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Abstract

This paper compares Ramsey optimal policy for the new-Keynesian model with public debt with its fiscal theory of the price level (FTPL) equilibrium. Both the fiscal theory of the price level and Ramsey optimal policy implies that a deficit shock is instantaneously followed by an increase of inflation and output gap. But each optimal policy parameters belongs in different sets with respect to FTPL. The optimal fiscal rule parameter implies local stability of public debt dynamics ("passive fiscal policy"). The optimal Taylor rule parameter for inflation is larger than one. The optimal Taylor rule parameter for output gap is negative, because of the intertemporal substitution effect of interest rate on output gap. Both Taylor rule optimal parameters implies the local stability of inflation and output gap dynamics.

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1 Introduction

The fiscal theory of the price level, in what follows FTPL, predicts that a deficit shock instantaneously implies a rise of inflation and of output gap in the new-Keynesian model with public debt (Woodford (1996; 1998), Bonam and Lukkezen (2019), Cochrane (2019)). The FTPL equilibrium is an alternative to new-Keynesian equilibrium (Cochrane (2019)).

But the local instability of public debt dynamics in the FTPL implies the policy-maker's lack of credibility. This is explained e.g. by Benassy (2009): "*The basic idea behind the FTPL is that the government pursues fiscal policies such that, in off-equilibrium paths, it will not satisfy its intertemporal budget constraint, and run an explosive debt policy. This leaves only one feasible equilibrium path. Now although such off-equilibrium paths are not observed in the model's equilibrium, it would be extremely optimistic to assume that in real life situations the economy would follow at every instant the equilibrium path while the government pursues such policies. As a result many people would be reluctant to advise such policies to a real life government.*"

Svensson (2003) and Cochrane (2011) describes Ramsey optimal policy as a benchmark: "*In most (Ramsey) analysis of policy choices,..., we think of governments choosing*

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policy configurations while taking first order conditions as constraints; we think of governments acting in markets." This paper shows that Ramsey optimal policy assuming a minimal credibility of the policy maker's commitment is an answer to the issue raised by Benassy (2009), using Chatelain and Ralf (2019a) algorithm.

As the FTPL, Ramsey optimal policy has the same key prediction for the transmission mechanism of the new-Keynesian model with public debt: *a deficit shock instantaneously implies a rise of inflation and of output gap.* By contrast to the FTPL, Ramsey optimal policy implies *the local stability of the equilibrium in the space of inflation, output gap and public debt.*

This local stability is obtained because each parameters in the optimal policy rule belongs in different sets that the ones of ad hoc policy rules for the FTPL. The optimal fiscal rule parameter implies local stability of public debt dynamics ("passive fiscal policy"). The optimal Taylor rule parameter for inflation is larger than one (instead of below one for the FTPL). The optimal Taylor rule parameter for output gap is negative, because of the intertemporal substitution effect of interest rate on output gap (Chatelain and Ralf (2019b)). Both Taylor rule optimal parameters implies the local stability of inflation and output gap dynamics.

Because of the policy maker's credibility of his commitment, there is no longer indeterminacy with multiple equilibrium path converging to long run equilibrium. Optimal initial condition are derived for inflation and output gap.

Section 1 solves Ramsey optimal policy model for the new-Keynesian model with public debt and compares it with the FTPL equilibrium. Section 2 concludes.

2 New-Keynesian model with public debt: Policy Transmission Mechanism

The fiscal theory of the price level applied on the new-Keynesian model with public debt can be found in Woodford (1996, 1998), Bonam and Lukkezen (2019) and Cochrane (2019). All variables as log-deviations of an equilibrium. In the representative household's intertemporal substitution consumption Euler equation, the expected future output gap $E_t x_{t+1}$ is *positively* correlated with the real rate of interest, equal to the nominal rate i_t minus *expected* inflation $E_t \pi_{t+1}$. The intertemporal elasticity of substitution (IES) $\gamma = 1/\sigma$ is a measure of the responsiveness of the growth rate of consumption to the interest rate, usually considered to be smaller than one. It is the inverse of σ, the relative fluctuation aversion or the relative degree of resistance to intertemporal substitution of consumption, which measures the strength of the preference for smoothing consumption over time.

$$x_t = E_t x_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + z_t \text{ with } \gamma > 0. \tag{1}$$

In the new-Keynesian Phillips curve, expected inflation $E_t \pi_{t+1}$ is negatively correlated with the current output gap x_t with a sensitivity $-\kappa < 0$.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \text{ with } 0 < \beta < 1 \text{ and } \kappa > 0.$$

The intertemporal budget constraint of the state for public debt B_t is:

$$B_{t+1} = (1+i_t) B_t - p_t s_t \Rightarrow \frac{B_{t+1}}{p_{t+1}} = (1+i_t) \frac{B_t}{p_t} \frac{p_t}{p_{t+1}} - s_t$$

The primary surplus is lump-sum tax income minus non-interest expenditures: $s_t = \tau_t - g$. Define a steady state with inflation equal to zero $\pi^* = 0$, with the real rate of interest equal to the discount rate so that $i^* - \pi^* = i^* = \beta^{-1} - 1$, with steady state public debt $\frac{B^*}{p} \geq 0$. In that steady state, the surplus pays the real interest cost of the debt: $s^* = \tau^* - g = (\beta^{-1} - 1) \frac{B^*}{p} \geq 0$. Hence, steady state surplus is a small proportion of the stock of public debt, of an order of magnitude of 1% per quarter. Woodford (1996, equation 2.9) log-linearize the public debt equation in deviation from its steady state:

$$b_{t+1} = \beta^{-1} (b_t - \pi_t) + i_t - \left(\beta^{-1} - 1\right) s_t$$

The marginal effect of funds rate on future public debt (equal to one) is around 100 times larger than the marginal effect of surplus (equal to the opposite of the discount rate) for quarterly periods. The dynamic system includes three policy targets (output gap, inflation, public debt) and two policy instrument (funds rate and primary surplus):

$$\begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ b_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{\gamma\kappa}{\beta} & -\frac{\gamma}{\beta} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & -\frac{1}{\beta} & \frac{1}{\beta} \end{pmatrix}}_{=\mathbf{A}} \begin{pmatrix} x_t \\ \pi_t \\ b_t \end{pmatrix} + \underbrace{\begin{pmatrix} \gamma & 0 \\ 0 & 0 \\ 1 & 1-\frac{1}{\beta} \end{pmatrix}}_{=\mathbf{B}} \begin{pmatrix} i_t & s_t \end{pmatrix} + \begin{pmatrix} \varepsilon_x \\ \varepsilon_\pi \\ \varepsilon_b \end{pmatrix} \quad (2)$$

Shocks ε_x, ε_π and ε_b are assumed to be independently and identically distributed without auto-correlation, with mean zero and a non-zero variance-covariance matrix. The variance covariance matrix of disturbances does not matter for seeking the optimal solution (Simon (1956) certainty equivalence result for linear quadratic regulator).

The three policy targets, the output gap x_t, inflation π_t and public debt b_t are two-time-step Kalman (1960) controllable by the two policy instrument (the interest rate i_t and surplus s_t) and only by the policy rate (using the first column $\mathbf{B}_{i.}$ of matrix $\mathbf{B}$) if $\gamma \neq 0$, $\kappa \neq 0$.

$$rank\left(\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}\right) = rank\left(\mathbf{B}_{i.}, \mathbf{AB}_{i.}, \mathbf{A}^2\mathbf{B}_{i.}\right) = 3 \text{ if } \gamma \neq 0 \text{ and } \kappa \neq 0.$$

The surplus instrument alone is only able to control public debt if $\beta \neq 1$, using the second column $\mathbf{B}_{.s}$ of matrix $\mathbf{B}$)

$$rank\left(\mathbf{B}_{.s}, \mathbf{AB}_{\mathbf{.s}}, \mathbf{A}^2\mathbf{B}_{.s}\right) = 1 \text{ if } \beta \neq 1.$$

We use $\beta = 0.99$ for quarterly periods and $\kappa = 0.1$, $\gamma = 0.5$ numerical values instead of Woodford (1996) numerical values $\beta = 0.95$ for yearly periods and $\kappa = 0.3$, $\gamma = 1$ which are oversized with respect to posterior estimations U.S.A. since the 1960s (Havranek (2015), Mavroeidis *et alii* (2014)).

The transmission parameter of funds rate on future debt is equal to 1 which is nearly 100 times larger in absolute value than the transmission parameter of the surplus (0.01). This implies that parameters of the surplus rule would have to be 100 times larger than the parameters of the Taylor rule to have the same size in the parameters of the closed loop system.

3 Ramsey Optimal Policy under Quasi-Commitment

In a monetary policy regime indexed by j, a policy maker may re-optimize on each future period with exogenous probability $1-q$ strictly below one, labeled "quasi commitment" (Schaumburg and Tambalotti, 2007 and Debortoli and Nunes, 2014)). Following Schaumburg and Tambalotti (2007), we assume that the mandate to minimize the loss function is delegated to a sequence of policy makers with a commitment of random duration. The degree of credibility is modelled as if it is a change of policy-maker with a given probability of reneging commitment and re-optimizing optimal plans. The length of their tenure or "regime" depends on a sequence of exogenous i.i.d. Bernoulli signals $\{\eta_t\}_{t\geq 0}$ with $E_t\left[\eta_t\right]_{t\geq 0} = 1-q$, with $0 < q \leq 1$. If $\eta_t = 1$, a new policy maker takes office at the beginning of time t. Otherwise, the incumbent stays on. A higher probability q can be interpreted as a higher credibility. A policy maker with little credibility does not give a large weight on future welfare losses. The policy maker j solves the following problem for regime j, omitting subscript j, before policy maker k starts:

$$V_0^j = -E_0 \sum_{t=0}^{t=+\infty} (\beta q)^t \left[\frac{1}{2} \left(Q_\pi \pi_t^2 + Q_x x_t^2 + Q_b b_t^2 + \mu_i i_t^2 + \mu_s s_t^2 \right) + \beta \left(1-q\right) V_t^k \right]$$

Inflation, output gap and public debt next period are an average between two terms. The first term, with weight q is the inflation, output gap and public debt that would prevail under the current regime upon which there is commitment. The second term with weight $1-q$ is the inflation and public debt that would be implemented under the alternative regime by policy maker k.:

$$q \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ b_{t+1} \end{pmatrix} + (1-q) \begin{pmatrix} E_t x_{t+1}^k \\ E_t \pi_{t+1}^k \\ b_{t+1}^k \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{\gamma\kappa}{\beta} & -\frac{\gamma}{\beta} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & -\frac{1}{\beta} & \frac{1}{\beta} \end{pmatrix}}_{=\mathbf{A}} \begin{pmatrix} x_t \\ \pi_t \\ b_t \end{pmatrix} + \underbrace{\begin{pmatrix} \gamma & 0 \\ 0 & 0 \\ 1 & 1-\frac{1}{\beta} \end{pmatrix}}_{=\mathbf{B}} \begin{pmatrix} i_t & s_t \end{pmatrix}$$

The optimal program for policy maker j is a discounted linear quadratic regulator (LQR) with a "credibility adjusted" discount factor βq. We apply Chatelain and Ralf (2019a) algorithm using $\sqrt{\beta q}\mathbf{A}/q$ and $\sqrt{\beta q}\mathbf{B}/q$ (Scilab code in the appendix).

Preferences of the policy maker are given by positive weights for the three policy targets $Q_x \geq 0$, $Q_\pi \geq 0$, $Q_b \geq 0$ ($\mathbf{Q} = diag(Q_x, Q_\pi, Q_b)$ in the Scilab algorithm). In order to insure the concavity of the LQR program, there are at least non-zero policy maker's preferences for *interest rate smoothing* and *primary surplus smoothing*, with *strictly* positive weights for these two policy instruments in the loss function: $\mu_i > 0$, $\mu_s > 0$. In the simulation grid on preferences of this paper, these strictly positive weights are also set down to 10^{-7} with respect to at least one other weight for policy targets set to 1. They are stacked in matrix $\mathbf{R} = diag(\mu_i, \mu_s)$ in the Scilab algorithm. In what follows, we consider the polar case of maximal credibility ($q = 1$). Results for any level of limited credibility $0 < q < 1$ can be found using the algorithm in appendix 1.

The policy maker seeks optimal linear feedback rules parameters stacked in matrix $\mathbf{F}$:

$$\begin{pmatrix} i_t \\ s_t \end{pmatrix} = \underbrace{\begin{pmatrix} F_x & F_\pi & F_b \\ G_x & G_\pi & G_b \end{pmatrix}}_{=\mathbf{F}} \begin{pmatrix} x_t \\ \pi_t \\ b_t \end{pmatrix}$$

Replacing feedback rules in the transmission mechanism leads to a closed loop system with transition matrix $\frac{\sqrt{\beta q}}{q}(\mathbf{A}+\mathbf{BF})$ for the algorithm:

$$\begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ b_t \end{pmatrix} = \frac{\sqrt{\beta q}}{q} \begin{pmatrix} 1+\frac{\gamma\kappa}{\beta}-\gamma F_x & -\frac{\gamma}{\beta}-\gamma F_\pi & -\gamma F_b \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ F_x-\left(\frac{1}{\beta}-1\right)G_x & -\frac{1}{\beta}+F_\pi-\left(\frac{1}{\beta}-1\right)G_\pi & \frac{1}{\beta}+F_b-\left(\frac{1}{\beta}-1\right)G_b \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \\ b_t \end{pmatrix}$$

When the Taylor rule does not include a reaction of funds rate to public debt ($F_b = 0$), the closed-loop matrix remains block triangular. In this case, we use these notations for the private sector closed-loop dynamics:

$$\mathbf{A}_{x\pi} + \mathbf{B}_{x\pi}\mathbf{F}_{x\pi} = \frac{\sqrt{\beta q}}{q} \begin{pmatrix} 1+\frac{\gamma\kappa}{\beta} & -\frac{\gamma}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix} + \frac{\sqrt{\beta q}}{q} \begin{pmatrix} -\gamma \\ 0 \end{pmatrix} \begin{pmatrix} F_x & F_\pi \end{pmatrix}$$

It is the transition matrix of the closed loop new-Keynesian model excluding public debt when shocks are not auto-regressive.

The case of the peg of policy instruments at their long-run value $\mathbf{F} = \mathbf{0}$ corresponds to an open loop system with transition matrix $\mathbf{A}$.

Varying the policy maker's preferences in all their range of possible values is equivalent to find the determinacy conditions on the rule parameters in the case of simple rules. Specific values of preferences for households welfare would be somewhere in this locus for policy rule. Tables 1 to 4 present solutions of the four polar cases of the policy maker's preferences.

Case 1: Maximal Inertia. Maximal inertia where the weight on the volatility of the policy targets is zero and the weight of each of the two policy instrument is any non-zero value which could be different for each of the two instruments, for example: $diag(\mathbf{Q}, \mathbf{R}) = (0, 0, 0, 1, 1)$. Changing the relative weight between the two instrument (10^7 to 10^{-7}) does not change the results.

The Hamiltonian system of the LQR with 3 state variables for the policy maker and 3 co-state variables has a transition matrix $\mathbf{H}_6$ of dimension six which is symplectic: its transpose is similar to its inverse. This implies that the list of eigenvalues of $\mathbf{H}_6$ (its spectrum $\Lambda_{\mathbf{H}}$) is such that all inverse of each eigenvalues belong the spectrum. Hence, if there is no eigenvalues exactly on the unit circle, there is an equal number (three) of eigenvalues inside the unit circle and outside the unit circle. Because of the requirement of local stability of the optimal solution, the optimal solution selects three stable eigenvalues available in the spectrum $\Lambda_{\mathbf{H}}$. This method has been extended to the solution of rational expectations models not necessarily derived from optimal behavior by Blanchard and Kahn (1980).

With maximal inertia, the spectrum includes the three eigenvalues of the open-loop transition matrix $\Lambda_{\mathbf{A}}$ where $\mathbf{F}=\mathbf{0}$ and their three inverse of these eigenvalues.

$$\Lambda_{\mathbf{H}} = \begin{pmatrix} 1.201 & 0.7481 & 1.005 & 1/1.201 & 1/0.7481 & 1/1.005 \end{pmatrix}$$
$$\Lambda_{\mathbf{A}} = \begin{pmatrix} 1.201 & 0.7481 & 1.005 \end{pmatrix}, \ \Lambda_{\mathbf{A+BF}} = \begin{pmatrix} 0.8322 & 0.7481 & 0.995 \end{pmatrix}$$

The open loop transition matrix $\mathbf{A}$ includes two unstable eigenvalues. In particular, debt dynamics is exploding without negative-feedback. Then, the maximal inertia eigenvalues $\Lambda_{\mathbf{A+BF}}$ includes the two inverse of these eigenvalues and keeps the eigenvalue inside the unit circle. For this reason, maximal inertia implies nonetheless non-zero optimal policy rule parameters. These policy parameters insures the local stability in dimension three of the closed-loop system.

$$\begin{pmatrix} i_t \\ s_t \end{pmatrix} = \begin{pmatrix} -0.742 & 1.898 & 0 \\ -2 & 1.2 & 1 \end{pmatrix} \begin{pmatrix} x_t & \pi_t & b_t \end{pmatrix}^T$$

It is optimal for the Taylor rule not to respond to public debt: there is no need to set the restriction $F_b = 0$. The optimal surplus rule parameter $G_b = 1$ is such that the public debt eigenvalue shifts from its diverging open loop value number $1/\sqrt{\beta}$ to its inverse $\sqrt{\beta}$. These eigenvalues include the factor $\sqrt{\beta}$ because the loss function is discounted.

$$\sqrt{\beta}\frac{1}{\beta} + \sqrt{\beta}\left(F_b - \left(\frac{1}{\beta}-1\right)G_b\right) = \sqrt{\beta} = \sqrt{0.99} = 0.995 < 1$$

This optimal surplus rule and the resulting public debt eigenvalue with least effort stabilization of public debt remains unchanged when the policy maker does not care about the volatility of public debt ($\mu_b = 0$) and for any non zero weights on inflation and output gap.

The Taylor rule parameter on inflation satisfies the Taylor principle ($1.898 > 1$). The output gap parameter is negative for Ramsey optimal policy with the new-Keynesian model (Chatelain and Ralf (2019b)). This is because a rise of funds rate increase future consumption and future consumption growth because of the intertemporal substitution effect for consumers. There is no income effect of funds rate nor cost of capital effect decreasing investment demand as in the investment saving equation of the IS-LM model. Hence, negative feedback aiming at decreasing future output gap implies a negative response of funds rate when current output gap is positive. Then, the intertemporal substitution mechanism implies a relative decrease of the growth rate of future consumption.

If one reverts the signs to Keynesian mechanism: κ accelerationist Phillips curve and γ (delayed cost of capital effect or cost of working capital), then the sign of the response of funds rate to output gap turns to be positive, with exactly the same numerical values ($+0.742$), as it was in the good old time of Keynesian economics.

The optimal surplus rule also responds to inflation and output gap. Because the marginal parameter in $\mathbf{B}$ of the surplus is nearly one hundred times smaller than the one of the funds, this leads to marginal changes of parameters relating future public debt with current inflation and current output gap than in the case of only an effect due to the Taylor rule.

The initial optimal jumps of the inflation and output gap on public debt are obtained from optimal marginal conditions. The optimal value of the loss function is:

$$L^{*}=\left(\begin{array}{ccc} x_0 & \pi_0 & b_0 \end{array}\right)\mathbf{P}_3\left(\begin{array}{ccc} x_0 & \pi_0 & b_0 \end{array}\right)^T$$
$$\text{with } \mathbf{P}_3=\left(\begin{array}{cc} \mathbf{P}_{x\pi} & \mathbf{P}_{x\pi,b} \\ \mathbf{P}_{b,x\pi} & \mathbf{P}_b \end{array}\right)$$

Where $\mathbf{P}_3$ is a square matrix of dimension three which is the solution of a discrete algebraic Riccati equation (DARE) given by the `lqr` instruction in Scilab. The marginal values of the loss function with respect to each state variables are equal to the co-state variable or Lagrange multiplier of each state variable. If state variables for the policy maker are jump variables of the private sector (here, inflation and output gap), there initial values is found optimizing the loss function at the initial date for each of these variables. These marginal conditions are:

$$\left(\begin{array}{c} (\partial L^*/\partial x)_{t=0} \\ (\partial L^*/\partial \pi)_{t=0} \end{array}\right)=\left(\begin{array}{c} \gamma_{x_0} \\ \gamma_{\pi_0} \end{array}\right)=\mathbf{P}_{x\pi}\left(\begin{array}{c} x_0 \\ \pi_0 \end{array}\right)+\mathbf{P}_{x\pi,b}b_0=\left(\begin{array}{c} 0 \\ 0 \end{array}\right)$$
$$\left(\begin{array}{c} x_0 \\ \pi_0 \end{array}\right)=-\mathbf{P}_{x\pi}\mathbf{P}_{x\pi,b}b_0=\left(\begin{array}{c} 0.653 \\ 0.255 \end{array}\right)b_0$$

An increase of initial public debt implies a proportional *increase* of initial output gap and inflation, as in Woodford (1996) model, a result which is obtained for $F_\pi < 1$. But, for the following periods, future values output gap and inflation do not depend on current values of public debt b_t for $t > 0$ because the parameters of marginal effects equal to zero in the closed loop matrix $\mathbf{A}+\mathbf{BF}$. In particular, because the funds rate do not react to public debt in the Taylor rule ($F_b = 0$). As seen below, this result changes when the cost of the volatility of public debt is taken into account by the policy maker ($\mu_b > 0$).

For Ramsey optimal policy with maximal inertia, the optimal initial jumps of forward-looking policy targets are chosen in order to minimize the volatility of policy instrument. They imply that the initial jumps of the policy instruments are equal to zero:

$$\left(\begin{array}{c} i_0/b_0 \\ s_0/b_0 \end{array}\right)=\left(\begin{array}{ccc} -0.742 & 1.898 & 0 \\ -2 & 1.2 & 1 \end{array}\right)\left(\begin{array}{c} 0.653 \\ 0.2551 \\ 1 \end{array}\right)=\left(\begin{array}{c} 0 \\ 0 \end{array}\right)$$

Even though the policy instrument are equal to zero, the dynamic system is no longer the open loop system with matrix $\mathbf{A}$ because this matrix includes eigenvalues outside the unit circle. It is the closed loop system $\mathbf{A}+\mathbf{BF}$ with $\mathbf{F}\neq\mathbf{0}$ so that there is local stability of the dynamic system in dimension three. The dimension three corresponds to the number of policy targets for the policy maker.

With Ramsey optimal policy, there is no indeterminacy with three eigenvalues inside the unite circle despite two forward-looking policy targets and one predetermined policy target. Optimal initial conditions implies that the co-states of the forward-looking policy targets are predetermined at zero: $\gamma_{x_0}=\gamma_{\pi_0}=0$. Because these co-state variables have a linear relation with the funds rate and its lag, these conditions can also be interpreted as if the funds rate and its lag are predetermined variables.

General results.

In all the following cases where the volatility of the public debt does has no weight in the loss function ($\mu_b = 0$, $\mu_x \geq 0$, $\mu_\pi \geq 0$, $\mu_i > 0$, $\mu_s > 0$), the surplus rule parameters do not change. The surplus rule achieves the maximal inertia (minimal effort) for stabilizing public debt eigenvalue. The percentage deviation of surplus is equal to the percentage deviation of public debt ($G_b = 1$). It is optimal that the funds rate do not respond to public debt in the Taylor rule ($F_b = 0$). The Taylor rule only controls the eigenvalues of output gap and inflation. The Taylor rule parameters (F_x,F_π) are identical to those found when assuming public debt is equal to zero at all periods ($b_t = 0$). However, an increase of the weight of the surplus $\mu_s > 0$ *increases* the parameter $\mathbf{P}_{x\pi,b}$ for the optimal loss function. Hence, it increases the absolute value of the optimal initial anchor of inflation and output gap, even though the Taylor rule parameters are unchanged when varying the surplus weight $\mu_s > 0$. For a negligible weight of the surplus $\mu_s = 10^{-7}$, inflation and output gap anchor are instantaneously anchored at zero.

Case 2: Minimize output gap volatility (table 1).

Output gap can be stabilized on the first period following a change of interest rate using the Euler consumption. The stabilization of inflation occurs on period two. The correlation of the policy instrument with output gap of period one is then transmitted on period two by the change of output gap correlated with expected inflation of period two. Hence, a policy maker only concerned with a single eigenvalue related to output gap. He may leave the second eigenvalue mostly related to inflation close to one, because the cost of inflation volatility is zero in this polar case.

When the cost of changing funds rate decreases from 1 to near-zero 10^{-7}, the output gap parameter in the Taylor rule increases in absolute value from -0.986 to -1.92 (whereas the inflation rule parameter decreases from 1.82 to 1.27). The output gap eigenvalue shifts from 0.54 to 0 (which means back to equilibrium in one period).

Table 1: Minimize output gap volatility.

μ_i / μ_s	Q_x / $\lvert\lambda_{x\pi}\rvert$	Q_π / $\lvert\lambda_{\pi x}\rvert$	Q_b / $\lvert\lambda_b\rvert$	F_x / G_x	F_π / G_π	F_b / G_b	$\frac{x_0}{b_0}$ / $\frac{\pi_0}{b_0}$	$\frac{i_0}{b_0}$ / $\frac{s_0}{b_0}$
1	1	0	0	-0.986	1.821	0	0.571	-0.338
1	0.542	0.918	0.975	-2	1.2	1	0.123	0.006
1	1	0	0	-0.986	1.821	0	0.000009	-0.0000005
10^{-7}	0.542	0.918	0.975	-2	1.2	1	0.000002	1
10^{-7}	1	0	0	-1.920	1.269	0	0.378	-0.969
1	0	0.995	0.975	-2	1.2	1	-0.201	0.002
10^{-7}	1	0	0	-1.920	1.269	0	0.00002	-0.00005
10^{-7}	0	0.995	0.975	-2	1.2	1	-0.00001	1

Case 3: Minimize inflation volatility (table 2).

Because of the sequence of stabilization: period 1, output gap then period 2, inflation, stabilizing inflation imposes to stabilize output gap first. Hence two eigenvalues of the block matrix of output gap and inflation need to be modify. This requires much more energy (volatility of the funds rate) to the policy maker.

When the cost of changing funds rate decreases from 1 to near-zero 10^{-7}, the output gap parameter in the Taylor rule increases in absolute value from -0.828 to -3.91 and the inflation rule parameter increases from 2.22 to... 21. The output gap eigenvalue shifts from 0.77 to 0 (which means back to equilibrium in one period).

Table 2: Minimize inflation volatility.

μ_i	Q_x	Q_π	Q_b	F_x	F_π	F_b	$\frac{x_0}{b_0}$	$\frac{i_0}{b_0}$
μ_s	$\lvert\lambda_{x\pi}\rvert$	$\lvert\lambda_{\pi x}\rvert$	$\lvert\lambda_b\rvert$	G_x	G_π	G_b	$\frac{\pi_0}{b_0}$	$\frac{s_0}{b_0}$
1	0	1	0	−0.828	2.218	0	0.613	−0.0088
1	0.772	0.772	0.975	−2	1.2	1	0.189	0.0009
1	0	1	0	−0.828	2.218	0	0.000001	0.00001
10^{-7}	0.772	0.772	0.975	−2	1.2	1	0.0000001	1
10^{-7}	0	1	0	−3.919	21.209	0	0.515	−1.486
1	0.006	0.006	0.975	−2	1.2	1	0.025	0.00001
10^{-7}	0	1	0	−3.919	21.209	0	0.0038	−0.011
10^{-7}	0.006	0.006	0.975	−2	1.2	1	0.0002	0.992

Locus of Taylor rule parameters when $\mu_b = 0$:

When $\mu_b = 0$ and $\mu_s > 0$ and our given numerical parameters of the transmission mechanism, we draw the locus of the reduced form values of Taylor rule parameters of Ramsey optimal policy. Table 3 and figure 1 provide the boundaries of the triangle of the linear quadratic regulator (LQR) reduced form Taylor rule parameters, obtained by a simulation grid, varying *the weights in the loss function in three dimensions* $\mu_x \geq 0$, $\mu_\pi \geq 0$, $\mu_i > 0$. The sides of the LQR triangle correspond to the cases where the central bank minimizes only the variance of inflation (inflation nutter) without taking into account the zero lower bound constraint on the policy interest rate ($\mu_i = 10^{-7} > 0$), or minimizes only the variance of output gap without taking into account the zero lower bound ($\mu_i = 10^{-7} > 0$), or seeks only maximal inertia of the policy rate ($\mu_i \to +\infty$). This is taken from Chatelain and Ralf (2019b).

Table 3: Taylor rule parameters for $\mu_b = 0$, $\mu_s > 0$, $\kappa = 0.1$, $\gamma = 0.5$, $\beta = 0.99$.

Minimize only:	Q_π	Q_x	μ_i	$\lvert\lambda_{x\pi}\rvert$	$\lvert\lambda_{\pi x}\rvert$	F_x	F_π
Inflation	1	0	10^{-7}	**7.10^{-5}**	**0.006**	**−3.92**	**21.21**
Inflation output gap	4	1	10^{-7}	4.10^{-7}	0.819	−2.27	4.76
Inflation output gap	1	1	10^{-7}	4.10^{-7}	0.905	−2.10	3.03
Inflation output gap	1/4	1	10^{-7}	4.10^{-7}	0.951	−2.01	2.10
Output gap	0	1	10^{-7}	**4.10^{-7}**	**0.995**	**−1.92**	**1.21**
Output gap interest	0	4	1	0.348	0.953	−1.31	1.70
Output gap interest	0	1	1	0.541	0.918	−0.98	1.83
Output gap interest	0	1/4	1	0.663	0.878	−0.82	1.87
Interest rate	0	0	$1(+\infty)$	**0.748**	**0.833**	**−0.74**	**1.89**
Inflation interest	1/4	0	1	0.784	0.784	−0.77	1.99
Inflation interest	1	0	1	0.772	0.772	−0.83	2.22
Inflation interest	4	0	1	0.742	0.742	−0.98	2.82

A similar analysis can be made for the alternative monetary policy transmission mechanism with $\gamma < 0$ and $\kappa < 0$, see figure 2. The numerical values of the Taylor rule parameters are the same in absolute value. But this time, the output gap rule parameter is *positive*. The new locus is symmetric with respect to the horizontal axis of the locus with opposite signs of the transmission mechanism.

Case 4: Minimize public debt volatility (table 4):

If $\mu_b > 0$ and μ_s sufficiently large with respect to μ_i, the policy maker's cares about public debt volatility and it is relatively costly for him to use the surplus instrument with respect to the funds rate instrument. In order to reduce a lot the persistence (auto-correlation and eigenvalue) of public debt, the Taylor rule is then called into action in addition to the surplus rule.

In this polar case, as there is no cost of inflation and output gap, the two first eigenvalues related to these variables remain relatively large.

When the cost of changing funds rate, surplus and public are all equal to 1, the funds rate decreases by -0.45 in proportion to public debt over its long run target and the surplus increases by 1.3. Because the marginal effect in the transmission mechanism of the funds rate is one hundred times the one of the surplus, the fall of the surplus autocorrelation from minimal effort 0.995 ($F_b = 0$, $G_b = 1$) to 0.358 ($F_b = -0.45$, $G_b = 1.3$) is nearly entirely due to the Taylor rule.

When the cost of changing surplus is near zero with respect to one for the cost of changing funds rate, the fall of the autocorrelation of public debt to nearly 0 is entirely due to the surplus rule. This implies a very large response of surplus by 100 times the deviation of debt from its long run value ($G_b = 99.8$), because the marginal transmission parameter is 100 times smaller than the one of funds rate in order to achieve the same decrease of the auto-correlation of public debt.

When the cost of changing surplus is one with respect to near zero for the cost of changing funds rate, the fall of the autocorrelation $|\lambda_3|$ down to zero is driven by a larger sensitivity of the funds rate to public debt ($F_b = 1$) whereas the surplus policy rule is identical to the one chosen when the policy maker does not care about public debt volatility ($G_b = 1$). The funds rate policy rule contributes to 95% of the fall of public debt persistence.

When the cost of changing surplus and changing funds rate are both near zero, the fall of the autocorrelation $|\lambda_3|$ down to zero, this leads to marginal changes with respect the previous case: $F_b = 0.99$ and $G_b = 1$. Because the marginal effect of funds rate on public debt is one hundred times the one of surplus in matrix $\mathbf{B}$, it is more efficient to use funds rate than surplus when there is a preference for a large fall of public debt persistence.

Table 4: Minimizing public debt volatility.

μ_i	Q_x	Q_π	Q_b	F_x	F_π	F_b	$\frac{x_0}{b_0}$	$\frac{i_0}{b_0}$
μ_s	$\lvert\lambda_{x\pi}\rvert$	$\lvert\lambda_{\pi x}\rvert$	$\lvert\lambda_b\rvert$	G_x	G_π	G_b	$\frac{\pi_0}{b_0}$	$\frac{s_0}{b_0}$
1	0	0	1	-0.424	1.448	-0.454	0.614	-0.443
1	0.992	0.929	0.358	-2.641	1.620	1.323	0.187	0.004
1	0	0	1	-0.742	1.897	-0.0007	1.124	-0.0005
10^{-7}	0.832	0.748	0.009	-73.46	87.84	99.833	0.440	55.843
10^{-7}	0	0	1	-0.020	1.022	-1	0.593	-0.853
1	0.995	0.995	10^{-8}	-2	1.2	1	0.155	0
10^{-7}	0	0	1	-0.030	1.029	-0.995	0.597	-0.849
10^{-7}	0.945	0.989	10^{-8}	-2.984	1.848	1.499	0.159	0.008

4 Ramsey optimal policy versus Fiscal Theory of the Price Level versus new-Keynesian Model

This section compares the three equilibrium based on the new-Keynesian transmission mechanism. It refers to the diagrams (appendix 2) in the plane of Taylor rule parameters (F_π, F_x) mapping eigenvalues (λ_1, λ_2) inside or outside the unit circle for given parameters of the monetary policy transmission mechanism (Chatelain and Ralf (2019b)). These eigenvalues (λ_1, λ_2) are the roots of the characteristic polynomial $p(\lambda)$ of the closed loop matrix $\mathbf{A}_{x\pi} + \mathbf{B}_{x\pi}\mathbf{F}_{x\pi}$ of new-Keynesian model with output gap and inflation:

$$\mathbf{A}_{x\pi}+\mathbf{B}_{x\pi}\mathbf{F}_{x\pi}=\begin{pmatrix} 1+\frac{\gamma\kappa}{\beta} & -\frac{\gamma}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}+\begin{pmatrix} -\gamma \\ 0 \end{pmatrix}\begin{pmatrix} F_x & F_\pi \end{pmatrix}$$

$$p(\lambda)=\det(\mathbf{A}_{x\pi}+\mathbf{B}_{x\pi}\mathbf{F}_{x\pi}-\lambda\mathbf{I}_2)=\lambda^2-T\lambda+D=0$$

$$p(1)=-T+D=(1-\lambda_1)(1-\lambda_2),\ p(-1)=T+D=(-1-\lambda_1)(-1-\lambda_2)$$

The Taylor rule parameters (F_x, F_π) are affine functions of the trace T and determinant D of the closed loop matrix $\mathbf{A}_{x\pi}+\mathbf{B}_{x\pi}\mathbf{F}_{x\pi}$. For dimension two dynamics, there is a mapping of eigenvalues in the plane of trace T and determinant D including a stability triangle for both eigenvalues of $\mathbf{A}_{x\pi}+\mathbf{B}_{x\pi}\mathbf{F}_{x\pi}$ are inside the unit circle (see e.g. Azariadis (1993)). Chatelain and Ralf (2019b) shows the affine-transformed mapping of eigenvalues (λ_1, λ_2) in the plane of Taylor rule parameters (F_x, F_π). This includes a stability triangle where both eigenvalues of of $\mathbf{A}_{x\pi}+\mathbf{B}_{x\pi}\mathbf{F}_{x\pi}$ are inside the unit circle. This stability triangle (ABC in figure 1) include real eigenvalues such that $-1<\lambda_1\leq\lambda_2<1$ (region 4.1 in figure 1) and complex conjugate eigenvalues such that $|\lambda_1|=|\lambda_2|<1$ (region 4.2 in figure 1). Its center Ω corresponds to the fully stabilized dynamic system with zero eigenvalues $\lambda_1=\lambda_2=0$.

This stability triangle implies that the Taylor rule parameter of inflation satisfies the Taylor principle. But it also implies that the Taylor rule parameter of output gap is strictly negative, because of the intertemporal substitution effect of the Euler equation. A rise of interest rate is positively correlated with future consumption and output gap. Hence, if output gap is currently positive, it is necessary to decrease interest rate so that future consumption will decrease.

As a thought experiment, if the sign of the transmission parameters are reverse ($\kappa<0$, $\gamma<0$), then the stability triangle is in the quadrant such that $F_x>0$ and $F_\pi>0$ (figure 2).

We locate on this diagram the policy rule parameters for the three following equilibria.

Ramsey optimal policy Equilibrium.

Chatelain and Ralf (2019b) using table 2 simulation grid show that the locus of Ramsey optimal policy rule parameter is a smaller triangle included into the larger stability triangle where both eigenvalues are inside the unit circle. In particular, negative eigenvalues $-1<\lambda_1\leq\lambda_2<0$ cannot be eigenvalues of optimal policy (figure 3 and a zoom in figure 5). Because Ramsey optimal policy parameter are located within the stability triangle, this implies that its Taylor rule parameter of inflation satisfies the Taylor principle. But it also implies that its Taylor rule parameter of output gap is strictly negative, because of the intertemporal substitution effect of the Euler equation.

As a thought experiment, if the sign of the transmission parameters are reverse ($\kappa<0$, $\gamma<0$), then the Ramsey triangle is within stability triangle which is this time in the quadrant such that $F_x>0$ and $F_\pi>0$ (figure 4).

With Ramsey optimal policy, optimal initial conditions are derived for inflation and output gap with co-state variables of inflation and output gap optimally predetermined at zero (transversality conditions). The negative-feedback values of the policy rules parameters implies that the number of stable eigenvalues inside the unit circle (three in this model) is equal to the number of forward-looking variables (output gap and inflation) and predetermined variable (public debt).

FTPL Equilibrium: Woodford (1996) assumes that surplus is a predetermined variable which follows an exogenous auto-regressive process $s_t=\rho s_{t-1}+\varepsilon_s$ instead of a feedback rule (its eigenvalue is $0 < \rho_s < 1$) so that $G_b = G_x = G_\pi = 0$. Woodford (1996) assumes that the funds rate does not respond to public debt so that $F_b = 0$. Then, the public debt eigenvalue is $\frac{1}{\beta} > 1$. It is outside the unit-circle and cannot be controlled by the policy maker.

Public debt is a predetermined variable. Output gap and inflation are forward-looking variables. The funds rate is assumed to be a forward-looking variable. Blanchard and Kahn (1980) determinacy condition implies that there should be one of the eigenvalues of the closed-loop matrix to be inside the unit circle and the other one outside the unit circle.

This corresponds to region 1 and region 2 of figure 1. These regions are limited by two lines. The first line includes the segment AC. It defines the new-Keynesian border of the Taylor principle, including the point: ($F_\pi = 1$, $F_x = 0$). It is a nearly vertical line such that at least one of the two roots is equal 1: $p(1) = 0$. The second line includes the segment BC. It is a negative slope line where at least one of the two roots is equal to -1: $p(-1) = 0$.

- Region 1 on the top-left is such that $-1 < \lambda_1 < 1 < \lambda_2$: The inflation Taylor rule parameter does not satisfy the Taylor principle ($F_\pi < 1$) and it can be negative without finite bound. Output gap rule parameter are positive without finite upper bound or negative with a finite lower bound. The regime is restricted by Woodford (1996) to positive and small values: $0 < F_\pi < 1$ and $0 < F_x < 1$ which corresponds to the FTPL near-square of figure 5, within region 1 of figure 1.

- Region 3 on the bottom-right is such that: $\lambda_1 < -1 < \lambda_2 < 1$. The inflation Taylor rule parameter satisfies the Taylor principle ($F_\pi < 1$) without a finite upper bound but the output gap rule parameter is always negative without a finite lower bound ($F_x < 1$). This regime is not mentioned by Woodford (1996), but the determinacy condition is indeed satisfied by these rule parameters.

New-Keynesian Equilibrium: The new-Keynesian model assumes that there is zero net supply of public debt: $b_t = 0$ at all periods. At this stage, there is no longer a predetermined variable. Output gap and inflation are forward-looking variables. The funds rate is assumed to be a forward-looking variable. Then, two auto-regressive shocks for the Euler consumption equation and for the new-Keynesian Phillips curve, with two specific eigenvalues $0 < \rho_x < 1$ and $0 < \rho_\pi < 1$. These exogenous eigenvalues are given and cannot be modified by policy rule parameters. The two exogenous shocks are not controllable. Then, in order to satisfy Blanchard and Kahn determinacy condition, it is necessary that the two roots of the closed loop block matrix for inflation and output gap dynamics are outside the unit circle.

This corresponds to the following regions in figure 1:

- On the top right, region 4.3 for complex conjugate eigenvalues such that $|\lambda_1| = |\lambda_2| > 1$ and regions 4.4 and 4.5 with real eigenvalues: $1 < \lambda_1 < \lambda_2$: The inflation Taylor rule parameter satisfies the Taylor principle without upper bound and some output gap rule parameter are positive and some are negative with a finite lower bound. The new-Keynesian regime is usually restricted to positive and small values: $1 < F_\pi < 2$ and $0 < F_x < 1$ which corresponds to new-Keynesian square of figure 5, within region 1 of figure 1.

- On the bottom left, region 2: $\lambda_1 < \lambda_2 < -1$ on the left of the Taylor principle line ($p(1) = 0$) and below the line ($p(-1) = 0$) including the segment CB. The inflation Taylor rule parameter does not satisfy the Taylor principle including negative value without bound. The output gap rule parameter is often negative, or the inflation Taylor rule parameter is negative when the output gap rule parameter is positive. These range of parameters are not mentioned in new-Keynesian theory, but the determinacy condition is indeed satisfied by these rule parameters.

Frictionless Endowment Economies

The Taylor principle is not satisfied in Woodford (1996) FTPL for the new-Keynesian model with public debt. It is the same condition than for the fiscal theory of the price level equilibrium in Leeper (1991) for the frictionless model of endowment economies.

But Chatelain and Ralf (2019c) found that Ramsey optimal policy for the frictionless model of endowment economies is an interest rate peg ($F_\pi = 0 < 1$) where the Taylor principle is not satisfied, by contrast to the result obtained in this paper for the new-Keynesian model with public debt.

The reason is that the monetary policy transmission mechanism of frictionless endowment economies (Fisher relation) reach the zero lower bound of inflation persistence for an interest rate peg to its steady state value, $i_t = 0$ and $F_\pi = 0 < 1$:

$$E_t \pi_{t+1} = 0.\pi_t + \beta i_t$$

By contrast, for an interest rate peg to its steady state value, $i_t = 0$ and $F_\pi = 0 < 1$, there is trend inflation $\frac{1}{\beta} > 1$ for the new-Keynesian Phillips curve transmission mechanism. Ramsey optimal policy leans against inflation spirals, so that an interest rate peg cannot be optimal in this case. Ramsey optimal policy rule parameters are within the stability triangle of figure 1. This implies its inflation Taylor rule parameter satisfies the Taylor principle and that its output gap rule parameter is strictly negative.

5 Conclusion

By construction, ad hoc policy rule for both the new-Keynesian equilibrium or the FTPL equilibrium do not model policy maker's rational behavior. In addition, surplus is assumed to be an exogenous auto-regressive process instead of a feedback rule in Woodford (1998). In the new-Keynesian model, public debt is assumed to be zero all the time. Because at least one predetermined variable is required in order to avoid a degenerate rational equilibrium without transitory dynamics, it is replaced by two auto-regressive exogenous shocks (cost-push shock and productivity shock). How economic agents select the FTPL versus the new-Keynesian equilibrium is not clear, in particular because there is no optimal behavior by the policy maker. The policy makers have no credibility to anchor expectations when their policy rule parameters imply local instability in the space of policy-maker's target variables.

On the other hand, the key issue with the Ramsey optimal policy is that it predicts the unpleasant result that the output gap Taylor rule parameter is always negative in the new-Keynesian model with or without public debt. The origin of the unpleasant sign of Ramsey optimal policy parameter is the substitution positive effect of interest rate rising future consumption. Ramsey optimal policy is contingent to the signs of the parameters of the transmission mechanism.

Alternative micro-foundations can restore the income effect of interest rate so that it offsets the intertemporal substitution effect. If one assumes that there is capital in the production, then, there can be a cost of capital effect decreasing future output. A working capital cost effect on future inflation can also be introduced in the new-Keynesian Phillips curve. Credit constrained households, limited asset market participation also increase the magnitude of the income effect. Reversing the sign of the transmission mechanism of interest rate (substitution versus income effect) reverts the sign of the output gap parameter which then turns positive in the Taylor rule.

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6 Appendix 1: Scilab code

Remark: Scilab code computes this transformation of the preferences of the policy maker, to be called in the LQR instruction. In our case, $\mathbf{S} = \mathbf{0}$ and $\mathbf{Q}$ and $\mathbf{R}$ are diagonal, so that $\mathbf{C}$ and $\mathbf{D}$ elements includes square roots of these diagonal elements of $\mathbf{Q}$ and $\mathbf{R}$.

$$\begin{pmatrix} C^T \\ D^T \end{pmatrix} \begin{pmatrix} C & D \end{pmatrix} = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$$

```
Qx=0;
Qpi=0;
Qb=0 ;
Ri=1;
Rt=1;
beta1=0.99; gamma1=0.5; kappa=0.1;
A1=[1-(kappa*gamma1/beta1) -gamma1/beta1 0 ; -kappa/beta1 1/beta1 0 ; 0 -1/beta1 1/beta1] ;
A=sqrt(beta1)*A1;
B1=[gamma1 0 ; 0 0 ; 1 1-(1/beta1) ];
B1
B=sqrt(beta1)*B1;
Q=[Qx 0 0 ; 0 Qpi 0 ; 0 0 Qb ];
R=[Ri 0 ; 0 Rt ];
Big=sysdiag(Q,R);
Big
[w,wp]=fullrf(Big);
w
```

```
wp
C1=wp(:,1:3);
C1
D12=wp(:,4:$);
D12
M=syslin('d',A,B,C1,D12);
[Fy,Py]=lqr(M);
A+B*Fy;
spec(A)
abs(spec(A+B*Fy))
A+B*Fy
A
B
B*Fy
A
spec(A+B*Fy)
abs(spec(A+B*Fy))
Fy
Py
Px=Py(1:2,1:2)
Pd=Py(1:2,3)
T0=-inv(Px)*Pd
T0(3,1)=1
T0(1:2,1)
Ins0=Fy*T0
Fy
abs(spec(A+B*Fy))
B(:,1)*Fy(1,:)
B(:,2)*Fy(2,:)
```

Appendix 2

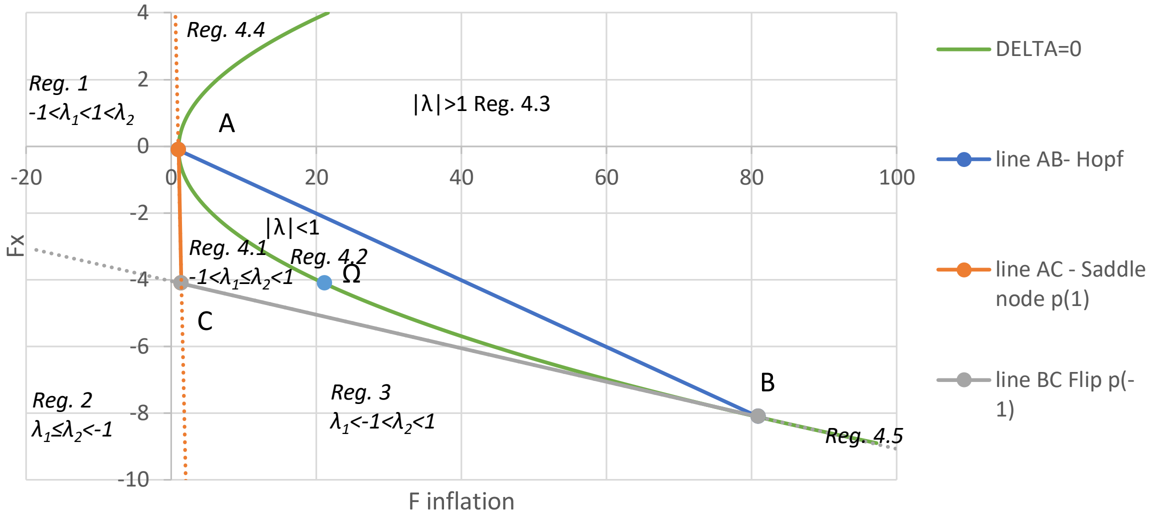


Figure 1: Stability triangle in the plane of policy parameters, γ=0.5, κ=0.1, β=0.99

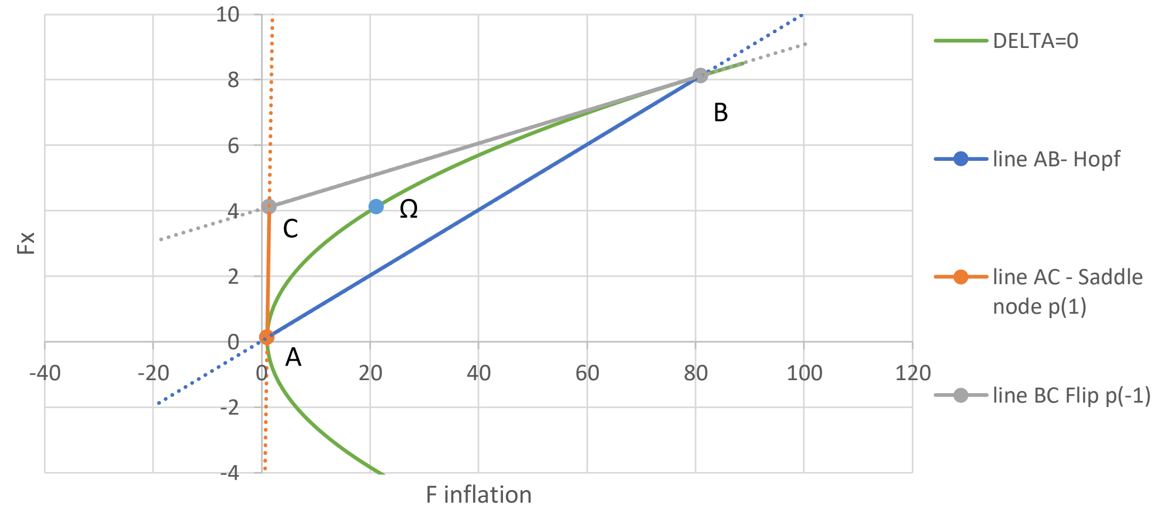


Figure 2: Stability triangle in the plane of policy parameters, γ=-0.5, κ=-0.1, β=0.99

Figure 3: LQR and stability triangle in the plane of policy parameters, γ=0.5, κ=0.1, β=0.99

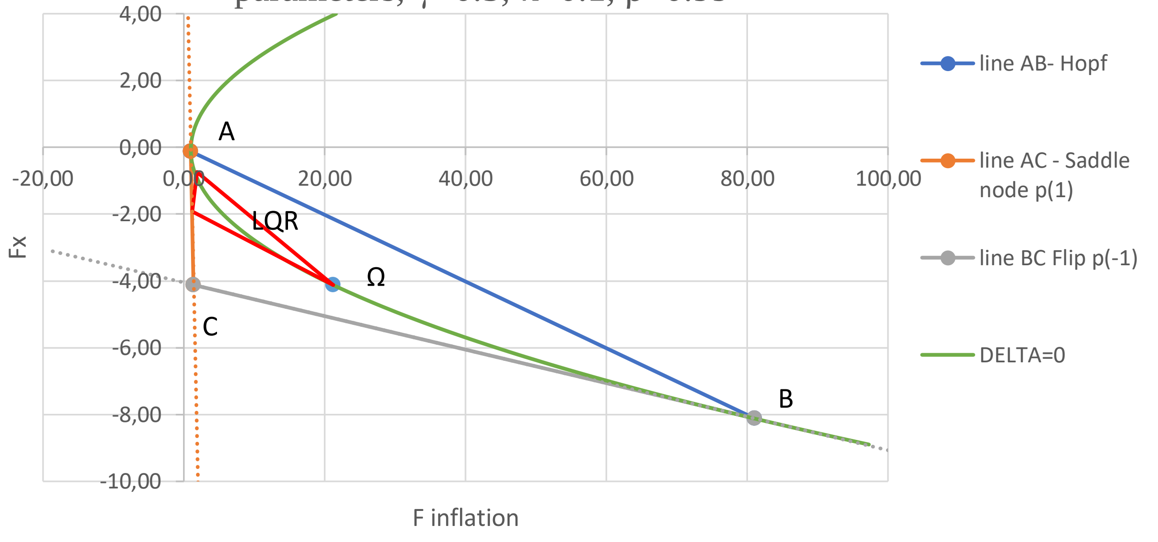
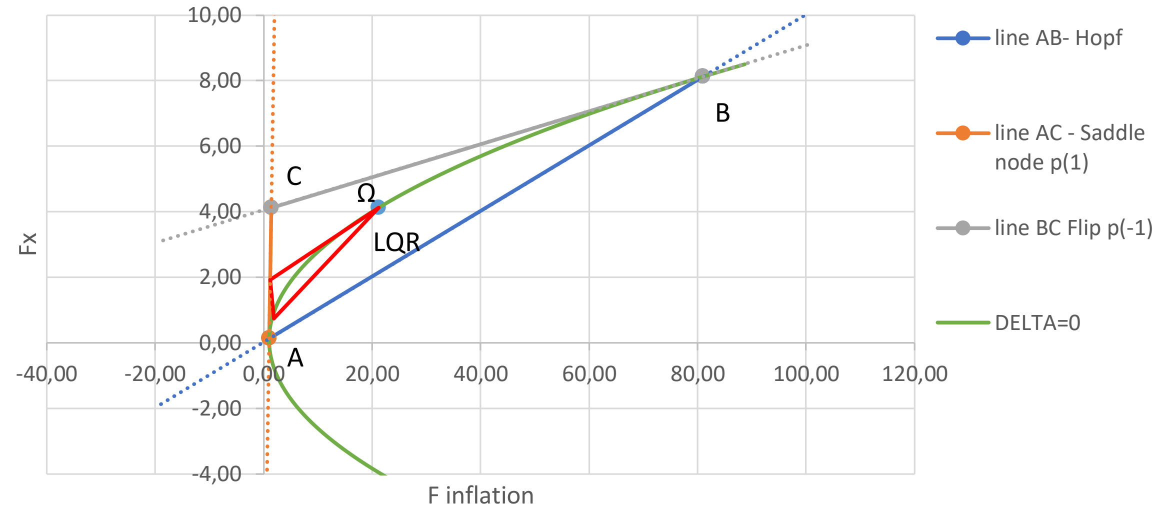


Figure 4: LQR and stability triangle in the plane of policy parameters, γ=-0.5, κ=-0.1, β=0.99



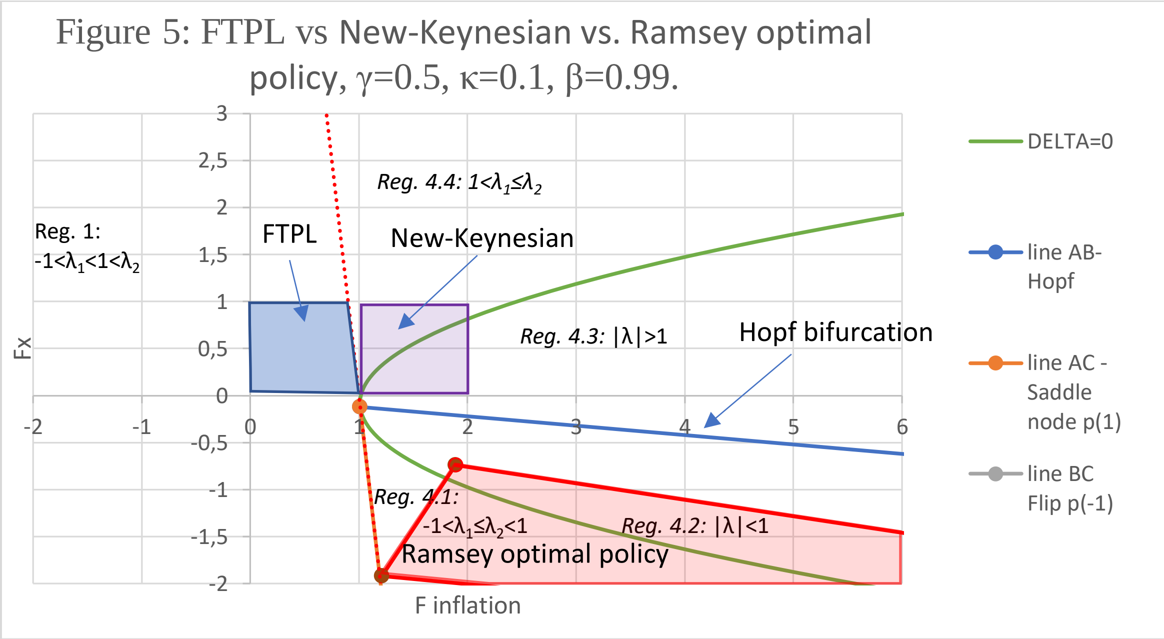


Figure 5: FTPL vs New-Keynesian vs. Ramsey optimal policy, γ=0.5, κ=0.1, β=0.99.
Reg. 4.4: 1<λ1≤λ2
Reg. 1:
-1<λ1<1<λ2
FTPL
New-Keynesian
Reg. 4.3: |λ|>1
Hopf bifurcation
Reg. 4.1:
-1<λ1≤λ2<1
Reg. 4.2: |λ|<1
Ramsey optimal policy
Fx
F inflation
3
2,5
2
1,5
1
0,5
0
-0,5
-1
-1,5
-2
-2
-1
0
1
2
3
4
5
6
DELTA=0
line AB-Hopf
line AC - Saddle node p(1)
line BC Flip p(-1)