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JEL Codes: E20, F22, J61

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Demographic Changes and the Labor Income Share*

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Abstract

In this article, we study the impact of demographic changes on the inequality between capital and labor. More precisely, we analyze the impact of exogenous changes in both the rate of natural increase and the net migration rate on the labor income as a share of total income. We estimate a structural vector autoregression (VAR) model on a panel of 18 OECD countries with annual data for 1985-2015. We obtain that the response of the labor income share to an exogenous change in the rate of natural increase is significantly negative a few years after the shock whereas its response to an exogenous change in the net migration rate is significantly positive. This suggests that inequality between capital and labor is reduced by international migration while fostered by the natural increase. We rationalize these findings in an original representative agent model where the rate of natural increase and the net migration rate are both modeled. The theoretical model reproduces the empirical findings and highlight the crucial roles of both the elasticity of substitution between capital and labor and the participation rate of migrants to the labor market. The model is then used to evaluate the dynamics consequences of permanent demographic changes and, most notably, reveals that in the long run, the labor income share is likely to fall with both the natural increase and the net migration.

JEL classification: E20, F22, J61.

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1 Introduction

In many developed countries, natural increase (difference between numbers of births and deaths) was for many years greater than net migration (difference between immigration and emigration). This has no longer been true since the end of the 1990s. Figure 1-(a) shows that on average, the migration accounted for most of the population growth in OECD countries over the last twenty years. There are, of course, differences between countries, but Figure 1-(b), showing the percentage of these countries where net migration is higher than natural increase, clearly demonstrates that this trend is upward.

In public and political discussion, international migration is often associated with other demographic variables. Some people, for example, think that migration may be a natural or necessary response to population aging. Others that migration must be reduced and fertility increased. Meanwhile, in academic economics publications, demographic variables are usually analyzed separately or, as in the textbook growth model, combined in a single variable: population growth rate. This article proposes a unified analysis of the empirical and theoretical effects of natural increase and net migration rates on incomes and inequalities. This is relevant as any increase in inequality may reinforce the opposition to globalization in general and international migration in particular. We focus here on the inequality between capital and labor, where the relationship with demography has been less well researched¹, despite the quality of the available data. Moreover, variation in this inequality probably correlates with disparity between the income of the richest and poorest, because capital is more concentrated in corporate profits, which are less equally distributed than wages (IMF, 2017).

Natural increase and net migration have one point in common: any increase causes an increase in population, which may lead to a dilution of capital if returns to scale in production are constant. However, the two components of population growth differ widely in their effects on population age structure: a rise in natural increase (*via* more births or

¹The literature has focused on the relationship between population structure and the disparity in income distribution among the population (Lam, 1986, 1987) and on the effect of immigration on wage inequalities among native workers (Borjas *et al.*, 1997; Lerman, 1999; Card, 2009; Dustmann *et al.*, 2013; Edo and Toubal, 2015).

fewer deaths) increases the number of dependants, whereas a rise in migration increases the number of working-age people. An increase in the share of the latter in population has a favorable effect on economic growth (Aksoy *et al.*, 2019) and thus international migration is likely to produce a demographic dividend (d’Albis *et al.*, 2019). However, productivity is affected by this increase in the number of workers, which may bear down on wages. The ultimate effect on the labor income as a share of total income is thus *a priori* an ambivalent one and depends on the extent of the responses of labor income and total income to demographic shocks.

We first empirically examine the effects of natural increase and net migration rates on per capita total income and the labor income share. We estimate a VAR model for a panel of 18 OECD countries from 1985 to 2015. This methodology controls for endogeneity between demographic and economic variables and has been used to examine the economic effects of international migration (Gross, 2002; Damette and Fromentin, 2013, d’Albis *et al.*, 2018, 2019) and the effects of the economy on birth and death rates (Eckstein *et al.*, 1985; Nicolini, 2007; Fernihough, 2013). We find that the labor income share falls some years after a natural increase shock but rises after a migration shock. This suggests that in addition to the factors usually adduced in the literature,² demographic ones play a role in the observed variation in the labor income share. But their effect counters the economic factors because the rise in the migration and fall in natural increase have had a mainly stabilizing effect on the labor income share.

These initial empirical findings are then interpreted using a simple, while original, model for analytically evaluating the macroeconomic effects of exogenous modifications in natural increase and net migration rates. This deterministic model is able to characterize the global dynamics brought about by demographic changes. Not least, it identifies two key parameters: the elasticity of substitution between capital and labor, and the employment response to a rise in migration. Our empirical findings can then be streamlined if elasticity of substitution is less than 1 and the employment rate response to a higher net

²The international division of labor (Elsby *et al.*, 2013), changes in the relative price of investment goods (Karabarbounis and Neiman, 2014), technology (Autor *et al.*, 2017; Aghion *et al.*, 2019) and variations in competition (Gutiérrez and Philippon, 2019; Philippon, 2019).

migration rate is sufficiently strong. We discuss these two conditions in detail and show that they are likely to be verified empirically. In particular, our findings suggest that the employment response to a rise in net migration is greater than 1. This “multiplier effect” is due to the spillovers from international migration on the labor market (see, e.g. Peri, 2016).

Our theoretical model is also used to analyze the dynamic effects of a permanent modification in the demographic parameters. We show, in particular, that natural increase and migration have the same effect on long-term labor income share, which (where the above conditions hold) falls when those variables rise. Thus, natural increase always negatively impacts the labor income share, while migration has a positive effect if the shock is temporary but a negative one if it is permanent. Consequently, international migration has an ambivalent effect on the inequality between capital and labor. This is because, with a temporary shock, productivity is barely affected, since the positive effects of migration on capital accumulation and employment make up for each other. However, if the net migration rate rise is permanent, productivity falls in the long term because of constant returns to scale.

The article is structured as follows. Section 2 provides the empirical facts. Section 3 introduces an original model that explains the mechanisms underlying our empirical results and evaluate the long run consequences of demographic shocks. Section 4 discuss the theoretical and empirical results. Section 5 concludes.

2 Empirical facts

2.1 Data

Our sample includes yearly observations from 1985 to 2015 for 18 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States. Data are obtained from Eurostat (2018) and OECD (2016, 2017, 2018) databases.

Main variables

We consider three main economic variables: total income, labor income and capital income. Total income is evaluated with the GDP, which is the source of the incomes generated within the country, used to remunerate the production factors. Labor income is evaluated with the compensation of employees, which consists of wages and employer's social contributions.³ Capital income is evaluated with the gross operating surplus and gross mixed income. This measure represents includes paid or received interest, rents or charges on financial or tangible non-produced assets. All economic variables are in real terms⁴ and are expressed in per capita terms using the annual average population.

We compute the labor income share as labor income divided by the sum of labor and capital incomes. We highlight that the latter sum corresponds to GDP minus taxes less subsidies on production and imports. Since taxes and subsidies could be used for a variety of purposes, the literature usually does not consider them when exploring the relative income shares in the domestic economy that are relevant for inequality (see Laurence, 2015, for more details).

We also consider the two components of population growth: natural increase and net migration. The natural increase is given by the difference between the number of live births and the number of deaths occurring in a year. Net migration is given by the difference between the population growth - the difference between the size of the population at the end and the beginning of a year- and the natural increase. Net migration then accounts for the difference between immigration into and emigration from the country during the year.⁵ Note that net migration data are the only annual data related to migration flows, that are available annually since 1985 for the 18 OECD countries we consider. Thus, we use it to measure the net flow of migrants, as in d'Albis *et al.* (2018, 2019). All the demographic variables are expressed in per thousand inhabitants by using the

³It was not possible to disentangle employers' social contributions from wages, given the limited availability of data in OECD (2018). Employers' social contributions are not available before 1990 for Australia, Germany or Sweden, and before 1995 for Austria, Belgium, Ireland, Italy, Japan, the Netherlands, Portugal, Spain or the United Kingdom.

⁴Economic variables in current prices are deflated using the GDP deflator.

⁵Note that population data from Eurostat (2018) make no distinction between nationals and foreigners before 2008.

population on 1 January.

Descriptive statistics

Table 1 provides the mean values of our main variables over the period 1985-2015.⁶

[Table 1]

Total income per capita ranges from \$19,529 for Portugal to \$77,152 for Norway. The sample averages of labor income per capita and capital income per capita range respectively from \$9,177 and \$8,019 in Portugal to \$34,928 and \$33,937 in Norway. Over the period 1985-2015, Denmark recorded the largest labor income share on average, followed by the United States and France (60%, 59.1% and 59%, respectively). The lowest labor income share was recorded in Italy and in Ireland (45% and 47%, respectively). Japan has the lowest net migration rate (-0.11‰), its population growth is driven by the rate of natural increase (1.37‰). Portugal has a low net migration rate (0.47‰) and a low rate of natural increase (0.56‰). The highest net migration rates are recorded in Australia, Canada and Spain (6.70‰, 5.95‰ and 4.59‰, respectively). Australia and Canada have also a high rate of natural increase (7.17‰ and 4.97‰, respectively). Over the period 1985-2015, Germany and Italy have witnessed an average natural decrease (-1.41‰ and -0.32‰, respectively), that was more than offset by the net migration rate (4.38‰ and 2.57‰, respectively).

[Figure 2]

Figure 2 displays the evolution of natural increase and net migration over time, for each country under consideration. Figure 2 shows important cross-country differences for the two sources of population growth and considerable variations over time.

⁶For the presentation of descriptive statistics, economic variables are expressed at constant PPPs, constant 2010 USD.

2.2 Empirical strategy

The model

Our empirical strategy relies on a structural VAR model that has been used to evaluate the economic responses of birth and death rates in e.g. Eckstein *et al.* (1985), Nicolini (2007), Fernihough (2013). For most countries, the accurate economic and demographic data are available annually over a limited time period. Thus, following d’Albis *et al.* (2018, 2019), we consider a panel framework that allows conducting an accurate analysis on annual data over the period 1985-2015. We consider the following panel VAR specification:

$$X_{it} = \sum_{s=1}^p \Gamma_s X_{it-s} + v_i + d_i t + f_t + \varepsilon_{it} \quad i = 1, \dots, N \text{ and } t = 1, \dots, T \quad (1)$$

where $X_{it} = (x_{it}^1, \dots, x_{it}^m)'$ is a m -dimensional vector of endogenous variables, the Γ_s are fixed $(m \times m)$ coefficient matrices, $v_i = (v_i^1, \dots, v_i^m)'$ is a vector of country fixed-effects, $d_i t = (d_i^1, \dots, d_i^m)'t$ represent country-specific time (linear) trends, $f_t = (f_t^1, \dots, f_t^m)'$ is a common time (year)-specific effect, and $\varepsilon_{it} = (\varepsilon_{it}^1, \dots, \varepsilon_{it}^m)'$ is a m -dimensional vector of errors that are assumed to satisfy $E(\varepsilon_{it}) = 0$ and $E(\varepsilon_{it}\varepsilon_{is}') = \Sigma \cdot \mathbb{1}\{t = s\}$ for all i and t .

We address the possible heterogeneity in our panel data by using a rather homogeneous sample of OECD countries, and by introducing country-fixed effects v_i and country-specific time trends $d_i t$. Moreover, to account for cross-country contemporaneous interdependence, we include year-specific effects f_t , as in d’Albis *et al.* (2018, 2019).

Given the sizes of the cross-sectional dimension N and the time dimension T of our panel data ($N = 18$ and $T = 31$), to remove the short- T dynamic panel data bias or the so-called Nickell (1981) bias, we employ the bias-corrected fixed-effects estimator developed by Hahn and Kuersteiner (2002).⁷ See, for example, Juessen and Linnemann

⁷This estimator is appropriate when T and N are of comparable sizes i.e. when $0 < \lim N/T < \infty$ (as here). Moreover, it does not require a preliminary consistent estimator and may then be understood as an implementable version of Kiviet’s (1995) bias-corrected fixed-effects estimator. Especially, it can be applied to VAR models with higher order $p > 1$ by rewriting the VAR(p) process in a VAR(1) form through imposing blockwise zero and identity restrictions (Hahn and Kuersteiner, 2002; Lütkepohl, 2005, p.15). Furthermore, Monte Carlo simulations implemented by Hahn and Kuersteiner (2002) showed that this bias-corrected estimator is often more efficient than GMM estimator in terms of mean squared error loss for the sample sizes.

(2012) and d’Albis *et al.* (2018, 2019) who applied this bias-correction in panel VAR frameworks.

Based on AIC (Akaike information criterion) and BIC (Bayesian information criterion), we set the VAR order p to two so as to remove any serial correlation in the errors. Using lag a length greater than two does not change our finding. Preliminary diagnostics (panel unit root tests) reject the null hypothesis of unit root for the detrended variables (with country-specific linear trend). Our VAR model then considers variables in log levels while controlling for country heterogeneity (by introducing country-specific effects and country-specific time trends) and cross-country interdependence (by introducing year-specific effects).

Baseline specification

To conduct a preliminary comparison of the economic impacts of natural increase and net migration rates, we consider the following baseline specification:

$$X_{it}^1 = [\log(1 + m_{it}), \log(1 + n_{it}), \log(y_{it})]',$$

where m_{it} is the net migration as a share of the population on 1 January, n_{it} is the natural increase as a share of the population on 1 January and y_{it} is the total income per capita. Because net migration and natural increase rates can be negative, we add one to express these variables in logarithm.

After estimating the VAR coefficients, we establish causal relationship between variables by identifying structural shocks based on Cholesky decomposition. This decomposition relies on the assumption that variables ordered first in the VAR can affect the other variables contemporaneously, whereas variables ordered later can affect those ordered first only with lags. Net migration is ordered first since it can contemporaneously affect natural increase (through births) and the economic performances of the host country, and it is assumed to respond to them only with a lag. The natural increase is ordered second and total income is ordered last, which means that natural increase may contemporaneously impact the economy and can respond to it with lag (as in Nicolini, 2007, for instance).⁸

⁸Nicolini (2007) made the assumption that economic variables do not affect demographic ones within

Impact on the labor income share

To empirically investigate how natural increase and migration shocks influence the labor income share, we take advantage of the income approach of GDP which represents all income generated by production activity and used to remunerate the production factors (see UN *et al.*, 2009 for more details). Thus, we consider the following system including labor and capital incomes:

$$X_{it}^2 = [\log(1 + m_{it}), \log(1 + n_{it}), \log(w_{it}), \log(r_{it})]',$$

where w_{it} is the labor income per capita, and r_{it} is the capital income per capita. To identify structural shocks in this system, net migration and natural increase are ordered first and second, as in the baseline specification. We put labor income before capital income. It is worth to note that, since demographic variables are ordered first in the identification scheme, the order between the other variables (labor income and capital income) does not matter for the analysis of the responses to a demographic shock.

The response of the labor income share, defined as $w_t/(w_t + r_t)$, is computed as:

$$\frac{w}{w+r} \left(\widehat{\log(w_t)} - \widehat{\log(w_t + r_t)} \right),$$

where $\widehat{\log(w_t)}$ and $\widehat{\log(w_t + r_t)}$ are the impulse responses of the logarithm of labor income per capita and of the calculated response of the logarithm of the sum of labor and capital incomes per capita.⁹ The ratio $w/(w + r)$ is approximated by the overall sample mean and is here equal to 0.547 (see Table 1). Note that the response of the labor income share is expressed in percentage-point change.

2.3 Empirical results

We analyze the macroeconomic impacts of demographic shocks. We first present our main results and then extend the analysis to provide some robustness checks and additional results. The size of each demographic shock is set to one person per thousand inhabitants.

the same year and argued that the endogenous responses of fertility are usually delayed (waiting time for conception and pregnancy).

⁹ $\widehat{\log(w_t + r_t)} = \frac{w}{w+r} \widehat{\log(w_t)} + \frac{r}{w+r} \widehat{\log(r_t)}$ where $\widehat{\log(r_t)}$ is the impulse responses of the logarithm of capital income per capita and the ratio $r/(w + r)$ is approximated by the overall sample mean.

2.3.1 Main results

First, we consider the dynamic consequences of natural increase and migration shocks. The responses are shown in Figure 3.

[Figure 3]

We first notice that natural increase monotonically responds to its own shock; the increase remains significant for approximately eight years. Second, the migration shock has a positive significant effect on the natural population change, from the year of the shock until at least ten years after the shock.

Concerning the economic effects, we observe that, following the natural increase shock, total income per capita does not respond significantly upon impact, and decreases significantly from one year to at least ten years after the shock, by 2.02 percent after one year and by 2.69 percent at the peak (after five years). On the contrary, as a response to a migration shock, total income per capita increases significantly by 0.25 percent on impact and by 0.31 percent at the peak (after one year). The increase remains significant for three years after the shock. It is interesting to note that the two sources of population growth have opposite short-run effects on total income per capita. Our findings are consistent with previous empirical studies on the effect of international migration (such as Boubtane *et al.*, 2016, Ortega and Peri, 2014, Clemens, 2011, Furlanetto and Robstad, 2016). With regards to the literature on the effect of demography on economic performance (see Bloom *et al.* (2001) for a survey) more broadly, our results are also consistent. For instance, Bloom and Williamson (1998) discussed the influence of population growth on economic growth and showed that an increase in the share of the working-age population has a positive effect on GDP per capita in a sample including the OECD countries. This finding was confirmed with a VAR estimation by a recent article by Aksoy *et al.* (2019). As international migration raises the share of the workforce in OECD countries (because migrants are young adults), it can induce a demographic dividend of economic growth as we showed in d’Albis *et al.* (2019).

The response of total income to demographic shocks can be decomposed into response of labor income and capital income response (see Figure 3). Indeed, a natural increase

shock leads to a significant decrease in labor income per capita from one year after the shock and until at least ten years after the shock. Moreover, in response to a natural increase shock, capital income per capita responds negatively and significantly between three and six years after the shock. Finally, in response to a natural increase shock, labor income share decreases significantly from seven years after the shock. On the contrary, migration shock leads to an immediately significant increase in both labor and capital incomes. The response of labor income become significantly negative after nine years the shock, while the response of capital income becomes significantly negative six years after the shock. Finally, in response to a migration shock, labor income share rises significantly from the third to the eighth year after the shock. To the best of our knowledge, there are no studies to date considering the effects of the components of population growth on labor income share¹⁰, but we may rely on two interesting literatures. On the one hand, there is a large literature analyzing the explanatory factors of the observed evolution of the labor income share (see Alvarez-Cuadrado *et al.*, 2018 and Cetto *et al.*, 2019 for a survey of the literature). Our results suggest that demography influences the labor income share in addition to the factors discussed in this literature, namely the international division of labor (Elsby *et al.*, 2013), the changes in relative price of investment goods (Karabarbounis and Neiman, 2014; Glover and Short, 2019), the technology (Autor *et al.*, 2017; Aghion *et al.*, 2019) and the variations in competition (Gutiérrez and Philippon, 2019; Philippon, 2019). On the other hand, there was an important literature in the 1980's on the effect of demography on income distribution across population that suggested that most population variables have increased inequality (see Pestieau, 1989, for a review that discusses the conceptual and methodological issues). However, there was a controversy on inequality measures¹¹ and whether demographic factors are exogenous or influenced by economic factors, which challenge the conclusions of this literature (Lam, 1987). We deal with two of these issues by considering labor

¹⁰It should be noted that Glover and Short (2018) estimate the effect of the age-distribution of earnings on labor income share. However, they propose a link between demographics and the strength of competition. Indeed, their microfoundation for the effect of interaction between age and labors share rests on monopsony power in bilaterally matched labor markets.

¹¹For instance, Lam (1986) found that most standard inequality measures yield conflicting signals in the presence of differential fertility.

income share rather than income classes indicators and by estimating a VAR that take into account the endogeneity of demographic variables. More recently, some studies have considered the effect of immigration on wage inequality. For example, Borjas *et al.* (1997) concluded that immigration in the US accounted for at most a small share of the increase in overall wage inequality. Nevertheless, Lerman (1999) showed that the estimated rise in wage inequality disappears when the evolution of the wages of recent immigrants is taken into account, and Card (2009) showed that immigration had a very small impact on wage inequality among native in the US. Dustmann *et al.* (2013) analyzed the effect immigration has on the distribution of native wages in the UK and find that immigration depresses native wages below the 20th percentile of the wage distribution but leads to slight wage increases in the upper part of the wage distribution. A recent article of Edo and Toubal (2015) find that immigration in France has decreased wage inequality between highly and lowly educated native workers.

2.3.2 Including unemployment rate

We first provide some robustness checks of our results by including unemployment. The inclusion of the unemployment rate in the models does not alter our results. From Figure 4, we see that effects of demographic shocks on total income per capita are roughly unchanged compared to the results of our baseline model.¹² Interestingly, we obtained that a migration shock significantly reduces the unemployment rate, while the natural increase shock has no significant impact on unemployment rate. This confirms the previous findings for migration effect obtained by Gross (2002), Damette and Fromentin (2013), d’Albis *et al.* (2018) and Esposito *et al.* (2019).

[Figure 4]

2.3.3 Alternative demographic variables

We complement our first analyzes by studying the dynamic responses with alternative demographic variables. We first consider the rate of population growth rather than the

¹²Our estimation of the response of the labor income share is also robust.

rate of natural increase and net migration rate in our models. The responses are shown in Figure 5.

[Figure 5]

We notice that population growth monotonically responds to its own shock; the increase remains significant for six years. Then, following the population growth shock, we observe that total income per capita increases significantly during three years after the shock. Moreover, Figure 5 shows that the shock induces a significant increase in both labor and capital incomes. Labor income rises significantly up to five years after the shock, whereas the positive response of capital income is significant up to two years following the shock. Interestingly, the population growth shock has no immediate impact on the labor income share during the first two year after the shock. Labor income share rise significantly from three years to nine years. It is interesting to note that the impulse responses to population growth shock are similar to the impulse responses to migration shock reported Figure 3. In the OECD countries over the 1985-2015 period, the population growth is driven by the international migration, which yields a demographic dividend on economic growth.

Second, we investigate the results by decomposing the natural increase into births and deaths. The estimations, presented in Figure 6, evaluate the responses to three demographic shocks: an increase in births, an increase in deaths and an increase in international migration.

[Figure 6]

Figure 6-(a) shows that birth rate monotonically responds to its own shock; the increase remains significant for at least ten years after the shock. Death rate also monotonically responds to its own shock; the increase remains significant for three years after the shock. Following a shock on birth rate, total income per capita does respond significantly on impact, and its response become significantly negative from two years to at least ten years after the years. In response to a shock on death rate, total income responds positively and significantly on impact, and until nine years after the shock. These results are

consistent with Nicolini (2007).¹³ We also observe that total income per capita increases significantly following the migration shock, consistent with our previous results presented in Figure 3.

Figure 6-(b) shows that the response of total income to each demographic shock is reflected in the responses of labor and capital incomes to the same shock. Birth shock has no significant impact on labor income share, while a shock on death rate leads to a significant increase a labor income share from seven years and until at least ten years after the shock. The responses to migration shock in this extended model are in line with our previous findings.

2.3.4 Alternative inequality measures

Income inequality between capital and labor is related to the disparity of income distribution among the population. We use the World Inequality (WID) database in order to consider the effects of demographic shocks on income distribution indicators.¹⁴ Specifically, we extend our baseline model in order to evaluate the responses of various income inequality indicators to natural increase and migration shocks. The results are presented in Figure 7.

[Figure 7]

We first notice that in reaction to a shock on natural increase, none of the income distribution indicators considered respond significantly. Concerning the migration shock, the share of the 1% of people with highest income and of the share of the 40% of people with the middle income do not respond significantly to a migration shock, but the share of the 10% of people with the highest income decreases significantly during five and seven years after the shock, while the share of the 50% with the bottom income increases significantly from the fourth year and until the eighth year after the shock. These results suggest that international migration has implied a more equal distribution of income across the population of the OECD countries, which is consistent with our finding based

¹³As noted by Nicolini (2007), it is difficult to gauge the influence of fertility and a model with many more lags is needed to capture the effect of the larger cohort entering the labor force. However, we can't do that here, given the limited time-dimension of our sample ($T = 31$).

¹⁴See Alvaredo *et al.* (2018) for more details on the WID database

on the responses of labor income share to a migration shock.

3 Theory

This section proposes an extended Representative Agent deterministic model to analyze the effects of changes in demographic variables on macroeconomic ones. In particular, it distinguishes the effects of a change in the rate of natural increase from those induced by a change in the net migration rate. Both short run and long run effects are studied, the former being compared to the estimates provided in the previous section.

3.1 The basic framework

Time is discrete and is denoted by $t = 0, 1, 2, \dots$. We consider a model where the social planner maximizes a total utilitarianism criteria over an infinite horizon. The objective writes:

$$\max_{\{c_t\}} \sum_{t=0}^{+\infty} \gamma^t P_t U(c_t), \quad (2)$$

where $\gamma \in (0, 1)$ is the discount rate, P_t is the size of the population, $U(\cdot)$ is an increasing and concave utility function and c_t is the consumption per capita that satisfies the following resource constraint:

$$c_t = \frac{F(K_t, L_t) - G(K_{t+1}, K_t)}{P_t}, \quad (3)$$

where K_t is the capital stock, L_t is the labor involved in production, $F(\cdot, \cdot)$ is a constant return-to-scale production function that is increasing and concave in both arguments, and $G(\cdot, \cdot)$ is a constant return-to-scale investment function that is increasing with respect to K_{t+1} and decreasing with respect to K_t . The function $G(\cdot, \cdot)$ features a general representation of the investment process that encompasses most specific functions used in the literature, and, in particular, the linear and separable case given by $K_{t+1} - (1 - \delta) K_t$ with $\delta \in (0, 1)$ as the depreciation rate, or the non separable case given by $K_{t+1}^{\frac{1}{1-\delta}} K_t^{-\frac{\delta}{1-\delta}}$.

The main novelty concerns the demographic side of the model that distinguishes

the rate of natural increase from the net migration rate. The size of the population, evaluated at the beginning of period t , is denoted by N_t , and is below referred as the “initial population”. The net flow of migrants during period t is denoted I_t and $\lambda_t > -1$ is the net migration rate as a proportion of the initial population ($\lambda_t = I_t/N_t$). Similarly, we denote by $\beta_t > -1$ the natural increase (births minus deaths) that applies not only to initial population but also to migrants. Thus, the rate of natural increase as a proportion of the initial population is given by $\beta_t (1 + \lambda_t)$. This modelling choice is consistent with the variable ordering we made for the identification of our empirical model and, more importantly, with the estimated response of the rate of natural increase to the net migration rate. We indeed see in Figure 3 that the former immediately respond to a shock on the latter. The evolution of the initial population is thus given by:

$$N_{t+1} = (1 + \beta_t) (1 + \lambda_t) N_t, \quad (4)$$

the growth rate of N_t being indeed the sum of the rate of natural increase and the net migration rate. However, in most empirical works in macroeconomics –including the one we presented in the previous section– economic indicators are not divided by what we named the “initial population” but rather by an “average population”, which is computed by national statistics agencies and international organizations as the average of populations at 1st January of two successive years. As this statistical convention is crucially important when evaluating the effects of demographic changes (see d’Albis *et al.*, 2019), we have adopted a general definition of the population that writes:

$$P_t = \mu N_{t+1} + (1 - \mu) N_t = [\mu (1 + \beta_t) (1 + \lambda_t) + (1 - \mu)] N_t, \quad (5)$$

where $\mu \in [0, 1]$ reflects the weight given to a particular population definition. Limit cases are the following: if $\mu = 1$, it’s the end of year definition of a population that is chosen (and, then *ceteris paribus*, c_t decreases with demographic parameters β_t and λ_t) while if $\mu = 0$, it’s the beginning of year definition that is chosen (and, then, the denominator of the ratio defining c_t does not depend on β_t and λ_t). In most empirical estimates, one

considers $\mu = 1/2$ whereas in most theoretical models, one considers $\mu = 0$.

Finally, we assume that the total workforce during period t , denoted L_t , is given by:

$$L_t = N_t + \eta I_t = (1 + \eta \lambda_t) N_t, \quad (6)$$

where η evaluates the direct effect of migrants to the workforce. With parameter η , we highlight a key difference between the inflow of migrants and the inflow of children, as the latter are not part of the workforce. It's obvious that for a larger η , the effect of international migration will be more likely to be positive on the economy. Assuming that parameter η belongs to $(0, 1)$ suggests a lower participation of migrants to the labor market than natives (that can be e.g. explained by the fact that they arrive during the considered period), while assuming that η is larger than 1 would characterize the fact that complementarity between migrants and natives overcompensate the latter effect. If it is assumed that $\eta = 0$, then β_t and λ_t will have exactly the same effects on the endogenous variables of the model. The empirical findings presented in the previous section suggest it is not the case, which drive us to assume that $\eta > 0$.

By replacing equations (3) and (6) in the objective function (2), we observe that the Social Planner problem can be written as:

$$\max_{\{K_t\}} \sum_{t=0}^{+\infty} \gamma^t P_t U \left(\frac{F(K_t, (1 + \eta \lambda_t) N_t) - G(K_{t+1}, K_t)}{P_t} \right), \quad (7)$$

subject to equations (4) and (5) and to initial conditions $K_0 > 0$ and $N_0 > 0$. To ensure that the objective is finite, we moreover assume a upper bound to the growth rate of the initial population N_t :

$$\lim_{t \rightarrow +\infty} (1 + \beta_t) (1 + \lambda_t) < \frac{1}{\gamma}. \quad (8)$$

Let us define the capital per initial population as $k_t = K_t/N_t$, which is not studied in the empirical model¹⁵ but which is very useful to understand the dynamics of any other variable of the theoretical model. This variable has, in particular, the advantage to be (at

¹⁵The conceptual definition and the estimation of capital is one of the most difficult in National Accounts. We therefore excluded it from our estimations and rely on well-defined and measured variables.

date t) independent from β_t and λ_t . Using equation (5) and the homogeneity property of functions $F(.,.)$ and $G(.,.)$, the problem (7) can be written again as:

$$\max_{\{k_t\}} \sum_{t=0}^{+\infty} \gamma^t P_t U \left(\frac{F(k_t, (1 + \eta\lambda_t)) - G(k_{t+1} (1 + \beta_t) (1 + \lambda_t), k_t)}{[\mu (1 + \beta_t) (1 + \lambda_t) + (1 - \mu)]} \right), \quad (9)$$

subject to equations (4) and (5) and to initial conditions $k_0 > 0$ and $N_0 > 0$. The first order condition of this problem is:

$$\begin{aligned} & -U'(c_{t-1}) G'_1(k_t (1 + \beta_{t-1}) (1 + \lambda_{t-1}), k_{t-1}) \\ & + \gamma U'(c_t) [F'_1(k_t, (1 + \eta\lambda_t)) - G'_2(k_{t+1} (1 + \beta_t) (1 + \lambda_t), k_t)] = 0, \end{aligned} \quad (10)$$

where $F'_i(.,.)$ and $G'_i(.,.)$ indicate the first derivatives of functions $F(.,.)$ and $G(.,.)$ with respect to argument $i = \{1, 2\}$. The concavity of the problem is satisfied if $G''_{ii} \geq 0$ for $i = \{1, 2\}$, a condition that will be assumed in the subsequent analysis (and which is satisfied in the two specific cases we mentioned above). We note that equation (10) simplifies to a more familiar equation:

$$-U'(c_{t-1}) + \gamma U'(c_t) [F'_1(k_t, (1 + \eta\lambda_t)) + (1 - \delta)] = 0, \quad (11)$$

when $G(K_{t+1}, K_t) = K_{t+1} - (1 - \delta) K_t$.

Let us now consider the case such that demographic parameters are constant: $\beta_t = \beta$ and $\lambda_t = \lambda$. At the steady-state such that $k_{t+1} = k_t$ and $c_{t+1} = c_t$, the capital per initial population is obtained as the solution of the equation below, obtained with (10) and by using the homogeneity property of function $G(.,.)$:

$$-G'_1((1 + \beta) (1 + \lambda), 1) + \gamma F'_1(k, (1 + \eta\lambda)) - \gamma G'_2((1 + \beta) (1 + \lambda), 1) = 0 \quad (12)$$

Provided that $\lim_{x \rightarrow 0} F'_1(x, .) = +\infty$, $\lim_{x \rightarrow +\infty} F'_1(x, .) = 0$ and $G'_1(x, 1) + \gamma G'_2(x, 1) > 0$ for all $x > 0$, there exists a unique positive solution to (12) that is denoted k^* . Using

(3), the steady-state consumption per capita is then given by

$$c^* = \frac{F(k^*, (1 + \eta\lambda)) - G(k^*(1 + \beta)(1 + \lambda), k^*)}{[\mu(1 + \beta)(1 + \lambda) + (1 - \mu)]}, \quad (13)$$

which is positive provided that output is larger than investment.¹⁶

3.2 Short run effects of temporary changes in demographic parameters

Let us start with simple computations that aim at presenting the mechanics that may rationalize the empirical results presented in the previous section.

We consider an economy at steady-state (computed for $\lambda_t = \lambda$ and $\beta_t = \beta$) that faces a “one period shock” at date $t = 0$ that satisfies: $\beta_0 > \beta$ and $\beta_t = \beta$ for all $t = 1, 2, 3, \dots$, or $\lambda_0 > \lambda$ and $\lambda_t = \lambda$ for all $t = 1, 2, 3, \dots$. We are interested in the effects on total income per capita, denoted y_t , and on the labor income share, denoted α_t , which can be defined as:

$$y_t = \frac{F(K_t, L_t)}{P_t} = \frac{F(k_t, (1 + \eta\lambda_t))}{\mu(1 + \beta_t)(1 + \lambda_t) + (1 - \mu)}, \quad (14)$$

and

$$\alpha_t = \frac{L_t F'_2(K_t, L_t)}{F(K_t, L_t)} = \frac{F'_2\left(\frac{k_t}{(1 + \eta\lambda_t)}, 1\right)}{F\left(\frac{k_t}{(1 + \eta\lambda_t)}, 1\right)}. \quad (15)$$

Consider first the effects at date $t = 0$ of a shock on the rate of natural increase. By definition, k_0 is unchanged and is equal to its steady-state value k^* . We can see from definitions (14) and (15) that following our one period shock on β , the total income per capita decreases ($y_0 < y^*$) whereas the labor income share remains constant ($\alpha_0 = \alpha^*$). In the estimations presented in Figure 3, both effects are non significant. The second theoretical prediction is thus in line with the empirical finding. Concerning the first prediction, this might be due to the fact that having a $\mu = 1/2$ considerably reduces

¹⁶or technically, provided that:

$$\lim_{k \rightarrow 0} F'_1(k, (1 + \eta\lambda)) - (1 + \beta)(1 + \lambda) G'_1(k(1 + \beta)(1 + \lambda), k) - G'_2(1 + \beta)(1 + \lambda) > 0.$$

the magnitude of the shock, which turn to have an insignificant impact in the empirical analysis.

Consider now the effects at date $t = 0$ of a shock on the net migration rate. A simple derivation of the equation given in (14) reveals that the total income per capita is going to increase if the following condition is satisfied:

$$y_0 \geq y^* \Leftrightarrow \alpha^* \geq \frac{\left(\frac{1}{\eta} + \lambda\right)}{(1 + \lambda)} \frac{\mu(1 + \beta)(1 + \lambda)}{\mu(1 + \beta)(1 + \lambda) + (1 - \mu)}. \quad (16)$$

This latter condition, which is similar to the one presented in d'Albis *et al.* (2019), says that the effect of a change in migration is more likely to have a positive effect contemporaneously if the labor income share is high. We see from (16) that $y_0 > y^*$ for η sufficiently large and that $y_0 < y^*$ for $\eta = 0$, which suggests there exists a threshold $\bar{\eta}$ above which the contemporaneous impact of migration on total income per capita is positive. A quick analysis of (16) permit to go further by observing that for $\mu = 1/2$ and for small β and λ , it can be rewritten as: $y_0 \geq y^*$ if and only if $\eta\alpha_0 \geq 1/2$, and therefore that $\bar{\eta} \in (0, 1)$ for most countries. Estimations presented in Figure 3 indicate a strong and significant contemporaneous response of total income per capita to the net migration rate, which therefore suggests that $\eta > \bar{\eta}$. Concerning the effect on the labor income share, we have¹⁷:

$$\alpha_0 \geq \alpha^* \Leftrightarrow \varepsilon_{KL} \geq 1, \quad (17)$$

where $\varepsilon_{K,L}$ is the elasticity of substitution between capital and labor. Estimations presented in Figure 3 suggest that there is no effect at date $t = 0$, which might be rationalized by our model provided that ε_{KL} is close to 1.

In periods $t = 1, 2, \dots$, the capital per initial population is likely to be affected by the shocks that took place in $t = 0$. According to our empirical analysis, both y_t and α_t are lower than their steady-state values in the case of the shock on the rate of natural increase

¹⁷One indeed has

$$\frac{d \frac{F'_2(x,1)}{F(x,1)}}{dx} = \frac{F'_2(x,1)}{F(x,1)} \left[\frac{F''_{12}(x,1)}{F'_2(x,1)} - \frac{F'_1(x,1)}{F(x,1)} \right] = \frac{F'_1(x,1) F'_2(x,1)}{[F(x,1)]^2} \left[\frac{1}{\varepsilon_{K,L}} - 1 \right].$$

whereas they are both higher in the case of the shock on the net migration rate. Those two evidences can be rationalized provided that both the total income per capita and the labor income share increase with k , which imposes that the elasticity of substitution is below 1 and that the response of k to β is the opposite of that to λ . Let us summarize those three conditions as follows: (i) $k_1 < k_0$ after a shock on β ; (ii) $k_1 > k_0$ after a shock on λ ; (iii) $\varepsilon_{KL} < 1$.

In the subsequent analysis we analyze the effect of permanent shocks in our model and explain why the two first conditions can be satisfied. Concerning the third condition, we rely on the literature (see Section 4 for more details) and conclude that it is likely to be satisfied.

3.3 Dynamic analysis of permanent changes in demographic parameters

We now assume that demographic parameters are constant for all t . Below, we first establish the steady-state effects of demographic parameters and then turn to a geometrical representation of the dynamics.

Let us denote by $\varepsilon_{k^*,\beta}$ and $\varepsilon_{k^*,\lambda}$ the elasticities of the steady-state capital per capita, k^* , with respect to the factor of natural increase $(1 + \beta)$ and the net migration factor $(1 + \lambda)$, respectively. We obtain the following results:

Proposition 1. *The sign of effect of a permanent change in the rate of natural increase on steady-state capital per initial population is given by:*

$$\varepsilon_{k^*,\beta} \geq 0 \Leftrightarrow G''_{11}((1 + \beta)(1 + \lambda), 1) + \gamma G''_{12}((1 + \beta)(1 + \lambda), 1) \leq 0. \quad (18)$$

The effect of a permanent change in the net migration rate on steady-state capital per initial population is given by:

$$\varepsilon_{k^*,\lambda} = \varepsilon_{k^*,\beta} + \frac{\eta(1 + \lambda)}{(1 + \eta\lambda)}. \quad (19)$$

Proposition 1 shows that the impact of the rate of natural increase crucially depends on the parametric form of the investment function. In the standard linear and separable case, i.e. for $G(K_{t+1}, K_t) = K_{t+1} - (1 - \delta)K_t$, we deduce from (18) that $\varepsilon_{k^*, \beta} = 0$, which is the traditional textbooks result saying that population growth has no impact on steady-state capital per initial population (provided that the Social Welfare criteria is built on total utilitarianism). In particular, condition (18) reveals that the cross derivative $G''_{12}(\cdot, \cdot)$ plays a crucial role. If $G''_{12}(\cdot, \cdot) = 0$, then $\varepsilon_{k^*, \beta} \leq 0$. But if the function is non separable, and since we necessarily have $G''_{12}(\cdot, \cdot) < 0$ due to the other assumptions made on $G(\cdot, \cdot)$, the effect of the rate of natural increase on k^* cannot be signed without further assumptions. For instance, in the case $G(K_{t+1}, K_t) = K_{t+1}^{\frac{1}{1-\delta}} K_t^{-\frac{\delta}{1-\delta}}$, inequalities in (18) can be rewritten as:

$$\varepsilon_{k^*, \beta} \geq 0 \Leftrightarrow \frac{1}{\gamma} \leq (1 + \beta)(1 + \lambda), \quad (20)$$

which using condition (18) permit to conclude that $\varepsilon_{k^*, \beta} < 0$: the effect of the rate of natural increase on k^* is strictly negative.

Proposition 1 also shows that the elasticity of k^* with respect to migration is always larger than the elasticity of k^* with respect to natural increase (as long as the participation rate of migrants to the labor market is not zero). Thus, the long run impact of migration on capital per initial population is more likely to be positive. In particular, we obtain that $\varepsilon_{k^*, \lambda} > 0$ in the standard case of a linear and separable capital accumulation function. This result is obtained for any positive value of η , and therefore strongly differs from what can be obtained with a Solow-type model that would requires a $\eta > 1$ to obtain a positive impact of migration.

The theoretical analysis of k_t is very useful to understand the dynamics of the other relevant variables of the model. Let us start with labor productivity, denoted π_t , that is defined as $\pi_t = F(K_t/L_t, 1)$ or, using the definition (6), given by $\pi_t = F(k_t/(1 + \eta\lambda_t), 1)$. We immediately see that the sign of the effect of a change in the rate of natural increase on π^* is the same as the one on k^* , which was discussed in Proposition 1. Moreover, a simple derivation reveals that the sign of the effect of a change in the net migration rate on π^* is positive if and only if $\varepsilon_{k^*, \lambda} \geq \eta(1 + \lambda)/(1 + \eta\lambda)$, which, using (19), can be

rewritten as $\varepsilon_{k^*,\beta} \geq 0$. We immediately conclude that migration has no effect on π^* if $G(K_{t+1}, K_t) = K_{t+1} - (1 - \delta) K_t$ and has a negative effect if $G(K_{t+1}, K_t) = K_{t+1}^{\frac{1}{1-\delta}} K_t^{-\frac{\delta}{1-\delta}}$. We finally note that the capital to income ratio, defined as $1/F(1, (1 + \eta\lambda)/k^*)$, displays similar responses to demographic shock than productivity. Let us denote by $\varepsilon_{\pi^*,\beta}$ and $\varepsilon_{\pi^*,\lambda}$ the elasticities of π^* with respect to $(1 + \beta)$ and $(1 + \lambda)$, respectively, and summarize those latter findings in the following:

Corollary 1. *The sign of the effects of permanent changes in demographic parameters on steady-state productivity satisfies:*

$$\varepsilon_{\pi^*,\beta} \geq 0 \Leftrightarrow \varepsilon_{k^*,\beta} \geq 0, \quad (21)$$

and:

$$\varepsilon_{\pi^*,\lambda} \geq 0 \Leftrightarrow \varepsilon_{k^*,\beta} \geq 0. \quad (22)$$

The reasoning is similar for the labor income share, denoted α_t which is formally defined in (15), and which can be written at steady-state as:

$$\alpha^* = \frac{F'_2\left(\frac{k^*}{(1+\eta\lambda)}, 1\right)}{F\left(\frac{k^*}{(1+\eta\lambda)}, 1\right)}. \quad (23)$$

The effect of a demographic parameter on the labor income share is thus the same as the one it has on productivity if the elasticity of substitution between capital and labor is lower than one, while it is the opposite if the latter is larger than one. As we suggested above that the elasticity should be below but close to 1, the model predicts that the long run effects of demographic variables on the labor income share are likely to be negative, although small. Let us denote by $\varepsilon_{\sigma^*,\beta}$ and $\varepsilon_{\sigma^*,\lambda}$ the elasticities of α^* with respect to $(1 + \beta)$ and $(1 + \lambda)$, respectively, and recall that $\varepsilon_{K,L}$ stands for the elasticity of substitution between capital and labor. We summarize the effects in the following:

Corollary 2. *The sign of the effects of permanent changes in demographic parameters*

on steady-state labor income share satisfies:

$$\varepsilon_{\alpha^*,\beta} \geq 0 \Leftrightarrow (1 - \varepsilon_{K,L}) \times \varepsilon_{k^*,\beta} \geq 0, \quad (24)$$

and:

$$\varepsilon_{\alpha^*,\lambda} \geq 0 \Leftrightarrow (1 - \varepsilon_{K,L}) \times \varepsilon_{k^*,\beta} \geq 0. \quad (25)$$

The variable k that we have considered above is defined with a population evaluated at the beginning of the year and should therefore not be considered as the capital per capita, strictly speaking. If we use the general definition of the population given in (5), we obtain a capital per capita that is written as:

$$\frac{K_t}{P_t} = \frac{k_t}{[\mu(1 + \beta_t)(1 + \lambda_t) + (1 - \mu)]}. \quad (26)$$

The capital per capita is, obviously, more likely to decrease with β_t and λ_t than k_t as the demographic parameters now appear at the denominator. When population is not evaluated at the beginning of the period (i.e. for $\mu > 0$), we observe a kind of additional “capital dilution” effect of population. The larger μ , the lower the effect of demography on capital per capita. The effect is the same for the total income per capita, which is formally defined in (15) and which, at steady-state, can be written as:

$$y^* = \frac{F(k^*, (1 + \eta\lambda))}{\mu(1 + \beta)(1 + \lambda) + (1 - \mu)}. \quad (27)$$

Let us denote by ε_{y^*,k^*} , $\varepsilon_{y^*,\beta}$ and $\varepsilon_{y^*,\lambda}$ the elasticities of steady-state total income per capita with respect to k^* , $(1 + \beta)$ and $(1 + \lambda)$, respectively. We obtained the following results:

Proposition 2. *The effect of a permanent change in the rate of natural increase on steady-state total income per capita is given by:*

$$\varepsilon_{y^*,\beta} = \varepsilon_{y^*,k^*} \times \varepsilon_{k^*,\beta} - \frac{\mu(1 + \lambda)(1 + \beta)}{\mu(1 + \beta)(1 + \lambda) + (1 - \mu)}. \quad (28)$$

The effect of a permanent change in the net migration rate on steady-state total income per capita is given by:

$$\varepsilon_{y^*,\lambda} = \varepsilon_{y^*,\beta} + \frac{\eta(1+\lambda)}{(1+\eta\lambda)}. \quad (29)$$

Proposition 2 shows that $\varepsilon_{y^*,\beta} < 0$ if $\varepsilon_{k^*,\beta} \leq 0$. As explained above, the definition of population matters, and the choice of an average population as a benchmark in empirical studies reinforce the predicted negative effect of the rate of natural increase on total income per capita. As in Proposition 1, we see the effect of migration is more likely to be positive as it appears from equation (29) that $\varepsilon_{y^*,\lambda} > \varepsilon_{y^*,\beta}$. Moreover, there exists a threshold for η above which the impact of migration on steady-state total income per capita is positive. For instance, using equations (28) and (29), we can compute that if $\varepsilon_{k^*,\beta} = 0$, we have: $\varepsilon_{y^*,\lambda} \geq 0 \iff \eta \geq 1 / \left(1 + \frac{(1-\mu)}{\mu(1+\beta)}\right)$. Interestingly, the latter threshold is lower than 1 provided that the population definition is not based on a end-of-the-year convention (i.e. provided that $\mu < 1$). Also, if it is the beginning-of-the-year convention that is chosen (i.e. if $\mu = 0$), then we have $\varepsilon_{y^*,\lambda} > 0$.

At steady-state, the consumption per capita denoted c^* , is given by (13). Let us denote by $\varepsilon_{c^*,\beta}$ and $\varepsilon_{c^*,\lambda}$ the elasticities of c^* with respect to $(1+\beta)$ and $(1+\lambda)$, respectively. We obtained the following results:

Proposition 3. *The effects of a permanent change in the demographic parameters on steady-state consumption per capita satisfy:*

$$\varepsilon_{k^*,\beta} \leq 0 \Rightarrow \varepsilon_{c^*,\beta} < 0, \quad (30)$$

and

$$\varepsilon_{c^*,\lambda} > \varepsilon_{c^*,\beta}. \quad (31)$$

As shown by Proposition 3, the long run effects of demographic parameters crucially depend on their effect on k^* . Moreover, as long as migrants participate to the labor force, their impact on consumption is more favorable than the one of natural increase.

Those comparative statics at steady-state can be complemented by a phase diagram analysis in the plane (k, c) . The locus such that c is constant is given by (12), and is represented in the plane by a vertical line that defines k^* . The locus such that k is constant is given by (13), and is represented by a concave function whose intersection with the other locus defines c^* . It's easy to show that due to condition (8), k^* is lower than the level that would maximize consumption¹⁸ and that the steady-state is saddle path stable. The Figure 8 represents the phase diagram of our model.

[Figure 8]

Let us now consider the dynamics effects of an increase in the rate of natural increase, β . As stated in Proposition 1, the effect on k^* is ambiguous but let us suppose that it decreases, which would be e.g. the case if $G(K_{t+1}, K_t) = K_{t+1}^{\frac{1}{1-\delta}} K_t^{-\frac{\delta}{1-\delta}}$. In the Figure 9, the locus such that c is constant then moves to the left. Using (13), we also see that following an increase in β , the locus such that k is constant moves downward. As a consequence, the new steady-state is obtained for a lower c^* , which is consistent with inequalities (30) in Proposition 3.

[Figure 9]

Interestingly, Figure 9 represents the dynamics toward the new steady-state. We see that k_t monotonically decreases toward its new value, which provide a rationale for the decrease in both the total income per capita and the labor income share after a shock on the rate of natural increase (see Figure 3). Figure 9 also reveals that consumption per capital should also monotonously decrease toward its new steady-state but cannot conclude on the direction of the jump at the date of the shock. This is consistent with our empirical results. As a robustness check, we have indeed rerun our estimations by including the consumption per capita within the VAR. Impulse response functions reveal that following a shock, consumption generally declines although experiencing a (non significant) increase at the date of the shock.

Let us now consider the dynamics effects of an increase in the net migration rate, λ . Following Proposition 1, let us assume that η , the direct impact of migrants on labor

¹⁸Note that, as we have a general function G , the Golden Rule is not the usual one.

force, is sufficiently large and induces a positive response of k^* to an increase in λ . In the Figure 10, the locus such that c is constant then moves to the right. Concerning the locus such that k is constant, it could move upward or downward depending on conditions.¹⁹ In Figure 10, we consider the case such that it moves upward.

[Figure 10]

In this case, both k_t and c_t converge to a new steady-state that is characterized by higher values. The monotonic increase in the capital per capita is consistent with the empirical findings related to the behavior of both the total income per capita and the labor income share after a shock on the net migration rate (see Figure 3). The consumption is generally increasing but, again, the direction of its jump at the date of the shock is uncertain. This is still consistent with empirical facts as impulse response functions reveal that consumption significantly increase after one year and is not significantly modified the year of the shock. This dynamics also suggest that the locus such that k is constant is unlikely to move downward.

A Representative Agent model is thus able to reproduce the differential dynamics induced by unexpected shocks on both the rate of natural increase and the net migration rate by introducing an exogenous share of migrants that participates to the workforce at the date they enter the economy. Conditions for replicating the empirical facts are the following: the latter share should be sufficiently large and the elasticity of substitution between capital and labor should be lower but close to one. This model could be extended by, in particular, allowing for an immediate response of capital to demographic shocks in order to better fit with the empirical model that uses annual data.

4 General discussion

Our theoretical model evaluates the dynamic effects of population growth and distinguishes between those changes due to a natural increase and those due to international migration. It identifies two key parameters: the elasticity of substitution between capital and labor and the employment response to an increase in migration. If the former is lower

¹⁹Using (13), we have $dc^*/d(1+\lambda) \geq 0$ iff $\eta F'_2 \geq k^*(1+\beta)G'_1 + c^*\mu(1+\beta)$.

than 1 and the latter sufficiently great, the theoretical model reproduces the empirical findings in Section 2, which establish that in the short term per capita total income and labor income share increase with net migration and fall with natural increase. Applying the same conditions to our key parameters, the long-term effects predicted by the theoretical model are as follows: per capita total income increases with migration and falls with natural increase, whereas labor income share falls with both. Below we discuss empirically plausible values for our two key parameters. We draw on the recent literature to conclude that the elasticity of substitution between capital and labor is certainly slightly less than 1 and we propose an empirical analysis to quantify the employment response to a rise in migration.

The empirical literature that seeks to estimate the elasticity of substitution between capital and labor is extensive and appears to converge on a value less than 1. In particular León-Ledesma *et al.* (2015) find an elasticity close to 0.7 for the United States. A recent meta-analysis by Knoblich *et al.* (2019) of 77 studies published between 1961 and 2017 shows that mean elasticity is 0.54, and 0.77 when the precision of the estimates is taken into account. A notable exception in this literature is Karabarbounis and Neiman (2014) who estimate the relationship between the labor income share and the relative price of capital goods. Analyzing a large cross-section of countries, they find, *ceteris paribus*, a positive correlation between the two variables, from which one might conclude that the elasticity between capital and labor is greater than 1.²⁰ However, including the heterogeneity of the labor force and technical progress is sufficient to obtain a positive correlation between the labor income share and the relative price of capital goods where elasticity is less than 1 (Cette *et al.*, 2019). Moreover, Glover and Short (2019) estimate an elasticity near or below 1 using the same data set and theoretical framework than of Karabarbounis and Neiman (2014). They show that Karabarbounis and Neiman (2014)’s estimate might be biased upwards due to omitted variable bias because the latter use investment prices alone to proxy for the rental rate, whereas the growth model relates

²⁰A similar reasoning may be found in Piketty and Zucman (2014), who note that the return on capital has fallen less in the last 40 years than the capital-output ratio, which may also be due to an elasticity between capital and labor greater than 1.

rental rates to investment prices and consumption growth. This empirical finding gives some perspective to the theoretical finding we establish in Corollary 2: in the long term, the sign of the elasticity of the labor income share with respect to rises in natural increase or migration is therefore the same as that of the elasticity of per capita capital with respect to a rise in natural increase. Since the discussion after Proposition 1 suggests that the latter is negative, we deduce that in the long term the labor income share is likely to fall with both components of population growth.

Employment response to a rise in migration has not been given the same attention in the macro-economic literature. And yet it is a crucial parameter in our model, and explains the contrary effects of natural increase and net migration rates on per capita total income. We estimated three further VAR models including differing measurements of the rate of employment: total employment divided by the working age population, total employment divided by the population on 1 January, and total hours worked divided by the population on 1 January. The IRFs show that in all three models the employment rate responds positively to migration shock. Table 2 presents the estimated values for the increase in employment rate following a 1 percentage-point increase in the net migration rate. The values vary according to the specifications used and the number of years elapsed since the shock, but we note that they are generally high and in most cases exceed 1: this implies that one extra migrant causes an increase in employment of more than one person. This “multiplier effect” is a further evidence for the positive externalities generated by migrants on the labor market (see in particular, Peri, 2016). As can be seen from the discussion following Proposition 2, an employment rate response to migration shock greater than 1 implies that in the long term international migration is likely to have a positive effect on per capita total income. Conversely the IRFs show that a natural increase shock does not have a positive effect on employment rate. This is consistent with the assumptions of the theoretical model and consequently explains the opposite effects of natural and net migration rates on per capita total income.

[Table 2]

From the values obtained for our key parameters, the theoretical model produces an

interesting result: the labor income share rises following a temporary migration shock but falls in the long term after a permanent rise in net migration rate. This is because in the former case productivity is barely affected, because the positive effects on capital accumulation and employment balance out. However, with a permanent rise in international migration productivity falls in the long term as a result of constant returns to scale. Clearly, if this assumption is replaced by the one of increasing returns, the positive effects of the demographic variables on the economy will be enhanced.

5 Conclusion

In this article we have analyzed the effects of demographic variables on the labor income share by distinguishing between natural increase and migration. We have shown empirically that these two variables have opposite effects on the economy: natural increase reduces per capita total income and the labor income share, whereas migration increases per capita total income and the labor income share. These empirical findings are analyzed with a neoclassical growth model showing that these opposite results can be explained as long as the elasticity of substitution between capital and labor is less than 1 and the employment rate response to a rise in net migration is sufficiently high. A review of the literature confirms the first condition, and our own estimates would appear to confirm the second. With these parameters, the theoretical model predicts that a permanent rise in the rate of natural increase will have a negative long-term effect on both per capita total income and labor income share, whereas a permanent rise in the net migration rate will have a positive effect on per capita total income but a negative one on labor income share.

This research could be improved in various ways. In particular, it would be instructive to go beyond the inequality between capital and labor and examine the effects of demographic variables on the disparity of income distribution among the population. One challenge for macro-economic analysis is the availability of data. In this article we have used WID data, but the temporal dimension of the database is somewhat limited. In the-

ory, the challenges are to adapt heterogeneous agent models to the specific questions of population growth. It would be particularly useful to integrate within a macro-economic framework the complementarities between migrants and natives on the labor market.

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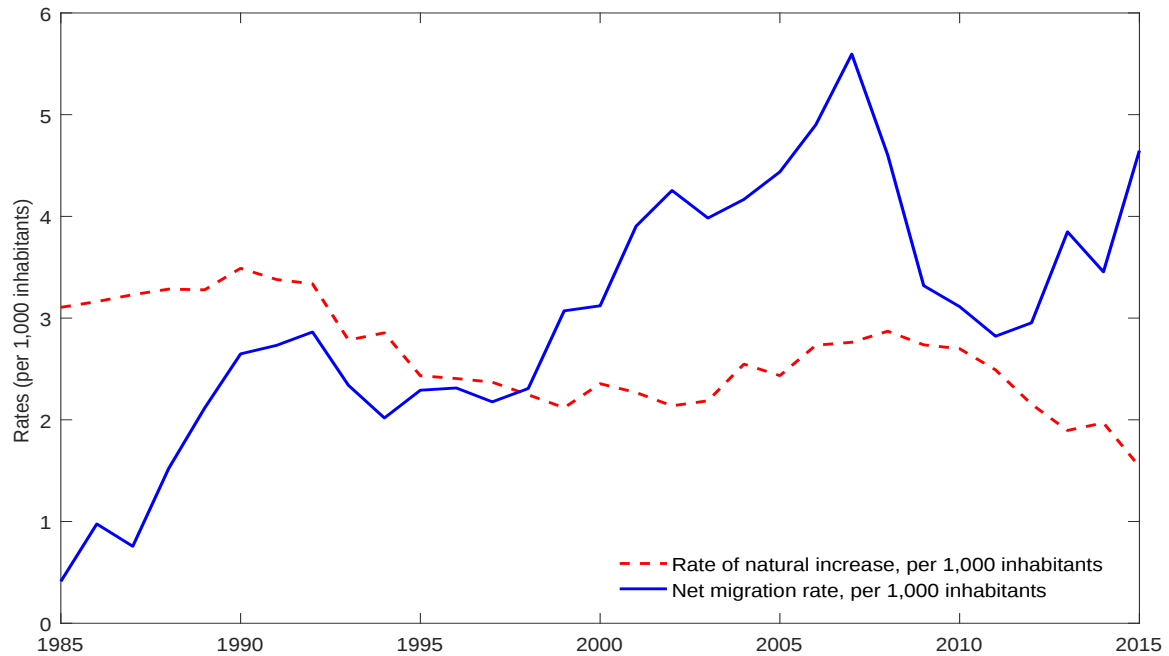
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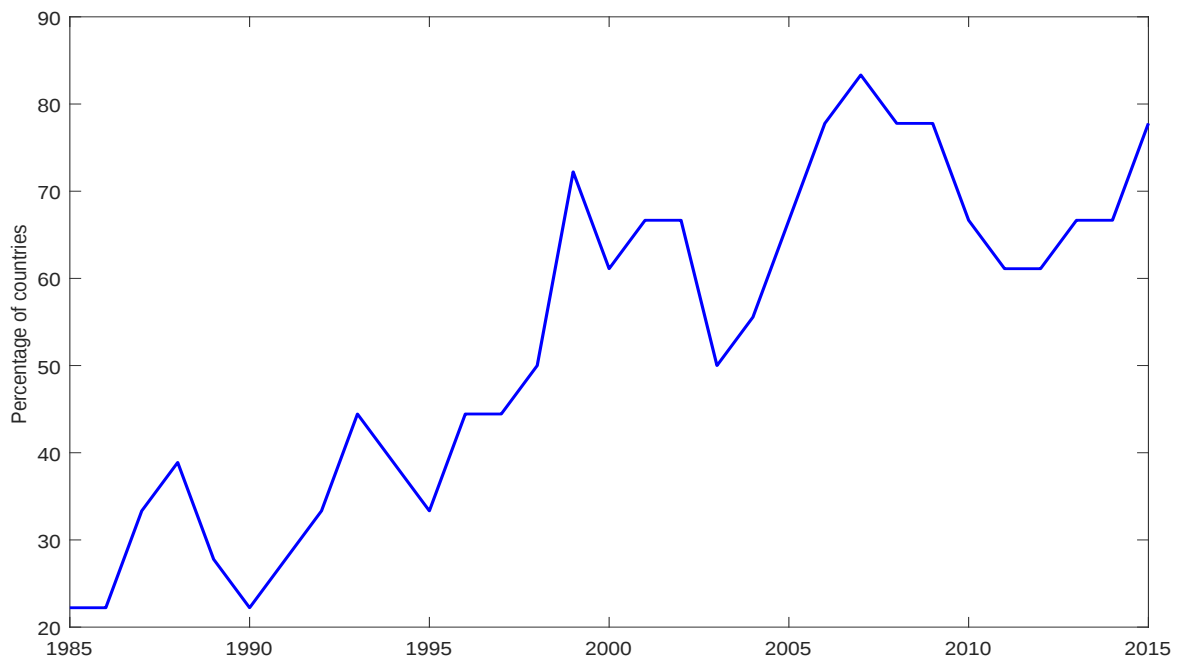
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6 Figures and tables

Figure 1: The source of population growth
 (a) *Population growth by component, OECD average*



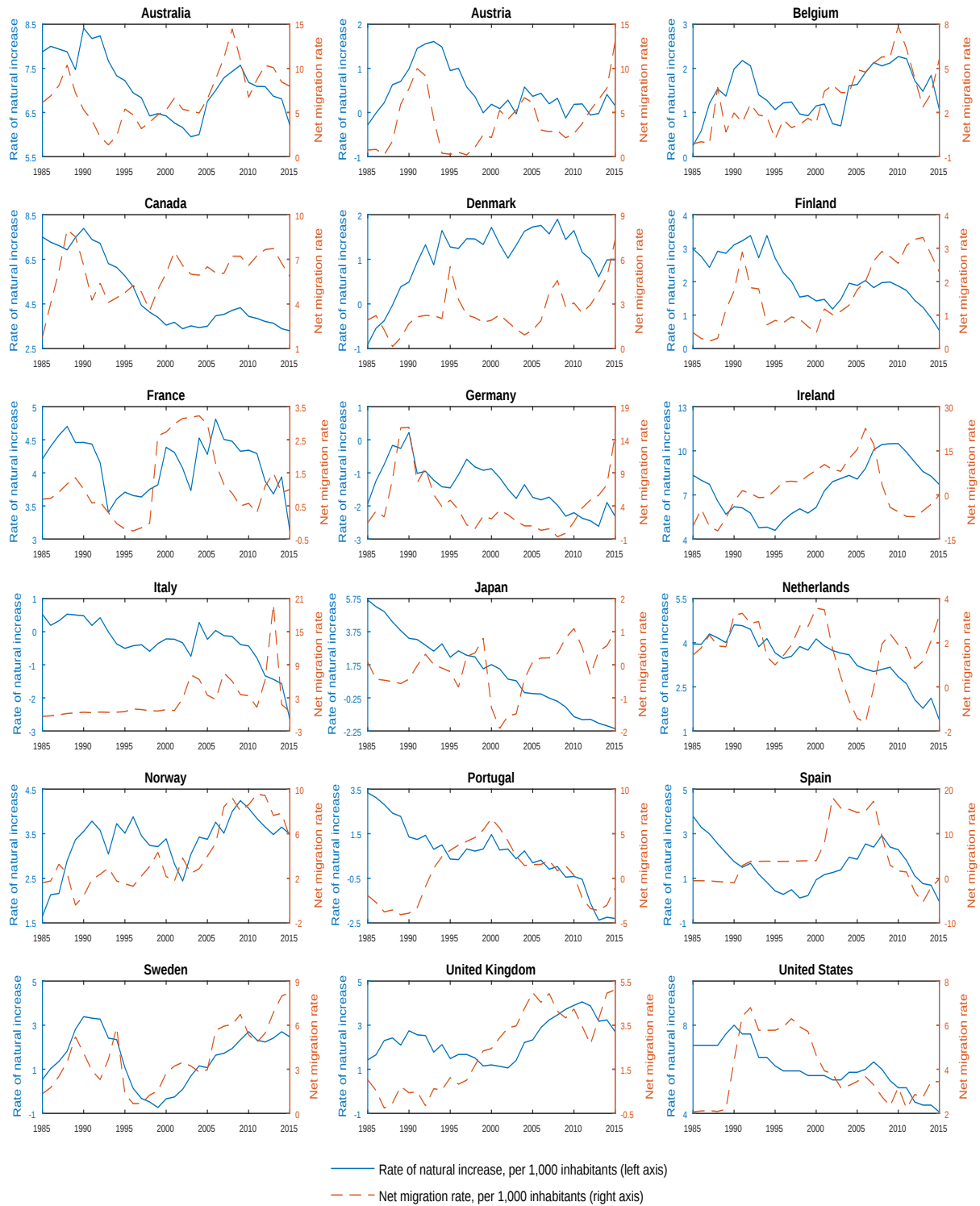
(b) *Percentage of OECD countries with net migration rate exceeding the rate of natural increase*



Source: Eurostat (2018) and OECD (2017) databases, Authors' computations.

Note: 18 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States.

Figure 2: Natural increase and net migration (per 1,000 inhabitants)



Source: Eurostat (2018) and OECD (2017) databases, Authors' computations.

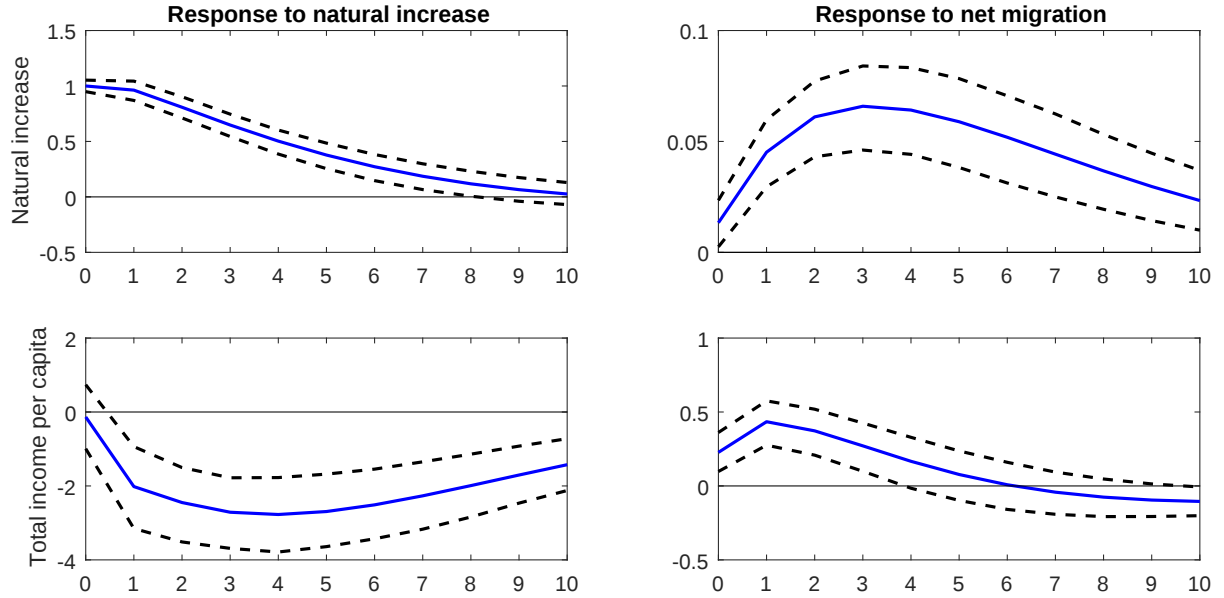
Table 1: **Summary statistics, averages per country over the sample period (1985-2015)**

Country	Pop. change (per 1,000)	Natural increase (per 1,000)	Net migration (per 1,000)	Total income per capita (PPP, 2010 USD)	Labor income per capita (PPP, 2010 USD)	Capital income per capita (PPP, 2010 USD)	Labor income share (in %)
Australia	13.8	7.14	6.71	49335	24754	20847	54.5
Austria	4.50	0.47	4.04	40479	19674	15946	55.6
Belgium	4.38	1.46	2.93	38632	19485	15211	56.2
Canada	10.9	4.98	5.96	42383	21603	15946	57.8
Denmark	3.57	1.05	2.53	52833	27226	18156	60.0
Finland	3.71	2.09	1.62	38956	19195	15192	56.2
France	5.32	4.12	1.21	36829	18877	13136	59.0
Germany	2.98	-1.41	4.39	37483	19816	15060	57.0
Ireland	8.96	7.44	1.52	38848	15776	19238	46.6
Italy	2.26	-0.32	2.57	33640	13292	16606	44.6
Japan	1.26	1.37	-0.11	41215	21453	18368	53.9
Netherlands	5.21	3.47	1.74	43184	21696	17437	55.6
Norway	7.43	3.34	4.10	77152	34928	33937	51.2
Portugal	1.04	0.57	0.48	19529	9178	8019	53.4
Spain	6.17	1.58	4.60	26567	12797	11327	52.9
Sweden	5.39	1.52	3.87	44176	20465	15807	56.3
United Kingdom	4.74	2.35	2.39	34378	16934	13586	55.3
United States	9.97	6.04	3.93	42968	23604	16472	59.1
OECD-18	5.65	2.63	3.03	41033	20042	16683	54.7

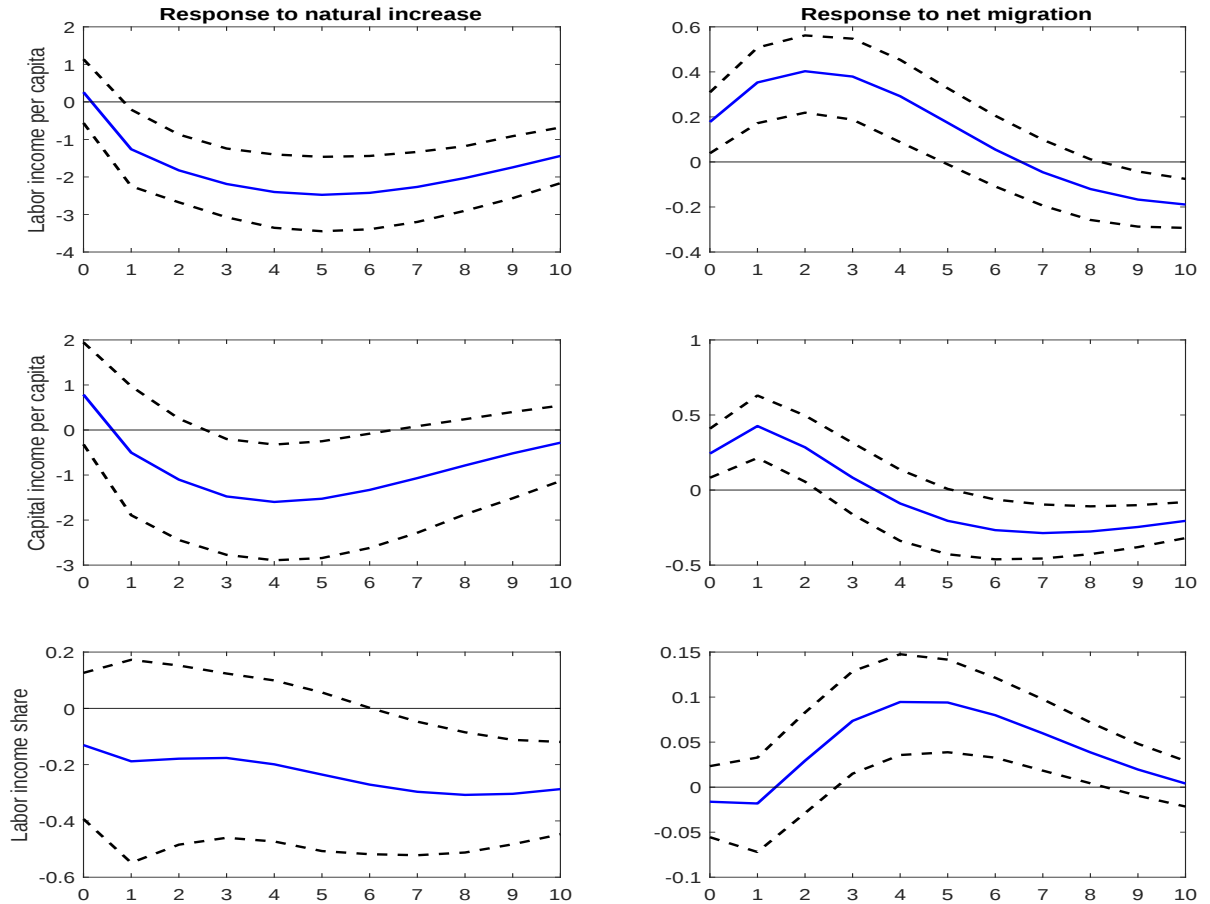
Source: Eurostat (2018) and OECD (2016, 2017, 2018) databases, authors' computations of the main variables.

Figure 3: Responses to natural increase and migration shocks

(a) *Model with total income*

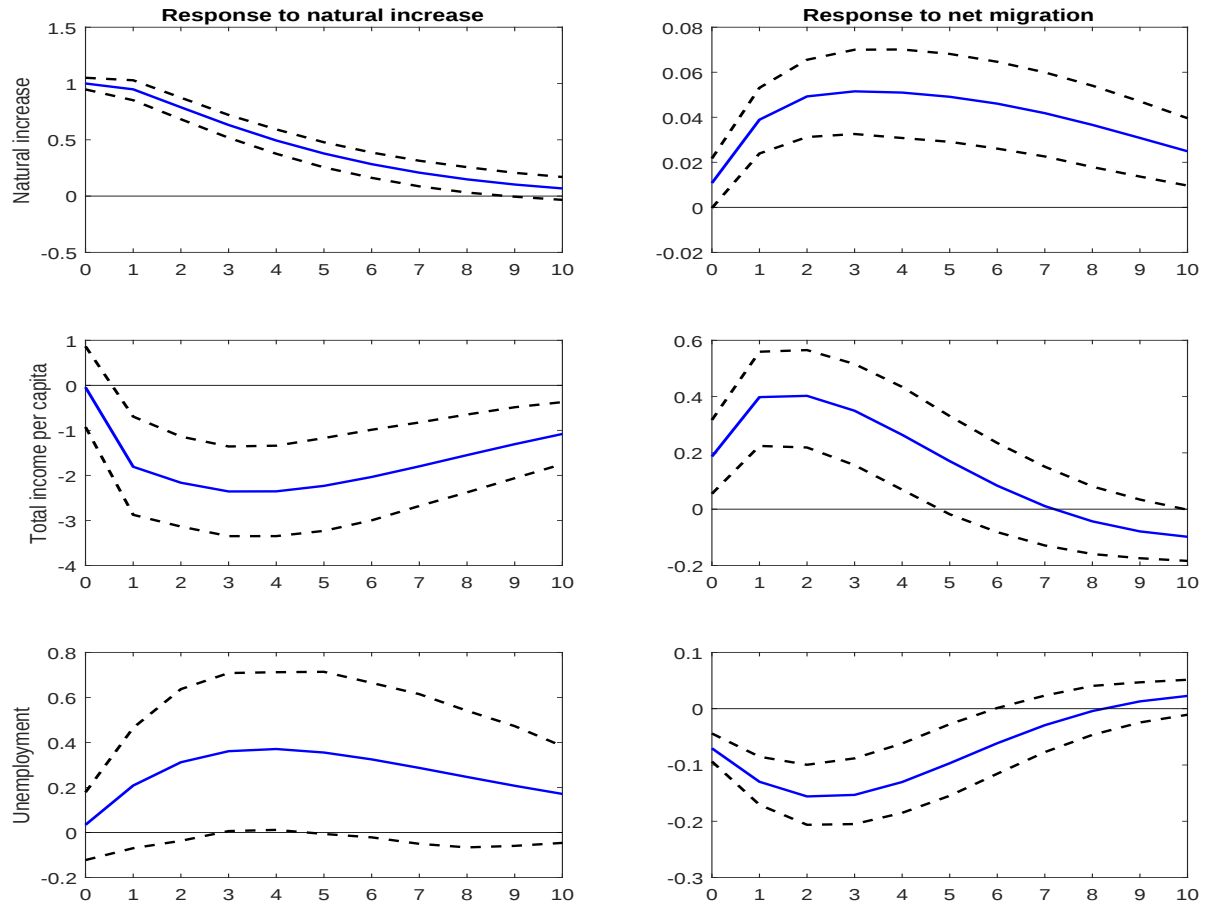


(b) *Model with labor and capital incomes*



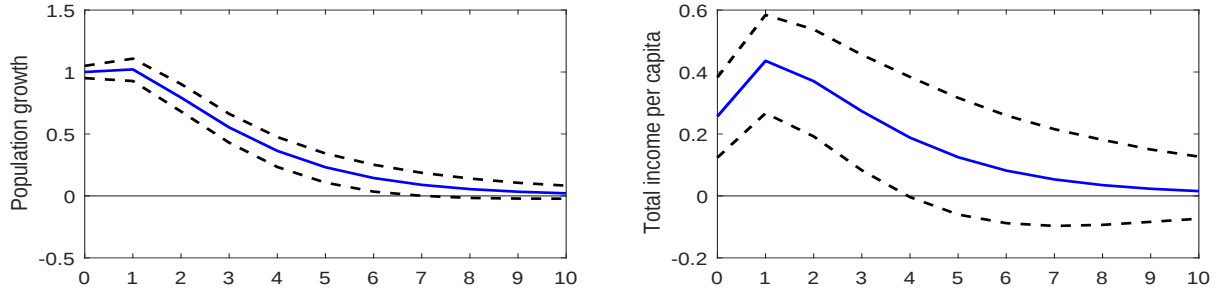
Notes: The solid line gives the estimated impulse response. Dashed lines give the 90% confidence intervals generated by Monte Carlo simulations with 5000 repetitions. The size of the shock is set to 1 per 1,000 inhabitants. The response of natural increase is in per 1,000 points change. The responses of per capita variables are in percentage change. For the labor income share, the response is in percentage points change.

Figure 4: Responses to natural increase and migration shocks in model with unemployment

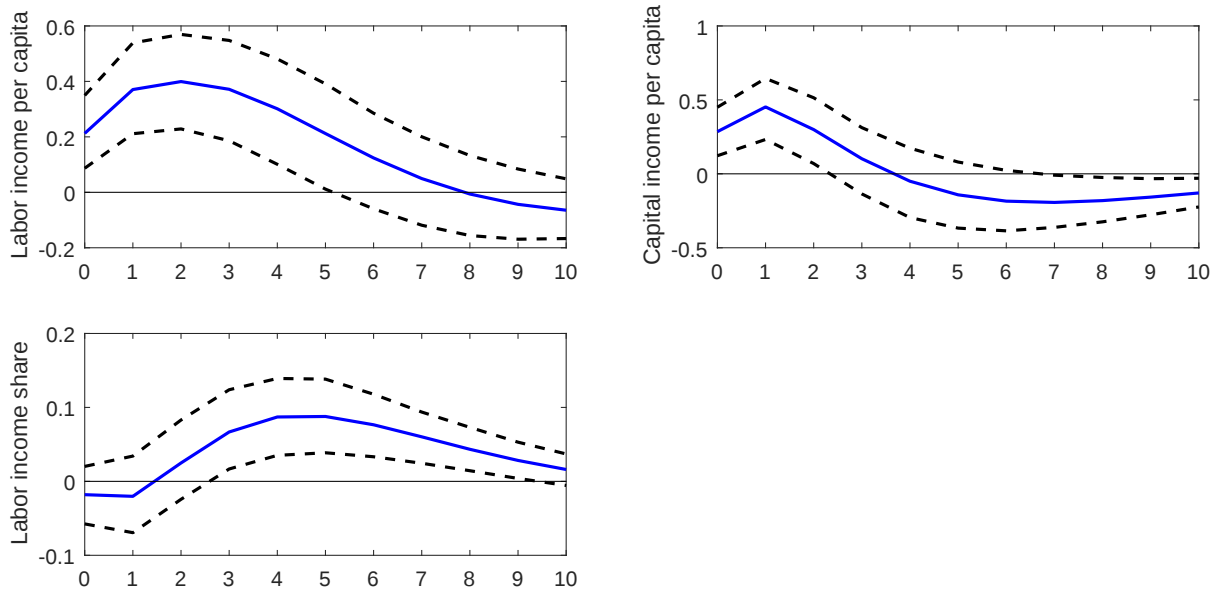


Notes: The solid line gives the estimated impulse response. Dashed lines give the 90% confidence intervals generated by Monte Carlo simulations with 5000 repetitions. The size of the shock is set to 1 per 1,000 inhabitants. The response of natural increase is in per 1,000 points change. The responses of income per capita are in percentage change. For unemployment rate, the responses are in percentage points change.

Figure 5: Responses to population growth shock
 (a) *Model with total income*



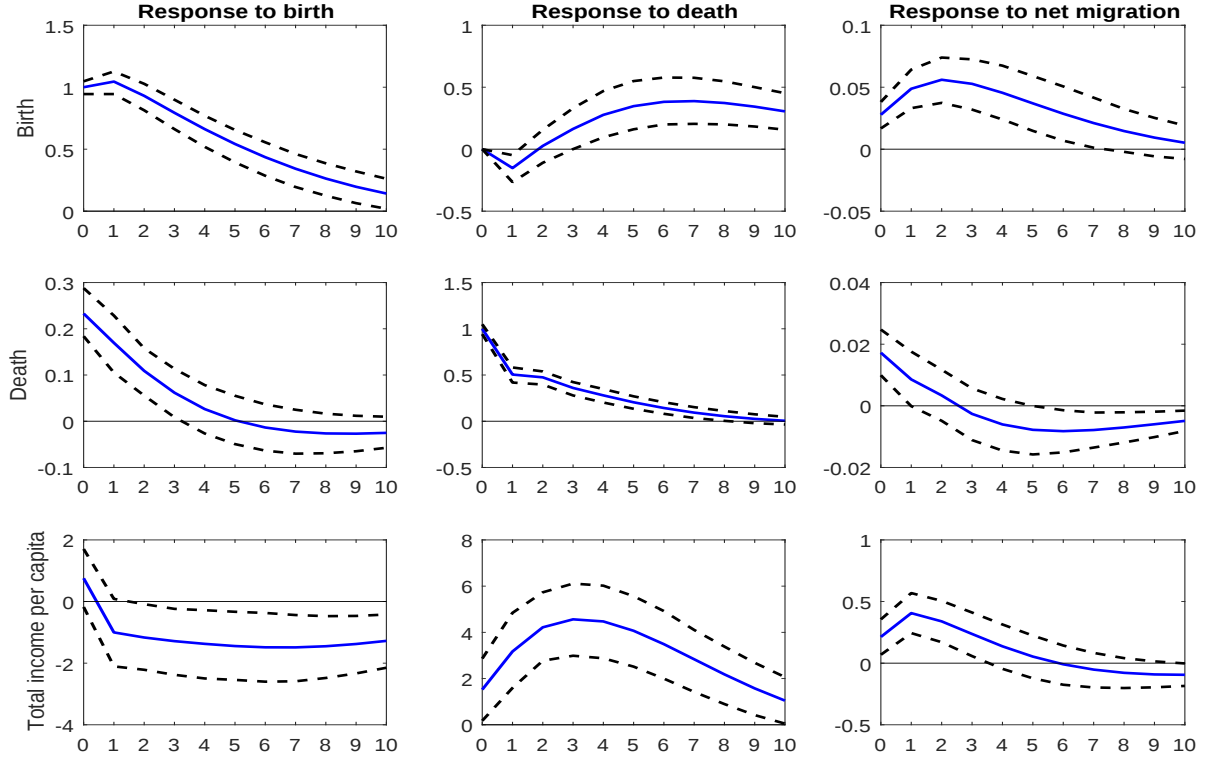
(b) *Model with labor and capital incomes*



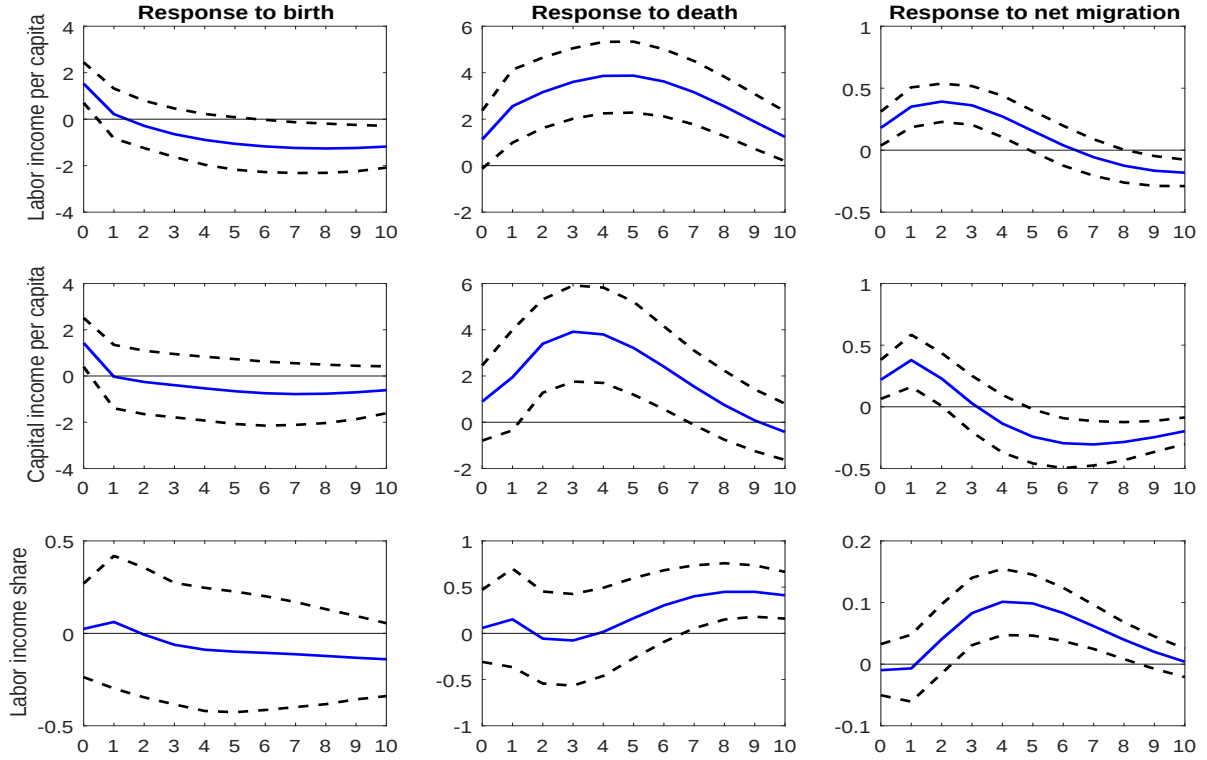
Notes: The solid line gives the estimated impulse response. Dashed lines give the 90% confidence intervals generated by Monte Carlo simulations with 5000 repetitions. The size of the shock is set to 1 per 1,000 inhabitants. The response of population growth is in per 1,000 points change. The responses of per capita variables are in percentage change. For the labor income share, the response is in percentage points change.

Figure 6: Responses to demographic shocks

(a) *Model with total income*



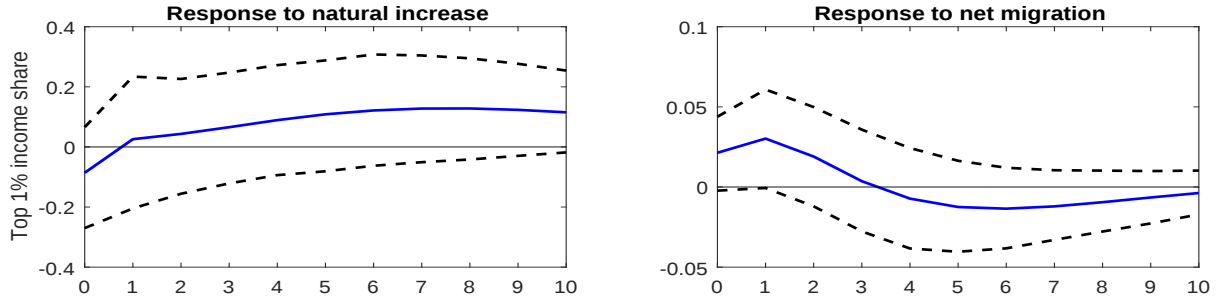
(b) *Model with labor and capital incomes*



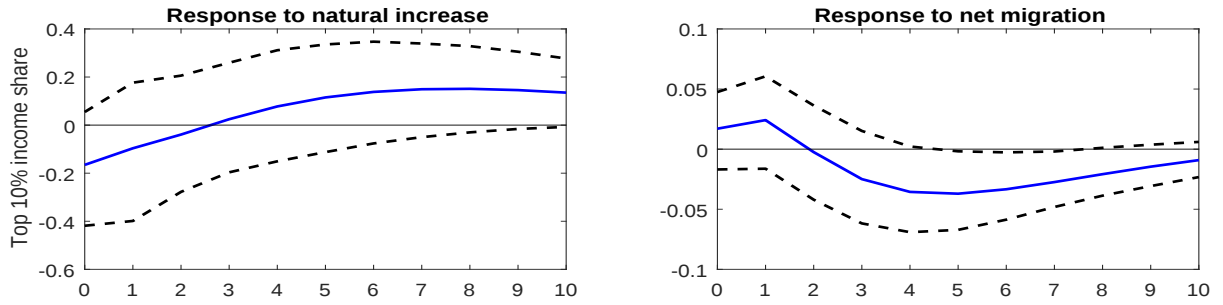
Notes: The solid line gives the estimated impulse response. Dashed lines give the 90% confidence intervals generated by Monte Carlo simulations with 5000 repetitions. The size of the shock is set to 1 per 1,000 inhabitants. The responses of demographic variables are in per 1,000 points change. The responses of per capita variables are in percentage change. For the labor income share, the response is in percentage points change.

Figure 7: Alternative inequality measures

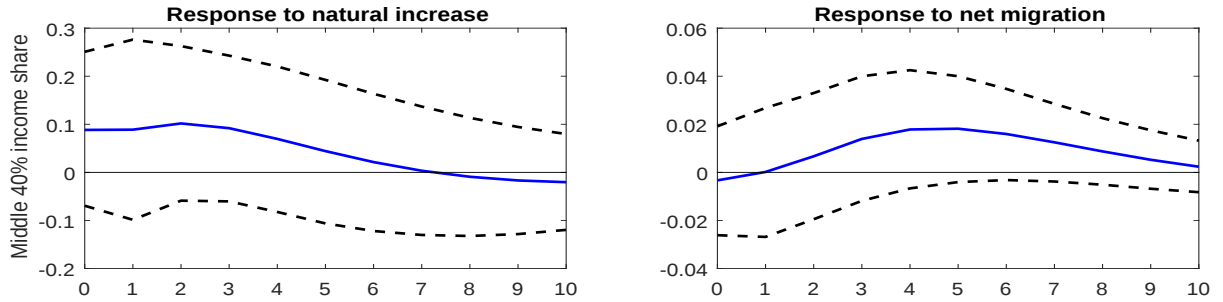
(a) *Top 1% income share*



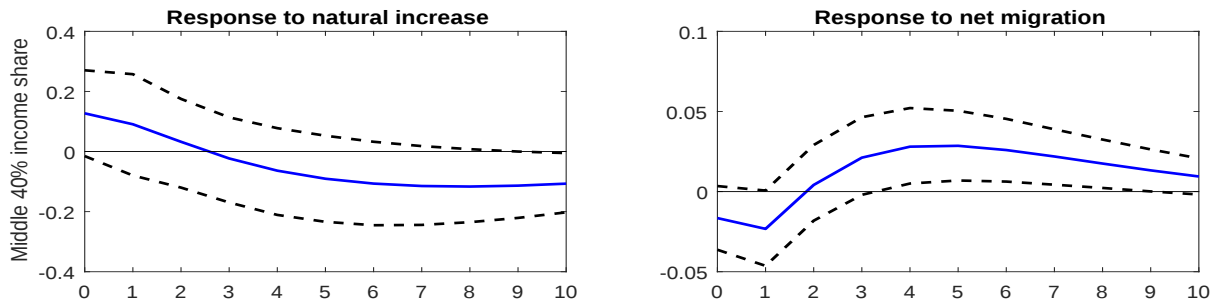
(b) *Top 10% income share*



(c) *Middle 40% income share*



(d) *Bottom 50% income share*



Notes: The solid line gives the estimated impulse response. Dashed lines give the 90% confidence intervals generated by Monte Carlo simulations with 5000 repetitions. The responses are in percentage points change.

Figure 8: Phase diagram with a steady-state denoted (k^*, c^*) .

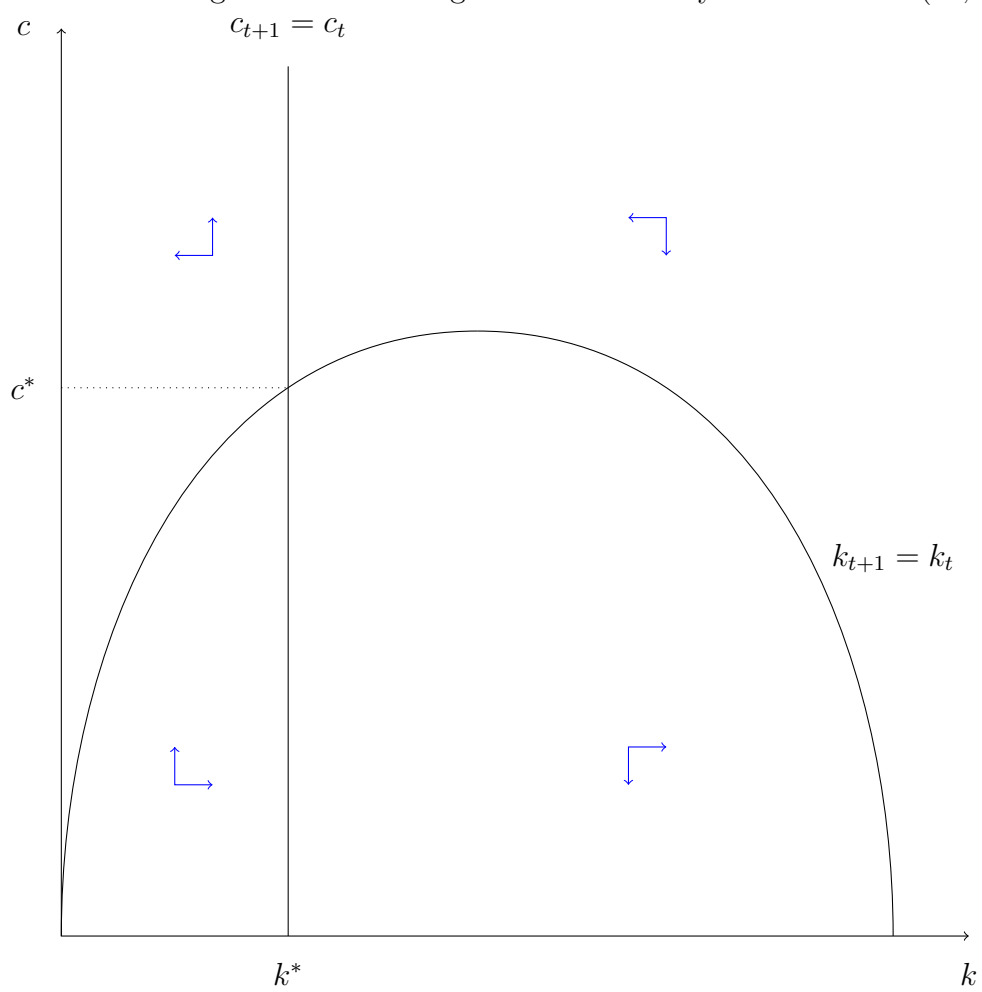


Figure 9: Dynamic consequences of a change in β that reduces steady-state capital and consumption. New locus for constant capital and consumption are in blue. Trajectory after the change is in red.

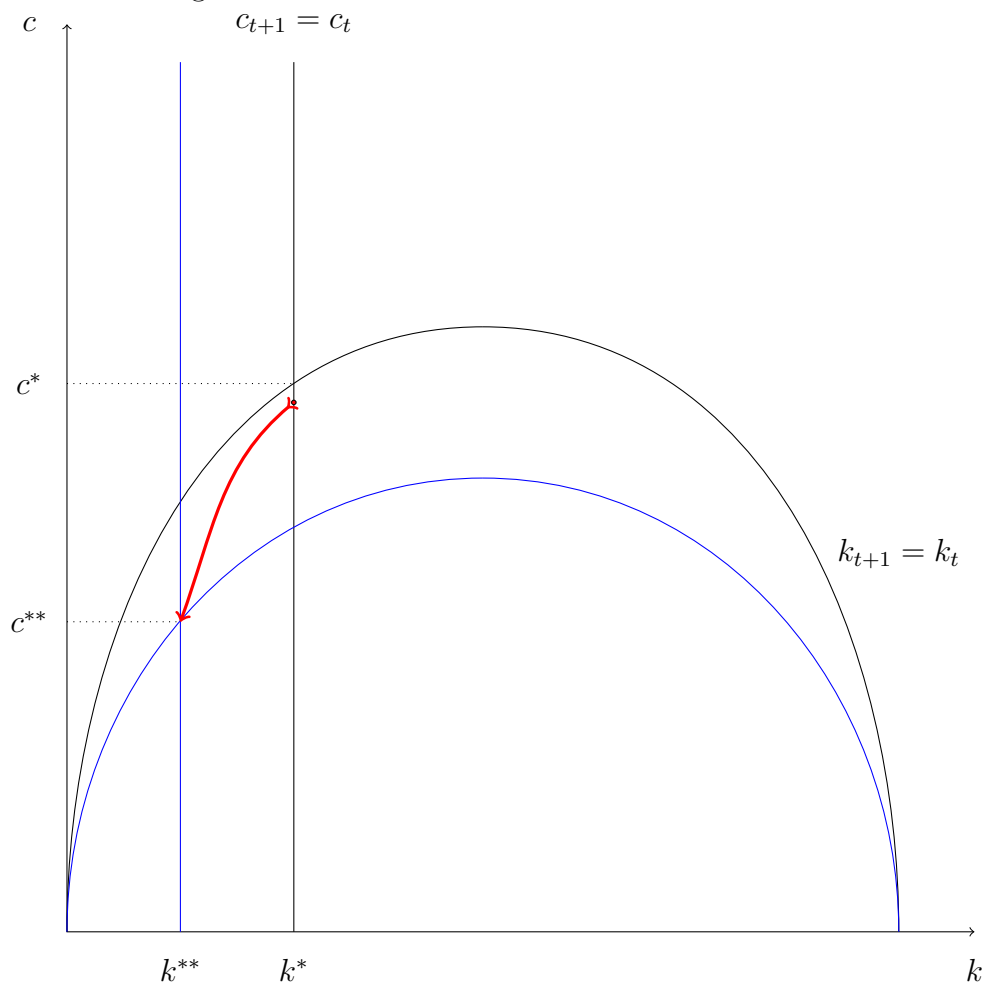


Figure 10: Dynamic consequences of a change in λ that increases steady-state capital and consumption. New locus for constant capital and consumption are in blue. Trajectory after the change is in red.

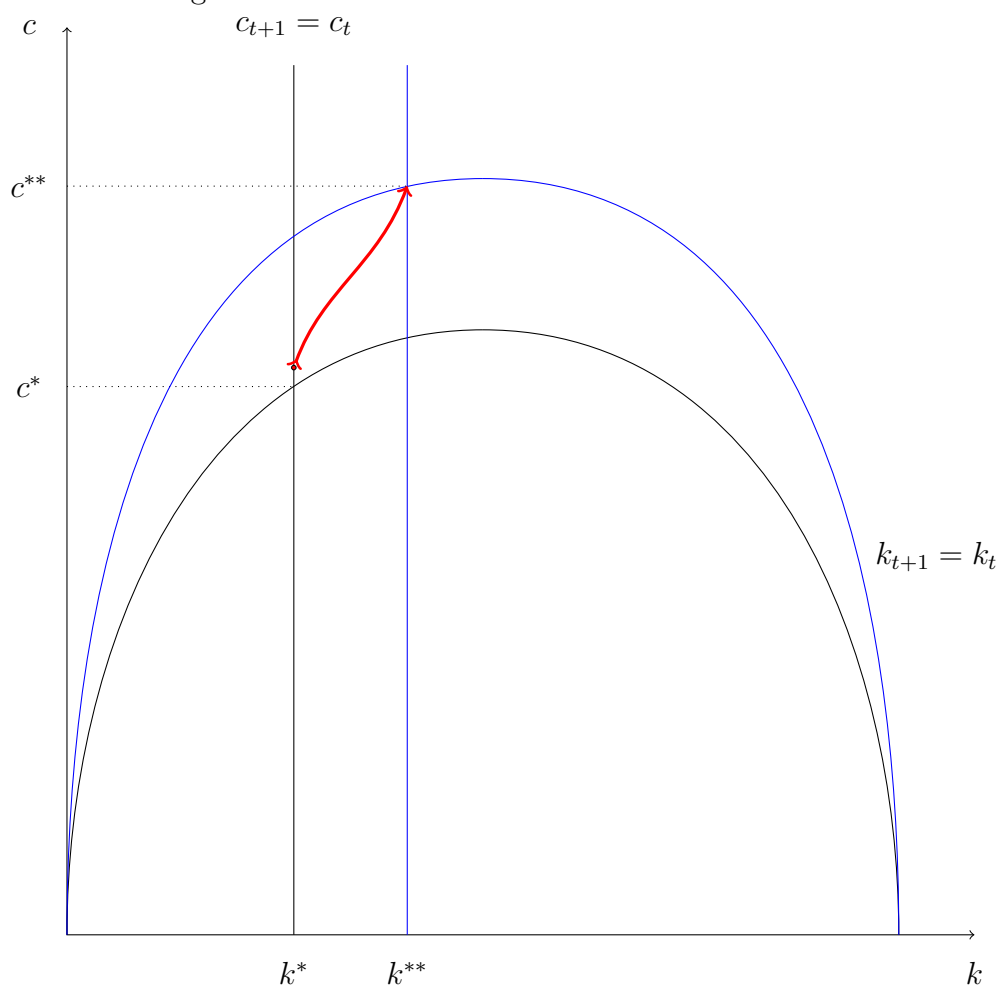


Table 2: Responses of employment to a migration shock (evaluation of η)

	Year 0	Year 1	Year 2	Year 5	Year 10
Model with empl./working-age pop.	1.16*	1.73*	2.19*	1.80*	0.44*
Model with empl./pop.	1.33*	1.88*	2.12*	1.43*	0.25
Model with hours worked/pop.	1.26*	1.99*	2.19*	1.42*	0.11

Notes: Year 0 stands for the year of the shock. * denotes statistical significance at the 10% level. The size of the migration shock is set to 1 percentage point. The responses are in percentage points change.