Measuring Well-Being and Lives Worth Living

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Abstract

We study the measurement of well-being when individuals have heterogeneous preferences, including different conceptions of a life worth living. When individuals differ in the conception of a life worth living, the equivalent income can regard an individual whose life is not worth living as being better off than an individual whose life is worth living. In order to avoid that paradoxical result, we reexamine the ethical foundations of well-being measures in such a way as to take into account heterogeneity in the conception of a life worth living. We derive, from simple axioms, an alternative measure of well-being, which is an equivalent income net of the income threshold making lifetime neutral. That new well-being index always ranks an individual whose life is not worth living as worse-off than an individual with a life worth living.

Keywords: Well-being, measurement, equivalent income, lifetime, value of life.

JEL classification codes: I31, J17.

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Introduction

In the recent years, the equivalent income has become an increasingly used indicator of well-being, with various applications for well-being measurement and comparisons within and between countries.[1] Defined as the hypothetical income which, combined with references on non-monetary dimensions of life, would make an individual indifferent with respect to his current situation, the equivalent income constitutes a preference-based indicator of well-being that is inclusive of (potentially) all non-monetary dimensions of well-being (see Fleurbaey and Blanchet 2013, Fleurbaey 2016). Its specificity is to allow for the weighting of the different dimensions of life, while being respectful of how individuals evaluate the different dimensions of their life.

Early uses of the equivalent income approach, such as Usher (1973, 1980), focused on a simple (income, lifetime) space. Given that lifetime is a central dimension of human well-being (Sen 1998), it does not come as a surprise that many studies using the equivalent income approach have focused on the (income, lifetime) space, or on the (income, life expectancy) space. Such studies include, in a historical context, Williamson (1984), Crafts (1997) and Costa and Steckel (1997), as well as Nordhaus (2003) and Becker et al (2005).

In order to compute equivalent incomes based on real-world data, most existing studies assume some structure for individual preferences in the (income, lifetime) space. The postulated functional form for individual utility represents not only how a representative individual weights life-years against income, but, also, defines a threshold level for income below which life is not worth living, and above which life is worth living (see Becker et al 2005). More precisely, those studies define a critical income level, above which lifetime is a desirable good, and below which lifetime is an undesirable good.[2]

Assuming the existence of a critical income level making lifetime neutral makes a lot of sense. If such a critical income level did not exist, then it would be the case either that *any* life, whatever living conditions are (even extremely miserable conditions), would be worth living, or, alternatively, that *any* life, whatever standards of living are (even excellent living conditions), would be not worth living. Such a corollary is implausible. Hence it makes sense to assume that there exists a critical income level making lifetime neutral, and which defines income intervals where more life-years is either desirable, or undesirable.

Whereas assuming a critical income level making lifetime neutral is justified, the existing literature on equivalent income did not, so far, examine all corollaries of that particular assumption for the measurement of well-being. A particularly interesting implication concerns the comparison of well-being between individuals who have *unequal* critical income levels, and, thus, who do not share the same conception of a life worth living. It is actually possible that the equivalent income associated to the life of a person who regards his life as not

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[1] Recent applications include Decoster and Haan (2015), Carpentier and Sapata (2016), Decancq and Neumann (2016), Ponthiere (2016), Decancq et al (2017) and Onder et al (2018).

[2] The concept of critical income making lifetime neutral is the equivalent, in income terms, of Broome's concept of a utility level neutral for the continuation of existence (Broome 2004).
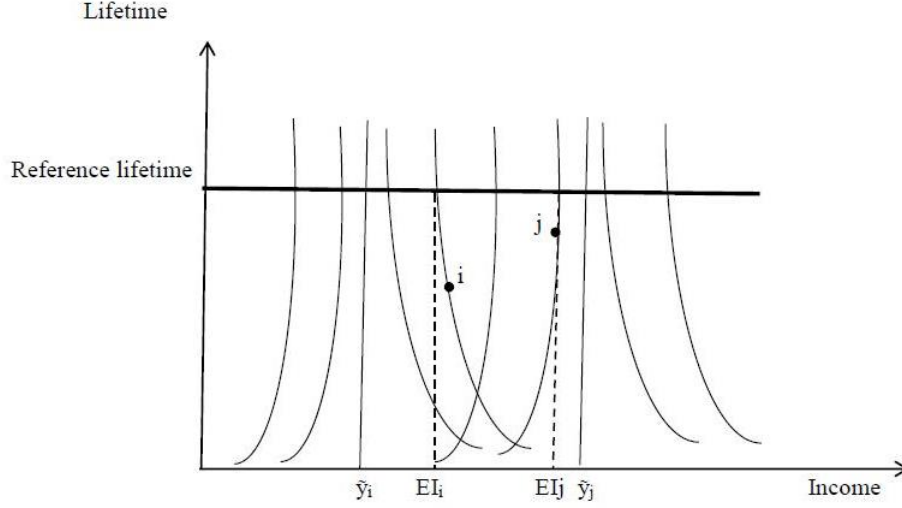
Figure 1: A paradox for the equivalent income.

worth living can exceed the equivalent income associated to the life of another person, who regards his life as worth living. That anomaly, first identified in Onder et al (2018), is illustrated on Figure 1.

Individuals i and j have distinct preferences, represented, on Figure 1, by distinct indifference maps in the (income, lifetime) space. In particular, those two individuals have different critical income levels $\widetilde{y_i}$ making lifetime neutral, and thus at which a vertical indifference line is defined. Individual i benefits from lower living standards than individual j, but he regards his life as worth living. On the contrary, individual j considers that his life is not worth living. Indeed, although individual j benefits from better living conditions, his conception of a life worth living is more demanding, and at the bundle represented on Figure 1, individual j considers his life not worth living. Figure 1 shows also the equivalent income (computed while defining the maximal lifetime as the reference lifetime) takes a higher level for individual j than for individual i, despite the fact that the former regards his life as not worth living, whereas the latter regards his life as worth living. Thus the equivalent income considers that the individual whose life is not worth living is *better off* than the one whose life is worth living.

That result is somewhat paradoxical. Regarding his life as worth living or not is definitely a central component of individual preferences. Hence, it is paradoxical that a preference-based indicator of well-being such as the equivalent income regards a person whose life is not worth living as strictly better off than

3

a person whose life is worth living. Of course, this paradox does not arise when individuals share the *same* conception of a life worth living. But there is no reason to assume *a priori* that all individuals share the same conception of a life worth living. If there exist as many preferences as there are individuals, for sure this aspect of life valuation is not uniform across all individuals.

Hence, in the light of that paradoxical result, one may want to reexamine the construction of equivalent income well-being indexes in the context of distinct conceptions of a life worth living. Note that, when facing that problem, one may simply argue that the person with a more demanding conception of a life worth living exhibits some form of "expensive tastes", and that a well-being measure should not be sensitive to such expensive tastes. But that response, too, invites a revision of the well-being measure that is used in the present context of unequal conceptions of life worth living.

The goal of this paper is precisely to reexamine the construction of a measure of well-being when individuals differ regarding their conception of a life worth living. For that purpose, we consider a model of the human life cycle, where individuals differ in preferences on bundles in the (income, lifetime) space, in particular concerning the definition of a life worth living. Then, we propose to build a well-being index on the basis of several intuitive properties.

Anticipating our results, we first show that the Pareto Principle, the Conditional Priority axiom and the Translation axiom suffice to fully characterize a new measure of well-being, which is the equivalent income index net of the critical income making lifetime neutral. That new well-being measure is, by construction, immunized against the paradoxical result highlighted at the very outset of this paper. Two new axioms are used for his characterization. On the one hand, Conditional Priority concerns interpersonal well-being comparisons under a shared conception of the life worth living. That axiom states conditions on bundles under which the fact of carrying more about longevity than about income makes a person either better off or worse off than another person sharing the same bundle but having different preferences. On the other hand, the Translation axiom concerns interpersonal well-being comparisons when individuals differ in their conception of a life worth living, their indifference maps being translations from each other. That axiom states that, when measuring well-being, what matters is the distance between the bundle under comparison and the vertical indifference line defined at the critical income level characterizing a neutral life. In a second stage, we characterize an alternative equivalent income index, also net of the critical income making lifetime neutral, but relying on different reference lifetime levels. That alternative well-being index relies on a variant of the Conditional Priority axiom, which regards individuals more concerned with longevity as always worse off than individuals less concerned with longevity, contrary to the initial Conditional Priority axiom.

This paper is related to several branches of the literature. First, it is related to the general literature on the measurement of well-being (see Adler and Fleurbaey 2016). In particular, it is related to the increasingly large literature on the construction of equivalent incomes (see Fleurbaey and Blanchet 2013, Decancq and Neumann 2016, Decancq and Schokkaert 2016, Onder et al 2018).

4

Second, given that the construction of well-being indexes is often a first stage in the design of optimal policies under heterogeneous preferences, this paper is also related to the normative literature on the fair allocation of resources under unequal lifetime, such as Fleurbaey and Ponthiere (2013) and Fleurbaey et al (2014). Those papers relied on consumption equivalents under heterogeneous preferences, and may be subject to the paradoxical result discussed at the very outset of this paper.

This paper is organized as follows. The model is presented in Section 2. Section 3 presents the axioms. The characterization of the equivalent income net of the critical income making lifetime neutral is developed in Section 4. Section 5 examines the characterization of an alternative well-being index, based on a variant of the Conditional Priority axiom. Section 6 concludes.

The model

The population is a set N of individuals $i, j, \dots$ Given that we want to examine interpersonal well-being comparisons, we assume that the number of individuals in N is at least two.

Each individual life is characterized by a (constant) income per period $y_i \in \mathbb{R}_+$ and a lifetime $L_i \in \left[0, \overline{L}\right]$.

Each individual i has well-defined preferences $\preceq_i$ on bundles (y, L) that are composed of (constant) income per period y and of lifetime L. Let $\mathcal{R}$ denote the set of preferences that are monotonic in income and convex.

As usual, the preference relation $\preceq_i$ is assumed to be complete, reflexive and transitive. Strict preference is denoted by $\prec_i$, while indifference is denoted by $\sim_i$.

An indifference curve passing through the basket is defined as follows:

$$IC\left(y, L, \preceq_i\right) = \left\{(y', L') : (y', L') \sim_i (y, L)\right\}.$$

We say that $IC\left(y, L, \preceq_i\right)$ is steeper than $IC\left(y, L, \preceq_j\right)$ if for all $(y', L') \in IC\left(y, L, \preceq_i\right)$, one has $(y', L') \succ_j (y, L)$ if $L' > L$ and $(y', L') \prec_j (y, L)$ if $L' < L$. Note that this implies single-crossing of the two curves: $IC\left(y, L, \preceq_i\right) \cap IC\left(y, L, , \preceq_j\right) = \{(y, L)\}$.

For each individual i, the indifference map $IM(\preceq_i)$ is defined as the set of all indifference curves $IC\left(y, L, \preceq_i\right)$. The indifference maps $IM\left(\preceq_i\right)$ is a translation of $IM\left(\preceq_j\right)$ whenever there is $z \in \mathbb{R}_+$ such that for all $(y, L), (y', L')$, one has $(y, L) \succeq_i (y', L')$ if and only if $(y + z, L) \succeq_j (y' + z, L')$.

For an individual i, we define the lower contour set at the bundle (y, L) as:

$$\mathbb{L}(y, L, \preceq_i) = \{(y', L') : (y, L) \succeq_i (y', L')\}$$

For an individual i, we define the upper contour set at the bundle (y, L) as:

$$\mathbb{U}(y, L, \preceq_i) = \{(y', L') : (y, L) \preceq_i (y', L')\}$$

For each individual i, there exists a critical income level $\widetilde{y_i} > 0$ such that for all bundles with $y > \widetilde{y_i}$, lifetime is a desirable good, whereas for all bundles with $y = \widetilde{y_i}$, lifetime is a neutral good, whereas for all bundles with $y < \widetilde{y_i}$, lifetime is an undesirable good. Observe that the subset $\{(y, L) : y = \widetilde{y_i}\}$ is an indifference curve for every i.

The critical income level $\widetilde{y_i} > 0$ captures the idea of *a life worth living*. If such a critical income level did not exist, then either all lives would not be worth living (lifetime being always an undesirable good), or all lives would be worth living (lifetime being always a desirable good). Those two possibilities are implausible. Clearly, when living conditions are excellent, it is hard to see why a life would not be worth living. Moreover, when living conditions are extremely miserable, it is hard to see why a life would necessarily be worth living. Thus, it makes sense to assume that there exists a particular critical income level, which captures a critical quality of life, below which lifetime becomes an undesirable good, whereas above that threshold, lifetime is a desirable good. That assumption is largely made in the literature (see, for instance, Becker et al 2005). One can also interpret the critical income level making lifetime neutral as the equivalent, in terms of income, of Broome's concept of the "utility level neutral for the continuation of existence" (see Broome 2004).

In the rest of the paper, we want to construct a measure of individual well-being. Such a measure, denoted by $M(y, L, \preceq_i)$, assigns a real number to all bundles (y, L) in such a way as to describe those bundles from the perspective of well-being. We thus have: $M(y, L, \preceq_i) : \mathbb{R}_+ \times [0, \overline{L}] \times \mathcal{R} \to \mathbb{R}$. That measure is defined for all bundles, and for all individuals (actually, for all possible preferences in the domain $\mathcal{R}$), and allows for the comparison of well-being across individuals with potentially different bundles and different preferences.

Axioms

This section presents some properties that we will impose on the well-being measure $M(y, L, \preceq_i)$.

PARETO PRINCIPLE $\forall i \in N$, if $(y, L) \sim_i (y', L')$ then $M(y, L, \preceq_i) = M(y', L', \preceq_i)$, and if $(y, L) \succ_i (y', L')$ then $M(y, L, \preceq_i) > M(y', L', \preceq_i)$.

The Pareto Principle is standard: it merely states that a measure of well-being should respect individual preferences. It states that if one considers bundles that lie on the same indifference curve as the bundle (y_i, L_i), then the measured well-being should remain the same for all those bundles between which individual i is indifferent. However, if a bundle lies on a higher indifference curve, this should be assigned a higher level of well-being.

The next axiom, Conditional Priority, specifies some requirements that the measure $M(y, L, \preceq_i)$ should satisfy when considering interpersonal comparisons of well-being for individuals who enjoy the same bundle but have different preferences (except the critical income level making lifetime neutral, which is here assumed to be the same for all individuals under comparison).

Conditional Priority $\forall i,j \in N$, with $\widetilde{y_i} = \widetilde{y_j} = \widetilde{y}$, $\exists \overline{L^1}, \overline{L^2} > 0$ such that:

- if $L_i = L_j = L \leq \overline{L^1}$ and if $y_i = y_j = y < \widetilde{y}$, then if $IC(y, L, \preceq_i)$ is steeper than $IC(y, L, \preceq_j)$ at (y, L), then $M(y, L, \preceq_i) \leq M(y, L, \preceq_j)$.

- if $L_i = L_j = L \geq \overline{L^1}$ and if $y_i = y_j = y < \widetilde{y}$, then if $IC(y, L, \preceq_i)$ is steeper than $IC(y, L, \preceq_j)$ at (y, L), then $M(y, L, \preceq_i) \geq M(y, L, \preceq_j)$.

- if $L_i = L_j = L \leq \overline{L^2}$ and if $y_i = y_j = y > \widetilde{y}$, then if $IC(y, L, \preceq_i)$ is steeper than $IC(y, L, \preceq_j)$ at (y, L), then $M(y, L, \preceq_i) \geq M(y, L, \preceq_j)$.

- if $L_i = L_j = L \geq \overline{L^2}$ and if $y_i = y_j = y > \widetilde{y}$, then if $IC(y, L, , \preceq_i)$ is steeper than $IC(y, L, \preceq_j)$ at (y, L), then $M(y, L, \preceq_i) \leq M(y, L, \preceq_j)$.

Conditional Priority states that, when comparing individuals enjoying the same bundle but having different preferences (except the critical income level making lifetime neutral), whether social priority should be given to individuals who are more concerned with income or with lifetime depends on whether their life is worth living (and thus on whether income is above or below some critical level), and on whether lifetime is above or below some critical threshold.

More precisely, when we consider individuals enjoying the same bundle, but whose lives are not worth living, there exists a threshold for lifetime above which the individual who cares more about lifetime is regarded as the worst-off, and below which the individual who cares more about consumption is regarded as the worst-off. On the contrary, when lives are worth living, there exists another threshold for lifetime above which the individual who cares more about consumption is regarded as the worst-off, and below which the individual who cares more about lifetime is regarded as the worst-off.

The two lifetime thresholds $\overline{L^1}$ and $\overline{L^2}$ are ethical parameters that have a clear meaning. When considering lives not worth living, one can acknowledge that, when the life is sufficiently short, the worst-off individual is the one who cares more about income, and less about lifetime, whereas the opposite holds when the life not worth living is too long. The ethical parameter $\overline{L^1}$ acts as a threshold separating what is regarded as a life not worth living that is "sufficiently short" (so that social priority goes to the individual with more concern for income), or definitely "too long" (so that social priority goes to the individual with more concern for lifetime). In a similar way, the threshold $\overline{L^2}$ separates, on the one hand, a life worth living that is "insufficiently long", and which leads thus to give priority to individuals who care more about lifetime, and, on the other hand, a life worth living that is "sufficiently long", so that social priority must be given to individuals who care more about income.[3]

Figure 2 illustrates the requirements of Conditional Priority in terms of well-being comparisons. Figure 2 shows the four areas of the (income, lifetime) space

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[3] Obviously those two lifetime thresholds are ethical parameters, whose levels reflect value judgments. One could think, for instance, about levels such as 30 years and 70 years. 30 years would be the threshold below which lives not worth living are sufficiently short, whereas 70 years would be the threshold above which lives worth living are sufficiently long.
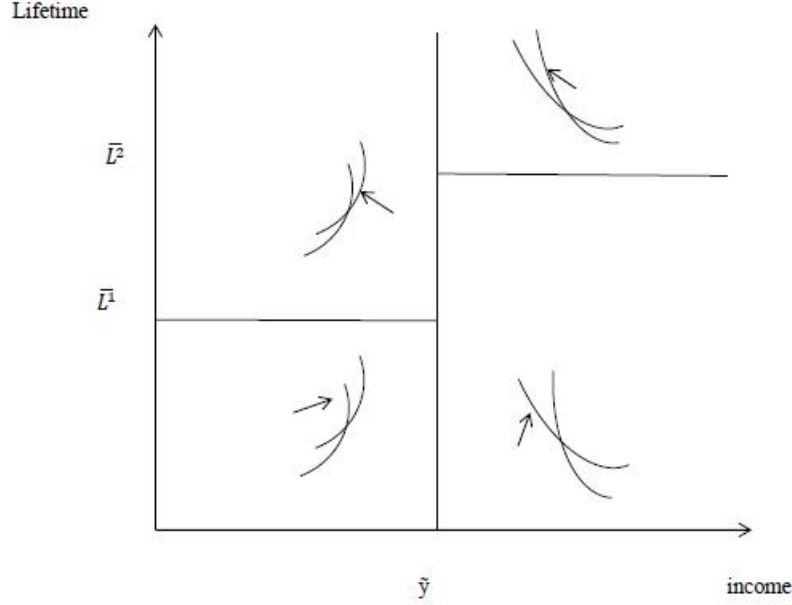
Figure 2: Conditional Priority

that are defined in the Conditional Priority axiom. Those areas are delimited by the critical income making lifetime neutral, as well as by the two thresholds for lifetime $\overline{L^1}$ and $\overline{L^2}$. The little arrow indicates the individual who is considered to have social priority under Conditional Priority.

Let us now consider a third axiom, the Translation axiom, which concerns the comparison of well-being between individuals who differ regarding the critical income level making lifetime neutral, but share all other dimensions of the indifference map. In other words, the Translation axiom concerns interpersonal well-being comparisons of individuals sharing the same indifference map up to a translation.

TRANSLATION $\forall i, j \in N$ such that $IM(\preceq_i)$ is a translation of $IM(\preceq_j)$, with $\widetilde{y_j} = \widetilde{y_i} + x$ with $x > 0$, if $L_i = L_j$ and if $y_i - \widetilde{y_i} = y_j - \widetilde{y_j}$, then $M(y_i, L_i, \preceq_i) = M(y_j, L_j, \preceq_j)$.

The Translation axiom states that, when the indifference map of an individual is a translation of the indifference map of another individual, then if their bundles exhibit the same lifetime and lie at the same distance of their individual-specific critical income levels making lifetime neutral, then those individuals are regarded as exactly *as well-off as* each other. The Translation axioms states that, when measuring well-being of individuals whose indifference maps are mere
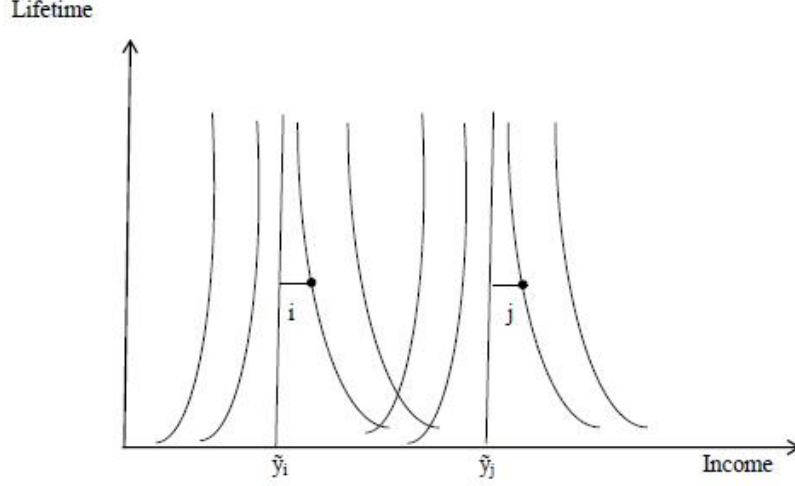
8

Figure 3: Translation axiom

translations, what matters is the distance to the vertical indifference line associated to the critical income level. Having a larger income does not give any advantage to an individual if the differential with respect to the critical income making lifetime neutral remains the same.

Figure 3 illustrates the Translation axiom in a simple two-person case. The indifference map of individual j is a mere translation of the indifference map of individual i. Since the bundles of individuals i and j involve the same lifetime and are equi-distant with respect to their critical income making lifetime neutral (respectively $\widetilde{y_i}$ and $\widetilde{y_j}$), the Translation axiom considers that individuals i and j are equally well-off.

Characterization of the well-being index

Theorem 1 states the logical implications of imposing that the well-being measure $M(y, L, \preceq i)$ satisfies the axioms Pareto Principle, Conditional Priority and Translation.

Let the equivalent income $EI(y, L, \preceq_i)$ be defined as follows:

- If $y_i < \widetilde{y_i}$, $(y_i, L_i) \sim_i (EI(y_i, L_i, \preceq_i), \overline{L^1})$;

- If $y_i = \widetilde{y_i}$, $EI(y_i, L_i, \preceq_i) = \widetilde{y_i}$;

- If $y_i > \widetilde{y_i}$, $(y_i, L_i) \sim_i (EI(y_i, L_i, \preceq_i), \overline{L^2})$.

9

Note that the equivalent income is not defined for indifference curves that everywhere above the thresholds $\overline{L^1}, \overline{L^2}$.

THEOREM 1 A well-being measure $M(y, L, \preceq_i)$ satisfies the axioms Pareto Principle, Conditional Priority and Translation if and only if, up to an increasing transform, it takes the form:

$$M(y, L, \preceq_i) = EI(y, L, \preceq_i) - \widetilde{y_i}$$

for all $(y, L, \preceq_i)$ for which $EI(y, L, \preceq_i)$ is defined.

PROOF: See the Appendix.

Theorem 1 states that a well-being measure that satisfies the three axioms presented above - the Pareto Principle, the Conditional Priority axiom and the Translation axiom - must necessarily take (up to an increasing transform) the form of the equivalent income net of the critical income making lifetime neutral, the equivalent income being defined for two particular reference lifetime levels (one for lives worth living, and one for lives not worth living).

In comparison to the standard equivalent income, the well-being index proposed here differs on two main grounds. First, the reference lifetime level is no longer unique for all lives, but is specific to whether the life is worth living or not. This difference comes from the Conditional Priority axiom, which defines a threshold for lifetime $\overline{L^1}$ separating lives not worth living that are sufficiently short from the ones that are definitely too long, as well as a second threshold $\overline{L^2}$ separating lives worth living that are insufficiently long from the ones that are sufficiently long. A second difference lies in the subtraction of the critical income making lifetime neutral. That second difference is clearly related to the Translation axiom. Subtracting the (individual-specific) critical income making lifetime neutral is a way to avoid the counter intuitive result of Figure 1: individuals with higher equivalents may not necessarily be regarded as better off: at the end of the day, what matters is the gap between the equivalent income and the critical income making lifetime neutral. In some sense, material achievements are here valued not absolutely, but relatively to the conceptions of a life worth living that individuals have.

With respect to the paradoxical result discussed at the very outset of this paper, an important property satisfied by the new measure of well-being characterized in Theorem 1 is that under this new measure of well-being, a person who regards his life as not worth living is always regarded as *worse-off* than a person who regards his life as worth living. Thus the measure of well-being characterized in Theorem 1 is clearly immunized against the problem discussed at the very outset of this paper. The intuition behind that result is that, under that well-being index, the equivalent income associated to a life not worth living is necessarily lower than the critical income making lifetime neutral. Hence, as a consequence, the well-being measure takes, for lives not worth living, a negative value, which is necessarily smaller than the values taken by the well-being index in case of lives worth living (since in that case the well-being index takes a strictly positive level).

Note that the characterization result stated in Theorem 1 is not the only possible way to deal with the problem of well-being measurement when individuals differ in their conception of the life worth living. In the next section, we propose to consider an alternative well-being measure, which is still based on the Pareto Principle and on the Translation axiom, but relies on a modified form of the Conditional Priority axiom.

Characterization of an alternative index

This section explores the characterization of an alternative measure of well-being, which is also an equivalent income net of the critical income level making lifetime neutral, but that relies on alternative reference lifetime levels, and, hence, defines social priority in a different manner.

For that purpose, we propose to replace the Conditional Priority axiom by another axiom, Conditional Priority II, which states that, whatever the level of lifetime, and whatever individuals are in a life worth living or not worth living, individuals who care more about longevity are the worst-off.

Conditional Priority II $\forall i, j \in N$, with $\widetilde{y_i} = \widetilde{y_j} = \widetilde{y}$, if $L_i = L_j = L$ and if $y_i = y_j = y \lessgtr \widetilde{y}$, then if $IC(y, L, \preceq_i)$ is steeper than $IC(y, L, \preceq_i)$ at (y, L), then $M(y, L, \preceq_i) \geq M(y, L, \preceq_j)$.

This modified version of the Conditional Priority axiom states that, at any point where a single crossing of indifference curves occurs, the individual who cares more about lifetime (and thus, whose indifference curve is less steep in the (income, lifetime) space), is regarded as the worst-off, and thus is given social priority.

The intuition behind that axiom goes as follows. When the life is not worth living (i.e. $y_i < \widetilde{y_i}$), then lifetime is undesirable, and so it makes sense to suppose that individuals who care more about their lifetime (to reduce it) are the worst-off. Moreover, when life is worth living (i.e. $y_i > \widetilde{y_i}$), then the least well off is the individual who cares more about increasing longevity.

In comparison to Conditional Priority, Conditional Priority II assigns social priority is a quite different way, by giving always priority to individuals who are more concerned with their lifetime. One can regard Conditional Priority II as specifying that the lifetime threshold $\overline{L^1}$ defining a life not worth living that is too long is equal to 0, whereas the threshold $\overline{L^2}$ defining a life worth living that is sufficiently long is here set to the maximum lifetime $\overline{L}$. As a consequence, there remain only two areas in the (income, lifetime) space, separated by the critical income level making lifetime neutral, and on each side of that threshold priority is given to individuals who care more about their lifetime. Figure 4 illustrates the Conditional Priority II axiom. The little arrows indicate the individual who, at a given bundle, is regarded as the worst-off. Based on that axiom, the worse-off is the person who cares more about lifetime (i.e. whose indifference curve is less steep at the bundle under study).
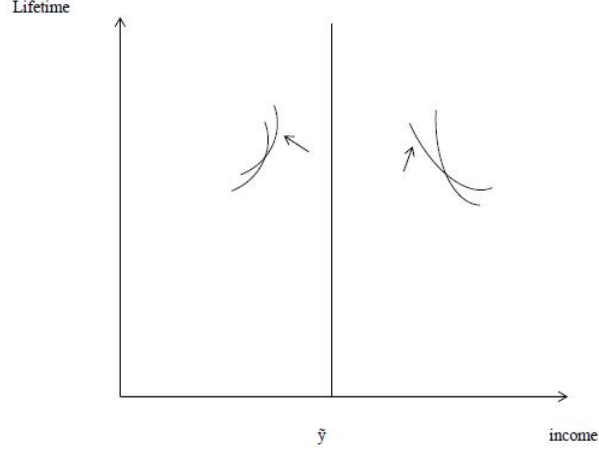
Figure 4: Conditional Priority II

Let us now introduce another axiom, which states that if one changes all indifference curves except the one on which the bundle (y, L) lies, this does not affect the level of measured well-being assigned to that particular bundle.

INDEPENDENCE WITH RESPECT TO IRRELEVANT PREFERENCES $\forall i, j \in N, \forall (y, L) \in \mathbb{R}_+ \times \mathbb{R}_+$, if $IC(y, L, \preceq_i) = IC(y, L, \preceq_j)$, then $M(y, L, \preceq_i) = M(y, L, \preceq_j)$.

The Independence with Respect to Irrelevant Preferences is a relatively weak requirement imposed on the well-being measure.

However, that axiom, when joined with the Pareto Principle, and Continuity, implies the axiom of Nested Contour Priority (see Fleurbaey and Maniquet 2011).

NESTED CONTOUR PRIORITY $\forall i, j \in N, \forall (y, L), (y', L') : int\left[\mathbb{L}(y, L, \preceq_i) \cap \mathbb{U}(y', L', \preceq_j)\right] = \varnothing$, if $\widetilde{y}_i = \widetilde{y}_j = \widetilde{y}$,

if $y < y' < \widetilde{y}$ and $L > L'$, then $M(y, L, \preceq_i) \leq M(y', L', \preceq_j)$;

if $y < \widetilde{y} < y'$ and $L \gtrless L'$, then $M(y, L, \preceq_i) \leq M(y', L', \preceq_j)$;

if $\widetilde{y} < y < y'$ and $L < L'$, then $M(y, L, \preceq_i) \leq M(y', L', \preceq_j)$.

Note that, since lifetime can be either desirable or undesirable, the definition of Nested Contour Priority requires to separate lives worth living from lives not worth living. When considering lives not worth living, Nested Contour Priority implies that an individual whose bundle lies on a *higher* indifference curve $IC(y, L, \preceq_i)$ is (weakly) worse-off than individuals whose bundles lie on lower

12

indifference curves (in the absence of crossing with $IC(y, L, \preceq_i)$). When considering lives worth living, Nested Contour Priority implies that an individual whose bundle lies on a *lower* indifference curve is (weakly) worse-off than individuals whose bundles lie on higher indifference curves (again in the absence of crossing).

To prove that the Pareto Principle, jointly with the Independence with Respect to Irrelevant Preferences, implies Nested Contour Priority, we need to introduce the following Continuity property.

CONTINUITY $\forall i \in N, \forall (y, L), \forall M(y, L, \preceq_i)$:

$$lim_{\epsilon \to 0, \zeta \to 0} M(y + \varepsilon, L + \zeta, \preceq_i) = M(y, L, \preceq_i)$$

The following lemma states an important intermediate result: any well-being measure $M(y, L, \preceq_i)$ that satisfies the axioms Pareto Principle and Independence with Respect to Irrelevant Preferences satisfies also, under Continuity, Nested Contour Priority.

LEMMA 1 A well-being measure $M(y, L, \preceq_i)$ that satisfies the Pareto Principle, Continuity and Independence with Respect to Irrelevant Preferences satisfies also Nested Contour Priority.

PROOF: See the Appendix.

Lemma 1 plays an important role in the characterization of the alternative well-being index based on Conditional Priority II. Indeed, that lemma allows us to use Nested Contour Priority to show that individuals with the same income being at reference lifetime levels have the same well-being levels.

Let the equivalent income $\widehat{EI}(y_i, L_i, \preceq_i)$ be defined as follows:

- If $y_i < \widetilde{y}_i$, $(y_i, L_i) \sim_i (\widehat{EI}(y_i, L_i, \preceq_i), 0)$;

- If $y_i = \widetilde{y}_i$, $\widehat{EI}(y_i, L_i, \preceq_i) = \widetilde{y}_i$;

- If $y_i > \widetilde{y}_i$, $(y_i, L_i) \sim_i (\widehat{EI}(y_i, L_i, \preceq_i), \overline{L})$.

The following theorem provides the characterization of an alternative measure of well-being in the context of unequal conceptions of the life worth living. Contrary to Theorem 1, Theorem 2 relies on Conditional Priority II, which amounts to give social priority to individuals who care more about their lifetime, whatever the level of lifetime is, and whatever the life is worth living or not.

THEOREM 2 A well-being measure $M(y, L, \preceq_i)$ satisfies the axioms Pareto Principle, Independence with Respect to Irrelevant Preferences, Conditional Priority II and Translation if and only if, up to an increasing transform, it takes the form:

$$M(y, L, \preceq_i) = \widehat{EI}(y, L, \preceq_i) - \widetilde{y}_i$$

for all $(y, L, \preceq_i)$ for which $\widehat{EI}(y, L, \preceq_i)$ is defined.

PROOF: See the Appendix.

The measure of well-being characterized in Theorem 2 shares some structural similarity with the well-being measure characterized in Theorem 1, in the sense that both well-being indexes consist of an equivalent income index from which the critical income making lifetime neutral is subtracted. Hence, that alternative measure of well-being has also the intuitive property of being immunized against the somewhat paradoxical result highlighted at the very outset of this paper. Under the well-being index characterized in Theorem 2, an individual whose life is not worth living is necessarily ranked as worse-off than an individual whose life is worth living.[4]

However, there is an important difference between the two well-being indexes. Whereas the well-being measure of Theorem 1 is based on an equivalent income index under reference lifetimes given by $\overline{L^1}$ under a life not worth living and by $\overline{L^2}$ under a life worth living, the well-being measure characterized in Theorem 2 is based on an equivalent income index under reference lifetimes equal to 0 under a life not worth living, and equal to the maximum lifetime $\overline{L}$ under a life worth living.

Those differences in terms of reference lifetime levels may seem benign, but these actually make a crucial difference for interpersonal well-being comparisons. To see this, take the case of two individuals with lives worth living, individual i being at $(y, \overline{L^2})$, whereas individual j is at $(y', \overline{L})$, with $y > y'$. Assume further that individuals i and j share the same critical income making lifetime neutral, and that their associated indifference curves intersect once for a lifetime level that lies between $\overline{L^2}$ and $\overline{L}$. Under the well-being measure characterized in Theorem 1, individual i is regarded as strictly worse off than individual j, whereas under the well-being measure of Theorem 2, individual i is regarded as strictly better off than individual j. Thus differences in reference lifetime levels matter for the measurement and the comparison of well-being across individuals, since reference lifetime levels incorporate value judgments relative to the social priority to be given to individuals in various situations. The same kind of well-being ranking reversals also arises when comparing the well-being levels of individuals whose life is not worth living.

Concluding remarks

This paper started from a paradoxical result for standard equivalent income indexes in the (income, lifetime) space: when individuals differ in their conception of a life worth living, it is possible that the equivalent income takes a higher

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[4]Indeed, the well-being index takes a negative value in case of a life not worth living, against a positive value for a life worth living.

level for individuals who regard their life as not worth living, in comparison to its level for individuals who regard their life as worth living. That paradoxical result comes from the fact that the standard equivalent income abstracts from an important aspect of individual preferences - their conception of a life worth living -. As a consequence, standard equivalent incomes could potentially lead to somewhat counter-intuitive results when making well-being comparisons between individuals with lives worth living or not worth living.

Starting from that somewhat paradoxical result, this paper proposed to provide foundations for an alternative equivalent income measure of well-being, which takes into account individual's conceptions of a life worth living. For that purpose, we developed a life cycle model with heterogeneous preferences, including heterogeneous conceptions of a life worth living, and we showed that simple axioms - the Pareto Principle, Conditional Priority and the Translation axiom - suffice to characterize a new index of well-being: an equivalent income net of the critical income making lifetime neutral, the equivalent income being defined for reference lifetime levels that differ depending on whether lives are worth living or not. We also characterized a variant of that well-being index, which relies on an alternative version of the Conditional Priority axiom, and, as a consequence, is based on distinct reference lifetime levels, which lead to different assignments of social priority.

The two new well-being indexes share an intuitive property: these are both immunized against the counter-intuitive result highlighted at the very outset of this paper: those well-being indexes always take a lower value for a life not worth living, in comparison to the measured well-being in case of a life worth living. Hence, under those well-being indexes, how individuals evaluate whether their life is worth living or not definitely matters, and is fully taken into account when measuring well-being. Since how individuals conceive a life worth living is a central aspect of their preferences, doing justice to that dimension of individual preferences contributes to improve the accuracy of well-being indexes.

Quite interestingly, accounting for individual conceptions of a life worth living does not only immunize well-being indexes against the paradoxical result emphasized above. This has also the virtue to provide a more accurate account of how social priority should be assigned in the context of heterogeneous preferences in the (income, lifetime) space. Actually, distinguishing between lives worth living and lives not worth living allows us to define reference lifetime levels that are conditional on that distinction, and, hence, to provide a more accurate account of how priorities should be assigned in economies where individuals differ in income and lifetime. When a life is not worth living, lifetime is an undesirable good, so that, when lifetime is sufficiently long, priority should be given to individuals who care more about their lifetime. On the contrary, when a life is worth living, lifetime is a desirable good, so that, when lifetime is not sufficiently long, priority should also be given to individuals who care more about their lifetime. Our paper explored two distinct ethical accounts of what "sufficiently long" means in those distinct contexts, and these account are directly translated into reference lifetime levels that depend on whether lives are worth living or not, and which have a clear ethical significance.

15

To conclude, it should be stressed that this paper has implications that go far beyond the mere measurement of well-being. From a policy perspective, governments often face difficult trade-offs, between allocating resources among groups differing in income and in survival conditions. In order to solve those policy trade-offs, the first stage is to assign priority to some groups, and this can only be done by measuring and comparing the well-being of different individuals. From that perspective, this paper suggests that one could hardly ignore, in that preliminary stage, how individuals conceive a life worth living.

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Appendix

0.1 Proof of Theorem 1

The proof proceeds in two steps. We first consider the proof of the statement that a well-being measure satisfying Pareto, Conditional Priority and Translation takes the form presented in Theorem 1. Then, in the second stage, we will prove that this measure of well-being satisfies indeed the three axioms.

FIRST STAGE (SUFFICIENCY).

The proof is organized in two stages.

We first focus on individuals whose preferences differ, but who have the same critical income level $\widetilde{y_i}$. We first show that if the indifference curves of two distinct individuals cross at the lifetime threshold $\overline{L^1}$ or $\overline{L^2}$, then the measure of well-being assigns the same well-being level to those two individuals.

Then, in the second stage, we focus on individuals whose preferences are such that these do not share the same critical income level $\widetilde{y_i}$.

Consider first the case where two individuals i, j enjoy the same bundle $(y, \overline{L^1})$.

Suppose that their indifference curves of i, j intersect only once (single crossing), precisely at that bundle. Moreover, suppose, as on Figure 5, that the indifference curve of i is steeper, at $(y, \overline{L^1})$, than the indifference curve of j . We thus have that, at that bundle, individual i cares less about lifetime, and more about income, in comparison with individual j.

Then, by the Conditional Priority axiom, we obtain that, given $L_i = L_j \geq \overline{L^1}$, that the well-being of individual j cannot exceed the well-being than individual i. However, since $L_i = L_j \leq \overline{L^1}$, we have also that the well-being of
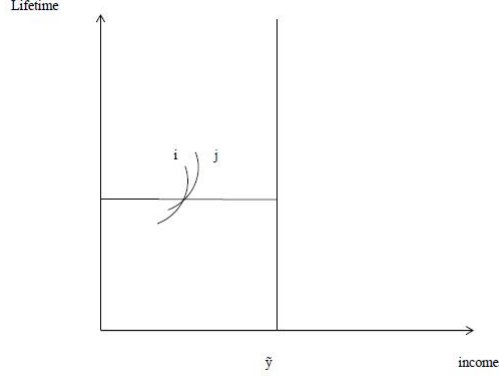
17

Figure 5: Well-being comparisons at $L = \overline{L^1}$

individual i cannot exceed the well-being than individual j. Hence it follows that the measured well-being must be equal for individuals i and j.

Consider now the case of no single crossing, that is, the indifference curves of individuals i, j intersecting at the bundle $(y, \overline{L^1})$ also intersect somewhere else in the space. That case can be dealt with as above. Indeed, in that case, it is possible to draw another indifference curve (let us say, for individual k) that intersects the indifference curves of i, j at the same bundle, but intersect these only once. Then, by the argument developed above, we have, given the single crossing, that individual k is exactly as well-off as individuals i and j. Hence, by transitivity of equality, the level of measured well-being must also be equal for i and j in that case as well.

A similar argument can be developed for the case where indifference curves of i, j intersect only once (single crossing) at the bundle $(y, \overline{L^2})$. In that case, the same argument holds, and applying the Conditional Priority axiom implies that the same well-being level must be assigned to individuals i, j.

It follows from this that individuals with distinct preferences but same critical income can be ranked quite easily, in terms of well-being, whatever their bundle is. Clearly, for any bundle (y_i, L_i) on an indifference curve that crosses somewhere either the horizontal line at $L = \overline{L^1}$ in case of a life not worth living, or that crosses the horizontal line at $L = \overline{L^2}$ in case of a life worth living, we know, by the Pareto Principle, that the well-being measured at a bundle (y_i, L_i) is necessarily equal to the measured well-being of a hypothetical bundle located at the threshold lifetime, either $L = \overline{L^1}$ or $L = \overline{L^2}$, while remaining on the same indifference curve. For all those bundles, the measurement of well-being can thus be carried out by focusing on the equivalent income associated to the threshold lifetime $L = \overline{L^1}$ when the life is not worth living, or to the threshold $L = \overline{L^2}$ when life is worth living.

In other words, we have shown so far that individuals who have the same

18

equivalent income and the same critical income level have the same level of well-being. Moreover, provided all individuals share the same critical income making lifetime neutral, the well-being of all bundles that lie on an indifference curve that intersects the thresholds $L = \overline{L^1}$ or $L = \overline{L^2}$ can be measured by the equivalent income associated to one of those reference levels, depending on whether life is worth living or not.

Note, however, that the proof is not complete yet, since we have focused only on well-being measurement across individuals sharing the same critical income level $\widetilde{y}_i$.

When the critical income level is not shared, it is nonetheless possible to rank individual well-being, by using the Translation axiom.

Take the case of two individuals i, j with different preferences $\preceq_i, \preceq_j$, represented by different indifference maps, including different critical income levels. One can define a third indifference map $IM(\preceq_k)$, which is a translate of the indifference map of individual i $IM(\preceq_i)$ and has the same critical income level as the one on the indifference map of individual j, i.e. $\widetilde{y}_j$. That indifference map being a translation of $IM(\preceq_i)$, we can use the Translation axiom, which states that any bundle on an indifference curve of individual i leads to the same well-being level as the bundles on the corresponding, translated indifference curve on $IM(\preceq_k)$. The Translation axiom allows us to assign a well-being level to all bundles located along the translated indifference curves.

But since $IM(\preceq_k)$ shares the same critical income level as $IM(\preceq_j)$, one can use the Conditional Priority axiom to assign also well-being levels to all bundles along the indifference curves of $IM(\preceq_j)$, by using the same arguments as above.

It follows from all this that, thanks to the Translation axiom, the well-being measure derived above, which can defined conditionally on indifference maps having the same critical income level, can be extended to the comparison of well-being for any indifference maps, even those that do not share the same critical income level making lifetime neutral.

Second stage (Necessity).

It is straightforward to show that the measure $EI(y, L, \preceq_i) - \widetilde{y}_i$ satisfies the Pareto Principle. Indeed, it ranks as equally good bundles that lie on the same indifference curve for a given individual. Moreover, it assigns a higher value to a bundle that lies on a lower indifference curve in case of a life not worth living, and a higher value to a bundle that lies on a higher indifference curve in case of a life worth living.

One can also show that this index satisfies Conditional Priority. To see this, take the case of two individuals whose indifference curves cross above $\overline{L^1}$, and assume that the indifference curve of person i is steeper than the one of person j. When moving along those two indifference curves in the direction of $\overline{L^1}$, we see that the indifference curve of i will intersect the horizontal line drawn at $L = \overline{L^1}$ for a higher level of income, leading to a higher equivalent income, and a higher measured well-being level than the one of person j. This is clearly in line with what the Conditional Priority axiom requires.

Concerning the Translation Axiom, it is easy to see that, if two individuals i, j with translated indifference maps up to a distance $x > 0$ have the same

lifetime, and lie at the same distance of their critical income level, then the associated measured well-being levels are equal, since the equivalent income of individual i is equal to the equivalent income of individual j $+x$. As a consequence, it follows that $EI(y_i, L_i, \preceq_i) - \widetilde{y_i} = EI(y_j, L_j, \preceq_j) + x - \widetilde{y_i} = EI(y_j, L_j, \preceq_j) + x - (\widetilde{y_j} + x) = EI(y_j, L_j, \preceq_j) - \widetilde{y_j}$, so that the Translation axiom is satisfied.

Proof of Lemma 1

Consider two individuals i and j who enjoy bundles (y, L) and (y', L') respectively. Suppose that $\widetilde{y} < y < y'$, $L < L'$ and that $int\left[\mathbb{L}(y, L, \preceq_i) \cap \mathbb{U}(y', L', \preceq_j)\right] = \emptyset$.

Suppose that the two associated indifference curves do not intersect, that is, $IC(y, L, \preceq_i) \cap IC(y', L', \preceq_j) = \emptyset$.

Then, we can modify individual i's preferences into $\preceq_i'$ such that $IC(y, L, \preceq_i') = IC(y, L, \preceq_i)$ and $IC(y', L', \preceq_i') = IC(y', L', \preceq_j)$. We know that $(y', L') \in \mathbb{U}(y, L, \preceq_i)$, therefore, by the Pareto Principle, we have that $M(y, L, \preceq_i') < M(y', L', \preceq_i')$. Moreover, from Independence with respect to Irrelevant Preferences, we have that: $M(y, L, \preceq_i') = M(y, L, \preceq_i)$ and $M(y', L', \preceq_i') = M(y', L', \preceq_j)$. Hence, by transitivity, we obtain: $M(y, L, \preceq_i) < M(y', L', \preceq_j)$, which is in line with Nested Contour Priority.

We thus have proved that the Pareto Principle, jointly with the Independence of Irrelevant Preferences, implies Nested Contour Priority when the two indifference curves associated associated with the two bundles under comparison do not intersect.

Consider now a second, more complex, case, where the two indifference curves $IC(y, L, \preceq_i)$ and $IC(y', L', \preceq_j)$ do not cross, but nonetheless intersect in some point. Suppose also that the upper contour set of $IC(y, L, \preceq_i)$ includes, as a subset, the upper contour set of $IC(y', L', \preceq_j)$.

Note that, in that case, one cannot use the Independence with respect to Irrelevant Preferences, since we can no longer modify $IC(y', L', \preceq_i)$ into $IC(y', L', \preceq_j)$ without reaching a contradiction, since two indifference curves of a given individual cannot intersect. However, it is possible to prove that $M(y, L, \preceq_i) \leq M(y', L', \preceq_j)$.

We show this by starting from a situation similar to the one considered above, with $y'' = y' + \varepsilon$ and $L'' = L' + \zeta$. We have, by the same argument as above, $M(y', L', \preceq_j) \leq M(y'', L'', \preceq_j)$ and $M(y, L, \preceq_i) \leq M(y'', L'', \preceq_i)$, because indifference curves do not cross.

Then, consider that the parameters ε and ζ become smaller and smaller. By the previous reasoning, we still have, for all $\varepsilon, \zeta > 0$: $M(y, L, \preceq_i) \leq M(y' + \varepsilon, L' + \zeta, \preceq_j)$. This implies, thanks to Continuity, that $lim_{\varepsilon \to 0, \zeta \to 0} M(y' + \varepsilon, L' + \zeta, \preceq_j) = M(y', L', \preceq_j) \geq M(y, L, \preceq_i)$. Indeed, if one had $M(y', L', \preceq_j) < M(y, L, \preceq_i)$, there would be ε, ζ small enough such that $M(y' + \varepsilon, L' + \zeta, \preceq_j) < M(y, L, \preceq_i)$. This would contradict the fact that for all $\varepsilon, \zeta > 0$: $M(y, L, \preceq_i) \leq M(y' + \varepsilon, L' + \zeta, \preceq_j)$.

A similar proof can be made when considering both lives not worth living. Note that, when comparing a life worth living with a life not worth living, the two associated indifference curves cannot intersect, so that only the first part of the proof is needed in that case.

Proof of Theorem 2

The proof proceeds in two steps. Using Lemma 1, we first consider the proof of the statement that a well-being measure satisfying Pareto Principle, Nested Contour Priority, Conditional Priority II and Translation takes the form presented in Theorem 2. Then, in the second stage, we will prove that this measure of well-being satisfies indeed the four axioms.

FIRST STAGE (SUFFICIENCY).

The proof is organized in two stages.

We first focus on individuals whose preferences differ, but who have the same critical income level $\widetilde{y}_i = \widetilde{y}_j = \widetilde{y}$. We first show that if the indifference curves of two distinct individuals cross at the lifetime threshold 0 or $\overline{L}$, then the measure of well-being assigns the same well-being level to those two individuals.

Consider first the case where individuals have the same bundle $(y, 0)$ with $y < \widetilde{y}$. Suppose that the indifference curve of i is steeper, at $(y, 0)$, than the indifference curve of j . We thus have that, at that bundle, individual i cares less about lifetime, and more about income, in comparison with individual j. By Conditional Priority II, we know that individual i cannot be strictly worse off than individual j, that is: $M(y, 0, \preceq_i) \geq M(y, 0, \preceq_j)$. But notice that the indifference curve of i lies above the indifference curve of j in the (income, lifetime) space. Hence, by Nested Contour Priority, we have that individual i cannot be strictly better off than individual j, that is, $M(y, 0, \preceq_i) \leq M(y, 0, \preceq_j)$. Hence, given $M(y, 0, \preceq_i) \geq M(y, 0, \preceq_j)$ (by Conditional Priority II) and $M(y, 0, \preceq_i) \leq M(y, 0, \preceq_j)$ (by Nested Contour Priority), it follows that: $M(y, 0, \preceq_i) = M(y, 0, \preceq_j)$.

Note that, whereas the Conditional Priority II axiom presupposed single crossing of indifference curves, one can extend the above results to the case where indifference curves meeting at the bundle $(y, 0)$ intersect more than once. To see this, take three indifference curves, denoted 1, 2, 3, that intersect only at $(y, 0)$ with the indifference curve 1 being above the indifference curve 2, which is itself above the indifference curve 3. Then draw another indifference curve, called c, which also passes through $(y, 0)$, but intersect indifference curve 2 also at another point, but without intersecting indifference curves 1 and 3 except at $(y, 0)$. By Nested Contour Priority, we have that $M(y, 0, \preceq_1) \leq M(y, 0, \preceq_c) \leq M(y, 0, \preceq_3)$. But by the argument developed in case of single-crossing (combination of Nested Contour Priority and Conditional Priority II), we have also that: $M(y, 0, \preceq_1) = M(y, 0, \preceq_3)$. Moreover, by Nested Contour Priority, we have also: $M(y, 0, \preceq_1) \leq M(y, 0, \preceq_2) \leq M(y, 0, \preceq_3)$. Hence it follows that: $M(y, 0, \preceq_1) = M(y, 0, \preceq_2) = M(y, 0, \preceq_3)$. As a consequence, we obtain that: $M(y, 0, \preceq_2) = M(y, 0, \preceq_c)$.

Hence the above argument can be extended to cases where indifference curves meeting at $(y, 0)$ intersect more than once.

A similar argument can be developed for the case where indifference curves of i, j intersect only once (single crossing) at the bundle $(y, \overline{L})$. In that case, the same argument holds, and applying the Conditional Priority II axiom with Nested Contour Priority implies that the same well-being level must be assigned to individuals i, j.

It follows from this that individuals with distinct preferences but same critical income can be ranked quite easily, in terms of well-being, whatever their bundle is. Clearly, for any bundle (y_i, L_i) on an indifference curve that crosses somewhere either the horizontal line at $L = 0$ in case of a life not worth living, or that crosses the horizontal line at $L = \overline{L}$ in case of a life worth living, we know, by the Pareto Principle, that the well-being measured at a bundle (y_i, L_i) is necessarily equal to the measured well-being of a hypothetical bundle located at the threshold lifetime, either $L = 0$ or $L = \overline{L}$, while remaining on the same indifference curve. For all those bundles, the measurement of well-being can thus be carried out by focusing on the equivalent income associated to the threshold lifetime $L = 0$ when the life is not worth living, or to the threshold $L = \overline{L}$ when life is worth living.

In other words, we have shown that individuals who have the same equivalent income and the same critical income level have the same level of well-being. Moreover, provided all individuals share the same critical income making lifetime neutral, the well-being of all bundles that lie on an indifference curve that intersects the thresholds $L = 0$ or $L = \overline{L}$ can be measured by the equivalent income associated to one of those reference levels, depending on whether life is worth living or not.

Note, however, that the proof is not complete yet, since we have focused only on well-being measurement across individuals sharing the same critical income level $\widetilde{y}_i$.

When the critical income level is not shared, it is nonetheless possible to rank individual well-being, by using the Translation axiom.

Take the case of two individuals i, j with different preferences $\preceq_i, \preceq_j$, represented by different indifference maps, including different critical income levels. One can define a third indifference map $IM(\preceq_k)$, which is a translate of the indifference map of individual i $IM(\preceq_i)$ and has the same critical income level as the one on the indifference map of individual j, i.e. $\widetilde{y}_j$. That indifference map being a translation of $IM(\preceq_i)$, we can use the Translation axiom, which states that any bundle on an indifference curve of individual i leads to the same well-being level as the bundles on the corresponding, translated indifference curve on $IM(\preceq_k)$. The Translation axiom allows us to assign a well-being level to all bundles located along the translated indifference curves.

But since $IM(\preceq_k)$ shares the same critical income level as $IM(\preceq_j)$, one can use the Conditional Priority II axiom to assign also well-being levels to all bundles along the indifference curves of $IM(\preceq_j)$, by using the same arguments as above.

It follows from all this that, thanks to the Translation axiom, the well-being

measure derived above, which can defined conditionally on indifference maps having the same critical income level, can be extended to the comparison of well-being for any indifference maps, even those that do not share the same critical income level making lifetime neutral.

SECOND STAGE (NECESSITY).

It is straightforward to show that the measure $\widehat{EI}(y, L, \preceq_i) - \widetilde{y_i}$ satisfies the Pareto Principle. Indeed, it ranks as equally good bundles that lie on the same indifference curve for a given individual. Moreover, it assigns a higher value to a bundle that lies on a lower indifference curve in case of a life not worth living, and a higher value to a bundle that lies on a higher indifference curve in case of a life worth living.

One can also show that this index satisfies Conditional Priority II. To see this, take the case of two individuals whose indifference curves cross above $L = 0$, and assume that the indifference curve of person i is steeper than the one of person j. When moving along those two indifference curves in the direction of $L = 0$, we see that the indifference curve of i will intersect the horizontal line drawn at $L = 0$ for a higher level of income, leading to a higher equivalent income, and a higher measured well-being level than the one of person j. This is clearly in line with what the Conditional Priority II axiom requires.

Concerning the Translation Axiom, it is easy to see that, if two individuals i, j with translated indifference maps up to a distance $x > 0$ have the same lifetime, and lie at the same distance of their critical income level, then the associated well-being measured are equal, since the equivalent income of individual i is equal to the equivalent income of individual $j + x$. As a consequence, it follows that $\widehat{EI}(y_i, L_i, \preceq_i) - \widetilde{y_i} = \widehat{EI}(y_j, L_j, \preceq_j) + x - \widetilde{y_i} = \widehat{EI}(y_j, L_j, \preceq j) + x - (\widetilde{y_j} + x) = \widehat{EI}(y_j, L_j, \preceq_j) - \widetilde{y_j}$, so that the Translation axiom is satisfied.