Scientific Competition between Countries: Did China Get What It Paid for?

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Scientific Competition between Countries: Did China Get What It Paid for?

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Abstract

This paper examines the rise of China's relative standing in the global academic science marketplace. We first develop a simple theoretical model, based on the aggregation of individual knowledge production functions. This model predicts the existence of a stable power (scaling) law, relating the world share of countries' scientific production to their world share of public investment in scientific research. We test and confirm this prediction, using bibliometric cross-country longitudinal data for OECD and non-OECD countries, over the 1996-2015 period. This analysis allows for China's impressive catch-up, and for the West's decline to be accounted for, in the science marketplace, over the last two decades.

JEL codes: O38, P5

Keywords: economics of science, knowledge production function, international ranking.

1 Introduction

The economic rise of China is most often perceived through its spectacular performance in terms of exports (Chakraborty and Henry, 2019; Che et al., 2018), supported by the implementation of spatially-targeted public programs (Chen et al., 2017). In this paper, we pay attention to another striking aspect of China's rise to power, namely its place in the scientific marketplace. The academic science marketplace is now a global playing field, characterized by a fierce competition between institutions and countries to train and attract researchers, and to convert scientific activity into visible outputs (publications, participation in scientific conferences, project or manuscript peer-reviews, etc.). During the last decade, the relative place of China in this field has evolved dramaticaly. This article aims to identify the main drivers of this unique catch-up.

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In economics, there has been long-standing recognition of the importance of academic research for growth and social welfare (Jaffe, 1989; Adams, 1990; David, 1992; Romer, 1992; Stephan, 1996; Weinberg, 2011). While academic research contributes to enhancing human capital and informing policy-makers, it also has a direct impact on the development of a number of industries in which incorporated technologies draw on a large base of fundamental research (Grupp, 2000). Examples of "science-driven markets" are numerous, all of which have been and still are important growth providers: the chemistry and pharmaceutical industries since the end of the 19th century, the laser and high-tech electronic industries in the second half of the 20th century, or more recently the biotech industry. In all these markets, technologies have depended greatly on the production of scientific knowledge, which is ultimately grounded in work in universities and government laboratories (Mansfield, 1991; Salter and Martin, 2001; Cohen et al., 2002; Jong, 2006; Fabrizio, 2007).

The performance of a nation in the academic science marketplace has also taken on a great symbolic and political importance during the last decade: countries signal their value in the global knowledge economy through efforts to improve their relative position in this science marketplace (Docampo et al., 2015). International ranking of universities (such as the Academic Ranking of World Universities by Shanghai Jiao Tong University, Webometrics, or the Good University Guide by *The Times* of London) all include assessments of the performance of academic research. In a number of countries, the annual "Shangai Ranking" is anxiously observed and commentated on by news media and political circles, and serves as a major justification for structural reforms of public research systems (Hazelkorn, 2015). Meanwhile, a major shift within the global, worldwide ranking of scientific output has occurred in recent years. China has emerged as a leading producer of scientific output to the detriment of western OECD countries (Zhou and Leydesdorff, 2006; Leydesdorff and Wagner, 2009; Wang, 2016; Confraria et al., 2017). In this article we seek to examine how this unique and striking catch-up by China is to be accounted for. More broadly, we examine what the main determinants of the worldwide ranking in scientific production are.

Public investments in research and higher education constitute the main leverage for governments to develop, sustain and secure a successful system of scientific production. Yet, we face a puzzle. While the United States of America (US) has constantly and substantially increased the public funding of its scientific research (+150% over the 1996-2015 period), its world share of publications and citations has decreased dramatically. Observing this puzzling trend, Zhou and Leydesdorff (2006) suggested that the rules of the academic science marketplace have evolved over time, to the detriment of western countries: *"The US greatly increased intramural funding of R&D within government agencies after '9/11', but this increase has not been reflected in an increase of its world share of publications. One of the reasons might be that the emergence of other scientific countries like China and South Korea has put pressure on traditional advantages*". In sum, the basic principles of scientific knowledge production may have changed at the turn of the century, giving a new comparative advantage to East-Asian countries.

In this paper, we provide an alternative economic explanation for this puzzle. Using bibliometric country-level data and data on public investment in basic research, we document the existence of a remarkably stable power law, relating the world share of scientific publications and citations to the world share of public investment in basic research. This empirical power law solves the puzzle: the decline of the US in the latter is driven by the decline of the US in the former. More generally, these results also hold for Western Europe and Japan. By contrast, the success of China has been primarily driven by its constant public effort to invest in basic scientific research - more than other countries. There is thus no need to call for new (and somewhat ill-defined) comparative advantages to account for observable regularities. Furthermore, we propose a simple theoretical model to account for this scaling relationship. Our model basically rests on the aggregation of (individual) knowledge production functions (KPFs), as initially formalized by Griliches (1979). These KPFs establish a (power-law) relation between the amount of (monetary) resources allocated to research and the production of knowledge (at the individual level). Convenient aggregation properties of power laws, together with a few other assumptions, allow generating a macro power law, that we bring to the data[1].

This paper builds on and ties up with two important lines of research.

First, it contributes to the literature on the production of knowledge, which is a centerpiece in the economics of innovation. More precisely, it contributes to the branch examining the properties of KPF at different scales of aggregation: micro (Fortin and Currie, 2013), meso (Arora et al., 1998; Adams and Griliches, 2000) and macro (Crespi and Geuna, 2008). A common finding in this literature is that the exponent of the production function varies according to the level of agregation. In a nutshell, at the individual/micro level, most papers observe decreasing returns to scale, while at a more aggregate level, they report evidence of (almost) constant returns to scale. Measurement errors and/or spillover effects are usually put forward to account for these differences. Our model, based on the aggregation of power laws, nicely captures these scale effects.

Second, our paper relates to the burgeoning bibliometric or scientometric analysis of country performance in the academic science marketplace (Zhou and Leydesdorff, 2006; Leydesdorff and Wagner, 2009; Das et al., 2013; Wang, 2016; Confraria et al., 2017). Research activity is not limited to publishing papers, and entails a number of other important tasks (knowledge diffusion, organization of workshops, seminars and conferences, PhD supervisions, fund raising, direct or indirect contributions to local economies, etc.). Moreover, bibliometric indicators (number of publications, citations) may be biased, favoring articles written in English, or reflecting the non-neutral coverage of databases. However, they are increasingly used in the literature on research, as well as by politicians, media and evaluation agencies when assessing scientific activity: they provide a simple, objective measure (i.e. verifiable by anyone) of research performance, allowing cross-country comparisons to be made (for a discussion, see e.g. Wang, 2016). The literature has identified a number of institutional, cultural or organizational factors associated with country performance, such as the level of international collaboration or having English as a native language (Confraria et al., 2017). We complement this literature by showing that at the first order, the standing of a country in the global playing field primarily depends on the amount of resources allocated to public research. From a policy point of view, the message is clear: the only way for a country to maintain its position in the world ranking is to increase its effort more than its competitors.

The rest of this article is organized as follows. We first present our research question and review the literature. Then we develop the theoretical model that we test using cross-country longitudinal

[1]By doing so, we follow Gabaix (2016) who noted that *"All economists should become familiar with power laws and the basic mechanisms that generate them because power laws are everywhere. [...] power laws can guide the researcher to the essence of a phenomenon"* (p. 201).

data. Finally, we discuss our main results and conclude.

2 The Research Question and Related Literature

2.1 China's rise in power

Using the database from Elsevier's Scopus (SCImago, 2007), Figure 1, Panel A, plots the ratio (in log scale) of domestic scientific publications to the world total number of scientific publications for several countries, over the 1996-2015 period. Following the SCImago (2007) methodology, a publication is related to a country through the location of the authors' institutions. The Figure shows that, in the mid-1990s, the US had a solid leadership, with a world share of publications of 35%. Japan and the United Kingdom scored respectively second and third, with less than 10% of the world publications, followed by Germany and France. In sum, the Triad (the US, Japan and Western Europe) largely dominated the landscape. China, with 3% of world publications, stood behind Italy (4%). Between 1995-2005, China steadily increased its share, overtaking all its competitors except the US. In 2015, China accounted for 18% of world publications, slightly below the US (22.3%), but far above the other Triad members (ranging between 4 to 6.5%). The ranking in terms of citations, another commonly used metric of scientific output, shows a similar pattern, with a slight lag: a decrease in the world share of the Triad members, and a steady growth in the world share of China. In 2015, the US accounted for 20% of citations worldwide, followed by the UK (8.8%) and China (8.1%). While the magnitude of the Chinese catch-up is unique, we also observe that two other emerging countries - South Korea and Turkey - have succeeded in improving their position in the global standing over the period.

This remarkable achievement has been supported in China by a strong public commitment to scientific research, since the mid-1980s. In 1986, the 863 Program was initiated, followed in 1997 by the 973 Program (also known as the National Basic Research Program). These two programs explicitly aimed to accelerate China's scientific and technological development, including a strengthening of basic research in a few strategic fields (agriculture, energy, information, environment, health, etc.).

In 2006, a 15-year "Medium- and Long-Term Program for Science and Technology Development" (2006-2020) was launched, setting out eight distinct objectives. Objective 7 proposed "*establishing a world-caliber contingent of scientists and research teams, attaining high-impact innovative achievements in the mainstream of science development, bringing the technological level in such frontier areas as information, biology, materials, and space to world advanced levels*", while Objective 8 intended "*establishing a number of world-class research institutes and universities, and world-competitive industrial R&D centers so that a fairly comprehensive national innovation system of Chinese characteristics can take shape.*"(State Council, 2006).

To meet these objectives, national gross expenditures on R&D were planned to rise to 2.5% or more of gross domestic product (GDP), by the end of the Program (2020). Large budgets were allocated to several government agencies that offered science and technology research funding to researchers. The major funding agencies are the Ministry of Science and Technology (MoST), the National Natural Science Foundation of China (NSFC), the Chinese Academy of Sciences (CAS),

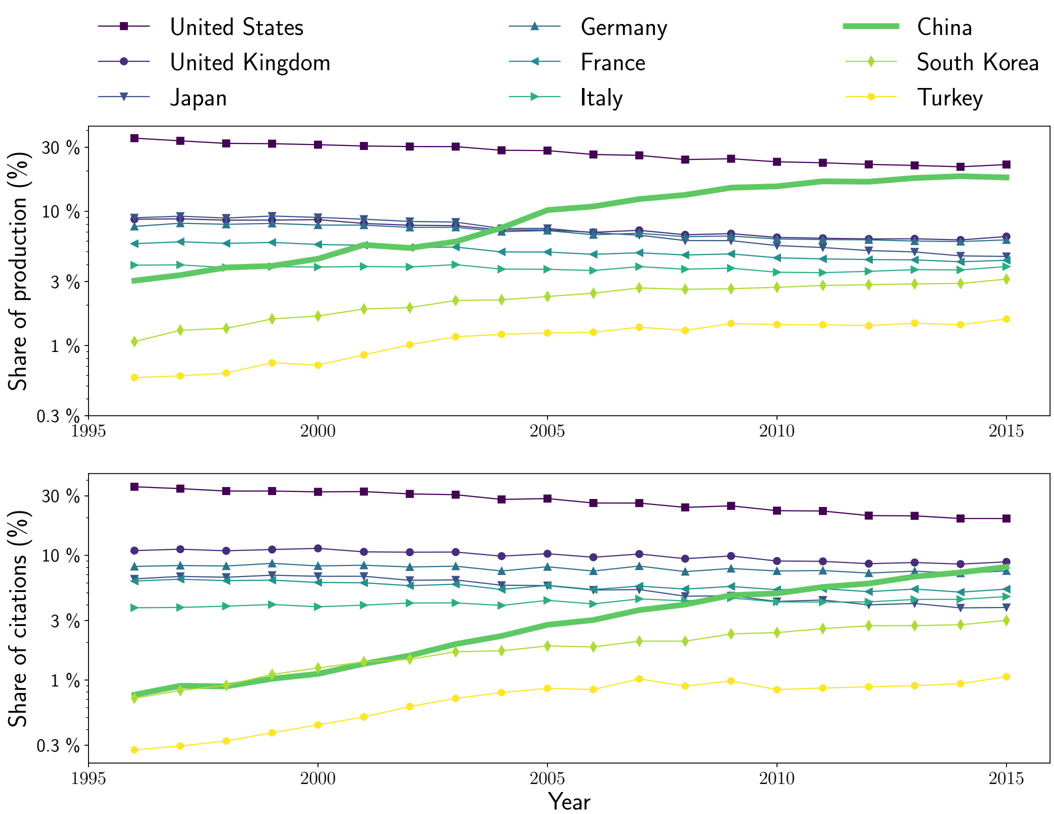

Figure 1: *Evolution of the world share of scientific production for selected countries, 1996-2015.*

and the China Scholarship Council (CSC) affiliated to the Ministry of Education (MoE). In 2008, a new program was implemented (the Recruitment Program for Innovative Talents - Long Term), targeted to bring overseas top (academic) talents to China in the coming years. The program offered attractive conditions (in terms of working conditions, salary and benefits like support for family arrangement) to foreign researchers under 55 years of age, willing to work in China on a full-time basis.

Using the Revealed Comparative Advantage method for the 2000-2013 period, Wang (2016) shows that these efforts have been associated with a consolidation of China's comparative advantage in a number of hard science fields (Physics and Astronomy, Chemistry, Computer Science, Chemical Engineering). By contrast, no improvement has apparently occured in Social Sciences, Economics, Management, Humanities or Psychology, where China's performances remained quite low.

A similar pattern of strong ambition in basic scientific research sustained by substantial public fundings has also been observed in South Korea (Zastrow, 2016) and in Turkey, with the Turkish Research Area, launched in 2004, and explicitly aimed at boosting the share of R&D expenditures in GDP and the number of qualified R&D personnel. In the rest of this paper, we show that these public efforts in quantitative terms are sufficient to account for the trends observed in Figure 1.

2.2 The production of knowledge

Studies by Griliches (1979) and Pakes and Griliches (1984) were probably the first to model explicitly the common intuition that there exists a stable relation between financial resources allocated to research activity and scientific output. More precisely, they hypothesized that the production of scientific knowledge is related to expenditure in research activity (or research funding), plus a disturbance term. They called this input-output relationship the Knowledge Production Function (KPF). A number of papers have since investigated the precise functional form of this KPF, as a way to measure returns in scientific activity.

A common result in the literature is that the rise in funding correlates with a rise in scientific output. At a high level of aggregation (country-level or scientific field-level regressions), the rise is almost linear (constant returns to scale or constant marginal productivity), but at a lower level of aggregation (research unit-, university- or individual-level), the rise is not linear (decreasing returns to scale).

Crespi and Geuna (2008) have studied the relationship between Higher education Expenditure on R&D (HERD) and scientific output (publications and citations) for 14 OECD countries, over the 1981-2002 period. Basic estimates showed decreasing returns to scale (the elasticitiy of publications to public funding is 0.47 and the elasticity for citations is 0.45). In richer specifications, Crespi and Geuna (2008) introduced knowledge spillover effects, as domestic publications are likely to be related to domestic HERD but also to HERD in the main scientific partner countries (identified with a weighting, computed as the number of international co-authorships between countries i and j, divided by the total number of co-authorships in i excluding j). In this case, they found that the sum of the estimated coefficients on domestic HERD and on spillovers is close to 1, indicating that constant returns to scale occur at the global level. This study therefore suggests that the elasticities of the KPF are scale-dependant (rising with scale and approaching 1 at the most aggregated level).

This result is supported by several other studies, at the meso and micro-levels.

At the meso-level, Adams and Griliches (2000) have used data for eight research fields in 109 universities, in the US, for the 1981-1989 period. At the field level, they regressed the number of scientific articles and the number of citations (in logs) against lagged values of R&D funding (in logs). The fitted coefficient is close to 1 (except for research in mathematics and agriculture): a 10% increase in funding is associated with a 10% increase in scientific production. However, when running regressions at the university-field level (i.e. at a lower level of aggregation), the results are different: with a sample of 40 universities, the elasticity of research output with respect to research funding is in between 0.6 and 0.7 (diminishing returns to scale). Arora et al. (1998) have examined the relationship between funding and scientific production in the Italian biotechnology and bio-instrumentation sector over the 1989-93 period. The level of analysis is that of the research unit (i.e. laboratory). They used data on applications to a research program sponsored by the Italian national research council (CNR). Their final sample consisted in 797 units, of which 347 were selected for funding through this program. They concluded that the elasticity of scientific production to funding is close to 0.6.

Arora and Gambardella (2005), Jacob and Lefgren (2011), and Fortin and Currie (2013) have estimated the KPF at the individual level. Arora and Gambardella (2005) used U.S. data on applications to the National Science Foundation (NSF) grants in economics during the 1985-1990 period. Their sample includes 1,473 distinct researchers (or applications), among whom 414 have successfully applied to the program - therefore benefiting from an additional budget. The estimated elasticities are low (the effect of NSF award on publications *"is very close to zero for senior professors"*), and the highest estimate (0.64) is observed for young researchers. The study by Jacob and Lefgren (2011) used a large sample of 54,741 applications to US National Institute of Health (NIH) grants, between 1980 and 2000, corresponding to 18,135 distinct researchers from 18 research units. 39,294 applications were finally awarded, based on a minimal score. Using a regression discontinuity design (together with IV estimates) they observe that *"NIH research grants do not have a substantial impact on total publications or citations"* (point estimates for elasticities are not significantly different from 0). One possible explanation lies in the "displacement" hypothesis: estimates do not capture other sources of funding, that may be allocated to high value projects irrespective of the success to any particular NIH research grant. Fortin and Currie (2013) have also performed individual level regressions using Canadian data. Public funding is captured by the total amount awarded by the Natural Sciences and Engineering Research Council (NSERC, a public agency) to a number of researchers in three disciplines (120 individuals in biology, 109 in chemistry and 139 in ecology), in two four-year periods (2002-2005 and 2006-2009). Considering scientific publications, they also observed that the estimates for elasticities are always inferior to 1 (0.4 in biology, 0.84 in chemistry and 0.39 in ecology).

Taken together, these results strongly suggest that the production of a researcher scales like funding to a degree lower than 1. In other words, the relation is of a power law type at the individual level, with an exponent inferior to unity. They also indicate that the exponent rises with the level of aggregation, as spillover effects or displacement effects are taken into account.

3 The Model

We start with an individual knowledge production function relating research funding and scientific publications, and which fulfills the above constraints and derives the optimal allocation of public funding for a given country. We then relate scientific publications to citations, in order to predict the relationship between funding and citations. Finally, we normalize the model to study longitudinal cross-country datasets, exhibiting a general growth over time of public investment in research, of publications and of citations.

3.1 From individual to country-level knowledge production functions

Let p_i and f_i be the scientific production (in terms of publications) and funding of researcher i within a country employing N researchers (hence $i \in \{1, 2, .., N\}$). We assume that these two quantities are related by a production function of the form:

$$p_i = a_i f_i^{\alpha_i}, \tag{1}$$

where α_i is the marginal productivity of the funding and a_i is a constant corresponding to the total factor productivity of a usual Cobb-Douglas production function.

Assumption 1: Researcher homogeneity.

$$\alpha_i = \alpha \; \forall i \tag{2}$$
$$a_i = a \; \forall i \tag{3}$$

Following (2) and (3), the elasticity of production (α) is assumed to be the same for every researcher, as is total factor productivity (a). This is not only a matter of simplicity and analytical tractability. Because of the internationalization of research activity and the increasing normalization of working conditions and publication procedures (such as the wide-spread use of English, the generalization of peer-review processes and the use of standardized methods), production ability tends to be similar across researchers, irrespective of their location. This assumption runs counter to the intuition that there are some "scientific stars" who may play the role of prestigious principal instigators within research units. For instance, Arora et al. (1998) attribute the few elasticities in knowledge production functions that approach 1 as resulting from the influence of these "scientific stars". However in our modelling, the point is not to contest productive heterogeneity across researchers, but only to take into account the evidence that because of the globalization of the scientific marketplace and the high mobility of researchers, the assumption that there is no difference - on average - between the elasticity of production (α) of different researchers operating in different countries is a credible assumption.

Assumption 2: Decreasing marginal productivity. The marginal productivity of a researcher dp/df decreases (decreasing returns to scale) - an assumption supported by the literature on KPF (see §2.2).

$$\alpha \in [0, 1[\tag{4}$$

By definition, the funding f_i received by researcher i is a fraction ϵ_i of the total funding (at the country level) F:

$$f_i = \epsilon_i F \tag{5}$$

with

$$\sum_{i=1}^{i=N} \epsilon_i = 1. \tag{6}$$

Equation (5) shows that the funding f_i in (1) is not limited to project grants but includes all sources of funding (hence wages, investment, recurrent expenses, and grants). The total production of a country is defined as the aggregate production of its researchers:

$$\sum_{i=1}^{i=N} p_i = P, \tag{7}$$

and together with (1) and (5) it can be written as

$$P = a \left(\sum_{i=1}^{i=N} \epsilon_i^{\alpha} \right) F^{\alpha}. \tag{8}$$

Assumption 3: the country's production is optimal. We assume that government and public agencies funding are allocated efficiently, i.e. to maximize the country's total scientific output. With N researchers (and given Assumptions 1 and 2), it can be shown that the optimal production of a country is obtained through an equal division of the available funding across scientists:

$$\epsilon_i^* = \frac{1}{N}, \forall i \in \{1, 2, ...N\}, \tag{9}$$

(the details of the calculation can be found in Appendix A).

Inserting (9) into (8) the optimal country's production then reads

$$P^* = aN^{1-\alpha} F^{\alpha}. \tag{10}$$

Assumption 4: The number of scientists N in the country is a power-law function of total funding. Research is, by definition, made up of non-routine analytic and interactive tasks, that ultimately involve skilled labor (in complementarity with capital). For this simple reason, there is a strong functional relationship between total research funding F and the number of researchers N, at the country level. We assume that this relationship can be approximated with a power law:

$$N = bF^{\beta} \tag{11}$$

where $\beta \in [0, 1]$ and b is a proportionality constant. β and b can be interpreted as parameters defining the labor demand (by government) for researchers: holding the price and the quantity of scientific capital constant, a government willing to spend F will demand (and hire) N researchers.

Again, we assume that this labor demand is similar across countries. In the empirical section, we show that this assumption regarding the functional form of the labor demand is supported by our data.

Using (11) together with (10) finally leads to the following power law between aggregate funding (F) and scientific optimal output at the national level:

$$P^* = cF^\gamma, \tag{12}$$

where

$$c = ab^{1-\alpha}, \text{ and } \gamma = \alpha + \beta(1-\alpha). \tag{13}$$

γ is an increasing function of β, with two limit cases: $\gamma = \alpha$ if $\beta = 0$ and $\gamma = 1$ if $\beta = 1$. Thus we have:

$$\gamma \in [\alpha, 1]. \tag{14}$$

Equation (12) relates directly the scientific output of a country to the parameters of the individual production function (a and α) and to the parameters of the labor demand for researchers (b and β).

3.2 From publications to citations

As generally in the bibliometric literature, we consider the number of citations per paper, as a proxy of research quality. The production p_i of researcher i corresponds to a number of citations

$$c_i = \sum_{k=1}^{k=p_i} c_{i,k}, \tag{15}$$

where $c_{i,k}$ represents the number of citations of the k-th publication of researcher i.

Different distribution functions for the number of citations of a given paper have been proposed in the literature (Redner, 1998; Radicchi et al., 2008). The number of citations of a particular article can be very large or very small. However, when the number of researchers and the number of publication are large enough, the law of large numbers applies, and the total number of citations approximates the number of publications P, multiplied by the mean of the distribution μ. Hence

$$C = \sum_{i=1}^{i=N} c_i = \mu P^* = \mu c F^\gamma. \tag{16}$$

The scientific impact of a country (in terms of citations) therefore follows the same scaling law as its production (number of articles) but with a different prefactor.

3.3 Worldwide ranking

We now extend our analysis to a set of J countries, engaged in a worldwide competition for publications and citations over a period of T years. Let $F_{j,t}, P_{j,t}, C_{j,t}$ be the investment, the number of citable documents, and number of citations a country j holds in year t. We define the world share of investment, production and citations of a given country and for a given year as:

$$\hat{F}_{j,t} = \frac{F_{j,t}}{F_{J,t}} \tag{17}$$

$$\hat{P}_{j,t} = \frac{P_{j,t}}{P_{J,t}} \tag{18}$$

$$\hat{C}_{j,t} = \frac{C_{j,t}}{C_{J,t}} \tag{19}$$

where $P_{J,t} = \sum_{j=1}^{J} P_{j,t}$, $F_{J,t} = \sum_{j=1}^{J} F_{j,t}$, and $C_{J,t} = \sum_{j=1}^{J} C_{j,t}$. Then, (12) and (16) can be rewritten as two power laws, relating the world share in funding to the world share in publications and citations:

$$\hat{P}_{j,t} = \left(c \frac{F_{J,t}^{\gamma}}{P_{J,t}} \right) \hat{F}_{j,t}^{\gamma} \tag{20}$$

$$\hat{C}_{j,t} = \left(\mu c \frac{F_{J,t}^{\gamma}}{C_{J,t}} \right) \hat{F}_{j,t}^{\gamma} \tag{21}$$

After simplification, the world share of citations then reads:

$$\hat{C}_{j,t} = \hat{P}_{j,t} \tag{22}$$

the share of citations equals the share of publications. In the next section, we use data from two different sources to test the predictions of (20), (21) and (22).

4 Empirical Analysis

4.1 Data source

We match two country-level databases to obtain a comparable but larger sample than previous macro studies on the subject (Crespi and Geuna, 2008) in terms of time span and the number of countries covered. The first dataset relating to scientific output is provided by the Scimago Lab company (SCImago, 2007). The database draws on the Elsevier scopus database (see Burnham, 2006, for a presentation). As of March 2018, the database runs from 1996 to 2016. The number of countries varies from 213 in 1996 to 233 in 2016. For each country and year, the database indicates the total number of citable documents published (articles, reviews and conference papers) and the (cumulative) number of citations by the (citable) documents published in previous years. In order to have a better proxy of the knowledge production disseminated across the scientific community, our measure of citations exclude self-citations. We have dropped 2016, as the figures had not stabilized at the time of writing this article.

The second dataset is the Main Science and Technology Indicators (MSTI) provided by the OECD (Organisation for Economic Cooperation and Development, 2017; Eurostat, 2005). We aggregate two of the MSTI indicators to measure national investment in scientific research in the public sector. The first indicator (HERD, for Higher Education Expenditure on R&D) corresponds to intramural R&D investment in the higher education sector.[2] By definition, such an indicator excludes spending for education. However, the very implication of the highest tertiary degrees in research projects (i.e. the fact that students directed by their professors may participate in some research at master's degree level for instance research assistants) may introduce a bias in the estimations: the "real" public R&D investment in the higher education sector may be higher than what we measure and may also differ across countries. For instance, this indicator is clearly better for some countries that have a higher education system which is more targeted to vocational education and/or bachelor's degrees. But it may also blur the real level of investment of some countries with a highly integrated level of public higher education and public R&D.

The second indicator (GOVERD, for Government intramural Expenditure on R&D) corresponds to the intramural investment in the public sector (all parts of central, regional or local government and all non-profit institutions that are controlled by government organizations; excluding organisations providing higher education services and state-owned companies are excluded). Values are given in millions of dollars at PPP. Data are available for the 35 OECD Member States and seven non-members. The Main Science and Technology Indicators also report the total number of researchers (in the higher education and the governmental sectors) per country and per year, in full-time equivalent (FTE) employment.

The number of sampled countries in the Main Science and Technology Indicators amounts to only 18% of the countries covered by the Scimago database. It may therefore be asked whether our sample is representative of worldwide scientific production. Table 1 shows the fraction of documents, citable documents, and citations that our 42 sampled countries account for, on a yearly basis, in the Scimago dataset. We observe that, over the 1996-2016 period, these countries accounted for, on average, 90% of the total world production of citable documents, and 93% of the citations. We are therefore confident that the analysis we perform and the results we obtain are representative of the worldwide situation. It may be noted, however, that the share of scientific output related to our sampled countries slightly decreases over the period, from 93% of citable documents in 1996 to 84% in 2016, and from 97% of citations to 88%. This trend is to a large extent driven by the rise of India: from 1996 to 2016, the India's share of citable documents more than doubled (from 1.9% to 4.2%). Unfortunately we do not have any information on India's investment in public scientific research, and, therefore, we cannot include it in our analysis.

4.2 Publications, citations and public investment in scientific research

Before testing (20), (21) and (22), we check that hypothesis 4, which assumes a scaling relationship between funding and the number of scientists at the country level (see (11)), is supported by our

[2]Note that intramural spending in public R&D include both public and private expenditures (Eurostat, 2005). For example a private donation to a university research center would be registered in HERD.

Year	Total documents	Citable documents	Citations
1996	0.93	0.93	0.97
1997	0.93	0.93	0.96
1998	0.93	0.93	0.96
1999	0.93	0.93	0.96
2000	0.93	0.93	0.96
2001	0.92	0.92	0.96
2002	0.92	0.92	0.95
2003	0.92	0.92	0.95
2004	0.92	0.92	0.95
2005	0.92	0.91	0.94
2006	0.91	0.91	0.94
2007	0.90	0.90	0.93
2008	0.89	0.89	0.93
2009	0.89	0.88	0.92
2010	0.88	0.88	0.92
2011	0.87	0.87	0.91
2012	0.87	0.86	0.91
2013	0.86	0.86	0.90
2014	0.85	0.85	0.89
2015	0.85	0.84	0.88
2016	0.84	0.84	0.88
Average	0.90	0.90	0.93

Table 1: *Fraction of the Scimago indicators corresponding to our 47 sampled countries*

data. To do this we plot (for the countries we sample) the number of scientists in the higher education and government sectors (N, in $\log_{10}$) against the total of investment in HERD and GOVERD (F, in $\log_{10}$), and the regression line (Figure 2).

We can see that the points gather around a clear trend, with a R-squared of 0.801. The point estimate for F is 0.779, with a (robust) standard error (clustered at the country level) of 0.057 (significant at the 1% level):

$$N = 51.167 F^{0.779}. \tag{23}$$

Equation (23) clearly supports assumption 4. As already noted the number of researchers in public institutions in OECD countries is given in full-time equivalents and not in headcount numbers. The way full-time equivalents are defined and computed is not necessarily similar across countries (see Section 4.1 for a discussion). This may introduce measurement errors in our analysis, accounting for the scatter we observe in Figure 2.

Figure 3 plots the world yearly share (in $\log_{10}$) of a country's publications (total number of citable documents) against the world yearly share (in $\log_{10}$) in total public investment (in scientific research). The correlation is striking and extends over more than three decades. In line with our first prediction, (20) this relationship follows a power law of the following form (see Table 2, column 1, for the details of regression, with robust standard errors clustered at the country level):

$$\hat{P}_{j,t} = 1.19 \hat{F}_{j,t}^{0.89}, \tag{24}$$

Our univariate model explains more than 90% of the variance in the world share of publications (the R-squared is 0.938). It should be noted that we obtain exactly the same estimate for the power law exponent (0.892) if we regress the number of publications in country j for year t (in $\log_{10}$)

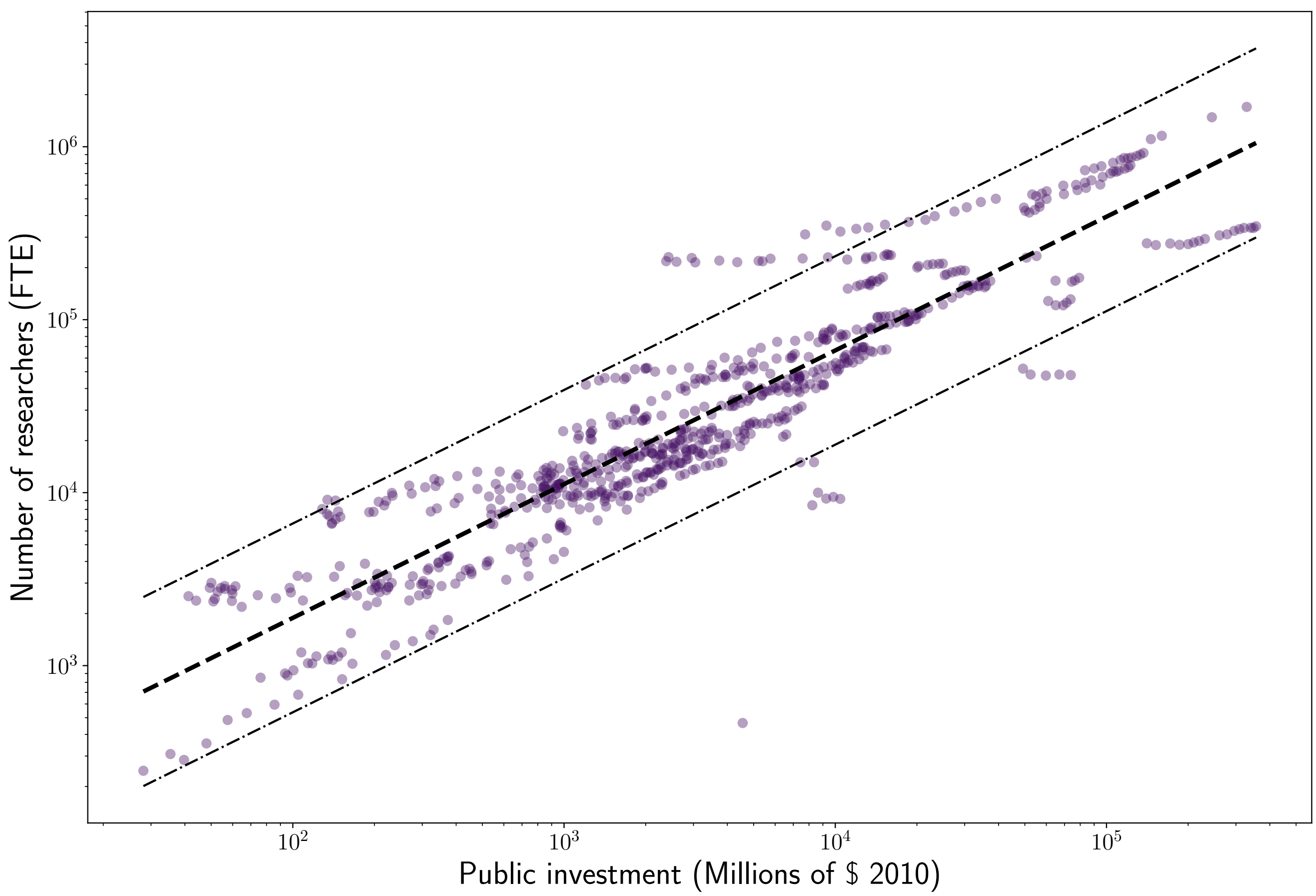

Figure 2: *The relationship between the number of researchers (counted in FTE, in* $\log_{10}$*) and investment in public research (HERD and GOVERD, in* $\log_{10}$*) in 42 (OECD) countries. Each point in the graph represents a country-year observation. The regression line, in black, is given by equation* (23)*. The dash-dotted lines indicate the 95% confidence interval.*

on the amount of funding (in j for t), plus a year fixed effect (as dividing our variables by the yearly world total of publications or funding is equivalent to incorporating an aggregate temporal trend). We also check that the estimates of the constant and of the exponent of the power law are never statistically different from one year to another, at the 99% confidence level (i.e. a Fisher test indicates for instance that for 1997 and 2007, the null hypothesis that the estimates are equal cannot be ruled out). The relation predicted by (20) and observed did not therefore evolve significantly over 20 years. In particular, the slope of the relation (the exponent) is systematically close to, yet lower than, 1.

Performing the same analysis on the world share of citations leads to the same conclusions (Figure 4). Regression results are displayed in Table 2, column 2 (with robust standard errors, clustered at the country level). We observe that the point estimate on $\hat{F}$ is positive (0.868) and significant at the 1% level, and the R-squared is equal to 0.836. As predicted by our theoretical

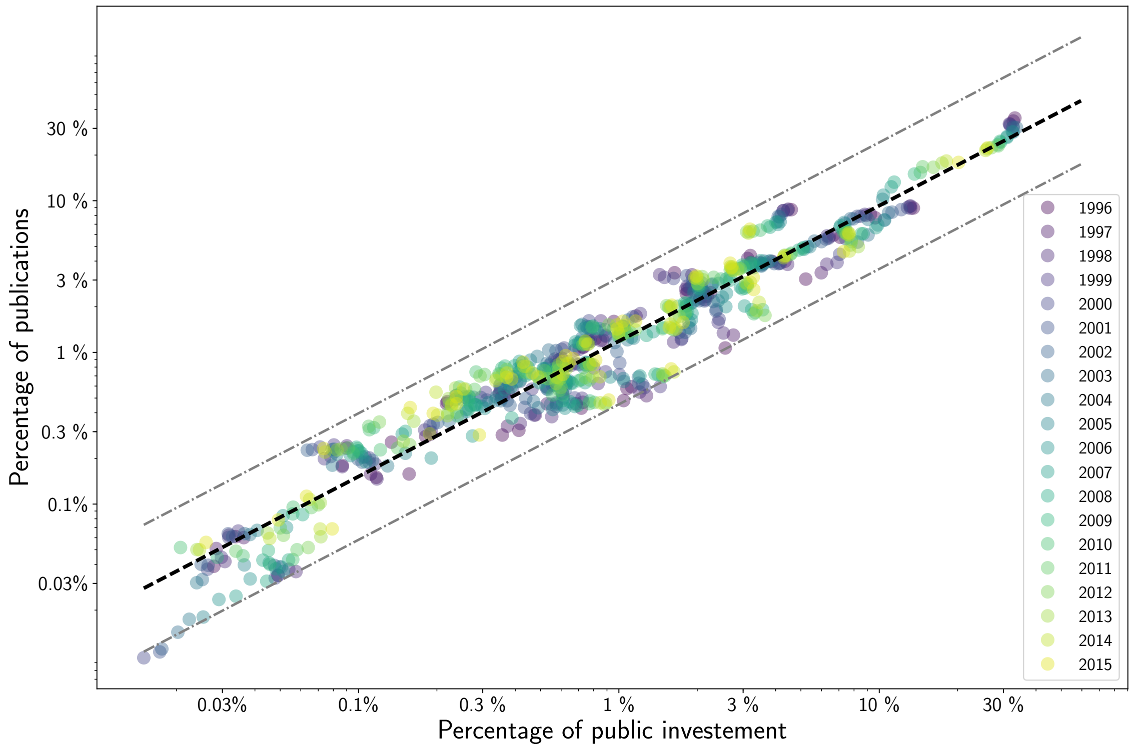

Figure 3: *The relationship between the world share of publications (in* $\log_{10}$*) and the world share of investment in higher education and research (in log ten), for OECD countries over the 1996-2016 period. Each point represents one country-year observation, the dashed line shows the best-fit regression, while the dash-dotted lines show the 99% confidence interval of the regression.*

model, a test of equality of the fitted coefficient on publications (0.892) and of the fitted coefficient on citations (0.868) does not allow the equality hypothesis to be rejected at the conventional level (p-value of 0.598). Ultimately, we obtain the following power law:

$$\hat{C}_{j,t} = 1.16\hat{F}_{j,t}^{0.87}, \tag{25}$$

in agreement with (21). Out of 380 potential pairs of years, 371 present no statistical differences in terms of estimates. The few exceptions (nine cases) are reported in Appendix B Table 3. We see that 2015 is somewhat particular, presenting a significant difference with the first eight years (1996 to 2003), and that 2014 differs from year 2000. As time passes, the number of citations for a given publication rises until it reaches a maximum value after approximately 10-11 years (Figure 5). Like in 2016, citations in 2015 are probably not fully stabilized (at the time of writing). This could explain why 2015 slightly departs from the rest of the data set. Also, let us note that, in all cases, differences concern the intercepts, and not the exponents (Table 3). Finally, a comparison between the two regressions confirms that at the 99% confidence interval (25) and (24) are the same, so

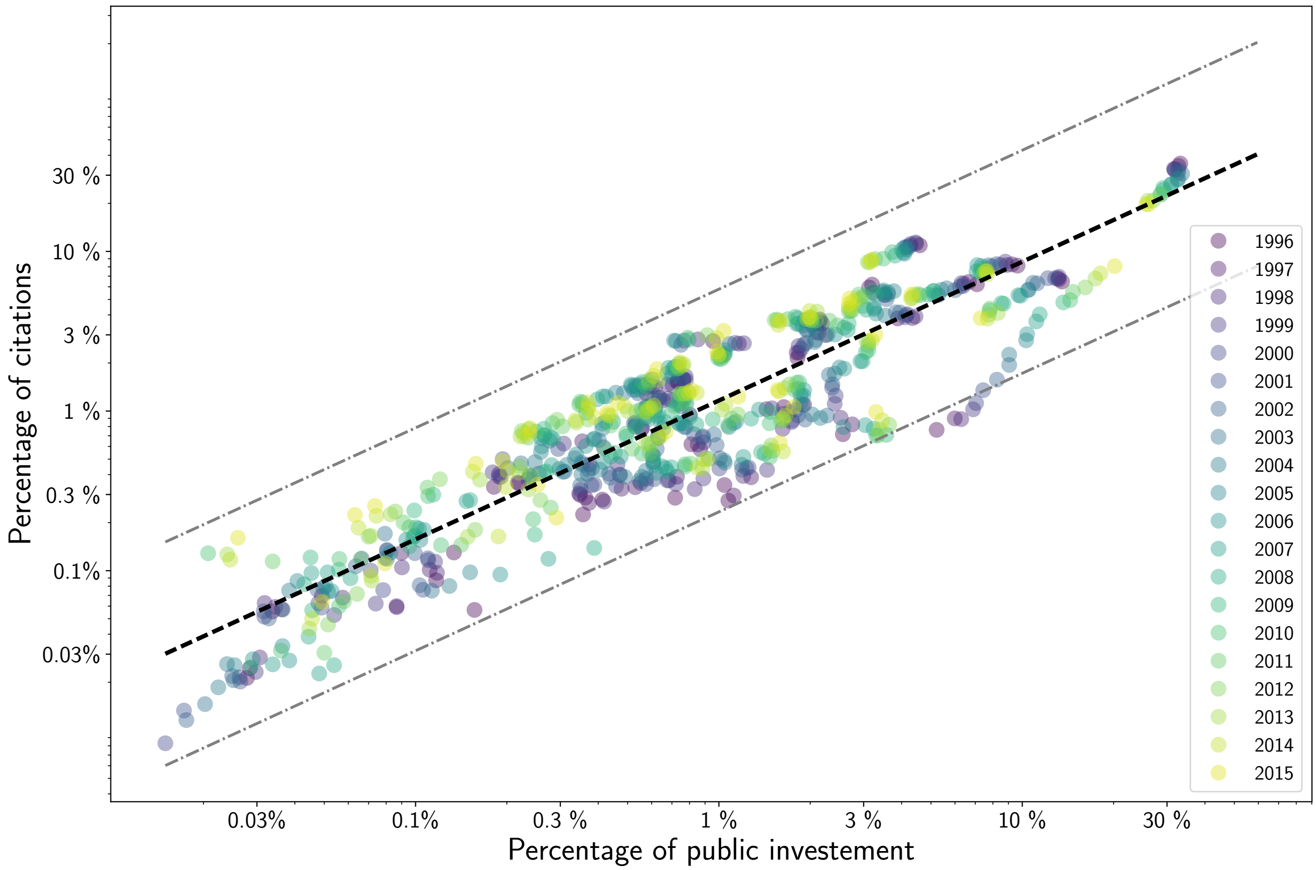

Figure 4: *The relationships between the world share of citations (in log ten) and the world share of investment in higher education and research (in logs), for OECD countries over the 1996-2015 period. Each point represents one country-year observation; the dashed line shows the best-fit regression, and the dash-dotted lines show the 99% confidence interval of the regression.*

that the prediction of (22) is also confirmed.

4.3 Robustness checks

As a robustness check we first re-estimate our regressions with a 1 year lag for the share of investment in scientific research. Results are very similar: the point estimate on lagged investment is 0.886 (with a standard error of 0.031) for publications, and 0.864 (standard error of 0.051) for citations.

Second, we introduce country fixed effects in our regressions. These effects capture unobserved heterogeneity across countries that may account for part of the relationship we have previously estimated between the relative scientific production and the relative research funding. We then obtain the following two elasticities (power law exponents): 0.640 for publications and 0.818 for citations (see Table 2, columns 3 and 4). While the former is significantly different (lower) than the pooled OLS estimate (see columns 1 and 3), it is not the case of the latter (see columns 2

	(1) Share of publications ($\log_{10}$)	(2) Share of citations ($\log_{10}$)	(3) Share of publications ($\log_{10}$)	(4) share of citations ($\log_{10}$)
Intercept	0.074**	0.065	0.275*	0.162
	(0.021)	(0.041)	(0.143)	(0.169)
Share of funds ($\log_{10}$)	0.892***	0.868***	0.640***	0.818***
	(0.032)	(0.054)	(0.160)	(0.189)
Observations	768	768	768	768
Country FE	No	No	Yes	Yes
R2	0.938	0.836		
R2-adj			0.985	0.974

Table 2: *Ordinary least square and fixed effects regressions in base ten logarithms of the world share of publications, and citations as a function of the world share of public investment in R&D. Robust standard errors (clustered at the country level) in parentheses; significance level* *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

and 4). In contrast with pooled OLSs, a test of equality of the fitted coefficient on publications (0.640) and of the fitted coefficient on citations (0.818) does not support the equality hypothesis (p-value of 0.001). This difference in the power law exponents of the publication and citation equations shows that accounting for unobserved heterogeneity across countries has more impact on publications than on citations. This suggest that some confounding factors influence both the ability of a country to raise funds for basic research and the ability of its researchers to publish scientific articles, while leaving unaffected the quality of this research (as measured by citations). This point deserves further research in the future. The fact that our estimation result is robust to controlling for unobserved heterogeneity, at least for citations, is to be contrasted with Adams and Griliches (2000), Fortin and Currie (2013) and Jacob and Lefgren (2011): in these three (individual-level) analyses, controlling for such heterogeneity (using long-difference regressions in the first two, and regression discontinuity design in the third one) drives the estimates for citations to zero. This indicates that unobserved heterogeneity is mainly a problem at the micro-level, where the existence of alternative sources of funding is usually not taken into account.

Third, while our data do not allow us to estimate directly the value of the power law exponent at the micro-level (α in equation 1), we can use (23) together with (24) to compute the value of α. We obtain:

$$\alpha \simeq 0.5 \tag{26}$$

This value is in line with the studies reviewed in Section 2.2, giving us confidence in the robustness of our results. At the university or laboratory-level, Adams and Griliches (2000) and Arora et al. (1998) report elasticities of around 0.6. At the micro-level, results are more divergent,

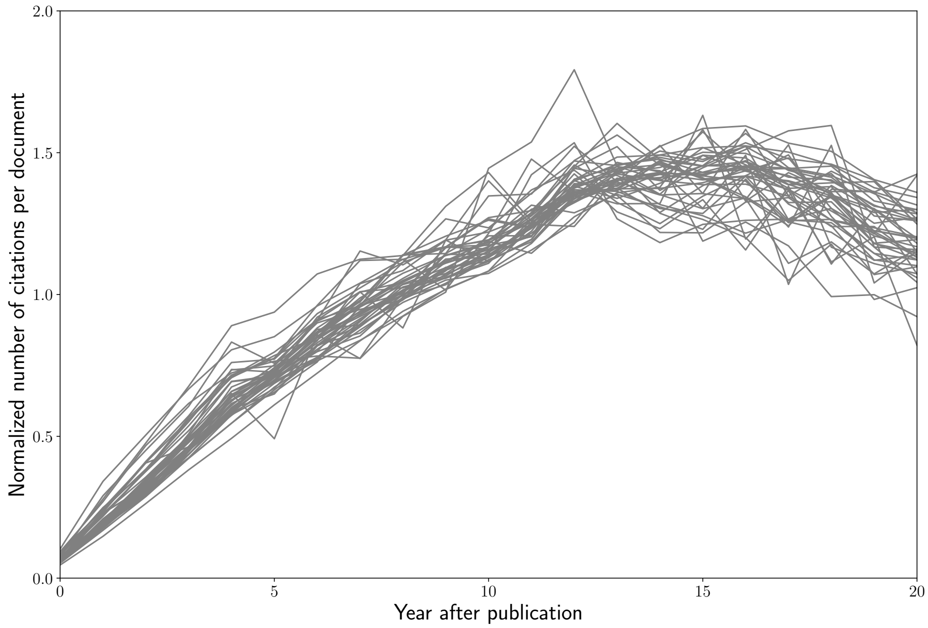

Figure 5: *The evolution of the average number of citations per document, for OECD countries, as a function of time after publication. Each line represents one country. Each country dataset is normalized by the number of citations per document of the country, averaged over the time span of the data.*

reporting value comprised between 0 and 0.8 with, for instance, 0.64 for young researchers in Arora and Gambardella (2005), and 0.4 in biology or ecology in Fortin and Currie (2013).

5 Discussion

Our main finding is the observation of a stable power law relating the world share of public funding in basic research to the world share in scientific publications and citations, with scaling exponents approximately equal to 0.9. This finding has a number of interesting implications.

First, the convergence of all countries around a single curve, both for scientific production and citations, implies that the rank of any country in the world-wide scientific competition is primarily determined by its financial investment, as predicted by equations (20), and (21). At the first order then, countries with the same order of magnitude of investment perform equally well. The decline of the Triad (the US, Japan and Western Europe) in the world share of publications and citations over the 1995-2015 period is mirrored by a decline in their world share of public investment. Likewise, the rise of China, South Korea and Turkey in publications and citations is primarily driven by their constant effort to increase their public investment, more than other countries (Figure 6). The rules of the game, at the international level, appear to be simple: stagnation in public investment leads to downgrading, while increasing a country's relative position in scientific output implies increasing its budget more than competitors. This is precisely what Chinese authorities have done over the last two decades, with a tangible and observable pay-off in global scientific competition.

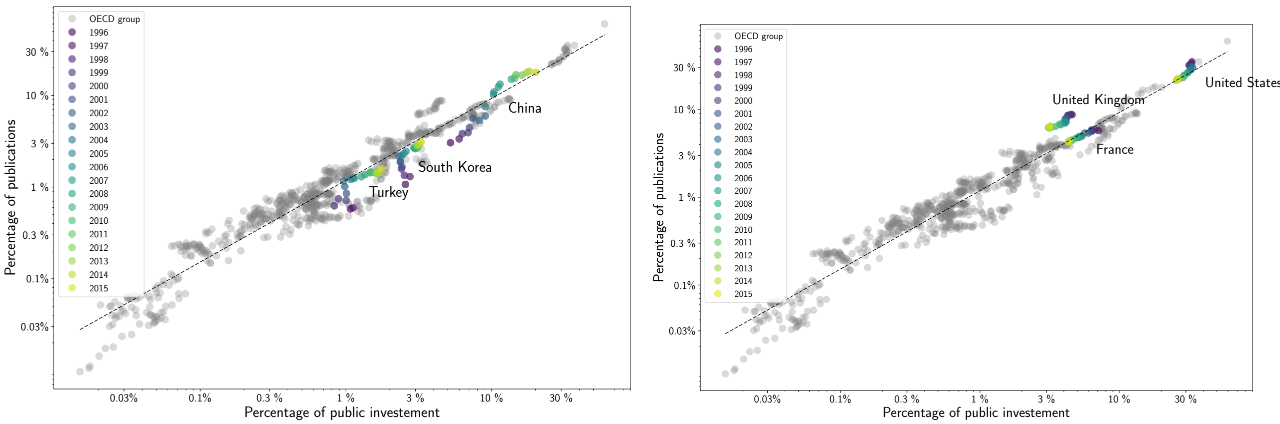

Figure 6: *The time evolution of the share of investment and publications for emerging scientific countries (left) and traditional scientific countries (right).*

Second, the world shares of publications and citations grow less than linearly with the rise of the relative investment, as indicated by the values of the exponent of equations (24) and (25). Accordingly, as investment rises, the marginal production and impact (in terms of citations) of a country decreases: smaller systems (in budgetary terms) are more productive than larger ones. Turning to China, what is interesting in the Chinese trajectory has been its financial capacity to generate a very high target level for investment in public research for two decades, so that it has progressively moved from being a small actor to being a large one in the global scientific marketplace. Further research should investigate in more depth the impact of China's political system has been on its capacity to secure such levels of investment (which could be a sort of comparative advantage in the very competitive marketplace for science).

Third, the empirical verification of (22) implies that a country's relative share in citations equals its share in publications. Thus the idea that the impact (in terms of citations) of scientific production matters more than the volume of production (as measured by the number of citable documents) seems incorrect - on average. Some authors have stressed the fact that while China was successfully sizing up its position in terms of publications, it was not so effective in terms of citations (Zhou and Leydesdorff, 2006). Our analysis shows this is no longer the case; in line with Confraria et al. (2017), Figure 7 shows that over the period China approached the 1 to 1 line.

Note that our model predicts this by assuming that countries exhibit no strong specialization and that the proportion of publications in each field is similar across countries. Concerning the modeling, in further research it could be interesting to relax assumption 1, stating that the exponent α of the production function (1) is independent of the research field. This could be tested in the future through a more complete analysis of the Scopus or WOS databases with OECD data broken down by research fields.

Fourth, our analysis suggests that the production system is optimal at the country level, which, here, means that funding is approximately equally shared between researchers (at the national level). This optimality may seem at first sight to be in contradiction with the widespread use of competitive grant systems, such as those promoted and implemented, for instance, by the National Science Foundation or the National Institute of Health in the US, the Natural Science Foundation of China, the Natural Environment Research Council in UK, or the Agence Nationale de la Recherche (ANR) in France. Although a complete analysis remains to be performed in future studies, we can partly resolve this apparent contradiction for the French case. In 2019, the budget of the ANR was a little less than €1 billion, whereas total public expenditures in research amount to more than €13.3 billion. In the higher education system, approximately 74% of government funding goes to paying both faculty and administrative staff, 9% to capital investment, and 18% to operational, or recurrent, expenditures (Algava et al., 2017). Hence most of the funding allocated to individual researchers escapes the grants system. As funding includes remuneration in equation (1), the value of f_i only varies marginally. The verification of our predictions suggests at first sight that this is also the case for other countries, but should be investigated in future research. For policy-makers, a direct consequence of optimality is that excessively favoring a competitive system to the detriment of a recurrent funding system would be counter-productive, as it would lead to a less efficient production system.

Finally, although the regressions are highly significant and the correlation coefficients are very high, the convergence of data around the trend is not perfect. There are several possible explanations for the scatter of country data around the trend, each of which could be investigated in future studies. First, the scatter may be related to "country effects". The fact that the objective of publication in ISI journals has not always been the norm in many countries, for example, may in part explain the variations around the trend for the number of publications and, even more, for the number of citations. Second, researchers of a given country may favor papers published by fellow citizens when citing articles (Larivière et al., 2018) - providing advantages to large countries or introducing some noise if patriotism is unequally distributed across countries. Third, our analysis assumes that countries do not specialize in specific fields of research. This specialization may introduce variations around the trend.

6 Conclusion

We started our paper with a simple, striking observation: the catch-up of China in the global academic science marketplace (and to a lesser extent, of South Korea and Turkey). Over the last two decades, China has steadily improved its position, whether in terms of publications and citations, to the detriment of the Triad members (the US, Japan and Western Europe). In 2015, China accounted for 18% of world publications (second only to the US with 22.3%) and for 8.1% of citations (behind the US and the UK, with respectively 10% and 8.8% of world citations).

While this unique catch-up has been already reported by a few studies (Zhou and Leydesdorff, 2006; Leydesdorff and Wagner, 2009; Wang, 2016; Confraria et al., 2017), none has offered a clear, simple explanation (mentioning for instance the new competitive advantages of the Eastern world). We have done so by arguing and showing that this trajectory in the world shares of publications and citations is driven by a similar trajectory regarding the world share of public investment in scientific research.

We first proposed a simple model, based on individual knowledge production functions. In

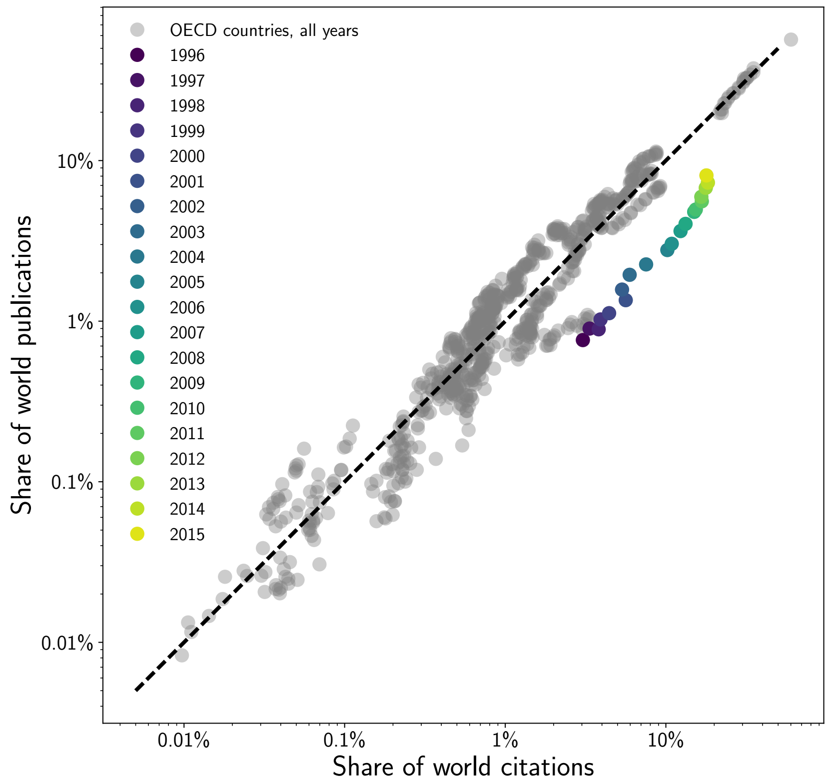

Figure 7: *The relationships between the world share of citations (in logs) and the world share of publications (in logs), for OECD countries (in gray) over the 1996-2016 period. Each point represents one country-year observation. China is represented in colors. The dashed line is the 1-1 line.*

accordance with the literature, we have assumed that the scientific production of a researcher is related to funding, under the form of a power law with decreasing returns to scale. The aggregation (at the country level) of these power laws, together with a few other assumptions, allows generating a macro-power law that keeps its functional form when translated in relative terms, at the international level. Ultimately, our model predicts that the shares of world scientific publications and citations follow power laws of the world share of public investments in research, with the exponents being less than or equal to 1.

We have tested this model with data, using cross-country longitudinal statistics for OECD countries and for a few non-OECD countries, for the 1996-2015 period. Our estimations support our predictions. More precisely, we have shown that there exists a stable power law relating the world shares under examination. For both publications and citations, the estimated exponent of the power law is approximately equal to 0.9.

Our main conclusion, then, is the following. The primary reason why China has overtaken or almost caught up with its main competitors is not because it has successfully redefined the rules of the game in the recent period: it is simply because it has increased its budgets more than others. And if Western countries aim to secure or improve their relative positions in the near future, they need to increase their efforts in budgetary terms. In the first instance, the key is money, more than structures or organizations.

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A Optimal scientific production

We want to calculate the optimal value of a country's scientific production P for a given investment F given that

$$P = a \left(\sum_{i=1}^{i=N} \epsilon_i^{\alpha} \right) F^{\alpha} \tag{27}$$

where a and $\alpha < 1$ are the production constant and exponent of the individual researcher's production function, and ϵ_i is the fraction of F received by researcher i. Thus we want to maximise

$$S = \left(\sum_{i=1}^{i=N} \epsilon_i^{\alpha} \right) \tag{28}$$

under the constraints that

$$\forall i \in \{1, 2, .., N\}, \epsilon_i \in [0, 1] \tag{29}$$
$$\sum_{i=1}^{i=N} \epsilon_i = 1 \tag{30}$$

Maximising (8) is equivalent to maximizing

$$M(\epsilon_i, \lambda) = \frac{P}{aF^{\alpha}} = \sum_{i=1}^{i=N} \epsilon_i^{\alpha} + \lambda \left(\sum_{i=1}^{i=N} \epsilon_i - 1 \right), \tag{31}$$

where $\lambda \in \mathbb{R}$ is a Lagrange multiplier. We look for solutions of $dM = 0$, hence

$$\forall i \in \{1, 2, .., N\}, \left(\alpha \epsilon_i^{\alpha - 1} + \lambda \right) d\epsilon_i = 0. \tag{32}$$

(32) implies that $\forall (i, j) \in \{1, 2, .., N\}^2, \epsilon_i = \epsilon_j$ and

$$\epsilon_i = \frac{1}{N} \tag{33}$$

The optimal scientific production of a country composed of N scientists is achieved when funding is shared equally between them.

B Funding and citations, year analysis

year	γ	c	year to be compared	H_0 slope	H_0 intercept	H_0 regression
1996	0.95 (0.22)	-0.04 (0.16)	2015	T	F	F
1997	0.90 (0.19)	-0.01 (0.14)	2015	T	F	F
1998	0.91 (0.19)	-0.01 (0.14)	2015	T	F	F
1999	0.91 (0.18)	-0.00 (0.13)	2015	T	F	F
2000	0.94 (0.17)	-0.00 (0.13)	2015	T	F	F
2001	0.92 (0.16)	0.02 (0.12)	2015	T	F	F
2002	0.92 (0.15)	0.03 (0.12)	2015	T	T	F
2003	0.91 (0.15)	0.05 (0.11)	2015	T	T	F
2000	0.94 (0.17)	-0.00 (0.13)	2014	T	T	F

Table 3: Fisher statistics for citations. *Comparison between yearly regressions $\hat{C} = c\hat{F}^{\gamma}$ ($p = 0.01$). Test of the null hypothesis for citations (H_0): the regression/slope/intercept cannot be differentiated from the regression/slope/intercept of 2015. T: H_0 cannot be ruled out; F: H_0 is rejected. Confidence intervals at the 99% confidence level in parentheses.*