Recent trends in Okun's law : eleven countries from 1995 to 2017

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RECENT TRENDS IN OKUN'S LAW: ELEVEN COUNTRIES FROM 1995 TO 2017

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ABSTRACT

We present an update of the Okun's coefficient using quarterly data for eleven major countries from 1995 to 2017. We find that Okun's relationship is still valid but with significant variability across countries and not at the same level than before: one percentage point of variation in the unemployment rate tends to be associated with less variation in the growth rate than previous studies on earlier periods have pointed out. We test for a structural break in 2008 and find that Okun's coefficient has been increased in some countries, left unchanged in some others. Taking advantage of important GDP fluctuations in recent years, we test for asymmetry over the various phases of the business cycle. Asymmetry is present in some countries and is revealing domestic differences in labour management during cyclical downturns. The paper emphasizes the need to link the unemployment-growth relationship with the reverse growth-unemployment relationship for a comprehensive view on international differences.

Keywords: Okun's Law, growth, unemployment, business cycles.

JEL classification numbers: E24, E32.

I. INTRODUCTION

We propose a new estimation of the Okun's coefficient for a set of major OECD countries over the past twenty years.

A motivation for this work is to check if the stylized facts highlighted in the early 60's by Arthur Okun (Okun, 1962) and throughout the following decades by numerous other studies, are still relevant. Past estimates of the Okun's coefficient have shown that changes in the unemployment rate and changes in the output growth rate were strongly linked and gave some benchmark for the relationship between both. But economy has changed all along the last twenty years and, because of the financial crisis, the macroeconomic situation has been very instable within the 2008-2017 years. Most major economies (Australia is a notable exception) have experienced a marked decline in GDP levels in 2008. Has this marked alteration of economic growth had an impact on unemployment in the proportion indicated by past evaluations of the Okun's coefficient or, on the contrary, have recent years been specific and has the 'Great Recession' radically changed the situation in this regard? If so, for all countries or only for some of them?

A new evaluation of the Okun's coefficient is not only of academic interest but is also important for economic policies, especially when countries are stucked in mass unemployment as they were in recent years. Then, for economic policies, the 'minimum threshold of growth to reduce unemployment' is a key issue.

Section 2 presents the analytical framework for the so-called 'Okun's Law'. Section 3 reports econometric results. We find that Okun's law is valid in numerous countries but the Okun's coefficient is less than 2 (the value which is generally cited in economic textbooks) in every country of the sample, with significative differences between them. Estimations also show that the financial crisis contributed, sometimes temporarily, to increase the coefficient. Assymmetry in good and bad times is observed in some countries but not all . Section 4 concludes.

II. IS THERE REALLY A LAW?

Okun's relationship is the measurable relationship which exists in an economy between change in the output and change in the unemployment rate. The production function, which links the amount of labour units implemented to the amount of output, is the theoretical support for this relationship. All things being equal, increasing the quantity of labour increases the overall product, which can be done either by increasing the number of workers or the number of hours worked per employee, or a combination of both. The increase in the number of workers affects the level of unemployment which decreases, notwithstanding the changes that may occur in labour supply behaviour.

This can be seen using an accounting identity, taking into account the output and the various elements related to labour. Considering Y the output in real terms, H the number of hours worked, E the number of people in employment, N the working-age population, we can write the following identity:

$$Y \equiv \frac{Y}{H} \times \frac{H}{E} \times \frac{E}{N} \times N \tag{1}$$

The total output is given by the output per working hour (the hourly productivity that reflects the state of the technology) multiplied by i) the number of hours worked per person in employment, ii) the employment rate of the working-age population, iii) the level of the latter.

The working-age population includes inactive (I) and active (A) people. Active population includes both employed (E) and unemployed (U) people, hence: $N = I + A = I + E + U$. Using the employment ratio: $e = \frac{E}{N}$, the labour market participation ratio: $p = \frac{A}{N} = \frac{E+U}{N}$ and the unemployment rate: $u = \frac{U}{E+U}$, the relationship between the three ratios is defined considering:

$$p = e + \frac{U}{N} = e + \frac{u(E+U)}{N}$$
$$p = e + up$$
$$e = p(1-u)$$

Substituting in (1):

$$Y \equiv \frac{Y}{H} \times \frac{H}{E} \times A \times (1-u) \tag{2}$$

and, after a logarithmic transformation:

$$\ln Y \equiv (\ln Y - \ln H) + (\ln H - \ln E) + \ln A + \ln(1-u) \qquad (3)$$

Differenciating the above equation and using lower case for logarithms yields:

$$\Delta y \simeq (\Delta y - \Delta h) + (\Delta h - \Delta e) + \Delta a - \Delta u \qquad (4)$$

It brings up the various forces acting for a change in the level of output. The output growth rate Δy is thus resulting from i) the rate of change in hourly productivity $(\Delta y - \Delta h)$ through improvements in production processes by technological progress and higher level of skills of workers, ii) the rate of change in hours worked per employee $(\Delta h - \Delta e)$, iii) the rate of change in the number of active people, due to population dynamics and the labour supply behaviour dictated by individual trade-offs between work and leisure or by expected jobs opportunity, iv) the percentage change in the unemployment rate.

This simple decomposition illustrates the conditions which must be gathered to consider the growth-unemployment relationship as a "law", i.e. a relationship that would have some historical stability and spatial universality.

First, there must be a stable relationship over time between Δu and Δy, i.e. a given change in the unemployment rate must contribute with some regularity to the variation of the output. This deserves a new evaluation. Econometric estimates by Arthur Okun for the US between 1947 and 1960 led him to consider that one percentage point reduction in the unemployment rate was accompanied by 3.3 points of additional GDP growth. The estimate was later revised and reduced to 2. Is this estimate still valid? A really big change in output and unemployment levels occured in the post-2007 years and this can be an influential source for a modified Okun's relationship.

Second, the universality of the Okun's relationship implies a good quality of statistical models and their estimates for all countries and no big difference in the associated coefficient from one country to another. One can guess that significant differences may exist due to domestic specificities, including the functioning of the labour market. This will happen, for example, if a change in the output growth rate is essentially accompanied with changes in working hours in some countries and, in some others, with the hiring or the firing of employees. Likewise it is conceivable that, from one country to another, the labour supply behaviour differs according to economic conditions, being stable in some, changing in others.

All these conditions must be met simultanously and this hypothesis casts doubt on the possibility to call this relationship a Law. The study by Lee (2000) for 16 OECD countries over the period 1955-96 pointed out that '*the quantitative estimates differ remarkably across countries*' and that '*substantial differences (exist) in estimates for the US and other OECD countries*' (p. 352). Regarding the stability over time, the author underlines that '*besides cross-country heterogeneity, the OECD data reveal strong evidence of structural change in the Okun's relationship*".

The specification of the relationship between output and unemployment is based on the assumption that there are cyclical fluctuations around equilibrium levels of output and unemployment rates and that less unemployment is accompanied by higher output. These fluctuations are linked by the following relationship:

$$y_t - y_t^* = \beta(u_t - u_t^*) + \nu_t \tag{5}$$

where y_t and y^* are the log of, respectively, current and potential GDP, u_t the observed unemployment rate and u^* the NAIRU, ν_t a disturbance term. $(y_t - y_t^*)$ is representing the output gap and $(u_t - u_t^*)$ is the short run component of unemployment rate. Both NAIRU and potential GDP are not fixed, which explained the time index for these two variables. The β parameter, whose value is expected to be negative, is called the Okun's coefficient.

The equilibrium values are not known and must be estimated. Several methods can be considered. Some can be a purely empirical estimation of a stochastic trend using a Hodrick-Prescott (H-P) filter for example, whereas some others can have theoretical foundations, using a production function to build up time series of potential output. Mixed theoretical and empirical methods can also be used, for example estimating the NAIRU (or potential output) as part of an unobservable component model involving an augmented Phillips curve and using the Kalman filter technique. Each of these methods has drawbacks. It has been shown that data filtering using H-P method induces significant cyclical fluctuations which can be considered exaggerated. Conversely, the estimation of a production function needs data on capital stock time series whose availability is limited and evaluation questionable. Likewise, the estimation of a Phillips curve gives low quality results in times of turbulence, as in the current period, where changes in unemployment and in prices level are strongly disconnected.

An alternative is provided by the statistical estimate of the relationship between changes in the unemployment rate and in GDP, expressed by the following model using first differences:

$$\Delta y_t = \alpha + \beta \Delta u_t + \varepsilon_t \tag{6}$$

According to this specification, in the absence of any change in the unemployment rate, the economy is growing at the rate α and β ($\beta < 0$) is measuring the effect on GDP growth rate of a change in the unemployment rate. One additional point in the unemployment rate causes a β decline in the growth rate. In this study, the evaluation will be conducted using this formulation, avoiding the pitfall of prior estimation of NAIRU and potential GDP.

In some papers the specification is reversed and the unemployment rate becomes the dependent variable of the evaluated relationship:

$$\Delta u_t = \alpha' + \beta' \Delta y_t + \eta_t. \tag{7}$$

α' is the observed change in the unemployment rate in the absence of growth and β' the change in the unemployment rate associated with a given variation in the growth rate. $\frac{\alpha'}{-\beta'}$ is the growth threshold associated with a stable unemployment rate.

Both formulations are meaningful for economic policy. The first makes output dependent on the amount of labour and emphasizes the loss of wealth that may result from underutilization of the workforce (the cost of underemployment). This formulation can be used to justify labour[2] market reforms to gain jobs, to reduce unemployment and, in this way, to stimulate growth. The second focuses on the unemployment increase due to insufficient growth. It can be used as a justification of goods market reforms in order to boost growth and, in this way, to reduce unemployment.

In his article, Okun gave an estimation of equation (7) but, in an unfortunate way, he used it to estimate β which he considers equal to $\frac{1}{\beta'}$ while the correct value is $\beta = \frac{1}{\beta'} \times \rho^2$ with ρ^2 the square of the correlation coefficient between the two variables. Estimating $\beta' = -0.3$, he concluded that β was equal to -3.3. In fact, given a reported correlation coefficient of -0.79, the true value can be estimated to be equal to -2.1.[1]

III. ESTIMATION AND RESULTS

We consider, for each i country, the following two time-series: u_t, the unemployment rate in percent and y_t, the real GDP expressed in log form and premultiplied by 100, from 1995Q1 to 2017Q4. The unemployment rate is the OECD harmonized unemployment rate. Data cover 11 countries and are taken from the OECD quarterly national accounts database. Both series are seasonally adjusted.

Past research showed that GDP growth rate is not stationary most of the time and that unemployment rate can be too. In this case, serious bias can affect the estimated coefficients in equation (6) or (7). Moreover, co-integration of the two series is possible. Attfield and Silverstone (1997) reassesed Prachowny's finding (1993) of a very low Okun's coefficient for the US using data from 1967Q2 to 1986Q2. They found non stationarity in the data and '*a strong evidence of a cointegrating relationship*' (p. 329). They showed that using an adapted framework, the coefficient would be much higher (in absolute value). The same authors (Attfield and Silverstone, 1998) have presented some empirical results for the United Kingdom from 1959 to 1995 and clearly found a unit root process for both series. A test for cointegration provided more ambiguous evidence but was conclusive enough to allow the authors to estimate a Vector Error Correction Model. In his already cited work, Lee used the same framework after concluding for cointegration for eleven on twelve countries of his panel. Prior estimation, unit root and cointegration tests have been conducted to avoid biased estimates. They are presented in Appendix.

III.1 Level and dispersion of Okun's coefficients

The estimates for equations (6) and (7) are shown in Table 1. The α coefficient (column 2) shows that productivity gains, changes in the number of hours worked, the behaviour regarding the labour force participation and the population growth have had a significant impact on the output growth rate (except for Italy, Japan and Spain). For Germany and, to a lesser extent, for France the impact of these variables on growth has been weaker than in most other countries.

The Okun's coefficient (column 3), which measures the effect on the output growth rate of a change in the unemployment rate is less than 2 in all countries, the value which is generally cited in economic textbooks. Canada, Germany, the

[1] See Plosser and Schwert (1979) for a modified estimation.

Netherlands, UK have the highest coefficient $(1.4-1.6)^2$. Okun's coefficient is lower in France, USA, Sweden, Italy $(1.1-1.3)$. Adjusted R^2 is very small in Australia, Japan and Spain. For this last country, first differences estimation would have show high correlation but with a high stationarity bias.

Table 1

Estimation results for variation in GDP growth rates and in unemployment rates

Country	α	-β	α'	$-\beta'$	$\frac{\alpha'}{-\beta'}$	$\overline{R}^2$
Australia	0.77*	0.35	−0.00	0.05	0.00	0.00
	(13.36)	(−1.20)	(−0.02)	(−1.20)		
Canada	0.54*	1.55*	0.11*	0.24*	0.46	0.37
	(10.33)	(−7.28)	(3.76)	(−7.28)		
France	0.37*	1.11*	0.08*	0.23*	0.35	0.25
	(8.85)	(−5.56)	(3.25)	(−5.56)		
Germany	0.28*	1.53*	−0.01	0.10*	−0.1	0.15
	(3.47)	(−4.07)	(−0.54)	(−4.07)		
Italy	0.14**	1.05*	0.02	0.17*	0.12	0.16
	(2.11)	(−4.25)	(0.83)	(−4.25)		
Japan	0.24**	1.39**	0.005	0.03**	0.17	0.04
	(2.31)	(−2.08)	(0.34)	(−2.08)		
Netherlands	0.44*	1.48*	0.05**	0.18*	0.28	0.26
	(7.07)	(−5.69)	(1.98)	(−5.69)		
Spain[a]	0.01	0.21**	0.002	0.44*	0.00	0.08
	(0.04)	(−2.97)	(0.03)	(−2.97)		
Sweden	0.59*	1.29*	0.06***	0.14*	0.43	0.17
	(6.80)	(−4.35)	(1.82)	(−4.35)		
U.K.	0.44*	1.46*	0.06**	0.21*	0.29	0.33
	(8.46)	(−5.76)	(2.43)	(−6.70)		
U.S.A.	0.58*	1.12*	0.14*	0.26*	0.54	0.29
	(10.56)	(−6.11)	(3.90)	(−6.11)		

Notes: α, β are estimated coefficients from the regression equation: $\Delta y_t = \alpha + \beta \Delta u_t + \varepsilon_t$ and α', β' are estimated coefficients from the regression equation: $\Delta u_t = \alpha' + \beta' \Delta y_t + \varepsilon'_t$. t-statistics are in parenthesis. *,**,*** are for statistical significance at 0.01, 0.05, 0.1 levels, respectively. [a] denotes the use of second-differenced data for y_t and u_t (see Appendix).

For the US, the estimated value is clearly quite lower than Okun's estimate over the period from 1947Q2 to 1960Q4 and lower than the 1.84 value found by Lee over the 1955-1996 period.[3] Ball *et al.* (2017) used quarterly data from 1948Q1 to 2013Q4 and provided an indirect assessment of the Okun's coefficient. They found $-\beta' = 0.289$ that can be transformed into a roughly $-\beta = 1.7$ (with $\overline{R}^2 = 0.484$ and $N = 263$). To go deeper on this issue, we use available OECD quarterly data for pre-1995 quarters and expand our sample taking the first quarter of 1960 as a starting point. We find $-\beta = 1.6$ and, for sub-periods, $-\beta_{1960Q1-1995Q1} = 1.8$ and $-\beta_{1995Q1-2015Q4} = 1.15$ (as mentioned earlier for the latter). Excluding the 2008

[2] In absolute value, here and after.

[3] Lee used annual and not quarterly data.

crisis and limiting the period to 1995Q1-2007Q4, we get a 1.3 value with a fairly wide confidence interval $(0.6 - 2.0)$. These results lead us to the conclusion that the Okun's coefficient for the US has been lower in recent years than it was before.

The estimated value for Canada is the highest of the panel. As for the US, we have been able to expand the sample to an earlier period. For 1960-95, we find $-\beta = 1.3$, a lower value than the estimated 1.6 for the period 1995-2015. Unlike the US, the ratio would have increased in Canada. However, this finding must be shaded because if we consider the pre-crisis years (1995-2007) $-\beta$ is only 0.8 (again with a fairly wide confidence interval between 0.09 and 1.4).

For the United Kingdom we find 1.46, which is similar to the value obtained by Attfield and Silverstone (1998) and close to 1.39, the estimate in Lee's study. For France the coefficent is 1.11, which is identical to the US coefficient and is well below the 2.91 estimated by Lee. For Italy, the coefficient is small, close to 1 and without much apparent change (1.09 in Lee's study). For other European countries, the Okun's coefficient stands at a high level for Germany (1.53), for the Netherlands (1.48) and lower in Sweden but again with values notably lower than in Lee's study[4]. Also note that the Okun's relationship is not verified or very weak for Australia and Japan as evidenced by low $\overline{R}^2$s and βs not significantly different from zero. This result differs notably from Lee's findings ($-\beta = 1.53$ for Australia and 4.41 for Japan). More investigation has been conducted for these two countries using annual data[5]. For Australia, Okun's coefficient is still not different from zero. For Japan, estimation is of good quality with a high level of coefficient ($-\beta = 2.72$). Tachibanaki and Sakurai (1991) emphasize the specificity of female labor supply in Japan to explain the small cyclical changes in unemployment rates and the high value of the Okun's coefficient.

It follows from these estimates that the Okun's coefficient shows prominent differences from one country to another. Remarkably, coefficient is low in France, Italy and the US. Using preceding estimates from older research, signs of a lowering of this ratio over time could be found, rather convincingly for the US, with more uncertainty if we consider the whole panel.

Columns 4-6 present the estimates from the reverse relation, with the underlying idea that output growth boosts employment and reduces unemployment. In this framework, α' is reflecting the change in the unemployment rate, irrespective of GDP growth. Column 4 shows that it is not significantly different from 0 in many countries of the panel. Canada, France and USA are exceptions, with an approximately 0.1 increase each quarter in the unemployment rate without GDP growth. For the US, it must be noticed that the estimated value is twice lower than in Okun's paper.

[4] 2.47 for Germany and 1.87 for Sweden.

[5] Estimation of equation 6 on annual data (1995-2014) gives the following results: i) For Australia: $\Delta y_t = 3.11 - 0.74\Delta u_t$ $[t_{\hat{\beta}} = -1.89, \overline{R}^2 = 0.126]$; ii) For Japan: $\Delta y_t = 0.95 - 2.72\Delta u_t$ $[t_{\hat{\beta}} = -3.30, \overline{R}^2 = 0.354]$.

We also used a 'gap' model (equation 5) with quarterly data. Long run levels u_t^* et y_t^* have been estimated using an Hodrick-Prescott filter.

For Australia: $(y_t - y_t^*) = -0.81(u_t - u_t^*)$ with $t_{\hat{\beta}} = -4.44$, $\overline{R}^2 = 0.182$. Considering $t_{\hat{\beta}}$, $\beta \neq 0$ is not rejected at the 5% level in this 'gap' model (it was rejected in the first differences model).

For Japan : $(y_t - y_t^*) = -3.48(u_t - u_t^*)$ with $t_{\hat{\beta}} = -7.89$, $\overline{R}^2 = 0.422$.

From column 5, we see that the estimated Okun's coefficient is 0.26 for the US, close to the 0.30 original Okun's study and to the 0.29 previously mentioned in Ball *et al.*'s study over the period from 1948Q1 to 2013Q4. As we can see, the used specification is not neutral. The above mentioned change over time in the US coefficient no longer appears in the reverse specification (or much less clearly). A second illustration lies in the comparison between Canada and the US: column 6 shows very close values in both countries (0.24 and 0.26) while they emerge clearly different in column 3 (1.55 and 1.12). The explanation lies in differences into the overall quality of the model since, with equivalent β, β' will vary with the $\overline{R}^2$ value.

According to this reverse specification, we can oppose countries where unemployment strongly reacts to changes in growth to those where the reaction is rather weak. The first group includes Canada, USA, France, the UK, Spain[6] and the second Australia and Japan. An intermediate group includes Germany, Italy, the Netherlands and Sweden.

In column 6, the $\frac{\alpha'}{-\beta'}$ ratio shows the quarterly growth rate associated with no change in the unemployment rate. It thus provides a growth target for economic policy when unemployment is high. In the US, 0.5 percentage point of quarterly growth is at least needed to reduce unemployment. This is less than before since it stood at 1 in Okun's paper.[7] USA, Canada and Sweden have similar values, higher than in other countries. Germany is a special case with a required quarterly growth rate equal to 0.

III.2 Structural change

Had the 2008 crisis a great impact on the unemployment-output relationship? If yes, how and in which countries? To answer these questions, we test the hypothesis of a change using the following equation:

$$\Delta y_t = \alpha + \beta_1 \Delta u_t + \beta_2 I_{2008Q1} + \beta_3 (I_{2008Q1} \times \Delta u_t) + \varepsilon_t \qquad (8)$$

where I_{2008Q1} is a dummy variable equal to 1 from 2008Q1 to 2017Q3 and 0 before, which appears twice in the equation, one time by itself and a second time crossed with Δu_t. The β_2 coefficient, whose expected value is negative, indicates a possible change in the growth rate of the economy, regardless of that induced by the change in unemployment. The Okun's coefficient value is β_1 until the beginning of the crisis and $(\beta_1 + \beta_3)$ for the whole period.

The presence or absence of a structural break in 2008 can be tested using β_2 and β_3. The nullity of these coefficients indicates that no break can be detected on that date. In contrast, a non-zero β_2 coefficient implies that a change in the pace of growth happened independently of any change in unemployment, and a non-zero β_3 demonstrates a specific effect of unemployment on growth after 2008. We use a Wald test to compare the models with and without structural change. Results are shown in Table 2, column 5. The F-statistic indicates that the H_0 hypothesis of the joint nullity of the coefficients can be rejected at the 5% for all countries except for

[6] Based on the second differences, estimation for Spain puts this country in the first group with a coefficient of 0.44. It is not significantly different from 0 in other countries if we use the same methodology.

[7] Ball *et al.* estimates for 1948Q1-2013Q4 are : $\Delta y_t = 0.241 - 0.289 \Delta u_t$, hence $\frac{\alpha'}{-\beta'} = 0.83$.

Australia, Japan and Spain. β_2 coefficient (column 3) is negative for every country but Japan, as expected in connection with the 2008 crisis and show that the break in the growth rate was significant. β_3 coefficients (column 4) show that the Okun's coefficient has been significantly higher since 2008 in Canada, Germany, Sweden and the UK. The nullity of the coefficients cannot be rejected for the remaining countries, including France, Italy, US and the Netherlands.

To go further into the analysis, equation 6 is estimated by rolling regressions over a 40 quarters interval. The first estimate is over the period 1995Q1-2004Q4 and the last over the period 2007Q4-2017Q3. The estimated values of β are plotted in Figure 1, panel A. The impact of the 2008 crisis is clear. We can see the rise of Okun's coefficient in the pre-cited countries when that year is included in the interval. A higher β-value reflects a stronger than before GDP reaction to changes in unemployment. GDP is overreacting downward during this period. We can also distinguish the countries where the effect is still lasting during the latest period, which leads to a staircase-looking curve, and those with a transient effect[8].

The right-hand part of Table 2 provides the estimates for the reverse relationship. The change in the post-2008 period is validated by the Wald test (column 10) for some countries but is not so strongly than before put in evidence (comparing columns 10 and 4, the number of countries where the post-2008 effect cannot be seen is higher in column 10). The constrained model, with no structural change, is better adapted for many countries meaning the post-2008 growth rate had no specific effect (compared to the pre-2008) on unemployment.

The impact of output growth on unemployment since 2008 is given by β'_3 coefficients (column 9). It has increased in several countries (Canada, Spain, the UK, USA) but not in the others where the change is not significant. The Netherlands are a noticeable exception since, in this country, the Okun's coefficient is lower: one point of change in GDP growth has had a lower impact on the unemployment rate after 2008 than prior that date.

Figure 1 panel B shows, in the same manner, the β' estimated coefficient of equation 7. Time-varying estimation are consistent with the results from Table 2 (column 9). We can see the higher level of Okun's coefficient after 2008 for Canada, Spain, the UK, USA[9]. In the US in particular, it goes from 0.2 to around 0.4. We can also see the decline of the coefficient for Netherlands.

[8] Alternatively, we have used a regression with a 50-quarter window. No change in results is observed except for Germany where the coefficient is not growing, leading on the figure to a flat line similar to that of Canada.

[9] The same change is observed for France and Italy but we previously see that β'_3 was not significantly different from 0 (Table 2, column 11).

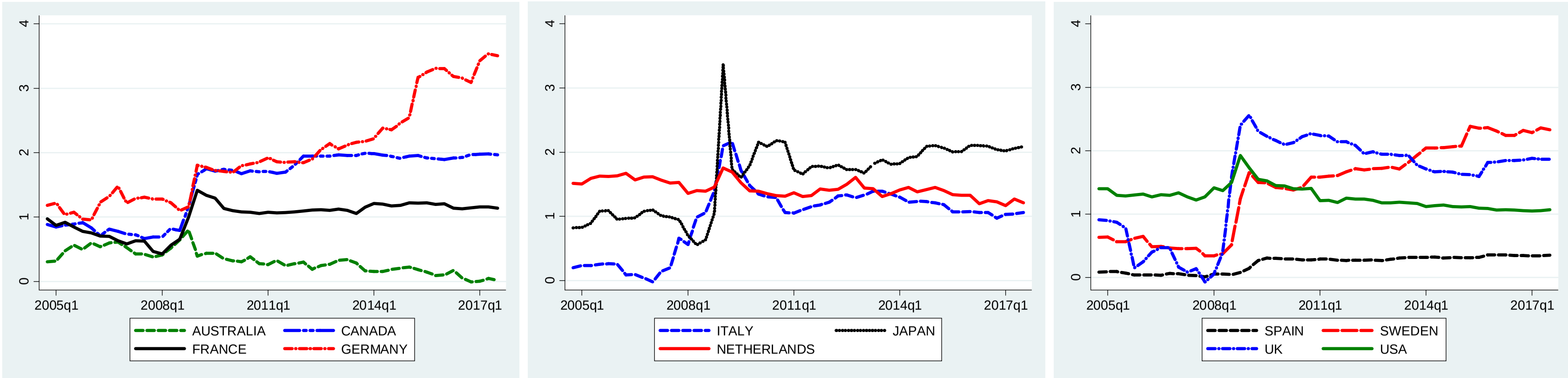
Table 2

Estimation results for variation in GDP growth rates and in unemployment rates with structural break

Country	$-\beta_1$	$-\beta_2$	$-\beta_3$	*F-stat*	$\overline{R}^2$	$-\beta'_1$	$-\beta'_2$	$-\beta'_3$	*F-stat*	$\overline{R}^2$
Australia	0.32	0.25**	−0.31	2.44***	0.04	0.03	−0.09	−0.03	3.46**	0.06
	(−0.75)	(−2.05)	(0.52)			(−0.61)	(1.23)	(0.28)		
Canada	0.75**	0.26**	1.22*	6.37*	0.44	0.13**	−0.11***	0.18**	3.25**	0.40
	(−2.31)	(−2.52)	(−2.92)			(−2.36)	(1.86)	(−2.55)		
France	0.55***	0.28*	0.74***	7.38*	0.35	0.16	−0.06	0.09	0.66	0.25
	(−1.96)	(−3.37)	(−1.90)			(−2.17)	(1.10)	(−0.98)		
Germany	1.11*	0.53*	2.44*	6.25*	0.24	0.16*	0.16*	−0.08	6.51*	0.24
	(−2.60)	(−3.07)	(−2.87)			(−3.80)	(−3.61)	(1.47)		
Italy	−0.33	0.32*	0.73	3.18**	0.20	0.05	−0.18*	0.13	4.90*	0.23
	(0.76)	(−2.22)	(−1.37)			(−0.74)	(3.03)	(−1.53)		
Japan	−0.73	0.24	1.42	1.22	0.04	0.03	0.05	−0.004	1.14	0.04
	(0.75)	(−1.18)	(−1.06)			(−1.02)	(−1.40)	(0.10)		
Nether-lands	1.45*	0.42*	−0.30	6.45*	0.34	0.28*	0.08	−0.15**	2.26	0.28
	(−4.03)	(−3.43)	(0.60)			(−4.84)	(−1.37)	(2.12)		
Spain[a]	0.06	0.01	0.29**	2.20	0.11	0.13	0.04	0.64**	2.51***	0.11
	(−0.64)	(−0.25)	(−2.09)			(−0.63)	(−0.51)	(−2.18)		
Sweden	0.55	0.35**	1.81*	6.81*	0.26	0.15**	0.002	−0.03	0.09	0.15
	(−1.56)	(−2.10)	(−3.10)			(−2.35)	(−0.02)	(0.34)		
U.K.	0.32	0.48*	1.53*	14.35*	0.48	0.05	−0.09	0.26*	7.27*	0.41
	(−0.77)	(−5.09)	(−3.15)			(−0.74)	(1.64)	(−3.50)		
U.S.A.	1.31*	0.43*	−0.24	9.24*	0.40	0.18**	0.006	0.21**	5.50*	0.36
	(−3.60)	(−4.22)	(0.59)			(−2.65)	(−0.08)	(−2.31)		

Notes: βs are coefficients from the regression equation: $\Delta y_t = \alpha + \beta_1 \Delta u_t + \beta_2 I_{2008Q1} + \beta_3 (I_{2008Q1} \times \Delta u_t) + \varepsilon_t$ and β's from: $\Delta u_t = \alpha' + \beta'_1 \Delta y_t + \beta'_2 I_{2008Q1} + \beta'_3 (I_{2008Q1} \times \Delta y_t) + \varepsilon'_t$. t-statistics are in parenthesis. *,**,*** are for statistical significance at 0.01, 0.05, 0.1 levels, respectively. F-stat is a Wald test for $H_0 : \beta_2 = \beta_3 = 0$ and $H'_0 : \beta'_2 = \beta'_3 = 0$. [a] denotes the use of second-differenced data for y_t and u_t. αs are not reported.

Panel A. $-\beta$ coefficient from Equation 6



Panel B. $-\beta'$ coefficient from Equation 7

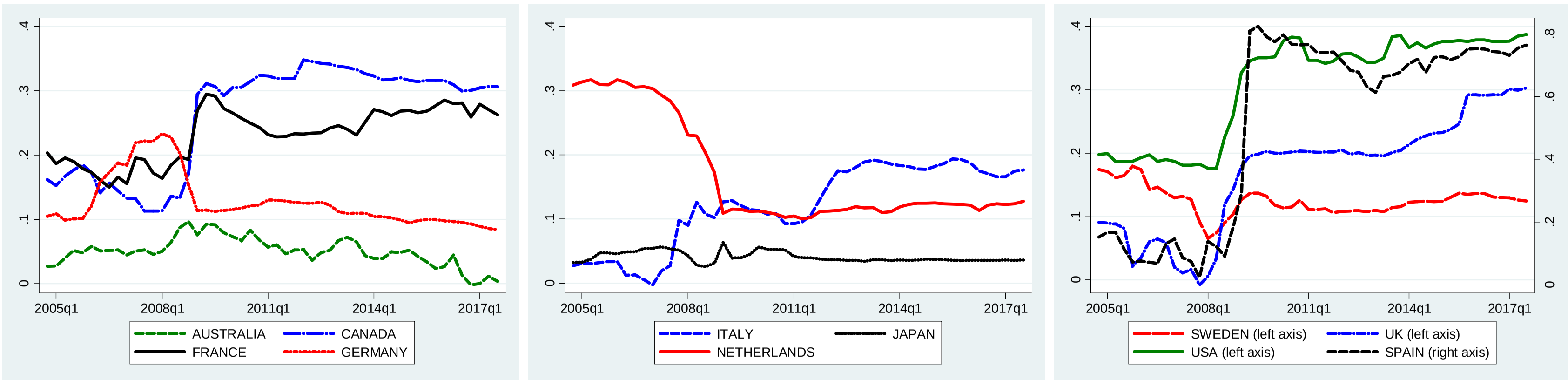


Figure 1. Time varying estimation of the Okun's coefficient

Notes: Rolling OLS regressions on 40 semestres. End of the period on the x-axis. First period is 1995Q1 to 2004Q4; last period is 2007Q4 to 2017Q3.

III.3 Asymmetries

A set of hypotheses can be formulated to explain possible asymmetries in the reaction of unemployment to a positive or a negative business shock as, for example, institutional restrictions to the labour market functioning, assumption of pessimism from employers, investment in specific training for employees, etc. Silvapule *et al.* (2004) give a detailed review on the subject and show that unemployment reacts more to a negative than to a positive output gap for the US.[10] Lee (2000) concludes in the same way for the US and Japan but in a reverse direction for Canada, France and the Netherlands. The importance of business fluctuations induced by the financial crisis offers the opportunity to reassess these results.

To test this hypothesis, we identify quarters with positive and negative change in the unemployment rate with I_t^{u+} and I_t^{u-} dummies and we build up the $\Delta^+ u_t$ and $\Delta^- u_t$ variables as follows:

$$\begin{aligned} \Delta^+ u_t \equiv I_t^{u+} \Delta u_t \; &= \Delta u_t \text{ if } \Delta u_t \geq 0 \\ &= 0 \text{ if } \Delta u_t < 0 \\ \Delta^- u_t \equiv I_t^{u-} \Delta u_t \; &= \Delta u_t \text{ si } \Delta u_t < 0 \\ &= 0 \text{ if } \Delta u_t \geq 0 \end{aligned} \tag{9}$$

Introducing both variables in the Okun's relationship, we get:

$$\Delta y_t = \alpha + \beta^+ \Delta^+ u_t + \beta^- \Delta^- u_t + \nu_t \tag{10}$$

Columns 2-5 from Table 3 provide the estimates for both coefficients, the values for an F-test of equality between them (H_0: $\beta^+ = \beta^-$ vs H_1: $\beta^+ \neq \beta^-$) and for $\overline{R}^2$s. The F-test in column 4 shows that only Canada, France, the UK and USA are really concerned by asymmetry with β^+ significantly higher (in absolute value) than β^- which means an Okun's coefficient higher in bad times than in good times.

Columns 6-9 show the results for the reverse relationship which assumes a dependency from changes in unemployment to changes in GDP.[11] As columns 2 and 3, columns 6 and 7 relate to economic slowdowns and recoveries. The F-test includes now Netherlands and excludes France from the group of countries where unemplyment does not react with symetry to increasing or decreasing growth change. The coefficients associated with positive GDP variation are usually lower than those associated with a negative change, which is consistent with results from our previous estimation. Therefore, on average, unemployment is decreasing at a slower pace in recoveries than it is increasing in slowdowns.

Interestingly, we see the difference between the Netherlands and the other countries. In this country, the coefficient is lower during downturns than it is in recoveries. This reflects the fact that the speed of unemployment increase when times are worsening is smaller than the one of unemployment decrease when the economy get better. This can be interpreted as sign of a specific model of labour management,

[10] See also Harrris and Silverstone (2001), Cuaresma (2003).

[11] $\Delta^+ y_t$ and $\Delta^- y_t$ are built up as previously using I_t^{y+} and I_t^{y-}, two dummy variables. The estimated equation is $\Delta u_t = \alpha' + \beta'^+ \Delta^+ y_t + \beta'^- \Delta^- y_t + \nu_t$.

very different from the Anglo-Saxon model illustrated there by Canada, the UK and the USA. In this specific model, unemployment is not the first answer for labour adjustment to slowing growth.

Table 3
Estimation results for variation in GDP growth rates and in unemployment rates with asymmetries

Country	$-\beta^{+}$	$-\beta^{-}$	$F(1,T)$	$\overline{R}^2$	$-\beta'^{-}$	$-\beta'^{+}$	$F(1,T)$	$\overline{R}^2$
Australia	0.27	0.43	0.03	0.00	0.33	0.03	1.04	0.00
	(−0.55)	(−0.82)			(−1.17)	(−.61)		
Canada	2.01*	0.80***	3.76**	0.39	0.53*	0.10**	25.19*	0.50
	(−6.35)	(−1.81)			(−8.22)	(−2.38)		
France	1.84*	0.31	6.76**	0.30	0.35*	0.17*	2.44	0.26
	(−5.41)	(−0.84)			(−4.11)	(−2.85)		
Germany	2.28**	0.89	1.05	0.15	0.08***	0.14*	0.75	0.15
	(−2.78)	(−1.11)			(−1.98)	(−2.97)		
Italy	1.54*	0.29	2.32	0.17	0.25*	0.06	2.43	0.17
	(−3.68)	(−0.55)			(−3.84)	(−0.84)		
Japan	2.39**	0.17	1.42	0.04	0.09*	0.04	8.21	0.11
	(−2.22)	(−0.11)			(−3.59)	(1.33)		
Nether-	1.26**	1.71*	0.17	0.25	0.04	0.28*	7.35*	0.32
lands	(−2.14)	(−2.71)			(−0.74)	(−5.91)		
Spain[a]	0.14	0.28**	0.22	0.08	0.16	0.68**	1.21	0.08
	(−1.12)	(−2.18)			(−0.55)	(−2.60)		
Sweden	1.92*	0.55	2.25	0.18	0.14**	0.13**	0.01	0.16
	(−3.76)	(−0.96)			(−2.49)	(−2.49)		
U.K.	2.46*	0.43	8.90*	0.39	0.32*	0.12	4.64**	0.36
	(−6.63)	(−0.97)			(−5.51)	(−2.39)		
U.S.A.	1.64*	−0.24	9.68*	0.35	0.55*	0.12**	11.69*	0.37
	(−6.79)	(0.47)			(−5.89)	(−2.02)		

Notes: βs are estimated coefficients from the regression equation: $\Delta y_t = \alpha+\beta^{+}\Delta^{+}u_t+\beta^{-}\Delta^{-}u_t+\varepsilon_t$ and β's are estimated coefficients from the regression equation: $\Delta u_t = \alpha' + \beta'^{-}\Delta^{-}y_t + \beta'^{+}\Delta^{+}y_t + \varepsilon'_t$. t-statistics are in parenthesis. *,**,*** are for statistical significance at 0.01, 0.05, 0.1 levels, respectively. [a] denotes the use of second-differenced data for y_t and u_t. F-stat is a Wald test for $H_0 : \beta^{-} = \beta^{+}$ and $H'_0 : \beta'^{-} = \beta'^{+}$.

IV. CONCLUSION

The paper has provided a renewed assessment of Okun's relationship, using quarterly data over twenty years, including the disruption from the major crisis experienced in 2008. The main conclusion is the validation of the relationship between unemployment and growth in most major economies. This result is important because it reinforces policies acting either on the labour market to boost growth through better utilization of manpower resources or, conversely, on markets for goods and services to solve unemployment problems.

Previous studies had already reported fairly significant differences across countries. Our results confirm it. Despite these differences, a benchmark for the Okun's coefficient emerged from past studies. This benchmark was 2. We show that, in developed countries, over the last twenty years, the cost for growth of one more point in the unemployment rate lies more, from one country to another, in a range between 1.2 and 1.6 and we propose this interval should become the new benchmark for the Okun's coefficient.

This research has also showed the interest of considering simultaneously the output-unemployment relationship and the reverse one since there is no theoretical evidence for the supremacy of one relation to the other. Depending on the view, a different picture of the international levels of the coefficient is obtained.

Given the severity of the crash which occurred in 2008, the paper has tried to assess its effect on the Okun's coefficient. It appears from the test of structural break and from rolling regression that the coefficient is on the rise since 2008 in some countries (Canada, Germany, Sweden, the UK, France and Spain too but with less intensity) meaning that growth is reacting more significantly to changes in unemployment,but not for the whole sample.

The significant cyclical fluctuations which occurred during the period offered the opportunity to test for a possible asymmetry in the coefficient. Econometric results showed that, in Anglo-Saxon countries the labour market is more responsive to a negative change in the goods market than it is to a positive change. It means that firms use unemployment as a favorite weapon to react to sales difficulty. Conversely, they magnified Netherland as an opposite model of institutional organization of the labour market.

Finally, the obtained results show that one should be cautious when referring to that relationship as a 'law'. To the initial question asked in section 2, the answer is rather negative both on temporal and spatial consideration, because we saw that the coefficient can vary significantly from one period to another and from one country to another. Even if we share many conclusions with Ball and alii' s recent study, we are a little bit reluctant to conclude, as they do, that "*Okun's law has earned its name*".

APPENDIX

In the following lines we present results for stationarity and cointegration tests. Unit root tests have been conducted on the basis of the Augmented Dickey-Fuller test as follows:

i) First, we regress $u_{i,t}$ and $y_{i,t}$ on past values including a time trend t and a constant α_i, using the following regression model (model A):

$$\Delta x_{i,t} = \alpha_i + \gamma_i t + \phi_i x_{i,t-1} + \sum_{j=1}^{j=p_i} \beta_{i,j} \Delta x_{i,t-j} + \varepsilon_{it} \tag{11}$$

where $x_{i,t}$ is for $u_{i,t}$ or $y_{i,t}$, Δ is the first difference operator and p_i is the optimal number of lags to use for country i given by Akaike information criterion, Hannan-Quinn information criterion and Schwartz criterion (in the few cases where two different numbers of lags were given, we test the two solutions). If $H_0 : \gamma_i = 0$ is rejected at 5% in favor of $H_a : \gamma_i < 0$ following critical values given by D-F tables, model A is chosen.

ii) If H_0 cannot be rejected, reestimation is made without a time trend (model B). If $H_0 : \alpha_i = 0$ is rejected, model B is chosen.

iii) if H_0 cannot be rejected, we estimate equation 11 without a constant (model C).

Existence of a unit root process is conditioned by $\phi_i = 0$ and the test $H_0 : \phi_i = 0$ versus $H_1 : \phi_i < 0$ is applied to the chosen A, B, or C model. Diagnosis of unit root is made using the t-statistic for ϕ_i and the presence of unit root is accepted if H_0 cannot be rejected at the 5% critical value given by the Dickey-Fuller tables.

TABLE A1

ADF-tests for u_{it} and y_{it}

Country	*N° of lags*	*ADF t-stat for ϕ_u* †	*N° of lags*	*ADF t-stat for ϕ_y* ††
Australia	2	−1.32	1	−3.38**
Canada	2	−1.14	2	3.74
France	3	−0.55	2	2.45
Germany	3	−1.10	2	−3.48**
Italy	3	−0.41	2	0.89
Japan	2	−0.44	1	1.82
Netherlands	3	−3.56*	2	2.35
Spain	2	−0.94	2	1.27
Sweden	3	−3.28**	2	3.00
UK	3	−1.16	4	2.65
USA	3	−0.73	2	2.99

Notes: † Adequate model is model C for all countries, except for the Netherlands and Sweden (model B). Critical 5% DF-value for $t_{\hat{\phi}}$ is −1.95 for model C and −2.90 for model B. †† Adequate model is model C for all countries, except for Australia (model B) and Germany (model A). Critical 5% DF-value for $t_{\hat{\phi}}$ for model A is −3.46. *,** are for statistical significance at 0.01, 0.05 levels, respectively.

For the unemployment rate (see Table A1, column 3 and note) model C is well suited for all countries except for the Netherlands and Sweden. Unit root tests show that the unemployment rate is not stationary, except for these two countries.

For the real GDP time series, model C is again more appropriate for most countries as Table 1, column 5 shows. Exceptions are Australia and Germany. The existence of a unit root for real GDP time series cannot be explicitely rejected for most countries. Australia and Germany are exceptions.

All series can be first differenced to obtain stationarity. Model A' is tested. It differs from model A by replacing $x_{i,t-1}$ with $\Delta x_{i,t-1}$ and $\Delta x_{i,t}$ with $\Delta^2 x_{i,t}$ in equation (11) (and similarly in models B' and C'). Table A2 gives the result of unit-root tests for first differenced variables. Concerning unemployment, model C' is well adapted and leads to stationarity for all countries. For the GDP series, model C' is again well adapted, except for Australia (model A') and Sweden (model B'). First differenced series are stationary, except for Spain. Differencing again Spanish GDP and using four lags does not allow to reject stationarity (ADF t-stat = −5.89 in a model with neither trend nor intercept).

Prior exploration shows that differencing data can lead to stationary time series and avoid spurious regression. Nonetheless, if $u_{i,t}$ and $y_{i,t}$ are cointegrated, a regression using $\Delta y_{i,t}$ and $\Delta u_{i,t}$ can suffer for a bias, omitting the cointegrating terms (Engle and Granger, 1987, p. 259). Then we need to test for cointegration before using those differenced variables.

TABLE A2

ADF-tests for Δu_{it} and Δy_{it}

Country	*N° of lags*	*ADF t-stat for* $\phi_{\Delta u}$ †	*N° of lags*	*ADF t-stat for* $\phi_{\Delta y}$ ††
Australia	1	-4.81^{*}	0	-11.54^{*}
Canada	1	-4.62^{*}	1	-4.98^{*}
France	2	-3.46^{*}	2	-3.60^{*}
Germany	2	-2.82^{**}	1	-5.11^{*}
Italy	2	-2.89^{**}	1	-4.02^{*}
Japan	1	-4.51^{*}	0	-7.82^{*}
Netherlands	1	-2.30^{**}	2	-3.26^{**}
Spain	1	-2.84^{**}	4	-1.53
Sweden	1	-3.82^{*}	4	-4.95^{*}
UK	1	-3.25^{*}	3	-2.37^{**}
USA	2	-2.93^{*}	2	-3.58^{*}

Notes: † Adequate model is model C' for all countries. Critical 5% DF-value for $t_{\hat{\phi}}$ is -1.95. †† Adequate model is model C' for all countries, except for Australia (model A'), France, Germany, Netherlands, Sweden and USA (model B'). Critical 5% DF-value for $t_{\hat{\phi}}$ is -1.95 for model C', -2.90 for model B' and -3.46 for model A'. *,** are for statistical significance at 0.01, 0.05 levels, respectively.

We know from Granger (1981) that, if two series are $I(1)$, a linear combination of the two which is stationary $(I(0))$ can possibly exists. In this case, the two series can evolve in different directions in the short-run, due to their non-stationarity but they share a common and stable long-run relation. The test of cointegration implemented by Engle and Granger (1987) for two $I(1)$ variables is a two-step process which first gives an estimate of the long run relation using the following regression:

$$y_{i,t} = \beta_{0i} + \beta_{1i} u_{i,t} + \eta_{i,t} \tag{12}$$

and then applies a unit-root test on lagged residuals obtained at the first step. An ADF derived form of this test is :

$$\Delta\hat{\eta}_{i,t} = \phi\hat{\eta}_{i,t-1} + \sum_{j=1}^{j=p_i} \theta_{i,j} \Delta\hat{\eta}_{i,t-j} + \nu_{i,t} \tag{13}$$

If $t_{\phi} <$ critical value, the H_0 hypothesis of no cointegration is rejected and the two variables are cointegrated. If $t_{\phi} \geq$ critical value, H_0 is accepted and the two variables are not cointegrated.

As reported in Table A3, t-stat is always greater than the 5% critical value. No cointegration relation can be detected for any country of our panel which includes ten countries of Lee's panel and his conclusion that "output and unemployment share at least one cointegrating relation" (p. 343) cannot be confirmed in our study.[12] With more recent, quarterly data, we do not find such evidence.

TABLE A3

Test for cointegration between u_{it} *and* y_{it}

[12] Spain is not to be found in Lee's panel.

Country	*N° of lags*	*ADF t-stat for* $\phi_{\hat{\eta}}$ †
Australia	3	3.98
Canada	2	2.95
France	2	−1.79
Germany	2	−1.90
Italy	3	−0.44
Japan	2	1.17
Netherlands	2	−1.42
Spain	2	−1.06
Sweden	2	−1.96
UK	2	−1.94
USA	3	−1.13

Notes: † Critical 5% *t*-stat given by McKinnon (1991) is −3.34.

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