planner's feasibility and human capital production constraints binding, which provides the following necessary characterisation of a planner's steady state.

Definition 1. *An* **egalitarian planner's steady state** *of the economy characterised by population dynamics, preferences, and production of consumption as well as of human capital represented by n, u, F and H —under the assumptions stated*

[15]Indeed, by being able to choose consumption profiles for all generations while focusing on stationary ones, the altruism effect boils down to a mere scaling factor in the planner's objective, with no impact on the optimal allocation.
[16]The assumptions on F and H guarantee that the non-negativity constraints on the choice variables cannot be binding at the solution.

in Section 2— is a profile c_0, c_1, k, e, h such that

$$\begin{aligned} \frac{u_0(c_0,c_1)}{u_1(c_0,c_1)} &= n = F_K(\frac{k}{n},h) \\ \frac{n - nH_h(e,h)}{H_e(e,h)} &= F_L(\frac{k}{n},h) \\ c_0 + \frac{c_1}{n} + k + ne &= F(\frac{k}{n},h) \\ h &= H(e,h) \end{aligned} \tag{7}$$

Note that, given the assumptions on u, F, and H, at an egalitarian planner's steady state, all c_0, c_1, k, e, and h are strictly positive.

The next proposition —the proof of which is straightforward— establishes that, whenever the human capital production function is concave, there can only be one planner's steady state

Property 1. The utilitarian planner's steady state is unique, for an economy characterised by population dynamics, preferences, and production of consumption as well as of human capital represented by n, u, F and H —under the assumptions stated in Section 2— if H is moreover concave.

4. The market allocations

When interacting through markets, households choose their consumption profiles in order to maximise their utilities given their labor income when young and the returns to their savings when old. Intergenerational transfers can take place both from parent households to their offspring and vice-versa. Indeed, households born at t can transfer resources to their offspring by (1) bequeathing to each of the n of them some amount b^t of physical capital when old, or (2) investing when young some amount e^t into the human capital formation of each of their children households. At the same time, households can hold real balances M^t/p_t of monetary savings M^t —where p_t is the price level of consumption at t— with which to pay when old for consumption bought from the young which —when chosen by all generations— effectively results in resources flowing from children households to their parent households.[17] Finally, households born at t can also save by means of lending some amount k^t of physical capital to firms at a gross rental rate or return factor r_{t+1} to be paid next period $t+1$.

The representative household born at t makes then bequest, education, saving, and consumption choices —i.e. b^t, e^t, k^t, M^t, c_0^t and c_1^t when young and old respectively— from which it derives a direct utility $u(c_0^t, c_1^t)$ to which it adds the overall utility obtained by each of its n children, weighted by the altruism factor $\gamma \in (0,1)$. As a result of this altruism towards its children households, the representative household's overall utility is defined recursively as shown in the next section.

4.1 Household's optimal choice.

Given the physical capital bequest b^{t-1} received, and the human capital it is endowed with as a result of the education investment received from its parents in combination with their own —that is to say e^{t-1} and h^{t-1} respectively— the period t representative household aims at maximising —with respect to its consumption profile c_0^t, c_1^t, saving choices (in capital and money) k^t, M^t, educational effort e^t, and bequest b^t (and under the first and second period budget constraints determined by the consumption and factors prices $x_t \equiv (p_t, p_{t+1}, w_t, r_{t+1})$, as well as under the human capital formation technology constraint)— its overall utility $V(e^{t-1}, h^{t-1}, b^{t-1}; \mathbf{x}_t)$ —where $\mathbf{x}_t \equiv \{x_\tau\}_{\tau \geq t}$ is all future prices—[18] comprising the utility it derives from its consumption profile $u(c_0^t, c_1^t)$, plus the maximum overall utility $V(e^t, h^t, b^t; \mathbf{x}_{t+1})$ of each of its n children, weighted by the altruism factor γ, that is to say

$$
\begin{aligned}
V(e^{t-1}, h^{t-1}, b^{t-1}; \mathbf{x}_t) = \max_{0 \leq c_0^t, c_1^t, k^t, M^t, e^t, h^t, b^t} & u(c_0^t, c_1^t) + n\gamma V(e^t, h^t, b^t; \mathbf{x}_{t+1}) \\
c_0^t + k^t + \frac{M^t}{p_t} + ne^t &\leq w_t h^t + b^{t-1} \\
c_1^t + nb^t &\leq r_{t+1} k^t + \frac{M^t}{p_{t+1}} \\
h^t &\leq H(e^{t-1}, h^{t-1})
\end{aligned}
\tag{8}
$$

[17]Equivalently, the amount $\frac{M^t}{p_t}$ can also be identified with rolled-over public debt with a return $\frac{p_t}{p_{t+1}}$ paid to the old by means of the proceeds of its re-sale to the contemporary young.
[18]Future prices are assumed to be known with perfect foresight.

for given consumption and factor prices $\mathbf{x}_t \equiv \{x_\tau\}_{\tau\geq t} \equiv \{p_t, p_{t+1}, w_t, r_{t+1}\}_{\tau\geq t}$, parent choices $e^{t-1}, h^{t-1}, b^{t-1}$, and a population growth factor n satisfying $n < \frac{1}{\gamma}$, given the altruism weight γ.[19]

The first-order conditions necessarily characterising the household choice are[20]

$$\begin{pmatrix} u_0(c_0^t, c_1^t) \\ u_1(c_0^t, c_1^t) \\ 0 \\ 0 \\ n\gamma V_e(e^t, h^t, b^t; \mathbf{x}_{t+1}) \\ n\gamma V_h(e^t, h^t, b^t; \mathbf{x}_{t+1}) \\ n\gamma V_b(e^t, h^t, b^t; \mathbf{x}_{t+1}) \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ n \\ -w_t \\ 0 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ 0 \\ 0 \\ n \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$
$$+ \nu_M^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \nu_e^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \nu_b^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \tag{9}$$

for some $\lambda_0^t, \lambda_0^t, \mu^t, \nu_M^t, \nu_e^t, \nu_b^t \geq 0$, along with the constraints binding, for all t, where

[19]The extent to which households *can* be altruistic is linked to the population growth factor. Should the altruism factor γ exceed the reciprocal of the population growth factor $\frac{1}{n}$, there would not exist a value function V allowing to define the representative household's problem —specifically, the right-hand side would fail to be a contraction of the space containing V, and the existence of the (fixed point) V allowing for the representative household's problem to be well-defined is not guaranteed. Alternatively, when fertility is endogenous, the altruism discount factor can be assumed to decrease fast enough in the population growth factor —see Becker, Murphy and Tamura (1994). Also, although it is obvious from the first period budget constraint that, at the solution, the third constraint is always binding, the recursive way in which human capital is formed requires h^t to be included in t's problem *as if* it was a variable of choice (which is actually none) since it is determined by e^{t-1} and h^{t-1}, that is to say by $e^{t-1}, e^{t-2}, e^{t-3}, \ldots$

[20]Ignoring the (at a solution, non-binding) non-negativity constraints for c_0^t, c_1^t (because of the differentiably strictly increasing and differentiably strictly quasiconcave assumptions on u), for k^t (because, at equilibrium, the returns to k^t and M^t will be positive and equal, and hence the composition of the necessarily positive —because of the differentiably strictly increasing and differentiably strictly quasiconcave assumptions on u— optimal savings *portfolio* is indeterminate in the household's choice, so that one of the non-negativity constraints on k^t and M^t can be dropped), and for h^t (because of the differentiably strictly increasing assumption on u, unless $H(e^{t-1}, h^{t-1}) = 0$ itself).

—from the envelope theorem—[21]

$$
\begin{aligned}
V_e(e^t, h^t, b^t; \mathbf{x}_{t+1}) &= \mu^{t+1} H_e(e^t, h^t) \\
V_h(e^t, h^t, b^t; \mathbf{x}_{t+1}) &= \mu^{t+1} H_h(e^t, h^t) \\
V_b(e^t, h^t, b^t; \mathbf{x}_{t+1}) &= \lambda_0^{t+1}
\end{aligned} \tag{10}
$$

that is to say

$$
\begin{aligned}
\begin{pmatrix} u_0(c_0^t, c_1^t) \\ u_1(c_0^t, c_1^t) \\ 0 \\ 0 \\ n\gamma\mu^{t+1} H_e(e^t, h^t) \\ n\gamma\mu^{t+1} H_h(e^t, h^t) \\ n\gamma\lambda_0^{t+1} \end{pmatrix} &= \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ n \\ -w_t \\ 0 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ 0 \\ 0 \\ n \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \\
&\quad + \nu_M^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \nu_e^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \nu_b^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}
\end{aligned} \tag{11}
$$

or, equivalently —eliminating the multipliers of the budget constraints, and taking into account the non-negativity of money savings, educational investment, and

[21] The derivatives of the value $V(e^t, h^t, b^t; \mathbf{x}_{t+1})$ of the Lagrangian of the problem faced by generation $t+1$, that is to say,

$$
\begin{aligned}
u(c_0^{t+1}, c_1^{t+1}) &+ n\gamma V(e^{t+1}, h^{t+1}, b^{t+1}; \mathbf{x}_{t+2}) \\
&- \lambda_0^{t+1}\big(c_0^{t+1} + k^{t+1} + \frac{M^{t+1}}{p_{t+1}} + ne^{t+1} - w_{t+1}h^{t+1} - b^t\big) \\
&- \lambda_1^{t+1}\big(c_1^{t+1} + nb^{t+1} - r_{t+2}k^{t+1} - \frac{M^{t+1}}{p_{t+2}}\big) - \mu^{t+1}\big(h^{t+1} - H(e^t, h^t)\big)
\end{aligned}
$$

with respect to e^t, i.e. $V_e(e^t, h^t, b^t; \mathbf{x}_{t+1})$, is indeed $\mu^{t+1} H_e(e^t, h^t)$, and similarly for $V_h(e^t, h^t, b^t; \mathbf{x}_{t+1}) = \mu^{t+1} \cdot H_h(e^t, h^t)$, and with respect to b^t, i.e. $V_b(e^t, h^t, b^t; \mathbf{x}_{t+1})$, is λ_0^{t+1}.

bequests—

$$\begin{aligned}\frac{u_0(c_0^t,c_1^t)}{u_1(c_0^t,c_1^t)} &= r_{t+1} \geq \frac{p_t}{p_{t+1}} \ (= \text{if } M^t > 0)\\ \gamma\mu^{t+1}H_e(e^t,h^t) &\leq u_0(c_0^t,c_1^t) \ (= \text{if } e^t > 0)\\ n\gamma\mu^{t+1}H_h(e^t,h^t) &= \mu^t - w_t u_0(c_0^t,c_1^t)\\ \gamma u_0(c_0^{t+1},c_1^{t+1}) &\leq u_1(c_0^t,c_1^t) \ (= \text{if } b^t > 0)\end{aligned} \tag{12}$$

so that the period t representative household's **optimal choice** $c_0^t, c_1^t, k^t, M^t, e^t, b^t$ and human capital endowment h^t are, whenever $e^t > 0$,[22] necessarily characterised by —eliminating the multipliers μ^t, μ^{t+1} for the human capital formation technology—

$$\begin{aligned}\frac{u_0(c_0^t,c_1^t)}{u_1(c_0^t,c_1^t)} &= r_{t+1} \geq \frac{p_t}{p_{t+1}} \ (= \text{if } M^t > 0)\\ \frac{1}{H_e(e^t,h^t)}\frac{1}{\gamma}\frac{u_0(c_0^t,c_1^t)}{u_0(c_0^{t+1},c_1^{t+1})} &= w_{t+1} + n\frac{H_h(e^{t+1},h^{t+1})}{H_e(e^{t+1},h^{t+1})}\\ \frac{u_1(c_0^t,c_1^t)}{u_0(c_0^{t+1},c_1^{t+1})} &\geq \gamma \ (= \text{if } b^t > 0)\\ c_0^t + k^t + \frac{M^t}{p_t} + ne^t &= w_t h^t + b^{t-1}\\ c_1^t + nb^t &= r_{t+1}k^t + \frac{M^t}{p_{t+1}}\\ h^t &= H(e^{t-1},h^{t-1})\end{aligned} \tag{13}$$

4.2 Market equilibria.

At a market equilibrium, capital and labor are remunerated by their marginal productivities, so that

$$\begin{aligned}w_t &= F_L(\frac{k^{t-1}}{n}, h^t)\\ r_{t+1} &= F_K(\frac{k^t}{n}, h^{t+1})\end{aligned} \tag{14}$$

and the allocation is feasible if, and only if,

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t + ne^t = F(\frac{k^{t-1}}{n}, h^t) \tag{15}$$

[22]Focusing on optimal choices with —like at the planner's steady state— positive educational investment.

which follows from collapsing the budget constraints of the agents alive at any given period t

$$\begin{aligned} c_0^t + k^t + \frac{M^t}{p_t} + ne^t &= w_t h^t + b^{t-1} \\ \frac{c_1^{t-1}}{n} + b^{t-1} &= r_t \frac{k^{t-1}}{n} + \frac{M^{t-1}}{p_t}\frac{1}{n} \end{aligned} \tag{16}$$

whenever

$$M^{t-1} = nM^t \tag{17}$$

which is the money market equilibrium condition.

Thus, taking into account the households' optimal behaviour characterised in the previous section, the conditions necessarily characterising a market equilibrium allocation in which households' educational investment is positive are those provided next.

Definition 2. *A* **market equilibrium** *with positive educational investments is any allocation $c_0^t, c_1^t, k^t, M^t, e^t, h^t, b^t$ and prices p_t such that, for all t,*

$$\begin{aligned} \frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} &= F_K(\frac{k^t}{n}, h^{t+1}) \geq \frac{p_t}{p_{t+1}} \ (= \text{if } M^t > 0) \\ \frac{1}{H_e(e^t, h^t)} \frac{1}{\gamma} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &= F_L(\frac{k^t}{n}, h^{t+1}) + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})} \\ \frac{u_1(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &\geq \gamma \ (= \text{if } b^t > 0) \\ c_0^t + k^t + \frac{M^t}{p_t} + ne^t &= F_L(\frac{k^{t-1}}{n}, h^t) h^t + b^{t-1} \\ c_1^t + nb^t &= F_K(\frac{k^t}{n}, h^{t+1}) k^t + \frac{M^t}{p_{t+1}} \\ h^t &= H(e^{t-1}, h^{t-1}) \\ M^{t-1} &= nM^t \end{aligned} \tag{18}$$

For the sake of comparing it to the planner's, the next section characterises the steady state market allocations with positive educational investment.

4.3 Market steady state.

A specific instance of market equilibrium allocation characterised in Definition 2 is any stationary allocation that treats all households equally, as characterised in the following definition.

Definition 3. *A* **market steady state** *is a profile $c_0, c_1, k, m, e, h,$ and b such that*

$$\begin{aligned} \frac{u_0(c_0,c_1)}{u_1(c_0,c_1)} &= F_K(\frac{k}{n},h) \geq n \ (= \text{if } m>0) \\ \frac{\frac{1}{\gamma} - nH_h(e,h)}{H_e(e,h)} &= F_L(\frac{k}{n},h) \\ \frac{u_1(c_0,c_1)}{u_0(c_0,c_1)} &\geq \gamma \ (= \text{if } b>0) \\ c_0 + k + m + ne &= F_L(\frac{k}{n},h)h + b \\ c_1 + nb &= F_K(\frac{k}{n},h)k + nm \\ h &= H(e,h) \end{aligned} \tag{19}$$

If $m > 0$ the steady state is referred to as a **monetary** *market steady state.*

It follows straightforwardly that, at a market steady state, intergenerational transfers take place only in one direction, either from young to old through the acceptance of fiat money, or from old to young through bequests.

Property 2. *For a steady state market allocation either $m = 0$ or $b = 0$.*

Proof. Should $m > 0$ and $b > 0$, then

$$\frac{1}{\gamma} = \frac{u_0(c_0,c_1)}{u_1(c_0,c_1)} = n \tag{20}$$

which cannot be since $n\gamma < 1$. □

The inability of the model to exhibit, at a steady state, simultaneously bequests and the kind of transfers from young to old —either in the form of holding public debt, or funding a pay-as-you-go pension scheme, or accepting fiat money— that are observed follows from the representative agent assumption and the absence of uncertainty, two assumptions to be relaxed in subsequent research.

The same holds true, close enough to the limit, for any equilibrium converging to a market steady state, as the following corollary of Property 1 states.

Corollary. *For a market allocation converging to a market steady state it holds that, from some period t onwards, either $m^t = 0$ or $b^t = 0$.*

Proof. Should, for all t, $m^t > 0$ and $b^t > 0$, then

$$\begin{aligned} 1 &= \lim_{t\to\infty} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} \\ &= \lim_{t\to\infty} \frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} \cdot \lim_{t\to\infty} \frac{u_1(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} = \lim_{t\to\infty} \frac{p_t}{p_{t+1}} \cdot \gamma \\ &= n \cdot \gamma \end{aligned} \tag{21}$$

which is a contradiction since $n\gamma < 1$. $\square$

The next section compares the allocations that the market can deliver at a steady state, with the steady state that a planner would choose.

5. Market vs planner steady states

In order to be able to obtain unambiguous comparisons between the planner's and the market steady states, it is necessary to assume that the human capital production function is, moreover, close enough to be linear in, at least, a neighbourhood of the market and planner's steady states.

Property 3. *For a sufficiently close to linear human capital production technology in a neighbourhood of the market and planner's steady states, the market steady state delivers*

(1) *a too high wage rate per efficient unit of labor or, equivalently,*

(2) *a too high net output per efficient unit of labor, if monetary,*[23] *relative to the egalitarian planner's.*

Proof. From the market and planner's steady state definitions above, it follows that they differ in that they are characterised respectively by

$$\begin{aligned} \frac{\frac{1}{\gamma} - nH_h(\bar{e},\bar{h})}{H_e(\bar{e},\bar{h})} &= F_L(\frac{\bar{k}}{n},\bar{h}) \\ \frac{n - nH_h(e^*,h^*)}{H_e(e^*,h^*)} &= F_L(\frac{k^*}{n},h^*) \end{aligned} \tag{22}$$

or, equivalently,

$$\begin{aligned} \frac{1}{\gamma} &= F_L(\frac{\bar{k}}{n},\bar{h})H_e(\bar{e},\bar{h}) + nH_h(\bar{e},\bar{h}) \\ n &= F_L(\frac{k^*}{n},h^*)H_e(e^*,h^*) + nH_h(e^*,h^*) \end{aligned} \tag{23}$$

Note that in the case in which H is close enough to linear around the steady states —*i.e.* when $H(e^{t-1},h^{t-1}) \simeq \alpha e^{t-1} + \beta h^{t-1}$, so that the approximation error is close enough to zero, for all e^{t-1}, h^{t-1} in at least a neighbourhood containing the market and planner's steady states— it turns out that from (23) it follows

$$F_L(\frac{k^*}{n},h^*) \simeq \frac{1}{\alpha}(n - n\beta) < \frac{1}{\alpha}(\frac{1}{\gamma} - n\beta) \simeq F_L(\frac{\bar{k}}{n},\bar{h}) \tag{24}$$

since

$$n < \frac{1}{\gamma} \tag{25}$$

so that the market steady state delivers a too high wage rate per efficient unit of labor.

Moreover, since for both the planner's and a *monetary* market steady states it holds respectively $F_K(\frac{\bar{k}}{n},\bar{h}) = n = F_K(\frac{k^*}{n},h^*)$, then

$$\begin{aligned} \frac{F(\frac{k^*}{n},h^*) - k^*}{h^*} &= \\ F_L(\frac{k^*}{n},h^*) &< F_L(\frac{\bar{k}}{n},\bar{h}) \\ &= \frac{F(\frac{\bar{k}}{n},\bar{h}) - \bar{k}}{\bar{h}} \end{aligned} \tag{26}$$

[23] That is to say, with zero bequests, according to the corollary in Section 4 —or, in the parlance of the literature, when the bequest motive is not operative.

so that a *monetary* market steady state delivers a too high net output per efficient unit of labor. □

6. Decentralising the planner's steady state

Consider now a policy consisting of (1) taxing/subsidizing[24] household t's labor income at a rate τ_t, while (2) transferring/taxing a lump-sum T_{t+1} when old. The representative household faces then the problem

$$\begin{aligned} V(e^{t-1}, h^{t-1}, b^{t-1}; \mathbf{x}_t) = \max_{0 \leq c_0^t, c_1^t, k^t, M^t, e^t, h^t, b^t} & u(c_0^t, c_1^t) + n\gamma V(e^t, h^t, b^t; \mathbf{x}_{t+1}) \\ c_0^t + k^t + \frac{M^t}{p_t} + ne^t &\leq (1+\tau_t) w_t h^t + b^{t-1} \\ c_1^t + nb^t &\leq r_{t+1} k^t + \frac{M^t}{p_{t+1}} + T_{t+1} \\ h^t &\leq H(e^{t-1}, h^{t-1}) \end{aligned} \tag{27}$$

for given policies τ_t, T_t, consumption and factor prices $\mathbf{x}_t \equiv \{x_\tau\}_{\tau \geq t} \equiv \{p_t, p_{t+1}, w_t, r_{t+1}\}_{\tau \geq t}$, parent choices $e^{t-1}, h^{t-1}, b^{t-1}$, and a population growth factor n satisfying $n < \frac{1}{\gamma}$, given the altruism weight γ.

The first order conditions necessarily characterising the household choice are

$$\begin{aligned} \begin{pmatrix} u_0(c_0^t, c_1^t) \\ u_1(c_0^t, c_1^t) \\ 0 \\ 0 \\ n\gamma V_e(e^t, h^t, b^t; \mathbf{x}_{t+1}) \\ n\gamma V_h(e^t, h^t, b^t; \mathbf{x}_{t+1}) \\ n\gamma V_b(e^t, h^t, b^t; \mathbf{x}_{t+1}) \end{pmatrix} &= \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ n \\ -(1+\tau_t)w_t \\ 0 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ 0 \\ 0 \\ n \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \\ &+ \nu_M^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \nu_e^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \nu_b^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \end{aligned} \tag{28}$$

[24]Depending on the sign of the rate.

along with the budget and human capital production constraints binding, where —from the envelope theorem,

$$\begin{aligned} V_e(e^t, h^t, b^t; \mathbf{x}_{t+1}) &= \mu^{t+1} H_e(e^t, h^t) \\ V_h(e^t, h^t, b^t; \mathbf{x}_{t+1}) &= \mu^{t+1} H_h(e^t, h^t) \\ V_b(e^t, h^t, b^t; \mathbf{x}_{t+1}) &= \lambda_0^{t+1} \end{aligned} \tag{29}$$

that is to say —eliminating the multipliers of the budget constraints, and taking into account the non-negativity of money savings, educational investment, and bequests—

$$\begin{aligned} \frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} &= r_{t+1} \geq \frac{p_t}{p_{t+1}} \ (= \text{if } M^t > 0) \\ \gamma \mu^{t+1} H_e(e^t, h^t) &\leq u_0(c_0^t, c_1^t) \ (= \text{if } e^t > 0) \\ n\gamma \mu^{t+1} H_h(e^t, h^t) &= \mu^t - (1+\tau_t) w_t u_0(c_0^t, c_1^t) \\ \gamma u_0(c_0^{t+1}, c_1^{t+1}) &\leq u_1(c_0^t, c_1^t) \ (= \text{if } b^t > 0) \end{aligned} \tag{30}$$

so that, necessarily, the household's optimal choice is characterised —whenever $e^t > 0$— by

$$\begin{aligned} \frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} &= r_{t+1} \geq \frac{p_t}{p_{t+1}} \ (= \text{if } M^t > 0) \\ \frac{1}{H_e(e^t, h^t)} \frac{1}{\gamma} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &= (1+\tau_{t+1}) w_{t+1} + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})} \\ \frac{u_1(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &\geq \gamma \ (= \text{if } b^t > 0) \\ c_0^t + k^t + \frac{M^t}{p_t} + ne^t &= (1+\tau_t) w_t h^t \\ c_1^t &= r_{t+1} k^t + \frac{M^t}{p_{t+1}} + T_{t+1} \\ h^t &= H(e^{t-1}, h^{t-1}) \end{aligned} \tag{31}$$

At a market equilibrium, capital and labor are remunerated by their marginal productivities, so that

$$\begin{aligned} w_t &= F_L(\frac{k^{t-1}}{n}, h^t) \\ r_{t+1} &= F_K(\frac{k^t}{n}, h^{t+1}) \end{aligned} \tag{32}$$

and the allocation is feasible if, and only if,

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t + ne^t = F(\frac{k^{t-1}}{n}, h^t) \tag{33}$$

which requires the equilibrium in the money market, that is to say

$$\frac{M^t}{M^{t+1}} = n \tag{34}$$

if the intervention is balanced, i.e. if τ_t and T_t are such that

$$0 = \tau_t w_t h^t + \frac{1}{n} T_t \tag{35}$$

A market equilibrium with positive educational investments under the policy $\{\tau_t, T_t\}_t$ is therefore any collection of sequences for $c_0^t, c_1^t, k^t, M^t, e^t, h^t$ and p_t such that, for all t,

$$\begin{aligned}
\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} &= F_K(\frac{k^t}{n}, h^{t+1}) \geq \frac{p_t}{p_{t+1}} \text{ (= if } M^t > 0) \\
\frac{1}{H_e(e^t, h^t)} \frac{1}{\gamma} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &= (1 + \tau_t) F_L(\frac{k^t}{n}, h^{t+1}) + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})} \\
\frac{u_1(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &\geq \gamma \text{ (= if } b^t > 0) \\
c_0^t + k^t + \frac{M^t}{p_t} + e^t &= (1 + \tau_t) F_L(\frac{k^{t-1}}{n}, h^t) h^t \\
c_1^t &= F_K(\frac{k^t}{n}, h^{t+1}) k^t + \frac{M^t}{p_{t+1}} + T_{t+1} \\
h^t &= H(e^{t-1}, h^{t-1}) \\
n &= \frac{M^t}{M^{t+1}} \\
0 &= \tau_t F_L(\frac{k^{t-1}}{n}, h^t) h^t + \frac{1}{n} T_t
\end{aligned} \tag{36}$$

and a market steady state —under the policy τ, T— is a profile c_0, c_1, k, m, e, h such that

$$\begin{aligned}
\frac{u_0(c_0,c_1)}{u_1(c_0,c_1)} &= F_K(\frac{k}{n},h) \geq n \ (= \text{if } m>0) \\
\frac{\frac{1}{\gamma} - nH_h(e,h)}{H_e(e,h)} &= (1+\tau)F_L(\frac{k}{n},h) \\
\frac{u_1(c_0,c_1)}{u_0(c_0,c_1)} &\geq \gamma \ (= \text{if } b>0) \\
c_0 + k + m + ne &= (1+\tau)F_L(\frac{k}{n},h)h \\
c_1 &= F_K(\frac{k}{n},h)k + nm + T \\
h &= H(e,h) \\
0 &= \tau F_L(\frac{k}{n},h)h + \frac{1}{n}T
\end{aligned} \tag{37}$$

from where the following policy supporting the planner's steady state follows.

Proposition 4. *The policy* τ, T *decentralises the planner's steady state* c_0, c_1, k, e, h *satisfying* (7) *if, and only if, labor income is subsidised at a rate*

$$\tau = \frac{1}{H_e(e,h)F_L(\frac{k}{n},h)}\left(\frac{1}{\gamma} - n\right) > 0 \tag{38}$$

through a second-period lump-sum tax

$$T = -\frac{nh}{H_e(e,h)}\left(\frac{1}{\gamma} - n\right) < 0 \tag{39}$$

Proof. It follows straightforwardly from the comparison of the market steady state equations under the policy above with those of the planner's steady state that, for the policy to decentralise the latter through the market, the product of the subsidy rate with the labor productivity in the right-hand side must offset the altruistic term in the left-hand side and replace it with the population growth factor, both relative to the productivity of educational investment in the production of human capital, i.e. it must hold that

$$\tau F_L(\frac{k}{n},h) = \frac{\frac{1}{\gamma}}{H_e(e,h)} - \frac{n}{H_e(e,h)} \tag{40}$$

The lump-sum transfer T follows from substituting the value of τ resulting above into the government balanced budget condition. The signs follow from the condition guaranteeing that the maximisation problems are well defined, namely $n\gamma < 1$. □

7. Concluding remarks

A couple of final remarks are in order. Firstly, the comparison above of two steady states (the market and the planner's) starting from *different* initial conditions should —given that they can be Pareto-ranked but the transition between between them cannot— be considered under the light of the questions it answers, namely, first, can the best possible steady state (from the viewpoint of an egalitarian planner) ever be a market outcome under laissez-faire? The fact that the answer to this question has been shown above to be negative prompted the next question: can the best possible steady state (from the viewpoint of an egalitarian planner) then be a market outcome under some policy? That the answer to this second question has instead been shown to be positive is of interest in itself, even if the implementation of the best steady state might require a shift of the initial condition to the planner's steady state that prevents it from being Pareto improving. Indeed, that the best steady state can be decentralised through the market is of interest, independently of whether —in a political economy expansion of the model— society is willing or not to (make some generation) pay the price necessary to move to it. Should it had turned out that the best steady state could not have been decentralised, there wouldn't be room to even ask the political economy question.

If, alternatively, one bound oneself to compare only market and planner allocations with the *same* initial conditions, then either the market or the planner's would not be stationary, so that the very question of how do market and planner steady states compare cannot even be posed under the common initial condition requirement.

Finally, it is worth clarifying that the solution to the planner's problem in (5) is the limit of the steady state solution to the problem of a planner discounting exponentially future generations' utility by a factor $\eta \in (0,1)$, as $\eta \to 1$, while disregarding their altruistic preferences,[25] i.e.

$$\begin{aligned} &\max_{0\le c_0^t, c_1^t, k^t, e^t, h^t} \sum_{t=1}^{\infty} \eta^{t-1} u(c_0^t, c_1^t) \\ &c_0^t + \frac{c_1^{t-1}}{n} + k^t + ne^t \le F(\frac{k^{t-1}}{n}, h^t) \\ &h^t \le H(e^{t-1}, h^{t-1}) \end{aligned} \tag{41}$$

whose solutions are necessarily characterised by

$$\begin{aligned} \frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} &= F_K(\frac{k^t}{n}, h^{t+1}) = \frac{n}{\eta} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} \\ \frac{1}{H_e(e^t, h^t)} \frac{n}{\eta} \frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+1}, c_1^{t+1})} &= F_L(\frac{k^t}{n}, h^{t+1}) + n \frac{H_h(e^{t+1}, h^{t+1})}{H_e(e^{t+1}, h^{t+1})} \\ c_0^t + \frac{c_1^{t-1}}{n} + k^t + ne^t &= F(\frac{k^{t-1}}{n}, h^t) \\ h^t &= H(e^{t-1}, h^{t-1}) \end{aligned} \tag{42}$$

Note that the limit of a steady state of this dynamics as $\eta \to 1$ is the steady state of the egalitarian planner, as defined in Section 3.

[25]See Bernheim (1989) for the impossibility, in general, for a well-defined objective for the planner that captures the different generations' altruism

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