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Cooperation with asymmetric environmental valuation and responsibility in a dynamic setting

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Idea of the paper

- Two countries/regions differently affected by & differently responsible for an environmental stock externality.
- A cooperative agreement on emissions reduction.
- Goal of the paper: propose a sharing mechanism that guarantees cooperation (time consistency of the agreement).

  with two properties:

  1. A benefit-pay-principle: the greater the benefit from cooperation, the greater must be share of the cost.
  2. A responsibility axiom: the higher its responsibility, the greater must be share of the cost.

For example ...

- Rise in temperatures caused by global warming: Northern colder regions vs. countries with warmer climate.
- Agreement to mitigate the environmental problem (reduce greenhouse gas emissions) within $[0, T]$, each region would be differently benefited.
- Countries have different responsibility for the state of the environment (CO_2 concentration).

Outline

1. Introduction ✓
2. Cooperation versus non-cooperation dilemma
3. A time-consistent IDP
4. Different gains from cooperation and different responsibility
5. Different axioms for the sharing rule
6. A proposal for a sharing mechanism
7. An example

A simple formulation

The maximization under the cooperative agreement reads:

$$\max_{E^1, E^2} \sum_{i=1}^{2} \left\{ \int_0^T w(E^i(\tau)) e^{-\rho\tau} d\tau - e^{-\rho T} D^i(P_c(T)) \right\},$$

$$\text{s.t.: } \dot{P} = E^i + E^{-i} - \delta P, \quad P(0) = P_0,$$

Countries have:

- identical instantaneous profits from emissions $w^i(E^i) = w(E^i)$,
- different damage from global warming, collected by the scrap values $-D^i(P_c(T))$, with $(D^i)'(P) > 0$.

A simple formulation

If cooperation halts, at t, a non-cooperative game from t till T.
Each player solves:

$$\max_{E^i} \int_t^T w(E^i(\tau))e^{-\rho(\tau-t)}d\tau - e^{-\rho(T-t)}D^i(P_{\mathsf{N}}(t;T)),$$

$$\text{s.t.: } \dot{P} = E^i + E^{-i} - \delta P, \quad P_t = P_{\mathsf{C}}(t).$$

Non-cooperative game starting at time t, for $\tau \in [t, T]$:

$E^i_{\mathsf{N}}(\tau;t)$: Feedback NE, optimal non-cooperative emissions

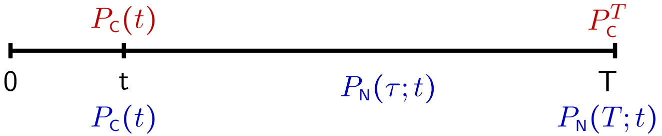
$P_{\mathsf{N}}(\tau;t)$: Optimal pollution stock path under the NE.

On time consistency

At time $t \in [0, T)$ shall players maintain cooperation?

maintain coop.

$$W^i_{\mathsf{C}}(t) = \int_t^T w(E^i_{\mathsf{C}}(\tau)) e^{-\rho(\tau - t)} d\tau \quad - \quad D^i(P_{\mathsf{C}}(T)) e^{-\rho(T-t)}$$



halt coop.

$$W^i_{\mathsf{N}}(t) = \int_t^T w(E^i_{\mathsf{N}}(\tau, t)) e^{-\rho(\tau - t)} d\tau \; - \; D^i(P_{\mathsf{N}}(T; t)) e^{-\rho(T-t)}$$

On time consistency

- At T: Absolute gains from cooperation from t to T.

$$B^i(t) \equiv [D^i(P_N(T;t)) - D^i(P_C(T))]e^{-\rho(T-t)}.$$

- Within $[t,T)$: Cost of (contribution to) cooperation:

$$C^i(t) = \int_t^T w(E_C^i(\tau))e^{-\rho(\tau-t)}d\tau - \int_t^T w(E_N^i(\tau,t))e^{-\rho(\tau-t)}d\tau.$$

Assumption 1: A positive global surplus to go: gap between total costs and total benefits.

$$S(t) = \sum_{i=1}^{2}\left(C^i(t) + B^i(t)\right) \geq 0, \quad \forall t \in [0,T].$$

The time consistent imputation distribution procedure

Define an IDP $\pi^i(t)$ to distribute the cooperative payoffs obtained within the interval $[0, T)$ in such a way both players prefer to follow the cooperative behavior when:

1. Cooperation represents a cost within the cooperative period.
2. Benefits from cooperation comes at the end of this period.
3. These benefits are asymmetric among regions.

The time consistent imputation distribution procedure

Let $\pi^i(t)$ be a payoff distribution procedure.

Definition 1

Given $\pi^i(\tau)$, the payoff to go for player i from t on reads:

$$W_\pi^i(t) = \int_t^T \pi^i(\tau) e^{-\rho(\tau - t)} d\tau - D^i(P_{\mathsf{C}}(T)) e^{-\rho(T-t)}.$$

This IDP is time consistent under condition:

$$W_\pi^i(t) = W_{\mathsf{N}}^i(t) + \phi^i(t) S(t) \qquad \forall t \in [0, T],$$

with $\phi^i(t)$ a differentiable function satisfying:
$\phi^i(t) \in [0, 1]$ and $\phi^i(t) + \phi^{-i}(t) = 1 \quad \forall t \in [0, T]$.

The time consistent imputation distribution procedure

Proposition 1

Consider $\phi^i(t)$ *any differentiable function satisfying* $\phi^i(t) \in [0,1]$ *and* $\phi^i(t) + \phi^{-i}(t) = 1$ *for all* $t \in [0,T]$.

$$\pi^i(t) = w^i_N(t) + \phi^i(t)s(t) - (\phi^i)'(t)S(t) + \phi^i(t)\Theta^{-i}_N(t) - \phi^{-i}(t)\Theta^i_N(t),$$

with $s(t) = w^i_C(t) + w^{-i}_C(t) - w^i_N(t;t) - w^{-i}_N(t;t)$ *and*

$$\Theta^i_N(t) = \int_t^T \dot{w}^i_N(\tau;t)e^{-\rho(\tau-t)}d\tau - \left(D^i\right)'(P_N(T;t))\dot{P}_N(T;t)e^{-\rho(T-t)}.$$

$$\boxed{\pi^i(t) + \pi^{-i}(t) = w^i_C(t) + w^{-i}_C(t), \quad \forall t \in [0,T]}$$

Standard approach:

1. Choose a bargaining approach (NBS, Shapley, Core...) and its corresponding solution
2. Compute the benchmark payoff i.e., the non cooperative solution
3. Decompose over time the total individual cooperative payoffs according to the chosen bargaining approach and subject to the satisfaction of the condition of time consistency.

$\phi^i(t)$ depends on the chosen bargaining approach.
For example, for the Nash bargaining solution: $\phi^i = 1/2$.

How to define $\phi(t)$

In this paper we do not choose a particular bargaining approach. We try to define the sharing rule, $\phi^i(t)$, so that the IDP verifies some axioms:

1. Time consistency
2. Benefits-pay-principle (BPP)
3. Polluter-pay-principle (PPP)- Responsibility

How to define $\phi(t)$

Gains from cooperation:

$$B^i(t) = D^i(P_N(T;t)) - D^i(P_C(T)) \text{ cannot be distributed.}$$

Contribution to cooperation of region i within $[t, T)$:

$$C^i_\pi(t) = \int_t^T [w^i_N(\tau) - \pi^i(\tau)] e^{-\rho(\tau - t)} d\tau.$$

1. Different valuation of a less polluted environment:

$$\hat{B}^i(t) \equiv \frac{B^i(t)}{B(t)} > \frac{B^{-i}(t)}{B(t)} \equiv \hat{B}^{-i}(t).$$

2. Different responsibility from past emissions:

$$R^i = \frac{r^i D^{-i}(P_0) - r^{-i} D^i(P_0)}{D^i(P_0) + D^{-i}(P_0)} = r^i - \hat{D}^i(P_0) = -R^{-i}.$$

with r^i the $\%$ of all past emissions accrued to region i.

Axioms

Axiom 1: Benefits-pay-principle (BPP)
The greater $\hat{B}^i(t)$ the greater must be $\hat{C}^i_\pi(t)$.

$$\left.\frac{\partial \hat{C}^i_\pi(t)}{\partial \hat{B}^i(t)}\right|_{B(t)=Cte} > 0.$$

Axiom 2: Responsibility with respect to past emissions (PPP)
The greater R^i the greater must be $\hat{C}^i_\pi(t)$.

$$\frac{\partial \hat{C}^i_\pi(t)}{\partial R^i} > 0.$$

Proposition of a value for $\phi^i(t)$

Define $\phi^i(t)$ as:

$$\boxed{\phi^i(t) = \hat{B}^i(t) - \alpha R^i.}$$

ϕ^i is a combination between the valuation of a less polluted environment and the responsibility from past emissions.
Conditions $\phi^i(t) + \phi^{-i}(t) = 1$ and $\phi^i(t) \in [0,1], \ \forall t \in [0,T]$, imply:

$$\alpha \leq \max\left\{\frac{\hat{B}^1}{R^1}, \frac{\hat{B}^2}{R^2}\right\} \equiv \alpha_{\max}.$$

The sharing rule satisfies:

1. Axiom 1 Benefits-pay-principle.
2. Axiom 2 Responsibility with respect to past emissions.

Egalitarian rule, $\phi^i(t) = 1/2$

Defining $\phi^i(t)$ as:

$$\boxed{\phi^i(t) = \hat{B}^i(t) - \alpha R^i.}$$

$$\frac{\partial \phi^i}{\partial \hat{B}^i} > 0, \qquad \frac{\partial \phi^i}{\partial R^i} < 0.$$

if $\hat{B}^i(t) > 1/2$ and $R^i < 0\,(R^{-i} > 0)$ then

$$\phi^i(t) > \frac{1}{2},\ \phi^{-i}(t) < \frac{1}{2}, \quad \forall \alpha \in [0, \alpha_{\max}].$$

Proposition 2

If the region which benefits most from the agreement is less responsible from past emissions the proposed IDP never leads to the egualitarian rule regardless of α.

Example: $\hat{B}^1(t) < \hat{B}^2(t)$, $R^1 > 0$.

A Linear-Quadratic example:

$$w(E) = aE - \frac{E^2}{2}, \quad D^i = d_1^i P^2(T),$$

$$a = 1,\ d^1 = 0.07,\ d^2 = 0.11,\ r^1 = .72,\ (r^2 = .28),\ \ \delta = .1,\ P_0 = 1,\ \rho = .03.$$

Region 2 higher relative gains:

$$\hat{B}^1 = \frac{0.07}{0.18} = 0.39 \quad < \quad \hat{B}^2 = \frac{0.18}{0.1} = 0.61$$

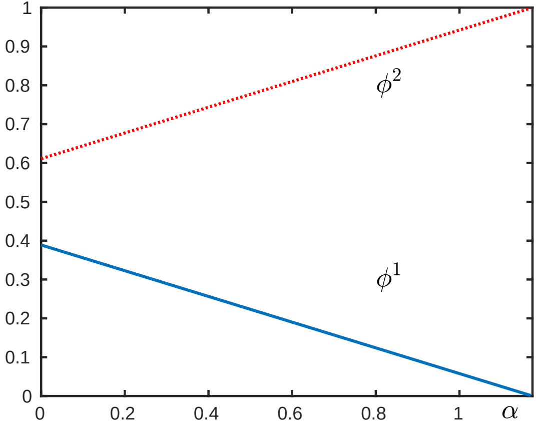
Region 1 higher responsibility:

$$R^1 = r^1 - \hat{D}^1(P_0) = 0.33 > 0, \quad R^2 = r^2 - \hat{D}^2(P_0) = -0.33 < 0$$

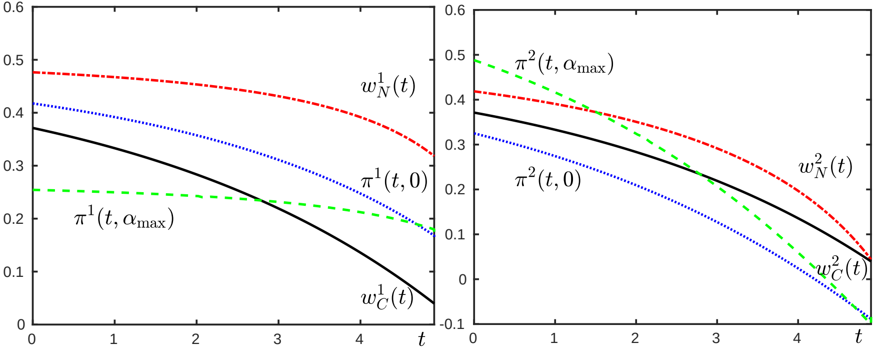
Example: $\hat{B}^1(t) < \hat{B}^2(t)$, $R^1 > 0$, $R^2 < 0$.

Simplifying assumptions:

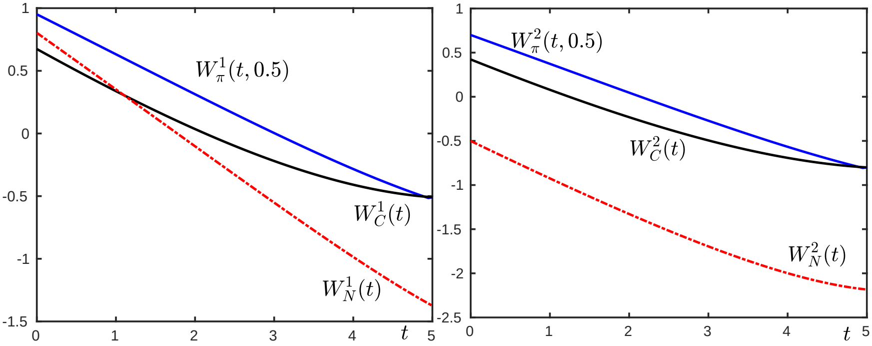
- Relative damage $\hat{D}^i(P)$ is independent of P.
- $\phi^1(t) = 0.39 - 0.33\alpha$, $\phi^2(t) = 0.61 + 0.33\alpha$ not time dependent.



Example: $\hat{B}^1(t) < \hat{B}^2(t)$, $R^1 > 0$, $R^2 < 0$.



Example: $\hat{B}^1(t) < \hat{B}^2(t)$, $R^1 > 0$, $R^2 < 0$.



Conclusion and extensions

...to summarize

- This paper propose a time consistent sharing mechanism satisfying three properties:
  1. the agreement is time consistent,
  2. the greater the benefit one country gets from cooperation, the greater is its share of the burden,
  3. the higher is its responsibility, the strongly must be the burden.
- Our proposal does not include the egalitarian rule.
- Particularize for a concrete example.

to be done...

- Can we find other ϕ^i?
- Our definition of ϕ^i has a free parameter α. Can we impose additional axioms?