Topological analysis of a Coupled Weighted Panic Control Reflex (CWPCR) model in the context of terrorist attack

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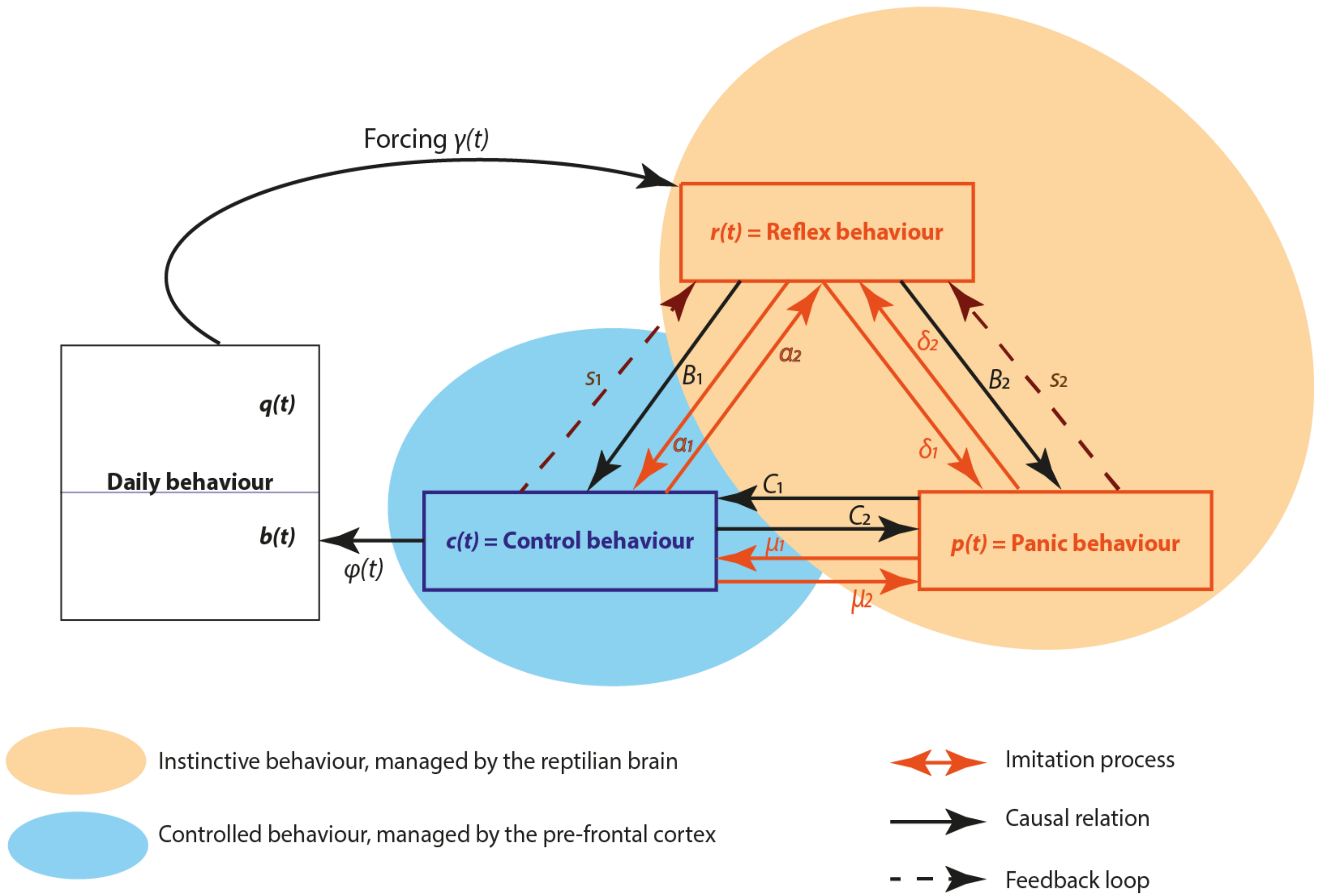
The Panic Control Reflex model



@ From : D. Provitolo, E. Dubos-Paillard, N. Verdière, V. Lanza, R. Charrier, C. Bertelle et M.A. Aziz-Alaoui, 2015,
« Les comportements humains en situation de catastrophe : de l'observation à la modélisation conceptuelle et mathématique », Cybergeo

Equations of the model

The variables $r(t), c(t), p(t), q(t), b(t)$ denote respectively the densities of people being in a reflex, control, panic, daily (quotidian) or back to daily behaviour

$$
\begin{cases}
\dot{r}(t) = \gamma(t)q(t)\left(1-\dfrac{r(t)}{r_m}\right) - (B_1 + B_2)r(t) + s_1(t)c(t) + s_2(t)p(t) \\
\qquad + F(r(t),c(t))r(t)c(t) + G(r(t),p(t))r(t)p(t) \\
\dot{c}(t) = -\varphi(t)c(t)(1-b(t)) + B_1 r(t) + C_1 p(t) - C_2 c(t) - s_1(t)c(t) \\
\qquad - F(r(t),c(t))r(t)c(t) + H(c(t),p(t))c(t)p(t) \\
\dot{p}(t) = B_2 r(t) - C_1 p(t) + C_2 c(t) - s_2(t)p(t) \\
\qquad - G(r(t),p(t))r(t)p(t) - H(c(t),p(t))c(t)p(t) \\
\dot{q}(t) = -\gamma(t)q(t)\left(1-\dfrac{r(t)}{r_m}\right) \\
\dot{b}(t) = \varphi(t)c(t)(1-b(t))
\end{cases}
$$

Parameters and imitation functions

The imitation functions *F*, *G* and *H* are real valued functions defined on $\mathbb{R} \times \mathbb{R}$

$$F(r(t),c(t)) = -\alpha_1 f_1\left(\frac{r(t)}{c(t)+\varepsilon}\right) + \alpha_2 f_2\left(\frac{c(t)}{r(t)+\varepsilon}\right)$$

$$G(r(t),p(t)) = -\delta_1 g_1\left(\frac{r(t)}{p(t)+\varepsilon}\right) + \delta_2 g_2\left(\frac{p(t)}{r(t)+\varepsilon}\right)$$

$$H(c(t),p(t)) = \mu_1 h_1\left(\frac{c(t)}{p(t)+\varepsilon}\right) - \mu_2 h_2\left(\frac{p(t)}{c(t)+\varepsilon}\right)$$

The initial values verify $r(t_0)+c(t_0)+p(t_0)+q(t_0)+b(t_0)=1$

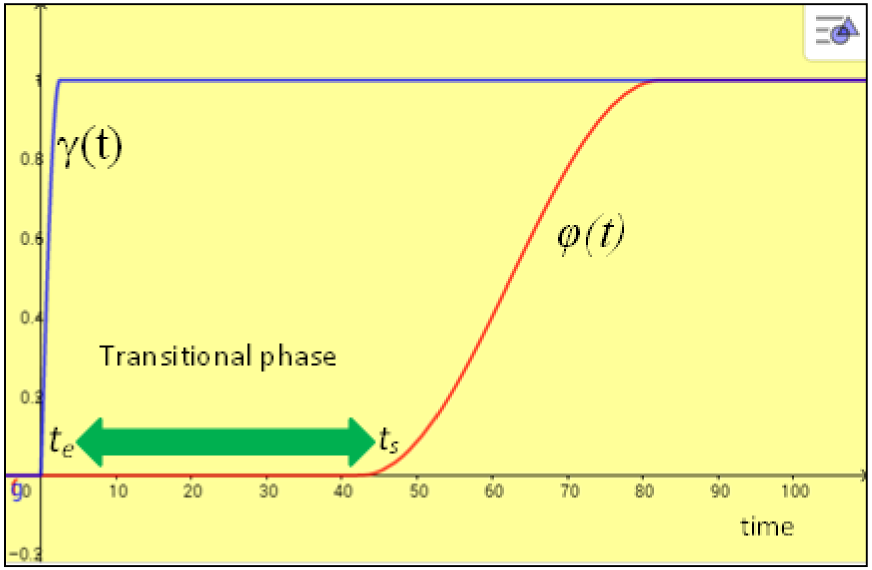
Because the model does not take into account the mortality rate
The parameter are real positive coefficients

$$B_i \geq 0,\ C_i \geq 0,\ \alpha_i \geq 0,\ \delta_i \geq 0,\ \mu_i \geq 0, \quad i=1,2$$

and $s_i(t) \geq 0, \quad i=1,2$ are the domino effect functions

Transitional dynamics

Both functions $\gamma(t)$ and $\varphi(t)$ model respectively the beginning of the attack (or the disaster) and the return to a quiescent daily behaviour

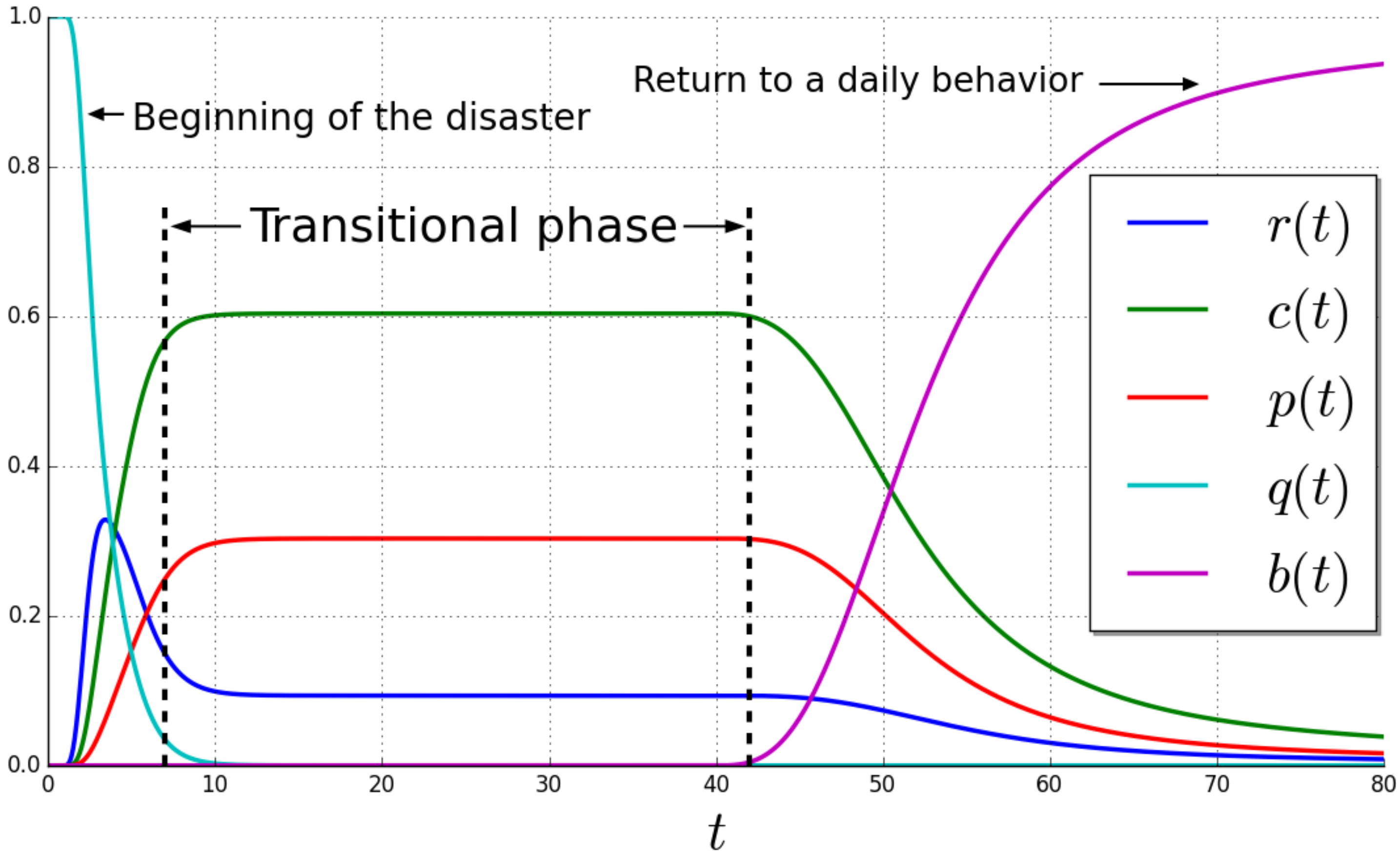


Therefore one can suppose that a terror attack that we will consider is shaped by two characteristic times: t_s(for start) and t_e (for end) for which

$$\begin{cases} \gamma(t) = 1 \,, \forall t \geq t_s \\ \varphi(t) = 0 \,, \forall t < t_e \end{cases} \qquad t \in I_{trans} = [t_s, t_e]$$

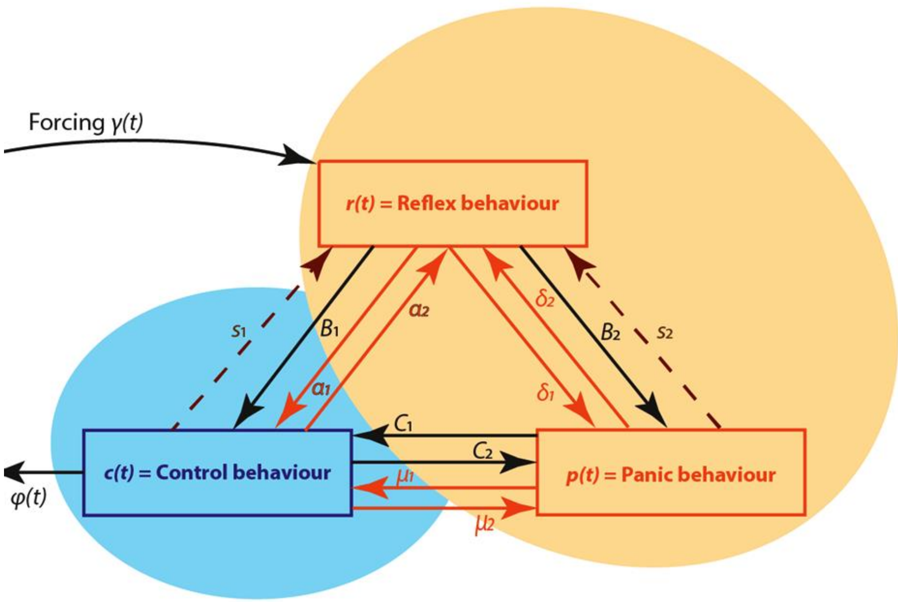
Transitional phase

During this transitional phase the densities $r(t), c(t), p(t),$ converge towards fixed points

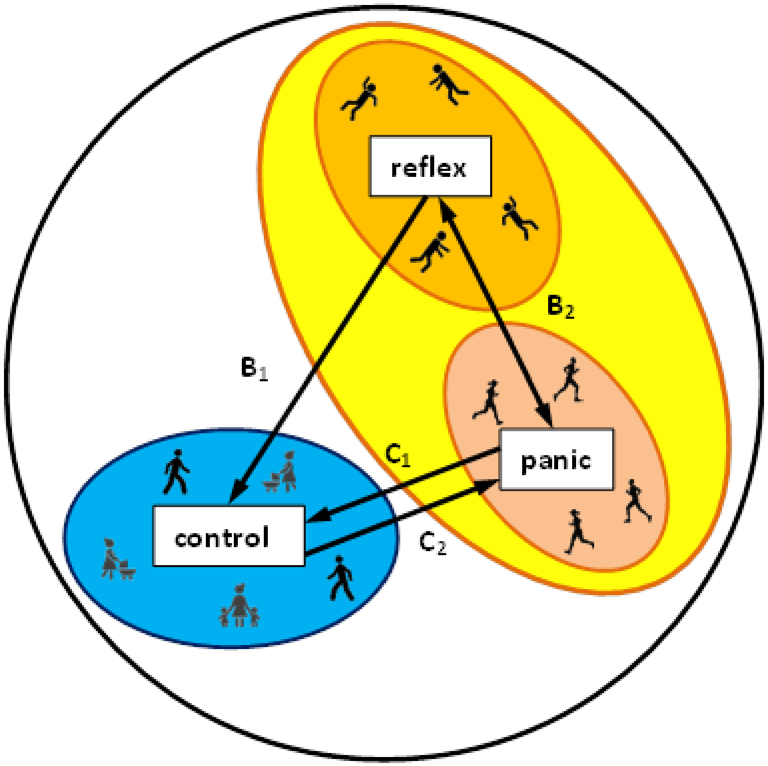


Core of PCR model

We focus now on the core of the PCR which models the transitional dynamics in the aftermath of the terror attack and before the return to the “normal life”



graphic PCR model



schematic representation

Equations of the core model

Only three variables are considered : $r(t), c(t), p(t),$

$$\begin{cases} \dot{r}(t) = -(B_1 + B_2)r(t) + s_1(t)c(t) + s_2(t)p(t) \\ \qquad + F(r(t), c(t))r(t)c(t) + G(r(t), p(t))r(t)p(t) \\ \dot{c}(t) = B_1 r(t) + C_1 p(t) - C_2 c(t) - s_1(t)c(t) \\ \qquad - F(r(t), c(t))r(t)c(t) + H(c(t), p(t))c(t)p(t) \\ \dot{p}(t) = B_2 r(t) - C_1 p(t) + C_2 c(t) - s_2(t)p(t) \\ \qquad - G(r(t), p(t))r(t)p(t) - H(c(t), p(t))c(t)p(t) \end{cases}$$

We suppose that there is no imitation mechanism $F \equiv H \equiv G \equiv 0$
and no domino effect (only one attack) $s_i(t) = 0, i = 1, 2$

The PCR core model is then reduced to

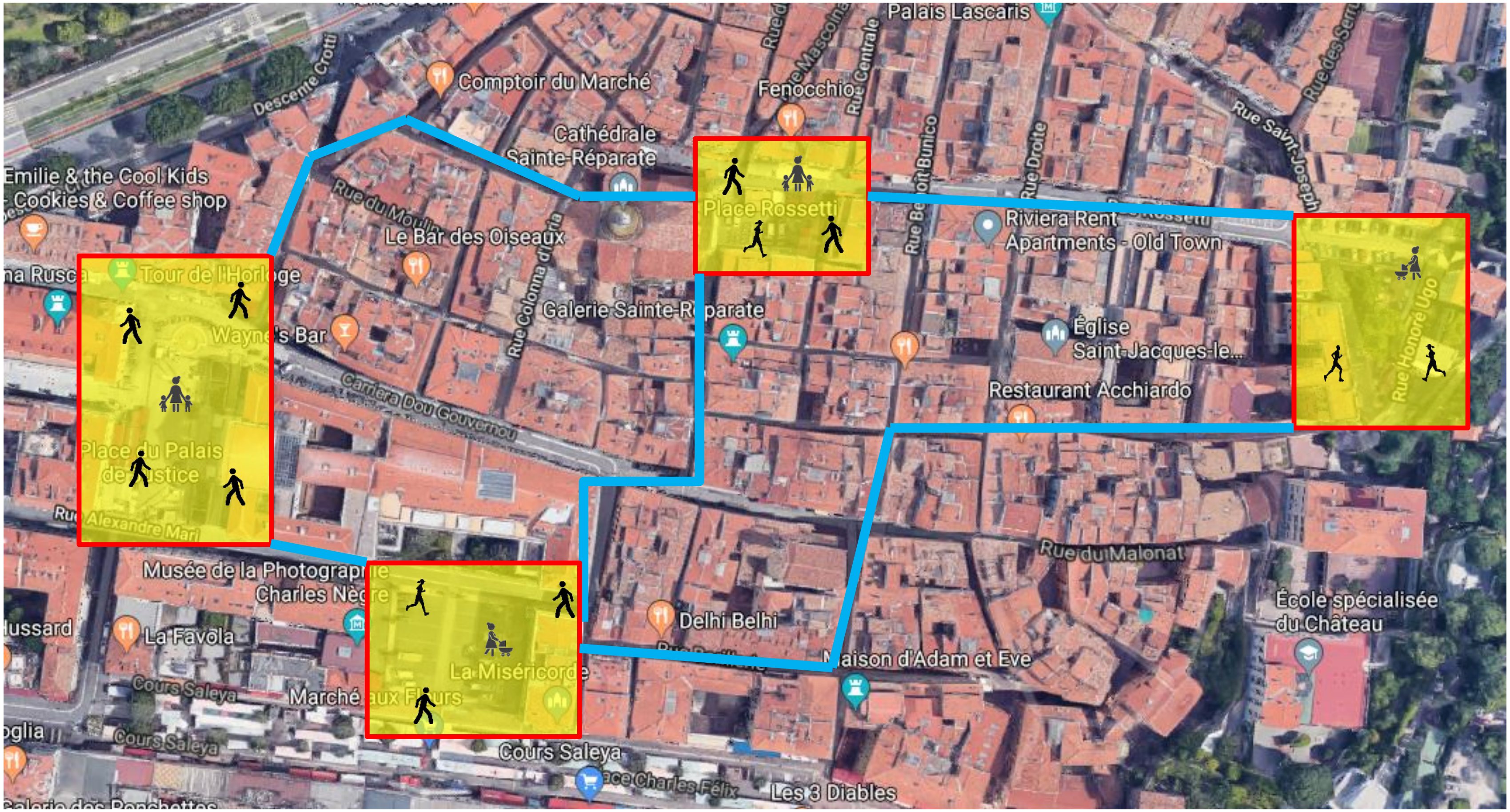
$$\begin{cases} \dot{r}(t) = -(B_1 + B_2)r(t) \\ \dot{c}(t) = B_1 r(t) + C_1 p(t) - C_2 c(t) \\ \dot{p}(t) = B_2 r(t) - C_1 p(t) + C_2 c(t) \end{cases}$$

The Coupled Weighted Panic Control Reflex (CWPCR) model

Motion of a crowd in an old city, like old Nice

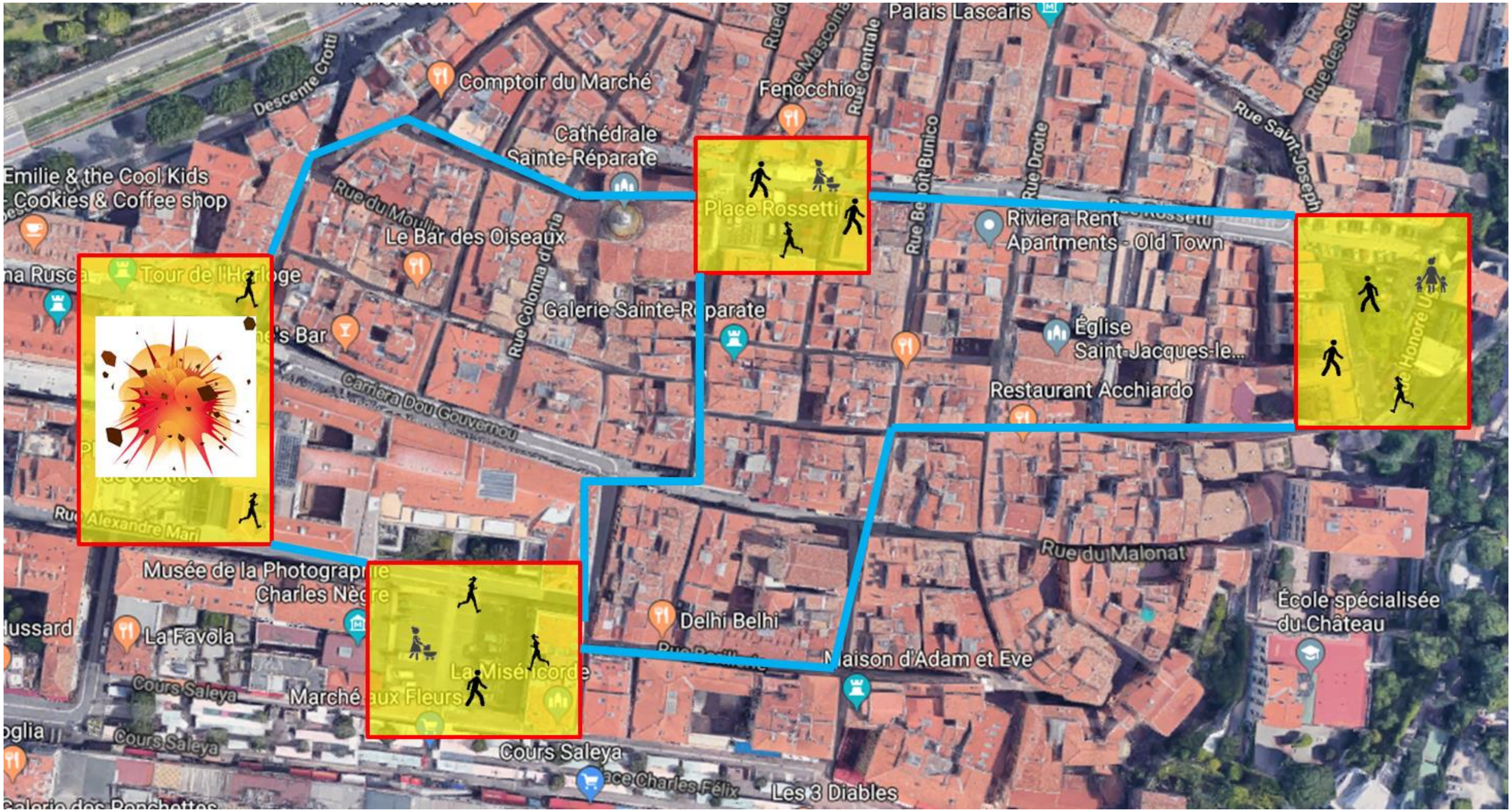
network of four places and five streets

We consider the network of four places and five streets, with people wandering in the places



Terror attack (a blast in one place)

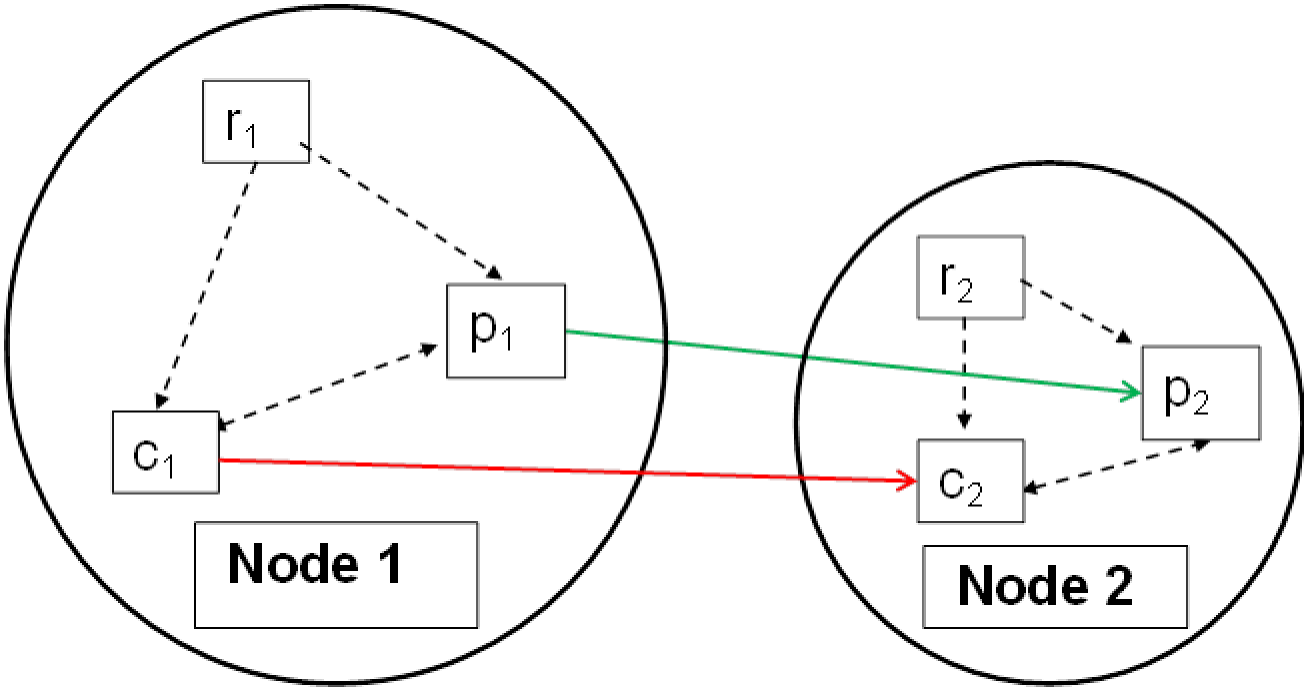
When the bomb blows up in one place, the crowd if rushing through streets towards other the places



CWPCR model step 1: coupling two places

Places in the city are called nodes in the network, *streets* are edges. We suppose that people in reflex behaviour cannot move.

Moreover we consider that at each time the number of people in one node is limited by the capacity of the node W_1 or W_2 (maximum number of people which can stay in a place).



How to couple two nodes?

The three variables in node 1 are $r_1(t)$, $c_1(t)$, $p_1(t)$, they are no more densities, but actual number of people in each behaviour

$$\begin{cases} \dot{r}_1(t) = -(B_1 + B_2) r_1(t) \\ \dot{c}_1(t) = B_1 r_1(t) + C_1 p_1(t) - C_2 c_1(t) \\ \dot{p}_1(t) = B_2 r_1(t) - C_1 p_1(t) + C_2 c_1(t) \end{cases}$$

With initial condition: $r_{1,0} + c_{1,0} + p_{1,0} = V_{1,0} \leq W_1$

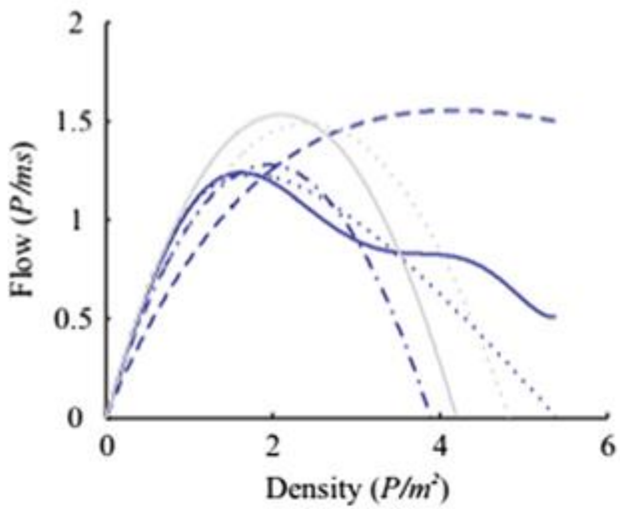
The three variables in node 2 are $r_2(t)$, $c_2(t)$, $p_2(t)$,

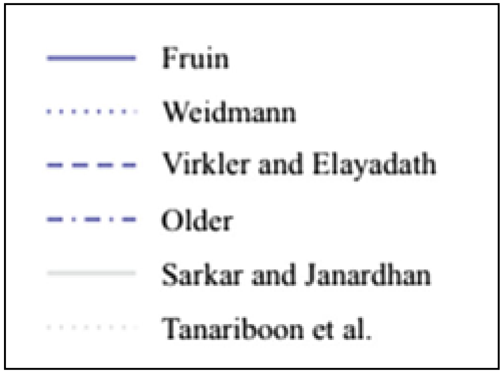
$$\begin{cases} \dot{r}_2(t) = -(B_1 + B_2) r_2(t) \\ \dot{c}_2(t) = B_1 r_2(t) + C_1 p_2(t) - C_2 c_2(t) \\ \dot{p}_2(t) = B_2 r_2(t) - C_1 p_2(t) + C_2 c_2(t) \end{cases}$$

With initial condition : $r_{2,0} + c_{2,0} + p_{2,0} = V_{2,0} \leq W_2$

Motion of a crowd through a narrow street

When people are rushing in a narrow street, the flow of the crowd is non linear and follows a logistic-like dynamics

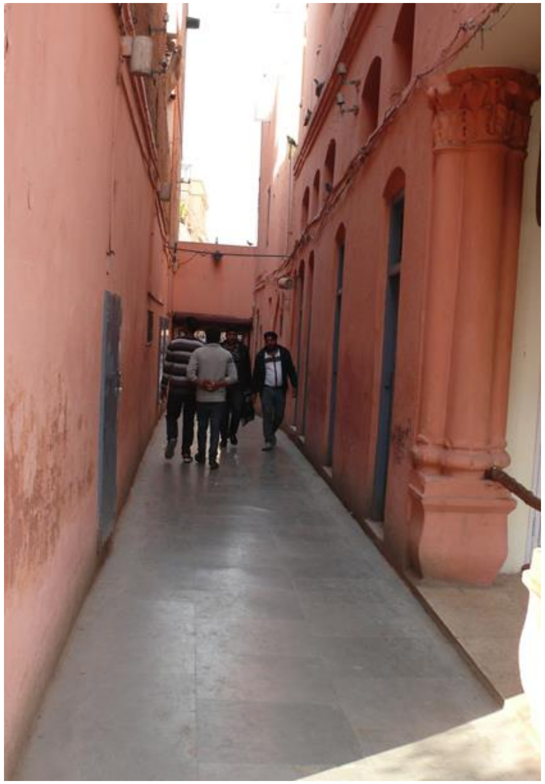




From several experiments

Barcelona

Jallianwala Bagh



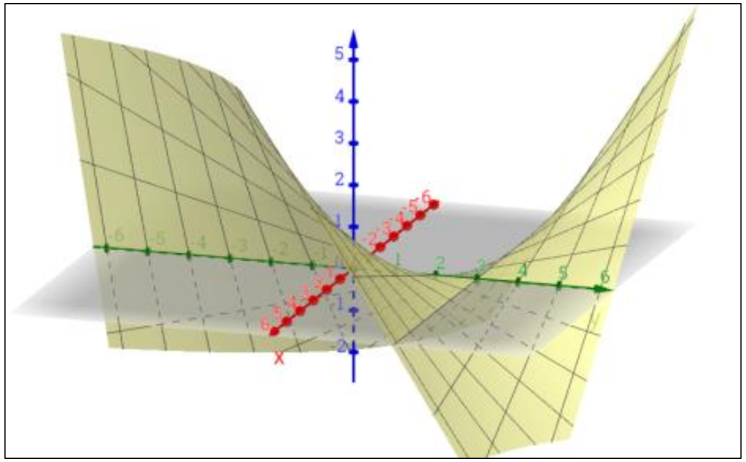
Bottleneck coupling

We introduce a special coupling to model the motion in narrow street: the bottleneck coupling

$$\begin{cases}
\dot{r}_1(t) = -(B_1 + B_2) r_1(t) \\
\dot{c}_1(t) = B_1 r_1(t) + C_1 p_1(t) - C_2 c_1(t) - \eta_{c,1,2} c_1(t)(W_2 - c_2(t) - p_2(t) - r_2(t)) \\
\dot{p}_1(t) = B_2 r_1(t) - C_1 p_1(t) + C_2 c_1(t) - \eta_{p,1,2} p_1(t)(W_2 - c_2(t) - p_2(t) - r_2(t)) \\
\dot{r}_2(t) = -(B_1 + B_2) r_2(t) \\
\dot{c}_2(t) = B_1 r_2(t) + C_1 p_2(t) - C_2 c_2(t) + \eta_{c,1,2} c_1(t)(W_2 - c_2(t) - p_2(t) - r_2(t)) \\
\dot{p}_2(t) = B_2 r_2(t) - C_1 p_2(t) + C_2 c_2(t) + \eta_{p,1,2} p_1(t)(W_2 - c_2(t) - p_2(t) - r_2(t))
\end{cases}$$

Which correspond to the following mapping from

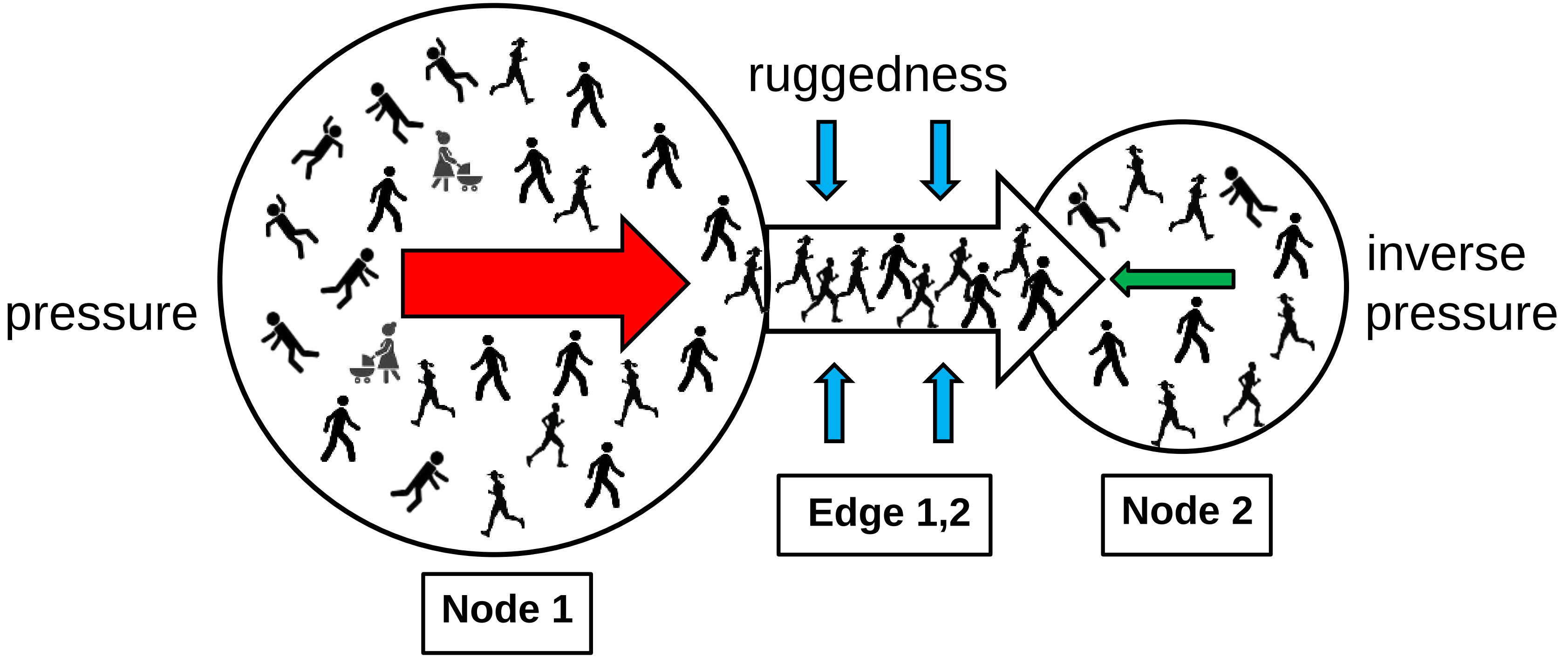
$$\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$



What are the components of this coupling?

$$c_1(t) \times \eta_{c,1,2} \times (W_2 - c_2(t) - p_2(t) - r_2(t))$$

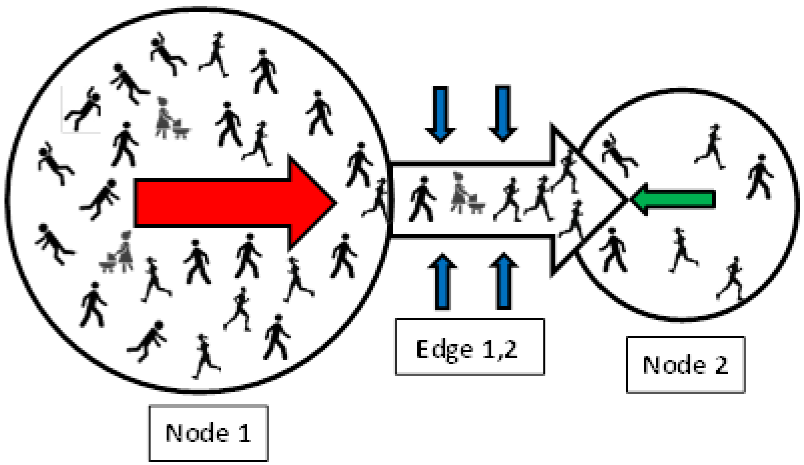
$$c_1(t) \times \eta_{c,1,2} \times (W_2 - V_2(t))$$



How bottleneck coupling is working versus number of people in node 2?

$$c_1(t) \times \eta_{c,1,2} \times (W_2 - V_2(t))$$

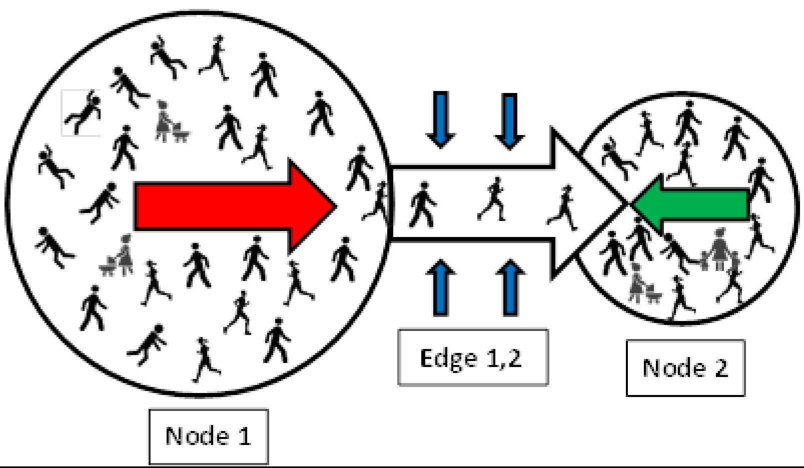
medium flow



few people in node 2

same capacity of node 2

weaker flow

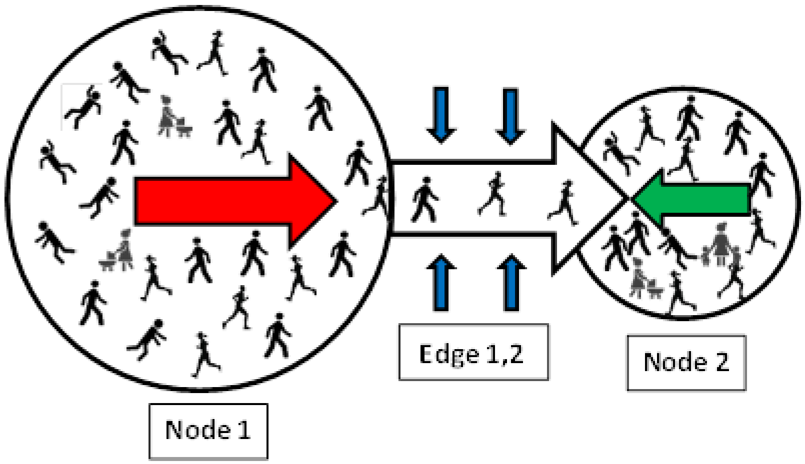


more people in node 2

How bottleneck coupling is working versus capacity of node 2 ?

$$c_1(t)\times\eta_{c,1,2}\times(W_2 - V_2(t))$$

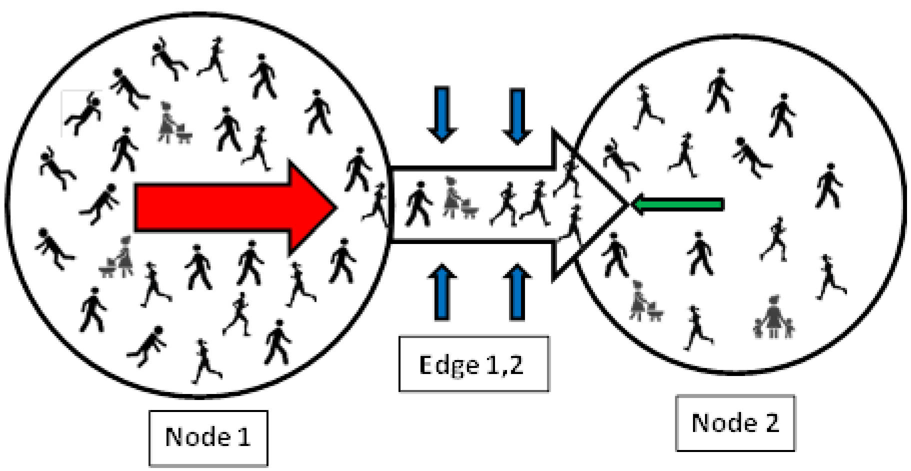
weak flow



small capacity of node 2

same number of people in node 2

stronger flow



larger capacity of node 2

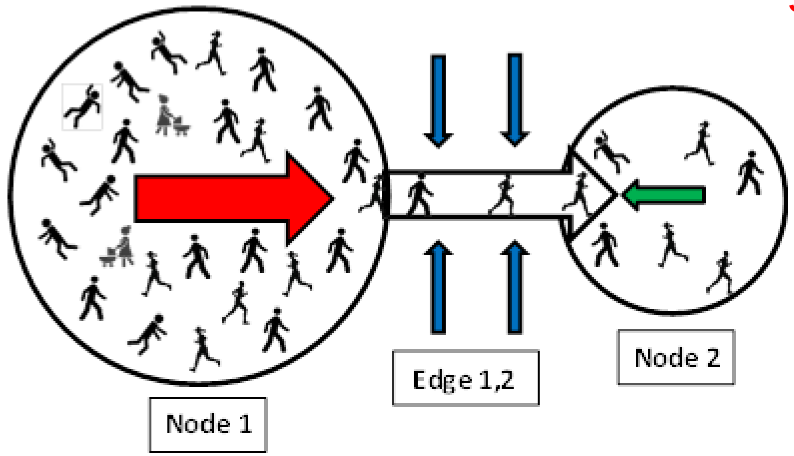
How bottleneck coupling is working versus the width of the street ?

$$c_1(t) \times \eta_{c,1,2} \times (W_2 - V_2(t))$$

Narrow street

strong ruggedness

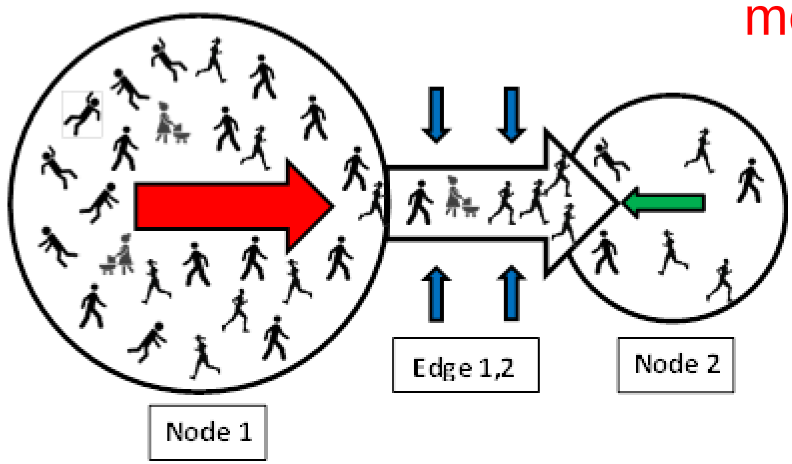
weak flow



Medium street

medium ruggedness

medium flow



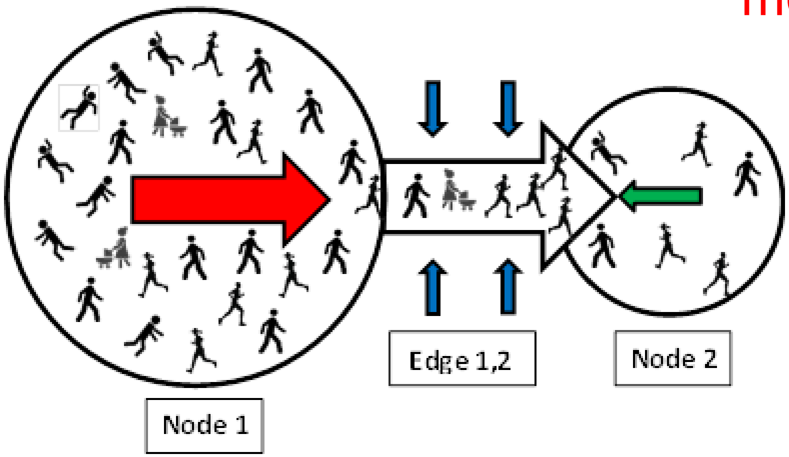
How bottleneck coupling is working versus the width of the street ?

$$c_1(t) \times \eta_{c,1,2} \times (W_2 - V_2(t))$$

medium street

medium ruggedness

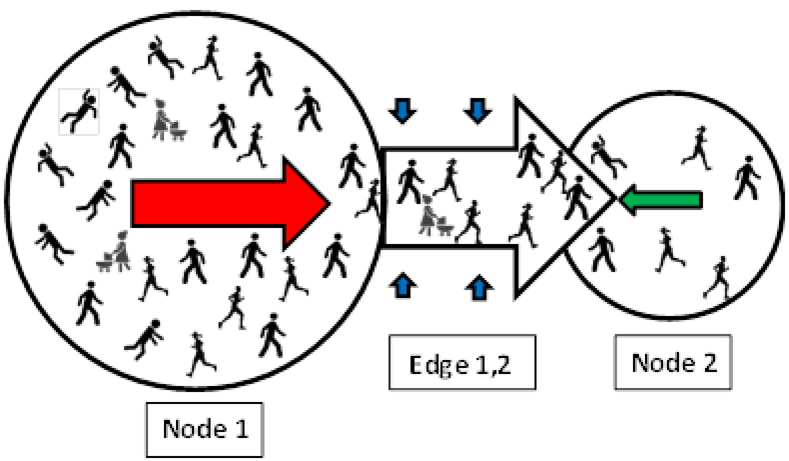
medium flow



large street

weak ruggedness

strong flow



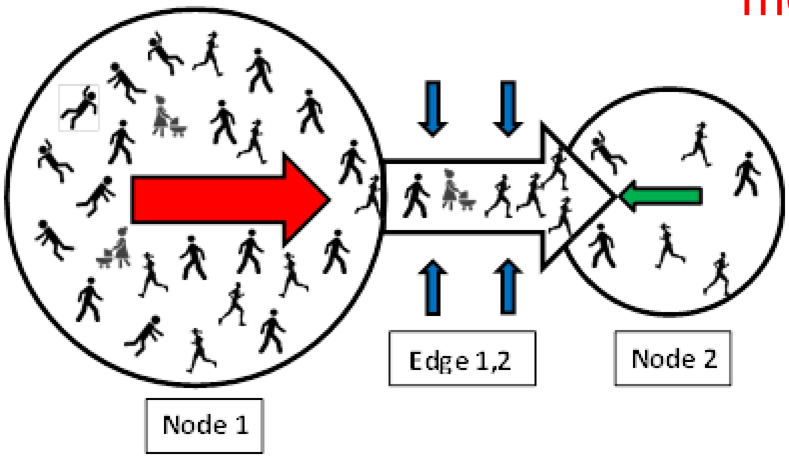
How bottleneck coupling is working versus the width of the street ?

$$c_1(t) \times \eta_{c,1,2} \times (W_2 - V_2(t))$$

medium street

medium ruggedness

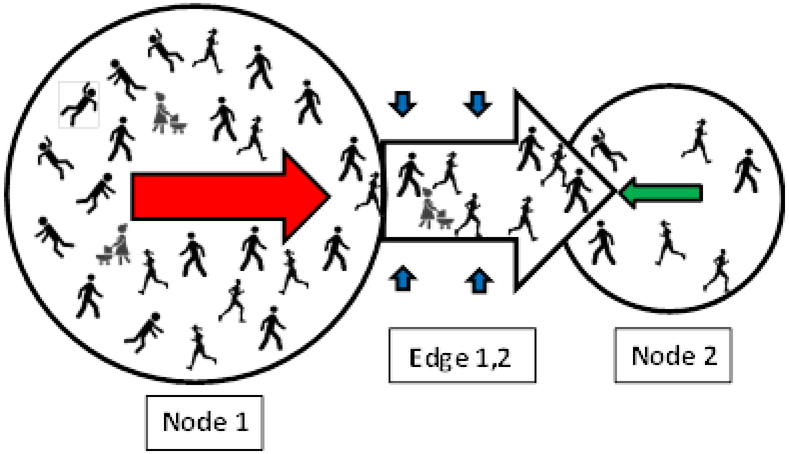
medium flow



large street

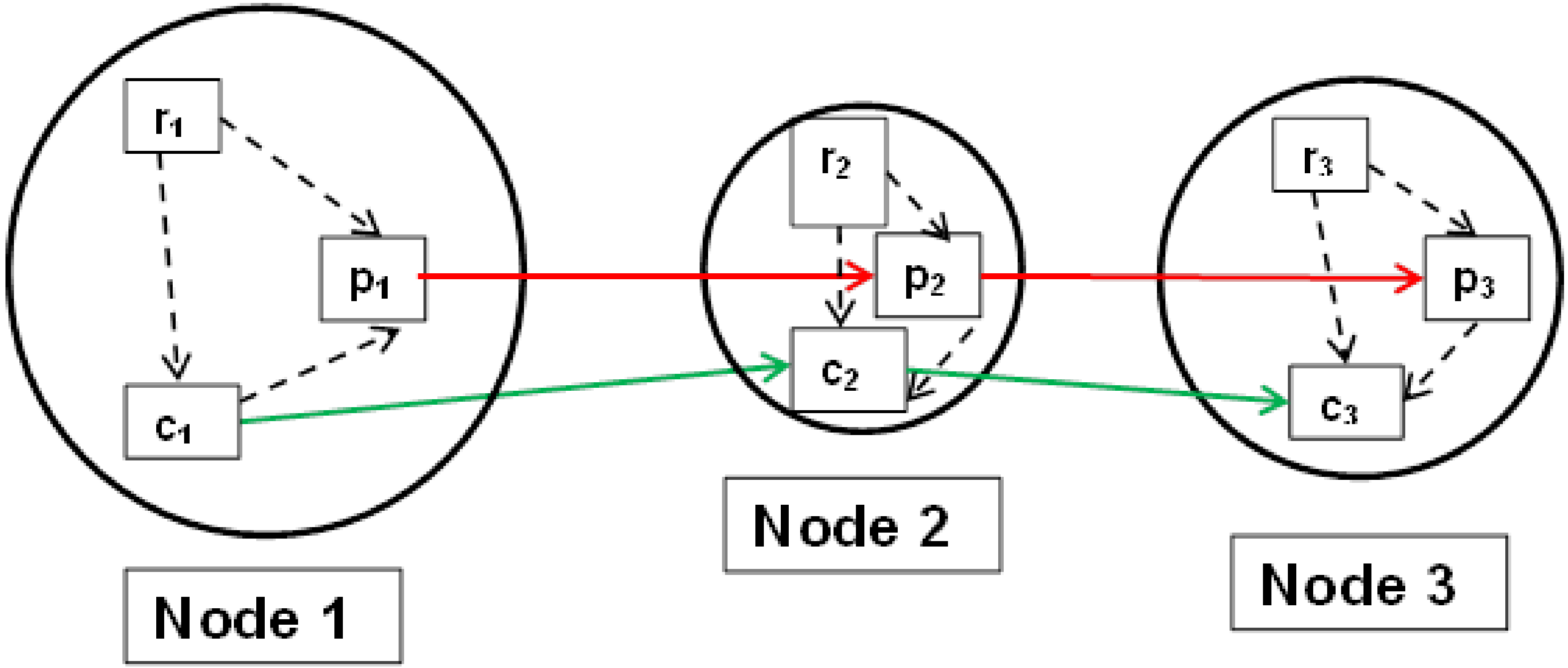
weak ruggedness

strong flow



Motion and changing behavior of the crowd in a three nodes network

Consider a part of a global network with 3 places linked by 2 nodes



Equations of the 3 nodes model

Bottleneck coupling :
in blue : controlled behaviour people from node 1 to node 2
in red : people in panic behaviour from node 1 to node 2
in yellow : controlled behaviour people from node 1 to node 2
in green : people in panic behaviour from node 1 to node 2

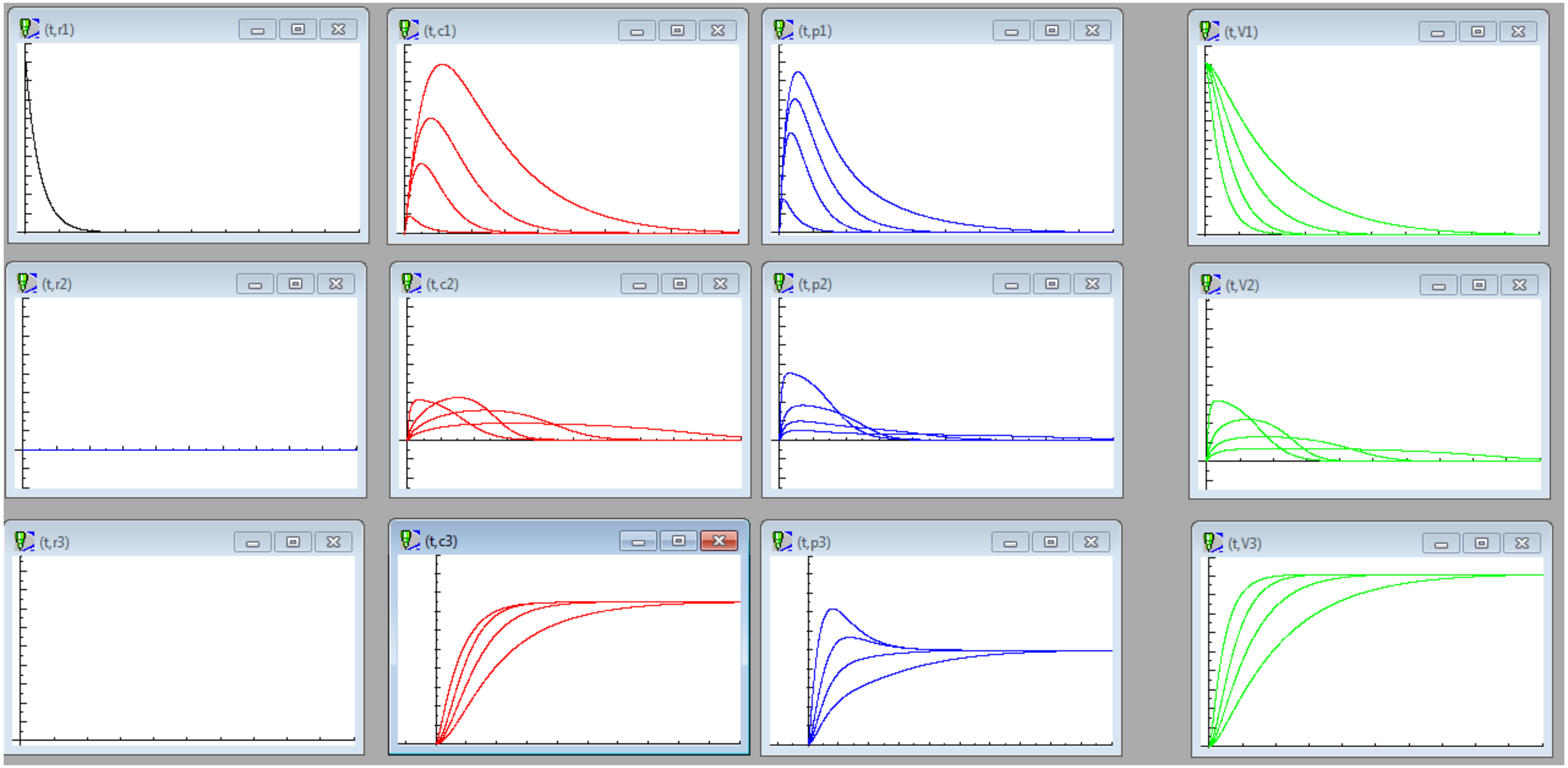
$$
\begin{cases}
\dot{r}_1 = -(B_1 + B_2) r_1 \\
\dot{c}_1 = B_1 r_1 + C_1 p_1 - C_2 c_1 - \eta_{c,1,2} c_1 (V_2 - c_2 - p_2 - r_2) \\
\dot{p}_1 = B_2 r_1 - C_1 p_1 + C_2 c_1 - \eta_{p,1,2} p_1 (V_2 - c_2 - p_2 - r_2) \\
\dot{r}_2 = -(B_1 + B_2) r_2 \\
\dot{c}_2 = B_1 r_2 + C_1 p_2 - C_2 c_2 + \eta_{c,1,2} c_1 (V_2 - c_2 - p_2 - r_2) - \eta_{c,2,3} c_2 (V_3 - c_3 - p_3 - r_3) \\
\dot{p}_2 = B_2 r_2 - C_1 p_2 + C_2 c_2 + \eta_{p,1,2} p_1 (V_2 - c_2 - p_2 - r_2) - \eta_{p,2,3} p_2 (V_3 - c_3 - p_3 - r_3) \\
\dot{r}_3 = -(B_1 + B_2) r_3 \\
\dot{c}_3 = B_1 r_3 + C_1 p_3 - C_2 c_3 + \eta_{c,2,3} c_2 (V_3 - c_3 - p_3 - r_3) \\
\dot{p}_3 = B_2 r_3 - C_1 p_3 + C_2 c_3 + \eta_{p,2,3} p_2 (V_3 - c_3 - p_3 - r_3)
\end{cases}
$$

Numerical simulation

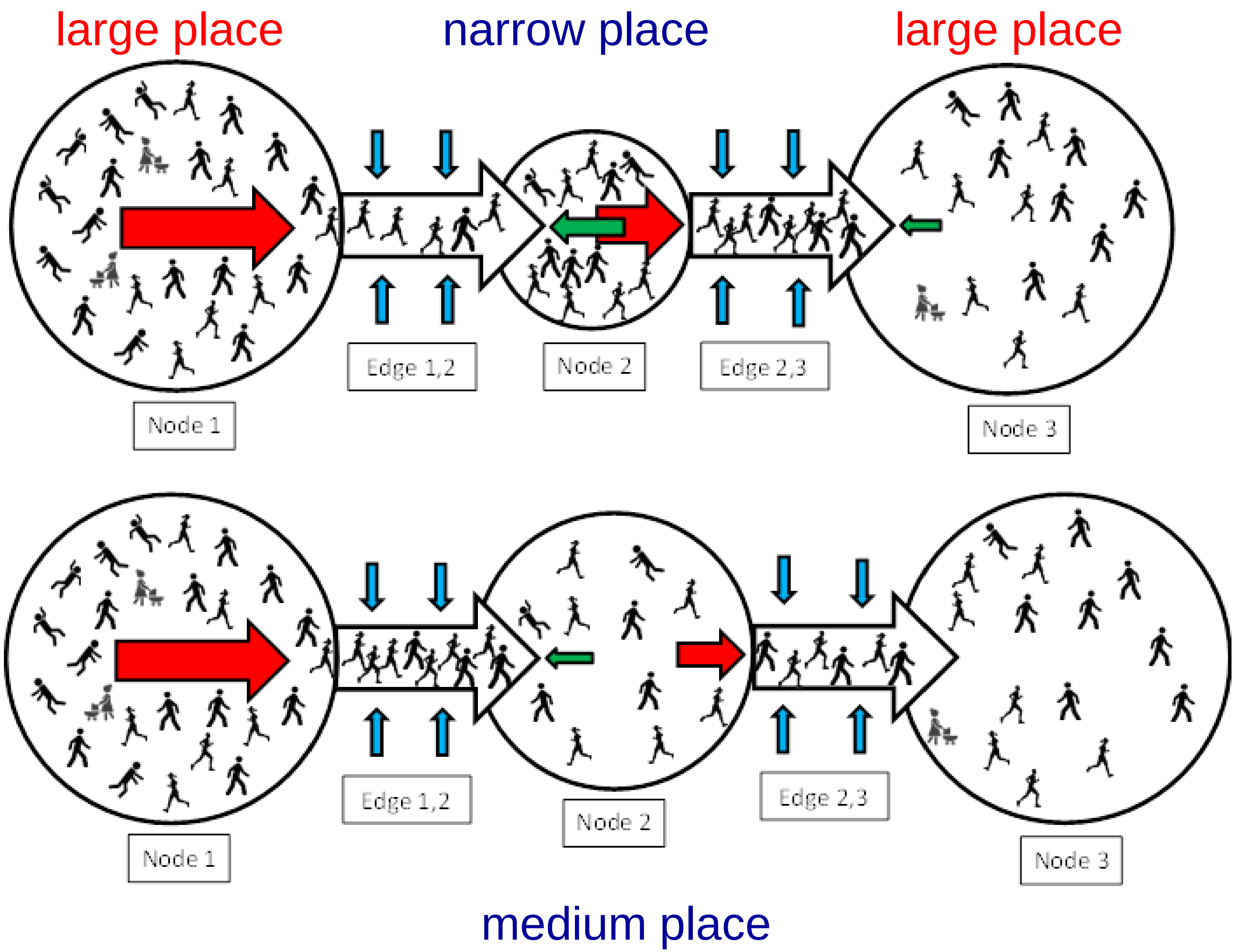
B_1= 0.2 B_2= 0.4 C_1= 0.3 C_2= 0.1 V_1= 20,000 W_1= 30,000 W_3= 20,500

$\eta_{c1,2}$= 0.005 $\eta_{p1,2}$=0.005 $\eta_{c2,3}$= 0.005 $\eta_{p2,3}$= 0.005

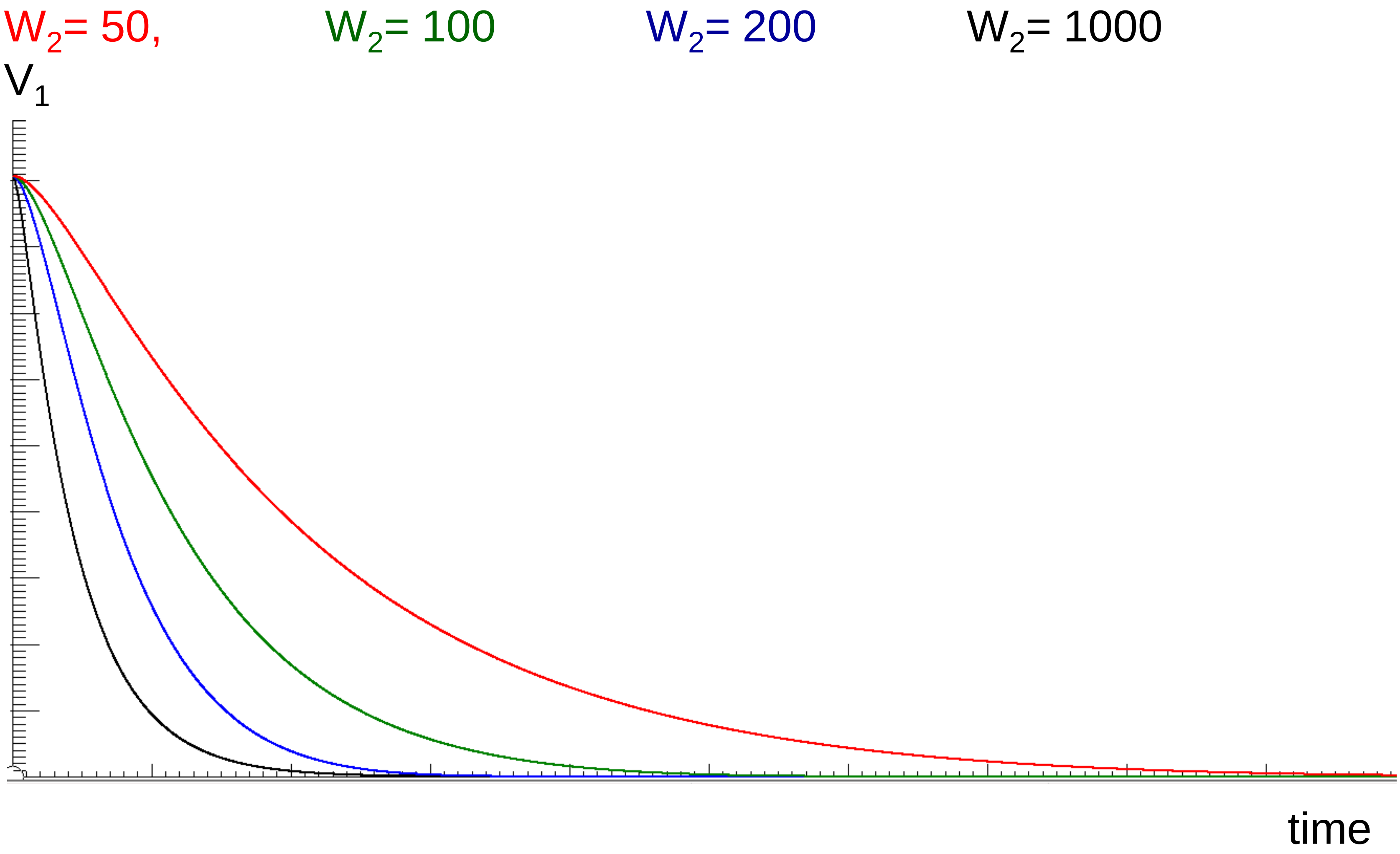
W_2= 50, 100, 200, 1000



Importance of the size of the intermediate node



Evacuation from node 1



Thank you for your attention

ધ"यवाद? आपकी जानकारी . ਲਏ ਤ"ਹਾਡਾ

ਧ'ਨਵਾਦ? ਤ"ਹਾਡਾ ਧਆਿਨ ਲਈ