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INVERSE SIMULATION OF SUNLIGHTING : A GEOMETRICAL FRAMEWORK FOR ARCHITECTURAL AND URBAN DESIGN

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ABSTRACT

We propose a framework based on integral geometry that enables to express and to solve all sunlighting problems : the direct problems for assessing sunlighting in a scene, and the inverse ones for achieving sunlighting constraints in a design context. The heart of our method is the sunlighting volume that we denote ∏(P, T). We describe this notion and we show how to implement it in an architectural and urban design process.

RÉSUMÉ

Nous proposons un formalisme géométrique qui permet d'exprimer et de résoudre tous les problèmes d'ensoleillement : les problèmes directs pour évaluer l'ensoleillement d'une scène, et les problèmes inverses pour satisfaire des contraintes d'ensoleillement en situation de conception. Le cœur de notre formalisme est la pyramide d'ensoleillement. Nous décrivons cette notion et nous précisons sa mise en œuvre pour la conception architecturale ou urbaine.

INTRODUCTION

We distinguish the *direct* sunlighting problems from the *inverse* ones. The former consist in determining the sunlighting states of a given scene using both shadows and shadings ; the latter consist in resolving the geometrical conditions that enable a scene to achieve a given sunlighting constraint in a design context (sunlighting as formgiver [1]).

In this paper, we propose a geometrical framework based on integral geometry [2] for managing all sunlighting problems. This framework offers three main benefits : it gives homogeneous expressions and solutions of the direct sunlighting problems of shadings and shadows ; it enables the generalisation of these problems either in space or time ; it offers solutions to the inverse problems of sunlighting.

The heart of our framework is the sunlighting volume that we denote ∏(P, T). We define this notion in the first section, within a set of geometrical notions that make up our method. In the second and third sections, we give solutions to the direct and inverse problems. Finally, we show how these methods can be implemented in an existing CAD system, and why they offer a powerful framework to cope with sunlighting problems in architectural and urban design.

1. THE INTEGRAL GEOMETRY OF SUNLIGHTING

Let ∑ be a geometrical scene located at the latitude □. We denote *p* any point of ∑ and P any continuous set of points in ∑. We consider D□ the set of all the apparent solar directions at the latitude □, and we denote *t* any direction included into D□ (i.e. any instant of the solar year). We define a time period T as any continuous set of instants *t*, that is consequently a composition of intervals of instants among days and months. T can likewise be seen as a geometrical patch of D□. Finally, we define the sunbeam *R(p, t)* as the half line starting from *p* in the direction of t. We define the inverse sunbeam $R(p,-t)$ starting from p in the opposite direction of t (which is the perceived natural sunbeam).

1.1 Sunlighting volumes

Let us consider now sunlighting volumes settled as continuous set of sunbeams. First, we define the sunlighting pyramid $\pi(p, \mathrm{T})$, that is the set of sunbeams starting at the point p for all the instants t of the time period T : $\pi(p, \mathrm{T}) = \{R(p,t), t \in \mathrm{T}\}$. In the same way, we define the sunlighting prism □(P, t), that is the set of sunbeams defined for all the points p of P and for the single instant t : □(P, t) = $\{R(p,t), p \in \mathrm{P}\}$. Finally, we define the complex sunlighting volume $\prod(\mathrm{P}, \mathrm{T})$ as the set of sunbeams starting from all the points p of the set P for all the directions t of the time period T, that is : $\prod(\mathrm{P}, \mathrm{T}) = \{R(p,t), p \in \mathrm{P}, t \in \mathrm{T}\}$. Notice that $\prod(\mathrm{P}, \mathrm{T})$ can be equally considered as the union of simple sunlighting pyramids $\pi(p, \mathrm{T})$ when p draws P, or as the union of prisms □(P, t) when t draws T, that is (fig. 1) :

$$\prod(\mathrm{P}, \mathrm{T}) = \bigcup_{p \in P} \pi(p, \mathrm{T}) = \bigcup_{t \in \mathrm{T}} □(\mathrm{P}, t) \qquad (1)$$

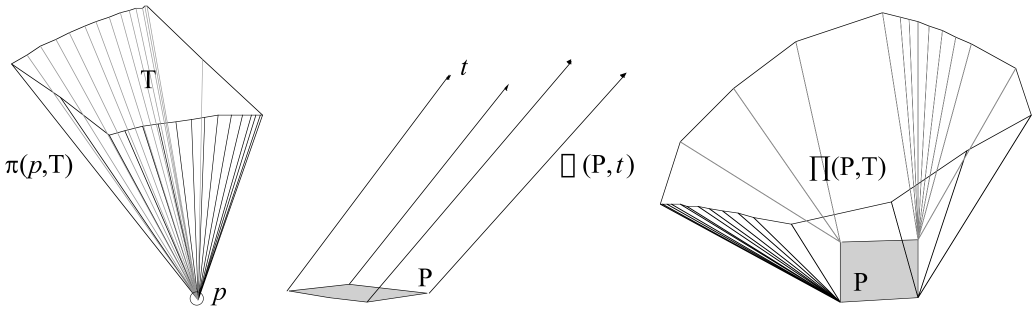

Figure 1: The three kinds of sunlighting volumes

1.2 Cores of sunlighting volumes (solar envelopes)

We define $\partial\prod(\mathrm{P}, \mathrm{T})$ the core of $\prod(\mathrm{P}, \mathrm{T})$ as the intersection of the sunlighting prisms □(P, t) when t draws T, that is :

$$\partial\prod(\mathrm{P}, \mathrm{T}) = \bigcap_{t \in \mathrm{T}} □(\mathrm{P}, t) \qquad (2)$$

The core $\partial\prod$ embodies the set of all the points of $\sum$ that are affected by all the sunbeams defined by all points of P during all instants of T. In other terms, any point of $\sum$ within $\partial\prod$ will have its shadow within P whatever is the instant of T. This means that the shadows of any volume built in $\partial\prod$ will never spill over the edge of P during T (fig. 2). This is the definition of Ralph Knowles' solar envelopes, as a container to regulate development within limits derived from the sun's relative motion [3]. We offer here a geometrical framework for defining and using this notion usually handled with empirical geometrical methods.

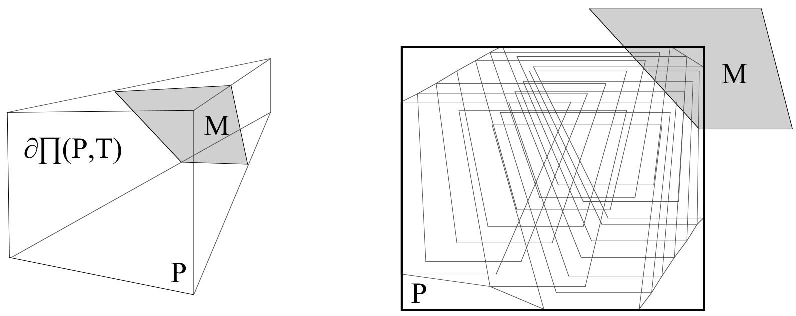

Figure 2: M defined in the core of a sunlighting volume (left) and its shadows during T (right)

2. DIRECT SUNLIGHTING PROBLEMS : EXPRESSION AND SOLUTIONS

Direct problems of shadows and shadings can be easily expressed, generalised either in time and space, and geometrically solved using integral geometry.

2.1 Generalised shadows

Let Ω(P, *t*) be the shadow of P in the scene ∑ for the instant *t*. By definition, Ω(P, *t*) is the intersection between the sunlighting prism $\square$(P,-*t*) and ∑, that is : Ω(P, *t*) = $\square$(P,-*t*) ∩ ∑. If we consider the time period T instead of the instant *t*, we can express the generalised shadow of P during T, that is the union of all the shadows of P for all the instants *t* of T :

$$\Omega(P, T) = \bigcup_{t \in T} \Omega(P, t) = \bigcup_{t \in T} (\square(P,-t) \cap \Sigma) = \prod(P,-T) \cap \Sigma \qquad (3)$$

Therefore, the generalised shadow of P during T is given by the intersection between the sunlighting volume ∏(P,-T) and the objects of ∑. This is the exact set of points of ∑ that are exposed to the shadow of P during at least one instant of T.

In the same way, we prove that the exact set of points of ∑ that are exposed to the shadow of P during all the instant of T are those that are in the core of the sunlighting volume ∂∏(P,-T). Then we denote the core of shadow : ∂Ω(P, T) = ∂∏(P,-T) ∩ ∑ (fig. 3 left).

Obviously, if P is an opening instead of a shading, then the generalised shadow is the union of all sunspots generated by P during T (generalised sunspot), and the core of shadow is the set of points that are always sunlit through P during T (core of sunspot, see fig. 5).

2.2 Generalised shading

Let M(*p*, T) be the shadings of the scene ∑ on the point *p* for the period T. M(*p*, T) is the intersection between the simple sunlighting pyramid π(*p*, T) centred in *p* and the volumes of the scene : M(*p*, T) = π(*p*, T) ∩ ∑. If we want to determine the shadings of the scene ∑ on the set of points P for the time period T, that is all the points of ∑ that may shade at least one point of P during T, then we obtain :

$$M(P, T) = \bigcup_{p \in P} M(p, T) = \bigcup_{p \in P} (\pi(p, T) \cap \Sigma) = \prod(P, T) \cap \Sigma \qquad (4)$$

The generalised shadings on P during T are given by the intersection between the sunlighting volume ∏(P, T) and the objects of the scene (fig. 3 right). We also prove that any point within the core of ∏(P, T) shade at least one point of P for all the instants of T. In other words, any object within ∂∏ is a permanent shading on P during T.

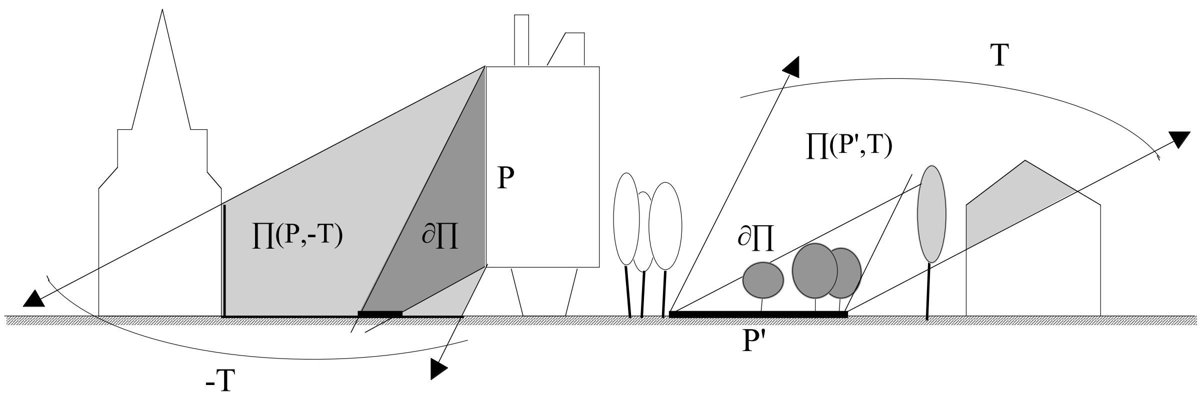

Figure 3: Generalised shadows (left) and generalised shadings (right)

2.3 Discussion

Geometrically speaking, sunlighting problems have to be considered in a 4D space (3D plus time). However, graphical simulation means have led to solve these problems by reducing them to 3D volumes, using a discretization of space or of time, and then, by projecting this 3D volumes onto 2D charts (axonometric projections for shadows at a discrete instant, and perspective projections for shadings on a discrete point). When computer is used, the same methods are usually transposed in a digital manner.

We have shown in this section that both problems of shadows and shadings *i)* can easily be generalised either in time and space, and *ii)* can be expressed and solved in an homogeneous way, as an intersection between the scene and a given volume ∏(P, T). Whatever are P and T (single point, single instant or continuous sets of points and instants), we define the sets of points of ∑ that involve simultaneously the points of P and the instants of T. Only the sign of T changes (the direction of the sunbeams).

3. INVERSE PROBLEMS OF SUNLIGHTING

Direct simulation tools developed for integrating solar constraints within architectural and urban design appraise the solar properties of geometrical forms that are fully ascertained. Supposing the forms do not achieve previous fixed solar goals, the process of simulation must be fully repeated until a good solution is found. One should consider two major drawbacks of such an analytic approach : it imposes a trial and error process inappropriate for design purpose ; it can neither provide the best solution that may achieve the goals, neither indicate the whole set of solutions if any, neither specify if there are no solution.

The paradigm of inverse simulation has been proposed in order to overtake these drawbacks [4]. The inverse problems of sunlighting can be expressed in a general way : given a set of points P and a time period T, these consist in achieving the following constraint : « P must check on the sunlighting state S during T ». In a more formal way, let us consider a sunlighting constraint as a logical proposal (P, S, T). Solving the inverse problem consists in making the sunlighting proposal true. Therefore, the solutions of a proposal (P, S, T) are the transformations of the scene ∑ that make P be S during T. These are openings and closings of the scene, that is geometrical transformations handled by architects during the design process. Let's study the solutions for the elementary proposals (P, *sunlit*, T) and (P, *sunless*, T).

3.1 Elementary proposals (P, *sunlit*, T)

The solutions for these proposals are the openings of the scene ∑ such as all the points of P become *sunlit* for all the instants of T. Therefore, the set of solutions of the proposal (P, *sunlit*, T) is the set of all the openings of the scene that contain the volume ∏(P, T). It exists a single minimal solution O = ∏(P, T) ∩ ∑, that is the opening by the exact intersection between the scene and the sunlighting volume. Obviously, if this intersection is already empty, the proposal is already true and the opening has no effect.

3.2 Elementary proposals (P, *sunless*, T)

The solutions for these proposals are the closings of the scene ∑ such as all the points of P become *sunless* for all the instants of T. There exists an infinite set of solutions that are all the objects that intersect all the sunbeams of ∏(P, T) (fig. 4). Therefore, as well as a generator of sunlighting volumes, an implementation of the method within a 3D CAD engine should consider a method for exploring this set of solutions. In [5], we propose such a method by handling a plane that interactively intersects the volume ∏(P, T) and displays the shadings.

A point should be discussed here. If P is exactly the same for all the closings M defined with ∏(P, T), we notice that each generalised shadow Ω(M, T) is distinct. Obviously, the shadings M close to the plane of P create generalised shadows close to P, while the shadings far from the plane of P create very large generalised shadows. That means that each solution M of the proposal (P, *sunless*, T) can be distinguished by its own generalised shadow (intersection between ∏(M,-T) and the scene ∑). This gives a useful means for checking up the effects of each closing achieving a (P, *sunless*, T) proposal while exploring the set of its solutions.

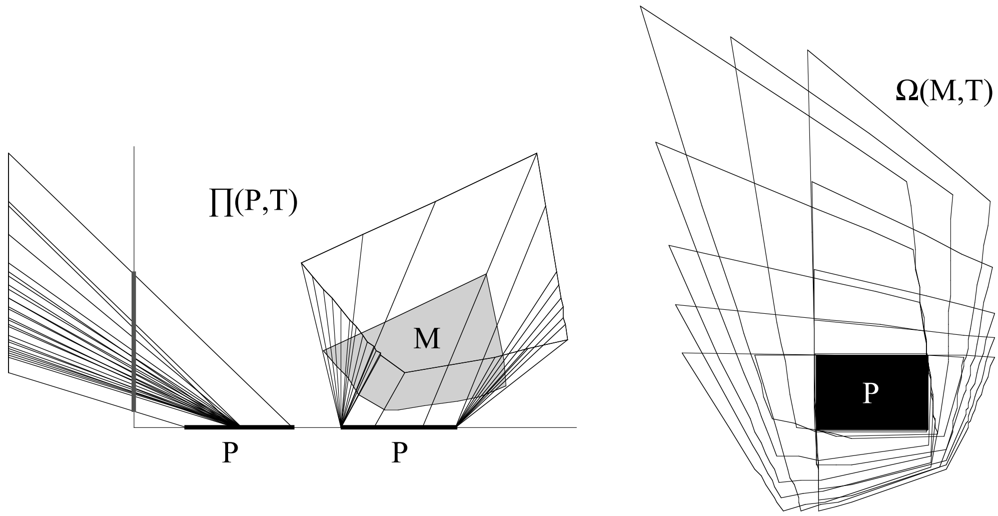

Figure 4: M is a section of ∏(P,T) (left and middle). The intersection of the shadows of M for all instants of T is equal to P (right).

4. Implementation

4.1 Computing 3D boundaries of sunlighting volumes

This geometrical problem is not self-evident. In [5] we propose a solution for computing the boundary of ∏(P, T) when P is a convex plane polygon. In that case, we show that any section between ∏(P, T) and a plane Q parallel to the plane of P can be figured as the Minkowski sum between P and the projection of T onto Q (that is, a computable geometrical operation). Moreover, the intersection between ∏(P, T) and Q owns a geometrical structure that enables to determine all the external beams of ∏. Using this approach, we have developed a generator of 3D sunlighting volumes usable in a CAD system [5].

4.2 Using sunlighting volumes within a CAD system

A suitable CAD system for helping designers to cope with sunlighting problems only needs to compute the intersections between some defined sunlighting volumes and the objects of the scene. No more charts or geometrical constructions are needed. Only intuitive orders or questions are used such as : « Is this face sunlit during the afternoon in winter ? », or « What is the shadings of these buildings in the morning in summer ? », and « I want this room to be sunless during T ». Such a system can make a diagnosis and it can provide an intelligent way for managing sunlighting constraints throughout the evolution of the design. The constraints are visualised as any other 3D volumes and their solutions can be explored interactively.

For instance, figure 5 shows how to design a window by controlling is total sunspot within a room for a given time period. Moreover, we can simultaneously adjust the whole limit of the spot given by the window on T (generalised sunspot) and the limit of the set of points that are always sunlit by the window during T (core of sunspot).

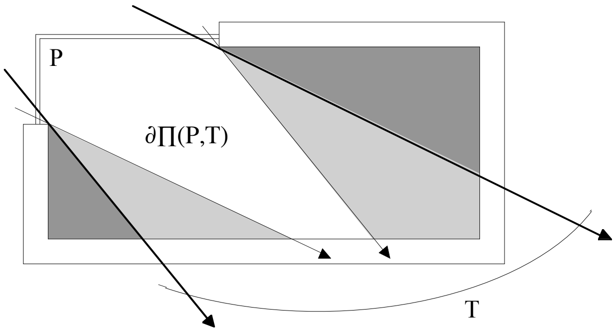

Figure 5: Interactive design of a window considering the generalised sunspot and its core

In addition, the framework enables to solve the composition of sunlighting proposals in a geometrical way. The point is to consider that the volumes $\prod(P_i, T_k)$ associated to the proposals (P_i, S_j, T_k) embody a geometrical representation of the corresponding constraints. The set of solutions of two constraints is found at the intersection or in the geometrical difference between the associated volumes. In figure 6 (left), we show an opening O defined such as P is sunlit for all instants of T, and a shading M such as the same P is sunless during T' (disconnected from T). We verify the achievement of these constraints for all instants of T and T' with direct simulation (figure 6, middle and right).

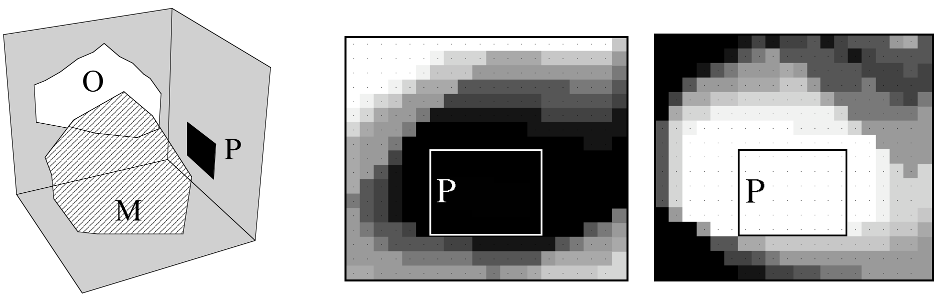

Figure 6: Solutions of two sunlighting proposals (left) and verifications (middle and right)

4.3 Rendering and meshing

Other possibilities of the framework should be mentioned. For instance, it can help the pre-processing for rendering very large scenes, like urban scenes. Supposing that T is the entire solar year, and P any face of the scene, then only the faces that intersects $\prod(P, T)$ have to be taken into account for computing shadows on P (whatever is the considered instant). In the same way, the sunlighting volumes may be helpful for computing the discontinuity meshing of a scene. Indeed, the volume $\prod(P, T)$ gives the limit of the points that are always exposed to P during T (the core) and the limit of the points that are never exposed to P during T. For solar assessment, the only required meshing is between these two limits.

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