Myopia and learning applied to water management

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Context of the paper

Dynamic learning model with general kind of conjectures. Convergence of the process.

The goal: Deduce economical and environmental consequences in a water management problem

How to proceed:

- Learning model with different conjectures
- Benchmarks: two dynamic myopic behaviors. In the first one players play Nash at each period in the second one players behave cooperatively and maximize at each period the joint profit.
- Application: groundwater exploitation problem.

Some related works

Conjectures

- Ch. Figuières, A. Jean-Marie, N. Quérou, M. Tidball, (2004) « Theory of Conjectural Variations ». In Monograph series in Mathematical Economics and Game Theory, World Scientific Publishing.
- A. Jean-Marie, M. Tidball, (2006) « Adapting behaviors through a learning process ». *Journal of Economic Behavior and Organization*.
- N. Quérou, M. Tidball, (2010) « Incomplete information, learning, and natural resource management ». *European Journal of Operational Research*.

Water management (usual groundwater problem).

The outline of the paper

- Learning models and benchmarks
- Some results at the steady state
- Application to the groundwater problem
- Numerical simulations
- Conclusions and extensions

The general learning model

Each player (two players) maximizes at each period t

$$\max_{w_i^t} F_i(w_i^t, w_j^t, H^{t+1}), \quad H^{t+1} = G(w_i^t, w_j^t, H^t).$$

Each player at each period makes a conjecture about the behavior of the other player

$$w_j^{t,c} = \chi_i(w_i^t, H^t).$$

At each period player i solves the following optimization problem

$$\max_{w_i^t} F_i(w_i^t, \chi_i(w_i^t, H^t), G(w_i^t, \chi_i(w_i^t, H^t), H^t)).$$

The general learning model

The first order condition

$$\frac{\partial F_i}{\partial w_i} + \frac{\partial F_i}{\partial w_j}\frac{\partial \chi_i}{\partial w_i} + \frac{\partial F_i}{\partial H}\left[\frac{\partial G}{\partial w_i} + \frac{\partial G}{\partial w_j}\frac{\partial \chi_i}{\partial w_i}\right] = 0.$$

Call the optimal solution for player i ($i = 1, 2$), w_i^* and the corresponding conjectured solution of the other player $w_j^{c*} = \chi_i(w_i^*, H)$.

This conjecture is in general different of w_j^* (at time t). Then player i actualizes his conjecture

$$\chi_i^{t+1} = (1-\mu)\chi_i^t + \mu\, w_j^{t*}.$$

At the steady state

Assuming limits exist ...

$$\chi_i^\infty(w_i^\infty, H^\infty) = w_j^\infty,$$

$$H^\infty = G(w_i^\infty, w_j^\infty, H^\infty),$$

+ first order condition

Benchmarks

Nash myopic (dynamic) equilibrium: at each time players solve

$$\max_{w_i} F_i(w_i, w_j, G(w_i, w_j, H)).$$

Cooperative myopic (dynamic) solution: they maximize the joint profit at each period

$$\max_{w_i, w_j} \left[F_i(w_i, w_j, G(w_i, w_j, H)) + F_j(w_i, w_j, G(w_i, w_j, H))\right].$$

Learning Nash

$$\text{If} \quad \frac{\partial \chi_i}{\partial w_i} = 0, \; i = 1, 2$$

$$\text{then} \quad w_i^{c\infty} = w_i^{N\infty}, i = 1, 2, \; H^{c\infty} = H^{N\infty},$$

where we denote with $c\infty$ the conjectural steady state and with $N\infty$ the Nash steady state.

Learning Pareto

If at the steady state

$$\frac{\partial F_i}{\partial w_j} = \frac{\partial F_j}{\partial w_i},\ \frac{\partial \chi_i}{\partial w_i} = 1,\ i = 1, 2 \text{ and } \frac{\partial F_j}{\partial H} = \frac{\partial F_i}{\partial H},\ \ \frac{\partial G}{\partial w_i} = \frac{\partial G}{\partial w_j},$$

then

$$w_i^{c\infty} = w_i^{P\infty}, i = 1, 2,\ H^{c\infty} = H^{P\infty},$$

where we denote with $c\infty$ the conjectural steady state and with $P\infty$ the cooperative steady state.

A groundwater exploitation problem

Water extraction is the only input in the production process of the farmers, and the dynamics is given by the evolution of the level of the water table.
Unitary cost increase when the level of the water table is low.
Players can also take into account the state of the resource and have an extra profit of maintaining the resource, in this case ρ is the discount factor and γ_i is his resource preference.

$$F_i(w_i^t, w_j^t, H^t) = P_i(w_i^t) - w_i C(H^{t+1}) + \rho \gamma_i H^{t+1},$$

such that

$$H^{t+1} = G(w_i^t, w_j^t, H^t) = H^t + R - \alpha(w_i^t + w_j^t).$$

The "usual" linear conjecture

$$w_j^c = \bar{w}_j + \beta_i(w_i - \bar{w}_i) := \chi_i(w_i), \quad i \neq j.$$

$\bar{w} = (\bar{w}_1, \bar{w}_2)$ are given and $\beta = (\beta_1, \beta_2)$ are going to evolve with the learning process.

Player i looks at real action of player j and "realizes" that his conjecture could be

$$w_j^* = \bar{w}_j + \beta_i'(w_i^* - \bar{w}_i),$$

that is the observed β_i is

$$\beta_i' = \frac{w_j^* - \bar{w}_j}{w_i^* - \bar{w}_i},$$

and he revises his learning procedure as follows

$$\beta_i^{t+1} = (1-\mu_i)\beta_i^t + \mu_i \frac{w_j^{t*} - \bar{w}_j}{w_i^{t*} - \bar{w}_i}, \quad \beta_i^0, \text{ given}, \quad i = 1, 2, j \neq i.$$

The “usual” linear conjecture

In the symmetric case, with this type of conjecture the learning process converges to the cooperative solution (Pareto). Because we can prove that in general (non symmetric case),

$$\beta_1^\infty \beta_2^\infty = 1.$$

Remember that any conjecture depending only on H gives a learning processes that converges to the Nash equilibrium

An example

$$F_i(w_i^t, w_j^t, H^{t+1}) = P_i(w_i^t) - w_i C(H^{t+1}) + \rho\gamma_i H^{t+1},$$

such that

$$H^{t+1} = G(w_i^t, w_j^t, H^t) = H^t + R - \alpha(w_i^t + w_j^t).$$

We consider

$$P_i(w_i) = a_i w_i - \frac{b_i}{2}(w_i)^2, \quad C(H) = c_0 - c_1 H,$$

we consider that we are in cases where $c_0 - c_1 H^t > 0$ for all t.

Conjectures

- The usual linear conjecture: $w_j^c = \beta_i w_i$,
- The case of conjecture that is only a function of H

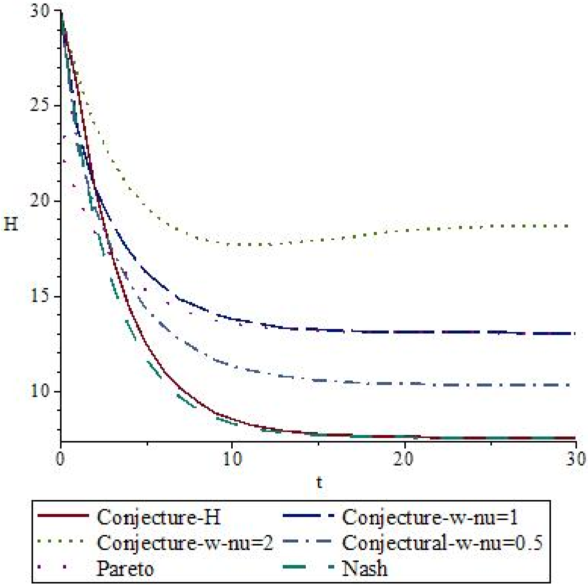
$$w_j^c = \beta_i(H_t + R), \quad i \neq j.$$

- The non-linear conjecture in w

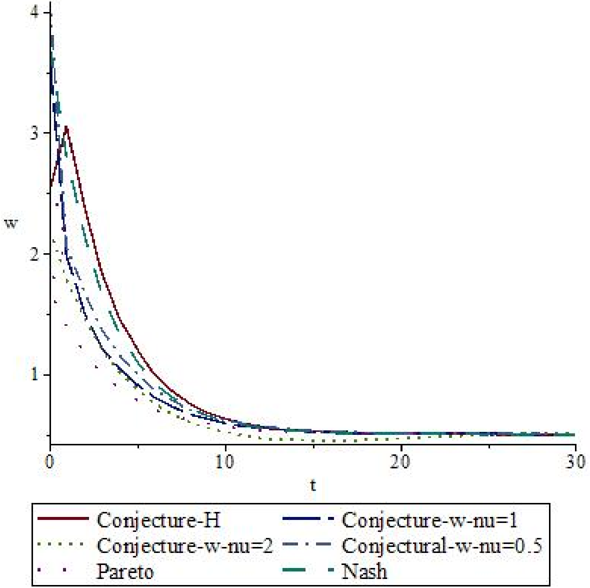
$$w_j^c = \beta_i w_i^{\nu}.$$

Where ν can represent what player i feels with respect to the aggressive behavior of player j.

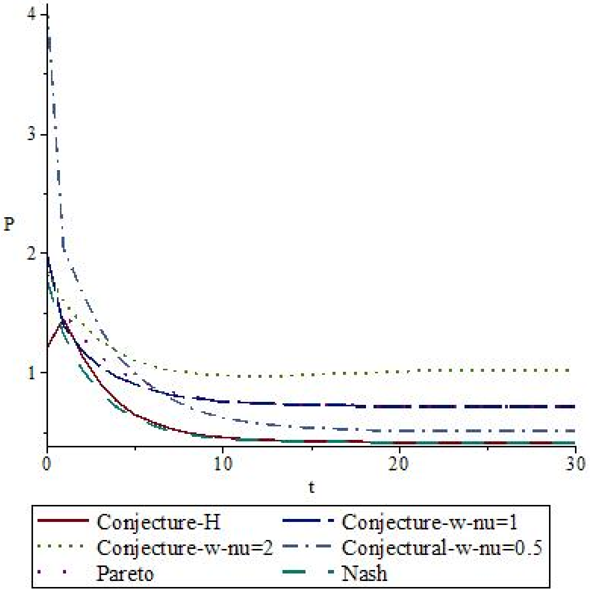
Simulation: H^t



Simulation: w^t



Simulation: profit at each time t



Simulation: accumulated profit

Accumulated profit after 30 periods

$$P^N = 16.97 < P^{cH} = 20.95 < P^P = 26.16 < P^{\nu=0.5} = 27.23 <$$
$$P^{\nu=1} = 32.85 < P^{\nu=2} = 43.52$$

Conclusions

- We have considered a general learning processes with conjectures.
- We have identified conditions for learning myopic Nash and myopic cooperative behaviors.
- We have proposed another type of conjecture that can represent an idea of aggressive behavior.

Work in progress ans extensions

- Interpretation of results with non-linear conjectures.
- Study the non symmetric case (Can we have with linear conjectures a Pareto solution with some weight?)
- Compare with the dynamic game and the dynamic cooperative solution, where the profit for each player is

$$\sum_{t=0}^{\infty} e^{-\rho t} \left[a_i w_i^t - \frac{b_i}{2}(w_i^t)^2 - (c_0 - c_1 H^{t+1}) w_i^t \right],$$

$$H^{t+1} = H^t + R - \alpha(w_i^t + w_j^t).$$

- ...