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Abstract

This paper investigates the sustainability of public finances in the European countries since 2002. We provide evidence of heterogenous behaviors among the EU countries and show that, even if they had been forced to focus their fiscal efforts on correcting the deviations of debt from their ceiling -through a correcting mechanism such as the recent TSCG rule-, this would not necessarily have changed the likelihood that debt and deficits become more sustainable. Sources of deviations from stable debt and deficits are related to the macroeconomic environment: the interest-growth differential, momentum dynamics in the sovereign bond markets, how markets react to rising debt.

Keywords: Fiscal rules, euro area, quantile regression, stability .

JEL Classification: C14, C51, C61

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1. Introduction

This paper proposes a behavioral approach to study the conditions of sovereign debt sustainability in the European countries. Almost a decade after the 2010 public debt crises in the eurozone, the issue of the sustainability of public finances continues to give rise to debates in the policy arena. The data provided by the European Semester 2019 document show that a number of countries with significant economic weight in the EU are in a situation of high risk of debt unsustainability illustrated by their high public debt ratio: Belgium (101.3%), France (99%), Italy (133.7%), Spain (96.3%) and Portugal (119.5%). The Greek sovereign debt has even reached a level around 180% of GDP, despite a high structural budget surplus around 5%. The sustainability criterion used by the European Commission is the S1 indicator, which measures the cumulative 5-year structural fiscal adjustment effort required for the debt ratio to reach 60% of GDP in 2032. In the Italian case, it would require cumulative surpluses of 9% of GDP, 5.2% in Spain, 4.2% in Belgium, France and Portugal. The Commission also looks at another medium-term sustainability criterion, S2, which measures the adjustment needed to stabilize the debt ratio over an infinite period including the costs of aging. These two criteria of sustainability are based on the idea that governments must be solvent, which implies that the debt ratio should not explode over a finite time horizon (for S1) and infinite time horizon (for S2).

In the theoretical literature on public finance, several concepts of fiscal sustainability have been proposed. The sustainability of debt has been related to intertemporal solvency (when governments satisfy their intertemporal budget constraint), debt liquidity (when governments are able to service their debt at any time). It is also common to define sustainable debt limits (sometimes corresponding to steady state values), fiscal limits (a deficit beyond which fiscal insolvency is likely to happen). For a survey of the different approaches, the reader can refer toAguiar and Amador (2014), Bouabdallah et al.(2017).

The empirical assessment of debt sustainability is usually done using cointegration techniques to investigate the joint dynamics of revenue and spending (see, among many others, Bohn 2007 and, for a recent paper, Beqiraj et al. 2018), balance-sheet micro-based models (see Giammarioli et al. 2007), signal extraction models (see Dufrénot et al. 2016, Savona and Vezzoli 2015), stochastic debt sustainability models (see Aguian and Amador 2014, Consiglio and Zenios 2017, Goedl and Zwick 2017), distribution models (see Dufrénot and Paret 2019, Medeiros 2012).

This paper proposes a behavioral approach that relates debt sustainability to the behavior of both fiscal authorities and financial markets. Some examples of previous papers using behavioral models are Collignon (2012), Gosh et al. 52013), Paret (2017). The idea is to go beyond the standard accounting debt equation to better account for the role of fiscal policy and market reactions. Specifically, the sustainability of public finance is related to the way governments respond to rising deficits by stepping up fiscal retrenchment (through adjusting their primary balance). Financial markets reactions also determine the sustainability of public finance through the reactions of investors to new fiscal developments when deciding upon the sovereign yield spreads.

Testing the sustainability of sovereign debts and deficits in the EU using a behavioral approach is appealing given the debates raised in the policy-making arena since the 2010 sovereign debt crises. On important concern was whether the higher sovereign bond spreads were caused by self-fulfilling expectations in the markets, or whether this was a consequence of the deterioration of the countries' economic fundamentals (see, for instance, Ayres et al. 2018, De Grauwe and Ji, 2013). We study here the ability of the fiscal rules of the Stability and Growth Pact (SGP) to stabilize the long-term debt ratio, given the prevailing financial conditions - e.g. the sensitivity of interest rate premiums to debt ratios and macroeconomic fundamentals. In addition, we investigate whether debt unsustainability was due to fiscal mismanagement -the way governments have increased their primary balance in response to overall deficits.

With regard to the role of fiscal authorities' reaction in promoting the sustainability of public debt, governments usually seek to avoid two pitfalls. The first is the risk of unnecessarily restrictive fiscal policies. For example, excessive fiscal consolidations in response to increased budget deficits may ultimately be ineffective in lowering the debt ratio, if the effects on growth are negative. This happened after the 2008 financial crisis due to the under-estimation of fiscal multipliers (see Blanchard and Leigh, 2013, Fatás, 2018). The other risk is to adjust the primary balance insufficiently to lower the debt ratio, which happens for instance when there exists a procyclical fiscal bias, or when the policies are not counter-cyclical enough (see Egert 2012 and, more recently, the European Fiscal Board 2019). In this paper, we define a condition for debt sustainability, based on the magnitude of the adjustment coefficient in the fiscal rule in response to the evolution of deficits and debt. We highlight the existence of bounds that depend on the parameters of the fiscal reaction function and on the evolution of sovereign premia in response to changes in the ratio of public debt and macro-financial variables.

In addition to the role of fiscal policy, we examine the conditions that determine automatic changes in the public debt ratios due to the dynamics of the growth-adjusted interest rates. In the literature, it is common practice to link debt sustainability to the Keynes-Ramsey rule, which implies that the adjusted interest rate (interest rate minus growth) must be positive (this condition, known as dynamic efficiency, is necessary to satisfy the transversality condition of the intertemporal budgetary constraint). From empirical studies, we know that debt ratios can, however, decrease sharply even in situations of dynamic inefficiency when the adjusted interest rate is low (see, for example, Blanchard 2019). This debate has been re-energized in the current context of low nominal interest rates in the industrialized countries (see Basseto and Cui 2017, Blanchard and Summers 2017, Mehrotra 2017). In this paper, we show that the important point for debt sustainability is that the adjusted interest rate should not exceed a threshold. In a context of heterogeneous fiscal policies and different premiums, the threshold varies across countries and over time.

Our paper contributes to two strands of literature.

First, many authors have interpreted the concept of fiscal sustainability as a situation in which the intertemporal budget constraint holds. This has led to empirical works based on cointegration tests between fiscal revenues and public spending (see the seminal papers by Bohn 1995, 1998, Trehan and Wash 1988). Other papers also study cointegration between structural primary surpluses and debt-to GDP ratio (see Beqiraj et al., 2018). This interpretation has, however, been challenged. Bohn (2007) shows that countries may not satisfy the intertemporal budget constraint and, yet, have sustainable public finances. An alternative literature has therefore proposed the following interpretation. We first need to know whether there exists, over a long time period, steady-state levels of sovereign debts, primary balances and overall deficits. And, if yes, the issue is whether, following a shock, these public finance variables deviate persistently from their steady-state levels. If the answer is positive, then they are unsustainable. Otherwise, they must be considered as sustainable. Collignon (2012) proposes an empirical model where debt ratios and primary balances are modelled jointly and, he analyzes the properties of the Jacobian matrix to investigate whether the steady state variables are stable or not. He concludes that public debts have been highly sustainable in the EU countries over the period from 1978 to 2009. Gosh et al. (2013) relate the steady state debt ratio to fiscal space defined as the difference between the current debt ratio and a debt limit. Using data on 23 advanced countries, they show that governments have become less reactive to rising debt since the latter has been very high (around $90\% - 100\%$ of GDP). Dufrénot et al. (2018) propose a model where debt ratios steady states are co-determined with the inflation rate resulting from monetary policy under fiscal dominance. They conclude that, during the transition to the steady state, fiscal deficits can be "locally" unstable, but "globally" stable if fiscal policy is Ricardian.

This paper extends previous approaches in two ways. First, we consider that debt dynamics depends, not only on fiscal reaction functions, but also on the dynamics of debt service. We therefore make the interest rate endogenous and specify an equation of the determinants of risk premium. Steady-state stability conditions therefore depend on the coefficients of fiscal reaction functions and, on those of the determinants of interest rate spreads. The main advantage of our approach is to link fiscal sustainability to both fiscal governance and to the macro-financial conditions summarized by a dynamic efficiency condition. Second, we explicitly take into account fiscal heterogeneity and the fact that financial markets may differenciate the countries when determining the interest rate premiums. These aspects need to be adressed given the lack of fiscal integration in the EU (the idea of issuing Eurobonds as the main instrument of sovereign debt remains a challenge due to probable moral hazard problems). We show that regressions based on the assumption of heterogenous coefficients lead results that differ significantly from standard OLS regressions.

Secondly, our paper contributes to a recent literature on how to model heterogeneous fiscal reactions (slope heterogeneity) across countries, when the estimations are done in a panel framework. Some papers suggest to use nonlinear specifications. For instance, nonlinear panel models allow to discriminate beween fiscal fatigue and a ”normal” positive reaction of primary balance to higher debt ratio (see Egert, 2015, Everaert and Jansen 2018, Nickel and Tudyka 2014). Other papers use dynamic CCE mean group estimator (Golinelli et al. 2018). Bouthevillain and Dufrénot (2016) propose quantile regression estimations to account for fiscal policy heterogenous reactions in the Euro area. We extend their methodology by ”crossing” the information in the conditional quantile distributions of primary balance changes and interest rate spreads. We account for four ”clusters” of countries and time that are distinguished with respect to two criteria : small/large fiscal adjustments and low/high intensity changes in yield spreads.

Our main findings are the following. First, we find little evidence of debt and primary balance sustainability in the EU since 2002, when the fiscal policy functions relate adjustments in the primary balance to overall deficit and debt *gaps* (difference between the level of these variables and the policy objectives defined by the Stability and Growth Pact -SGP-). The estimates reveal that there have been little efforts of fiscal policies to react to debt gaps. This may have been a source of the unsustainability of sovereign debts. We also account for differences between governments regarding the role of market spreads. For some of them, the negative contribution of excessively high ajusted interest rates is added to that of fiscal policy insufficient reaction to rising debts to make debt and primary balances unsustainable. We also find that, even in the case where the countries would have adopted the recent debt corrective mechanism of the TSCG, few of them would have had sustainable public finances.

The rest of the paper is structured as follows. Section 2 presents our behavioral model of debt sustainability. Section 3 contains the empirical estimation and the investigation of the stability properties of the steady state. Finally, Section 4 concludes.

2. A behavioral model of debt sustainability

2.1. A 3-dimension system to investigate debt dynamics

The concept of sustainability used here is *not* based on the examination of governments' intertemporal budget constraint. Rather, we investigate the stability properties of a steady-state equilibrium in a system of equations in which public finance variables move together. A formulation was proposed by Collignon (2012). His model includes two endogenous variables: primary balance and debt ratio. We extend this approach by making the interest rate endogenous. This adds a dimension to the system to be studied.

- Debt accumulation

The first equation of our system is standard and explains how a country's debt ratio - outstanding stock of debt over GDP - changes according to the initial level of debt ratio, primary balance (revenues minus expenditure excluding interest payment), the real GDP growth rate and real interest rate:

$$\Delta d_t = (r_t - g_t)d_{t-1} - s_t, \tag{1}$$

where $\Delta x_t = x_t - x_{t-1}$, d_t is the ratio of sovereign debt over GDP, s_t is primary balance, g_t and r_t are respectively real GDP growth rate and the real interest rate (government bond yield).

- Overall deficit

The second equation is also standard in the literature:

$$def_t = (r_t + \pi_t)d_{t-1} - s_t, \tag{2}$$

where def_t is the ratio overall deficit (expenditure -including interest spending - minus fiscal revenue over GDP) and π is the inflation rate.

- Fiscal reaction functions in a European context

We specify a fiscal reaction function, e.g. a behavioral equation showing how fiscal authorities adjust a fiscal instrument - here the primary balance - according to various macroeconomic variables. This type of function is described in the literature by reduced-form equations, which are assumed to derive from both governments' preferences and from the fiscal policy constraints they face. The latter are usually interpreted as fiscal rules. The rules are defined according to different criteria : short- or medium-term targets, public spending, debt ratios, etc. In the EU, a new fiscal compact was voted in 2011 and launched in the countries in 2012/2013 (the new TCSG -Treaty on Coordination, Stability and Growth-). It reforms the SGP by adding new dimensions of macro-fiscal surveillance (automatic correction mechanisms, macroeconomic imbalance procedure, etc). The period under examination in this paper (2002-2018) covers the SGP (10 years) and only 7 years of implementation of the TCSG. To avoid under-sizing the samples in the empirical study, we do not split our sample in two sub-periods. We retain the common denominator of the targets for these two periods. These are the 3% ceiling on the budget deficit and 60% on the debt ratio. This leads us to model a short-term fiscal reaction function. Our specification of the fiscal reaction function is the simplest. Governments adjust the primary balance according to the overall deficit of the previous period and to the debt ratio of the previous period's. The new TCSG may have changed the governments' fiscal behavior. In this case, this should change the value of the coefficients that we suppose here to vary across countries and over time (this point is discussed in the empirical section).

Our basic specification is:

$$\Delta s_t = \alpha(def_{t-1} - \tau_1) + \beta(d_{t-1} - \tau_2) + \epsilon_t^s,\ \tau_1 = (3\%)/4,\ \tau_2 = (60\%)/4. \quad (3)$$

α and β measure the sensitivity of primary balance changes to, respectively, overall deficit and, debt ratio. ϵ_t^s is a noise component. We divide de targets by 4 by assuming that an annual 3% target is equivalent to 0.75% quarterly target (the same for the debt ratio target: using quarterly data, we define a quaterly target as one-fourth ofthe annual target).

- The dynamics of the yield spread

Changes in the interest rate are governed by yield spreads in the sovereign bond markets. There are empirical evidence in the literature that the following factors are key determinants of the yield spreads: international risk captured by the VIX, monetary policy (represented by the short-term interest rate SR_t), the financial cycle (measured by housing prices, HP_t, stock prices, SP_t and credit to the private sector, $credit_t$) and macroeconomic imbalances (public debt ratio, current account balance, CAB_t, unemployment rate, UNR_t, real effective exchange rate, $REER_t$, private debt ratio, pd_t). For a survey of the literature on the determinants of sovereign bond yield spreads in the EU, the reader can refer to Afonso et al. 2015, Afonso and Nunes 2015, Georgoutsos and Migiakis 2013). Our equation of the interest rate differential is therefore

$$\Delta(r_t - r_t^*) = \beta_0(r_{t-1} - r_{t-1}^*) + \beta_1 d_{t-1} + \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'}_t + \epsilon_t^r, \tag{4}$$

where $\boldsymbol{X_t} = (VIX_t, SP_t, SR_t, UNR_t, CAB_t,)$. $\overline{\Delta\boldsymbol{X}'}_t$ means that we smooth the first-difference by using its 3-year moving average. $(r_t - r_t^*)$ is the yield spread defined as the difference between a country's interest rate and the yield of a riskless asset.

From these equations, the joint evolution of debt, primary balance and interest rate spreads is described by a 3-dimensional system of equations :

$$\begin{cases} \Delta d_t = (r_t - r_t^* - g_t)d_{t-1} + r_t^* d_{t-1} - s_t, \\ \Delta s_t = d_{t-2}\alpha(r_{t-1} - r_{t-2}^* + \pi_{t-1}) + \beta d_{t-1} + \alpha r_{t-2}^* d_{t-2} - \alpha s_{t-1} - \\ (\alpha\tau_1 + \beta\tau_2) + \epsilon_t^s, \\ \Delta(r_t - r_t^*) = \beta_0(r_{t-1} - r_{t-1}^*) + \beta_1 d_{t-1} + \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'}_t + \epsilon_t^r. \end{cases} \tag{5}$$

2.2. Stability properties of the steady-state

We rewrite the system of equations in continuous time:

$$\begin{cases} \dot{d(t)} = [r(t) - r^*(t) - g(t)]d(t) + r^*(t)d(t) - s(t), \\ \dot{s(t)} = \alpha[d(t)(r(t) + \pi(t)) - \tau_1 - s(t)] + \beta(d(t) - \tau_2) + \epsilon^s, \\ \dot{r(t)} - r^*(t) = \beta_0[r(t) - r^*(t)] + \beta_1 d(t) + \boldsymbol{\beta_2}\overline{\Delta \boldsymbol{X}'(t)} + \epsilon^r. \end{cases} \tag{6}$$

The dots mean time derivative. The steady-state corresponds to the fixed point:

$$\begin{cases} \overline{s} = \overline{d}[\overline{r} - g], \\ \overline{d} = \frac{\alpha\tau_1 + \beta\tau_2}{\alpha(\pi+g)+\beta}, \\ \overline{r} = r^* \frac{(\beta_0 - 1)}{\beta_0} - \frac{\beta_1}{\beta_0}\left[\frac{\alpha\tau_1+\beta\tau_2}{\alpha(\pi+g)+\beta}\right] - \frac{\beta_2}{\beta_0}\overline{\Delta \boldsymbol{X}'}. \end{cases} \tag{7}$$

The stability conditions are obtained from the Jacobian matrix of the linearized system and by applying the Routh-Hurwitz theorem. Details are provided in Appendix. The criterion for the asymptotic stability of the steady-state amounts to examining whether the characteristic polynomial obtained from the Jacobian matrix is stable. The conditions are based on the second method of Lyapunov and a generalization of the so-called interlacing property for polynomials of any order [3]

This leads us to two propositions. Proposition 1 below refers to the ability

[3]The seminal papers by Routh (1877) and Hurwitz (1895), Lyapunov (1896) and Liénard and Chipart (1914) show that only half of the original Routh-Hurwitz conditions are needed for the stability of a characteristic polynomial.

of a government to service its debt in the short term: its borrowing conditions should not tighten too much. Proposition 2 below defines conditions to gauge the fiscal space of a government. The latter is defined in terms of fiscal behavior as the "minimum" reaction of primary balance to overall deficits and debt, below which public finances sustainability is in doubt.

Proposition 1. A fiscal policy focused on debt and deficit targeting ($\alpha \neq 0$ and $\beta \neq 0$), is sustainable, if a government can service its debt, and therefore if it is liquid. In this case, the following condition must hold:

$$(\bar{r} - g) < (\alpha - \beta_0). \tag{8}$$

See the proof in Appendix.

Proposition 1 defines a maximum threshold for the growth-adjusted interest rate. To ease the repayment of debt service and avoid a situation whereby governments are "forced" by the markets to adjust their primary balance because the risk premia are too high, there must be a ceiling on the growth-adjusted interest rate. Two types of factors influence the determination of the ceiling. The first factor is fiscal policy (captured by α). The second factor is momentum dynamics in the sovereign bond markets, which characterizes the persistence of the interest rate spreads (here β_0).

Proposition 2. In the medium/long-term, an increase in the debt ratio and/or public deficits can happen without appreciable risk of unsustainability, if the government fiscal reaction is sensitive enough to changes in debt and deficits. The government must counteract the higher debt and deficits by increasing sufficiently its primary balance. In this case, the following two inequalities must be satisfied:

$$\alpha > \frac{\beta_0 \beta}{\beta_1(\bar{d} - 1) - \beta_0(\pi - g)}, \tag{9}$$

$$\beta > \alpha(\beta_0 + \pi - g) + (\beta_1 - \beta_0). \tag{10}$$

See the proof in Appendix.

Proposition 2 provides a minimum bound to α and β. For public finances to be sustainable, the government needs to be reactive enough to overall deficits and debt. Debt and deficits might become unsustainable, if a government adopts a laissez-faire policy. This can happen if it decides that higher deficits do not elicit and the reaction to changes in deficits and debt is too weak. "Weak" means that α and β remain below a given threshold. Sustainability implies here mix targets on deficits *and* debt, which means that the reactions are inter-dependent. For example, a government that puts a great emphasis on the debt ratio target tends to adjust its primary balance more strongly (which is reflected here by the fact that the threshold defined on α increases with the value of β). Similarly, in the second inequality, the threshold of β is an increasing value of α: a government tends to react significantly to the debt ratio, if it is ready to adjust more sharply its primary balance.

The threshold values depend on several factors : a) how the yield spreads change in reaction to a higher debt ratio (β_1), b) the macroeconomic environment (inflation π and real GDP growth g) and c) the persistence of the yield spreads (β_0). If a more persistent risk is priced by the financial markets (β_0 increases), this can reduce a government's incentive to pursue a "laissez-faire" fiscal policy, or a countercyclical fiscal policy, when the overall deficit increases. Indeed, when β_0 increases, the threshold value is larger.

3. Empirical Evidence

3.1. Data and econometric methodology

We now estimate the parameters of Equations 3 and 4. The sources of data are presented in Appendix.

In Equation 3, the endogenous variable is the first-difference of the primary balance ratio. The exogenous variables are the lagged overall deficit (in % of GDP) and the lagged debt ratio (Government consolidated gross debt over GDP). To capture "true" policy reaction and neutralize changes in the primary balance coming from automatic stabilizers, we add the output-gap as a control variable. This variable is measured using HP filter.

In Equation 4, the endogenous variable is the sovereign yield spread measured as follows. We take the long-run interest rate (EMU convergence criterion bond yields) and we substact the riskless risk computed as zero-coupon yield curve spot rate (AAA-rated euro area central government bonds) with a maturity of 30 years. The explanatory variables are the current account balance as percentage of GDP, harmonized unemployment rate, share price indices (prices of common shares of companies traded on national or foreign stock exchanges), short-term rates (short-term borrowing rates between financial institutions or short-term government paper rates issued or traded in the market). We further consider a financial stress indicator (country-Level Index of Financial Stress -CLIFS-).

Data are taken from OECD Statistics (OECD.Stat), the European Statistical Office (Eurostat) and the ECB Statistical Data Warehouse. We consider a panel of 15 countries of the European Union[4], over the period from 2002 to 2018 at quarterly frequency.

Both Equations 3 and 4 are estimated using panel quantile regressions to account for heterogeneity across time and countries. We use the estimator proposed by Machado and Santos Silva (2019). We briefly outline the methodology and refer the reader to the authors paper for details.

Consider a generic model with the couple (y_{it}, X_{it}) where y_{it} is the endogenous variable and X_{it} is the vector of explanatory variables. We consider N countries $(i = 1..., N)$ and T quarters $(t = 1, ..., T)$. In standard OLS regression, one estimates the conditional mean of the variable y by considering ther regression equation:

$$y_{it} = \gamma_i + X_{it}^{'}\delta + \sigma\ \epsilon_{it},\ \epsilon_{it} = y_{it} - E[y_{it}/X_{it}]. \tag{11}$$

for $i = 1, ...N$ and $t = 1, ...T$. $\epsilon_{it} \approx iid(0,)$. σ is a scale parameter used for instance to account for heteroscedastic residuals. The conditional mean

[4]Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and United Kingdom

equation cannot account for the conditional behavior of y when samples are characterized by strong heterogeneities. In this case, the conditional expectation is replaced by a conditional quantile:

$$y_{it}(\tau) = \gamma_i(\tau) + X_{it}^{'}\delta(\tau) + \epsilon_{it}(\tau),\ \epsilon_{it}(\tau) = y_{it} - Q_{\tau}[y_{it}/X_{it}]. \tag{12}$$

The τ-th conditional quantile is defined by:

$$Q_{\tau}[y_{it}/X_{it}] = inf\{y_{it}/F(y_{it}/X_{it} \geq \tau\},\ \tau \in (0,1). \tag{13}$$

$F(.)$ denotes the conditional distribution of the endogenous variable and is strictly increasing. Equation 12 is robust to distribution with heavy tails and outliers. Quantile regressions have become widely used since the 1990s in the empirical economics literature. For an overview of the theory and applications, the interested reader can refer to Koenker et al. (2017).

Consider the check function $\rho_{\tau}(\epsilon_{it}) = (\tau - I(\epsilon_{it} < 0))$. $I(x)$ is the heaviside function. A typical - consistent and asymptotically Normal- quantile estimator is defined by

$$(\hat{\gamma}_i(\tau), \hat{\delta}(\tau)) \in \underset{\gamma_i,\delta}{\text{argmax}} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \rho_{\tau}(y_{it} - \gamma_i - X_{it}^{'}\delta). \tag{14}$$

Machado and Santos Silva (2019) relate the heterogeneous behaviors in a sample to heteroscedasticity. They propose the following model:

$$y_{it} = \gamma_i + X_{it}^{'}\delta + [\mu_i(\tau) + Z_{it}^{'}\omega(\tau)]\epsilon_{it},\ \epsilon \approx iid(0,1). \tag{15}$$

Z is the group of variables that are the source of heteroscedastic residuals. If $Z_{it} = X_{it}$, the conditional quantile function is written as:

$$Q_\tau[y_{it}/X_{it}] = \Theta_i^1(\tau) + X_{it}^{'}\Theta_i^2(\tau), \tag{16}$$
$$\Theta_i^1(\tau) = \gamma_i + \mu_i(\tau),\ \Theta_i^2(\tau) = \delta + \omega(\tau).$$

The authors propose a GMM estimator to obtain the estimations of the coefficients.

Heteroscedasticity in empirical public finance is a topic that is usually not treated by most papers. However, there are several reasons why we consider this aspect here.

First, as overall deficits, debt ratios or output-gaps change, the variability of primary balance adjustment can vary substantially accross countries and time. In a context of lack of coordination of fiscal policies in the EU, the primary balance sample is likely to contain "outliers". The latter can be a cause of heteroscedasticity in the residuals of Equation 3.

Second, the EU does not meet the traditional conditions of an optimum currency area, because of heterogeneous macroeconomic fundamentals (inflation differentials, differences in external and internal balances), asymmetric shocks (incomplete risk-sharing between countries) and different resiliences to shocks (reflected for instance by differences in financial stress indicators across countries and time). This is a source of variability in the risk premia because financial markets view the EU as a fragmented market. Equation 4 is therefore also likely to have heteroscedastic residuals.

3.2. Estimation results

Equation 3 - policy reaction function - is estimated as follows

$$\Delta s_{it}(\lambda_1) = \mu_i^1(\lambda_1) + \alpha(\lambda_1)\boldsymbol{w_{1it}} + \beta(\lambda_1)\boldsymbol{w_{2it}} + \omega_3(\lambda_1)gap_{it} + \epsilon_{it}^s(\lambda_1), \quad (17)$$

$$w_{1it} = (def_{it-1} - \tau_1) \text{ and } w_{2it} = (d_{it-1} - \tau_2),\ \ \lambda_1 \in (0,1).$$

gap is the output-gap. μ_i^1 is a country fixed effect. The estimation results are shown in Table 1. Quantile regressions are estimated at nine different quantiles (λ_1 = 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90), with the estimates reported in the first nine columns of the table. The standard errors are reported in parentheses. For comparison, the results from ordinary least-squares regression (OLS) are also shown in the last column. Figure 1 presents a summary of the estimations. The solid line represents the point estimates with the shaded gray area depicting a 95% pointwise confidence band.

An increase in the gap between the overall deficit and its target leads governments to strengthen their adjustment of the primary balance. Since the endogenous variable is already the variation of the primary balance, any change reflects a higher intensity of variation. The coefficients α are positive, sinificant, and increase accross quantiles. Therefore, countries that usually managed to have high increases in their primary deficits (they are located at the high quantiles of the endogenous variable Δs_{it}) tend to be more reactive than the others in reducing new overall deficits. Conversely, countries that usually do not reduce their primary deficits widely (those located at the lowest quantiles) are less reactive in adjusting their primary balance in response to new overall deficits.

With regard to the debt ratio coefficient, the quantile estimates suggest that governments do not react to changes in the debt ratio *gap* by adjusting their primary budget surplus. This suggests that fiscal policy has been non-Ricardian, in the sense that no fiscal consolidation has taken place in order to maintain the debt ratio below the threshold. It is important to notice that we are testing the reaction of primary balances, not to the debt ratio per se, but to the debt ratio *gaps*. In this formulation, what matters for fiscal reaction is debt target. There are several reasons why governments may not be reactive to an annual debt target at 60%.

One reason is that the target that matters to governments is not the institutional threshold, set by the European authorities. They are likely to be more sensitive to other debt thresholds, for instance those that trigger a debt overhang (when public debt starts to have negative externalities such as crowding out effects for private and public investment, Ricardian equivalence behavior, excessive risk-premium). Empirical estimates of the so-called Laffer debt curve suggest that, in the advanced countries, negative effects of higher indebtedness are in place when a threshold of 90% of the debt ratio is reached, e.g. much higher than 60% (see Lee et al. 2017, Checherita-Westphal and Rother 2012, Reinhart et al. 2012). Eberhardt and Presbitero (2015) show evidence of heterogeneous -country-specific - peaks of the debt ratio in the Laffer debt curves, between 7% and 150%. In this context, governments are likely to adjust the primary balance only in relation to the debt ratio from which public debt becomes detrimental to economic activity.

Another reason why the coefficients β are statistically insignificant is that the threshold τ_2 that matters is the governments' level of tolerance for public debt. This level is endogenenous and likely to vary accross countries and time. In this case, by imposing the yearly constraint $\tau_2 = 60\%$ in the regression, there is a risk that the estimated coefficients are inconsistent. The fact that the quantile coefficients equal zero at all quantiles suggest that fiscal policy does not appear to adjust in order to satisfy the governments' budget constraint.

The response of fiscal policy to the output-gap is positive - and the coefficients are statistically significant at 5% level of significance-, with a widening output-gap leading to an improved primary balance. The coefficients vary from single to triple when one moves from the lowest to the highest quantiles, between 0.08 and 0.27.

	Quantile Regression									OLS
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
α	0.576***	0.628***	0.67***	0.711***	0.742***	0.772**	0.797***	0.824***	0.863***	0.637***
	(0.098)	(0.076)	(0.06)	(0.05)	(0.047)	(0.049)	(0.054)	(0.063)	(0.078)	(0.03)
β	0.0013	-0.0002	-0.0015	-0.0028	-0.0037	-0.0046	-0.0054	-0.0063	-0.0075	- 0.028***
	(0.017)	(0.013)	(0.01)	(0.008)	(0.008)	(0.008)	(0.009)	(0.011)	(0.013)	(0.004)
gap	0.088	0.1217**	0.149***	0.176***	0.197***	0.216***	0.233***	0.251***	0.276***	0.162***
	(0.076)	(0.058)	(0.046)	(0.038)	(0.036)	(0.037)	(0.041)	(0.048)	(0.06)	(0.035)

Table 1: Quantile Regression for the Fiscal Function Reaction

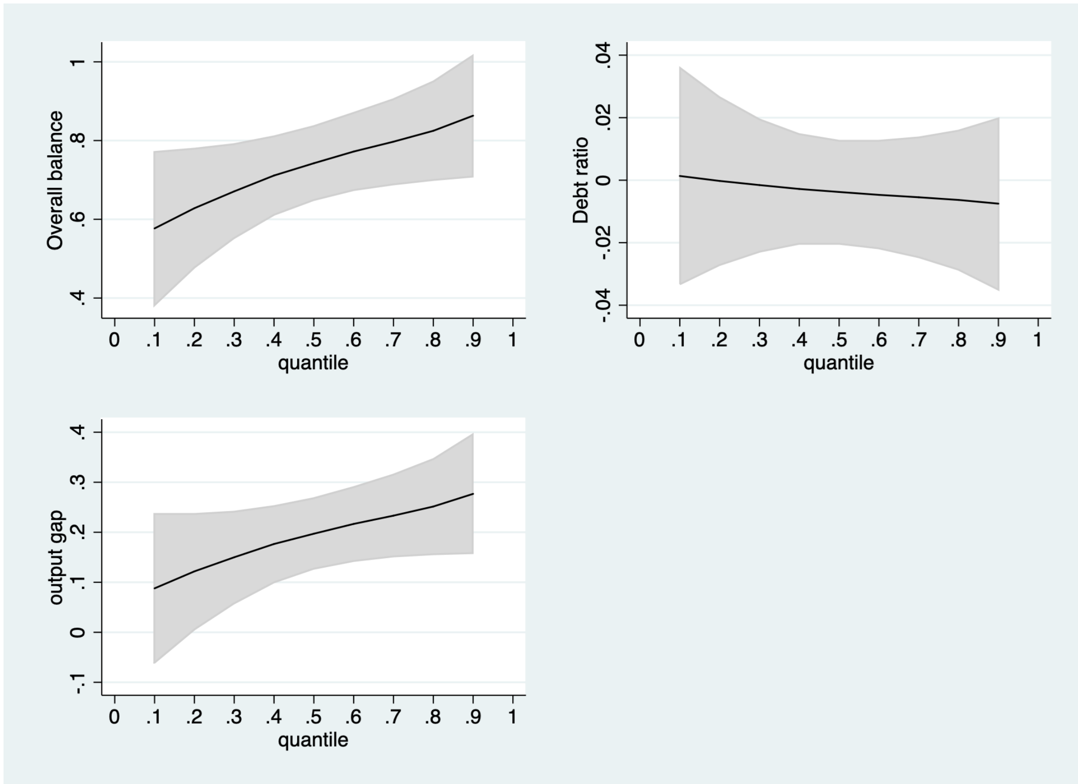

Figure 1: Quantile regression for the Fiscal Function Reaction

As for the estimates of the yield spread equation, several interesting findings emerge (see Table 2 and Figure 2). We estimate the following quantile regression:

$$\Delta\left[r_{it}-r^{*}_{it}\right](\lambda_2) = \mu^{2}_{i}(\lambda_2)+\beta_0(\lambda_2)\left[r_{it-1}-r^{*}_{it-1}\right]+\beta_1(\lambda_2)d_{it-1}+\boldsymbol{\beta_2}(\lambda_2)\overline{\Delta \boldsymbol{X}'_{it}}+\epsilon^{r}_{it}(\lambda_2). \tag{18}$$

$\lambda_2 \in (0,1)$ is a quantile. $\mu^{2}_{i}(\lambda_2)$ is a country fixed effect.

We find that the macroeconomic fundamentals -debt, unemployment and current account - do not matter in the pricing of default risk in yield spreads.

Neither does the global risk (FSI perceived by the markets). Indeed, all quantile coefficients are insignificant for these variables at 5% significance level. There are two potential explanations to this result.

On the one side, this evidence is in line with previous studies suggesting that financial markets may have been irrational - in the sense that the determination of premia have relied on self-fulfilling expectations and have been driven by shift in market sentiments, rather than on fundamentals - in pricing the risks of sovereign defaults (see, among others, Ayres, 2019, Chang and Leblond 2015, De Grauwe and Ji 1013, 2014). This weakens the fiscal discipline coming from the markets, as can be seen from the interval of variation of α in Proposition 2. Indeed, when the value of β_1 becomes small, the fiscal adjustment needed to reach the sustainability of public finances becomes stronger (the upper bound of the interval is larger). And when the effort for adjusting becomes stronger, governments are more likely to avoid it.

An alternative explanation is that investors pay a greater attention to liquidity risk than to solvency risk. Macroeconomic fundamentals are important to investigate whether countries are able to repay their debt at all points in the future. But, for shorter time horizons, other factors matter like the market value of liquid assets available to meet the maturing liabilities, or market short-term interest rate since they influence debt servicing.

As is seen from Table 2 and Figure 2, the coefficients of share prices are statistically significant at quantiles around the median. The negative signs can be interpreted as follows. A decrease in share prices imply higher risk of equity investing. And, higher equity risk imply a steeper slope of the risk premia. The results also show statistically significant coefficients for the short-term interest rates at quantiles around the mean. Because they can increase the burden of debt service, higher short-term rates lead to steeper risk premium slope in sovereign bonds markets.

	Quantile Regression									OLS
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
β_0	-0.1617	-0.0732	-0.0207	0.0251	0.0655	0.1064	0.1500**	0.2008**	0.2744**	0.0400***
	(0.1308)	(0.0955)	(0.0789)	(0.0685)	(0.0651)	(0.0679)	(0.0766)	(0.0921)	(0.1191)	(0.0100)
β_1	-0.0040	-0.0038	-0.0037	-0.0036	-0.0035	-0.0035	-0.0034	-0.0033	-0.0031	-0.0014**
	(0.0049)	(0.0035)	(0.0029)	(0.0025)	(0.0024)	(0.0025)	(0.0028)	(0.0034)	(0.0045)	(0.0005)
Share prices	-0.0067	-0.0062	-0.0059**	-0.0057**	-0.0054**	-0.0052**	-0.0050	-0.0047	-0.0043	-0.0054***
	(0.0045)	(0.0033)	(0.0027)	(0.0023)	(0.0022)	(0.0023)	(0.0026)	(0.0032)	(0.0042)	(0.0013)
Current account balance	0.0090	0.0060	0.0049	0.0034	0.0022	0.0009	-0.0004	-0.0020	-0.0043	0.0024
	(0.006)	(0.0043)	(0.0035)	(0.0030)	(0.0029)	(0.0030)	(0.0034)	(0.0042)	(0.0055)	(0.0037)
Unemployment	-0.1460	-0.0773	-0.0366	-0.0009	0.0303	0.062	0.0958	0.1352	0.1923	0.0630
	(0.1569)	(0.1137)	(0.0929)	(0.0805)	(0.0764)	(0.0798)	(0.0908)	(0.1102)	(0.1450)	(0.0443)
Financial stress index	0.2032	0.2971	0.3528	0.4015	0.4443	0.4877	0.5340	0.5879	0.666	0.4748*
	(0.8508)	(0.6159)	(0.5031)	(0.4358)	(0.4135)	(0.4321)	(0.4922)	(0.5976)	(0.7871)	(0.2852)
Short interest rate	0.1707	0.1556*	0.1466**	0.1388**	0.1319**	0.1249**	0.1175*	0.1088	0.0963	0.1430***
	(0.1229)	(0.0890)	(0.0727)	(0.0629)	(0.0597)	(0.0624)	(0.0711)	(0.0863)	(0.1137)	(0.0462)

Table 2: Quantile Regression for the dynamics of the yield spread

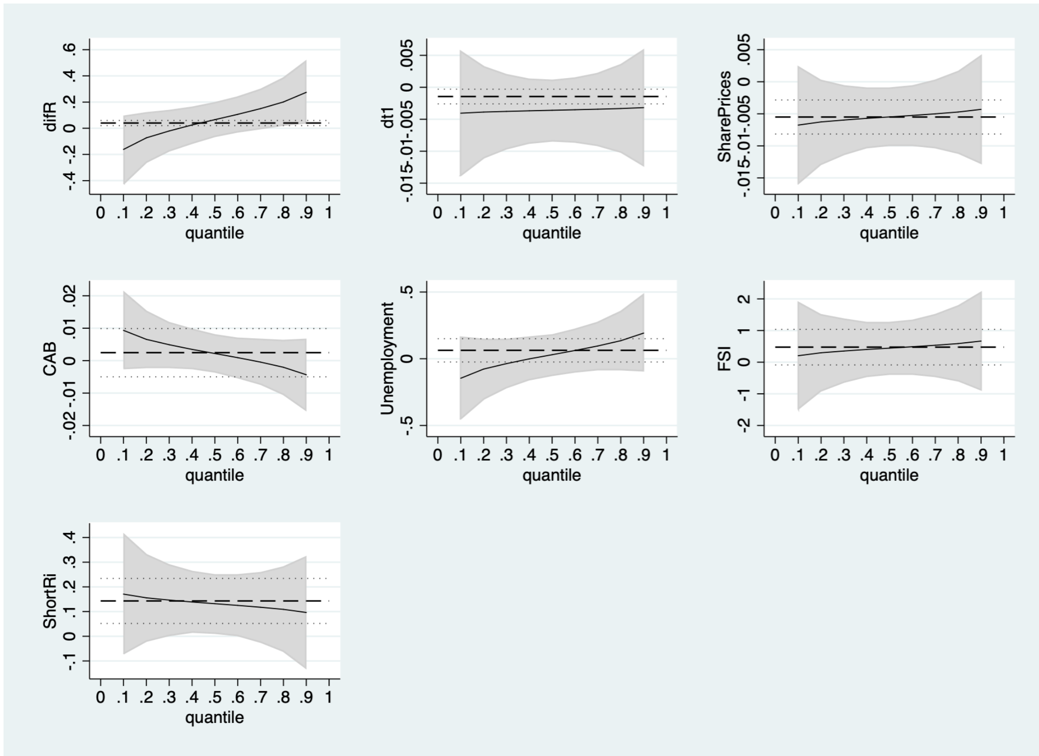

Figure 2: Quantile regression for the Dynamics of the Yield Spread

3.3. Implications for fiscal sustainability

Using the econometric estimates, we now investigate whether the conditions in Propositions 1 and 2 are satisfied. We consider the joint distribution of both endogenous variables in Equations 3 and 4, e.g. $\Delta s_t(\lambda_1)$ and $\Delta\big[r_t - r_t^*\big](\lambda_2)$. Δs_t measures the size of fiscal adjustment (large-scale fiscal adjustments or low-scale fiscal adjustments). The size of an adjustment can be related to quantiles. The adjustment is smaller at lower quantiles and larger at higher quantiles. Similarly, as one moves from the lowest to the highest quantiles of the distribution of $\Delta\big[r_t - r_t^*\big](\lambda_2)$, changes in the risk premium becomes greater. We therefore differentiate between low and high intensity changes in the risk premium. Crossing both variables, we delimitate four situations depending on λ_1 and λ_2 (see Table 3).

<table>
<tr><th>λ_2 / λ_1</th><th>0.1</th><th>0.2</th><th>0.3</th><th>0.4</th><th>0.5</th><th>0.6</th><th>0.7</th><th>0.8</th><th>0.9</th></tr>
<tr><td>0.1</td><td colspan="4" rowspan="4">Small fiscal adjustments
Low intensity changes in yield spreads
(Region I)</td><td colspan="5" rowspan="4">Small fiscal adjustments
High intensity changes in yield spreads
(Region II)</td></tr>
<tr><td>0.2</td></tr>
<tr><td>0.3</td></tr>
<tr><td>0.4</td></tr>
<tr><td>0.5</td><td colspan="4" rowspan="5">Large fiscal adjustments
Low intensity changes in yield spreads
(Region III)</td><td colspan="5" rowspan="5">Large fiscal adjustments
High intensity changes in yield spreads
(Region IV)</td></tr>
<tr><td>0.6</td></tr>
<tr><td>0.7</td></tr>
<tr><td>0.8</td></tr>
<tr><td>0.9</td></tr>
</table>

Table 3: Delimitation of regions according to the values of the quantiles

In Table 4, we report the values of the upper bound of the interest rate-growth differential as defined in Proposition 1, e.g. $\hat{\alpha}(\lambda_1) - \hat{\beta}_0(\lambda_2)$ (a "hat" means that we consider the estimates of α and β). The literature has extensively discussed the implications of the sign of this differential on debt dynamics. When it is positive, a primary surplus is needed to stabilize the debt ratio. When it is negative, the debt ratio can be reduced even with primary deficits. Proposition 1 does not refer to "zero" as the benchmark for the differential, but to a threshold value that depends on $\hat{\alpha}$ (fiscal policy stance) and $\hat{\beta}_0$ (degree of inertia of yield spreads). A country can sustain higher debt, with no fiscal cost -without adjusting -, even if the interest rate-growth differential is positive. The larger the value of the bound, the lower the likelihood of observing a snowball effect in the debt ratio dynamics. In the table, we see that, for given quantiles λ_2, the bound $\hat{\alpha}(\lambda_1) - \hat{\beta}_0(\lambda_2)$ increases as λ_1 varies from small to high values.

λ_1 \ λ_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.739	0.650	0.598	0.552	0.511	0.470	0.427	0.376	0.302
0.2	0.790	0.701	0.649	0.603	0.563	0.522	0.478	0.427	0.354
0.3	0.833	0.744	0.692	0.646	0.605	0.565	0.521	0.470	0.397
0.4	0.873	0.785	0.732	0.686	0.646	0.605	0.561	0.510	0.437
0.5	0.904	0.816	0.763	0.717	0.677	0.636	0.592	0.542	0.468
0.6	0.934	0.845	0.793	0.747	0.707	0.666	0.622	0.571	0.498
0.7	0.959	0.870	0.818	0.772	0.732	0.691	0.647	0.596	0.523
0.8	0.987	0.898	0.846	0.800	0.759	0.718	0.675	0.624	0.550
0.9	1.025	0.936	0.884	0.838	0.798	0.757	0.713	0.662	0.589

Table 4: Table 4 : $\alpha(\lambda_1) - \beta_0(\lambda_2)$

In Table 1, we have seen that $\hat{\alpha}(\lambda_1)$ was an increasing value of λ_1, meaning that larger fiscal adjustments reflect stronger reactions of the primary balance to changes in the overall deficit gap. The increasing values of the bound thereby suggest that the stronger the fiscal reaction to overall deficits, the lower the likelihood to endure a snowball effect in the debt ratio. Now, for given values of λ_1, the bound $(\hat{\alpha}(\lambda_1) - \hat{\beta}_0(\lambda_2)$ decreases as λ_2 increases. Since, we have seen in Table 2 that $\hat{\beta}_0(\lambda_2)$ is an inceasing value of λ_2, the results shown in the table imply that a higher persistence in the yield spread increases the likelihood of a snowball effect - because the value of the upper bound becomes smaller.

For purpose of illustration, we select some countries which belong to the different regions. We plot their interest rate-growth differential along with the average limit $\hat{\alpha}(\lambda_1) - \hat{\beta}_0(\lambda_2)$ of the region (dotted line). Figures 3, 4 and 5 show the cases of France, Germany, Italy, the Netherlands, Portugal and Spain. We see that, except in times of crises (during the beginning $2000s$, the 2008 financial crisis, or during the European debt crises from 2010 onwards), situations of interest rate-growth differentials below the ceiling leading to unsustainable public finances has been the historical norm. During crises, recessions push the growth-adjusted interest rates down in all countries (this is a common factor to all real interest rates).

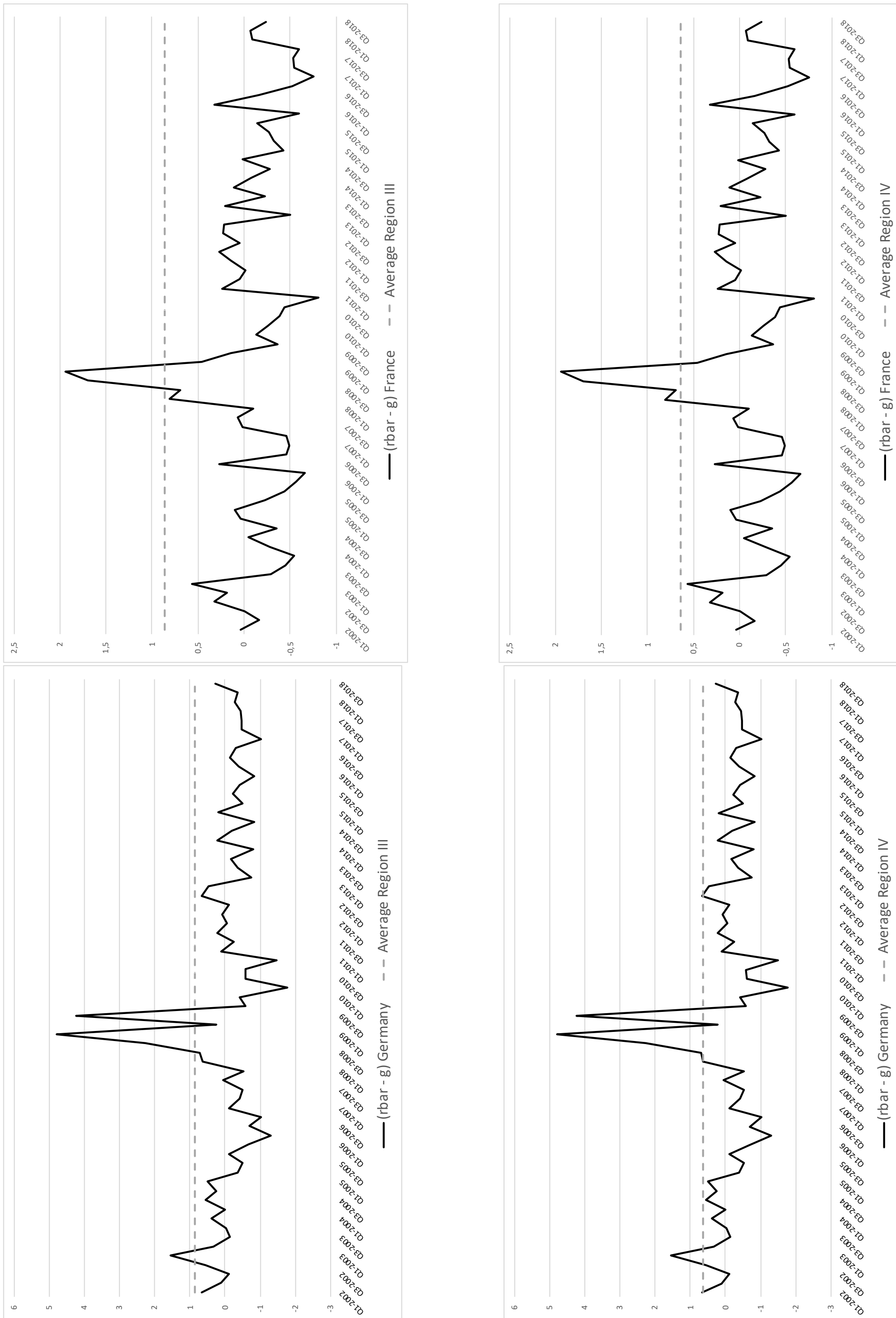

Figure 3: Interest-growth differential - France and Germany

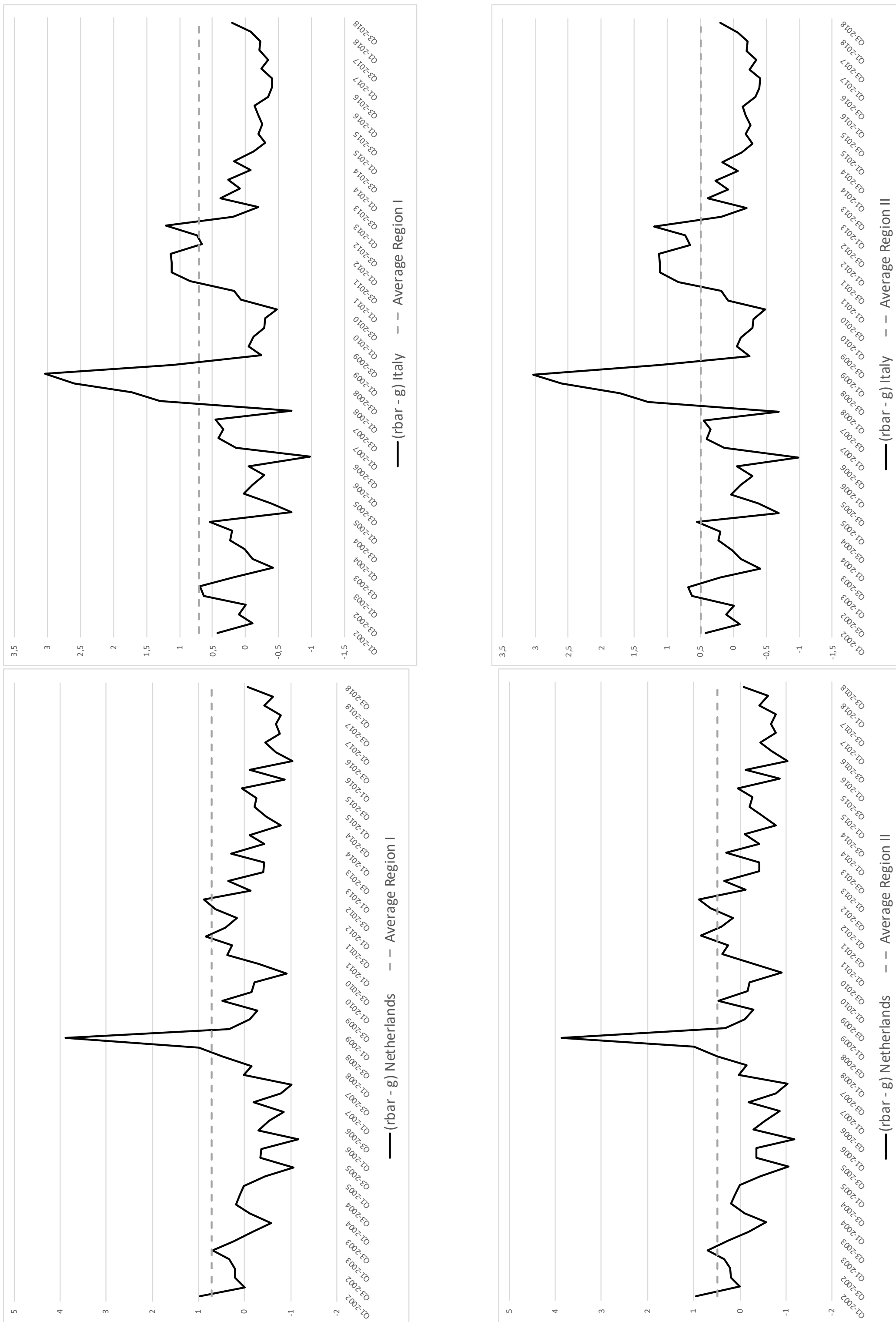

Figure 4: Interest-growth differential - Italy and the Netherlands

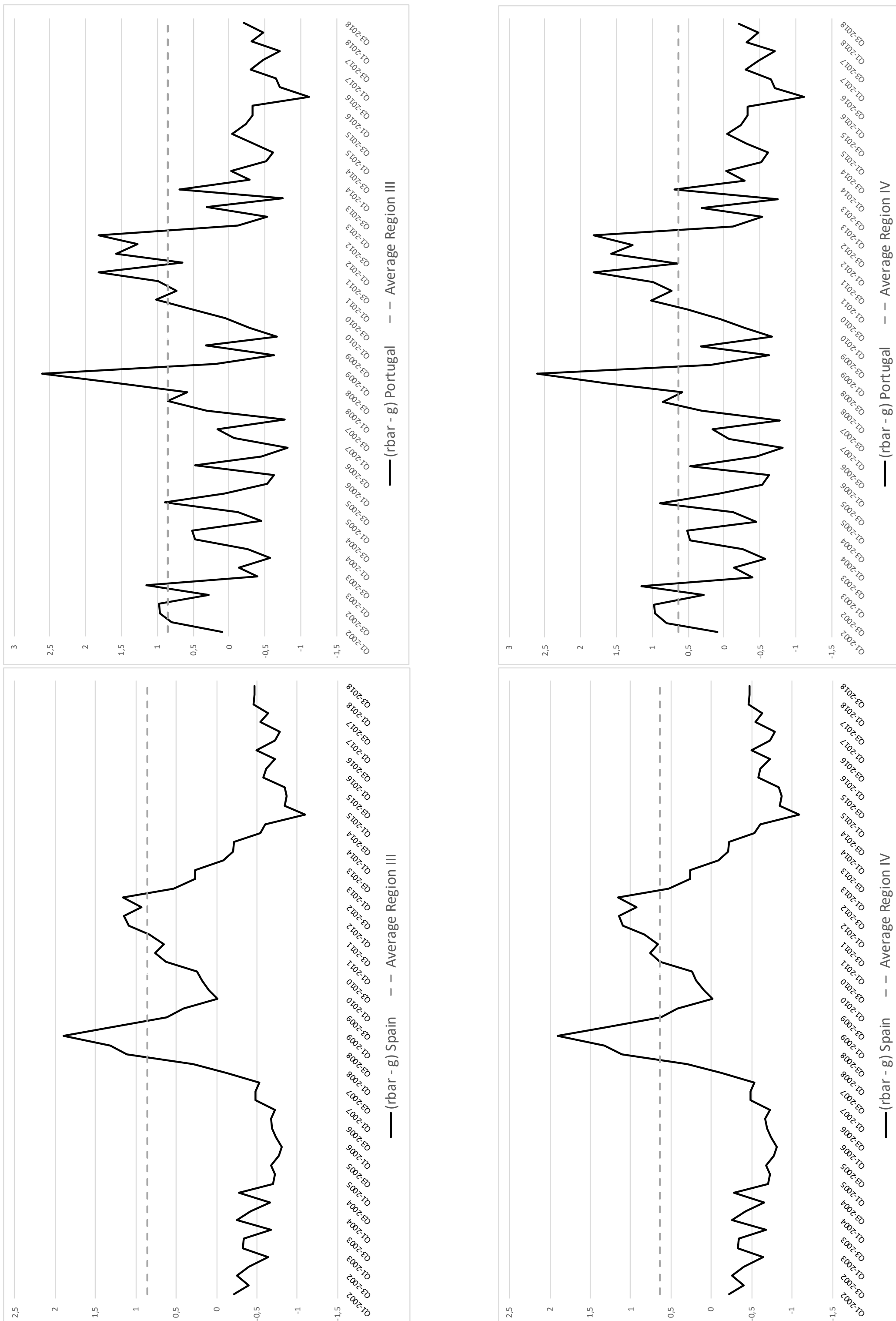

Figure 5: Interest-growth differential - Portugal and Spain

However, the role of the interest rates is uneven accross crountries. Some economies like Portugal and Spain have suffered from more frequent episodes of "high" nominal interest rates (for instance during the EU debt crises), thereby implying that the growth-adjusted interest rate crosses the threshold value more frequently than in other countries. $(\alpha - \beta_0)$ can be thought as a "cut off" limit of the growth adjusted interest rate which guarantees that indebtedness in sovereign bond markets would not tend to rise debt service - and therefore overall deficits - to unsustainable levels. Figures 3, 4 and 5 suggest that, even though $(\alpha - \beta_0)$ sometimes evolves above the limit, such a deviation is temporary.

We now look at Proposition 2. Since $\hat{\beta}$ is statistically insignificant for all quantiles, the assumption to be tested on α is:

$$\begin{cases} H_0^{\alpha}: \ \alpha(\lambda_1) = 0 \text{ (sustainable public finances)}, \\ \text{against} \\ H_1^{\alpha}: \ \alpha(\lambda_1) > 0 \text{ (unustainable public finances)}. \end{cases}$$

This amounts to testing whether α is statistically significant. In the quantile regressios, from Table 1, we see that the null hypothesis is systematically rejected at 1% level of significance. However, this is not enough to conclude that debt and deficits are stable in the medium/long term. As stated by Proposition 2, a prerequesite to create a safe fiscal regime where public finances remain stable in the face of external shocks is that governments are reactive enough to changes in both overall deficits and debt ratio. So, satisfying the constraint on β is also necessary. Since β_1 is statistically insignificant in Table 2, the inequality on β amounts to testing:

$$\begin{cases} H_0^{\beta}: \ \beta(\lambda_1, \lambda_2) \leq \beta^{H_0}(\lambda_1, \lambda_2) = [(\alpha - 1)]\beta_0(\lambda_1, \lambda_2) + \alpha(\lambda_1)(\pi - g), \\ \text{against} \\ H_1^{\beta}: \ \beta(\lambda_1, \lambda_2) > \beta^{H_0}(\lambda_1, \lambda_2). \end{cases}$$

The inequality on β in proposition 2 is satisfied if β^{H_0} is rejected. Table 5 reports the values of $\beta^{H_0}(\lambda_1, \lambda_2)$. The values of $(\pi - g)$ used are the average for each quantile λ_1. The stars denote the cases for which the null hypothesis is rejected. They are quite a few (7% of all possibilities, e.g. 4 out of 61). In the vast majority of cases, H_0^{β} cannot be rejected. The rejection occurs in very specific cases: when the coefficient of β are negative, e.g. when primary deficits could increase a little, high changes in the risk premium. Only in these rare cases, can the null hypothesis be rejected.

	λ_1								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\lambda_2 \in [0.1, 0.6]$	0.06	0.09	0.21	0.3	0.38	0.51	0.64	0.78	0.99
$\lambda_2 = 0.7$	-0.06*	0.03	0.16	0.25	0.34	0.47	0.60	0.75	0.97
$\lambda_2 = 0.8$	-0.02*	0.01	0.15	0.24	0.33	0.46	0.53	0.74	0.96
$\lambda_2 = 0.9$	-0.06*	-0.014*	0.123	0.219	0.315	0.446	0.58	0.73	0.95

Table 5: Values of β^{H_0}

The restrictions on α and β in Proposition 2 suggest that achieving deficits and debt targets, through adjusting primary balance, should be done jointly. According to our estimates, this does not seem to resonate with the EU experience. There are several ways of interpreting our findings. A first reason may be that debt-related fiscal rules in practice do not focus on the sustainability of public finances. Even under prudent fiscal policies, sovereign debts are contracted to finance public spending for the purpose of maximizing growth. Second, up until the recent TSCG (Treaty on Stability, Coordination and Governance), there were no rules for returning to the 60% target once the latter was breached.

3.4. Anchoring fiscal behaviors to debt target?

To make the sustainability of public finances more effective, a new rule has been introduced by the TSCG in 2013, imposing a corrective mechanism : countries must reduce by one-twentieth each year any excess of debt over the 60% benchmark. By constraining the use of fiscal discretion, this new rule amounts to setting a debt anchor to governments. A motivation is that imposing a target on headline fiscal deficits alone is not effective in achieving debt sustainability, because the channels from fiscal balances to debt involves many factors that are not under the direct control of governments (the macroeconomic environment prevailing before, during and after fiscal efforts take place). Another reason is the predominant view among European policymakers that the new rule represent a permanent constraint on fiscal policy, that avoid creating self-fulfilling debt crises. A corrective mechanism can help sending a signal to sovereign bond markets and avoid excessively increasing risk premia.

We specify a new rule for fiscal policy defined as follows:

$$\Delta s_{it} = \alpha \; (def^{T}_{it-1} - \tau_1) + \beta \; (d_{it-1} - \tau_2) + \epsilon^{s}_{it}, \tag{19}$$

where def_{it}^{T} is the overall balance anchor, defined as the headline fiscal balance ratio that stabilizes the debt ratio at time t in country i, under the TSCG corrective rule. Targeting the overall balance for which the debt ratio remains stable is not enough, since the condition $\Delta d_{it} = 0$ is compatible with an infinity number of debt ratios. For the EU Commission, a good fiscal situation is not only to stabilize the debt ratios, but also to avoid that deficits become too high. Therefore defining a limit τ_1 on overall deficit is also necessary.

Using the standard equation of debt dynamics, it can be easily shown that, under the corrective mechanism, we have:

$$def_{it}^{T} = [g_{it} + \pi_{it}]d_{it-1} - \frac{\tau_2 - d_{it-1}}{80}. \tag{20}$$

We divide by 80, assuming that an annual $(1/20)^{th}$ reduction of the excess debt is equivalent to a $(1/80)^{th}$ reduction each quarter. g_t and π_t are the real GDP growth rate and inflation at time t. $\tau_2 = 60\%/4$. Thought the rule is recent, we investigate its implications on public finance sustainability, had it be followed by the EU countries.

We first re-estimate the primary balance equation with the new rule, using again quantile estimators and, adding the output-gap as a control variable (see Table 6). * and ** indicate that a coefficient is statistically significant at, respectively, 10% and 5%. *gap* is the output gap. The new estimates show several differences compared with those in Table 1. For almost all quantiles (except at the lowest and highest) the primary balance is now reactive to changes in the debt ratio and the coefficient does not vary that much across quantiles. The correcting mechanism imposed by the TSCG would have forced the countries to behave in a similar way. However, heterogeneous behaviors remain with regard to the coefficient α. Indeed, the countries that adjust their debt-stabilizing overall deficit to the threshold are those for which the required fiscal effort would be weak (small adjustments of the primary balance at the lowest quantiles).

	Quantile Regression									OLS
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
α	0.0041**	0.0033*	0.0026*	0.0018	0.0008	60.0002	-0.001	-0.0017	-0.0024	0.0001
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.0012)
β	0.021	0.021	0.022*	0.022**	0.022**	0.022**	0.022*	0.022	0.022	0.006
	(0.017)	(0.014)	(0.012)	(0.010)	(0.010)	(0.011)	(0.013)	(0.015)	(0.017)	(0.005)
gap	0.093	0.095	0.096*	0.098**	0.102**	0.106**	0.108*	0.111	0.114	0.09**
	(0.077)	(0.065)	(0.056)	(0.048)	(0.045)	(0.049)	(0.058)	(0.067)	(0.078)	(0.043)

Table 6: Quantile Regression with the TSCG rule

We now examine the conditions under which the new rule would have led to sustainable public finances. The system to study is (we omit the index i):

$$\begin{cases} \Delta d_t = (r_t - g_t)d_{t-1} - s_t, \\ \Delta s_t = \beta d_{t-1} + \alpha(g_{t-1} + \pi_{t-1})d_{t-2} - \alpha(\frac{\tau_2 - d_{t-2}}{80}) \\ \qquad -(\alpha\tau_1 + \beta\tau_2) + \epsilon_t^s, \\ \Delta(r_t - r_t^*) = \beta_0(r_{t-1} - r_{t-1}^*) + \beta_1 d_{t-1} + \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'_t} + \epsilon_t^r. \end{cases} \tag{21}$$

The steady state of the system is given by

$$
\begin{cases}
\overline{s} = \overline{d}(\overline{r} - g), \\
\overline{d} = \frac{\alpha\tau_1 + \beta\tau_2 + \frac{\alpha}{80}\tau_2}{\beta + \alpha(g+\pi) + \frac{\alpha}{80}}, \\
\overline{r} = r^* \frac{\beta_0 - 1}{\beta_0} - \frac{\beta_1}{\beta_0}\overline{d} - \frac{\beta_2}{\beta_0}\overline{\Delta \boldsymbol{X}'},
\end{cases} \tag{22}
$$

This allows us to write the Jacobian matrix evaluated at the fixed point $(\overline{d}, \overline{s}, \overline{r})$:

$$
J = \begin{pmatrix} (\overline{r} - g) & -1 & \overline{d} \\ \beta + \alpha\big(g + \pi + \frac{1}{80}\big) & 0 & 0 \\ \beta_1 & 0 & \beta_0 \end{pmatrix} \tag{23}
$$

We now write the Ruth-Hurwitz conditions for the stability of the fixed point under the following assumptions that summarize our findings in the econometric estimations: $\alpha = 0$ (this coefficient is insignificant for a majority of quantiles in Table 6) and $\beta_1 = 0$ (from the estimates of this coefficient in Table 2). The sign restrictions on the trace and determinant of the Jacobian matrix lead the following inequalities:

$$
\begin{cases}
trace(J) < 0 \implies \overline{r} - g < \beta_0, \\
det(J) < 0 \implies \beta_0\ \beta < 0, \\
trace(J^2) - [(trace(J)^2] < 0 \implies \beta > -\beta_0(\overline{r} - g).
\end{cases} \tag{24}
$$

These three conditions can only be satisfied simultaneously if, at least, $\beta_0 < 0$. Negative values of this coefficient means that the yield spread dynamics is non-monotonic but sinusoidal. A sine wave indicates that the spreads are "mean-reverting", which is not compatible with momentum dynamics in the sovereign bond markets. As is seen from the first condition in the system, a negative β_0 implies that the growth-adjusted interest rate must be negative. Looking at the preceding figures of the time varying $(r - g)$ (Figures 3, 4, 5), we see that historical interest-growth differentials have frequetly been positive in the EU since 2002. Further, looking again at the regressions in Table 2, we observe that for the highest quantiles $\lambda_2 = 0.7, 0.8, 0.9$, β_0 is statisticially significant and positive (countries located in Regions II and IV in Table 3). In this case, the third inegality of the Routh-Hurwitz condition is violated.

In conclusion, calibrating a correction mechanism so that the debt remains below the annual ceiling of 60% of GDP, would not necessarily have helped the EU countries to avoid non-sustainable public finance, even in the most optimistic situation where governments would have chosen, on their side, to target their debt stabilizing overall deficits near the limit of 3%. Amongst the factors that could have endangered fiscal sustainability, the macro-financial environment plays a decisive role through the influence of financial market rates, real growth rates, or momentum dynamics in sovereign bond markets.

4. Conclusion

In this paper, we have proposed a behavioral approach to investigate the conditions of the sustainability of public finances in the EU since the beginning 2000s. The overall picture is that debt ratios and primary deficits have been far from being sustainable. Fiscal reaction functions reveal that governments have not paid attention to the debt gap when adjusting their primary balances, even when they benefited from a favorable financial environment when the interest rates -adjusted for growth - were below the levels implying a snowball effect. We provide evidence that, even if they had been forced to focus their fiscal efforts on correcting the deviations of debt from their ceiling -through a correcting mechanism - this would not necessarily have changed the likelihood that debt and deficits become more sustainable. Our main message is, however, that governments should not, necessarily, be blamed for not being rigorous enough in their fiscal efforts to avoid rising debts and deficits. We have seen that reducing primary deficits in response to rising overall deficits has been important for the EU fiscal authorities. Indeed, the coefficients $\hat{\alpha}$ in the first regression is statistically significant for all quantiles. The picture here is the opposite of the so-called phenomenon of fiscal fatigue. We have seen that $\hat{\alpha}(\lambda_1)$ increases with λ_1. This suggests that countries have been more reactive in adjusting their primary balance, in response to a change in the overall deficit, if they were already used to adjust their primary balance more strongly than other countries.

This papers could be extended in several ways. First, it would be interesting to investigate other fiscal reactions functions based on expenditure, revenue, structural primary balance, golden rule, etc. Second, there are potential applications to other geographical areas, for instance by examining the case of OECD countries. Third, it could be interesting considering a more general decomposition of the debt dynamic equations, by investigating how international factors such as the exchange rate, foreign interest rates contribute directly to debt increases through their impact on debt service.

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5. Appendix

5.1. Steady State

$$d(\dot{t}) = 0 \; ; \; then$$

$$(\overline{r} - r^* - g)\overline{d} + r^*\overline{d} - \overline{s} = 0$$

$$\overline{s} = [\overline{r} - r^* - g] + r^*\overline{d}$$

$$\overline{s} = \overline{d}[\overline{r} - r^* - g +$$

$\overline{s} = \overline{d}[\overline{r} - g]$

$$\dot{s(t)} = 0 \text{ ; } then$$

$$\alpha[\overline{d}(\overline{r} + \pi)\pi - \tau_1 - \overline{s}] + \beta(\overline{d} - \tau_2) = 0$$

$$\alpha\overline{d}(\overline{r} + \pi) - \alpha(\tau_1 + \overline{s}) + \beta\overline{d} - \beta\tau_2 = 0$$

$$\alpha\overline{d}(\overline{r} + \pi) + \beta\overline{d} = \alpha(\tau_1 + \overline{s}) + \beta\tau_2;\ and\ \overline{s} = \overline{d}(\overline{r} - g)$$

$$\alpha\overline{d}(\overline{r} + \pi) + \beta\overline{d} = \alpha(\tau_1 + \overline{d}(\overline{r} - g)) + \beta\tau_2$$

$$\alpha\overline{d}(\overline{r} + \pi) + \beta\overline{d} - \alpha\overline{d}(\overline{r} - g) = \alpha\tau_1 + \beta\tau_2$$

$$\overline{d}[\alpha\overline{r} + \alpha\pi + \beta - \alpha\overline{r} + \alpha g] = \alpha\tau_1 + \beta\tau_2$$

$$\overline{d} = \frac{\alpha\tau_1 + \beta\tau_2}{\alpha(\pi + g) + \beta} \tag{A.2}$$

$$\dot{r(t)} = 0 \text{ ; } then$$

$$\dot{r(t)} - r^*(t) = \beta_0[r(t) - r^*(t)] + \beta_1 d(t) + \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'(t)} = 0$$

$$\dot{r(t)} = \beta_0[r(t) - r^*(t)] + \beta_1 d(t) + \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'(t)} + \boldsymbol{r^*(t)} = 0$$

$$\beta_0\left[\overline{r} - r^*\right] + \beta_1\overline{d} + \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'} + r^* = 0$$

$$\beta_0\overline{r} = \beta_0 r^* - \beta_1\overline{d} - \boldsymbol{\beta_2}\overline{\Delta\boldsymbol{X}'} - r^*$$

$$\beta_0\overline{r} = r^*(\beta_0 - 1) - \beta_1\left[\frac{\alpha\tau_1 + \beta\tau_2 - \epsilon^s}{\alpha(\pi + g) + \beta}\right] - \beta_2\overline{\Delta\boldsymbol{X}'} - \epsilon^r$$

$$\overline{r} = r^*\frac{(\beta_0 - 1)}{\beta_0} - \frac{\beta_1}{\beta_0}\left[\frac{\alpha\tau_1 + \beta\tau_2 - \epsilon^s}{\alpha(\pi + g) + \beta}\right] - \frac{\beta_2}{\beta_0}\overline{\Delta\boldsymbol{X}'} \tag{A.3}$$

To sum up, the analytical expression of the steady state is:

$$\begin{cases} \overline{s} = \overline{d}[\overline{r} - g] \\ \\ \overline{d} = \frac{\alpha\tau_1+\beta\tau_2}{\alpha(\pi+g)+\beta} \\ \\ \overline{r} = r^* \frac{(\beta_0-1)}{\beta_0} - \frac{\beta_1}{\beta_0}\left[\frac{\alpha\tau_1+\beta\tau_2}{\alpha(\pi+g)+\beta}\right] - \frac{\beta_2}{\beta_0}\overline{\Delta \boldsymbol{X}'} \end{cases}$$

This allows us to write the Jacobian matrix evaluated at the fixed point $(\overline{d}, \overline{s}, \overline{r})$:

$$J = \begin{pmatrix} (\overline{r} - r^* - g) + r^* & -1 & 1 \\ [\alpha(\overline{r} - r^* + \pi) + \beta] + \alpha r^* & -\alpha & \alpha\overline{d} \\ \beta_1 & 0 & \beta_0 \end{pmatrix}$$

$$J = \begin{pmatrix} (\overline{r} - g) & -1 & 1 \\ \alpha(\overline{r} + \pi) + \beta & -\alpha & \alpha\overline{d} \\ \beta_1 & 0 & \beta_0 \end{pmatrix}$$

Routh-Hurwitz conditions Let A be a square matrix of order 3 with arbitrary coefficients as the one given. The characteristic polynomial is given by:

$$P(\lambda) = \lambda^3 - trace(A)\lambda^2 - \frac{1}{2}\big(trace(A^2) - (trace(A))^2\big)\lambda - det(A)$$

According to the Routh-Hurwitz conditions the necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts (the roots of $P(\lambda)$, or the eigenvalues of the matrix A), and thus that the dynamic system characterized by the matrix A is stable. The conditions can be summarized as follows:

$$\begin{cases} trace(A) & < 0 \\ trace(A^2) - \big(trace(A) - (trace(A))^2\big) & < 0 \\ det(A) & < 0 \\ \frac{1}{2}trace(A)\big(trace(A^2) - (trace(A))^2\big) & > det(A) \end{cases}$$

The last condition is satisfied if the first two are also satisfied, so that:

$$trace(A)\big(trace(A^2) - (trace(A))^2\big) > 0 > det(A)$$

5.2. Stability conditions

First condition: $trace(J) < 0$

$$\overline{r} - g - \alpha + \beta_0 < 0$$

$$\overline{r} - g + \beta_0 < \alpha$$

Second condition: $trace(J^2) - \big(trace(J)\big)^2 < 0$

$$(\overline{r}-g)^2 - 2\alpha(\overline{r}-\pi) - 2\beta + 2\beta_1 + \alpha^2 + \beta_0^2 - (\overline{r}-g)^2 + 2\alpha(\overline{r}-g) - 2\beta_0(\overline{r}-g)$$
$$-\alpha^2 + 2\alpha\beta_0 - \beta_0^2 < 0$$
$$2\beta_1 - 2\alpha(\overline{r}-\pi) - 2\beta + 2\alpha(\overline{r}-g) - 2\beta_0(\overline{r}-g) + 2\alpha\beta_0 < 0$$
$$\beta_1 - \alpha(\overline{r}-\pi) - \beta + \alpha(\overline{r}-g) - \beta_0(\overline{r}-g) + \alpha\beta_0 < 0$$
$$\alpha[\overline{r} - g - \overline{r} + \pi + \beta_0] + \beta_1 - \beta - \beta_0 < 0$$
$$\alpha[\beta_0 + \pi - g] - \beta - \beta_0 + \beta_1 < 0$$
$$\beta > \alpha(\beta_0 + \pi - g) + (\beta_1 - \beta_0)$$

Third condition: $det(J) < 0$

$$\beta_1\alpha + \beta_0[\alpha(\overline{r}+\pi) + \beta] - \alpha\beta_0(\overline{r}-g) - \beta_1\alpha\overline{d} < 0$$
$$\beta_1\alpha(1-\overline{d}) + \beta_0[\alpha(\overline{r}+\pi) + \beta - \alpha(\overline{r}-g)] < 0$$
$$\beta_1\alpha(1-\overline{d}) + \beta_0[\alpha(\overline{r}+\pi-\overline{r}+g) + \beta] < 0$$
$$\beta_1\alpha(1-\overline{d}) + \beta_0[\alpha(\pi+g) + \beta] < 0$$
$$\beta_1\alpha(1-\overline{d}) + \beta_0\alpha(\pi+g) + \beta_0\beta < 0$$
$$\alpha[\beta_1(1-\overline{d}) + \beta_0(\pi+g)] + \beta_0\beta < 0$$
$$\alpha[\beta_1(1-\overline{d}) + \beta_0(\pi+g)] < -\beta_0\beta$$
$$-\alpha[\beta_1(1-\overline{d}) + \beta_0(\pi+g)] > \beta_0\beta$$
$$\alpha[\beta_1(\overline{d}-1) - \beta_0(\pi-g)] > \beta_0\beta$$
$$\alpha > \frac{\beta_0\beta}{\beta_1(\overline{d}-1) - \beta_0(\pi-g)}$$

5.3. Data description

Name	Description	Source	Period Covered
Current account balance (CAB)	Current account balance as % of GDP.	OECD.Stat	2002Q1 - 2018Q3 Except for: Belgium 2003Q1 - 2018Q3 Netherlands 2002Q2 - 2018Q3
Financial Stress Index (FSI)	Country-Level Index of Financial Stress (CLIFS) (monthly)	ECB Statistical Data Warehouse	2002Q1 - 2018Q3
Real GDP	Quarterly growth rates of real GDP, change over previous quarter	OECD.Stat	2002Q1 - 2018Q3
Debt ratio	Government consolidated gross debt	Eurostat	2002Q1 - 2018Q2
Inflation rate	Inflation, average consumer prices annual percentage change (monthly)	Eurostat	2002Q1 - 2018Q3
Long-term interest rate	EMU convergence criterion bond yields (percentage per annum)	Eurostat	2002Q1 - 2018Q3
Short-term interest rate	% per annum	OECD Data Stat	2002Q1 - 2018Q3
Deficit ratio	Net lending (+) / net borrowing (-) (Unadjusted data (i.e. neither seasonally adjusted nor calendar adjusted data)	Eurostat	2002Q1 - 2018Q2
Riskless interest rate	Zero-cupon yield curve spot rate (AAA-rated euro area central government bonds); maturity 30 years.	Eurostat	2004Q4 - 2018Q3 Same for all countries
Share prices	Index	OECD Data Stat	2002Q1 - 2018Q3
Primary Balance ratio	Own estimation = (100)(Net lending/100)+ (nominal interest rate/100 * debt ratio (t-1)/100*1/(1+growth rate)		
Unemployment rate	Harmonised unemployment rate	OECD Data Stat	2002Q1 - 2018Q3
Potential GDP	Potential gross domestic product		

Primary balance

The primary balance (in % of GDP) is constructed as follows:

Primary balance (% GDP) = Overall balance (% GDP) + interest payments (% GDP)

$$s_t = \textit{net lending (\% GDP)} + i_{t-1}\ d_{t-1}\ \frac{1}{(1+\tilde{g}_t)}$$

where i_t is nominal interest rate, d_t is debt ratio and $\tilde{g}_t$ is nominal GDP growth rate between quarters $t-1$ and t.

Output-gap

The output gap us defined using HP filter, and we define:

$$\text{Output gap} = \frac{y^0 - y^p}{y^p}$$