Jules Vuillemin on the Aristotelian Notion of the Possible and the Master Argument

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1 Introduction

Jules Vuillemin (1984) well-known classification of philosophical systems is a result of his own reconstruction of Diodorus's take on the Megarian Master Argument (MA) – i.e. Diodorus attempt to show that it is incompatible to assert that a given proposition is possible, even it is neither true now nor it will ever be, and at the same time endorse the irrevocability of the past and Aristotle's principle(s) of possibility.

Vuillemin's (1977, 1979, 1984, 1996) approach to the MA which is quite different to Arthur Prior's (1950) modal formalization, follows his own formal interpretation of Aristotle's notion of possibility as potentiality and makes this notion the central concept of the argument.

Unfortunately, the lack of a thorough systematic syntactic and semantic development makes Vuillemin's formalization, difficult to grasp, and lead to harsh criticisms, not only from a logical but also from a historic and philosophical point of view. A general feature of Vuillemin's perspective on the MA, is that, unconvinced of the style of modal analysis of ancient logic launched by Prior's work, he looks for some alternative approach with a richer expressive power. What he comes out with however, is a mainly syntactic system that combines the contemporary modal (propositional) operators, with first-order quantification over temporal moments and also with second-order quantification over propositional variables – though a systematic account of all these components of his system is missing.

Actually, letting by side for the moment Vuillemin's own formal reconstruction, his criticism of Priorian-style modalities is well-grounded. Indeed, recent literature in ancient philology and philosophy strongly indicates that possible world semantics does not seem to provide a suitable reconstruction of modality and temporality within the ancient tradition. Moreover, coming back to the specific subject of the present paper, I think it is nowadays quite safe to say that Prior's style of formalization does not offer a reliable framework for the understanding of the concept of modality at work in the Master Argument.

In fact, despite Vuillemin's own puzzling formalizations, it must be acknowledged that he pinpointed important issues underlying the debates on the Master Argument such as:

- The importance of associating the notion of possibility with references to specific time moments. In line to his understanding, the logical analysis of the MA requires what he calls a "double indexation" – it is quite difficult to grasp what he has in mind, but it seems to come close to Hans Reichenbach's (1948, § 51) distinctions of a time of speech, of reference and of the event itself.

- The importance of linking the notion of possibility at work in the MA with Aristotle's inferential principles displayed in *De Caelo* I, 283 b 6-17.

Indeed, as stressed by Seel (2005, p. 113) the latter is a particularly remarkable contribution and involves both *Aristotle's possibility principle* **PP** (i.e., *If it is necessary that if A then B, then B is possible if A is*) and to what Jacob Rosen and Marko Malink (2012), based on Kit Fine (2011), call the *possibility* [inference] *rule* **PR** (*If A is possible, and if B follows from the assumption of A, then B is is possible*).[1]

The general approach I will follow is at the crossing were inferentialism meets dialogical logic, since, like Vuillemin, I am not at all convinced that possible world semantics is a suitable framework for understanding what ancient modalities are about.[2] However, I am not claiming that a less radical reconstruction of Vuillemin's views on the Aristotelian notion of possibility can be developed with the help of a more sophisticated model-theoretic approach than the one that was available to him. Indeed, one might pass over the fact that possible world semantics is not at the root of the conceptual framework underlying ancient modalities and deploy it anyway focusing on obtaining the set of logical validities prefigured by the system under examination,[3] perhaps, in the context of the MA with the help of a kind of hybrid-logic that introduces explicit names for time moments.[4] Nevertheless, this is not the path I wish to go. Furthermore, according to my view, it is not at all clear if a model-theoretic reconstruction of Vuillemin's take on the Master Argument, even if sound, provides a solid grounding to his classification of philosophical systems.

The present study is structured as follows:

- I will start recalling Prior's formalization of the Master Argument.
- I will continue by addressing some of the most salient problems with Vuillemin's own reconstruction.
- I will proceed by studying Seel's (2017) reconstruction.
- I will then suggest different alternatives for an inferentialist approach to the Master Argument motivated by Vuillemin's remarks on the relevance of Aristotle's notion of possibility. However, I will rather pursue Seel's (2017) views on the matter.
- The inferentialist approach, should set the basis for a framework where rights to ask for reasons and duties to provide them, unfold a dialectical understanding of the Aristotelian notion of possibility. The dialectical approach offers a way to

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[1] **PP** and **PR** are both stated in *Prior Analytics* , I, 15, where Aristotle justifies the latter by the validity of the former. Both are also involved in some of the passages in the *De caelo* mentioned by Vuillemin. However there are also other places such as in the *Physics, De generatione et corruptione*, and the *Metaphysics*, where Aristotle make ample use of **PP** and **PR**.[1] It is not clear if Vuillemin distinguishes **PP** and **PR**, but as pointed out by Malink and Rosen (2012, p. 186) many other have also overlooked this distinction. Let us mention that in the context of type-theory, Pfenning, F. and Davies, R. (2001) state a rule closely related to Fine's **PR**.

[2] For an insightful paper on *the proper intepreration of modal logic* from a inferentialist point of view see Sundholm (2003) . Relevant too is Malink (2013) books where he develops a reconstruction of Aristotle's Modal Syllogism, without any appeal to possible world semantics.

[3] Cf. Hermann Weidemann (2008), who suggests that Aristotle's notion of possibility at work in the MA should be understood as underlying some kind of possible-world framework. Perhaps the msot accurate possible-world reconstruction of Aristotle's modal logic is the one of Ulrich Nortmann (1996, however it focuses on Aristotle's Modal Syllogistics rather than on the modality of events relevant for the MA.

[4] See Blackburn (2001).

implement Seel's (2017) remark, that, strictly speaking the reasoning pattern at work in the Master Argument does not feature the premisses-conclusion structure typical of a (syllogistic) argument.[5] According to this view, what the so-called MA displays is Diodorus' dialectical reasoning on the (incompatible) commitments undertook by those who endorse the assertions and principles that constitute the MA.

These considerations constitute the content of the last sections of the paper where I suggest how to reconstruct the dialectical framework underlying Diodorus reasoning in line with Seel's (2017) outline. I will set the main steps towards such a dialectical reconstruction by means of Paul Lorenzen and Kuno Lorenz (1978) Dialogical Logic. The dialogical reconstruction should bestow the basis for extending Mathieu Marion's (2020) work on *Dialectical Bouts* in Aristotle's Dialectic to modalities. However, this research is still work in progress. The general point being is that a dialectical study of the MA provides an instance of a general pattern for reasoning, that I call *Dialectical Bouts of Material Admissibility*, aimed at examining the rationale behind an argumentation rule against the background of the commitments undertaken by endorsing a specific set of assertions – in contradistinction to the more abstract "admissibility" of imperfect syllogistic forms developed in the *Analytica priora*.[6] This general point can be seen as underlying Vuillemin's project of linking the study of the MA with the classification of philosophical systems.

2 Arthur Prior's Reconstruction of the Master Argument

William Kneale and Martha Kneale (1962, p. 119) quote the following description of Epictetus report on the Master Argument:

The Master Argument (MA) seems to have been formulated with some such starting points as these. There is an incompatibility between the three following propositions,

1. ***"Everything that is past and true is necessary,"***
2. ***"The impossible does not follow from the possible,"*** *and*
3. ***"What neither is nor will be is possible."***

Seeing this incompatibility, Diodorus used the convincingness of the first two propositions to establish the thesis.

3* Nothing is possible which neither is nor will be true.

The first two premisses can both be found in Aristotle, but not the conclusion: Aristotle believed that something could happen even though it never come to be actual does – e.g. this cloak could be cut up even though it never is.

There is good reason to think that Diodorus's argument is a sharpened version of the fatalistic argument Aristotle had considered in *De Interpretatione* 9.

[5] Marion (2020) also suggests that the MA should be studied as featuring a dialectical rather than a syllogistic form.

[6] For the study of reduction of imperfect to perfect syllogism as an *admissibility* procedure see Ebbinghaus (1964).

Let us now reconstruct Diodorus's conclusion using the tools of standard modal and temporal logic in the style of Arthur Prior (1950), despite the fact that it quite dubious that it is compatible with the notion of modality of ancient logic. [7] The presentation of Prior's reconstruction below, follows the notational "updating" of Fitting and Mendelson (1998, pp.38-40).

Let us symbolize Diodorus's first premiss, *Everything that is past and true is necessary*, as

1 $\mathbf{P}A.\supset \mathbb{L}\mathbf{P}A$

where "**P**", stand for at least once in the past; and " $\mathbb{L}$" *stands for "necessary"*

- ***Comment:*** This might sound quite odd. What seems more natural is to think that past is irrevocable. However, this is better formalized as $\mathbf{P}A.\supset \mathbf{GP}A.$, but then the argument does not go through, or, if substitute all modalities by temporal operators, it becomes trivial.

2 There are a number of ways in which to interpret the second premiss *The impossible does not follow from the possible. The point is that if A is possible and B impossible, A cannot follow from B.* Or if B is impossible, and if it follows that if $A\supset B$ is necessary, then this implies the impossibility of A

Here *"*$\mathbb{M}$*"* stands for possible

$$\frac{\sim\mathbb{M}B}{\mathbb{L}(A \supset B) \supset \sim\mathbb{M}A}$$

or
$$\frac{\sim\mathbb{M}B \wedge \mathbb{L}(A \supset B)}{\sim\mathbb{M}A}$$

or
$$\frac{\mathbb{L}(A \supset B)}{\sim\mathbb{M}B \supset \sim\mathbb{M}A}$$

or
$$\frac{\mathbb{L}(A \supset B)}{\mathbb{M}A \supset \mathbb{M}B}$$

We can a read it as contradictory propositions are not possible. Or positively, logical truths (the negation of a contradiction) is possible.

This yields the following inference rule, that is the contemporary formulation of Aristotle's (*Pr. An*, I, 15, 34[a] 5-7) possibility principle **PP**, mentioned above.

[7] For a defence of Prior's -style reconstruction see Denyer (1981, 1998, 1999, 2009).

$$\vdash A \supset B$$

$$\mathbb{M}A \supset \mathbb{M}B$$

For the third proposition to be demonstrated, constituted by the negation of the consequent of the original (*What neither is nor will be is **possible***) we have

*What neither is nor will be is **impossible***

3* $(\sim A \wedge \sim \mathbf{F}A) \supset \sim \mathbb{M}A$

where "**F**", stand for at least once in the future

Let also involve the following implicit premiss (Arthur Prior makes a similar assumption):

4 *Time is infinite and discrete:* If a proposition is not true in the present and it is not the case that it will be ever true, there must be a time in the past, relative to which this proposition will never be true. [8]

$$(\sim A \wedge \sim \mathbf{F}A) \supset \mathbf{P} \sim \mathbf{F}A$$

Demonstration

Let us show that if we assume the antecedent of the third premiss, given the other 2 premisses and assumption of discrete time, the consequent of 3***** follows.

5 $\sim A \wedge \sim \mathbf{F}A$ (*a*ssumption: the antecedent of 3*).

6 $\mathbf{P}\sim\mathbf{F}A$ (*modus ponens applied to 5 and 4*).

7 $\mathbf{P}\sim\mathbf{F}A. \supset \mathbb{L}\mathbf{P} \sim \mathbf{F}A$ (*substitution A with* ~**F***A in the first premiss*).

8 $\mathbb{L}\mathbf{P} \sim \mathbf{F}A$ *(modus ponens applied to 6 and 7*).

9 $\sim\mathbb{M} \sim\mathbf{P}\sim\mathbf{F}A$ (*definition of "necessary A" as "it is impossible that ~A*).

10 *It is a logical truth that if A is true then never in the past it was the case that A will never be true:*

$\vdash A. \supset \sim\mathbf{P} \sim\mathbf{F}A$

11 *Since 10 is a logical truth, it is possible*. That is applying 2 to 10 we obtain

$\mathbb{M}A. \supset \mathbb{M}\sim\mathbf{P}\sim\mathbf{F}A$

12 $\sim\mathbb{M}A$. *(modus Tollens (contraposition) applied to 9 and 11)*

Quod erat demonstrandum.

The assumption of discreteness might be a problem. Particularly so if we are aiming at render a version of the argument compatible with Aristotle's views. Hermann Weidemann

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[8] If time is **not discrete**, following happens: take it that there is an instant where we assume that after it the proposition *A* will never be true. Thus; take it that the instant is the instant t-1, before t0, where $\sim A \wedge \sim \mathbf{F}A$ is true. Now if time is not discrete; we cannot be sure that there is no instant squeezed between t-1 and t0 where *A* is true after all. Thus if $\sim A \wedge \sim \mathbf{F}A$ is the case and time is not discrete $\mathbf{P} \sim \mathbf{F}A$ cannot be verified.

(2008) develops another version without assuming discreteness, however he reinterprets the first premiss as stating $A \supset \mathbb{M}(A \vee \mathbf{P}A)$, a very plausible premiss, though it does not seem to be found in the sources.

More generally, there are certainly other problems too with Prior's style of reconstruction:

- On one hand Sussane Bobzien (1993) strongly suggests that Megarian and/or Stoic modalities should not be understood as propositional operators but as being closer to some form of second-order predication over propositions. Moreover, the Diodoran notion of possible proposition seems to amount to establishing that a proposition is possible if it is either true now or it will be. But in this case the conclusion of MA follows by *vim terminorm*. Indeed, if we substitute in 3* $\mathbb{M}A$ *with* $\mathbf{F}A \vee A$, we obtain $(\sim A \wedge \sim\mathbf{F}A) \supset \sim(\mathbf{F}A \vee A)$, i.e. $(\sim A \wedge \sim\mathbf{F}A) \supset (\sim A \wedge \sim\mathbf{F}A)$. This does not only make the rest of the argument redundant it is not compatible with the notion of possibility to be contested, namely, that *A* is possible is true proposition though *A* never realizes.
- On the other the work of Marko Malink (2013) shows that Aristotle modal syllogistic should be understood as involving modal term-relations, (e.g. *Rational is necessarily said of Man*), rather than operators with an underlying possible-world semantics. Furthermore, as pointed out by Rosen & Malink (2012) and Malink & Rosen (2013) a second approach to modality, very much deployed by Aristotle's in his methaphysics and natural philosophy is rooted in **PP** and **PR**. This "second approach" does not naturally invite to a model-theoretical semantics either, but rather to an inferentialist view on the syllogistic as the one stressed by the work of Kurt Ebbinghaus (1964), based on Paul Lorenzen's (1955) *Operative Logik* and by John Corcoran (1972), based on natural-deduction.[9]

Jules Vuillemin (1977, 1979, 1984, 1996) openly rejects Prior's assumption of the discreteness of time – though in some passages it looks as if he assumes it, particularly so since he deploys quantification over time moments (rather than over intervals) instead of Priorian operators Vuillemin also rejects the use of the modal axiom $|\text{-}\ A. \supset \sim\mathbf{P} \sim\mathbf{F}A$, which he finds incompatible with Aristotle's view on truth, since according to Vuillemin this axiom involves retrogradation of truth. However, as point out by Vidal-Rosset (2011), there is retrogradation of refutation not of truth: a past truth remains true for ever, or the existence in the past of a proof that *A* will never be refuted. Furthermore, Vuillemin indicates that the temporal dimension involved in the MA requires both reference to some temporal index constituting a propositional function and a second temporal reference for the saturated propositional function.

3 Jules Vuillemin's Reconstruction of the Master Argument

It looks as if the first result of Vuillemin's reflections on the Master Argument was a paper published in 1977 in Brazil under the title "La puissance selon Aristote et le possible selon Diodore". This was further developed into the very well-known paper "L'argument dominateur" published 1979.

[9] Lion and Rahman 2018) compare the inferentialist approaches of Ebbinghaus and Corcoran. M. Crubellier, M. Marion. Z. McConaughey and S. Rahman (2019) advocate for a dialogical interpretation of the syllogistic.

The title of the first paper, is telling; since indeed, as just mentioned above, Vuillemin's contribution to the discussion is the proposal to focus on the notion of possibility as potentiality. Indeed, if possibility is associated to potentiality, it concerns the future and excludes the past. Accordingly, a possibility cannot be actualized in the past – this is how Vuillemin interprets the first principle of the MA on the irrevocability of the past. Similarly, he extends this approach to the necessity of (actualizations in) the present: if something is actual in the present it cannot enjoy potentiality at the same time and in the same respect. Furthermore, if possibility is potentiality, the possibility of a proposition *A*, excludes the actualization of its negation at the very same (span of) time (and under the same circumstances) under which this proposition is being actualized – this is what is behind Vuillemin's reading of the second principles of the MA that denies that something impossible follows from the possible.

Vuillemin sets himself high standards for a reconstruction of the Master Argument. namely:[10]

- The reconstruction must show that the MA is logically sound – here he coincides with Prior's approach.
- The reconstruction must focus on the point that the MA is purported to refute Aristotle's claim that there are possible propositions that will never be actualized.
- Accordingly, if new principles have to be added to the logical reconstruction, they must be compatible with Aristotle's theory of the possible. This is his main complain, against approaches such as the one of Prior. Particularly so Prior's assumption that the flow of time enjoys discreteness.

I will start by pointing out some of the difficulties in Vuillemin's approach. Then I will try to disscuss some of the philosophical points of his reconstruction that, on my view, constitute interesting insights into the matter.

3.1 Some formal and philosophical difficulties

Unfortunately, Vuillemin's formal system lacks of both a systematic presentation of the syntactic formation rules he uses and the underlying semantics he has in mind, despite the fact that it abounds in informal discussions on the underlying motivations for his formal choices. Gerhard Seel (2005, p. 115), renders the following table of the last version of Vuillemin's reconstruction of the MA. We will follow this one since the last version is the most mature one – whereas "A", "B", "C", stand for the original premisses of Diodorus number "2" indicates Vuillemin's second formal reconstruction.

Vuillemin deploys the following notational conventions: "$\mathbb{C}$" stands for the contingency-operator, "N" for the indexical now, and "$\mathbb{L}$" and "$\mathbb{M}$" stand for the usual notation of those days for the propositional operators of necessity and possibility. Universal quantification, for example over temporal moments, carries the notation "$(t)\ p_t$", "$\equiv$" stands for bi-implication. Vuillemin uses a dot for the conjunction, for the sake of readability I will use "$\wedge$" instead (this is the only change of the original notation I use). Moreover, in order to unify the notation I will use the notation "$A(t)$" instead of "p_t.

[10] Cf. Seel (2005, p. 84).

- **Vuillemin's reading of the first premiss of Diodorus**

  The possible cannot be realized in the past (A2)

  $(A)\ (t)\ \{[\mathbb{C}_{\mathbf{N}}A(t) \wedge t<\mathbf{N} \equiv (t<\mathbf{N} \wedge \mathbf{N}=t)] \wedge \sim(\mathbb{C}_{\mathbf{N}}A(t) \wedge t<\mathbf{N})\ \}.$

- **Vuillemin's reading of the second premiss of Diodorus**

  The possible realization of the possible as the Syncronic Contraction of Possibility (B2, S'2)

  $(A)\ (t)\ \{\mathbb{M}_{\mathbf{N}}A(t) \supset \sim(t_1)\ [(t \leq t_1 < \mathbf{N} \vee \mathbf{N} \leq t \leq t_1) \supset \sim \mathbb{M}_{\mathbf{N}}A(t_1)]\}.$[11]

- **Vuillemin's reading of the third premiss of Diodorus**

  It is now possible that p (at some time t' in the present or in the future) even if neither it is the case that p now nor will ever be the case that p (C2)

  $\exists A\ \exists t'\ [(\sim\mathbb{L}_{\mathbf{N}}\sim A(t') \wedge \mathbf{N} \leq t') \wedge (t)\ (\mathbf{N} \leq t \supset \sim A(t))]$

  Additional principle: Conditional Necessity ($\mathbf{C_N}$)

  $(t)\ \mathbb{M}_t A(t) \supset A(t)$

Syntactic issues

After a first look at these formulae one might wonder what is a well-formed formula in the system. Let us have a closer look.

There is second-quantification of propositional variables in the first premisses, but not over relations. If I rightly understood, the rationale of deploying second order quantification is to allow to have as axiom a proposition such as C2, that is not generally valid for any substitution of its propositional variables, but only for some –Seel (2017) speaks of relying on a *concrete* example. Are we then in presence of a second-order logic over propositional variables? Not quite, since in the last formula *p* occurs "free".

Furthermore, propositional variables seem to be formulated as predicates over temporal moments. Events are predicates of time-moments, thus, it looks its formation-rule is $A(t) : prop\ (t : Time)$. But this does not apply to relations such as $t_1 < \mathbf{N}$, that are a-temporally true. But this applies only to atomic propositions, so, it looks as if $(p_{t1} \wedge q_{t1})(t_2)$ is not well formed.

However, if the propositional variable (i.e. the monadic predicate) is within the scope of a modal operator then, the new formula is temporally indexed. Thus, double indexation results from adding a modality to a temporal predicate (the event). Thus the formation of $\mathbb{M}_{\mathbf{N}}A(t)$ is $\mathbb{M}_{\mathbf{N}}A(t)$: $prop\ (t, \mathbf{N}: Time)$, or more precisely $(\mathbb{M}A(t))_{(\mathbf{N})} : prop\ (t, \mathbf{N}: Time)$, thus, of the present time $\mathbf{N}$ is said that A_t is possible. Again, is $\mathbb{M}_{t1}(A(t_1) \wedge B(t_1))$ well formed? If we look at S'2 complex formulae formed by negation are well formed, namely $\mathbb{M}_{\mathbf{N}}\sim A(t_1)$ - perhaps double indexation only applies to literals (positive and negative). What about iterated modalities?

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[11] I used the alternative formulation recorded by Seel (2005, p. 94).

Perhaps double indexation is close to what nowadays is known as nominals in hybrid logic, i.e. propositional operators for names of temporal or worlds such as $\mathbb{M}A(t)@_{\mathbf{N}}$ – see for example Blackburn (2001)). More generally, though there are temporal indexes in the object language there are no indexes for worlds. So, either indexes for worlds are left to the meta-language or perhaps, modalities do not assume a set of possible worlds but only times. So, there are indexed Priorian temporal modalities or propositional functions, in the sense of Bobzien.

True, Vuillemin (1996, p. 26) distinguishes *real modalities* from *logical modalities*, while the first involve temporality, the second ones not: the double temporal indexation in $\mathbb{M}_t A(t)$ and the lack of it in $\mathbb{M}A(t)$ should manifest syntactically that the first one is a real modality and the second one a logical one. However, as mentioned above, since Vuillemin admits the implication $\mathbb{M}_t A(t) \supset \mathbb{M}A(t)$ this makes it hard to see what the underlying semantics might be. This leads us to the following remarks.

Semantic issues.

As mentioned above Vuillemin's underlining of the temporal underpinnings of the Aristotle's notion of (natural) possibility, including time reference is certainly a valid point. However, his way of implementation is not convincing, neither from the systematic nor from the historical point of view. Indeed, Vuillemin assumes temporal moments as individuals over which predicates expressing events can be applied. However, quantification over time moments is incompatible with Aristotle's view on time, since for Aristotle time is not a substance.

A general problem with the lack of an explicit semantics is related to the syntactic issues mentioned above – letting by side his second-order quantification over propositions. Semantically speaking the problem is that we do not know if the modal formulae are to be evaluated in relation to just temporal moments or a pair of world and temporal indexes. In fact, possible worlds never occur in a formula, so either they are at the metalanguage, or the semantics does not include possible worlds at all (or there is just the actual world). Moreover, real modalities look much more like predicates than operators. However, since all this is not explicit, it is difficult to figure out what the formal system underlying Vuillemin's reconstruction looks like. Vuillemin (1996) suggests that possibility, should be reduced to temporal modality:

> *If the possible is said to be that which is or will be, the actual cannot be derived from the possible, for one member of a disjunction (that which is) does not follow logically from the disjunction. Likewise, if the necessary is said to be that which is true and will not be false, the necessary cannot be derived from the actual, for a conjunction does not follow logically from one of its conjuncts (that which is true).* Vuillemin (1996, p. 45).

> This is puzzling. If *possible A* reduces to the disjunction, *A is true now or in the future,* then inferring that *A is impossible*, from the premisses that *A is not true now nor it will ever be,* requires only one step, namely distributing the negation over the disjunction. Notice that double indexation is not a way out if CN is assumed. Clearly, if say, $\mathbb{M}_{\mathbf{N}}A(\mathbf{N})$ reduces by CN to $A(\mathbf{N})$, then the searched contradiction follows immediatelty. Similar for any t in the future.

Seel (1982), who has been inspired by Vuillemin's take on the MA deploys three kind of modalities: namely (i) general universal and existential quantifiers over temporal moments (ii) universal and existential quantifiers for the past and the future and (iii) modalities linked to some specific indexical date (e.g. being possible *now* that A at some time t). But is not clear if Vuillemin shares a classification of modalities similar to the one of Seel. If so, then, I guess, Seel's indeterminate modalities would correspond to Vuillemin's logical ones.

Vuillemin's reading of the principles.

- **On the necessity of past facts.** Vuillemin's reading of the first principle is quite unorthodox. As pointed to me by Vincent Wistrand, this reading follows the interpretation of possibility as potentiality: what is potential cannot be actualized. This leads Vuillemin to understand the first principle of the MA as establishing the irreversibility of time. Accordingly, the contingent cannot be realized in the past: what is potential can only be actualized in the future. More precisely, the formula should express the idea that contingency of the past is equivalent to the collapse of time-order, that is, (i) the contingency of the past is equivalent to $t < t$ and (ii) no proposition is now contingent if it has been actualized before now. Actually, in his second reconstruction Vuillemin (1986, 1996) reduces the role of the first principle to establishing the irrevocability of time – thus, there is no need to link past facts with the future in the present involved in the second and third principles.
  Now, if we provide a temporal reading to the modalities, then the necessity of the past expresses that it will always be the case of a past fact that it once happened. Strictly speaking this assumes the transitivity of the time relation, not its irreversibility.[12]
  Independently of this, though, it is certainly true that Aristotle defended the irreversibility of time, it is historically implausible that this is what the first principle of the MA expresses – for a detailed discussion of the objections see Seel (2005).

- **On the second principle**. As mentioned above, one of the most interesting features of Vuillemin's take on the MA is stressing the relevance of Aristotle's inferential notion of possibility. In Vuillemin's terms the point of this notion is, to put it bluntly, that a proposition is said to be possible if the hypothesis of being actualized does not lead to contradiction – here again possibility is linked to potentiality. Vuillemin gives as reference *De Caelo* I, 283 b 6-17. This notion of possibility constitutes indeed the core of the second principle of the MA, that, I may recall, establishes that the impossible does not follow from the possible. Vuillemin's take on **PP** (or perhaps **PR**) , that he calls *the possible realization of the possible*; is surprising. In its first version Vuillemin (1979), he formalizes this principle by means of what he calls the *synchronic contraction of necessity* and the *synchronic contraction of possibility* (SCP). Roughly, SCP allows transferring the actual possibility of a future fact (that takes place at some instant t) to the possibility at some t_1 (equal of before t and equal or after now) that this very fact happens at t_1. The transference of the possibility at some t_0 to the possibility t_1 in order to obtain formulae of the form $\mathbb{M}_{t'}A(t')$ is called

[12] Irreversibility amounts to asymmetry, i.e. irreflexivity and transitivity, or if reflexivity is assumed, as Vuillemin sometimes does, then anti-symmetry is required – it seems that left part of the conjunction in Vuillemin's renderings of the first principle, aims at establishing anti-symmetry. It is well-known that there is no axiom of standard temporal logic that expresses irreflexivity and/or connectedeness, i.e. there is no axiom of standard temporal logic that characterizes irreflexive frames.

a *contraction*. Mutatis mutandis similar applies to the contraction of the necessity. SCP is counterintuitive. Indeed, Seel (2005, p. 100) points out that it might the case that **now**, it is possible that at some future t A is the case, but it could well happen that this possibility is lost at some time before t, **even** if **now** there is no obstacle preventing A to be realized at t. Vuillemin's (1986, pp. 22-23) attempt to infer it from the possibility of the possible is not sound – see Gaskin (1998, pp. 628-629). Thus, there is not obvious logical way to link SCP and Aristotle's notion of possibility and in fact as such it is quite of an implausible principle. Now, Vuillemin introduces contraction in order to pass from $\mathbb{M}_{t'}A(t')$ to the actualization of $A(t')$ at t'. This is what the astonishing principle of conditional necessity is purported to accomplish. The CN principles states $\mathbb{M}_{t'}A(t\ t')$ implies $A(t')$. Let me quote Vuillemin (1996, p. 23):

> *It might be objected that this proposed formulation is entirely inadequate for expressing the possible realization of the possible. To synchronically contract a possible by saying that it is possible at t_1 that p at $t_1$is surely not to realize that possible or to say that p at t_1. But, we have forewarned that Aristotle did not separate the clause of synchronic contraction from the clause of conditional necessity. Should it turn out that this latter clause signifies or implies that a contracted possibility is none other than a realized one, all the requisites of the interpretation will have been satisfied.*

- **On Conditional Necessity**: Let us turn to (t) $\mathbb{M}_t A(t) \supset A(t)$, that in its contrapositive form Vuillemin calls principle of *conditional necessity* (CN). CN has been harshly criticized by commentators such as Denyer (1998, pp. 222-223), who took this axiom as trivializing all of Vuillemin's formal system. Indeed, if we deploy the implication $\mathbb{M}_{t1}A(t_1) \supset \mathbb{M}A(t_1)$., the interpretation of the implication seems to suggest that *For any time t, if something is possible at t, it is true at this time.* In fact, Vuillemin explicitly justifies this implication by observing that if a real (temporal) modal proposition is true, so it is its logical (non-temporal) modality:

> *If it is possible now that p at t then it is a fortiori logically possible that p at t, since the synchronic possibility puts added strictures on the logical possibility.* Vuillemin (1996, p. 23).

Clearly, from $\mathbb{M}A(t_1) \supset A(t_1)$ and $\mathbb{M}A(t_1)$ we obtain a contradiction with the premisses of the last principle, for the case that t_1 is either equal to **N** or to some moment in the future we can. Thus, if we have $\mathbb{M}A(t) \supset A(t)$, nothing else is needed to prove the conclusion – this has been also pointed out by Vidal-Rosset (2011).[13] Is there a confusion between inference and implication? Certainly, necessitation, $A \vdash \mathbb{L}A$, is a standard modal axiom of normal modal logics, but not its implicative form! Necessitation requires a categorical as premiss not an open assumption.[14]

Vuillemin's own motivation for introducing CN is that it links the result of applying contraction, a formula of the form $\mathbb{M}_{t1}A(t_1)$, with one of the premisses of the third principle. For short, the task of CN is to instantiate the modal formula in such a way to produce the contradiction.

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[13] Notice, that such a trivialization does not apply to Prior's reconstruction. Indeed, one cannot gather $\sim A \supset \sim \mathbf{M}A$ from Prior's $\mathbf{P}A. \supset \sim\mathbf{M}\sim\mathbf{P}A$, by substitution, since $\mathbf{P}A \,/\, \sim A$ is an incorrect substitution

[14] See Klev (2016, section 3).

Be that as it may, Vuillemin's who is aware of the awkwardness of this axiom, explicitly recognizes that it can be seen as the collapse of the possible and the real. In order to advocate for CN, Vuillemin comments that the antecedent of the implication, and the double indexation, somehow mitigates the collapse, since the consequent is true in the "context" of the antecedent:

> *Le principe de nécessité conditionnelle produit-il un effondrement des modalités ? Oui, en ce que, **durant** que p, on peut conclure de p à la nécessité de p. Mais cette conclusion ne saurait être détachée de sa condition, ce qui limite la portée de l'effondrement. Vuillemin* (1979, p. 242).

But, unless the implication is defined otherwise, this does not really help. If every proposition that is now true becomes necessary, then modal collapse threatens. Nevertheless, if we understand the formulation *while p*, in the Avicenna-way, as i*f p is true during the span of time d, then it is contradictory to negate p during exactly this span of time*, there is some way to render a sensible re-formalization of CN.

3. 2 Vuillemin's General Strategy

After the discussion above the general strategy underlying Vuillemin's (second) reconstruction should be clear.[15]

If, according to the last principle some A is now possible at some time t, then it can be actualized either in the present, in the past or in the future. However, if t is the present, by contraction we obtain $\mathbb{M}_{\mathbf{N}}A(\mathbf{N})$. but then by CN, we obtain $A(\mathbf{N})$, and this contradicts the third principle. If t is some moment in the future, then by contraction and CN we obtain that there is some time t' in the future at which $A(t')$ is true and this again contradicts the third principle. If t is some moment in the past, then A would actualize in the past; but this is impossible because of Vuillemin's first reading. In his first reconstruction the first principle only has the role to limit the possible to the future or the present.

Now, independently of the formal issues, whereas the historic accuracy of Vuillemin's interpretation of the first principle is dubious, his reading of the second principle as following from Aristotle's notion of possibility is definitively implausible. Moreover, the addition of CN either trivializes the whole reconstruction or makes one or two of the other principles redundant.

Nevertheless, perhaps, if we are prepared to oversee switches from the metalogical level to the object-language level a more friendly presentation of Vuillemin's idea might be possible. Assume that possible A means, that there is some time in the future or in the present at which A is the case and, further more that synchronic contraction is in fact a kind of metalogical device that, by means of an equality, fixes the bounded variable (it provides a precise *date*). If we follow this thought, the result synchronic contraction yields

Given $\exists t\mathbb{M}_{\mathbf{N}}A(t)$, one of the following alternatives is the case

$\exists t\,(A(t) \wedge t{=}\mathbf{N})$ -$_{\mathrm{df}}$- $\mathbb{M}_{\mathbf{N}}A(\mathbf{N})$ contraction of $\exists t\mathbb{M}_{\mathbf{N}}A(t)$ to the present moment $\mathbf{N}$.

$\exists t\,(A(t) \wedge t{>}\mathbf{N} \wedge t{=}\,t_1)$ -$_{\mathrm{df}}$- $\mathbb{M}_{t1}A(t_1)$ contraction of $\exists t\mathbb{M}_{\mathbf{N}}A(t)$ to the future moment t_1.

[15] For a reconstruction with a similar general view on MA to that of Vuillemin, though is critical with some of Vuillemin's formal developments see Gaskin (1995, 1996, 1998, 1999).

$\exists t\,(A(t) \wedge t{<}\mathbf{N} \wedge t{=}\,t_2)$ -df- $\mathbb{M}_{t2}A(t_2)$ contraction of $\exists t\mathbb{M}_{\mathbf{N}}A(t)$ to the past moment t_2.

If this is what Vuillemin had in mind, CN is not "grotesque" anymore, to use Denyer (2009, p. 37) devastating criticism.[16] It only amounts to elimination of the possibility operator followed by substitution of identicals.

$\exists t\,(A(t) \wedge t{=}\mathbf{N}) \supset A(\mathbf{N})$ -df- $\mathbb{M}_{\mathbf{N}}A(\mathbf{N}) \supset A(\mathbf{N})$

$\exists t\,(A(t) \wedge t{>}\mathbf{N} \wedge t{=}\,t_1) \supset A(t_1)$ -df- $\mathbb{M}_{t1}A(t_1) \supset A(t_1)$

$\exists t\,(A(t) \wedge t{<}\mathbf{N} \wedge t{=}\,t_2) \supset A(t_2)$ --df- $\mathbb{M}_{t2}A(t_2) \supset A(t_2)$

However, we might obtain all this without contraction and simply by eliminating the existential to *now* or to the future or to the past. More generally, this reconstruction looks quite different to the original formulation of Master Argument, and it is not clear how this is compatible with Aristotle's notion of the possible as that which might rest for ever unactualized, unless we add some other (world-) dimension to the one of time. If we add possible worlds, then CN comes out again as highly implausible.

Gerhard Seel (2017), takes Vuillemin's desiderata at face value. In fact, he even strengthens Vuillemin's conditions for a logical reconstruction of the MA in the sense that no additional principles should be allowed. However, his new reconstruction suggests both, that that additional principles have to be assumed after all and that the argument is logically unsound. The core of Seel's development is that he takes Vuillemin's double indexation as involving the three temporal dimensions of Reichenbach (1948, § 51). Moreover, as my formal delineation below shows, according to Seel's (2017) analysis the source of the logical failure is indeed the conflation of these three dimensions.

4 Gerhard Seel's Fresh Start

In view of the difficulties and objections to Vuillemin's approach and the one of other attempts, and according to some philosophical and historical consideration Seel (2017) observes that a faithful reconstruction of Master Argument must fulfil the following conditions

1. *The first principle – in the restricted form* [i.e., facts concerning the past] - *must be given a crucial role in the argument.*
2. *The second principle must be applied to a concrete example.*
3. *This example must consist of two different states of affaires one implying the other.*
4. *The validity of this implication should be evident and accepted by all the ancient philosophers engaged in the dispute.*
5. *The example must provide that the first principle in the restricted version has an impact on the third principle.*
6. *No other principles should be added.*

This list comes out as a result of observing that some earlier attempts, either do not stick to the three-principles condition, such as the attempts of Arthur N. Prior (1950), David Sedley (1977) and Hermann Weidemann (2008), do not comply with 6, not only because they add principles such as discreteness of time, and/or other theorems of contemporary modal logic, but

[16] I think that this is the gist of Seel's (2005, p. 103) answer to Denyer's (2009) critcisim.

because the use of the second principle is not based on the application of a concrete example; or do stick to three principles but the demonstration of the inconsistency makes one of the principles quite (or even totally) irrelevant, such as in the early reconstructions of Jules Vuillemin (1984, 1996), Richard Gaskin (1995) and Seel (1982) himself.

The point of condition 5 is that whereas the first principle concerns the past the third principle involves the present and the future. Thus, some kind of bridge must be built. The idea of a fresh start is to build such a bridge without introducing new principles and the proposal is to consider the introduction of a concrete example that instantiates the Aristotelian definition of the possible as that the hypothesis of its actualization does not lead to a contradiction.

Following a remark of Cicero in *De fato*, VII, 3, Seel (proposes) to

> […] *surmise something like 'Cypselus is ruling over Korinth'. Now, what other state of affairs does this one inevitably imply? No doubt, if Cypselus is now ruling over Korinth, he must have ascended the throne some time before. On this basis we get a valid implication which has the required property of linking a present tense sentence to a past tense sentence.* Seel (2017).

Now, if we wish to understand the definition of Aristole's as allowing the distribution of the possibility over antecedent and consequent, we need to consider the following formulation of the example just mentioned:

- *If Cypselus rules over Korinth then (necessarily) for some time before that ruling, Cypselus ascended the throne*

Seel's (2017) informal presentation of the reasoning and its failure is excellent. In the following paragraphs I will propose some formalization, that might help to see his overall strategy and will also provide that source of my own analysis in the next section. Let me start with some notational conventions

Notational conventions: In order to simplify the notation and in order to prepare for the notation for temporal logic developed by Ranta (1994, chapter 5) – that, as I will argue further on, is closer to the original texts – I will deploy the following quantifiers:

$$(\exists t{>}\mathbf{N})A(t)$$

instead of the standard first-order notation

$$\exists t(A(t) \wedge t{>}\mathbf{N})$$

$$\mathbb{M}_{\mathbf{N}}\,(\exists t{\geq}\mathbf{N})A(t) \text{ – i.e., } \mathbb{M}_{\mathbf{N}}(A_{\mathbf{N}} \vee (\exists t{>}\mathbf{N})A(t)),$$

instead of the standard first-order notation

$$\mathbb{M}_{\mathbf{N}}\,\exists t(A(t) \wedge t{\geq}\mathbf{N})$$

As usual in such frameworks, I will not bind the point of reference **now** with a quantifier. However, let us recall that the now is an identity mapping, that yield the present Speech-time for any instant *t*,

$$t: \mathit{Time}$$

$\mathbf{N}(t)$: *Time*
$\mathbf{N}(t) = t$: *Time*

More generally, quantification over time involved in perfective forms, is often restricted to time anterior or posterior to given reference time $\boldsymbol{t_0}$ – see Ranta (1994, section 5.7).

I shall use $\boldsymbol{t}$ ("t" bold) for an arbitrary but fixed temporal reference time in the future given by the context of the concrete example mentioned above.

In fact Seel, deploys such a reference $\boldsymbol{t}$ moment such that $\mathbf{N}>\boldsymbol{t}$**,** for the time at which the antecedent A of the implication will be tested.

Accordingly

It is possible now that Cypselus rules over Korinth at future time $\boldsymbol{t}$*,*
carries the notation

$\mathbb{M}_{\mathbf{N}}\, A(\boldsymbol{t})$. Provided $\mathbf{N} < \boldsymbol{t}$

It is impossible now that Cypselus ***will have ascended*** *to the throne some time before* $\boldsymbol{t}$
carries the notation

$\sim\mathbb{M}_{\mathbf{N}}\,(\exists t'<\boldsymbol{t})B(t')$. Provided $\mathbf{N}<t'<\boldsymbol{t}$

Instead of the standard first-order notation

$\sim\mathbb{M}_{\mathbf{N}}\,(\exists t'\,(B(t') \wedge \mathbf{N}<t'<\boldsymbol{t}\,)$.

The Steps

I The third principle affirms that there is at least one state of affairs that neither is nor will ever be the case and nevertheless possibly is or will be the case. When applied to the test example, this yields the following assertions:

I.1 $\vdash\sim A_{\mathbf{N}}$ *It is not the case that Cypselus rules over Korinth*.
Provided $A(t)$: *prop* (t: *Time*), $\mathbf{N}$: *Time.*

I.2 *It will ever be the case that Cypselus rules over Korinth*.
$\vdash\sim(\exists t>\mathbf{N})A(t)$.

I.3 *That Cypselus rules over Korinth in some future time* $\boldsymbol{t}$*, is (now) possible.*
$\vdash\mathbb{M}_{\mathbf{N}}\, A(\boldsymbol{t})$. Provided $\mathbf{N}<\boldsymbol{t}$

We restricted the temporal variable within the scope of the possibility to the specific case of $\boldsymbol{t}$ which lies in the future. Certainly, there is also the case of $\mathbb{M}_{\mathbf{N}}\, A(\mathbf{N})$. However, this is the "easy" case, at least according to my reconstruction.

II According to the assumed example, *If Cypselus rules over Korinth neither now nor in the future, then Cypselus will not have ascended the throne before the time of that ruling.*

II.1 $\vdash(\sim A_{N} \wedge \sim A(\boldsymbol{t})) \supset \sim(\exists t'<\boldsymbol{t})B(t')$.

Provided, $\mathrm{N}<t'<\boldsymbol{t}$. After inferring $\sim A(\boldsymbol{t})$ from $\sim(\exists t>\mathrm{N})A(t)$

II.2 $\vdash\sim(\exists t'<\boldsymbol{t})B(t')$, provided, $\mathrm{N}<t'<\boldsymbol{t}$.

From I.1 and I.2 and II.1 –after introduction of conjunction, and by elimination of implication.

- **Remark.** This move suggests that the main connective of the assumed example is not material implication. In any case some kind of implication that establishes that the consequent obtains necessarily from the antecedent, in the sense that, if the antecedent does not obtain, neither the consequent does. Recall that according to Aristotle, if we stick to the well formation and inference rules for the figures of syllogism, a false conclusion follows necessarily from false premisses. The implication above seems to transpose Aristotle's view on false premisses to the propositional level. However, Seel's reconstruction does not assume that every implication between events must be of this kind; only that such a link is part of the meaning of the concrete example.

  We can in fact; skip this step and substitute by adding $\sim(\exists t'<\boldsymbol{t})B(t')$ as a subsidiary premiss involved in the meaning of the example. However, as discussed in section 5, there is a natural approach to the implicative form if we really stick to the hypothetical feature of Aristotle's notion of contingency.

III The first principle establishes that every past fact is necessary. If we apply it to II.2, we obtain:

III.1 $\vdash\sim(\exists t'<\boldsymbol{t})B(t') \supset \mathrm{L}_{\mathrm{N}}\sim(\exists t'<\boldsymbol{t})B(t')$, provided $\mathrm{N}<t'<\boldsymbol{t}$.

III.2 $\vdash\mathrm{L}_{\mathrm{N}}\sim(\exists t'<\boldsymbol{t})B(t')$. Provided $\mathrm{N}<t'<\boldsymbol{t}$, or

$\vdash\sim\mathrm{M}_{\mathrm{N}}(\exists t'<\boldsymbol{t})B(t')$

From II.2 and III1 (by elimination of implication)

Thus, *It is impossible now that Cypselus* ***will have ascended*** *to the throne some time before t*

- **Remark.** As discussed below, Seel (2017) contests that the first principle applies to $\exists t'<\boldsymbol{t})B(t')$ since the principle that past facts are necessary does not apply to future state of affairs. Moreover, according to Seel (2017) this unduly application of the first principle spoils the inference underlying the Master Argument. Moreover, according to Seel (2017), $\mathrm{M}_{\mathrm{N}}(\exists t'<\boldsymbol{t})B(t')$ should be read as *It is impossible now that Cypselus will ascend to the throne at t'*. Thus, Seel does not read the expression within the scope of the possibility operator in III.2 as concerning the future perfect, but the simple future. Nevertheless, his objection is justified, since, if $\sim(\exists t'<\boldsymbol{t})B(t')$ is the case **now** and given that $\mathrm{N}<t'<\boldsymbol{t}$, this proposition concerns the future not the past.

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IV The point of Seel (2017) is to test if the possibility of A at some time t leads or not to a contradiction. For my formalization of Seel (2017) I will deploy **PP** (Aristotle's Possibility Principle), though it might also be seen as implementing **PR**. (Aristotle's Possibility Rule). However, as I will discuss in 6.2 **PR**, requires taking if A, then B, as, to put it in the words of Rosen and Malink (2012, p. 187), an inference *subordinate* to the possibility of *A*. Aristotle states the possibility principle in the following terms:

First, it must be said that if it is necessary for B to be when A is, then it will also be necessary for B to be possible when A is possible. (*Pr. An*, I, 15, 34[a] 5-7).

Seel (2017) applies **PP** to $\mathbb{M}_{\mathbf{N}}A(\boldsymbol{t})$, in order to display his reconstruction of Diodorus reasoning. Anticipating a point, the relevance of which we will develop further on, one can present informally the implementation of **PP** in form of a dialogue. We follow here Marion's (2020) work on *Dialectical Bouts* in Aristotle. Very briefly, a *bout* would start with *Answerer* committing to a thesis, say *A*, and *Questioner*'s role would be to elicit through a series of questions *Answerer*'s commitment to further theses that would then be shown to entail, when taken together with *A*, a contradiction:

Answerer, conceded that if $A(\boldsymbol{t}) \supset (\exists t'{<}\boldsymbol{t})B(t')$ is asserted, he is further committed to the assertion $\mathbb{M}_{\mathbf{N}}A(\boldsymbol{t}) \supset \mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')$.
Diodorus, the Questioner, reminds the antagonist that the testing example, endorsed by answerer, is constituted by the assertion $A(\boldsymbol{t}) \supset (\exists t'{<}\boldsymbol{t})B(t')$, and so **Answerer** is committed to the further assertion mentioned above.
Answerer fulfils his commitment and asserts $\vdash \mathbb{M}_{\mathbf{N}}A(\boldsymbol{t}) \supset \mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')$,
Diodorus recalls the antagonist that with the third principle the **Answerer** also asserted the antecedent of this implication, and thus; he must also assert the consequent.
Answerer fulfils his commitment by asserting the consequent $\mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')$
Diodorus observes that this contradicts the assertion III.2

A description of the underling inference-schema at work can be casted as follows:

IV.1 $\vdash A(\boldsymbol{t}) \supset (\exists t'{<}\boldsymbol{t})B(t')$. Provided $\mathbf{N}{<}t'{<}\boldsymbol{t}$
------------------------------ **PP**
$\vdash \mathbb{M}_{\mathbf{N}}A(\boldsymbol{t}) \supset \mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')$,

IV.2 $\vdash (A(\boldsymbol{t}) \supset (\exists t'{<}\boldsymbol{t})B(t')$. Povided $\mathbf{N}{<}t'{<}\boldsymbol{t}$
Assumed example.

IV.3 $\vdash \mathbb{M}_{\mathbf{N}}A(\boldsymbol{t}) \supset \mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')$, from IV.1 and IV.2

IV.4 $\vdash \mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')$, I.3 and IV.3

IV.5 If put III.2 and IV.5 together we obtain the contradiction

$$\boxed{\sim\mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t') \wedge \mathbb{M}_{\mathbf{N}}(\exists t'{<}\boldsymbol{t})B(t')}$$

Hence, according to this reasoning $\mathbb{M}_{\mathbf{N}}A(\boldsymbol{t})$ is impossible after all.

So far so good, however as observed above, Seel (2017), contests that this inference is a faithful reconstruction of the original Master Argument since

1) It introduces principles beyond the ones that constitute the original formulation of Diodorus.
2) It is based on an unduly application of the first principle:
The problem is, however, that our demonstration does not show that already at [the present moment] *tp it is impossible that (a) at t. For at tp both (a) and (b) are future events. As a consequence the first principle of the 'kurieuon' doesn't apply to (b). Therefore, we cannot demonstrate the impossibility of (b) anymore and thus the second principle is fully respected.* Seel (2017).

In relation to the first observation this is certainly right, the example brought forward, does not only assume, Aristotle's possibility principle, but that time is discrete and more specifically if a state of affairs is the case in some future time t, but this was not the case an earlier time t', there must be a period of time between t' and t during which this state of affairs was generated. We might add as I will discuss further on that time moments are substances.

The second observation of Seel (2017), is also justified. Step III.1 is incorrect since it deploys as antecedent of the implication the future perfect tense, whereas the first principle requires the antecedent to concern the past. Seel (2017) points out that the unduly use of the first principle, stems from an ambiguity on the distribution of the references time **N** and $\boldsymbol{t}$ in the natural language formulation of the counterfactual. The point is that t' involves the past tense in relation to $\boldsymbol{t}$, but the future tense in relation to **N**. The proposition "*that Cypselus ascends to the throne*" occurring in the consequent of the implication; should not be read as "*Cypselus ascended to the throne some time before* ***t***" but "*Cypselus will have ascended to the throne some time before* ***t***".

In other words; the mistake comes from the fact that the conjunction

$$\sim\mathbb{M}_{N}(\exists t'<\boldsymbol{t})B(t') \land \mathbb{M}_{N}(\exists t'<\boldsymbol{t})B(t')$$

does it not make apparent the three temporal parameters involved, namely,

now (the speech time), **the time of the event** (the time t' at which B takes place) and **the reference time** (the time in the future $\boldsymbol{t}$, in reference to which t' is placed in the past).

So, one way to make the mistake apparent with help of Reichenbach's (1948, §51) is to point out that the first principle requires the moment of speech be after the time of the event, whereas the conjunction assume that it is after. More precisely the application of the first principle to the example at stake requires the structure

$t' < \mathbf{N} < \boldsymbol{t}$ '

Some **Event time** t' (some time at which B does not happen) $<\mathbf{N}$ (the time at which it is asserted that B did not happen) $< \boldsymbol{t}$ (the time in the future in which A is hypothesized to happen)

However, the real structure of the example and in particular of $\sim\mathbb{M}_{N}(\exists t'<\boldsymbol{t})B(t')$ is

$\mathbf{N} < t' < \boldsymbol{t}$

If, on the other hand, instead of $\boldsymbol{t}$, the present time $\mathbf{N}$ is taken as reference, then the contradiction follows, since this yields $\sim\mathbb{M}_{\mathbf{N}}(\exists t'<\mathbf{N})B(t')$ – by inferring $(\exists t'<\mathbf{N})B(t')$ from $\sim A(\mathbf{N})$ and applying the first principle to $(\exists t'<\mathbf{N})B(t')$), which is indeed in the past. This contradicts with the assumption that from $\mathbb{M}_{\mathbf{N}}\, A(\mathbf{N})$, follows $\mathbb{M}_{\mathbf{N}}(\exists t'<\mathbf{N})B(t')$. Thus, Diodorus "wins" if Answerer chooses the present time as reference for timing B in the past. However, the optimal move for Answerer is to choose some future time $\boldsymbol{t}$. In this setting if Answerer choses the later, Diodorus argument is based on an illegitimate move.

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Conclusion

The analysis of Seel (2017) is remarkable, though there two interrelated points that might be seen as unsatisfactory. Namely; the treatment of the modalities is rather syntactic and Aristotle's possibility principle is deployed metalogically to the concrete example in order to obtain the wished elimination of the implication. It might be desirable to integrate this definition explicitly in the argument and provide therewith the meaning explanation of the modality operator. Recall, that if understand the modality as some temporal operator without some explicit provisos, the third principle suffices for the demonstration.

Moreover, from a philosophical point of view; quantification over time moments assumes time moments as substances from which events are said to happen. However, this does not seem to be compatible with Aristotle's philosophical views on the matter. Aristotle's point is rather that, temporal moments *time* changes (such as events), rather than "saturating" incomplete expressions for events.

Another important point that will engage a longer discussion at the end of the paper, is that, on my view, the *third principle*, is not really a *principle*, it is rather a set of assertions in order to test the compatibility of Aristotle's notion of possibility against the background of the principles on the necessity of past and the possibility principle (or the associated possibility rule).

In the next sections I will have as background Seel's (2017) overall strategy but I will try to address the sticking points just mentioned. However, I have to concede that whereas the way of "timing" event without assuming that events are propositional functions over time moments is quite straightforward, the meaning explanation of Aristotle's notion of modality is a very difficult issue. I will be happy if I manage to launch a path for further reflection on the issue.

5 The Temporal Dimension of Events

In the present sections I propose an alternative reconstruction of Seel's (2017) recent take on the MA based on Martin-Löf's constructive type theory (CTT).[17] The CTT framework offers a natural way to overcome some of the most pressing criticisms to previous

[17] See Martin-Löf (1984). In what follows, a basic familiarity with CTT is assumed. All the background requirements can be found in Rahman *et al.* (2018, Chapter 2).

reconstructions, namely the way to associate time parameters and events and the rather syntactic approach to the notion of possibility.

In relation to the first criticism, there is not only the historical and philosophical point that Aristotle's philosophy does not seem to be compatible with having time instants as individual substances of which events can be said, but also the point that, to put in in John L. Austin's (1960) words one should not conflate the (e.g., temporal) circumstance of evaluation of a proposition, with the proposition to be evaluated in the context of that circumstance. More generally, semantic evaluation requires not merely a content to evaluate, but also a circumstance against which to evaluate that content. In the CTT-setting the possibility of distinguishing what makes a proposition true from the proposition made true offers a simple and clean way to avoid conflating the contextual elements with the proposition itself, or to use an Aristotelian terminology, to avoid fusing the measurement of a change (namely, time) with the subject of that change (namely, the event itself) and on the same time setting this measurement as the context against which the truth of the proposition is established. Moreover, the approach

In relation to the second criticism to be addressed in section **6**, I propose quite briefly to explore different approaches to the Aristotelian notion of possibility relevant for the Master Argument, namely

1) Modality as Causality and Avicennian Alternative on Spans of Time
2) Hypothetical Reasoning, Assumptions and the Possibility Rule: Towards a Dialogical Approach.

I will develop the second proposal more in detail as the first. Both are still first explorations and though there have been some work tying to link both approaches, such as in Rahman, Farid, Young (202), this is work in progress.

5.1 Displaying the Framework

In the following sections I shall

- show how the CTT framework allows to associate events and time without assuming that events are predicates of time moments
- show how the CTT framework offers a natural way to express the meaning dependence of the consequent upon the antecedent within Seel's (2017) example.

5.1.1 On *What* and *How*

The expression '*a* is *S*' can in principle encode two basic forms of predication: [18]

(1) The expression encodes **what** *a* is. For instance, if *S* is a genus of *a* or a category to which *a* belongs, then '*a* is *S*' encodes (at least partially) what *a* is; e.g. a human-being, a tree, a raining event

(2) On the other hand, the expression '*a* is *S*' encodes **how** *a* is. For instance, if *S* is a description which can be sometimes but not always true of *a*, then '*a* is *S*' encodes

[18] See Almog (1991, 1996). The analogy with Almog's distinctions stems from Klev, in Rahman, McConaughey, Klev, Clerbout (2018, p. 19). The present section is based on Rahman/Zarepour (2020).

(again, at least partially) how the a is, of which we already know what it is; e.g. intelligent (human being), leafy (tree), hard (raining).

These different forms of predication cannot be distinguished with the standard means of classical logic, where there is only one way to analyse the expression 'a is S', namely as the saturated propositional function $S(a)$. The CTT framework, on the contrary is sensitive to the difference between these two kinds of predication.

Indeed, suppose that 'a is A' expresses what a is. This can be captured in the framework of CTT as a's being a member of the category (or domain or type) A. The latter notion can be represented in the language of CTT as follows:

a: A

In "a: A" the colon ':' can be read as 'is' (in the sense of expressing the *what*) or, equivalently, as 'belongs to'. Moreover, suppose that 'a is B' expresses how a is. In other words, the expression describes a as having the property B and this expression constitutes a proposition. This can be captured in the language of CTT as follows:

$B(a)$: *prop*

In this expression '*prop*' represents the category of propositions. Accordingly, that a is an object of the type A which bears the description B can be expressed by combining the two previous expressions as follows:

$B(a)$: *prop*, given a: A.

More generally, $B(x)$ constitutes a proposition when an x that is of the category A, bear the description B. Formally,

$B(x)$: *prop* (x: A)

In fact, this expresses the well-formedness of the predicate $B(x)$, its meaning constitution.[19] It determines the domain upon which the predicate is defined. More explicitly, it clarifies that B is predicated upon the objects which belong to the set A. In general, the CTT formation rules for predicates are in accordance with Plato's observation that how something is cannot be asserted without presupposing what that thing is.[20] Once the well-formedness of a predicate has been established, we can produce formal structures expressing that the predicate is true of some objects. For example, that B is true of a, which is an arbitrary but fixed element of A, can be expressed by:

$B(a)$

[19] In CTT, the well-formation is not only syntactic but also semantic. Consider, for example, the predicate *Hungry*. The well-formedness of this predicate can be expressed by '*Hungry*(x) : *prop* (x : *Animal*)', which reveals not only the correct syntactical use of that predicate but also the semantic domain of the objects of which that predicate can be true.

[20] In their thorough and meticulous discussion of Plato's *Cratylus*, Lorenz and Mittelstrass (1967) highlight the distinction between *naming* (ὀνομάζειν)—as establishing *what* something is—and *stating* (λέγειν)—as establishing *how* something is. They (1967, p. 6) point out that "[t]he subject has to be effectively determined, i.e., it must be a thing correctly named, before one is going to state something about it".

Similarly, that some or all of the elements of *A* are *B* can be expressed, respectively, by the following expressions:

$(\exists x: A)\ B(x)$
$(\forall x: A)\ B(x)$

In all of the latter three expressions the formation rule $B(x)$: *prop* $(x : A)$ is presupposed.[21]

By employing this machinery, the subject-predicated structure of traditional propositions can comprehensively be reflected in the language of CTT. Consider the proposition 'some cobblers are good'. An oversimplified analysis of this proposition within the classical logic with unrestricted quantification would be as follows:

$(\exists x)\ [Cobbler(x) \wedge Good(\mathrm{x})]$

By contrast, in the language of CTT, 'some cobblers are good' could be formalized by this notation:

$(\exists x: Cobbler)\ Good(x)$

The latter translation restricts *Good* to the subject term of the proposition i.e., *Cobbler*. It singles out the set of those cobblers that are good insofar as they are cobblers.

Let us now provide a more systematic view on the elements of CTT to be deployed in the present paper.

5.1.2 Categoricals and Hypotheticals

One of the general philosophical tenets of CTT is linked to the task of avoid *keeping content and form apart* – Martin-Löf's (1984, p.2).[22] More precisely, within Per Martin-Löf's constructive type theory (for short CTT), both, logical and non-logical constants are interpreted through the Curry-Howard correspondence between propositions and sets. A proposition is interpreted as a set whose elements represent the proofs of the proposition. It is also possible to view a set as a problem description in a way similar to Kolmogorov's explanation of the intuitionistic propositional calculus. In particular, a set can be seen as a specification of a programming problem, the elements of the set are then the programs that satisfy the specification – Martin-Löf (1984, p. 7). Furthermore, in CTT sets are understood also as types so that propositions can be seen as data (or proof-)-types.[23]

Additionaly, in the CTT framework it is possible to express at the object-language level

A ***true*** or or $\vdash A$ *or*

[21] In CTT, the judgement that the proposition $B(a)$ is true is usually represented by '$B(a)$ *true*'. But as long as we are considering a proposition itself (without making any judgement that it is true) we do not really need to add '*true*'.
[22] Cf. Sundholm (1997, 2001)
[23] Cf. Nordström/Petersson/Smith (1990), Granström (2011).

which, when asserted by some individual **g**, conveys the information that this individual is in possession of some proof-object *a* for *A*, a proof-object that *makes A* true. Moreover, it can be rendered explicit by means of the **categorical judgement**

a: A,

which reads, *there is a proof-object a of A* – or the individual **g** can bring forward the proof-object *a* in support of his claim that *A* is ***true***.[24]

Summing up, within CTT a proposition is interpreted as a set the elements of which represent the proofs of the proposition, the solution to a problem, the fulfilments of an expectation. Accordingly,

Explicit	**Tacit**
a*: *A	***A true*** or ⊢***A***

can be read as

a is an element of the set *A*	*A* has an element
a is a proof of the proposition *A*	*A* is true, there is a truth-maker for *A*
a is a solution to the problem *A*	*A* has a solution
a fulfils the expectation *A*	*A* is fulfilled

One of the characteristic features of CTT is that it also allows, at the object-language level, expression of **hypothetical judgements** as a form of statement distinguishable from the assertion of the truth of an implicational proposition. Hypothetical judgements give rise to dependency structures in CTT, such as

$B(x)$ *prop* $(x: A)$

This is a formation rule for the hypothetical. More generally, formation rules, a distinctive feature of CTT, by the means of which semantic and syntactic features are processed together, establish well-typing rules.

The proof-object of an hypothetical judgement is a function:

$b(x): B(x) \ \ (x: A)$,

which reads:

$b(x)$ is a (dependent) proof-object of $B(x)$, provided x is a proof-object of the proposition A;
whereby the function b takes elements from the set A, and yields proof-objects for $B(x)$.

In other words, in this frame the dependence of the truth of *B* upon the truth of *A* amounts to the dependence of the proof-object of *B* upon the proof-object of *A*. And the dependence of

[24] See Martin-Löf (1984, pp. 9-10). For a short introductory survey see Rahman, McConaughey, and Klev (2018, chapter II).

the proof-object of B upon the proof-object of A is expressed by means of the function $b(x)$ (from A to B), where x is a proof-object of A and where the function $b(x)$ itself constitutes the dependent proof-object of B.

Thus, if we have $b(x)$: $B(x)$ (x: A) as a premiss, and we have as a second premiss the fact that indeed that there is some evidence a for the proposition A (i.e., if we have as premiss a: A), then we can infer that $b(a)$: $B(a)$.

In plain words, from the premisses
some x is a B, provided it is of the type A;

and
a is indeed of the type A (a: A);

we can infer:
performance a is a B ($b(a)$: $B(a)$).

$$\frac{a: A \qquad b(x): B(x)\ (x: A)}{b(a): B(a)}$$

- Notice that the assertion $b(a)$: $B(a)$, **presupposes** the formation $b(x)$: $B(x)$ (x: A).

In other words, the assertion $b(a)$: $B(a)$ or even the tacit formulation $\vdash B(a)$, presupposes a suitable set A that provides the element a occurring in the function that makes $B(a)$ true. It is the context of the assertion that determines what the suitable set is.

5.1.3 Saturation vs. Enrichment

There are at least two different approaches for dealing with temporal reference in the CTT-framework.[25] More precisely, a proposition which expresses the occurrence of an event (or fact) at some time (or witin a span of time) can be seen in at least two different ways. Such a proposition can be seen either

- as an incomplete propositional function that can be saturated by that specific time moment (or specific span of time), or
- as an event (or a fact) that can be timed by a timing function.

These two formal terminologies are translatable into each other. Nonetheless, there is a significant philosophical difference between these two approaches. In the first approach time is primitive. Temporal entities (i.e., singular moments of times or time spans) are independent entities which can be put as the arguments of propositional functions. So, ontologically speaking, complete propositions in some sense depend on these temporal entities. By contrast, in the second approach, events (or facts, or truth-makers of the propositions which express those events) are primitive individuals which can be put as the arguments of the timing functions.

[25] See Ranta (1994, sec. 5.4).

Thus, whereas according to the first approach time instants are primitive, within the second approach time, is dependent on events, in the sense that is constituted by functions the role of which is of timing the event. Inspired by François Recanati's terminology, we call these two approaches, respectively, 'saturation' and 'enrichment'.[26]

- According to the saturation approach, 'A occurs at some t of the time scale T' can be formalized as a propositional function A that is saturated by t. So:

  $A(t)$: *prop* $(t: T)$

  Which is the formalization adopted by Vuillemin (1984, 1996) and Seel (1982, 2005).

- By contrast, according to the enrichment approach A itself is a fully saturated proposition which is made true by different events (or facts) at different time moments (or spans of time).
  Equivalently, it has different truth-makers or proofs at different times. These truth-makers can be timed by a timing function. Informally speaking, the timing function operates upon the set of truth-makers (or justifications, or proofs) of A and determines the time (span) in which such a truth-maker is obtained.[27] So if x is a truth-maker of the proposition A (i.e., if x is an event or fact whose occurrence makes A true), then the timing function τ would determine the time in which x is obtained. So the role of τ can be defined as follows:[28]

  A : *prop*
  $\tau(x) : T \ \ (x : A)$

For example, that *Cypselus rules over Korinth* at some time t can be expressed by the saturation approach as follows:

Cypselus rules over Korinth(t): *prop* $(t$: *Time*$)$.

Quite differently, the enrichment approach yields:

$$\left\{\begin{array}{l} \textit{Cypselus rules over Korinth} : \textit{prop} \\ \tau(x)\text{: } \textit{Time}) \ \ (x : \textit{Cypselus rules over Korinth}) \end{array}\right.$$ [29]

In this formalization x is a truth-maker or evidence for the proposition *Cypselus rules over Korinth*; and τ is a timing function which determines the time at which x takes place. What makes the proposition *Cypselus rules over Korinth*, are (evidences of) actual rulings of

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[26] This terminology has been borrowed from Recanati (2007b, 2007a).

[27] Recall that as pointed out before, it is assumed that a proposition has different truth-makers (or proofs or justifications). In the present context this amounts to the assumption that a proposition has different truth-makers during different time spans. That a proposition is true in a specific time span is equivalent to that one of its truth-makers is obtained in that time span.

[28] See Ranta (1994, p. 108).

[29] In order to avoid notational complexity we omitted one variable within the timing function. Indeed, strictly speaking, the correct formalization must be $\tau(x, b(x)) = d : span(T)$ $(x : Human,\ b(x) : Running(x))$.

Cypselus over Korinth, it is these actual rulings, that are timed. To put it slightly different, the expression *Cypselus rules over Korinth* stands for a type, each instantiation of it, is the one that can be provided with a time at which it happens. Notice that, though this approach, is closer to Prior's temporal logic than to the first-order approach adopted by Vuillemin and others, it is still different. Both share the view, that say, *It is raining at 12hs* and *It is raining at 14hs* are constituted by the same proposition, namely *that It is raining,* true at 12h and, say, false as 14hs – in contradistinction to the saturation approach that considers them to be different propositions However, though nowadays, due to the work of Patrick Blackburn and others, we can extend Prior's temporal logic by adding names for specific moments of time, called *nominals*, the CTT-approach still distinguishes, at the object language level, the proposition from what makes the proposition true.

Borrowing Aristotelian terminology, we can say that in the enrichment approach time elements are measurements—i.e., timing operations—of (and, consequently, dependent on) events. Since I am not convinced that the saturation approach is compatible with the Artistotelian view on how to link time and events, I will follow the enrichment rather than the saturation approach, despite the fact that it is more cumbersome. Moreover, the enrichment approach is close to the Stoic notion of complete *Lekta* concerning time parameters -Rahman (2020). Accordingly, the first two assertions of principle three of the Seel's (2017) version of the MA, carry the notation

- *It is not the case that Cypselus rules over Korinth now*.

III.1 $\vdash \sim(\exists x$: (*It is the case that Cypselus rules over Korinth*)) $\tau(x)=_{Time}\mathbf{N}$

For short, if "*A*" stands for the proposition *Cypselus rules over Korinth*

III.1 $\vdash \sim(\exists x: A)\ \tau(x)=_{Time}\mathbf{N}$

- *It will be always the case that (in relation to the present*) *Cypselus does not rule over Korinth*.

III.2 $\vdash(\forall t>\mathbf{N})$ [($\exists y$: ~(*It is the case that Cypselus rules over Korinth*)) $\tau'(y)=_{Time}t$].

In its short form

III.2 $\vdash(\forall t>\mathbf{N})\ [(\exists x: \sim A)\ \tau'(y)=_{Time}t]$

Accordingly, a first formulation of the third assertion of principle three, focused in the future yields

III.3 $\vdash \mathbb{M}(\exists x: A)\ [(\exists t>\mathbf{N}: Time)\ \tau(x)=_{Time}t]$

Or if we restrict it to the test case $\boldsymbol{t}$, that eliminates the existential temporal quantifier

III.3 $\vdash \mathbb{M}(\exists x: A)\ \tau(x)=_{Time}\boldsymbol{t}>\mathbf{N}$.

As explained below, one way to express that A is a contingent proposition is to embed it within the scope of a universal quantifier as follows:

III.3 $\vdash (\forall x: A \vee \sim A)\ \mathbb{M}\{(\exists y: A)\ [(\mathbf{i}(y) =_{H} x) \wedge (\tau(y) =_{Time} \boldsymbol{t}_{\mathrm{i}} > \mathbf{N})]\}$.

This certainly requires the further elucidations of the next sections

5.1.4 Revisiting Seel's Example

One important feature of the concrete example proposed by Seel (2017) is that the meaning constitution of the implication is such that the consequent is dependent upon the consequent. Indeed, the point is that each act *of Cyspselus ascending to the throne* (B), is made dependent upon a specific act of *Cypselus ruling over Korinth,*. In other words, for each particular act x (or evidence for it) of *Cypselus Ruling over Korinth*, there is process, a function $b(x)$, that associates this act of ruling with an act of ascending to the throne. For short, the meaning constitution, or more precisely to deploy the language of CTT, *the meaning explanation* of the implication at stake in the example is the following:

Cyspselus ascending to the throne: *prop* (x: *Cypselus ruling over Korinth*)

B: *prop* (x: A)

The truth-condition of the resulting hypothetical judgement is a function. Thus, the truth of *Cyspselus ascending to the throne* is made dependent upon *Cypselu ruling over Konrinth*

B *true* (x: A), unfolds as

$b(x)$: B (x: A)

Now, *Cypselus ruling over Korinth* points out to contingent acts of such a ruling: They might or not happen. In such a context, third excluded cannot be in assumed but must be explicitly added.[30] This is one of our main motivations for deploying a constructive logic.

According, assuming that A might or not happen, and if it is A that is verified, i.e. if it is the left of the disjunction that is verified, then a verification of B can be found, i.e. an act of *Cypeslus ascending to the throne* can be brought forward.

$x: (A \vee \sim A)$ Assumption of Contingency of A

….

$d(x): (\exists y : A)\ (\mathbf{i}(y) =_{H} x) \supset B$ [31]

[30] For a (brief) discussion on future contingents within CTT see Martin-Löf (2014).

[31] **Notational conventions:** "H" is a short cut of hypothesis, that is the assumption $A \vee \neg A$. It indicates that the identity has been defined over the set $\{A \vee \neg A\}$. Thus, "$\mathbf{i}(y) =_{H} x$." stands for the three-folded relation identity relation $\mathbf{I}(A \vee \neg A, \mathbf{i}(y), x)$.

In CTT a canonical proof-object for a disjunction $A \vee B$ is a proof-object of one of the sides *and* the indication which is the side that makes the disjunction true. So from a: A, we infer $\mathbf{i}(a)$: $A \vee B$.. The function $\mathbf{i}(a)$, called a *selector*, indicates that the verified side is the first member of the disjunction. In other words, the selector is the injection $\mathbf{i}(a) = a$: A. Similar applies for obtaining the selector $\mathbf{j}(b)$: $A \vee B$.

However, the dependence must also respect the timing functions. That is the function that associates the antecedent to a specific moment $\boldsymbol{t}_{\mathrm{i}}$ in the future and the one that associates the consequent to some t before $\boldsymbol{t}_{\mathrm{i}}$ but after the present time $\mathbf{N}$.[32]

x: $(A \vee \sim A)$
….
$(\exists y\!: A)\ \{[(\mathbf{i}(y) =_{H} x) \wedge (\tau(y){=}_{Time}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})] \supset (\exists z\!: B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}\!: Time)\ \tau'(z){=}_{Time}\ t{>}\mathbf{N}]\}.$

If we put all together as one proposition, we obtain the following universal quantification:

II.1a $\vdash (\forall x\!: A \vee \sim A) \big\{(\exists y\!: A)\ \{[(\mathbf{i}(y) =_{H} x) \wedge (\tau(y){=}_{Time}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})] \supset (\exists z\!: B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}\!: Time)\ \tau'(z){=}_{Time}\ t{>}\mathbf{N}]\}\big\}.$

That can be glossed as

- Assuming that one of both, A or $\sim A$ happens, then, given some verification y of A, , if it counts as a verification of the first member of the disjunction A (i.e. if y is equal to a verification of the of the disjunction), and if it is associated to some future time $\boldsymbol{t}_{\mathbf{i}}$, then, there is some z such that verifies that B – i.e. Cypelus's ascending to the throne – happens at some time t before **ti** and after the present time

Clearly, if instead of A it is $\sim A$ that happens, that is; if it is the second member of the disjunction that is verified, we obtain the dual:

II.1b $\vdash (\forall x\!: A \vee \sim A) \big\{(\exists u\!: \sim A)\ \{[(\mathbf{j}(u) =_{H} x) \wedge (\tau(u){=}_{Time}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})] \supset (\exists w\!: \sim B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}\!: Time)\ \tau'(w){=}_{Time}\ t{>}\mathbf{N}]\}\big\}.$

We could bring both together as a big conjunction, so that if the A is verified, then B is verified as happening before the time at which A happens, **and** if it is rather $\sim A$ that is verified, then $\sim B$ is verified as happening some time before the time that $\sim A$ is verified. In fact, the meaning explanation of the example leads to such a conjunction. However, for the sake of readability we will split the example in the two assertions, one for the positive and the other for the negative case.

Thus II.1a and II.1b, is our reformulation of Seel's (2017) example. The **PP** as applied to our example yields:

II.2 $\vdash (\forall x\!: A \vee \sim A) \big\{(\exists y\!: A)\ \{[(\mathbf{i}(y) =_{H} x) \wedge (\tau(y){=}_{Time}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})] \supset (\exists z\!: B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}\!: Time)\ \tau'(z){=}_{Time}\ t{>}\mathbf{N}]\}\big\}$

[32] For the sake of notational complexity, I will not display the quantification over the time associated to A, but rather assume some specific instant $\boldsymbol{t}_{\mathrm{i}}$.

---.

$\vdash (\forall x\text{: } A \vee \sim A)\ \mathbb{M}\{(\exists y\text{: } A)\ \{[(\mathbf{i}(y) =_{H} x) \wedge (\tau(y) =_{Time} \boldsymbol{t}_{\mathrm{i}} > \mathbf{N})] \supset \mathbb{M}(\exists z\text{: } B)\ [(\exists t < \boldsymbol{t}_{\mathrm{i}}\text{: } Time)\ \tau'(z) =_{Time} t > \mathbf{N}]\}\}$

Accordingly, the first principle in the MA on the necessity of the past, when applied to our example might be set as follows:

I. $\vdash (\exists w\text{: } \sim B)\ [(\exists t < \boldsymbol{t}_{\mathrm{i}}\text{: } Time)\ \tau'(w) =_{Time} t > \mathbf{N}] \supset \mathbb{L}\ (\exists w\text{: } \sim B)\ [(\exists t < \boldsymbol{t}_{\mathrm{i}}\text{: } Time)\ \tau'(w) =_{Time} t > \mathbf{N}]$.

If we adopt this formalization the argument can follow the same main steps as in our first reconstruction of Seel's fresh start.

We can even come closer to "double indexation" notation of Vuillemin, by placing the whole event in the future: the rationale behind being. This amounts to a relativization of the necessity of the past, where the future is set in relation to some fixed moment:

- If ~*B* happened sometime *t* in the past, then necessarily, for all future **after *t*** it will be true that ~*B* happened at *t*.

In order to avoid further complexity, let us fix some arbitrary time $\boldsymbol{t}_{\mathrm{k}}$ in the past in relation to $\boldsymbol{t}_{\mathrm{i}}$ and in the future in relation to $\mathbf{N}$.

$\vdash (\exists w\text{: } \sim B)\ \tau'(w) =_{Time} \boldsymbol{t}_{\mathrm{k}} \supset \mathbb{L}\ \{\forall z\text{: } (\exists w\text{: } \sim B)\ \tau'(w) =_{Time} \boldsymbol{t}_{\mathrm{k}}\}\ (\forall t' > \mathbf{N}\text{: } Time)\ \tau''(\mathbf{fst}(z) =_{Time} t'$ – provided $(\boldsymbol{t}_{\mathrm{k}} > \mathbf{N} \wedge \boldsymbol{t}_{\mathrm{k}} < \boldsymbol{t}_{\mathrm{i}})$.

- This might take the sting away, form the "unduly" application to the first principle to an event happening in the future. The point of the proposed reconstruction is that it offers a way to render Vuillemin's "double indexation" as the explicit introduction of Reichenbach's three-folded time parameters mentioned above, without assuming events to be predicates of time instants.

Moreover, the scope of the necessity as only affecting the consequent does not invite to endorse the principle if casted in this form. What seems to be necessary is that if some event happened int the past then it will be true for any future that this very event happened. However, if we follow standard modal logic, this will prevent the argument to go through in these terms. As we will discuss in the next sections, here necessity should be understood as the categoric necessity, of the whole implication. This indicates that we need to delve into the meaning explanation of the modalities.

6 Approaches to Aristotelian Modality on Events: Towards a Dialogical Approach

Let us start recalling Seel's (2017) observation that the MA has not the form of syllogism. As suggested above, we take this as indicating that the Aristotelian test too must be understood as a dialectical rule. This can also help to elucidate the notion of possibility involved: If assume that the Opponent asserts that, given *any condition*, if there a proof of *A* timed at ***t***, then there is also a proof of *B* timed as happening before ***t;*** then he is committee to also assert that if given some *specific conditions* there is a proof of *A*, then under the same specific conditions there is also a proof of *B* (both occurring in accordance to their respective timing functions).

Thus, the idea here is to read premiss and conclusion as commitments to assert during a dialectical interchange and as involving modalities understood as stating conditions. This calls for the following rewriting – where we emulate Martin-Löfs (17,a,,b) formulation of dialogical logic.

II.2 **Answerer** $\vdash (\forall x\colon A \vee {\sim}A)\ \mathbb{L}\big\{(\exists y\colon A)\ \{[(\mathbf{i}(y) =_H x) \wedge (\tau(y) =_{Time} \boldsymbol{t}_{\mathrm{i}} > \mathbf{N})] \supset (\exists z\colon B)\ [(\exists t < \boldsymbol{t}_{\mathrm{i}}\colon Time)\ \tau'(z) =_{Time} t > \mathbf{N}]\}\big\}$

Diodorus $\mathbb{M}$?. The **Questioner,** reminds the antagonist that, if **Answerer** endorses that given the choice between $A \vee {\sim}A$, t the conditions are such that if any of them are satisfied then if *A* is the case at ***t***, so is *B* at some time before ***t***; then, according to **PP**, it must also endorse the claim that this hold for some subset of those conditioning both *A* and *B*.

Answerer $\vdash (\forall x\colon A \vee {\sim}A)\ \mathbb{M}\big\{(\exists y\colon A)\ \{[(\mathbf{i}(y) =_H x) \wedge (\tau(y) =_{Time} \boldsymbol{t}_{\mathrm{i}} > \mathbf{N})] \supset \mathbb{M}(\exists z\colon B)\ [(\exists t < \boldsymbol{t}_{\mathrm{i}}\colon Time)\ \tau'(z) =_{Time} t > \mathbf{N}]\}\big\}$

But what are those conditions encoded by the modalities $\mathbb{L}$ and $\mathbb{M}$? Possible worlds in disguise after all?

In the following sections I will explore different alternatives to the meaning explanation of the modalities involved in the MA. I will end up by suggesting if we cast Diodorus reasoning within a dialectical framework, whereas the possibility claim can interpreted as involving a hypothetical judgement whereby *falsity is not known to be true*, necessity requires *truth to be known.*[33]

This take on modality is not only germane to the Aristotle's possibility principle, it also furnishes the meaning explanation underlying the notion possibility at work in that principle.

6.1 Modality as Causality and an Avicennian Alternative on Spans of Time

6.1.1 Causality in degrees

According to chapter 9 of the *Peri Hermeneias* the notion of modality as applied to events, individualized by some time structure, amounts to a predicative relation between cause and

[33] Cf. Martin-Löf (1998), Sundholm (2003 ; p. 241).

effect[34]. However, in the context of natural necessity, Aristotle seems to think the relation from the event to the cause, rather than the other way round: *if there is rain, there is necessarily a cause (clouds),* but rain is not necessary! – for a lucid and thorough study on the subject see Crubellier (2010).

The Stoics, who arguably preserved the time structure, undertook the task of constituting the Cause-Effect link by means of a propositional connective. The Leibnizian project (at least in the contemporary version promulgated by Kripke and Hintikka), substituted the time structure with the abstract and meta-logical structure of possible worlds. However, in this move causes are, in a manner of speaking, absorbed by the conditions defining a possible world. More precisely, in such an approach modality is not attached to causality *per se.*

Rahman/Farid/Young (2020), undertook the task to restore the causality-effect link for tackling Aristotle's notion of modality in the realm of natural events. Informally, the proposal is to formulate necessity as the following hypothetical, where C stands for some set of causal conditions and **E** for the events as effects:

Necessity

.**If, given *C* or *not-C, E* is present when *C* is, then *E* happens as often as *C* does.**

Impossible

If, given *C* or *not-C*, not-*E* is present when *C* is, then, the number of absences of *E* equals the number of presences of *C*.

Near Possibility

.**If, given *C* or *not-C, E* is present when *C* is, then *E* is present more often than ~*E*.**

Far Possibility

.**If, given *C* or *not-C, E* is present when *C* is, then ~*E* is present more often than *E*.**

Even Possibility

If, given *C* or *not-C, E* or not-*E* are present when *C* is, then, when *E* is present, this presence equals the number of absences.

In order to formalize these notions, Rahman, Farid & Young (2020) deploy Sundholm's (1989) constructive generalized quantifiers *More* and *Most*. The point is that Aristotle's notion of degrees of possibility, amounts to a distribution on the cardinality of presences and absences of the effect (the event) in relation to the set of causal conditions

Relevant for the MA are the first three modalities or the first two and the last one.[35] According to this view, whereas the occurrence of $\mathbb{L}$, in the "premiss" can be seen as encoding a set of casual conditions, determining that if Cypselus rules over Korinth at ***t*,** then he ascended to the throne some time before; the occurrence of $\mathbb{M}$ in the follow up assertion, encodes some subset of those causal conditions with a lower degree of determination. So, if the **Answerer** asserts the necessity of the implication he is also forced to assert the distribution of the possibility.

[34] Bénatouïl (2017) reminds us that already Brochard (1892, 1912) and Hamelin (1901) discussed the notion of modality as involving a predicative cause-effect relation.

[35] It would be interesting to study if the different degrees of possibility have bearings on the development of the MA. Presumably not so long as the possibilities of antecdent and consequent have the same degree.

I shall not develop here the formalization further. The idea is substitute $\mathbb{L}$, and $\mathbb{M}$ with *C* or not *C*, and *E*, by all the implication, and then apply the generalized quantifier *More* in the way indicated in Rahman, Farid & Young (2020).

- The main general idea behind is that if given the question if *A* is or not true, and no information is available to decline for one of the sides of the disjunction, we might "hypothesize" that if any of some specific causal conditions obtain then if *A* is the case then so *B*, but if absent then the dual holds. Thus, modality is here understood as the result of hypothetizing that causal conditions (in different degrees of determination) obtain given the actual lack of information in relation to some question. From the dialogical point of view the **Questioner** will examine if **Answerer**'s endorsement of, say, a necessity claim on grounds of some specific causal conditions, is compatible to a possibility claim involving some (but not all) of the causal conditions of the former.

6.1.2 Avicenna and Spans of Time:

The framework also supports an alternative formalization of the implication under study, and follows by adding modality to Avicenna's *descriptional propositions*. If we pursue this alternative we obtain the following

Necessity

If, given *A* or *not-A*, *B* is present when *A* is, then for every span of time N< *t'*< *t*, *B* happens at some *t'*<*t* whenever *A* happens in *t*.

Possibility

If, given *A* or *not-A*, *B* is present when *A* is, then for at least one span of time N< *t'*< *t*, *B* happens at some *t'*<*t* whenever *A* happens in *t*.

This alternative, based on Rahman & Zarepour's (2020) reconstruction of Avicena's *descriptional propositions*, has the advantage that it is based on *intervals of time* rather than on instants. In fact, one might explore different other formulations, involving section of a span of

Moreover, it admits further variations such as ***most of the time***, ***hardly during that time,*** and so on.

This is still very brief and is meant to motivate a further development. However, we will pursue now a different path, that takes seriously the fact that the *Master Argument* constitutes a new form of *bout*, namely a hypothetical one.

6.2 Hypothetical Reasoning, Assumptions and the Possibility Rule. Towards a Dialogical Approach to the MA.

6.2.1 The Possibility Rule.

After having established the Possibility Principle Aristotle, states the following paragraph that Rosen and Malink (2012, pp. 185-187) take as the formulation of the Possibility (Inference) Rule:

Now that this has been shown, it is clear that if something false but not impossible is hypothesized, what follows because of the hypothesis will be false but not impossible. For example, if A is false but not impossible, and if when A is B is, then B will also be false but not impossible. Pr. An, I, 15, 34^{a} 25-9

Rosen and Malink (2012, p. 186), based on Fine (2011), cast **PR** in the following way:

- Given the premiss that *A* is possible, and given a deduction of *B* from *A*, you may infer that *B* is possible.[36]

The authors point out:

In order to supply the required deduction of B from A, it is not necessary first to establish that A is actually the case. It suffices to introduce A as an assumption, or hypothesis, which serves as the starting-point of the deduction, and on the basis of which we derive the consequence B. We will call such a deduction of B from A a subordinate deduction, and will refer to B as the conclusion of the subordinate deduction. A better clue to the difference between the possibility principle and what Aristotle says in … [in relation to the Possibiliy Rule, S.Rahman] *is his use of the verb 'hypothesize' (ὑποτίθεσθαι) in [vii]. This verb did not occur in Aristotle's formulation of the possibility principle. It is typically used in the description of proof procedures. For example, in proofs by reductio, 'hypothesize' is used to describe the step in which we assume the contradictory of the statement we want to establish.* Rosen and Malink (2012, p. 186).

Their example, is the following, whereby the *subordinate* inference is indented and prefixed by a vertical line

$\mathbb{M}\ A\mathrm{e}B$	That *A* belongs to no *B* is possible, is the premiss of an instance of **PR**
$\vert\ A\mathrm{e}B$	That *A* belongs to no *B*, is assumed
$\vert\ B\mathrm{e}A$	That *B* belongs to no *A*, follows by conversion from the former
$\mathbb{M}\ B\mathrm{e}A$	That *B* belongs to no *A* is possible, follows from the premiss and the subordinate inference

So far so good, and adapting it to Seel's (2017) example for MA is straightforward

$$\mathbb{M}_{N}A(\boldsymbol{t})$$

$$\left|\begin{array}{l} A(\boldsymbol{t}) \\ \ldots \\ (\exists t'<\boldsymbol{t})B(t') \\ \end{array}\right.$$

$$\mathbb{M}_{N}(\exists t'<\boldsymbol{t})B(t')$$

Or in the notation with timing functions

$$(\forall x\colon A \vee {\sim}A)\ \mathbb{M}\big\{(\exists y\colon A)\ \{[(\mathbf{i}(y) =_{H} x) \wedge (\tau(y)=_{Time}\boldsymbol{t}_{\mathrm{i}}>\mathbf{N})]\big\}$$
$$\ldots$$

[36] Cf. Pfenning, F. & Davies, R. (2001, section 5).

$(\exists y: A)\ \{[(\mathbf{i}(y) =_{H} x) \wedge (\tau(y)=_{Time}\boldsymbol{t}_{\mathrm{i}}>\mathbf{N})]$
…
$(\exists z: B)\ [(\exists t< \boldsymbol{t}_{\mathrm{i}}: Time)\ \tau'(z)=_{Time}\ t>\mathbf{N}]\}\}$

…

$\mathbb{M}(\exists z: B)\ [(\exists t< \boldsymbol{t}_{\mathrm{i}}: Time)\ \tau'(z)=_{Time}\ t>\mathbf{N}]\}\}$

However, there is not a great of progress either on the notion of possibility itself or on the difference to the hypothesize move in the first step of the subordinate inference. Furthermore, the setting does not make is explicit at the object language, that the conclusion $\mathbb{M}_{\mathbf{N}}(\exists t'<\boldsymbol{t})B(t')$ obtained is dependent upon the subordinate inference that is dependent upon the possibility of the antecedent $\mathbb{M}_{\mathbf{N}}A(\boldsymbol{t})$. Let tackle both of these issues at the same time.

6.2.2 Contexts and the Possible: Crubellier meets CTT.

6.2.2.1 Ranta's take on Modalities

Inspired by some lectures of Per Martin-Löf, Aarne Ranta's (1991, 1994) proposes a CTT-approach to modality quite germane to the inferential setting for Aristotle's notion of necessity and possibility as occurring in the texts studied by Vuillemin (1979, 1984, 1996) Fine (2011) and Rosen and Malink (2012); and Crubellier (2010).

To put it bluntly, Ranta's basic idea is that given a ***yes*** or ***no*** question in relation to the say the proposition *A*, one might posit that *A* (~*A*) is true *within* a *context* **H**. A context is a set of dependent assumptions A_{i}.:

$$x_1: A_1,\ \ldots,\ x_{\mathrm{n}}: A_{\mathrm{n}}(x_{1,\ \ldots,}\ x_{\mathrm{n}-1})$$

The context can be read as a list of *approximations by a progressive specification* to answers to a question on the holding of *A* – cf. Ranta (1994, pp. 86-88). In other words, the answer if *A* holds or not is made dependent upon answers to some list of questions (not yet answered) – see Dango (2016). For instance, the answer to the question if *Bucephalus wins the race,* might be set as dependent upon responses to a list of questions such as if *Bucephalus a fast horse?, Does Bucephalus compete with young horses*?... [37]

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[37] This approach naturally leads to belief contexts – see Ranta (1994, pp. 153-157). Consider an agent **a** stating judgements one after another, later ones possibly making reference to earlier ones. Let us assign to **a** a belief context, i.e. the beliefs or hypotheses he has adopted at some occasion.

$$\mathbf{H}^{\mathbf{a}} = x_1: A_1,\ \ldots,\ x_{\mathrm{n}}: A_{\mathrm{n}}(x_{1,\ \ldots,}\ x_{\mathrm{n}-1}).$$

Given a proposition $A(x_{1,\ \ldots,}\ x_{\mathrm{n},})$ we obtain

a believes $A(x_{1,\ \ldots,}\ x_{\mathrm{n},})$ is *true* or $A(x_{1,\ \ldots,}\ x_{\mathrm{n},})$ is *in* the set of beliefs of **a.**

i.e.,

$$a(x_{1,\ \ldots,}\ x_{\mathrm{n},}): A(x_{1,\ \ldots,}\ x_{\mathrm{n},})\ (\mathbf{H}^{\mathbf{a}});$$

which is a hypothetical judgement. The agent **a**, might move to another context $\mathbf{H}^{\mathbf{a}}\mathbf{i}$ where he modified some of his beliefs. In fact, **H**, might encode not only one progressive list but several ones, constituting different hypotheses. Each of the lists can be interpreted as a list of *epistemic alternatives* to some initial list of beliefs associated with the claim – cf. Ranta (1994, pp. 147-150). Assume for example that *A* is made dependent upon, among other assumptions, of the truth of a disjunction, then, there might be a list of assumptions that extends the first list assuming that the first member of the disjunction holds, and a second one where it does not. The point is that each list of assumptions constituting a belief claim is not taken to be exhaustive and leaves room for further explorations: that is why possible truth is hypothetical rather than categorical. Notice that whereas the specification

According to this view asserting that *A* is possible amount to positing that there is some set of dependent assumptions – including alternative sets of dependent assumptions – that provide a hypothetical explanation, with some increasing degree of specification, of why *A*.

Crubellier's (2010) point is that we should think of modality as a kind of what he calls *retrodiction*, if there is rain, there is necessarily a cause (condensation of clouds, a decrease in the atmospheric pression during autumn when the condensation takes place, …).

According to this view, a modally qualified judgement on *A:*

- *presupposes* a set **H** of assumptions upon which *A* is dependent– i.e. $A(x)$: prop (x: **H**)**,** such that

  (a) the assertion that *A is possible* (in relation to the background **H**) is justified, if the verification of $A(x)$ is grounded on **just one (and at least one)** verification of the assumptions in context **H**. Clearly, some of the verifications of the assumption in **H** might not ground the assertion $A(x)$.

  (b) the assertion that $A(x)$ *is necessary* (in relation to the background **H**) is justified, if the verification of $A(x)$ is grounded on **every** verification of the assumptions in context **H**.

A Kripke-Hintikka style minded reader, might associate this setting with the truth conditions in standard modal logic, whereby the truth of the possibility of *A* (at some world) just requires *A* to be true in at least on accessible world, and the truth of the necessity of *A* (at

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within each $\mathbf{H_i}$ is unidimensional, in the sense that it indicates *one way* the specification of the precedent assumption might take, the specification displayed by the alternative is multidimensional, i.e., it indicates *different ways* a specification can be conceived, or to use Ranta's (1994, pp. 145-147) own words different ways *to extend a context*. So, if $\mathbf{H^a_k}$ stands for some initial context, or shared context of beliefs, but at some stage two alternative extensions of this initial context develop in such a way that some new proposition *B* dependent upon the hypotheses in $\mathbf{H^a_k}$, is part of the hypotheses in $\mathbf{H^a_i}$ but not $\mathbf{H^a_j}$, where the new proposition *C* is present (but not in $\mathbf{H^a_j}$):

$$\mathbf{H^a_i} = \mathbf{H^a_k},\ y_1: B_1(x_{1,\ \dots,}\, x_n),\ \dots\ ,\ y_m: B_1(x_{1,\ \dots,}\, x_{n,}\, y_{1,\ \dots,}\, x_{m-1})$$
$$\mathbf{H^a_j} = \mathbf{H^a_k},\ z_1: C_1(x_{1,\ \dots,}\, x_n),\ \dots\ ,\ z_k: C_k(x_{1,\ \dots,}\, x_{n,}\, z_{1,\ \dots,}\, z_{k-1})$$

Ranta (1994, p. 141-42) suggests that this can also be interpreted as different alternatives to some initial shared context $\mathbf{H_k}$ brought forward by two agents during a dialogical interaction. The interlocutors are not required to have proofs in relation to the shared context, when making assertions. But they are committed to what they assert. If for example, in $\mathbf{H^b_i}$ agent **b** asserts that there is a lion in the garden, the interlocutor **c** might ask if it is male or female, despite that there is no such an assertion in her own set of beliefs $\mathbf{H^c_j}$.
Thus, in general **H** encodes the following structure where each $\mathbf{H_i}$ stands for a list of dependent assumptions that *extends* the ground list $\mathbf{H_k}$, and each arrow is a mapping to this ground context. In other words, each arrow is a set of functions from $\mathbf{H_i}$ to $\mathbf{H_k}$

$$\begin{array}{ccccc} & & \mathbf{H_k} & & \\ & \nearrow & \uparrow & \nwarrow & \\ \mathbf{H_i} & & \mathbf{H_j} & \dots & \mathbf{H_n} \end{array}$$

Let us assume that *A* is *true* in the context $\mathbf{H_k}$, i.e. $a(x)$: $A(x)$ ($\mathbf{H_k}$). If $\mathbf{H_i}$ extends $\mathbf{H_k}$, then we obtain judgments such as $a(f(y)/x)$: $A(f(y)$ (y: $\mathbf{H_i}$).

some world) requires A to be true in all of those accessible worlds. Indeed, Ranta (1991, 1994) is lead, so far it goes, by this interpretation whereby contexts play the role of possible worlds.[38]

Now, Crubellier's (2010) point is that necessity and possibility affect in principle the explanatory conditions – i.e. the set of assumptions. That the presence of clouds conditions the presence of rain, sets the presence of clouds as **the presupposition** for justifying assertions on rain. But this is presupposed by both necessity and possibility assertions. The distinction between possibility and necessity is about the quantification inside the context encoding those conditions, does every verification of the assumptions in a contexts furnish a justification of an assertion in that context or is just one verification of the assumptions enough? This matches with the approach of Ranta (1994, pp. 150-51) who, inspired by some lectures of Per Martin-Löf in 1990-91 on constructive mathematics, distinguishes possibility from necessity by introducing existential quantification (more generally Σ-type) in the set of assumption constituting a context; and universal quantification (Π-type) for the case of necessity:

- **Necessity**:

$$(\mathbf{LH})\ A(x_1, \ldots x_n) = (\forall\mathbf{H})\ A\ (x_1, \ldots x_n): prop$$

If we spell out the quantification within **H** we obtain

$$[(\forall x_1: A_1) \ldots (\ \forall x_n: A(x_1, \ldots x_{n-1})]\ A(x_1, \ldots x_n) : prop$$

Whereby "$[(\forall x_1: A_1) \ldots (\ \forall x_n: A(x_1, \ldots x_{n-1})]$" displays the constitution of $\forall\mathbf{H}$

The categorical judgement $(\forall\mathbf{H})\ A(x_1, \ldots x_n)$ *true*, is equivalent to the hypothetical judgement

$A(x_1, \ldots x_n)$ *true* **H**

Or more explicitly:

$$a(x_1, \ldots x_n): A(x_1, \ldots x_n)\ \ [(x_1: A_1) \ldots (\ x_n: A(x_1, \ldots x_{n-1})]$$

If we deploy the so-called *vector notation* $\mathbf{x}$ for the sequences of variables $x_1, \ldots x_n$ – Ranta (1994, p. 155)

we obtain the hypothetical

$a(\mathbf{x}): A(\mathbf{x})\ (\mathbf{x} : \mathbf{H})$

- **Possibility**:

$$(\mathbf{MH})\ A(x_1, \ldots x_n) = (\exists\mathbf{H})\ A\ (x_1, \ldots x_n): prop$$

[38] Pfenning, F. & Davies, R. (2001) develop an alternative type-theoretical version of modal logic, where contexts are also conceived as the proof-theoretical counterparts of worlds. However, possibility and necessity are defined in relation to *staks* of alternative contexts. Nevertheless it seems to be close to defining modality in relation to extensions of contexts, as Ranta's does.

If we spell out the quantification within **H** we obtain

$$[(\exists x_1: A_1) \ldots (\exists x_n: A(x_1, \ldots x_{n-1})]\ A(x_1, \ldots x_n) : prop$$

Whereby "$[(\exists x_1: A_1) \ldots (\exists x_n: A(x_1, \ldots x_{n-1})]$" displays the constitution of $\exists$**H**

The correspondent hypothetical judgement is

$A(\mathbf{fst}(z), \mathbf{snd}(z), \ldots \mathbf{n\text{-}th}(z))\ true\ \ [z : (\exists x_1: A_1) \ldots (\exists x_n: A(x_1, \ldots x_{n-1})]$[39]

Or again using the vector notation

$a(\mathbf{z}): A(\mathbf{z})\ (\mathbf{z} : \exists \mathbf{H})$

whereby **z** stands for the list of projections $\mathbf{fst}(z), \mathbf{snd}(z), \ldots \mathbf{n\text{-}th}(z)$

Hence, and coming back to **PR**, whereas the assumption of the subordinate inference is just an assumption of even some set of them, it does not have the structure of a **context.** This also gives an answer to what a subordinate inference is. Indeed, according to this view, whereas the conclusion of a subordinate inference is dependent upon an assumption (or set of them), the conclusion of the main inference is made dependent upon function composition.

Actually, as mentioned above, the meaning explanations of the modalities involved in the MA can be developed in a simpler way if we understand

- the possibility of *A* being *true* as, *A false* not known
- the necessity of *A* being *true* as, *A true* known[40],

and that the first is casted by a hypothetical and the second by a categorical.[41] Hence, in this setting, what the "contradiction" occurring in Diadorus take on the MA amounts to, is the incompability of asserting (categorically) that ~*B* is true (known to be true) and asserting that *B* is true given the hypothesis of the context **H**.

Thus, in relation to the first principle, I will take that the necessity involved is the one of a categorial judgment

I $(\exists w: \sim B)\ \tau'(w) =_{Time} \boldsymbol{t}_k \supset \{\forall z: (\exists w: \sim B)\ \tau'(w) =_{Time} \boldsymbol{t}_k\}\ (\forall t' > \mathbf{N}: Time)\ \tau''(\mathbf{fst}(z) =_{Time} t'$ – provided $(\boldsymbol{t}_k > \mathbf{N} \wedge \boldsymbol{t}_k < \boldsymbol{t}_i)$.

[39] In CTT elimination rules for an existential $c : (\exists x: A)\ B(x)$, introduce two operators, the projections $\mathbf{fst}(b) = a: A$ and $\mathbf{snd}(b(a)) = B(x)$, given that c is the pair $c = (a, b(a)): (\exists x: A)\ B(x)$. In other words, those projections select the first and the second element of the pair encoded by c.

[40] Cf. Sundholm (2003, p. 241).

[41] Pfenning and Davies (2001), also understand possibility-assertions as hypotheticals.

This approach follows a different path to the reading of the first principle by Prior (1950) and Denyer (1981, 1999, 2009) and also, if my formalization in section 4 is faithful, the one by Seel (2017).

If instead, we insert Ranta's necessity operator into the consequent and follow the reading on the necessity of the past,

$\vdash (\exists w{:}\ {\sim}B)\ [(\exists t< \boldsymbol{t}_{\mathrm{i}}{:}\ Time)\ \tau'(w)=_{Time}\ t>\mathbf{N}]\ \supset\ (\forall\mathbf{H}\ )\ (\exists w{:}\ {\sim}B)\ [(\exists t< \boldsymbol{t}_{\mathrm{i}}{:}\ Time)\ \tau'(w)=_{Time}\ t>\mathbf{N}]$

we obtain, *mutatis mutandis* Seel's (2017) reconstruction.

As already mentioned, the narrow take on the scope of the necessity in the first principle, as mentioned above, seems not be obvious at all.

However,, if we follow the idea that necessity is what is expressed by categoric assertions, then the MA becomes trivial since, according to the setting, it is known that ~*A* is true for some future time ***ti*** , but then the hypothetical judgement expressing the possibility of *A* being true at ***ti*** is in incorrect judgement.

Hence, we are left with the following choices

1) Declare the MA trivial
2) Deploy the narrow scope of the first principle after all, using Ranta's ∀**H** and ∃**H** operators.
3) Follow the idea that the MA is about testing the admissibility of **PR** given the conditions set by the MA. Thus, in a way similar to relevant approaches, it might be required that all of the assertions and principles are put into work.

Options 2 and 3 seems to be the interesting one. Thus, the testing should involve the use of all of the relevant assertions and principles. Option two, the closes to Seel's (2017) is the most logical one, since it fulfils this requirement in a straightforward manner. I will mainly follow the third option but as I shall show below the argumentation pattern is essentially the same. In fact,there is still a fourth option based on Pfenning and Davies (2013) idea that whereas possibility is linked to a hypothetical dependent upon a fixed context **H**, the context associated with necessity is a universal one. I will not purse in the present paper this fourth option.

Let us now revisit the **PR** in the context of CTT

6.2.2.2 Hypotheticals and the Possible

Let us reformulate Rosen and Malink's (2012) example quoted above:

c(**z**): *A*e*B* (**z**: **H**) That *A* belongs to no *B* is possible

| *y*: *A*e*B* That *A* belongs to no *B*, is assumed
| *b*(*y*): *B*e*A* That *B* belongs is not *A*, follows from the function *b*(*y*) transforming the putative proof *y* of *A*e*B* into a proof for its conversion

$b(c(\mathbf{z})/y){:}BeA$ That B belongs to no A is possible, follows from the possibility premiss and the subordinate inference. In other words the conclusion of the main inference, is, dependent upon its dependence on the assumption, that is in its turn dependent upon **H**. Hence the possibility of BeA, amounts to its dependence on the same **H,** on the grounds of which AeB is said to be possible.

For short subordination is the fact that the assumption leading to a conclusion is itself dependent upon **H.** Hence the conclusion is, is also dependent upon **H,** by function composition. This allows expressing subordination without need of graphical help

$c(\mathbf{z})$: AeB ($\mathbf{z}$: **H**)
$b(y)$: BeA (y: AeB)

$b(c(\mathbf{z})/y){:}BeA$ ($\mathbf{z}$: **H**)

If we apply this to Seel's (2017) test-example for MA we obtain

$a(\mathbf{x})$: $(\exists y{:}\ A)\ (\tau(y){=_{Time}}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})$ ($\mathbf{x}$: **H**) provided $\tau(y)$: $Time$ (y: A)
$v(u)$: $(\exists z{:}\ B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}{:}\ Time)\ \tau'(z){=_{Time}}\ t{>}\mathbf{N}]$ (u: $(\exists y{:}\ A)\ (\tau(y){=_{Time}}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})$)

$v(a(\mathbf{x})/u)$: $(\exists z{:}\ B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}{:}\ Time)\ \tau'(z){=_{Time}}\ t{>}\mathbf{N}]$ ($\mathbf{x}$: **H**)

Alternatively, if we stick to the formalization closer to Seel's (2017) notation on temporality we obtain:

$a(\mathbf{x}){:}\ A(\boldsymbol{t})$ ($\mathbf{x}$: **H**) provided $A(t){:}prop$ (t: $Time$)
$v(u)$: $(\exists t'{<}\boldsymbol{t})B(t')$ (u: $A(\boldsymbol{t})$)

$v(a(\mathbf{x})/u)$: $(\exists t'{<}\boldsymbol{t})B(t')$ (u: $A(\boldsymbol{t})$) ($\mathbf{x}$: **H**)

If we choose to use Ranta's operators we obtain:

$a(\mathbf{x})$: $(\exists \mathbf{H})\ (\exists y{:}\ A)\ (\tau(y){=_{Time}}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})$ provided $\tau(y)$: $Time$ (y: A)
$v(u)$: $(\exists z{:}\ B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}{:}\ Time)\ \tau'(z){=_{Time}}\ t{>}\mathbf{N}]$ (u: $(\exists y{:}\ A)\ (\tau(y){=_{Time}}\boldsymbol{t}_{\mathrm{i}}{>}\mathbf{N})$)

$v(a(\mathbf{x})/u)$: $(\exists \mathbf{H})\ (\exists z{:}\ B)\ [(\exists t{<}\ \boldsymbol{t}_{\mathrm{i}}{:}\ Time)\ \tau'(z){=_{Time}}\ t{>}\mathbf{N}]$

Let us now tackle another issue related to the use of **PR** and that will take us to the dialectical reconstruction.

6.3 Aristotelian Bouts and Immanent Reasoning

If we

6.3.1 Adding Assertions with a Subordinate Inference.

Rosen and Malink (2012, p. 189) observe that in the practice Aristotle introduces into the subordinate inference additional assumption without any restriction, and this might produce invalid inferences. Following a terminology usual in Fitch-style presentation of natural deduction they call it the *iteration problem.* The point is that Aristotle uses iterations without any restriction. Herewith the example taken from Rosen and Malink (2012, p. 189):

*A*i*B*	That *A* belongs to some *B,* is assumed.
M *A*e*C*	That *A* belongs to no *C* is possible, is the premiss of an instance of **PR**
| *A*e*C*	That *A* belongs to no *C,* is assumed
| *A*i*B*	That *A* belongs to some *B,* is iterated in the subordinate inference.
| *C*o*B*	By *Festino*, given the two precedent assumptions
M *C*o*B*	That *C* does not belong to some *B* is possible.

The problem is that if we take this as a pattern of inference we could also produce invalid inferences such as

*A*i*B*	
M *A*e*B*	
| *A*e*B*	Assumption of the subordinate inference
| *A*i*B*	iteration
| *B*o*B*	Also by *Festino,*
M *C*o*B*	That *C* does not belong to some *B* is possible.

Rosen and Malink (2012, p. 189) reject, and rightly so, Fine's (2011) strategy of restricting iterations to *necessary assertions.* However, after a thorough and fascinating study of some of the applications, including those that inspired Vuillemin's work on the MA, they concede that there is no evidence in the Aristotelian sources what a general restriction might be.

Now, if we read Seel's (2017) rejection of the standard reconstructions of the MA, as a plaidoyer for a dialectical reconstruction, the case of iteration can be handled in a pragmatic way: **Questioner**, might ask how to ground, to use our example, *B*o*B*, on *A*e*B*, and **Answerer** might then introduce the further assumption *A*i*B*.[42] Moreover, this view is germane to the contentual rather than syntactic feature of Aristotle's logic.

Indeed, if we cast the example of invalid iteration in an Aristotelian Bouts as worked out by Marion (2020), where **Questioner**, tests the consistency of the assumptions of **Answerer**, Questioner will force **Answerer** to concede that the additional iterated assumption *A*i*B* is contradictory to the assumption *A*e*B*. and the game will be over. Now, interesting is that Aristotle's main if not sole use of the ***PR*** is in *indirect proofs*. This might require a general examination of the rules for developing Bouts in a way suitable for the examination of the applications undertook by Aristotle.

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[42] Rahman/Iqbal/Soufi (2019) considered a similar dynamics while studying debates within the framework of medieval Islamic Jurisprudence.

6.3.2 The MA in a Dialectical Framework

In the present paper, I will restrict to the study of the case of the MA. I will slightly adapt Marion's (2020) reconstruction to the framework of *Immanent Reasoning* – see Rahman, McConaughey, Klev and Clerbout (2018), where the expressive power of CTT is combined with the dialogical conception of logic of Paul Lorenzen and Kuno Lorenz (1978). This should set the basis for future work on hypothetical argumentation within the framework of *Immanent Reasoning*.

- The general point being is that a dialectical study of the MA provides an instance of a general pattern for reasoning aimed at examining the rationale behind an argumentation rule against the background of the commitments undertaken by endorsing a specific set of assertions. According to this dialectical take on the MA, the primary interest is not so much of a *consistency maintenance* test,[43] but rather on seeking for accepting a rule that sets the meaning explanation of Aristotle's notion possibility against the background of some endorsed assertions. In other words according to this view; the point of the MA is to examine within a dialectical setting the admissibility of PR in view of some concrete (material) assertions endorsed by the **Answerer**.

Moreover, the endorsed assertion can be seen as proposals brought forward by the **Questioner**, in order to examine the meaning explanation of the possibility at work.

One of the core notions in Marion's (2020) reconstruction of Aristotelian dialectic is the so-called *Socratic Rule*, that in its contemporary formulation has its roots in Lorenzen and Lorenz (1978) *formal rule*, that restricts the moves (usually involving stating elementary propositions) of one of the players. Usually, in the original setting of Lorenzen and Lorenz (1978), the *formal rul*e only restricts the moves of the **Proponent** (**Answerer**). In the Aristotelian dialectical games it is the **Opponent** (**Questioner**). Let us formulate the following the restriction in a general form:

- The player restricted by the *Socratic Rule*, is not allowed to posit any statetement not already granted by the antagonist.

This allows to, what we might *call immanent refutation*, that is a refutation of player grounded only on his own statements – see Rahman.McConaughey, Klev and Clerbout (2018, pp. 280-281, though there it is simply called *refutation*)

- **X** ! *A* is immanently refuted if player **Y** can bring up a sequence of moves such that her moves (**Y**'s moves) are restricted by the Socratic Rule, these moves lead to **Y** winning the dialogue.

Aristotelian Bouts are those plays where **Questioner** aims at an immanent refutation of **Answerer**. Thus, within the framework of Aristotelian Bouts, **Questioner**'s moves are those that are restricted by the Socratic Rule.

[43] The expression *consistency maintenance* is taken from Duthil-Novaes (2005), where it is applied to the Medieval counterpart of dialectic, the *Obligationes*. In fact, at least in the context of the dialectical setting of the MA, the plays are closer to abuctive procedures for rules, called *Structure-Seeking Dialogues* – see Rahman and Keiff (2005, section 4.10) and Keiff (2007).

Let me now summarize Marion's rules (2020) for *Aristotle's Dialectical Bouts*.

R1. Bouts always involve two players, that are named *Questioner* and *Answerer*, in virtue of the role they assume at the beginning of the bout.

R2. A play begins with *Questioner* eliciting from *Answerer* his commitment to an assertion or thesis *A*.

R3. The play then proceeds through a series of alternate questions and answers. *Questioner* asks questions such that *Answerer* may give a 'short answer', ideally 'yes' or 'no.

R4. Proceeding thus, *Questioner* elicits further commitments from *Answerer* commitment to further assertions *B*1, *B*2, …, *Bn*. The set {*A*, *B*1, *B*2, …, *Bn*} is called *Answerer*'s 'scoreboard'or 'commitment store'.34

R5. *Questioner* cannot introduce in *Answerer's* scoreboard, or use, any premiss

R6. *Questioner* can then 'take together' or 'add up' (συλλογίζεσθαι) premisses from *Answerer*'s scoreboard to infer an impossibility (ἀδύνατον).39

R7. If *Questioner* infers an impossibility, *Questioner* wins, otherwise *Answerer* wins. (Winning rule)

In fact, for the sake of simplicity, we will assume that R3, has already been implemented and this led **Answerer** to endorse:

A) The test example:

$$v(u)\text{: } (\exists z\text{: } B)\ [(\exists t< \boldsymbol{t}_{\mathrm{i}}\text{: } \mathit{Time})\ \tau'(z)=_{\mathit{Time}}\ t>\mathbf{N}]\ \ (u\text{: } (\exists y\text{: } A)\ (\tau(y)=_{\mathit{Time}}\boldsymbol{t}_{\mathrm{i}}>\mathbf{N}))$$

and its dual

$$v'(u')\text{: } (\exists z'\text{: } {\sim}B)\ [(\exists t< \boldsymbol{t}_{\mathrm{i}}\text{: } \mathit{Time})\ \tau'(z')=_{\mathit{Time}}\ t'>\mathbf{N}]\ \ (u'\text{: } (\exists y'\text{: } {\sim}A)\ (\tau(y)=_{\mathit{Time}}\boldsymbol{t}_{\mathrm{i}}>\mathbf{N}))$$

Actually, for the sake of simplicity; we will drop the poof-object (in a concrete play we call it "local reason" and retain the assertion sign, that in a dialectical setting is expressed by an exclamation mark (see Rahman et al. (2018, section 6.3.2). I will also simplify further the complexity of the expression by fixing the moment at which *B* happens by some arbitrary $\boldsymbol{t}_{\mathbf{k}}$, as discussed in the precedent sections.

Answerer ! $(\exists z\text{: } B)\ \tau'(z)=_{\mathit{Time}}\ \boldsymbol{t}_{\mathrm{k}}\ \ (u\text{: } (\exists y\text{: } A)\ (\tau(y)=_{\mathit{Time}}\boldsymbol{t}_{\mathrm{i}}>\mathbf{N}))$ – provided $(\boldsymbol{t}_{\mathrm{k}}>\mathbf{N} \wedge \boldsymbol{t}_{\mathrm{k}}<\boldsymbol{t}_{\mathrm{i}})$

and its dual

Answerer ! $(\exists z'\text{: } {\sim}B)\ \ \tau'(z')=_{\mathit{Time}}\ \boldsymbol{t}_{\mathrm{k}} >\mathbf{N}\ \ (u'\text{: } (\exists y'\text{: } {\sim}A)\ (\tau(y)=_{\mathit{Time}}\boldsymbol{t}_{\mathrm{i}}>\mathbf{N}))$

B) The application of the Principle of the irrevocability of the past to the test example.

Answerer ! $(\exists w\text{: } {\sim}B)\ \tau'(w)=_{\mathit{Time}}\ \boldsymbol{t}_{\mathrm{k}}\ \supset\ \{\forall z\text{: } (\exists w\text{: } {\sim}B)\ \tau'(w)=_{\mathit{Time}}\ \boldsymbol{t}_{\mathrm{k}}\}\ (\forall t'>\boldsymbol{t}_{\mathrm{k}}\text{: } \mathit{Time})$
$\tau''(\mathbf{fst}(z)=_{\mathit{Time}}\ t'$ – provided $(\boldsymbol{t}_{\mathrm{k}}>\mathbf{N} \wedge \boldsymbol{t}_{\mathrm{k}}<\boldsymbol{t}_{\mathrm{i}})$.[44]

[44] Herewith I apply the test example to the *relativized* version of the **PNP**.

C) The assertions that *A* is possible, even it is neither true now nor it will ever be.

Answerer ! $(\exists x: \sim A)\ \tau(x) =_{\text{Time}} \mathbf{N}$

Answerer ! $(\forall t > \mathbf{N}: \text{Time})\ [(\exists x: \sim A)\ \tau'(x) =_{\text{Time}} t]$

Answerer ! $(\exists y: A)\ (\tau(y) =_{\text{Time}} \mathbf{t}_1 > \mathbf{N})\ (\mathbf{x} : \mathbf{H})$

In relation to **PR**, I will formulate it as rule that provides the dialectical meaning of the principle, setting what we call the *local-meaning* of the that principle. That is, the way to challenge and to defend an assertion by some player. In the case of **PR** (as applied to our example), it involves a player asserting both, that

(i) *B* is the case at some time before, under the assumption that *A* takes place at $\boldsymbol{t}_i$

(ii) It is Possible that *A* at some future time $\boldsymbol{t}_i$. In other word; *A* is the case at some future time $\boldsymbol{t}_i$, given the context **H**

It proceeds by the antagonist asking the interlocutor to live to his commitment of asserting that *B* is the case at some time before $\boldsymbol{t}_i$ given the same context **H**

It ends by the initial player fulfilling the required commitment.

Possibility Principle

Assertion	Challenge	Defence
Possibility Principle $\mathbf{X}$! $(\exists z: B)\ \tau'(z) =_{Time} \boldsymbol{t}_k\ (u: (\exists y: A)\ (\tau(y) =_{Time} \boldsymbol{t}_i > \mathbf{N}))$ … $\mathbf{X}$! $(\exists y: A)\ (\tau(y) =_{\text{Time}} \mathbf{t}_i > \mathbf{N})\ (v(\mathbf{x}) : \mathbf{H})$ Provided $(\boldsymbol{t}_k > \mathbf{N} \wedge \boldsymbol{t}_k < \boldsymbol{t}_i)$	**Y PR**?	$\mathbf{X}$! $(\exists z: B)\ \tau'(z) =_{Time} \boldsymbol{t}_k\ (v(\mathbf{x}) : \mathbf{H})$

We need also a rule of how to challenge and defend judgements conditioned by open assumptions and the usual dialogical rules for quantifiers, implication and negation. All of them described I Rahman et al (2017, chapters 4-6). In other words:

- The rules for judgements conditioned by open assumptions are similar to those for a universal quantifier: stating such judgement means that the challenger may choose any instance a_i of the assumption and request of the utterer to make his statement by instantiating every free occurrence of x with a_i. That is, the challenger chooses which proposition he wants the utterer to state.
- For the particular case that the judgement is a hypothetical one based on a **Context,** i.e. a list of dependent assumptions. It is the defender how chooses the instance for the context conditioning the hypothetical. Recall that under this interpretation a hypothetical judgment conditioned by a **Context**, stands for possibility – and as usual

in standard dialogical logic for modality, the player who states the possibility of a proposition is one who chooses the dialogical context where the proposition will be submitted to dialogical scrutiny – See Rahman/Keiff (2005).

- Stating a universally quantified proposition means that the challenger may choose means that the challenger may choose any instance a_i of the domain of quantification and request of the utterer to make his statement by instantiating every free occurrence of x with a_i. That is, the challenger chooses which proposition he wants the utterer to state.
- As expected, the rules for existential quantification are the dual of the universal: it is the defender who chooses the proposition he wants to state in response to the challenge.
- An implication is a statement that is challenged by stating the antecedent. The challenge is answered by stating the consequent. So to challenge an implication stated by the other player **X** ! $A \supset B$, the challenger must state the antecedent **Y** ! A; the defender must then state the consequent **X** ! B
- From a dialogical point of view, a negation is basically taken to be an implication of the form $A \supset \bot$, so that if the defender of **X** ! ~A cannot counter the antagonist's challenge **Y** ! A the defender is committed to giving up but stating **X** ! $\bot$. Marion (2020, section 2) integrates this notion of negation into the Dialectical Bouts.

Let us express all this with two further tables (in a simplified form).

Assertions conditioned by open Assumptions and by Contexts

Assertion	Challenge	Defence
***Open* assumption** **X** ! $B[x]$ (x: A) Whereby x may occur in B	**Y** a_i: A The challenger choses some instance a_i: of A For the sake of avoiding complexity we we might drop the instance and formulate the challenge as **Y** ! A Whereby the challenger commits to the claim that he can bring forward and instance if required	**X** ! $B[a_i]$ The defender states the consequent
Context H **X** ! $B[z(\mathbf{x})]$ ($z(\mathbf{x})$: **H**) Whereby $z(\mathbf{x})$ may occur in B	**Y** ? **H** The challenger requests **X** to choose an instance of context **H**	**X** ! $B[\boldsymbol{a}_i/\mathbf{x}_)]$ The defender states the consequent In its explicit form **X** $b[\boldsymbol{a}_i]$: $B[\boldsymbol{a}_i/\mathbf{x}_)]$

Still, the setting of the MA assumes ***permanence*** of past truth. If B is true at some time $\boldsymbol{t}_{\mathbf{k}}$ before, say $\boldsymbol{t}_{\mathrm{i}}$,, then at $\boldsymbol{t}_{\mathrm{i}}$, is true that B happened at $\boldsymbol{t}_{\mathbf{k}}$. In order to simply the notation I will deploy Ranta's (1994, p. 103) adverbial at @*t*, that adds a temporal dimension to a proposition. However ! B @ $\boldsymbol{t}_{1>}$ $\boldsymbol{t}_{\mathrm{k}}$, stands for the assertion B happened at $\boldsymbol{t}_{\mathbf{k}}$ is true at $\boldsymbol{t}_{\mathbf{1}}$.

In fact, the rule for ***permanence*** can be obtained by applying necessitation of the past to that assertion that B happened at $\boldsymbol{t}_{\mathbf{k}}$ but this rule shortens the development of the *Bout* for MA.

Permanence

Assertion	Challenge	Defence
Permanence **X** ! c: B …. **X** $\tau(c)=_{Time}$ $\boldsymbol{t}_{\mathrm{k}<}$ $\boldsymbol{t}_1$	**Y** @ $\boldsymbol{t}_{1>}$ $\boldsymbol{t}_{\mathrm{k}}$? The challenger asks the defender to establish the permanence of B in relation to $\boldsymbol{t}_{1>}$ $\boldsymbol{t}_{\mathrm{k}}$	**X** d: B @ $\boldsymbol{t}_{1>}$ $\boldsymbol{t}_{\mathrm{k}}$

Now; for the standard connectives

Implication, Negation and Quantifiers

Assertion	Challenge	Defence
X ! $A \supset B$	**Y** ! A	**X** ! B
X ! $\sim A$	**Y** ! A	**X** ! $\perp$ The defender gives up
X ! $(\forall x: A)\, B[x]$ Whereby x may occur in B	**Y** a_{i}: A The challenger choses some instance a_{i}: of A	**X** ! $B[a_{\mathrm{i}}]$ The defender states the consequent
X ! $(\exists x: A)\, B[x]$ Whereby x may occur in B	**Y fst**?: The challenger requests **X** to chose some instance a_{i}: of A. **Y snd**? The challenger requests **X** to state $B[x]$ according to his response to the first challenge	**X** a_{i}: A The defender choses some instance a_{i}: of A **X** ! $B[a_{\mathrm{i}}/x]$ The defender substitutes x with $a_{\mathrm{i}/}$ for any x that might occur in B

We have now all the ingredients to develop the Dialectical Bout for the MA

The Master Argument as a Dialectical Bout

Answerer			Diodorus		
0	**Answerer's Score Board** **A) Test Example** **A.1** ! $(\exists z: B)\ \tau'(z)=_{Time}\ \boldsymbol{t}_k\ \ (u: (\exists y: A)\ (\tau(y)=_{Time}\boldsymbol{t}_i>\mathbf{N}))$ **A.2** ! $(\exists z': \sim B)\ \tau'(z')=_{Time}\ \boldsymbol{t}_k>\mathbf{N}\ \ (u': (\exists y': \sim A)\ (\tau(y)=_{Time}\boldsymbol{t}_i>\mathbf{N}))$ Provided $(\boldsymbol{t}_k>\mathbf{N} \wedge \boldsymbol{t}_k<\boldsymbol{t}_i)$ **B) The Irrevocability of the Past** $(\exists w: \sim B)\ \tau'(w)=_{Time}\ \boldsymbol{t}_k \supset \{\forall z: (\exists w: \sim B)\ \tau'(w)=_{Time}\ \boldsymbol{t}_k\}\ (\forall t'>\boldsymbol{t}_k: Time)\ \tau''(\mathbf{fst}(z)=_{Time}\ t'$ – provided $(\boldsymbol{t}_k>\mathbf{N} \wedge \boldsymbol{t}_k<\boldsymbol{t}_i)$ **C) Assertions** **C.1** ! $(\exists x: \sim A)\ \tau(x)=_{\mathrm{Time}}\mathbf{N}$ **C.2** ! $(\forall t>\mathbf{N}: Time)\ [(\exists x: \sim A)\ \tau'(x)=_{\mathrm{Time}}t]$ **C.3** $a(\mathbf{x})$: $(\exists y: A)\ (\tau(y)=_{\mathrm{Time}}\boldsymbol{t}_i>\mathbf{N})\ \ (\mathbf{x} : \mathbf{H})$				
2	! $(\exists x: \sim A)\ \tau'(x)=_{\mathrm{Time}}\ \boldsymbol{t}_i$		0.**C.2**	$\boldsymbol{t}_i>\mathbf{N}$: *Time*	1
4	! $(\exists z': \sim B)\ \tau'(z')=_{Time}\ \boldsymbol{t}_k>\mathbf{N}$		0.**A.2**	! $(\exists x: \sim A)\ \tau'(x)=_{\mathrm{Time}}\ \boldsymbol{t}_i$	3
6	$\{\forall z: (\exists w: \sim B)\ \tau'(w)=_{Time}\ \boldsymbol{t}_k\}\ (\forall t'>\boldsymbol{t}_k: Time)\ \tau''(\mathbf{fst}(z)=_{Time}\ t'$		0.**B**	! $(\exists z': \sim B)\ \tau'(z')=_{Time}\ \boldsymbol{t}_k>\mathbf{N}$	5
8	! $(\forall t'>\boldsymbol{t}_k: Time)\ \tau''(\mathbf{fst}(b)=_{Time}\ t$		6	b: $(\exists w: \sim B)\ \tau'(w)=_{Time}\ \boldsymbol{t}_k$[45]	7
10	! $(\exists z: B)\ \tau'(z)=_{Time}\ \boldsymbol{t}_k\ \ (v(\mathbf{x}) : \mathbf{H})$		0.**A.1** , 0.**C.3**	? **PR**	9
12	$\boldsymbol{c}$: $(\exists z: B)\ \tau'(z)=_{Time}\ \boldsymbol{t}_k$:		10	? **H**	11
14	$c_1: B$		12	¸? **fst** ?	13
16	$\tau'(c_1)=_{Time}\ \boldsymbol{t}_k$		12	¸ **snd** ?	15
18	$c_1: B\ @\ \boldsymbol{t}_{i>}\ \boldsymbol{t}_k$			$@\ \boldsymbol{t}_{i>}\ \boldsymbol{t}_k$?	17
20	! $\tau''(\mathbf{fst}(b)=_{Time}\ \boldsymbol{t}_i$ Given the presupposition		8	$\boldsymbol{t}_i>\mathbf{N}$: *Time*	19

[45] Here, as usual in Aristotelian contexts it is assumed that the domain of quantification is not empty -see McConaughey's dialectical implementation in Crubellier et al. (2019).

	fst(b): ~B [46] (Notice that since **fst**(b) verifies that ~B happened at $\boldsymbol{t}_{\mathbf{k}}$, **Diodorus** forces **Answerer** to elicit the statement that the timing function τ'' *associates this event with* $\boldsymbol{t}_{\mathrm{i}}$; In other words, **Answerer** is forced to concede that the timing function establishes that at $\boldsymbol{t}_{\mathrm{i}} > \boldsymbol{t}_{\mathbf{k}}$ it is the case that ~B happened at $\boldsymbol{t}_{\mathrm{k}}$)				
24	⊥ **Answerer** gives up		20 (presupposition)	c_1: B @ $\boldsymbol{t}_{\mathrm{i}} > \boldsymbol{t}_{\mathbf{k}}$ **Diodorus** challenges the negation presupposed in move 20, Recalling that **Answerer** himself endorsed at move 18 the contradictory of that presupposition.	23

The table follows the usual notation of Dialogical Logic:

- the outmost columns indicate the number of move: odd number for the moves of **Questioner (Diodorus)** and even number for the moves of **Answerer**
- a challenge engages a new line in the table. Answers are stated in the same line the challenge has been launched.
- The inner column at the side of Diodorus indicates which move of Questioner is being challenged. The inner column in **Answerer**'s Score Board is emptly since in these kind of plays **Answerer** does not challenge.

The assertions in move 0 in the Score Board of Answerer can be thought as the result of successive applications of the SR3

Moves 1 and 2 are the result of Diodorus challenging the universal quantifier on time moments in order to commit Answerer to fix the result of the timing at t_i>N: Time,already present in A.1

Moves 3 to 4 Diodorus forces Answerer to the state that ~B happened at time $\boldsymbol{t}_{\mathbf{k}}$**.** before t_i

Moves 5 to 8 Diodorus uses Answerer's stating that ~B happened at time $\boldsymbol{t}_{\mathbf{k}}$**.** before $\boldsymbol{t}_{\mathbf{i}}$**.** in order to launch an application of the irrevocability of the past

Moves 9 to 18 Didorus forces to apply the **Possibility Principle** and then requests Answerer, to choose some instance that verify the hypothetical that B happened at $\boldsymbol{t}_{\mathbf{k}}$ and that this is true of for any future and in particular for $\boldsymbol{t}_{\mathbf{I}}$. In fact, the Bout could end there since, the hypothetical that B can happen is incorrect against the background that **Answerer** categorically asserted that ~B is that case at at $\boldsymbol{t}_{\mathbf{k}}$**.**.

The following moves simply spell at that If for any time in the future it is true that ~B happened at time t_k before t_i it cannot be that at precisely the future moment t_i B happened at $\boldsymbol{t}_{\mathbf{k}}$**.** Move 23 forces Answerer to give up

Though the conceptual setting at work in the precedent Bout for MA has rich historical and philosophical features on the general aims of such kind of debates – in the next section we will come to them, the logical structure underlying the MA it is quite disappointing. Indeed according to the existing logical reconstruction it does not prove at all or if it does, it requires either endorsing strong restrictions, as the one requiring all the principles and assertions to be

[46] There is a way to build up the preceeding rules so that the presupposition constitutes a separate move. Mainly using Zoe McConaughey's device for avoiding in non empty domains in Dialectical Bouts already mentioined, but I do not wish here to add more rules to the present framework – see Crubellier et al. (2019).

used even if two of them are patently contradictory, or an implausible “narrow” reading of the irrevocability of the past..

For the record, I set below the initial moves at the score board for an Aristotelian Bout for the MA based on a “narrow scope” reading of the principle of the irrevocability of the past. I shall leave the development for the diligent reader, keeping in mind that the dialectical meaning explanation of Ranta’s modal operators, are simply given by the rules for the universal and the existential quantifiers.

The Rule on the challenge on judgments conditioned by Contexts is of no use in this setting, since possibility is rendered by the operator ∃**H**. However, the Possibility Principle requires some (straightforward) re-writing:

Assertion	Challenge	Defence
Possibility Principle **X** ! $(\exists z\colon B)\ \tau'(z) =_{Time} \boldsymbol{t}_k\ \ (u\colon (\exists y\colon A)\ (\tau(y) =_{Time} \boldsymbol{t}_i > \mathbf{N}))$ … **X** ! $(\exists \mathbf{H})\ (\exists y\colon A)\ (\tau(y) =_{Time} \mathbf{t}_i > \mathbf{N})$ Provided $(\boldsymbol{t}_k > \mathbf{N} \wedge \boldsymbol{t}_k < \boldsymbol{t}_i)$	**Y PR**?	**X** ! $(\exists \mathbf{H})\ (\exists y\colon A)\ (\tau(y) =_{Time} \mathbf{t}_i > \mathbf{N})$ $(\exists z\colon B)\ \tau'(z) =_{Time} \boldsymbol{t}_k$

Answerer			Diodorus		
0	**Answerer’s Score Board** **A) Test Example** **A.1** ! $(\exists z\colon B)\ \tau'(z) =_{Time} \boldsymbol{t}_k\ \ (u\colon (\exists y\colon A)\ (\tau(y) =_{Time} \boldsymbol{t}_i > \mathbf{N}))$ **A.2** ! $(\exists z'\colon \sim B)\ \tau'(z') =_{Time} \boldsymbol{t}_k > \mathbf{N}\ \ (u'\colon (\exists y'\colon \sim A)\ (\tau(y) =_{Time} \boldsymbol{t}_i > \mathbf{N}))$ Provided $(\boldsymbol{t}_k > \mathbf{N} \wedge \boldsymbol{t}_k < \boldsymbol{t}_i)$ **B) The Irrevocability of the Past** $(\exists w\colon \sim B)\ \tau'(w) =_{Time} \boldsymbol{t}_k \supset (\forall \mathbf{H})\{\forall z\colon (\exists w\colon \sim B)\ \tau'(w) =_{Time} \boldsymbol{t}_k\}\ (\forall t' > \boldsymbol{t}_k\colon Time)\ \tau''(\mathbf{fst}(z) =_{Time} t'$ – provided $(\boldsymbol{t}_k > \mathbf{N} \wedge \boldsymbol{t}_k < \boldsymbol{t}_i)$ **C) Assertions** **C.1** ! $(\exists x\colon \sim A)\ \tau(x) =_{Time} \mathbf{N}$ **C.2** ! $(\forall t > \mathbf{N}\colon Time)\ [(\exists x\colon \sim A)\ \tau'(x) =_{Time} t]$ **C.3** ! $(\exists \mathbf{H})\ (\exists y\colon A)\ (\tau(y) =_{Time} \mathbf{t}_i > \mathbf{N})$				

				....	

7 Conclusions and the Work Ahead
An Invitation to the Study of *Dialectical Bouts of Material Admissibility*

The main idea animating the present paper is that the general aim of debates, such as the one involving the notorious case of the Master Argument, is the ponderation of logical principles by confronting them with some set of assertions and other endorsed principles on the meaning explanation of connectives, quantifiers and modality.

As suggested by Seel (2017), the point of the specific case of the MA is about examining Aristotle's notion of possibility – as implemented by the Possibility Principle (i.e., *If it is necessary that if A then B, then B is possible if A is*) and the Possibility Rule (i.e., *If A is possible, and if B follows from the assumption of A, then B is is possible*) in the context of a dialectical debate where it has been asserted that a given proposition is possible, even it is neither true now nor it will ever be, and where at the same time the irrevocability of the past has been endorsed. This suggests to study the MA debate in a setting based on Marion's (2020) reconstruction of *Aristotle's Dialectical Bouts*, adapted to propositions affected by modalities.

The setting of Dialectical Bouts for the MA proposed includes

- Propositions affected by Reichenbach's (1948, § 5) three-folded temporal reference, namely: the *point of speech S*, the *point of reference R* and the *point of the event E*. This constitutes our take on Vuillemin's (1977, 1979, 1984, 1996) suggestion of a "double indexation".
- The dialogical meaning explanation for modalities understood as involving hypotheticals.
- The dialogical rules that settle the commitments undertaken when endorsing Aristotle's Possibility Principle.

In fact, in relation to the notion of possibility at work in Vuillemin's (1996) reconstruction we explored different formulations of the Possibility Rule. Still, there remains a dearth of researches confronting the different formulations of the Possibility Rule with the texts studied by Crubellier (2010) and Rosen and Malink (2012).

More generally and coming back to my remarks above, one of the most fruitful insights that can be gained from Vuillemin's writings on the MA, is that such kind of debates provide an instance of a general pattern for reasoning that I shall call *Dialectical Bouts of Material Admissibility* (BMA).

The rationale behind such kind of argumentation-pattern is to examine the incorporation of new (epistemological, ontological or logical) principles against the background of the commitments undertaken by endorsing a specific set of assertions and meaning explanations. They display the kind of abductive procedure stressed by Crubellier's (2014) study of syllogism.

Under this perspective Ebbinghaus's (1964) casting of the inferential moves that reduce imperfect syllogisms to perfect ones as implementing an *admissibility* procedure, represents a logical generalization of the material procedures at stake in the Dialectical Bouts of Material Admissibility.

Furthermore if, as stressed by Duthil-Novaes (2005) *Medieval Obligations* represent a development of Aristotelian Bouts aimed at the ludic exercise of *consistency maintenance*, DMA should be rather linked to the Islamic tradition of dialectical theory [*jadal*, *munāẓara*, *ādāb al-baḥth*], aimed at the epistemological task of what Young (2017) calls the *forging* of principles, including legal and deontic ones.

In a more contemporary setting, DMA can be related to Brandom's (2000, pp. 66-77) considerations on *Non-Conservative Extensions*, where the point of dialectical procedures is put under public *Socratic scrutiny* the discovery and elucidation of concepts.

This general point on *Dialectical Bouts of Material Admissibility* can be seen as underlying Vuillemin's project of linking the study of the MA with the classification of philosophical systems.

The present paper does not contain a fully developed study on such Dialectical Bouts, but it is rather is an invitation to pick up the gauntlet that Jules Vuillemin threw into the ring.

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