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Abstract: We provide an evolutionary explanation for the well-established evidence of the existence of in-group favoritism in intergroup conflict. Using a model of group contest, we show that the larger the number of groups competing against one another or the larger the degree of complementarity between individual efforts, the more likely group members are altruistic towards their teammates under preference evolution.

Keywords: Indirect Evolutionary Approach; Evolutionary Stability; Groups; Altruism; Conflicts

JEL classification: C73; D64; D74

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1 Introduction

Since ancient times, a number of economic, social and political activities involve groups that are in opposition to one another. Intergroup conflicts have thus been extensively studied within different disciplines. For example, in social psychology, social conflicts are the main focus of the social identity theory (Tajfel and Turner, 1979). A central feature of this theory is the recognition that humans have a tendency to discriminate between "in-group" and "out-group" members even though "groups" are not formed according to some intrinsic characteristics or preferences but by random assignment. Furthermore, it is recognized that a stronger conflict with an "out-group" is likely to reinforce group cohesion by strengthening in-group favoritism (see Tajfel, 1982). More recently, social psychologists have conducted experimental studies in the laboratory which confirm that, in general, inter-group competition improves intra-group coordination in simple team games (see, e.g., Bornstein et al., 2002; Bornstein, 2003). There is also an important literature on human evolutionary biology that explains, often using dynamic games and simulations, that non-kin altruism towards members of one's own group together with out-group hostility – what is called "parochial" altruism – is a powerful force of the evolution of behavior in human societies (see, e.g., Choi and Bowles, 2007; Lehmann and Feldman, 2008; and Rush; 2014 or Glowaki et al. 2017, for a survey). As to the economic literature, the analysis of group conflicts is principally based on contest or rent-seeking games between groups (Katz et al. 1990 and Nitzan, 1991).[1] A number of relatively recent experimental studies aim at testing this type of games and generally conclude that subjects over contribute to group effort compared to the theoretical predictions (see, e.g., Abbink et al., 2010, 2012; Ahn, et al., 2011; and Sheremeta, 2018, for a survey).

In this paper, we provide an evolutionary and theoretical analysis of the emergence and stability of in-group favoritism in intergroup conflict by using a model of group contest. More precisely, there is large population of players who are randomly matched in groups that compete for an exogenous prize. We think of local (common-access) resources, each of which being targeted by a certain number of groups. The probability of winning the contest of a certain group is given by a Contest Success Function (CSF) that depends on the group members' efforts relative to members' efforts of competing groups. We consider that each group has two members and that the CSF has the standard *ratio*-form such that each group's probability of winning the prize is equal to the proportion of its collective effort out of the sum of collective efforts by all groups involved in the contest. However, in contrast to most analysis on group contests that assume a 'summation technology' with perfect substitutability between individual efforts, we consider that the effective level of group effort – its *impact function* – is given by a technology featuring a varying degree of complementarity between individual efforts. Indeed, as first pointed out by Alchian and Demsetz (1972), team or group production exists to the extent that it can exploit complementarities of inputs and this might be especially the case in a context of competition between groups (see also Kolmar and Rommeswinkel, 2013; and Brookins et al., 2015).

Another crucial feature of the present analysis is that each player has a *utility* – his/her "preference type" – that depends not only about his/her own material payoff but also on

[1] For a review of the literature on group contests, see Garfinkel and Skaperdas (2007, Section 7), Corchón (2007, Section 4.2), and Konrad (2009, Chapter 6).

that of his/her teammate. We first remain agnostic as to whether this concern is altruistic or spiteful, that is to whether each member puts a positive or negative weigh on the payoff of his/her teammate when deciding his/her contribution to collective effort. We then characterize the (pure strategy) Nash equilibria of this group contest game when group members have heterogeneous "other-regarding" preferences within and across groups. Next, we use the *indirect* evolutionary approach pioneered by Güth and Yaari (1992) to endogenize players' preferences. That is, evolution does not play *directly* at the level of strategies but *indirectly* at the level of preferences while players act rationally. In other words, preferences determine the players' actions which in turn determine (individual) *material payoff* – or "fitness" – and ultimately the evolutionary survival of preferences.[2] Thus, applying the concept of evolutionary stability (see Maynard Smith, 1982) to preferences – rather than to strategies – allows us to endogenize attitudes towards one's partner.

It is worth pointing out that a change in the preference type and thus in the equilibrium action of one player has strategic implications in that it induces a change in equilibrium actions of other players. The difficulty of the present analysis stems from the fact that there is a dual level of strategic interactions between players. The first one occurs within groups. Each group member decides on his/her contribution to group effort according to his/her preferences given the preference type and the resulting action of his/her teammate. The second level of strategic interactions occurs across groups. A change in the preference type of one or both members of the same group is passed on group effort which in turn induces a change in the winning probabilities for all groups. The evolutionary success of a certain preference type is the product of this dual level of strategic interactions. And we are particularly interested by the impact of the existence of within-group complementarities and by the intensity of the competition between groups, as measured by the number of competing groups, on the evolutionary selection process of preferences.

Using a notion of local evolutionary stability (Alger and Weibull, 2010), we can explicitly determine the evolutionarily stable degree to which a group member cares about the material payoff of his/her teammate. Clearly, a given group would be more successful in the group contest with more in-group altruism. But in the process of evolution, the question arises of whether a certain degree of altruism within groups is immune against "mutant" members with lower degrees of altruism. In fact, we show that preference evolution can equally result in altruism or spite within groups in intergroup conflict. Yet, we show that the larger the degree of complementarity between individual efforts or the larger the number of opponents, the more likely group members are altruistic towards each other. Furthermore, a further increase in the degree of complementarity between partners' efforts or in the intensity of competition usually tends to reinforces group cohesion in that it increases altruism towards

[2]This approach has been further developed by, among others, Bester and Güth (1998) and Sethi and Somanathan (2001). More recently, the literature on the evolution of preferences has focused on the informational structure in general settings. Heiftez et al. (2007a, 2007b) show that in a large class of games, agents with "biased" preferences, such as altruism, spite, reciprocity or fairness, may actually be more successful in terms of material payoff even though agents' preferences are not fully observable. The reason is that a bias in a player's objective function may be favorable by changing other players' equilibrium actions. However, if preferences are completely unobservable, payoff-maximization is evolutionary stable (see, e.g., Ok and Vega-Redondo, 2001; and Dekel et al., 2007). Alger and Weibull (2013) show that, under incomplete information, selfishness will indeed prevail if there is no assortative matching at all but that, with some degree of assortativity, preference evolution leads to a certain degree of *kantian* morality.

one's partner.

While there are several evolutionary analysis of conflicts between single players,[3] there are very few theoretical analysis applying the evolutionary approach to group conflicts. In fact, we are aware of only two papers. The first one is due to Eaton et al. (2011) who consider a production and conflict model with a large population of players. In each period, players are randomly matched to form groups of two members and each group competes for a common access resource with just one other group. After appropriating some of the common resource, the members of each group can spend some processing efforts to produce a consumption good. While the model is specific, they cannot prove the existence of evolutionarily stable preferences and have to rely on numerical simulations for endogenizing the preference parameters. The numerical results show that the evolutionary stable parameter on the payoff of one's teammate is positive – thus featuring intra-group altruism – while that on the payoffs of the out-group members is negative – thus featuring intergroup spitefulness.[4]

The other theoretical analysis is due to Konrad and Morath (2012). They consider a finite population of players with two groups of equal size competing for a prize (or a rent). Each player cannot observe the preference types of other players and, thus, they introduce the concept of *robust beliefs* such that any player with a certain preference type believes that all other players are of the same type (and have the same robust beliefs). This assumption greatly simplifies the analysis since it eliminates all strategic effects on behaviors of a change in preference types – i.e. a change in the weighs attributed to others' payoffs. Konrad and Morath (2012) then characterize the set of evolutionarily stable preferences, which involve a linear combination of in-group favoritism and out-group spite with the two traits being perfect substitutes. Indeed, the utility of each player is itself given by a linear combination of material payoffs of all players (including out-group members), while each player exerts only one level of effort. And this last can be increased by either more in-group altruism or more spiteful behavior towards the out-group.

Our contribution is that we demonstrate the existence and obtain an analytical solution for the evolutionary stable preference parameters in a simple model featuring strategic interactions within and across groups. Furthermore, we explicitly show that as stronger adversity, as measured by the number of groups competing for the same prize, reinforces in-group favoritism when group partners perform complementary tasks.

[3]The evolutionary analysis of contests conflict between single players started with Schaffer (1988) who adapted the notion of an Evolutionary Stable Strategy (ESS) by Maynard Smith (1982) to a finite population of players. Following Schaffer (1988), Hehenkamp et al. (2004) compare behaviors induced by an ESS to behaviors in Nash equilibrium and show that an ESS involves spiteful efforts in the contest between individuals. This in turn involves overdissipation of the rent compared to Nash equilibrium. Yet, in an infinite population, ESS behavior coincides to Nash equilibrium behavior. Finally, Leininger (2009) applies the *indirect* evolutionary approach to contests between single players and shows that it gives rise to spiteful preferences that induce the same aggressive behavior than in an ESS.

[4]In the second part of their paper, they analyze a rent-seeking game for a private prize and for a public prize with interdependent preferences. However they do not try to endogenize the preference parameters of the players in this part of the analysis.

2 The Framework

2.1 A simple group contest game

We consider a large population of players. In each period, players are randomly matched to form groups of two members playing a group contest game for a common access resource Y normalized to 2. As in Eaton et al. (2011), group contests are localized and isolated from each other in that each contest involves a certain number n of groups. The justification is that the common access resources are situated in geographically identified places.

Let (i,j), for $i=\{1,2\}$ and $j=\{1,2,...,n\}$, denote member i of group j. $e_{ij}\in\mathbb{R}_{+}$ is the amount of effort expended by player (i,j) and $\mathbf{e}_j=(e_{1j},e_{2j})\in\mathbb{R}_{+}^{2}$ is the vector of efforts in group j. The effort of group j depends on group members' efforts according to an *impact function* $G_j:\mathbb{R}_{+}^{2}\longrightarrow\mathbb{R}_{+}$, which has the CES form, that is

$$G_j\left(\mathbf{e}_j\right)=\left[e_{1j}^{\sigma}+e_{2j}^{\sigma}\right]^{\frac{1}{\sigma}},\ \text{for } j=\{1,2,...,n\}\,, \tag{1}$$

where $\sigma\in\{(-\infty,0)\cup(0,1]\}$ measures the degree of complementarity between individual efforts. The elasticity of substitution is $1/(1-\sigma)$. Thus, the lower σ, the lower is the elasticity of substitution or the higher is the degree of complementarity between individual efforts within groups. For $\sigma=1$, we have perfect substitutability between individual efforts and Eq. (1) becomes the standard 'summation technology', i.e. $G_j\left(\mathbf{e}_j\right)=\sum_i e_{ij}$. For $\sigma\longrightarrow-\infty$, we have perfect complementarity, i.e. $G_j\left(\mathbf{e}_j\right)=Min\{e_{ij}\}$ (referred as to the 'weakest-link' function). Finally, for $\sigma<0$ and $e_{ij}=0$, the impact function of group j is not well-defined. Hence we will take the limit of (1) as $e_{ij}\rightarrow 0$, which means $G_j\left(\mathbf{e}_j\right)=0$ in this case.[5]

Group efforts determine the division of the prize. The share allocated to group j is given by a contest success function $p_j:\mathbb{R}_{+}^{n}\rightarrow[0,1]$, which has the ratio-form[6]

$$p_j\left(\mathbf{e}_j,\mathbf{e}_{-j}\right)=\begin{cases}\dfrac{G_j\left(\mathbf{e}_j\right)}{\sum_{k=1}^{n}G_k\left(\mathbf{e}_k\right)} & \text{if } \sum_{k=1}^{n}G_k\left(\mathbf{e}_k\right)\neq 0,\\[2ex] \dfrac{1}{n} & \text{otherwise,}\end{cases} \tag{2}$$

where $\mathbf{e}_{-j}=((e_{11},e_{21}),...,(e_{1j-1},e_{2j-1}),(e_{1j+1},e_{2j+1}),...,(e_{1n},e_{2n}))$.

Each member in each group receives an equal amount of the share of the prize of total value equal to 2 – implying that each group member receives a share of a prize of value $Y/2=1$. In addition, it is assumed that each player incurs a constant marginal cost of effort. The *material payoff* to member (i,j), for $i=\{1,2\}$ and $j=\{1,2,...,n\}$, is given by an additively separable function $\Pi_{ij}:\mathbb{R}_{+}\times[0,1]\rightarrow\mathbb{R}_{+}$, that is

$$\Pi_{ij}\left(\mathbf{e}_j,\mathbf{e}_{-j}\right)=p_j\left(\mathbf{e}_j,\mathbf{e}_{-j}\right)-e_{ij}. \tag{3}$$

[5] Note that (1) is also discontinuous at $\sigma=0$. This case is excluded from our analysis.

[6] For an axiomatization of group contest success functions, see Munster (2009). And for a rather complete analysis of group contests – but without other-regarding preferences – where the *impact function* is given by a CES technology, see Kolmar and Rommeswinkel (2013).

2.2 Preference interdependence

Let consider that each player has a *utility* that depend not only on his/her own material payoff but also on that of his/her teammate. The utility of member (i, j), for $i = \{1, 2\}$ and $j = \{1, 2, ..., n\}$, is also given by an additively separable function $V_{ij} : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$, that is

$$V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = \Pi_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) + \theta_{ij}\Pi_{-ij}(\mathbf{e}_j, \mathbf{e}_{-j}), \tag{4}$$

where the subscript $-ij$ stands for the member other than i in team j. $\theta_{ij} \in (-1, R]$, with $R \geq 1$ and for $i = \{1, 2\}$ and $j = \{1, 2, ...n\}$, is the utility weight given by member i in group j to the material payoff of his/her teammate – with positive values representing "altruism" and negative values representing "spite".[7] Let $\Theta \in (-1, R]^{2n}$ be the vector of preference parameters in a localized contest that involves n groups of two members.

In contrast to Eaton et al. (2011) and Konrad and Morath (2012), the utility of a player does not depend – presumably negatively – on the material payoffs of out-group members. Again, in a simple rent-seeking game, this is not a restrictive assumption since players exert just one level of effort and this last could increase with either more in-group altruism or more spiteful behavior towards out-group members. Hence, player (i, j), for $i = \{1, 2\}$ and $j = \{1, 2, ..., n\}$ chooses his effort level e_{ij} so as to maximize his utility given by (4), which can be rewritten with (3) as

$$V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = (1 + \theta_{ij})p_j(\mathbf{e}_j, \mathbf{e}_{-j}) - (e_{ij} + \theta_{ij}e_{-ij}). \tag{5}$$

We have the following Lemma.[8]

Lemma 1: (i) *Given* $\Theta \in (-1, R]^{2n}$, *there exists a pure-strategy Nash equilibrium. In equilibrium, the effort of player* (i, j) *is characterized for* $i = \{1, 2\}$ *and* $j = \{1, 2, ..., n\}$ *by the following first-order condition*

$$(1 + \theta_{ij})\frac{p_j(\mathbf{e}_j, \mathbf{e}_{-j})[1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})]}{e_{ij}^{1-\sigma}(e_{ij}^{\sigma} + e_{-ij}^{\sigma})} \leq 1 \textit{ with equality for } e_{ij} > 0; \tag{6}$$

(ii) *In any equilibrium, a given group* j *is either fully active i.e.* $e_{ij} > 0$ *for* $i = \{1, 2\}$ *or fully inactive i.e.* $e_{ij} = 0$ *for* $i = \{1, 2\}$.

The proof consists in three steps. We first show that $V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})$ is strictly concave in e_{ij} so that the first-order condition (6) is necessary and sufficient for characterizing the best-response function of player (i, j). Next, we show that there cannot exist an equilibrium in which a corner solution holds for a member of one group while an interior solution holds for the other member of the *same* group (property (ii)). This important property allows us to reduce the group contest to a lottery contest between individual players with heterogeneous preferences and then use the existence theorems of Cornes and Hartley (2005). It is also worth pointing out that when $\sigma < 0$, there are multiple equilibria since if player (i, j) chooses

[7] We exclude the case $\theta_{ij} \leq -1$ since then it would prevent positive levels of efforts. However, degrees of altruism $\theta_{ij} \geq 1$ may exist, for instance, between biological parents or couples.

[8] All the proofs are given in the Appendix.

$e_{ij} = 0$, then $G_j(\mathbf{e}_j) = 0$ and thus player $(-i, j)$ cannot do better than choosing $e_{-ij} = 0$. In other words, the members of a given group can "coordinate" on participating or not participating to the group contest when $\sigma < 0$ (see also, Kolmar and Rommeswinkel, 2013).[9]

Suppose now that all players in the population except one player, whom we will call the *mutant,* have a certain degree of altruism $\theta \in (-1, R]$ towards their teammates. Without loss of generality, let assume that the mutant is player $(1, 1)$ – i.e. the member 1 of group 1 – and let $\theta_m \in (-1, R]$ be his/her degree of altruism/spitefulness. These weights are exogenous to the players.

The mutant with the preference parameter θ_m chooses e_{11} so as to maximize his/her utility given by

$$V_{11}(\mathbf{e}_1, \mathbf{e}_{-1}) = (1 + \theta_m)p_1(\mathbf{e}_1, \mathbf{e}_{-1}) - (e_{11} + \theta_m e_{21}). \tag{5}$$

Member 2 of group 1, with the preference parameter θ, chooses e_{21} so as to maximize his/her utility given by

$$V_{21}(\mathbf{e}_1, \mathbf{e}_{-1}) = (1 + \theta)p_1(\mathbf{e}_1, \mathbf{e}_{-1}) - (e_{21} + \theta e_{11}). \tag{6}$$

Finally, the utility of all other players are symmetric since they all belong to a group where the two members have the same preference parameter θ. Thus, player (i, j), for $i = \{1, 2\}$ and $j = \{2, 3, ..., n\}$, chooses e_{ij} to maximize his/her utility given by

$$V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = (1 + \theta)p_j(\mathbf{e}_j, \mathbf{e}_{-j}) - (e_{ij} + \theta e_{-ij}). \tag{7}$$

There are thus three distinct equilibrium effort levels that must satisfy the first order conditions for the two members of group 1 and the first-order condition for one member of any other group than group 1.

We have the following Proposition.

Proposition 1: *Let* $\Delta(\theta_m, \theta) = (1 + \theta_m)^{\frac{\sigma}{1-\sigma}} + (1 + \theta)^{\frac{\sigma}{1-\sigma}}$ *where* $\theta_m \in (-1, R]$ *is the preference parameter of the mutant and where* $\theta \in (-1, R]$ *is the incumbent preference parameter, then:*

(i) *There exists a unique interior pure strategy Nash equilibrium with* $e_{ij} > 0$ *for all* i *and* j *if and only if*

$$\left[\frac{\Delta(\theta_m, \theta)}{2}\right]^{\frac{1-\sigma}{\sigma}} \geq \frac{(n-2)(1+\theta)}{(n-1)}; \tag{8}$$

(ii) *If* (8) *holds, equilibrium effort levels are given by*

$$\begin{aligned}
e_{11}^*(\theta_m, \theta) &= \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)(1+\theta_m)^{\frac{1}{1-\sigma}}[(n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}} - 2^{\frac{1-\sigma}{\sigma}}(n-2)(1+\theta)]}{\Delta(\theta_m, \theta)[2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}]^2}, \\
e_{21}^*(\theta_m, \theta) &= \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)^{\frac{2-\sigma}{1-\sigma}}[(n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}} - 2^{\frac{1-\sigma}{\sigma}}(n-2)(1+\theta)]}{\Delta(\theta_m, \theta)[2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}]^2}, \\
e_{ij}^*(\theta_m, \theta) &= \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)^2[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}}{2[2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}]^2} \text{ for } i = \{1, 2\} \text{ and } j = \{2, ..., n\}.
\end{aligned} \tag{9}$$

[9] Thus, there also exists the corner equilibrium – that we ignore – with $e_{ij} = 0$ for all i and j.

As shown in the proof of this Proposition, there are two types of equilibrium (excluding those in which group members "coordinate" on not participating when $\sigma < 0$): either all groups are fully active or group 1 is the only inactive group. Observe that the inequality (8) is always verified for $\theta_m = \theta$ and that its left-hand term is increasing in θ_m. Therefore, the inequality is always verified for $\theta_m \geq \theta$. If however, given σ and n, θ_m is sufficiently small relative to θ so that the inequality (8) is reversed then, in equilibrium, the group with the mutant member is fully inactive because the utility weight given by the mutant to the material payoff of his/her partner is relatively too small (and potentially negative).

Suppose that (8) holds, then the material payoff – or *fitness* – of player (i, j) for $i = \{1, 2\}$ and $j = \{1, 2, 3, ..., n\}$, is

$$\Pi^*_{ij}(\theta_m, \theta) = p_j\left(\mathbf{e}^*_j(\theta_m, \theta), \mathbf{e}^*_{-j}(\theta_m, \theta)\right) - e^*_{ij}(\theta_m, \theta). \tag{10}$$

3 Evolutionarily Stable Preferences

To study the evolutionarily stability of altruism or spitefulness, we employ the *indirect* evolutionary approach pioneered by Güth and Yaari (1992). All players choose effort levels that maximize their utility and evolution pressure ensures the survival of preferences' parameters that induce equilibrium behavior providing the highest levels of material payoff.

Initially, all players have the same preference parameter θ and the question is whether this preference parameter is immune against invading mutant players with a preference parameter θ_m. Suppose a mutant is selected to play a n-group contest game as member 1 of group 1. The preference parameter θ is said to be *evolutionarily stable* if $\Pi^*_{11}(\theta, \theta) > \Pi^*_{11}(\theta_m, \theta)$ for *all* $\theta_m \neq \theta$. In other words, a degree of altruism θ is *evolutionary stable* if $\Pi_{11}(\theta_m, \theta)$ reaches its unique global maximum in $\theta_m = \theta$. Unfortunately, we will not be able to verify global stability or that the above inequality holds for *all* $\theta_m \neq \theta$. Hence, following Alger and Weibull (2010), we use the concept of local stability.

Definition 1 (Alger and Weibull, 2010): *A necessary and sufficient condition for a degree of altruism/spitefulness $\theta^* \in (-1, R]$ to be locally evolutionarily stable is* (i)-(ii) *where*

(i) $\dfrac{\partial \Pi^*_{11}(\theta_m, \theta^*)}{\partial \theta_m}|_{\theta_m = \theta^*} = 0,$

(ii) $\dfrac{\partial^2 \Pi^*_{11}(\theta_m, \theta^*)}{\partial \theta_m^2}|_{\theta_m = \theta^*} < 0.$

In other words, θ^* is locally evolutionarily stable if and only if $\Pi^*_{11}(\theta_m, \theta^*)$ has a strict local maximum in $\theta_m = \theta^*$.

We have the following Proposition.

Proposition 2: *If $\sigma \leq \bar{\sigma}$, then there exists a unique locally evolutionarily stable preference parameter $\theta^* \in (-1, R]$ given by*

$$\theta^* = \frac{(n^2 - 2)(1 - \sigma) - n}{(n^2 + 2)(1 - \sigma) - n(1 - 2\sigma)}. \tag{11}$$

and where

$$\bar{\sigma}=\frac{2n^3+3n^2-6n+4-n\sqrt{4n^3(n-1)+17n^2-20n+12}}{4(n-1)^2}, \tag{12}$$

which is increasing in n over $\left[5-3\sqrt{2},1\right)$.

Recall that the lower σ, the greater is the complementarity between group members' contributions. Thus, there may not exist stable evolutionary preferences if individual contributions are too substitutable. The limit value of $\bar{\sigma}$ is nevertheless increasing in the number of competing groups. It reaches a minimum in $n=2$ in which case $\bar{\sigma}\,|_{n=2}=5-3\sqrt{2}\approx 0.76$ and approaches 1 as n is going to infinity.

From this Proposition, we have the following Corollary.

Corollary 1: (i) *$\theta^* \gtreqless 0$ for $\sigma \lesseqgtr \hat{\sigma}$ where $\hat{\sigma}=\left(n^2-n-2\right)/\left(n^2-2\right)$;* (ii) *$\hat{\sigma}$ is increasing in n over $[0,1)$.*

Corollary 1 states that θ^* can be positive – featuring altruism – or negative – featuring spite – depending on the degree of complementarity between individual efforts within groups and on the number of groups competing against each other.[10] Indeed, a crucial feature of the contest game is that individual fitness is determined by the probability of success of the group and this last has the characteristics of a public good. Thus, increasing one's own contribution to group effort above the level corresponding to selfish behavior – as a result for example of altruism – can be advantageous vis-à-vis the players of the other groups but can also be detrimental vis-à-vis one's teammate because it could induce him/her to free-ride on one's own additional effort. Therefore, it is not clear whether altruism or spiteful behavior is more successful in terms of individual fitness. This depends on the technology for aggregating individual efforts into group effort and on the number of groups competing against each other for the resource.

Yet, if the degree of the complementarity between individual efforts σ is sufficiently strong – i.e. $\sigma\leq\hat{\sigma}$ – then evolution leads towards intra-group altruism. Furthermore, the requirement about the degree of complementarity becomes less stringent as the number of competing groups increases. Indeed $\hat{\sigma}$ is increasing in n, reaches a minimum in $n=2$ in which case $\hat{\sigma}\,|_{n=2}=0$, and approaches 1 as the number of groups competing against each other is going to infinity. As a result, preference evolution leads to intra-group altruism for any $\sigma<0$ independently of the number of competing groups and, in the limit, when the number of competing groups becomes very large, preference evolution leads to intra-group altruism for any $\sigma\leq 1$.

We can also determine how θ^* changes with the degree of complementarity σ between individual efforts. The derivative of θ^* with respect to σ is given by

$$\frac{\partial\theta^*}{\partial\sigma}=-\frac{2n^2(n-1)}{\left[(n^2+2)(1-\sigma)-n(1-2\sigma)\right]^2}. \tag{13}$$

We thus have the following Proposition.

[10] Since $\hat{\sigma}<\bar{\sigma}$ for any $n\geq 2$, $\theta^*\leq 0$ can indeed be locally evolutionarily stable. One can also easily verify that θ^* given by (11) is also strictly greater than -1.

Proposition 3: *The higher the degree of complementarity between individual efforts – i.e. the lower σ – the lower is the degree of spitefulness or the higher is the degree of altruism – i.e. the higher is θ^*.*

As already stated, the degree of complementarity σ must be sufficiently strong – i.e. $\sigma \leq \hat{\sigma}$ – for a positive degree of in-group altruism to be (locally) evolutionarily stable. In any case, an increase in the degree of complementarity between partners' efforts reinforces group cohesion in that it decreases spitefulness or increases altruism towards one's partner. Intuitively, higher degrees of complementarity make group effort and in turn the probability of success of a given group more sensitive to individual effort. As a result, "free-riding" on the contribution of one's teammate becomes detrimental to group success and in turn to individual fitness as the degree of complementarity increases.

There remains the question as to whether a larger number of competing groups increases the degree of intra-group altruism (or decreases the degree of intra-group spitefulness). Calculating the derivative of θ^* with respect to n, we obtain

$$\frac{\partial \theta^*}{\partial n} = \frac{2(1-\sigma)\left[2(2n-1) + \sigma(n^2 - 4n + 2)\right]}{\left[(n^2+2)(1-\sigma) - n(1-2\sigma)\right]^2}. \tag{14}$$

One can observe that this derivative is always positive for any $\sigma > 0$ while its sign is ambiguous for $\sigma < 0$. We can thus state the following Proposition.

Proposition 4: (i) *When $\sigma > 0$, the evolutionarily stable preference parameter θ^* is increasing in n and becomes positive from $n \geq \tilde{n}$ where $\tilde{n} = \left[1 + \sqrt{9 - 8\sigma(2-\sigma)}\right] / [2(1-\sigma)]$*; (ii) *When $\sigma < 0$, the evolutionarily stable preference parameter θ^* is not monotonous in n, but it is always positive.*

When the degree of complementarity between individual efforts is relatively low – i.e. when $\sigma > 0$ – preference evolution can lead to spitefulness within groups if few groups compete against each other. However, as the number of groups increases, it leads to the emergence of in-group altruism and this behavioral trait becomes increasingly strong as the number of competing groups keeps going up (property (i) of Proposition 4). This result echoes that of an increase in the degree of complementarity between individual efforts. When the intensity of the conflict between groups increases, the success of one's own group is becoming more and more decisive for the evolutionary success of a group member. In other words, the intensity of the conflict helps solving the tendency to "free-riding" – or selfishness – by developing in-group favoritism. When the degree of within-group complementarities is relatively large – i.e. when $\sigma < 0$ – preference evolution leads to in-group altruism independently of the number of groups. Yet, in this case, a larger number of competing groups can increase as well as decrease the evolutionary stable degree of altruism within groups (property (ii) of Proposition 4). It remains that $\sigma < 0$ always leads to a stronger degree of in-group altruism than $\sigma > 0$ for any given number of competing groups (Proposition 3).

Finally, we can characterize equilibrium levels of efforts and of individual fitness when players have locally evolutionarily stable preferences. When all players in society have the same preference parameter θ^*, they all exert the same level of individual effort i.e. – using

(9) – $e^* = (1+\theta^*)(n-1)/2n^2$. Substituting θ^* given by (11) into this expression, we obtain

$$e^* = \frac{(1-\sigma)(n-1)^2}{n\left[(n^2+2)(1-\sigma) - n(1-2\sigma)\right]}. \tag{15}$$

Furthermore, all groups have the same probability of success i.e. $1/n$, so that each player's *material payoff* induced by evolutionarily stable preferences is $\Pi^* = (1/n) - e^*$ or

$$\Pi^* = \frac{n+1-\sigma}{n\left[(n^2+2)(1-\sigma) - n(1-2\sigma)\right]}. \tag{16}$$

It can be easily verified that an increase in the degree of complementarity between individual efforts – i.e. a decrease in σ – unambiguously decreases material payoff. Indeed, the lower σ the higher is the evolutionarily stable level of in-group altruism and thus the higher the level of individual effort. As a result, the conflict becomes more severe. A larger number of competing groups has an ambiguous impact on the evolutionarily stable preference parameter and thus on equilibrium levels of efforts. Yet, a larger number of groups also unambiguously decreases individual payoffs simply because a larger number of players are involved in the conflict for a resource of a fixed amount.

4 Conclusion

The inclination of people to pull together in face of a common enemy appears to be a universal trait of human behavior and this is confirmed by several experimental studies in economics or social psychology. In this paper, we provide an evolutionary foundation for the emergence and stability of 'parochial' altruism in a context where several groups compete against each other for an exogenous resource. It is shown that both a strong degree of complementarity between individual efforts and a large number of competing groups reinforce in-group favoritism. Indeed, the success of a group crucially depends on its ability to contain the tendency to free-riding or selfishness. There is thus an evolutionary pressure towards in-group altruism because it increases the probability of success of the group in the conflict, which in turn increases individual fitness. The downside of this behavior is that it makes the conflict between groups more severe and potentially more destructive. A fundamental reason for this is that there is a fixed amount of resources at the origin of the conflict. Thus, it would be interesting to try to extend the analysis to a model where players have some endowments – for example units of time – that can be used for production or for rent-seeking. However, the present analysis shows that the existence and characterization of evolutionarily stable preferences can be very challenging even in a very simple framework.

5 Appendix

5.1 Proof of Lemma 1

The proof of this Lemma is conducted in three steps. The first one is to show that the first-order conditions given by (6) are necessary and sufficient for maximization.

The first derivative of $V_{ij}(e_j, e_{-j})$, given by (5), with respect to e_{ij} is given by

$$\frac{\partial V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial e_{ij}} = (1+\theta_{ij}) \frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} \frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} - 1. \tag{A1}$$

Using (1) and (2), we have

$$\frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} = \frac{\sum_{k \neq j} G_k(\mathbf{e}_k)}{\left(\sum_{l=1}^{n} G_l(\mathbf{e}_l)\right)^2}, \tag{A2}$$

and

$$\frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} = \left[e_{ij}^{\sigma} + e_{-ij}^{\sigma}\right]^{\frac{1}{\sigma}-1} e_{ij}^{\sigma-1} = \frac{G_j(\mathbf{e}_j)\, e_{ij}^{\sigma-1}}{e_{ij}^{\sigma} + e_{-ij}^{\sigma}}. \tag{A3}$$

Thus (A1) can be rewritten as

$$(1+\theta_{ij}) \frac{p_j(\mathbf{e}_j, \mathbf{e}_{-j}) \left[\sum_{k \neq j} p_k(\mathbf{e}_k, \mathbf{e}_{-k})\right] e_{ij}^{\sigma-1}}{\left(e_{ij}^{\sigma} + e_{-ij}^{\sigma}\right)} - 1. \tag{A4}$$

As a result, the first-order condition for $i = \{1, 2\}$ and $j = \{1, 2, ..., n\}$ is given by (6) since $p_j(\mathbf{e}_j, \mathbf{e}_{-j}) + \sum_{k \neq j} p_k(\mathbf{e}_k, \mathbf{e}_{-k}) = 1$.

The second derivative of $V_{ij}(e_j, e_{-j})$ with respect to e_{ij} is given by

$$\frac{\partial^2 V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial e_{ij}^2} = (1+\theta_{ij}) \left\{ \begin{array}{c} \left[1 - 2p_j(\mathbf{e}_j, \mathbf{e}_{-j})\right] \dfrac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_1)} \dfrac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} \dfrac{e_{ij}^{\sigma-1}}{\left(e_{ij}^{\sigma} + e_{-ij}^{\sigma}\right)} \\ \\ p_j(\mathbf{e}_j, \mathbf{e}_{-j}) \left[1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})\right] \dfrac{e_{ij}^{\sigma-2}\left[(\sigma-1)\, e_{-ij}^{\sigma} - e_{ij}^{\sigma}\right]}{\left(e_{ij}^{\sigma} + e_{-ij}^{\sigma}\right)^2} \end{array} \right\}. \tag{A5}$$

From (A2) and (A3), we also have

$$\frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} \frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} = \frac{p_j(\mathbf{e}_j, \mathbf{e}_{-j}) \left[1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})\right] e_{ij}^{\sigma-1}}{e_{ij}^{\sigma} + e_{-ij}^{\sigma}}. \tag{A6}$$

Substituting (A6) into (A5), we obtain

$$\frac{\partial^2 V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial e_{ij}^2} = \frac{(1+\theta_{ij}) p_j(\mathbf{e}_j, \mathbf{e}_{-j}) \left[1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})\right] \left[-2p_j(\mathbf{e}_j, \mathbf{e}_{-j})\, e_{ij}^{\sigma} + (\sigma-1)\, e_{-ij}^{\sigma}\right] e_{ij}^{\sigma-2}}{\left(e_{ij}^{\sigma} + e_{-ij}^{\sigma}\right)^2}, \tag{A7}$$

which is always negative for $i = \{1, 2\}$and $j = \{1, 2, ..., n\}$ since $\sigma \leq 1$.

As a result $V_{ij}(e_j, e_{-j})$, for $i = \{1,2\}$and $j = \{1,2,...,n\}$, is strictly concave and continuous in e_{ij} for $\sigma \in (-\infty, 0)$ or $\sigma \in (0,1]$. Thus, the first-order condition given by (6) is both necessary and sufficient for characterizing the best-response function of player (i,j) for $i = \{1,2\}$and $j = \{1,2,...,n\}$.

The second step of the proof is to show that if a group participates to the contest, then its two members produce positive levels of effort. In other words, a corner solution for player (i,j) – that is $e_{ij} = 0$ – and an interior solution for player $(-i,j)$ – that is $e_{-ij} > 0$ – cannot be mutual best responses for the two players of group j. Indeed, suppose first that $\sigma \in (0,1]$ with $e_{ij} = 0$ and $e_{-ij} > 0$, then we would have $p_j(\mathbf{e}_j, \mathbf{e}_{-j}) > 0$ but the denominator of the LHT of (6) would tend to 0, so that the LHT would approach infinity. Hence, (6) cannot be satisfied for $e_{ij} = 0$ and $e_{-ij} > 0$ when $\sigma \in (0,1]$. Now suppose that $\sigma \in (-\infty, 0)$ with $e_{ij} = 0$, then $G_j(\mathbf{e}_j) = 0$ and thus $V_{-ij}(e_j, e_{-j})$ is strictly decreasing in e_{-ij} so that player $(-i,j)$ cannot do better than $e_{-ij} = 0$ in this case. To conclude, if a group participates to the contest it fully participates with both members being active (see also Kolmar and Rommeswinkel, 2003). Furthermore, there cannot exist an equilibrium where all groups do not participate to the contest for $\sigma \in (0,1]$. Indeed, if all other groups do not enter the contest, the members of group j could win the prize with probability 1 in return for an arbitrarily small effort (exerted by both group members). As a result for $\sigma \in (0,1]$, at least one group is fully active. If $\sigma \in (-\infty, 0)$, $e_{ij} = 0$ and $e_{-ij} = 0$ are mutually best responses independently of the behavior of other groups. Hence, there may exist an equilibrium in which none the n groups participate to the contest.

The final step for demonstrating the existence of a pure strategy Nash equilibrium characterized by (6), is to reduce the group contest to a contest among individual players[11] and then use Theorem 1 of Cornes and Hartley (2005). Suppose that there are m active groups and consider the system of $2m$ first-order conditions (6) holding with equality. This system can be rewritten as (with (A1), (A2) and (A3))

$$\frac{\left(\sum_{k \neq j} G_k\right) G_j^{1-\sigma}}{\left(\sum_{l \in M} G_l\right)^2} = \frac{e_{ij}^{1-\sigma}}{(1+\theta_{ij})}, \tag{A8}$$

for $i = \{1,2\}$and j being an element of the set of active groups denoted by M.

For a given group j, the LHS of (A8) is the same for $i = \{1,2\}$and hence $e_{1j}(1+\theta_{2j})^{1/(1-\sigma)} = (1+\theta_{1j})^{1/(1-\sigma)} e_{2j}$. The aggregate output of group $j \in M$ can thus be written as a function of the effort of player 1, that is

$$G_j = \left[\frac{(1+\theta_{1j})^{\frac{\sigma}{1-\sigma}} + (1+\theta_{2j})^{\frac{\sigma}{1-\sigma}}}{(1+\theta_{1j})^{\frac{\sigma}{1-\sigma}}}\right]^{\frac{1}{\sigma}} e_{1j} \equiv \Psi_{1j}(\theta_{1j}, \theta_{2j}) e_{1j}. \tag{A9}$$

We thus have $e_{1j} = [\Psi_{1j}(\theta_{1j}, \theta_{2j})]^{-1} G_j$.

The system of of $2m$ first-order conditions (A8) can thus be reduced to a system of m equations in G_j for $j = \{1,2,...,m\}$, that is

$$\frac{\sum_{k \neq j} G_k}{\left(\sum_{l \in M} G_l\right)^2} = \frac{1}{\Phi_j(\theta_{1j}, \theta_{2j})}, \tag{A10}$$

[11] See also Brookins et al. (2015) in a model of group contest with CES impact functions and heterogeneous and convex cost functions.

where

$$\Phi_j(\theta_{1j},\theta_{2j}) = (1+\theta_{1j})\left[\Psi_{1j}(\theta_{1j},\theta_{2j})\right]^{1-\sigma} = \left[(1+\theta_{1j})^{\frac{\sigma}{1-\sigma}} + (1+\theta_{2j})^{\frac{\sigma}{1-\sigma}}\right]^{\frac{1-\sigma}{\sigma}}. \tag{A11}$$

In other words, the system of first-order conditions (A4) for the group contest with heterogeneous (other-regarding) preferences can be reduced to a system that is induced by a lottery contest of m individual players choosing G_j with heterogeneous (and constant) marginal costs given by the RHT of (A10). Applying Theorem 1 of Cornes and Hartley (2005), we can conclude that there exists a pure strategy Nash equilibrium G_j^* for $j \in M$. In turn, $e_{ij}^* = \left[\Psi_{1j}(\theta_{1j},\theta_{2j})\right]^{-1} G_j^*$ for $i = \{1,2\}$ and $j \in M$ satisfying (6) constitute an equilibrium in the contest between groups.

5.2 Proof of Proposition 1

(i) From the proof of Lemma 1, we know that if a group participates to the contest, then its two members produce positive levels of effort. Furthermore, the group contest with heterogeneous "other-regarding" preferences can be reduced to a lottery contest between individual players choosing G_j with heterogeneous marginal costs. Let $G = \sum_k G_k$. From (A10), the first-order condition for player j can be rewritten as

$$\frac{G - G_j}{G^2} - \frac{1}{\Phi_j(\theta_{1j},\theta_{2j})} \leq 0. \tag{A12}$$

It is non-positive at $G_j = 0$ for $\Phi_j(\theta_{1j},\theta_{2j}) \leq G$. Thus player/group j is fully inactive if $\Phi_j(\theta_{1j},\theta_{2j}) \leq G$. If, however $\Phi_j(\theta_{1j},\theta_{2j}) > G$, then player j is active and thus $G_j = G - (G^2/\Phi_j(\theta_{1j},\theta_{2j}))$. Again, let M be the set of the m active players in equilibrium. We have $G = \sum_{j\in M} G_j$, and hence

$$G = \frac{m-1}{\sum_{j\in M}\left(1/\Phi_j(\theta_{1j},\theta_{2j})\right)}. \tag{A13}$$

Suppose now that all group members have the same preference parameter θ except group member $(1,1)$ with the preference parameter θ_m. We thus have (from (A11)) $\Phi_{-1}(\theta) = 2^{\frac{1-\sigma}{\sigma}}(1+\theta)$ for all $j \neq 1$ and $\Phi_1(\theta_m,\theta) = \left[(1+\theta_m)^{\frac{\sigma}{1-\sigma}} + (1+\theta)^{\frac{\sigma}{1-\sigma}}\right]^{\frac{1-\sigma}{\sigma}}$. We first show that there cannot exist an equilibrium in which player (i.e. group) 1 with marginal cost $1/\Phi_1(\theta_m,\theta)$ is active and some $x > 1$ – but not all i.e. $x < n-1$ – other players (i.e. groups) with the common marginal cost $1/\Phi_{-1}(\theta)$ are inactive. If it were the case, we would have

$$G = \frac{n-x-1}{\left[1/\Phi_1(\theta_m,\theta)\right] + (n-x-1)\left[1/\Phi_{-1}(\theta)\right]} = \frac{(n-x-1)\Phi_1(\theta_m,\theta)\Phi_{-1}(\theta)}{(n-x-1)\Phi_1(\theta_m,\theta) + \Phi_{-1}(\theta)}. \tag{A14}$$

But for a player $j \neq 1$ to be inactive, we must also have $\Phi_{-1}(\theta) \leq G$ which is in contradiction with (A14). As a result, there are two possibilities. All groups $j \neq 1$ are either fully active or fully inactive. However, there cannot exist an equilibrium where all these groups are fully inactive while group 1 is fully active. Indeed, in that case, we would have that the LHT of (A12) is strictly negative for any positive level of $G_1 > 0$ so that this group would not play its best response. As a consequence, if player (i.e. group) 1 is active, then all players (groups) are active in equilibrium.

Now, suppose player 1 is fully inactive while some $x > 1$ – but not all i.e. $x < n-1$ – other players are also fully inactive. In this case, we would have

$$G = \frac{(n-x-2)\,\Phi_{-1}(\theta)}{(n-x-1)}. \tag{A15}$$

But again for a player $j \neq 1$ to be inactive, we must also have $\Phi_{-1}(\theta) \leq G$ which is in contradiction with (A15). Thus when player 1 is fully inactive, All groups $j \neq 1$ are either fully active or fully inactive. As stated in footnote 9, we ignore the equilibrium where all groups are fully inactive. Thus, suppose that all players $j \neq 1$ are fully active (still with player 1 being active). In this case, G is given by (A15) with $x = 0$. This is an equilibrium if $\Phi_1(\theta_m, \theta) \leq G$, or

$$\frac{\Phi_1(\theta_m, \theta)}{\Phi_{-1}(\theta)} \leq \frac{(n-2)}{(n-1)}. \tag{A16}$$

To conclude there are two types of equilibrium. If (A16) holds, group 1 is fully inactive while all other groups are active. If (A16) does not hold, then all groups are fully active. Let define

$$\Delta(\theta_m, \theta) = (1+\theta_m)^{\frac{\sigma}{1-\sigma}} + (1+\theta)^{\frac{\sigma}{1-\sigma}}, \tag{A17}$$

so that $\Phi_1(\theta_m, \theta) = [\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}$. Since $\Phi_{-1}(\theta) = 2^{\frac{1-\sigma}{\sigma}}(1+\theta)$, the necessary and sufficient condition for all groups being active is given by (8).

(ii) Suppose (8) holds. If $\theta_{ij} = \theta$ for $i = \{1, 2\}$ and $j = \{2, ..., n\}$, then first-order conditions given by (6) are symmetric for all $j \neq 1$ and thus all players – except the members of group 1 – exert the same level of individual effort that we denote $e_{ij} = e_S$ for $i = \{1, 2\}$ and $j = \{2, 3, ..., n\}$. As a result $p_j(\mathbf{e}_j, \mathbf{e}_{-j}) = p_S(\mathbf{e}_1, \mathbf{e}_{-1})$ for all $j = \{2, 3, ..., n\}$ and where $e_{-1} = \underbrace{((e_S, e_S), ..., (e_S, e_S))}_{n-1 \text{ times}}$.

The two members of group 1 differ – in terms of (other-regarding) preferences – and thus will exert different levels of efforts in equilibrium. This gives rise to a probability of success for group 1, i.e. $p_1(\mathbf{e}_1, \mathbf{e}_{-1})$, that differs from $p_S(\mathbf{e}_1, \mathbf{e}_{-1})$.

From these observations, there are three distinct equilibrium levels of effort that must satisfy the following first-order conditions,

$$(1+\theta_m)\frac{p_1(\mathbf{e}_1, \mathbf{e}_{-1})\,[(n-1)p_S(\mathbf{e}_1, \mathbf{e}_{-1})]\,e_{11}^{\sigma-1}}{e_{11}^{\sigma} + e_{21}^{\sigma}} = 1, \tag{A18}$$

for the mutant player $(1, 1)$, and

$$(1+\theta)\frac{p_1(\mathbf{e}_1, \mathbf{e}_{-1})\,[(n-1)p_S(\mathbf{e}_1, \mathbf{e}_{-1})]\,e_{21}^{\sigma-1}}{e_{11}^{\sigma} + e_{21}^{\sigma}} = 1, \tag{A19}$$

for player $(2, 1)$ who is the partner of the mutant in group 1, and

$$(1+\theta)\frac{p_S(\mathbf{e}_1, \mathbf{e}_{-1})\,[p_1(\mathbf{e}_1, \mathbf{e}_{-1}) + (n-2)p_S(\mathbf{e}_1, \mathbf{e}_{-1})]}{2e_S} = 1, \tag{A20}$$

for all players (i, j), for $i = \{1, 2\}$and $j = \{2, ..., n\}$.

Thus, using (A18) and (A19), we have

$$e_{21} = \left[\frac{1+\theta}{1+\theta_m}\right]^{\frac{1}{1-\sigma}} e_{11}. \tag{A21}$$

With (A19) and (A20), we have

$$2e_S p_1(\mathbf{e}_1, \mathbf{e}_{-1})(n-1)e_{21}^{\sigma-1} = [p_1(\mathbf{e}_1, \mathbf{e}_{-1}) + (n-2)p_S(\mathbf{e}_1, \mathbf{e}_{-1})](e_{11}^{\sigma} + e_{21}^{\sigma}). \tag{A22}$$

Using (1) and (2) and the fact that $G_j(\mathbf{e}_j) = G_S(\mathbf{e}_S) = 2^{1/\sigma} e_S$ for $j = \{2, 3, ..., n\}$ and that $p_1(\mathbf{e}_1, \mathbf{e}_{-1})$ and $p_S(\mathbf{e}_1, \mathbf{e}_{-1})$ have the same numerator, (A22) becomes

$$e_S\left[2G_1(\mathbf{e}_1)(n-1)e_{21}^{\sigma-1} - 2^{1/\sigma}(n-2)(e_{11}^{\sigma} + e_{21}^{\sigma})\right] = G_1(\mathbf{e}_1)(e_{11}^{\sigma} + e_{21}^{\sigma}). \tag{A23}$$

Since $G_1(\mathbf{e}_1) = (e_{11}^{\sigma} + e_{21}^{\sigma})^{1/\sigma}$, (A23) can be rewritten as

$$e_S\left[2(n-1)e_{21}^{\sigma-1} - 2^{1/\sigma}(n-2)(e_{11}^{\sigma} + e_{21}^{\sigma})^{(\sigma-1)/\sigma}\right] = (e_{11}^{\sigma} + e_{21}^{\sigma}). \tag{A24}$$

Let

$$\Delta(.) \equiv \Delta(\theta_m, \theta) = (1+\theta_m)^{\frac{\sigma}{1-\sigma}} + (1+\theta)^{\frac{\sigma}{1-\sigma}}. \tag{A25}$$

Then, using (A11) and (A13), (A14) can be rewritten has

$$e_S\left[2(n-1)[\Delta(.)]^{\frac{1-\sigma}{\sigma}} - 2^{\frac{1}{\sigma}}(1+\theta)(n-2)\right](1+\theta_m)^{\frac{1}{1-\sigma}} = (1+\theta)[\Delta(.)]^{\frac{1}{\sigma}} e_{11}. \tag{A26}$$

The first-order condition (A18) can also be rewritten as

$$\frac{(1+\theta_m)(n-1)G_1(\mathbf{e}_1)G_S(\mathbf{e}_S)e_{11}^{\sigma-1}}{(e_{11}^{\sigma} + e_{21}^{\sigma})[G_1(\mathbf{e}_1) + (n-1)G_S(\mathbf{e}_S)]^2} - 1 = 0. \tag{A27}$$

Since $G_S(\mathbf{e}_S) = 2^{1/\sigma} e_S$ and $G_1(\mathbf{e}_1) = (e_{11}^{\sigma} + e_{21}^{\sigma})^{1/\sigma}$, (A27) can be rewritten as

$$2^{\frac{1}{\sigma}}(n-1)(1+\theta_m)(e_{11}^{\sigma} + e_{21}^{\sigma})^{\frac{1-\sigma}{\sigma}} e_{11}^{\sigma-1} e_S = \left[(e_{11}^{\sigma} + e_{21}^{\sigma})^{\frac{1}{\sigma}} + 2^{\frac{1}{\sigma}}(n-1)e_S\right]^2 \tag{A28}$$

Using (A21) and (A25), (A28) becomes

$$2^{\frac{1}{\sigma}}(n-1)(1+\theta_m)^{\frac{2}{1-\sigma}}[\Delta(.)]^{\frac{1-\sigma}{\sigma}} e_S = \left[[\Delta(.)]^{\frac{1}{\sigma}} e_{11} + 2^{\frac{1}{\sigma}}(n-1)(1+\theta_m)^{\frac{1}{1-\sigma}} e_S\right]^2 \tag{A29}$$

Substituting e_S given by (A26) into (A29), we find after some tedious rearrangements, the equilibrium level of effort $e_{11}^*(\theta_m, \theta)$ by the mutant – agent 1 of group 1 – given in (9). The equilibrium level of effort $e_{21}^*(\theta_m, \theta)$ by the mutant's partner is obtained by using (A21). Finally, the common equilibrium level of effort for any player $i = \{1, 2\}$ of group j, for $j = \{2, 3, ..n\}$, that is $e_{ij}^*(\theta_m, \theta)$ in (9) is obtained by substituting $e_{11}^*(\theta_m, \theta)$ into (A26).

5.3 Proof of Proposition 2

We first determine condition (i) of Definition 1. We have

$$\begin{aligned}\frac{\partial \Pi_{11}^*}{\partial \theta_m} &= \frac{\partial p_1\left(\mathbf{e}_1^*, \mathbf{e}_{-1}^*\right)}{\partial G_1\left(\mathbf{e}_1^*\right)}\left[\frac{\partial G_1\left(\mathbf{e}_1^*\right)}{\partial e_{11}^*}\frac{\partial e_{11}^*}{\partial \theta_m} + \frac{\partial G_1\left(\mathbf{e}_1^*\right)}{\partial e_{21}^*}\frac{\partial e_{21}^*}{\partial \theta_m}\right] \\ &\quad + \sum_{j\neq 1}\frac{\partial p_1\left(\mathbf{e}_1^*, \mathbf{e}_{-1}^*\right)}{\partial G_j(\mathbf{e}_j^*)}\left[\frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{1j}^*}\frac{\partial e_{1j}^*}{\partial \theta_m} + \frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{2j}^*}\frac{\partial e_{2j}^*}{\partial \theta_m}\right] - \frac{\partial e_{11}^*}{\partial \theta_m}. \qquad \text{(A30)}\end{aligned}$$

Using the first-order conditions for the effort levels of the two members of group 1, that is (A1) with $\theta_{11} = \theta_m$ and $\theta_{21} = \theta$, we have

$$\begin{aligned}\frac{\partial \Pi_{11}^*}{\partial \theta_m} &= -\frac{\theta_m}{1+\theta_m}\frac{\partial e_{11}^*}{\partial \theta_m} + \frac{1}{1+\theta}\frac{\partial e_{21}^*}{\partial \theta_m} \\ &\quad + \sum_{j\neq 1}\frac{\partial p_1\left(\mathbf{e}_1^*, \mathbf{e}_{-1}^*\right)}{\partial G_j(\mathbf{e}_j^*)}\left[\frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{1j}^*}\frac{\partial e_{1j}^*}{\partial \theta_m} + \frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{2j}^*}\frac{\partial e_{2j}^*}{\partial \theta_m}\right]. \qquad \text{(A31)}\end{aligned}$$

We have

$$\frac{\partial p_1\left(\mathbf{e}_1^*, \mathbf{e}_{-1}^*\right)}{\partial G_j(\mathbf{e}_j^*)} = -\frac{G_1\left(\mathbf{e}_1^*\right)}{\left[G_1\left(\mathbf{e}_1^*\right) + \sum_{j\neq 1} G_j(\mathbf{e}_j^*)\right]^2} \text{ for } j \neq 1. \qquad \text{(A32)}$$

From (A3), we also have

$$\frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{ij}^*} = \left[(e_{ij}^*)^{\sigma} + (e_{-ij}^*)^{\sigma}\right]^{\frac{1-\sigma}{\sigma}}(e_{ij}^*)^{\sigma-1}. \qquad \text{(A33)}$$

Since $e_{ij}^* = e_S^*$ for $i = \{1, 2\}$ and $j \neq 1$, we have $G_j(e_j^*) = G_S\left(\mathbf{e}_S^*\right)$ and $\partial G_j(e_j^*)/\partial e_{ij}^* = 2^{(1-\sigma)/\sigma}$ for $i = \{1, 2\}$ and $j \neq 1$ and thus (A31) reduces to

$$\begin{aligned}\frac{\partial \Pi_{11}^*}{\partial \theta_m} &= -\frac{\theta_m}{1+\theta_m}\frac{\partial e_{11}^*}{\partial \theta_m} + \frac{1}{1+\theta}\frac{\partial e_{21}^*}{\partial \theta_m} \\ &\quad -\frac{2^{1/\sigma}(n-1)G_1\left(\mathbf{e}_1^*\right)}{\left[G_1\left(\mathbf{e}_1^*\right) + (n-1)G_S\left(\mathbf{e}_S^*\right)\right]^2}\frac{\partial e_S^*}{\partial \theta_m}. \qquad \text{(A34)}\end{aligned}$$

We now evaluate this expression at $\theta_m = \theta$.

Recall first that $e_{21}^* = [(1+\theta)/(1+\theta_m)]^{\frac{1}{1-\sigma}} e_{11}^*$. Therefore, the derivative of e_{21}^* with respect to θ_m is thus given by

$$\frac{\partial e_{21}^*}{\partial \theta_m} = \left(\frac{1+\theta}{1+\theta_m}\right)^{\frac{1}{1-\sigma}}\left[-\frac{e_{11}^*}{(1-\sigma)(1+\theta_m)} + \frac{\partial e_{11}^*}{\partial \theta_m}\right], \qquad \text{(A35)}$$

and hence,

$$\frac{\partial e_{21}^*}{\partial \theta_m}\Big|_{\theta_m=\theta} = -\frac{e^*}{(1-\sigma)(1+\theta)} + \frac{\partial e_{11}^*}{\theta_m}\Big|_{\theta_m=\theta}, \qquad \text{(A36)}$$

since at $\theta_m = \theta$, we have $e_{11}^* = e_{21}^* = e_S^* = e^*$.

Also using (9), the derivative of e_S^* with respect to θ_m is given (after tedious calculations and rearrangements) by

$$\frac{\partial e_S^*}{\partial \theta_m}=\frac{2^{\frac{1-\sigma}{\sigma}}(n-1)\,(1+\theta)^2\,[\Delta\,(.)]^{\frac{1-2\sigma}{\sigma}}\,[2^{\frac{1-\sigma}{\sigma}}(1+\theta)-(n-1)\,[\Delta\,(.)]^{\frac{1-\sigma}{\sigma}}]}{2\,(1+\theta_m)^{\frac{1-2\sigma}{1-\sigma}}\,[2^{\frac{1-\sigma}{\sigma}}(1+\theta)+(n-1)\,[\Delta\,(.)]^{\frac{1-\sigma}{\sigma}}]^3}, \tag{A37}$$

and hence

$$\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta}=-\frac{(n-2)(n-1)}{4n^3}. \tag{A38}$$

Substituting (A38) and (A36) into (A34), we have

$$\frac{\partial \Pi_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta}=\frac{1-\theta}{1+\theta}\frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta}-\frac{e^*}{(1-\sigma)(1+\theta)^2}+\frac{(n-1)^2(n-2)}{4n^5e^*}, \tag{A39}$$

since $G_1\left(\mathbf{e}_1^*\right)=G_S\left(\mathbf{e}_S^*\right)=2^{\frac{1}{\sigma}}e^*$.

Using (9), we also have

$$e^*=\frac{(1+\theta)(n-1)}{2n^2}. \tag{A40}$$

Substituting (A40) into (A39), one obtain

$$\frac{\partial \Pi_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta}=\frac{1-\theta}{1+\theta}\frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta}-\frac{(n-1)\,[\sigma(n-2)+2]}{2n^3(1-\sigma)(1+\theta)}. \tag{A41}$$

Also, after long and tedious calculations, we obtain

$$\begin{aligned}
\frac{\partial e_{11}^*}{\partial \theta_m} &= \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)(1+\theta_m)^{\frac{\sigma}{1-\sigma}}\,[\Gamma_1(.)+\Gamma_2(.)+\Gamma_3(.)]}{(1-\sigma)\,[\Delta\,(.)]^2\left[2^{\frac{1-\sigma}{\sigma}}\,(1+\theta)+(n-1)\,[\Delta\,(.)]^{\frac{1-\sigma}{\sigma}}\right]^3}\ \text{where} \\
\Gamma_1(.) &\equiv \Gamma_1\,(\theta_m,\theta)=-4^{\frac{1-\sigma}{\sigma}}(n-2)(1+\theta)^2\left[\Delta\,(.)-\sigma(1+\theta_m)^{\frac{\sigma}{1-\sigma}}\right], \\
\Gamma_2(.) &\equiv \Gamma_2\,(\theta_m,\theta)=(n-1)^2\,[\Delta\,(.)]^{\frac{2(1-\sigma)}{\sigma}}\left[\Delta\,(.)-(1+\theta_m)^{\frac{\sigma}{1-\sigma}}\right], \\
\Gamma_3(.) &\equiv \Gamma_3\,(\theta_m,\theta)=-2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)\,[\Delta\,(.)]^{\frac{1-\sigma}{\sigma}}\left[(n-3)\Delta\,(.)-[n(2-\sigma)-3]\,(1+\theta_m)^{\frac{\sigma}{1-\sigma}}\right].
\end{aligned} \tag{A42}$$

Therefore, we have

$$\frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta}=\frac{(n-1)\left[(n-1)^2-(n-2)(2-\sigma)-(n-1)\,(\sigma n-3)\right]}{4(1-\sigma)n^3}. \tag{A43}$$

Substituting into (A41) and setting it to 0 yields a unique value for θ^* given by (11). One can also easily verify that $\theta^*\geq -1$, since this inequality reduces to $n\geq -\sigma/(1-\sigma)$, which holds for any $\sigma\in\{(-\infty,0)\cup(0,1]\}$.

We now verify condition (ii) of Definition 1. (A34) can be rewritten as

$$\begin{aligned}
\frac{\partial \Pi_{11}^{*}}{\partial \theta_m} &= \Lambda_1(.) + \Lambda_2(.) + \Lambda_3(.) \text{ where} \\
\Lambda_1(.) &\equiv \Lambda_1(\theta_m, \theta) = -\frac{\theta_m}{1+\theta_m}\frac{\partial e_{11}^{*}}{\partial \theta_m}, \\
\Lambda_2(.) &\equiv \Lambda_2(\theta_m, \theta) = \frac{1}{1+\theta}\frac{\partial e_{21}^{*}}{\partial \theta_m}, \\
\Lambda_3(.) &\equiv \Lambda_3(\theta_m, \theta) = \frac{-2^{1/\sigma}(n-1)G_1(\mathbf{e}_1^{*})}{[G_1(\mathbf{e}_1^{*}) + (n-1)G_S(\mathbf{e}_S^{*})]^2}\frac{\partial e_S^{*}}{\partial \theta_m}.
\end{aligned} \tag{A44}$$

We have

$$\frac{\partial \Lambda_1(.)}{\partial \theta_m} = -\frac{1}{(1+\theta_m)^2}\frac{\partial e_{11}^{*}}{\partial \theta_m} - \frac{\theta_m}{1+\theta_m}\frac{\partial^2 e_{11}^{*}}{\partial \theta_m^2}. \tag{A45}$$

Now, let evaluate this expression at $\theta_m = \theta$. We have

$$\frac{\partial \Lambda_1(.)}{\partial \theta_m}\Big|_{\theta_m=\theta} = -\frac{1}{(1+\theta)^2}\frac{\partial e_{11}^{*}}{\partial \theta_m}\Big|_{\theta_m=\theta} - \frac{\theta}{1+\theta}\frac{\partial^2 e_{11}^{*}}{\partial \theta_m^2}\Big|_{\theta_m=\theta}. \tag{A46}$$

We also have

$$\frac{\partial \Lambda_2(.)}{\partial \theta_m} = \frac{1}{1+\theta}\frac{\partial^2 e_{21}^{*}}{\partial \theta_m^2}. \tag{A47}$$

Using (A35), the second derivative of e_{21}^{*} with respect to θ_m is given by

$$\frac{\partial^2 e_{21}^{*}}{\partial \theta_m^2} = \left(\frac{1+\theta}{1+\theta_m}\right)^{\frac{1}{1-\sigma}}\left[-\frac{2}{(1-\sigma)(1+\theta_m)}\frac{\partial e_{11}^{*}}{\partial \theta_m} + \frac{(2-\sigma)e_{11}^{*}}{(1-\sigma)^2(1+\theta_m)^2} + \frac{\partial^2 e_{11}^{*}}{\partial \theta_m^2}\right]. \tag{A48}$$

Substituting (A48) into (A47) and evaluating this last expression at $\theta_m = \theta$, we obtain

$$\frac{\partial \Lambda_2(.)}{\partial \theta_m}\Big|_{\theta_m=\theta} = \frac{1}{1+\theta}\left[-\frac{2}{(1-\sigma)(1+\theta)}\frac{\partial e_{11}^{*}}{\partial \theta_m}\Big|_{\theta_m=\theta} + \frac{(2-\sigma)e^{*}}{(1-\sigma)^2(1+\theta)^2} + \frac{\partial^2 e_{11}^{*}}{\partial \theta_m^2}\Big|_{\theta_m=\theta}\right]. \tag{A49}$$

Recalling that $e^{*} = (1+\theta)(n-1)/2n^2$, using (A46) and (A49) we obtain

$$\begin{aligned}
\frac{\partial(\Lambda_1(.) + \Lambda_2(.))}{\partial \theta_m}\Big|_{\theta_m=\theta} &= \frac{(2-\sigma)(n-1)}{2n^2(1-\sigma)^2(1+\theta)^2} - \frac{(3-\sigma)}{(1-\sigma)(1+\theta)^2}\frac{\partial e_{11}^{*}}{\partial \theta_m}\Big|_{\theta_m=\theta} \\
&\quad + \left(\frac{1-\theta}{1+\theta}\right)\frac{\partial^2 e_{11}^{*}}{\partial \theta_m^2}\Big|_{\theta_m=\theta}.
\end{aligned} \tag{A50}$$

Substituting (A43) into (A50), we have

$$\frac{\partial(\Lambda_1(.) + \Lambda_2(.))}{\partial \theta_m}\Big|_{\theta_m=\theta} = -\frac{(n-1)\left[3n^2 - 7n + 6 - \sigma(n^2 - 2n + 2)\right]}{4n^3(1-\sigma)(1+\theta)^2} + \left(\frac{1-\theta}{1+\theta}\right)\frac{\partial^2 e_{11}^{*}}{\partial \theta_m^2}\Big|_{\theta_m=\theta}. \tag{A51}$$

We now calculate the derivative of $\Lambda_3(.)$ with respect to θ_m. From (A44), $\Lambda_3(.)$ can be rewritten as

$$\begin{aligned} \Lambda_3(.) &= -2^{1/\sigma}(n-1)p_1\left(\mathbf{e}_1^*, \mathbf{e}_S^*\right)\Phi(\mathbf{e}_1^*, \mathbf{e}_S^*)\frac{\partial e_S^*}{\partial \theta_m} \\ &\quad \text{where} \\ \Phi(\mathbf{e}_1^*, \mathbf{e}_S^*) &= \frac{1}{G_1\left(\mathbf{e}_1^*\right) + (n-1)G_S\left(\mathbf{e}_S^*\right)}. \end{aligned} \tag{A52}$$

We have

$$\frac{\partial \Lambda_3(.)}{\partial \theta_m} = -2^{1/\sigma}(n-1)\left\{ \begin{array}{c} \dfrac{\partial p_1(.)}{\partial \theta_m}\dfrac{\partial e_S^*}{\partial \theta_m}\Phi(.) + p_1(.)\Phi(.)\dfrac{\partial^2 e_S^*}{\partial \theta_m^2} \\ +p_1(.)\dfrac{\partial e_S^*}{\partial \theta_m}\left[\begin{array}{c} \dfrac{\partial \Phi(.)}{\partial G_1(.)}\left(\dfrac{\partial G_1(.)}{\partial e_{11}^*}\dfrac{\partial e_{11}^*}{\partial \theta_m} + \dfrac{\partial G_1(.)}{\partial e_{21}^*}\dfrac{\partial e_{21}^*}{\partial \theta_m}\right) \\ +\dfrac{\partial \Phi(.)}{\partial G_S(.)}\dfrac{\partial G_S(.)}{\partial e_S^*}\dfrac{\partial e_S^*}{\partial \theta_m} \end{array} \right] \end{array} \right\}. \tag{A53}$$

We have

$$\frac{\partial \Phi(.)}{\partial G_S(.)} = (n-1)\frac{\partial \Phi(.)}{\partial G_1(.)} = -\frac{n-1}{\left[G_1(.) + (n-1)G_S(.)\right]^2}. \tag{A54}$$

Furthermore, from the first-order condition (A30), we have $\partial p_1(.)/\partial \theta_m = \partial e_{11}/\partial \theta_m$. Thus (A53) becomes

Thus (A44) becomes

$$\frac{\partial \Lambda_3}{\partial \theta_m} = -2^{1/\sigma}(n-1)\left\{ \begin{array}{c} \dfrac{\partial e_{11}}{\partial \theta_m}\dfrac{\partial e_S^*}{\partial \theta_m}\Phi(.) + p_1(.)\Phi(.)\dfrac{\partial^2 e_S^*}{\partial \theta_m^2} \\ +p_1(.)\dfrac{\partial e_S^*}{\partial \theta_m}\dfrac{\partial \Phi(.)}{\partial G_1(.)}\left[\begin{array}{c} \dfrac{\partial G_1(.)}{\partial e_{11}^*}\dfrac{\partial e_{11}^*}{\partial \theta_m} + \dfrac{\partial G_1(.)}{\partial e_{21}^*}\dfrac{\partial e_{21}^*}{\partial \theta_m} \\ +(n-1)\dfrac{\partial G_S(.)}{\partial e_S^*}\dfrac{\partial e_S^*}{\partial \theta_m} \end{array} \right] \end{array} \right\}. \tag{A55}$$

In a symmetric equilibrium, we have

$$\begin{aligned} p_1(.)\mid_{\theta_m=\theta} &= \frac{1}{n}; \quad \Phi(.)\mid_{\theta_m=\theta} = \frac{1}{2^{\frac{1}{\sigma}}ne^*}; \quad \frac{\partial \Phi(.)}{\partial G_1(.)}\mid_{\theta_m=\theta} = -\frac{1}{\left[nG(e^*)\right]^2} = -\frac{1}{\left[2^{\frac{1}{\sigma}}ne^*\right]^2}; \\ \frac{\partial G_S(.)}{\partial e_S^*}\mid_{\theta_m=\theta} &= 2^{\frac{1}{\sigma}}; \quad \frac{\partial G_1(.)}{\partial e_{11}^*}\mid_{\theta_m=\theta} = \frac{\partial G_1(.)}{\partial e_{21}^*}\mid_{\theta_m=\theta} = 2^{\frac{1-\sigma}{\sigma}}. \end{aligned} \tag{A56}$$

As a result, we have

$$\frac{\partial \Lambda_3(.)}{\partial \theta_m}\,|_{\theta_m=\theta} = -\frac{(n-1)}{ne^*}\left\{\begin{array}{c} \left(\frac{\partial e_{11}^*}{\partial \theta_m}\frac{\partial e_S^*}{\partial \theta_m}\right)|_{\theta_m=\theta} + \frac{1}{n}\frac{\partial^2 e_S^*}{\partial \theta_m^2}\,|_{\theta_m=\theta} \\ \\ -\frac{1}{2n^2e^*}\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta}\left[\begin{array}{c} \frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta} + \frac{\partial e_{21}^*}{\partial \theta_m}\,|_{\theta_m=\theta} \\ \\ +2(n-1)\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta} \end{array}\right] \end{array}\right\}. \tag{A57}$$

From (A35), we have

$$\frac{\partial e_{21}^*}{\partial \theta_m}\,|_{\theta_m=\theta} = -\frac{e^*}{(1-\sigma)(1+\theta)} + \frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta}\,. \tag{A58}$$

Substituting (A58) into (A57), we obtain

$$\begin{aligned} \frac{\partial \Lambda_3(.)}{\partial \theta_m}\,|_{\theta_m=\theta} \quad = \quad & -\frac{(n-1)}{ne^*}\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta}\left\{\begin{array}{c} \left[1-\frac{1}{n^2e^*}\right]\frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta} \\ \\ +\frac{1}{2n^2(1-\sigma)(1+\theta)} - \frac{(n-1)}{n^2e^*}\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta} \end{array}\right\} \\ & -\frac{(n-1)}{n^2e^*}\frac{\partial^2 e_S^*}{\partial \theta_m^2}\,|_{\theta_m=\theta}\,. \end{aligned} \tag{A59}$$

Substituting e^* given by (A40) into (A59), one obtain

$$\begin{aligned} \frac{\partial \Lambda_3(.)}{\partial \theta_m}\,|_{\theta_m=\theta} \quad = \quad & -\frac{2n}{(1+\theta)}\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta}\left\{\begin{array}{c} \left[\frac{(1+\theta)(n-1)-2}{(1+\theta)(n-1)}\right]\frac{\partial e_{11}^*}{\partial \theta_m}\,|_{\theta_m=\theta} \\ \\ +\frac{1}{2n^2(1-\sigma)(1+\theta)} - \frac{2}{(1+\theta)}\frac{\partial e_S^*}{\partial \theta_m}\,|_{\theta_m=\theta} \end{array}\right\} \\ & -\frac{2}{(1+\theta)}\frac{\partial^2 e_S^*}{\partial \theta_m^2}\,|_{\theta_m=\theta^*}\,. \end{aligned} \tag{A60}$$

Substituting (A38) and (A43) into (A60), we obtain after some trivial (but tedious) calculations

$$\begin{aligned} \frac{\partial \Lambda_3(.)}{\partial \theta_m}\,|_{\theta_m=\theta} \quad = \quad & \frac{(n-1)(n-2)\left[\begin{array}{c} (n^3-2)(1-\sigma)(1+\theta) - n^2(2-3\sigma)(1+\theta) \\ \\ +n(1-2\sigma+\theta(3-4\sigma)) \end{array}\right]}{8(1-\sigma)(1+\theta)^2n^5} \\ & -\frac{2}{(1+\theta)}\frac{\partial^2 e_S^*}{\partial \theta_m^2}\,|_{\theta_m=\theta}\,. \end{aligned} \tag{A61}$$

The second derivative of $\Pi_{11}^*(\theta_m, \theta)$ with respect to θ_m evaluated at $\theta_m = \theta$ is given by the sum of (A51) and (A61).

What remains to be done is the calculation of the second derivative of e_{11}^* and of e_S^* with respect to θ_m. Calculating the derivative of $\partial e_{11}^*/\partial\theta_m$ given by (A42) with respect to θ_m and evaluating the resulting expression at $\theta_m = \theta$, we obtain[12]

$$\frac{\partial^2 e_{11}^*}{\partial\theta_m^2}\,|_{\theta_m=\theta} = -\frac{(n-1)\left[n^3-5n^2+10n-6-2\sigma(2n^3-6n^2+8n-3)\right]}{8(1-\sigma)\,(1+\theta)\,n^4}. \tag{A62}$$

Similarly, calculating the derivative of $\partial e_S^*/\partial\theta_m$ given by (A37) with respect to θ_m and evaluating the resulting expression at $\theta_m = \theta$, we obtain

$$\frac{\partial^2 e_S^*}{\partial\theta_m^2}\,|_{\theta_m=\theta} = \frac{(n-1)\left[3n^2-10n+6-2\sigma(2n^2-6n+3)\right]}{8(1-\sigma)\,(1+\theta)\,n^4}. \tag{A63}$$

Substituting (A62) into (A51) to obtain $\left[\partial\left(\Lambda_1(.)+\Lambda_2(.)\right)/\partial\theta_m\right]|_{\theta_m=\theta}$ and (A63) into (A61) to obtain $\left[\partial\Lambda_3(.)/\partial\theta_m\right]|_{\theta_m=\theta}$ and adding the two terms, we obtain $\left[\partial^2\Pi_{11}^*/\partial\theta_m^2\right]|_{\theta_m=\theta}$ (see (A44)), that is

$$\frac{\partial^2\Pi_{11}^*}{\partial\theta_m^2}\,|_{\theta_m=\theta} = -\frac{(n-1)\left\{\begin{array}{c}(n-1)\left[6n^3-3n^2-6n+4-\theta(2n^3-7n^2+10n-4)\right]\\ -\sigma\left[5n^4-3n^3-12n^2+12n-4-\theta(5n^4-17n^3+26n^2-16n+4)\right]\end{array}\right\}}{8(1-\sigma)(1+\theta)^2n^5}. \tag{A64}$$

Substituting θ by θ^* given by (11) into this expression, we obtain

$$\frac{\partial^2\Pi_{11}^*}{\partial\theta_m^2}\,|_{\theta_m=\theta^*} = -\frac{\left[n^2-n+2-\sigma(n^2-2n+2)\right]\left[\begin{array}{c}2(n^3-n+1)+2\sigma^2(n-1)^2\\ -\sigma\,(2n^3+3n^2-6n+4)\end{array}\right]}{16(1-\sigma)^3n^5}. \tag{A65}$$

One can observe that the sign of this expression is always negative for any $\sigma \leq 0$. For $\sigma \geq 0$, one can easily show that $\left(\partial^2\Pi_{11}^*/\partial\theta_m^2\right)|_{\theta_m=\theta^*} = 0$ as three roots in σ, that is

$$\begin{aligned}\sigma_1 &= \frac{n^2-n+2}{n^2-2n+2},\\ \sigma_2 &= \frac{2n^3+3n^2-6n+4-n\sqrt{4n^3(n-1)+17n^2-20n+12}}{4(n-1)^2},\\ \sigma_3 &= \frac{2n^3+3n^2-6n+4+n\sqrt{4n^3(n-1)+17n^2-20n+12}}{4(n-1)^2}.\end{aligned} \tag{A66}$$

It can be easily verified that σ_1 and σ_3 are strictly larger than 1 for any $n \geq 2$. As a result, condition (ii) of definition is verified if and only if $\sigma \leq \sigma_2 \equiv \bar{\sigma}$.

[12] Since the mathematical expressions are very long, we only write the values of these expressions evaluated at $\theta_m = \theta$. We also used the Mathematical software to be sure not to have made any errors in the computation of these derivatives.

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