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WORKING PAPER N° 2019 – 41

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Equivalent income *versus* equivalent lifetime: does the metric matter?*

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April 23, 2019

Abstract

We examine the effects of the postulated metric on the measurement of well-being, by comparing, in the (income, lifetime) space, two indexes: the equivalent income index and the equivalent lifetime index. Those indexes are shown to satisfy different properties concerning interpersonal well-being comparisons, which can lead to contradictory rankings. While those incompatibilities arise under distinct indifference maps, we also explore the effects of the metric while relying on a unique indifference map, and show that, even in that case, the postulated metric matters for the measurement of well-being. That point is illustrated by quantifying, by those two indexes, the (average) well-being loss due to the Syrian War. Relying on a particular metric leads, from a quantitative perspective, to different pictures of the deprivation due to the War.

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JEL classification codes: I31*,* J17.

*The authors are most grateful to Franz Dietrich, Fabrice Etilé, Marc Fleurbaey, Stephan Klassen, Jean-François Laslier, Erik Schokkaert, Claudia Senik, Holger Strulik, Stephane Zuber, as well as two anonymous reviewers, for their comments on a previous version of this paper. We also thank participants of seminars at the University of Gottinghen and at Paris School of Economics.

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1 Introduction

Developed in the 1970s by Usher (1973) in the (income, life expectancy) space, the equivalent income is a preferences-based index of well-being, which can potentially include all non-monetary dimensions of standards of living. The equivalent income is defined as the hypothetical income, which, when combined with reference achievements on non-monetary dimensions of well-being, makes an individual indifferent between that hypothetical situation and his current situation. In the recent decades, the equivalent income approach has become increasingly used in the measurement of well-being across countries and epochs.[1]

As stated in Fleurbaey (2016), the equivalent income is an inclusive well-being index satisfying two properties. On the one hand, that index satisfies Respect for Preferences, since it assigns a larger value to a bundle that individuals regard, in the light of their preferences, as better. On the other hand, the equivalent income index satisfies Resourcism, a property according to which, when non-monetary dimensions of standards of living take their reference levels, the comparison of the well-being of two individuals can be carried out merely by comparing their income levels. Resourcism, when combined with Respect for Preferences, leads to constructing an index of well-being whose metric is money, in line with Pigou's (1920) definition of economic welfare ("the part of welfare that can be brought, directly or indirectly, with the measuring rod of money").

Among those properties, Respect for Preferences has a strong ethical appeal. When individual preferences are well-defined (and not anti-social), it is hard to see why the measurement of well-being should abstract from how individuals weight the different components of their living conditions.[2]

Resourcism is, from an ethical perspective, more difficult to assess. Using money as a metric for well-being measurement seems at first glance intuitive, since individuals are familiar with that metric. That point was made by Sen (1973) in an early attempt to adjust national income statistics in such a way as to incorporate non-monetary dimensions of standards of living (anterior to Sen's theory of functionings and capabilities). The familiarity with the money metric motivated Sen (1973) to normalize his measure of lifetime income, by dividing it by a reference level of life expectancy, in order to obtain an amount in monetary units, which is of the same order of magnitude as GDP per capita.

However, relying on the money metric can also be questioned. Following Fleurbaey (2016), an important criticism against Resourcism is that one may want, ideally, to measure well-being in terms of a metric that is a fundamental human functioning in Sen's sense, i.e. something that is necessary to realize one's conception of a good life, whatever that conception is. Fleurbaey considers that

[1] See Usher (1980), Williamson (1984), Crafts (1997), Costa and Steckel (1997), Murphy and Topel (2003), Nordhaus (2003), Becker et al (2005), Fleurbaey and Gaulier (2009), Decancq and Schokkaert (2016) and Ponthiere (2016).

[2] There exist, however, some cases where Respect for Preferences can be questioned. For instance, a child who is unable to read and write may not value schooling a lot, implying that well-being indexes should assign little weight to education. Note, however, that this criticism does not question Respect for Preferences *per se*, but, rather, the definition of the preferences to be taken into account when measuring well-being.

money is not such a fundamental functioning, which questions the attractiveness of Resourcism. Moreover, Sen (1998) argued that lifetime is a fundamental dimension of standards of living, since "being alive" is a necessary condition to achieve the goals that one pursues in life, whatever these goals are.[3] In the light of this, one may question the reliance on the income metric, and suggest that lifetime may be a more appealing metric for well-being measurement.

To what extent does the choice of a particular metric matter for well-being measurement? Does the choice of money or lifetime as a metric have some impact on well-being comparisons across individuals having different preferences? Alternatively, when considering the measurement of well-being under a unique indifference map, does the reliance on a particular metric matter?

This paper proposes to examine the impact of the postulated metric on the measurement of well-being, by comparing, in the (income, lifetime) space, two well-being indexes: on the one hand, the equivalent income index, and, on the other hand, the equivalent lifetime index.[4] The equivalent lifetime index is defined as the hypothetical lifetime (number of life-years) which, combined with the reference income level, would make the individual indifferent with respect to his current situation. The equivalent lifetime index is built while respecting the same kind of procedure as for the equivalent income index, but differs regarding the metric that is used: life-years instead of money.[5]

In order to examine the impact of the metric on well-being measurement, we develop a simple lifecycle model, where individuals have preference defined in the (income, lifetime) space, and we propose to compare, within that framework, the two equivalent indexes, which differ only regarding the postulated metric. Our comparison proceeds in three stages. First, we study the conditions under which the equivalent income index and the equivalent lifetime index exist. Second, we examine the properties satisfied by those two indexes, while paying a particular attention to interpersonal comparisons of well-being under distinct indifference maps. Third, we examine the extent to which the measurement of well-being is sensitive to the postulated metric, while assuming a unique indifference map (supposed to represent the preferences of a representative agent), as in most applied economic works using the equivalent income approach.[6]

Anticipating our results, we first show that the conditions under which the equivalent lifetime index exists are more restrictive than the ones under which the equivalent income index exists. Actually, the existence of an equivalent

[3] A natural corollary of this is that a premature death constitutes a major form of human deprivation. This motivated Sen (1998) to consider mortality as an indicator of economic success and failure.

[4] Our emphasis on 2 dimensions of well-being (instead of n dimensions) is made here for the simplicity of presentation. Introducing $n > 2$ dimensions would add complexity without bringing extra-value for the issue at stake.

[5] The life-year metric is not as widespread as the money metric. One exception is Veenhoven (1996), who developed the happy life expectancy index, which is defined as the product of life expectancy and happy scores normalized on a 0-1 scale. Another exception is the QALY index (see Abellan et al 2016).

[6] Recent exceptions include Decoster and Haan (2015), Carpentier and Sapata (2016), Decancq and Neumann (2016), Decancq et al (2017) and Akay et al (2017).

lifetime index requires, in addition to the usual conditions on preferences, that the reference income level and the actual income level are both either larger or smaller than the critical income level making life neutral (defined as the income per period making the individual indifferent between, on the one hand, further life with that income, and, on the other hand, death). However, we also show that it is possible to define an alternative equivalent lifetime index, for which existence conditions are as weak as for the equivalent income index.

At the qualitative level, we show that the equivalent income index and the (alternative) equivalent lifetime indexes satisfy different properties concerning interpersonal comparisons of well-being, so that the reliance on a particular metric matters for well-being measurement. Whereas the equivalent income index satisfies Resourcism, the (resp. alternative) equivalent lifetime index satisfies (resp. Alternative) Lifetimism. Those properties, when combined with Respect for Preferences, lead to interpersonal rankings that can be contradictory. Furthermore, we show that the alternative equivalent lifetime index violates, unlike the equivalent income and the standard equivalent lifetime, the Same Preference principle (which states that comparisons between persons with the same preferences should follow the ranking according to their common preferences), but is the only one, among the three indexes, to satisfy the Respect for Value of Life property (which states that a person whose life is worth being lived should be ranked as better off than an individual whose life is not worth being lived).

At the quantitative level, and assuming a single indifference map, we show that the postulated metric matters also: the measured absolute and relative variations in well-being are, in absolute value, larger under the equivalent income index than under the equivalent lifetime index. To illustrate that comparison, we use equivalent income and equivalent lifetime indexes to compute the (average) welfare loss due to the Syrian War. Our calculations show that, although these are constructed on the basis of the same indifference map, the two well-being indexes provide, from a quantitative perspective, very different pictures of the deprivation due to the War. This illustrates that the choice of the metric matters for the measurement of well-being not only when individuals have distinct preferences, but also when there is a unique indifference map.

This paper is related to several branches of the literature. First, it is related to the welfare economics literature on the strengths and weaknesses of the equivalent income approach (see Fleurbaey 2011, Fleurbaey and Blanchet 2013, Fleurbaey 2016).[7] This paper complements those works by focusing on the metric of the equivalent income index, and on its impact on well-being measurement. Second, this paper is related to the literature in economics and economic history using the equivalent income approach (see Usher 1980, Williamson 1984, Crafts 1997, Costa and Steckel 1997, Murphy and Topel 2003, Nordhaus 2003, Becker et al 2005, Fleurbaey and Gaulier 2009, Decancq and Schokkaert 2016, Ponthiere 2016). We complement those papers by exploring the effect of the

[7] Among the limitations under study, some attention was paid to whether the equivalent income index is too welfarist or not welfarist enough, to the difficult choice of reference levels for all non-monetary dimensions under study, and also to whether that indicator should take into account more subjective aspects of well-being.

money metric on the measurement of well-being. Our study is also related to the literature on fairness, such as Fleurbaey and Maniquet (2011), since the measurement of well-being, by involving ethical judgements on how to compare the situations of individuals, plays a key role in identifying who is the worst-off, and, hence, who should have priority when considering the allocation of resources. Finally, our study is also related to papers in development economics, such as Ravallion (2012), who showed the sensitivity of standards of living indexes to the postulated functional forms in a multidimensional setting.[8]

The rest of the paper is organized as follows. Section 2 presents our framework. The equivalent income index and the equivalent lifetime index are presented in Section 3. The existence of those indexes is studied in Section 4. Section 5 compares well-being indexes regarding their capacity to respect individual preferences. Then, Section 6 compares indexes concerning interpersonal comparisons of well-being under distinct indifference maps. Then, assuming a unique indifference map, Section 7 compares the absolute and relative variations in well-being measured under the equivalent income and the equivalent lifetime indexes. Section 8 illustrates our results by means of the measurement of the (average) welfare loss due to the Syrian War. Section 9 concludes.

2 The framework

Let us first introduce the lifecycle model on which our analysis is based. The economy is composed of N individuals, indexed with letters $i, j, \ldots$. For the sake of the presentation, we consider, throughout this paper, a simple two-dimensional model. In that model, a human life is reduced to two dimensions, which summarize, in a nutshell, the "quality" and the "quantity" of life.

The first dimension is income per period, denoted by $y_i \in \mathbb{R}^+$. Income is here assumed to be constant along the lifecycle. This income per period dimension is a proxy for the "quality" of each period of life.

The second dimension is the length of life $L_i \in \mathbb{R}^+$. This length of life captures the pure "quantity" of life.[9]

Individuals have well-defined preferences on the set of all bundles (y_i, L_i), which are represented by the utility function $U_i(y_i, L_i)$. It is assumed, as usual, that $U_i(\cdot)$ is continuous in its arguments y_i and L_i.

Throughout this paper, we assume that the function $U_i(\cdot)$ is (strictly) increasing in income y_i, that is, that $U_{iy}(y_i, L_i) > 0$. This assumption amounts to state that, whatever the length of life is, it is always strictly welfare-improving to increase income per period, which is here a proxy for the "quality" of life at

[8] Ravallion (2012) shows that, as a consequence of its multiplicative form, the new HDI assigns a lower weight to longevity achievements in poor countries, relatively to rich countries. Like the new HDI, the equivalent income and the equivalent lifetime indexes involve a multiplication of longevity achievements by a transform of income, which explains, in Section 8, the low willingness to pay, in money terms, for coming back to pre-conflict survival conditions, and the high willingness to pay, in life-year terms, for coming back to pre-War income.

[9] Note that we abstract here from individual's interests in joint survival as studied in Ponthiere (2016) using an equivalent consumption approach.

a given period. Note that this assumption of strict monotonicity rules out the case of perfect complementarity between income per period and lifetime.

Concerning the impact of lifetime L_i on well-being $U_i(\cdot)$, we follow the literature, and assume that additional lifetime is desirable only if the quality of life (here captured by income per period) is sufficiently high.[10] This amounts to assume that there exists an individual-specific critical income level $\tilde{y}_i > 0$ that makes individual i indifferent between, on the one hand, further life with that income, and, on the other hand, death.[11] If income is above $\tilde{y}_i$, then adding some extra life periods increases individual well-being. If, on the contrary, income is below $\tilde{y}_i$, then adding some extra life periods reduces individual well-being.

At first glance, assuming the existence of an income level $\tilde{y}_i$ making the individual indifferent between additional lifetime and death may look like a strong assumption. However, assuming, on the contrary, that such a neutral income level does not exist would be an even stronger assumption. This would amount to assume that either being alive - even in extreme misery, with zero income - would always be better, for an individual, than being dead, or, alternatively, that being alive - even with a very high income - would always be worse, for an individual, than being dead. Those two alternative assumptions are not plausible, which justifies the existence of a critical income level $\tilde{y}_i > 0$.[12]

Normalizing the utility of being dead to 0, we have thus $U_i(\tilde{y}_i, L_i) = 0$ for any L_i, as well as $U_i(y_i, L_i) > 0$ when $y_i > \tilde{y}_i$ and $U_i(y_i, L_i) < 0$ when $y_i < \tilde{y}_i$. We have also that: $U_{iL}(y_i, L_i) > 0$ when $y_i > \tilde{y}_i$, $U_{iL}(y_i, L_i) = 0$ when $y_i = \tilde{y}_i$ and $U_{iL}(y_i, L_i) < 0$ when $y_i < \tilde{y}_i$.[13]

Figure 1 shows an example of indifference map in the (y_i, L_i) space satisfying our assumptions. Indifference curves are decreasing when $y_i > \tilde{y}_i$, since in that area both income per period and lifetime are desirable goods. When $y_i = \tilde{y}_i$, lifetime is a neutral good, so that the indifference curve is a vertical line at $y_i = \tilde{y}_i$. Finally, when $y_i < \tilde{y}_i$, lifetime is an undesirable good, and indifference curves are increasing in the (y_i, L_i) space. Arrows in Figure 1 show the direction in which well-being increases in the two areas of the indifference map.

Finally, for the purposes of constructing our well-being indexes - equivalent incomes and equivalent life years - we assume that there exists some reference levels for the two dimensions of standards of living considered. We denote by $\bar{y} > 0$ the reference income per period level, and by $\bar{L} > 0$ the reference level of the length of life. Those two parameters are supposed to be unique (i.e. the same for all individuals), so that $(\bar{y}, \bar{L})$ constitutes a reference point for all.[14]

[10] See, for instance, Becker et al (2005).

[11] One can regard the critical level of income $\tilde{y}_i$ as the equivalent, in the money metric, of Broome's (2004) concept of utility level neutral for the continuation of existence.

[12] On the existence of that income threshold, see also Fleurbaey et al (2014).

[13] Note that such a normalization is not, strictly speaking, necessary for the purpose at hand. However, the economics literature on life and death usually normalizes the utility of the dead to zero (except in the presence of a bequest motive, as in Fleurbaey et al 2017).

[14] We will, in Sections 4 and 5, come back on the advantages and drawbacks of introducing *individual-specific* reference levels $\bar{L}_i$ and $\bar{y}_i$.

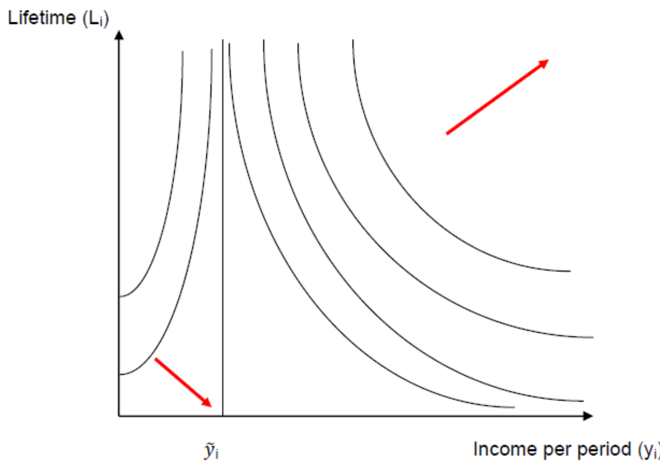

Figure 1. Indifference map in the (income per period, lifetime) space.

3 Two well-being indexes

Let us first introduce the equivalent income index, which has been widely studied in the recent years (see Fleurbaey and Blanchet 2013, Fleurbaey 2016). Suppose that an individual i has income y_i and lifetime L_i. In the present setting, the equivalent income $\hat{y}_i$ is defined as the hypothetical income level which, combined with the reference level for lifetime $\bar{L}$, would make the individual indifferent with respect to its bundle (y_i, L_i).

Definition 1 (equivalent income) *Suppose a reference level for the length of life $\bar{L}$. Suppose that an individual i has preferences represented by the utility function $U_i(y_i, L_i)$. For any bundle (y_i, L_i), the equivalent income index $\hat{y}_i$ is defined implicitly by the following equality:*

$$U_i\left(\hat{y}_i, \bar{L}\right) = U_i(y_i, L_i)$$

The equivalent income is an inclusive measure of well-being, since it includes not only the income dimension, but, also, the other dimension of well-being, here the length of life L_i. The equivalent income $\hat{y}_i$ is a function of income y_i and lifetime L_i, so that it can be rewritten as $\hat{y}_i = \hat{y}_i(y_i, L_i)$. The equivalent income index is increasing in y_i. Moreover, as long as $y_i > \tilde{y}_i$, so that $U_{iL}(y_i, L_i) > 0$, the equivalent income is also increasing in L_i. However, when $y_i < \tilde{y}_i$, the equivalent income is decreasing in L_i. Figure 2 illustrates the construction of the equivalent income index in the (income, lifetime) space.

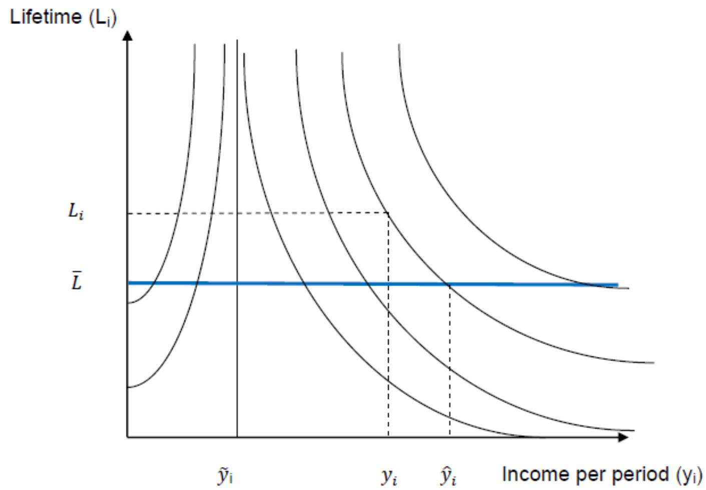

Figure 2. Construction of the equivalent income index.

The equivalent income measures well-being by using the income metric. Note, however, that one may want to proceed differently, and to construct an equivalent index while using not the income metric, but the lifetime metric. This is the intuition behind the equivalent lifetime index.

Consider, here again, an individual i with income y_i and lifetime L_i. The equivalent lifetime index $\hat{L}_i$ is defined as the hypothetical lifetime level which, combined with the reference level for income per period $\bar{y}$, would make the individual indifferent with respect to its bundle (y_i, L_i).

Definition 2 (equivalent lifetime) *Suppose a reference level for the income per period $\bar{y} > 0$. Suppose that an individual i has preferences represented by the utility function $U_i(y_i, L_i)$. For any bundle (y_i, L_i), the equivalent lifetime index $\hat{L}_i$ is defined implicitly by the following equality:*

$$U_i\left(\bar{y}, \hat{L}_i\right) = U_i(y_i, L_i)$$

Figure 3 below illustrates the construction of an equivalent lifetime index, using the same example of indifference map as above. From the definition of the equivalent lifetime index, one can rewrite the equivalent lifetime index as a function of income y_i and lifetime L_i, i.e. $\hat{L}_i = \hat{L}_i(y_i, L_i)$. It is easy to see that the equivalent lifetime index is increasing in income y_i and in lifetime L_i.

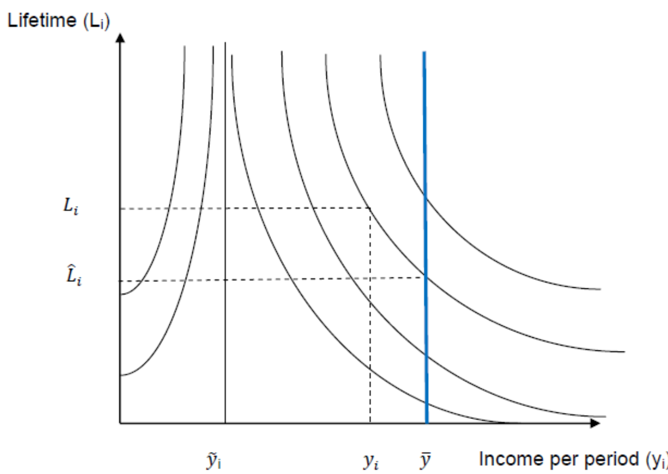

Figure 3. Construction of the equivalent lifetime index.

At first glance, the equivalent lifetime index seems to be very similar to the equivalent income index. Actually, both indexes are constructed on the basis of indifference maps, and both indexes consist of fixing a reference level for one dimension, and looking for the hypothetical level of either income or lifetime that makes the individual indifferent with respect to his bundle. Hence, both indexes look like quite similar inclusive measures of well-being, which synthesize standards of living in a single number. However, as we will argue in the rest of this paper, those two well-being indexes differ on various important aspects.

4 Existence

Consider first the existence of the equivalent income index. The existence of that index requires, in the (y_i, L_i) space, that the indifference curve on which a bundle lies must cross, at some point, the horizontal line drawn at $\bar{L}$. This is achieved when the following condition on preferences holds.

Proposition 1 (existence of equivalent income) *Conditionally on a reference level for lifetime $\bar{L} > 0$, the equivalent income index $\hat{y}_i$ exists if and only if, for any individual i, the utility function $U_i(y_i, L_i)$ satisfies the following property: $\forall (y_i, L_i) \in \mathbb{R}^+ \times \mathbb{R}^+, \exists x > 0$ such that: $U_i\left(x, \bar{L}\right) = U_i(y_i, L_i)$.*

Proof. See Figure 2. ■

Note that, in the case of perfect complementarity between income per period and lifetime, the above property is not satisfied, so that the equivalent income does not exist. That case is quite extreme, and is actually ruled out here by the strict monotonicity of preferences in income per period. Note also that, given our assumptions on preferences, the conditions of Proposition 1 also guarantee the uniqueness of the equivalent income index.

Let us now turn to the equivalent lifetime index. Actually, the existence of the equivalent lifetime index requires more restrictive conditions than the existence of the equivalent income index. More precisely, the existence of the equivalent lifetime index requires some restrictions on where the reference income per period must be fixed in comparison to the prevailing income per period.

Proposition 2 (existence of equivalent lifetime) *Assume a reference level for income $\bar{y} > 0$. Then, for any individual i with bundle (y_i, L_i):*

- *If $y_i \leq \tilde{y}_i$ and $\bar{y} > \tilde{y}_i$, or if $y_i = \tilde{y}_i$ and $\bar{y} \neq \tilde{y}_i$, or if $y_i \geq \tilde{y}_i$ and $\bar{y} < \tilde{y}_i$, the equivalent lifetime index does not exist.*
- *If $y_i > \tilde{y}_i$ and $\bar{y} > \tilde{y}_i$, the equivalent lifetime index exists if and only if the utility function $U_i(y_i, L_i)$ satisfies the following property: $\forall (y_i, L_i)$ with $y_i > \tilde{y}_i, \exists x > 0$ such that: $U_i(\bar{y}, x) = U_i(y_i, L_i)$.*
- *If $y_i < \tilde{y}_i$ and $\bar{y} < \tilde{y}_i$, the equivalent lifetime index exists if and only if the utility function $U_i(y_i, L_i)$ satisfies the following property: $\forall (y_i, L_i)$ with $y_i < \tilde{y}_i, \exists x > 0$ such that: $U_i(\bar{y}, x) = U_i(y_i, L_i)$.*
- *If $y_i = \tilde{y}_i$ and $\bar{y} = \tilde{y}_i$, the equivalent lifetime index exists but is not unique.*

Proof. See Figure 3. ■

The intuition behind that result goes as follows. Remind that the indifference map in the (y_i, L_i) space involves indifference curves that are decreasing when $y_i > \tilde{y}_i$, a vertical line at $y_i = \tilde{y}_i$, and increasing when $y_i < \tilde{y}_i$. As a consequence of that, the existence of an equivalent lifetime level requires that the reference income level $\bar{y}$ lies, with y_i, on the same side of the vertical line drawn at $\tilde{y}_i$. Otherwise, it is not possible, by moving along an indifference curve, to find the hypothetical lifetime level that, combined with the reference income, will make the individual indifferent with respect to his current bundle.[15]

For instance, if the current bundle involves a life not worth being lived, (i.e. $y_i < \tilde{y}_i$), and if $\bar{y} > \tilde{y}_i$, then it is impossible to find a hypothetical lifetime that would, jointly with the reference income level $\bar{y}$, make the individual as worse off as he is under his bundle, since the hypothetical life would, at worst, involve $\hat{L}_i = 0$, which would still be better than the life not worth being lived.

In the light of all this, a first, major difference between the equivalent income index and the equivalent lifetime index is that, whereas the existence of the former holds under general conditions on preferences, this is not true for the

[15] Note that, when the conditions for existence stated in Proposition 2 are satisfied, it is also the case that the equivalent lifetime index is unique. The only exception is the particular case where $y_i = \tilde{y}_i = \bar{y}$. In that case, the equivalent lifetime index exists, but is not unique. In that case, since income is equal to the critical income level making lifetime neutral, *any* level of lifetime, combined with the reference income level, makes the individual indifferent with respect to his situation. We are thus in a special case where $\hat{L}_i$ can take any positive value, that is, a multiplicity problem.

latter, whose existence requires additional conditions. Those additional conditions restrict the possible uses of the equivalent lifetime index with respect to the use of the equivalent income index. To illustrate this, take the case of a poor individual, whose initial income per period is *above* the neutral level for continuing existence. Then, a natural disaster arises, which reduces his income to a level that lies *below* the neutral level for continuing existence. Given that the initial bundle and the final bundle lie on two distinct sides of the critical income level making life neutral, one cannot, on the basis of a single reference income level, compute the equivalent lifetime index for *both* the pre-disaster and the post-disaster period. On the contrary, it is possible to compute the equivalent income index for both periods, since the horizontal line drawn at $\bar{L}$ must necessarily cross the two indifference curves along which the bundles lie.

When facing that problem, one solution is to redefine the equivalent lifetime index in an alternative way, to avoid the non-existence problem. Let us call this new index the alternative equivalent lifetime index.

Definition 3 (alternative equivalent lifetime) *Suppose two individual-specific reference levels for the income per period $\bar{y}_{i1}$ and $\bar{y}_{i2}$, with $0 < \bar{y}_{i1} < \tilde{y}_i < \bar{y}_{i2}$. Suppose that an individual i has preferences represented by the utility function $U_i(y_i, L_i)$. For any bundle (y_i, L_i), the alternative equivalent lifetime index $\breve{L}_i$ is defined as follows.*

- $\forall (y_i, L_i) : y_i < \tilde{y}_i : \breve{L}_i = -\hat{L}_i$, *where* $U_i\left(\bar{y}_{i1}, \hat{L}_i\right) = U_i(y_i, L_i)$;
- $\forall (y_i, L_i) : y_i = \tilde{y}_i : \breve{L}_i = 0$;
- $\forall (y_i, L_i) : y_i > \tilde{y}_i : \breve{L}_i = \hat{L}_i$, *where* $U_i\left(\bar{y}_{i2}, \hat{L}_i\right) = U_i(y_i, L_i)$.

The intuition behind the alternative equivalent lifetime index goes as follows. Non-existence problems for the standard equivalent lifetime index occur when the actual income y_i and the reference income $\bar{y}$ lie on different sides of the neutral income level $\tilde{y}_i$. Hence, one solution is to fix *two* reference levels for income $\bar{y}_{i1}$ and $\bar{y}_{i2}$, where $\bar{y}_{i1} < \tilde{y}_i < \bar{y}_{i2}$, and to construct the equivalent lifetime index while using the low reference level when income is below $\tilde{y}_i$, and the high reference level when income is above $\tilde{y}_i$. The construction of the alternative equivalent lifetime index is illustrated on Figure 4.

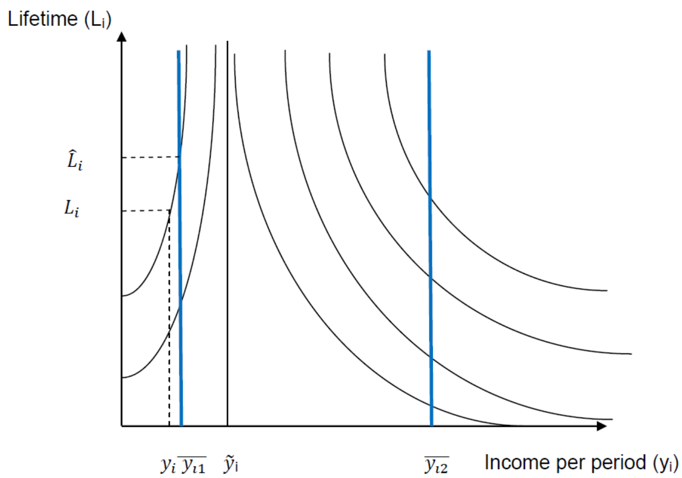

Figure 4. Construction of the alternative equivalent lifetime index.

Shifting from the standard to the alternative equivalent lifetime index has the advantage to simplify the conditions necessary for the existence of the index.

Proposition 3 (existence of alternative equivalent lifetime) *Assume two individual-specific reference levels for the income per period* $\bar{y}_{i1}$ *and* $\bar{y}_{i2}$, *with* $0 < \bar{y}_{i1} < \tilde{y}_i < \bar{y}_{i2}$. *Then, for any individual i with bundle* (y_i, L_i)*:*

- *If* $y_i < \tilde{y}_i$, *the alternative equivalent lifetime index* $\breve{L}_i$ *exists if and only if, for any individual i, the utility function* $U_i(y_i, L_i)$ *satisfies the following property:* $\forall (y_i, L_i) \in \mathbb{R}^+ \times \mathbb{R}^+, \exists x > 0$ *such that:* $U_i(\bar{y}_{i1}, x) = U_i(y_i, L_i)$.
- *If* $y_i = \tilde{y}_i$, *the alternative equivalent lifetime index* $\breve{L}_i$ *exists and is equal to zero.*
- *If* $y_i > \tilde{y}_i$, *the alternative equivalent lifetime index* $\breve{L}_i$ *exists if and only if, for any individual i, the utility function* $U_i(y_i, L_i)$ *satisfies the following property:* $\forall (y_i, L_i) \in \mathbb{R}^+ \times \mathbb{R}^+, \exists x > 0$ *such that:* $U_i(\bar{y}_{i2}, x) = U_i(y_i, L_i)$.

Proof. See Figure 4. ■

The alternative equivalent lifetime index can thus be regarded as a solution when facing the possible non-existence of the standard equivalent lifetime index.[16] Having stressed this, the reliance on the alternative equivalent lifetime index has its own drawbacks.

First, it requires to assume not one, but two reference levels for income, which must lie on both sides of the critical income level $\tilde{y}_i$. Given that $\tilde{y}_i$ is

[16] It also solves the non-uniqueness problem when $y_i = \tilde{y}_i$.

individual-specific, the two reference levels for income must also be *individual-specific*. Hence, one needs to replace the unique reference $\bar{y}$ by N individual-specific references $\bar{y}_{i1}$ and N individual-specific references $\bar{y}_{i2}$. Beyond the issue of complexity, such a shift constitutes a major departure with respect to the initial framework. Replacing a single ethical parameter $\bar{y}$ by $2N$ individual-specific parameters definitely modifies what a "reference level" is about.

Second, supposing that reference levels $\bar{y}_{i1}$ and $\bar{y}_{i2}$ satisfy $0 < \bar{y}_{i1} < \tilde{y}_i < \bar{y}_{i2}$ is far from a weak assumption. It may be the case that individuals are endowed with reference levels that are either both below or both above $\tilde{y}_i$. In those cases, the alternative equivalent lifetime index faces the same non-existence problems as the standard equivalent lifetime index. Thus one avoids non-existence problems by imposing strong conditions on reference income levels.

5 Respect for Preferences

Let us now examine the properties satisfied by the equivalent income index and the (alternative) equivalent lifetime index. A first, standard property to be studied is Respect for Preferences (see Fleurbaey 2011, 2016). That property states that, if a variation in y_i or L_i increases (resp. decreases) individual welfare, this will necessarily lead to increase (resp. decrease) the well-being index, and that any variation in the well-being index must necessarily coincide with a variation, in the same direction, of individual welfare.

Definition 4 (Respect for Preferences) *A well-being index $b_i(y, L)$ satisfies Respect for Preferences if and only if, for any individual i and any two bundles (y_i, L_i) and (y'_i, L'_i), we have:*

$$b_i(y'_i, L'_i) \gtreqless b_i(y_i, L_i) \iff U_i(y'_i, L'_i) \gtreqless U_i(y_i, L_i)$$

That ethical property states that moving an individual to a bundle that he considers to be better (resp. worse) must lead to a rise (resp. a fall) of the measured well-being for that person. That property is quite intuitive, and one may want that well-being indexes satisfy that property.

Proposition 4

- *The equivalent income index satisfies Respect for Preferences.*
- *Regarding the equivalent lifetime index,*
  - *if $y_i > \tilde{y}_i$ and $\bar{y} > \tilde{y}_i$, the equivalent lifetime index satisfies Respect for Preferences.*
  - *if $y_i < \tilde{y}_i$ and $\bar{y} < \tilde{y}_i$, the equivalent lifetime index does not satisfy Respect for Preferences, but Reverse Respect for Preferences (it takes a lower (resp. higher) value when the bundle is better (resp. worse)).*
- *The alternative equivalent lifetime index satisfies Respect for Preferences.*

Proof. See the Appendix. ■

The fact that the equivalent income index satisfies the Respect for Preferences property is not a new result (see Fleurbaey 2013, 2016). The major novelty in Proposition 4 concerns the equivalent lifetime index. It is stated there that the equivalent lifetime index satisfies Respect for Preferences only if the bundles under comparison involve an income that is higher than the critical income level making the individual indifferent between life and death. However, the equivalent lifetime index does not respect preferences in the case where a life is not worth being lived (i.e. the case where $y_i < \tilde{y}_i$ and $\bar{y} < \tilde{y}_i$). The intuition behind that violation goes as follows. When $y_i < \tilde{y}_i$, an individual who lies on a lower indifference curve is better off. Thus, when moving along indifference curves so as to cross the vertical line at $\bar{y}$, it appears that a bundle involving a higher level of well-being is being assigned a *lower* level of the equivalent lifetime index $\hat{L}_i$.

This violation may be qualified, since, when $y_i < \tilde{y}_i$, a lower lifetime implies a higher well-being. Thus assigning a lower value of the index when individuals are better off may not be so problematic; preferences are being respected, in the sense of another definition of "respecting preferences", which would consist of "assigning a higher level of a desirable good" to situations that are regarded as better by the individual. Lifetime being undesirable when $y_i < \tilde{y}_i$, "respecting preferences" can here be interpreted as the requirement of "assigning a lower level of the undesirable good" to situations that are regarded as better by the individual, which is indeed satisfied. One should thus not exaggerate the violation of Respect for Preferences, even though it may be disturbing, when interpreting measurement results, to see larger values of the index assigned to bundles that are actually regarded as worse by individuals.

Quite interestingly, the alternative equivalent lifetime index does not face those problems, and satisfies Respect for Preferences. Thus the alternative formulation of the equivalent lifetime index allows us not only to avoid non-existence problems, but, also, to satisfy Respect for Preferences.

6 Interpersonal well-being comparisons

6.1 Same Preference principle

Let us first consider well-being comparisons when individuals have the same preferences, i.e. when $U_i(\cdot) = U_j(\cdot)$. In that particular case, an intuitive property is the Same Preference principle (Decancq et al 2015). According to that principle, the comparison between persons with the same preferences should follow the ranking according to their common preferences.

Definition 5 (Same Preference principle) *Take two individuals i and j with same preferences, i.e. $U_i(\cdot) = U_j(\cdot) = U(\cdot)$. A well-being index $b_i(y, L)$ satisfies the Same Preference principle if and only if, for any two bundles (y_i, L_i) and (y_j, L_j), we have:*

$$b_i(y_i, L_i) \gtreqless b_j(y_j, L_j) \iff U(y_i, L_i) \gtreqless U(y_j, L_j)$$

The Same Preference principle is a quite intuitive property to be satisfied by well-being indexes. Clearly, if individuals share the same preferences, the comparison of their situations by means of well-being indexes should follow the ranking of those situations according to those common preferences.

Proposition 5 below states that, among the three indexes considered here, only the equivalent income index satisfies that property.

Proposition 5

- *The equivalent income index satisfies the Same Preferences principle.*
- *Regarding the equivalent lifetime index,*
  - *if $y_i > \tilde{y}_i$ and $\bar{y} > \tilde{y}_i \forall i$, the equivalent lifetime index satisfies the Same Preference principle.*
  - *if $y_i < \tilde{y}_i$ and $\bar{y} < \tilde{y}_i \forall i$, the equivalent lifetime index does not satisfy the Same Preference principle*
- *The alternative equivalent lifetime index does not satisfy the Same Preference principle.*

Proof. See the Appendix. ■

Proposition 5 provides a strong argument in favor of the income metric. Clearly, among the three indexes considered, only the equivalent income index satisfies the Same Preference principle. Whereas the violation of that principle by the standard equivalent lifetime index is not a surprise in the light of the previous section, the violation of the Same Preference principle by the alternative equivalent lifetime index is worth being stressed. An illustration of such a violation is given on Figure 5. Whereas the common preferences state that individual i is better off than individual j, the alternative equivalent lifetime index says the opposite. Indeed, since the reference income level for individual j is below his income, whereas the reference income level for individual i is above his income, the alternative equivalent lifetime index takes a higher value for individual j than for individual i, contradicting the Same Preference property.

The violation of the Same Preference principle by the alternative equivalent lifetime index comes from the fact that individuals with the same preferences do not necessarily have the same reference income levels. To avoid this problem, one could impose the restriction that two individuals i and j with the same preferences should have also the same reference income levels $\bar{y}_{i1} = \bar{y}_{j1}$ and $\bar{y}_{i2} = \bar{y}_{j2}$. But imposing same reference income levels under same preferences would make individual-specific reference income levels something close to preferences. This goes against what "reference levels" are supposed to be.

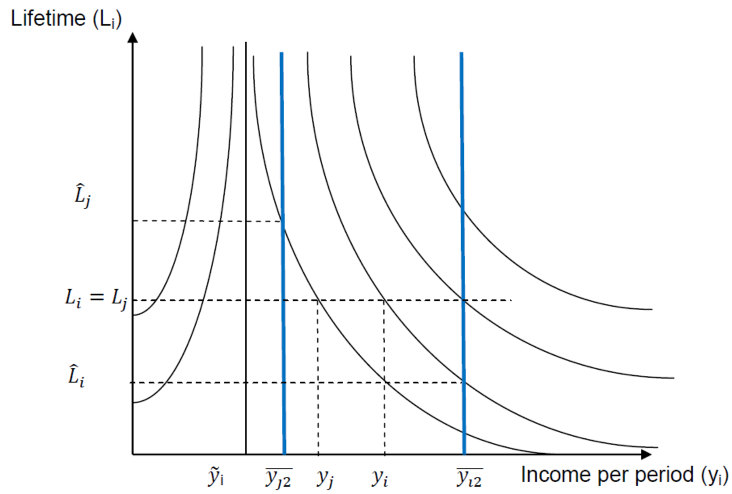

Figure 5. Violation of the Same Preference principle by the alternative equivalent lifetime index.

All in all, this section provides a negative result for the alternative equivalent lifetime index. This was defended above as a solution to the non-existence problems faced by the equivalent lifetime index. But that solution imposes a cost - the introduction of individual-specific reference income levels - that makes the alternative lifetime equivalent index violate the Same Preference principle.

6.2 Resourcism and Lifetimism

Let us now consider how the equivalent income index and the equivalent lifetime index compare individuals with different preferences. For that purpose, this section will focus on two properties, Resourcism and Lifetimism, which lead to distinct metrics for well-being measurement.

Resourcism states that, when comparing the well-being of two individuals, it is sufficient to consider the income level of those individuals when the non-monetary dimension takes its reference level.

Definition 6 (Resourcism) *A well-being index $b_i(y, L)$ satisfies Resourcism if and only if, when comparing the well-being of two individuals i and j, it is sufficient to consider the income level of those individuals when the non-monetary dimension - here L_i - takes its reference level $\bar{L}$ (for both individuals):*

$$\text{if } L_i = L_j = \bar{L}, \text{ then } b_i(y_i, \bar{L}) \gtrless b_j(y_j, \bar{L}) \iff y_i \gtrless y_j$$

Resourcism is ethically attractive when comparing two lives worth being lived, that is, for which $y_i > \tilde{y}_i$ and $y_j > \tilde{y}_j$. Indeed, in that case, it makes sense to suppose that, if those two lives involve the reference lifetime, the well-being index should take a higher value for the life with the largest income per period. Note also that Resourcism keeps its ethical appeal when comparing two lives not worth being lived. To see this, take two individuals i and j with incomes $y_i < y_j < \tilde{y}_i, \tilde{y}_j$ and with lifetimes $L_i = L_j = \bar{L}$. Resourcism ranks individual j as better off than individual i, which is intuitive, since, despite the fact that the two lives are not worth being lived, at least individual j enjoys a higher income.

Note, however, that the ethical appeal of Resourcism is less clear when considering two lives, one worth being lived, whereas the other is not worth being lived, that is, the case where $\tilde{y}_i < y_i < y_j < \tilde{y}_j$. In that case, if both individuals enjoy $\bar{L}$, Resourcism ranks individual j as better off than individual i (since $y_i < y_j$), even though individual i has a life worth being lived, whereas individual j has a life not worth being lived. That result is counterintuitive. Thus the ethical appeal of Resourcism is limited when comparing some lives worth being lived with lives not worth being lived.

Let us now introduce a second property, i.e. Lifetimism. Lifetimism states that, when comparing the well-being of two individuals at the reference income level, it is sufficient to compare their lifetimes.

Definition 7 (Lifetimism) *A well-being index $b_i(y, L)$ satisfies Lifetimism if and only if, when comparing the well-being of two individuals i and j, it is sufficient to consider the lifetime level of those individuals when the income takes its reference level $\bar{y}$ (for both individuals):*

$$\text{if } y_i = y_j = \bar{y} \text{ then } b_i(\bar{y}, L_i) \gtreqless b_j(\bar{y}, L_j) \iff L_i \gtreqless L_j$$

Lifetimism has some intuitive support when considering two individuals with lives worth being lived and incomes equal to the reference level, that is, when $y_i = y_j = \bar{y} > \tilde{y}_i, \tilde{y}_j$. In that case, it makes sense that the well-being index takes a higher value when the lifetime is larger. However, once lives under comparison are not worth being lived, the ethical appeal of Lifetimism becomes questionable. Take, for instance, two individuals i and j with incomes $y_i = y_j = \bar{y} < \tilde{y}_i, \tilde{y}_j$ and with lifetimes $L_i < L_j$. In that case, Lifetimism ranks individual j as better off than individual i, since he has a longer lifetime. However, since lifetime is, for such low income levels, an undesirable good, one may consider that individual i should be ranked as better off than individual j, contrary to what Lifetimism recommends. Moreover, Lifetimism leads also to counterintuitive results when comparing a life worth being lived with a life not worth being lived.

In the light of the lack of attractiveness of Lifetimism in case of lives not worth being lived, one may reformulate Lifetimism as follows.

Definition 8 (Alternative Lifetimism) *A well-being index $b_i(y, L)$ satisfies Alternative Lifetimism if and only if, when comparing the well-being of two individuals i and j, we have that:*

$$\begin{aligned}
\text{if } y_i &= \bar{y}_{i2} \text{ and } y_j = \bar{y}_{j2}, \text{ then } b_i(\bar{y}_{i2}, L_i) \gtreqless b_j(\bar{y}_{j2}, L_j) \iff L_i \gtreqless L_j \\
\text{if } y_i &= \bar{y}_{i2} \text{ and } y_j = \bar{y}_{j1}, \text{ then } b_i(\bar{y}_{i2}, L_i) \gtreqless b_j(\bar{y}_{j1}, L_j) \iff L_i \gtreqless -L_j \\
\text{if } y_i &= \bar{y}_{i1} \text{ and } y_j = \bar{y}_{j1}, \text{ then } b_i(\bar{y}_{i1}, L_i) \gtreqless b_j(\bar{y}_{j1}, L_j) \iff -L_i \gtreqless -L_j
\end{aligned}$$

Alternative Lifetimism states that, if individuals have incomes equal to their (individual-specific) reference income levels, then the comparison of their well-being can be made by focusing merely on their lifetime if lifetime is a good, and on minus their lifetime if lifetime is a bad.

Resourcism, Lifetimism and Alternative Lifetimism are three distinct approaches to interpersonal well-being comparisons. Under Respect for Preferences, those approaches are logically incompatible, since these lead to contradictory rankings. Let us first show this incompatibility for Resourcism and Lifetimism. To illustrate this, Figure 6 compares two individuals, a and b, who have different preferences. Those two individuals have the same lifetime (equal to the reference lifetime $\bar{L}$), but the income is larger for a than for b. When comparing a and b, Resourcism considers that individual a, who has a larger income than individual b, is better off than b. On the contrary, Lifetimism leads to the opposite result: individual a is, under Lifetimism, regarded as worse off than b. Thus, if one wants to respect preferences, Resourcism and Lifetimism lead to contradictory rankings.

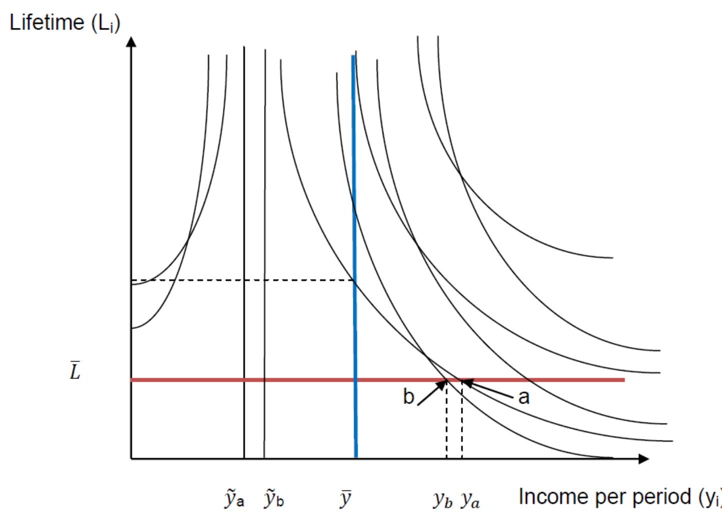

Figure 6: Incompatibility of Resourcism and Lifetimism under Respect for Preferences.

Actually, similar incompatibilities exist between, on the one hand, either Resourcism and Lifetimism, and, on the other hand, Alternative Lifetimism.

Proposition 6 *Under Respect for Preferences, Resourcism, Lifetimism and Alternative Lifetimism are not compatible.*

Proof. The incompatibility between Resourcism and Lifetimism under Respect for Preferences is illustrated by Figure 6.

The incompatibility between Resourcism and Alternative Lifetimism can be shown by considering the case of two individuals i and j with $\tilde{y}_j < y_j = \bar{y}_{j2} < y_i = \bar{y}_{i1} < \tilde{y}_i$ and $L_i = L_j = \bar{L}$. Resourcism ranks i as better off than j. Alternative Lifetimism, on the contrary, ranks i as worse off than j.

The incompatibility between Lifetimism and Alternative Lifetimism can be shown by considering two individuals i and j with $\tilde{y}_j < y_j = \bar{y}_{j2} = \bar{y} = y_i = \bar{y}_{i1} < \tilde{y}_i$ and $L_i > L_j$. Lifetimism ranks i as better off than j. But Alternative Lifetimism ranks i as worse off than j. ■

Proposition 7 states the properties satisfied by the equivalent income index, the equivalent lifetime index and the alternative lifetime index.

Proposition 7

- *The equivalent income index satisfies Resourcism.*
- *The equivalent lifetime index satisfies Lifetimism.*
- *The alternative equivalent lifetime index satisfies Alternative Lifetimism.*

Proof. See the Appendix. ■

The three well-being indexes under comparison rely on quite different approaches for the interpersonal comparisons of well-being. As a consequence, the ethical attractiveness of those indexes depends on the ethical attractiveness of the underlying approach for interpersonal well-being comparisons.

At first glance, there is an advantage for Resourcism over Lifetimism. The major problem with Lifetimism is that lifetime is not necessarily a desirable good: if the quality of life is very low (extreme misery), lifetime becomes an undesirable good. On the contrary, income is always a desirable good, in the sense that a higher quality of life is always more desirable than a lower quality of life for a given duration of life.[17] The fact that income is necessarily a desirable good - unlike lifetime - makes it a better candidate for being the metric of well-being measurement. That argument supports Resourcism against Lifetimism, and thus the equivalent income index over the equivalent lifetime index.

But Resourcism also faces some criticisms. As stated above, Resourcism may, in some cases, lead to the counterintuitive conclusion that a person considering his life not worth being lived may be ranked as better off than a person considering his life worth being lived. Quite interestingly, Alternative Lifetimism does not face that criticism: when comparing a life worth being lived with a life not worth being lived, it always ranks the former as better off than the latter. This provides some support for the alternative equivalent lifetime index. Note, however, that it does not follow from this that this index would necessarily dominate the other indexes, since, as stated above, the alternative equivalent lifetime index violates the - quite intuitive - Same Preference principle.

[17] More income is always better, since, even if there were satiation in consumption at the individual level, one could always find someone to give that extra income, which would generate a joy of giving. Giving is not modelled in our framework, but one may regard this as justifying the strict monotonicity of preferences in income per period.

6.3 Pigou-Dalton transfer principles

As stated in Fleurbaey and Maniquet (2011), the theory of well-being measurement is an essential component of a theory of fairness, since the identification of the worst-off is, in several cases, quite sensitive to the adopted approach for well-being measurement. Hence, from the perspective of those authors, the particular way in which well-being is measured already relies on ethical foundations, and cannot be separated from social welfare considerations, that is, from the construction of an index of social well-being.

When constructing an index of social well-being to be used for allocation problems, Fleurbaey and Maniquet (2011) rely extensively on Pigou-Dalton axioms, also known as transfer axioms. Those ethical axioms state conditions under which transferring resources from some individuals to other individuals leads to a social improvement. Such axioms play a key role in Fleurbaey and Maniquet's general theory of fairness.[18]

In the present two-dimensional setting, transfer principles based on the income metric or, alternatively, the lifetime metric, can be formulated as follows.

Definition 9 (Pigou-Dalton principle for reference lifetime) *Take two individuals i and j with $y_i > y_j$ and $L_i = L_j = \bar{L}$. A transfer of income from i to j is a social improvement.*

Definition 10 (Pigou-Dalton principle for reference income) *Take two individuals i and j with $y_i = y_j = \bar{y}$ and $L_i > L_j$. A transfer of lifetime from i to j is a social improvement.*

While those two principles look quite similar, there are important differences between these, and those two principles do not have the same intuitive appeal. Actually, there are two reasons to believe that the Pigou-Dalton principle for reference lifetime has more intuitive appeal.

A first difference between the two transfer principles lies in the fact that income is a desirable good, whereas lifetime may be either a desirable or an undesirable good, depending on individual-specific critical income levels. That difference plays a key role concerning the intuitive appeal of transfer axioms. Standard Pigou-Dalton axioms recommend progressive transfers of income, and it is clear, because of the desirability of income, that such transfers will make the worst off better off. However, this is not necessarily the case with life-year transfers. Indeed, if someone has a short life that is not worth being lived, transferring life-years to that person would make him worse off and not better off, contrary to what would motivate the transfer.

A second point follows directly from the previous one: if transferring life-years cannot allow for the compensation of the worst-off (which arises when the worst-off has a life that is not worth being lived), then it is not clear to see why we should base ethical judgement on the principle of progressive transfers

[18] In particular, those transfer axioms were used to examine the issue of the fair allocation of resources under unequal lifetime in Fleurbaey et al (2014).

of lifetime. From that perspective, the Pigou-Dalton principle for reference lifetime clearly dominates the other transfer principle.

In order to avoid those two criticisms, one may want to define a Pigou-Dalton principle for reference income levels in the following way.

Definition 11 (Pigou-Dalton principle for reference incomes) *Take two individuals i and j with $L_i > L_j$.*

- *If $y_i = \bar{y}_{i1}$ and $y_j = \bar{y}_{j1}$, a transfer of lifetime from i to j is a social improvement.*
- *If $y_i = \bar{y}_{i1}$ and $y_j = \bar{y}_{j2}$, a transfer of lifetime from i to j is a social improvement.*
- *If $y_i = \bar{y}_{i2}$ and $y_j = \bar{y}_{j2}$, a transfer of lifetime from i to j is a social improvement.*

That Pigou-Dalton transfer principle based on individual-specific reference income levels does not face the problems mentioned above. However, it still faces another criticism, which is common to Pigou-Dalton principles defined in terms of the lifetime metric. Actually, whereas income can be transferred from one individual to another one, life-years are non-transferable across individuals. Hence, even if one can, in theory, propose a transfer axiom stating conditions under which transferring life-years from one individual to another leads to a social improvement, the intuitive appeal of such transfer axioms is limited. Given that life-years are not transferable, it is not easy to see why we should base ethical judgements on a transfer principle defined in that metric.

In the light of this, it appears that, from the perspective of a fair allocation, the Pigou-Dalton principle for reference lifetime is more intuitive. This provides some support for the income metric, and, hence, for the equivalent income index (which is also based on a reference lifetime). It should be stressed, however, that the Pigou-Dalton principle for reference lifetime can be criticized on the grounds that it may, in some cases, consider transfers from individuals who regard their life as *not worth* being lived towards individuals who regard their life as *worth* being lived as a social improvement.[19] This is quite counterintuitive, and one may want to explore further how our well-being indexes compare individuals who disagree on whether their life is worth being lived or not.

6.4 Respect for Value of Life

All individuals have their own ideas of what makes a life worth being lived. This is captured, in our model, by the parameter $\tilde{y}_i$, the critical income level making life neutral for individual i. Considering one's own life as worth being lived, or, alternatively, as not worth being lived, is something that has strong significance, and one may want a well-being index to respect this. In particular, if two persons must be compared, one who regards his life as worth being lived,

[19] Note, however, that the Pigou-Dalton principle for reference incomes avoids that criticism.

whereas the second regards his life as not worth being lived, one may require that a well-being index ranks the first person as better off than the second person. That intuitive property can be coined as the Respect for Value of Life.

Definition 12 (Respect for Value of Life) *A well-being index $b_i(y, L)$ satisfies Respect for Value of Life if and only if, when comparing the well-being of two individuals i and j, where i regards his life as worth being lived, whereas j regards his life as not worth being lived, the index ranks i as better off than j:*

$$\text{if } y_i > \tilde{y}_i \text{ and } y_j < \tilde{y}_j \text{ then } b_i(y_i, L_i) > b_j(y_j, L_j)$$

Respect for Value of Life is intuitive, since it is hard to see how a well-being index could rank a person who regards his life as not worth being lived as better off than a person who regards his life as worth being lived. However, although intuitive, that property is not compatible with Resourcism and Lifetimism, but is only compatible with Alternative Lifetimism.

Proposition 8 *Under Respect for Preferences, neither Resourcism nor Lifetimism are compatible with Respect for Value of Life. On the contrary, Alternative Lifetimism is compatible with Respect for Value of Life.*

Proof. Consider first Resourcism and Respect for Value of Life. Assume that $\tilde{y}_i < y_i < y_j < \tilde{y}_j$. If both individuals enjoy $\bar{L}$, Resourcism implies that j is ranked better off than i (since $y_i < y_j$), against Respect for Value of Life.

Consider now Lifetimism and Respect for Value of Life. Assume that $\tilde{y}_i < y_i = y_j = \bar{y} < \tilde{y}_j$, whereas $L_i < L_j$. Lifetimism leads to $\hat{L}_i < \hat{L}_j$, against Respect for Value of Life.

Consider now Alternative Lifetimism. Take two individuals i and j with $\tilde{y}_j < y_j = \bar{y}_{j2} = \bar{y} = y_i = \bar{y}_{i1} < \tilde{y}_i$ and $L_i > L_j$. Alternative Lifetimism ranks j as better off than i, in line with Respect for Value of Life. Actually, since $\breve{L}_j = \hat{L}_j > 0 > -\hat{L}_i = \breve{L}_i$, it is always the case that Alternative Lifetimism ranks a life worth being lived as better off than a life not worth being lived. ■

If one believes in the intuitive appeal of Respect for Value of Life, Proposition 8 provides ethical support for Alternative Lifetimism.

Back to our well-being indexes, it is easy to show that the equivalent income index, which satisfies Resourcism, cannot satisfy Respect for Value of Life. In the same way, it follows also from above that the equivalent lifetime index, which satisfies Lifetimism, cannot satisfy Respect for Value of Life. However, the alternative equivalent lifetime index satisfies Respect for Value of Life.

Proposition 9
- *The equivalent income index and the equivalent lifetime index do not satisfy Respect for Value of Life.*
- *The alternative equivalent lifetime index satisfies Respect for Value of Life.*

Proof. Regarding the first item, take two individuals with $\tilde{y}_i < y_i < y_j < \tilde{y}_j$ and lifetimes $L_i = L_j = \bar{L}$. The equivalent income index ranks j as better off than i, since: $\hat{y}_j = y_j > y_i = \hat{y}_i$. This violates Respect for Value of Life.

Regarding the equivalent lifetime index, take two individuals with $\tilde{y}_i < y_i = y_j = \bar{y} < \tilde{y}_j$ and with $L_i < L_j$. The equivalent lifetime index ranks j as better off than i, since $\hat{L}_i = L_i < L_j = \hat{L}_j$. This violates Respect for Value of Life.

Finally, take the alternative equivalent lifetime index. Since $y_i > \tilde{y}_i$ and $y_j < \tilde{y}_j$, we have, no matter what L_i, L_j are, that $\breve{L}_i = \hat{L}_i > 0 > \breve{L}_j = -\hat{L}_j$, thus respecting the value of life. ■

All in all, this section provides some support for the alternative form of the equivalent lifetime index. Among our well-being indexes, only the alternative equivalent lifetime index satisfies Respect for Value of Life.

7 Well-being variations under same preferences

Let us now examine to what extent our well-being indexes provide different pictures of well-being and well-being variations under a *unique* indifference map. The reason why we would like to explore the sensitivity of well-being measurement to the postulated metric in that simplified context is that most applied studies using equivalent incomes assume, due to the lack of data at the microeconomic level, the existence of a representative agent.[20]

Under a single indifference map (and assuming that lives are worth being lived), the equivalent income index and the equivalent lifetime index (under the standard or the alternative form) rank any two situations in the same way, since these respect preferences and rely on the same indifference map. Having stressed this, one may want to know whether the reliance on a particular metric has, under a unique indifference map, a *quantitative* impact on the measurement of well-being variations, either in absolute terms or in relative terms.

To explore that issue, this section considers a representative agent model, whose preferences are given by the function $U(y, L)$, which has the same properties as the functions $U_i(y_i, L_i)$ studied above. There exists a critical income level $\tilde{y}$ making the representative individual indifferent between life and death.

Let us consider a shift from the initial situation (y', L') to the final situation (y'', L''). Let us assume that the initial situation constitutes the point of reference: $\bar{y} = y'$ and $\bar{L} = L'$. Using the equivalent income index, the measured absolute and relative variations of well-being are, respectively:

$$\Delta\hat{y} \equiv \hat{y}(y'', L'') - \hat{y}(y', L') \text{ and } \frac{\Delta\hat{y}}{\hat{y}(y', L')} = \frac{\hat{y}(y'', L'') - \hat{y}(y', L')}{\hat{y}(y', L')}$$

where $\hat{y}(y'', L'')$ is defined implicitly by the equality: $U(\hat{y}(y'', L''), \bar{L}) = U(y'', L'')$.

Using the equivalent lifetime index, the measured absolute and relative variations of well-being are, respectively:

$$\Delta\hat{L} \equiv \hat{L}(y'', L'') - \hat{L}(y', L') \text{ and } \frac{\Delta\hat{L}}{\hat{L}(y', L')} = \frac{\hat{L}(y'', L'') - \hat{L}(y', L')}{\hat{L}(y', L')}$$

[20]See, for instance, Usher (1980), Williamson (1984), Crafts (1997), Costa and Steckel (1997), Murphy and Topel (2003), Nordhaus (2003), and Becker et al (2005).

where $\hat{L}(y'', L'')$ is defined implicitly by the equality: $U\left(\bar{y}, \hat{L}(y'', L'')\right) = U(y'', L'')$.

Using the alternative equivalent lifetime index, the measured absolute and relative variations of well-being are, respectively:

$$\Delta\breve{L} \equiv \breve{L}(y'', L'') - \breve{L}(y', L') \text{ and } \frac{\Delta\breve{L}}{\breve{L}(y', L')} = \frac{\breve{L}(y'', L'') - \breve{L}(y', L')}{\breve{L}(y', L')}$$

where $\breve{L}(y'', L'') = \hat{L}(y'', L'')$ if $y'' > \tilde{y}$, $\breve{L}(y'', L'') = -\hat{L}(y'', L'')$ if $y'' > \tilde{y}$ and $\breve{L}(y'', L'') = 0$ if $y'' = \tilde{y}$, and where also $\breve{L}(y', L') = \hat{L}(y', L')$ if $y' > \tilde{y}$, $\breve{L}(y', L') = -\hat{L}(y', L')$ if $y' > \tilde{y}$ and $\breve{L}(y', L') = 0$ if $y' = \tilde{y}$.

At this stage, it should be stressed that there exist simple relations between welfare variations measured by means of the lifetime equivalent index and the alternative lifetime equivalent index.[21] In the light of those relations, we will, in this section, focus only on the comparison of measured well-being variations under the equivalent income and the standard equivalent lifetime indexes.

Without imposing further assumptions on $U(y, L)$, it is difficult to derive precise results concerning the comparison of $\Delta\hat{y}$ with $\Delta\hat{L}$, and of $\frac{\Delta\hat{y}}{\hat{y}}$ with $\frac{\Delta\hat{L}}{\hat{L}}$. To facilitate the comparison, let us assume that $U(y, L)$ takes the standard, time-additive, form: $U(y, L) = Lu(y)$, with $u'(y) > 0$ and $u''(y) < 0$. That formulation is standard in life-cycle theory. This amounts to look at the well-being of a life as the sum of temporal well-being levels.

Proposition 10 *Assume common preferences, with $U(y, L) = Lu(y)$, where $u'(y) > 0$ and $u''(y) < 0$. Consider a shift from (y', L') to (y'', L''). Consider equivalent income and equivalent lifetime indexes based on $\bar{y} = y'$ and $\bar{L} = L'$, with, without loss of generality, $\bar{y} > \bar{L}$.*

- *the measured absolute variation in well-being is, in absolute value, larger under the equivalent income index than under the equivalent lifetime index;*
- *the measured relative variation in well-being is, in absolute value, larger under the equivalent income index than under the equivalent lifetime index.*

Proof. See the Appendix. ■

Proposition 10 states that, whatever one measures absolute or relative variations in well-being, the equivalent income index leads to measured well-being changes that are, in absolute value, larger than the ones measured under the equivalent lifetime index.

Besides the measurement of the well-being variation by comparing the levels of indexes in situations (y', L') and (y'', L''), one may also want to compare, along a given metric, equivalent and standard variables for the final situation (y'', L''). The intuition is that the equivalent index defined along a dimension of

[21] Actually, when $y', y'' > \tilde{y}$, we have $\Delta\breve{L} = \Delta\hat{L}$. Moreover, when $y', y'' < \tilde{y}$, we have $\Delta\breve{L} = -\Delta\hat{L}$. Finally, if $y'' > \tilde{y} > y'$, we have that: $\Delta\breve{L} = \hat{L}(y'', L'') - \left(-\hat{L}(y', L')\right) = \Delta\hat{L} + 2\hat{L}(y', L') > \Delta\hat{L}$, whereas, if $y'' < \tilde{y} < y'$, we have: $\Delta\breve{L} = -\hat{L}(y'', L'') - \hat{L}(y', L') = \Delta\hat{L} - 2\hat{L}(y'', L'') < \Delta\hat{L}$.

well-being incorporates the change in the other dimension of well-being, unlike the latter. Hence this comparison allows us to quantity the extra-value brought by the composite index of well-being in comparison with raw indicators.

Proposition 11 *Assume common preferences, with $U(y, L) = Lu(y)$, where $u'(y) > 0$ and $u''(y) < 0$. Consider a shift from (y', L') to (y'', L'').*

- *If $L'' - L' > y'' - y' > 0$ or if $L'' - L' < y'' - y' < 0$, then the gap between the equivalent income and the standard income exceeds, in absolute value, the gap between the equivalent lifetime and the standard lifetime. Otherwise, the two gaps cannot be ranked.*
- *If there is a well-being gain with $\frac{L'y''}{L''y'} < 1$, or a well-being loss with $\frac{L'y''}{L''y'} > 1$, the relative gap between the equivalent income and the standard income exceeds, in absolute value, the relative gap between the equivalent lifetime and the standard lifetime. Otherwise, the two gaps cannot be ranked.*

Proof. See the Appendix. ■

Proposition 11 states that, when comparing the equivalent index with the standard (unadjusted) variable, the postulated metric also matters. Actually, when the variation in lifetime dominates the variation in income, the gap between the equivalent income and the standard income always exceeds the gap between the equivalent lifetime and the standard lifetime. That result is true whatever we consider absolute or relative gaps.

All in all, this section shows that, even if one assumes a representative agent (unique indifference map), the choice of a particular metric matters for the measurement of well-being variations. Adopting the equivalent income index or the equivalent lifetime index has an impact on the measured variations in well-being (in absolute and relative terms), as well as on the measured gap between equivalent and standard variables along the same dimension of well-being.

8 An application to the Syrian War

In order to further examine the sensitivity of the measurement of well-being to the postulated metric, this section takes the case of the measurement of well-being in the context of the Syrian War. The Syrian War (2011-2019) is at the origin of thousands of deaths and injured persons, and caused the displacement of thousands of refugees, a strong contraction of economic activity and massive destructions (including important cultural sites).[22]

	Before Conflict (2010)	Conflict (2016)
Population (inside Syria)	20.7 million	18.5 million
Per Capita Income (current $)	$2806	$1215
Life expectancy at birth	74.4 years	69.5 years

Table 1: Basic indicators, Syria, 2010 and 2016. Sources: World Bank.

[22] On the estimation of the number of deaths and injured persons, see the report of the Syrian Centre for Policy Research (2016). See also the report of the World Bank (2017).

Whereas the War affected numerous dimensions of life, we will, throughout this section, focus only on the two dimensions that were studied in the theoretical part of the paper, i.e. income per period and lifetime. Due to data limitation, we will abstract here from inequality among those two dimensions, and consider a representative agent framework. In this section, we will measure the first dimension by the income per capita (in current US$), denoted by y, and measure the second dimension by life expectancy at birth, denoted by L.[23]

Following Becker et al (2005), we assume that preferences on lotteries of life satisfy the expected utility hypothesis, that the utility of a life is additive in temporal utilities, and that temporal utility depends only on income, and takes a constant-elasticity form, so that preferences can be represented by:[24]

$$U(y, L) = L\left[\frac{(y)^{1-\sigma}}{1-\sigma} - \alpha\right] \tag{1}$$

where L is the life expectancy, while $\sigma > 0$ and $\alpha \lesseqgtr 0$.[25]

The function $U(y, L)$ is increasing in income per head y, but can be increasing or decreasing in lifetime L, depending on how large income per head is. There exists a threshold for income per head $\tilde{y} = [\alpha(1-\sigma)]^{\frac{1}{1-\sigma}}$ such that lifetime is a desirable good for $y > \tilde{y}$, whereas lifetime is an undesirable good for $y < \tilde{y}$, and a neutral good for $y = \tilde{y}$.[26]

Based on that functional form, the equivalent income index is equal to:

$$\hat{y} = \left[(1-\sigma)\left[\left(\frac{(y)^{1-\sigma}}{1-\sigma} - \alpha\right)\frac{L}{\bar{L}} + \alpha\right]\right]^{\frac{1}{1-\sigma}} \tag{2}$$

where $\bar{L}$ is the reference lifetime.

Moreover, the equivalent lifetime index is equal to:

$$\hat{L} = L\frac{\left[\frac{(y)^{1-\sigma}}{1-\sigma} - \alpha\right]}{\left[\frac{(\bar{y})^{1-\sigma}}{1-\sigma} - \alpha\right]} \tag{3}$$

where $\bar{y}$ is the reference income per period.

As far as the calibration of σ is concerned, we follow Blundell et al (1994) and take $\sigma = 0.83$. Concerning α, this can be calibrated using studies on the

[23] Throughout this section, we thus take life expectancy as an indicator of the average lifetime in the population, i.e. the lifetime of the representative individual. This consists of an approximation for the lifetime variable studied in the theoretical part of the paper. Unfortunately, cohort life tables are not available for the population under study.

[24] We abstract here from pure time preferences. Survival probabilities play here the role of biological discount factors.

[25] L is defined as $\sum_{k=0}^{T} s_{k+1}$ where T is the maximum lifespan, and s_k is the (unconditional) probability of survival to age k.

[26] As above, the utility of being dead is normalized to 0.

value of a statistical life (VSL), defined as the marginal rate of substitution between income and mortality risk:

$$VSL = -\frac{\frac{\partial U}{\partial d_0}}{\frac{\partial U}{\partial y_0}} = \frac{\frac{L}{s_0}\left[\frac{y_0^{1-\sigma}}{1-\sigma} - \alpha\right]}{s_0\left(y_0\right)^{-\sigma}} \tag{4}$$

where d_j is the probability of death at age j while $s_i = \prod_{j=0}^{i}(1-d_j)$ is the probability of survival to age i.

In order to calibrate α on the basis of VSL estimates, we rely here on the meta-analysis of VSL studies carried out by Miller (2000). Miller collected 68 studies estimating VSL across 13 countries, while using various methodologies (wage-risk studies, contingent valuation methods, behavioral studies), in order to estimate rules of thumb, which relate the VSL to the level of GDP per capita. The interest of those rules of thumb is the following. Most VSL studies have focused exclusively on rich countries, whereas for most countries there exists no direct VSL estimate. Hence, the rules of thumb estimated by Miller allow us to extrapolate VSL estimates for any country, by merely knowing the GDP per capita of that country. This is the case for Syria, for which there exists no direct VSL estimate. Thus Miller's rules of thumb allow us to have an indirect estimate of the VSL for Syria, and to use it for our calibration.[27]

Following Miller's (2000) rules of thumb, the VSL amounts to between 120 and 180 times GDP per capita. Hence, on the basis of the pre-conflict income per head (\$2806), we obtain two values for α: α equal either to 16.46 (lower bound of VSL) or to 13.35 (upper bound of VSL).[28] This implies that the critical income level $\tilde{y}$ is equal to \$424 (low VSL) or to \$123 (high VSL). Observed income levels being above those levels, this implies that, provided $\bar{y}_2 = \bar{y}$, the alternative equivalent lifetime index takes here the same level as the standard equivalent lifetime index. This section will thus concentrate on the comparison between the equivalent income and the equivalent lifetime indexes.

In order to compute equivalent income and equivalent lifetime indexes, we take, as reference levels for income per period and lifetime, the pre-War levels of y and L, which leads to $\bar{y} = 2806$ and $\bar{L} = 74.4$.[29] Figure 7 shows the equivalent income index for 2010 (pre-War) and 2016 (War), under low and high VSL, whereas Figure 8 shows the equivalent lifetime index for 2010 and 2016 (also under low and high VSL).

[27] Note that relying on rules of thumb constitutes an approximation. One limitation of using rules of thumb is that this assumes some form of stability of preferences concerning income-risk trade-offs across countries and time periods. Back to the case of Syria, if the War modified preferences in a particular way, this will not be captured by our calibrations based on Miller's rules of thumb.

[28] We take here, as a proxy, $s_0 \approx 1$.

[29] Obviously, other reference points could have been selected. However, for the sake of space, we will take the pre-War income and lifetime as references throughout this section, because the pre-War situation seems to be a natural reference point, unlike the War situation.

Figures 7 and 8 show the strong deterioration in standards of living due to the War. However, although the two indexes agree qualitatively, in the sense that these provide the same rankings, these lead to quite different pictures from a quantitative perspective. Two main differences should be highlighted.[30]

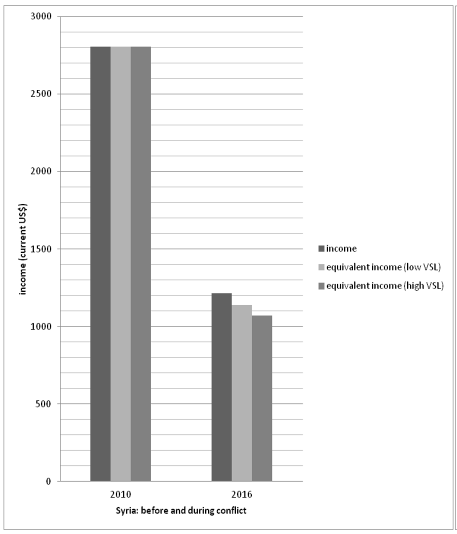

Figure 7: Income and equivalent income in Syria, 2010 and 2016.

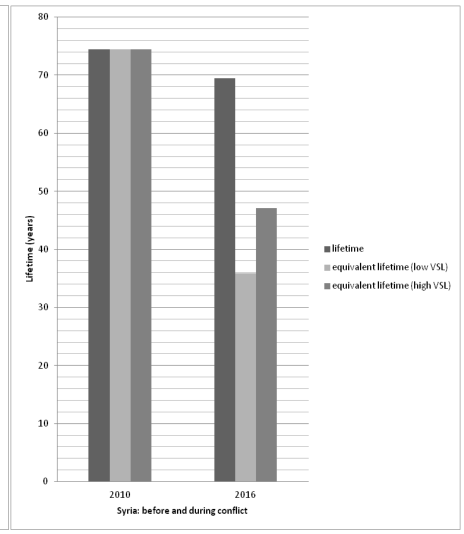

Figure 8. Lifetime and equivalent lifetime in Syria, 2010 and 2016.

A first important difference concerns the measurement of the well-being loss due to the War. Using the equivalent income index, the average well-being loss due to the War lies, in relative terms, between $\left|\frac{1140-2806}{2806}\right| = 0.593$ (under the low VSL) and $\left|\frac{1071-2806}{2806}\right| = 0.618$ (under the high VSL). However, when one uses the equivalent lifetime index, the measured (average) well-being loss due to the War lies, in relative terms, between $\left|\frac{47-69.5}{69.5}\right| = 0.323$ (under the high VSL) and $\left|\frac{36-69.5}{69.5}\right| = 0.482$ (under the low VSL). Note that those results are, from a qualitative perspective, in line with Proposition 10, which states that measured well-being variations are, under general conditions, larger under the equivalent income than under the equivalent lifetime. However, our application reveals that the adopting the income metric or the lifetime metric can have

[30] A third difference concerns the comparison of well-being indexes under the high and the low VSL estimates. Whereas the equivalent income takes lower levels when the high VSL estimate is adopted, it is the opposite for the equivalent lifetime index, which takes higher levels when the high VSL estimate is assumed. The intuition goes as follows. When a higher value is assigned to life in comparison to income, this means that the willingness to pay (WTP), in income terms, to come back to pre-conflict survival conditions goes up, leading to a lower equivalent income index. On the contrary, when a higher value is assigned to life in comparison to income, this tends to reduce the WTP, in life-year terms, to come back to pre-conflict income conditions, which leads to a higher equivalent lifetime index.

strong quantitative consequences, by leading to overestimate or underestimate the (relative) average well-being loss due to the War by about 50 %.

Second, whereas the equivalent income indexes during the War are close to the standard income, this is not the case when considering equivalent lifetime indexes, which exhibit much lower levels than the (unadjusted) lifetime.[31] Figure 8 shows that the hypothetical lifetime that would, combined with the pre-War income, make the representative individual indifferent with respect to the War situation is as low as 36 years (under the low VSL) and 47 years (under the high VSL). Thus the deprivation due to a lower income has been so strong that a representative individual would be willing to give up between 22.5 years (i.e. $69.5 - 47$) and 33.5 years (i.e. $69.5 - 36$) of life to go back to the pre-War income. In relative terms, the differential between the equivalent lifetime and the standard lifetime (between 32% and 48%) is much larger than the differential between the equivalent income and the standard income (between 6% and 12%).

Why is it the case that adopting Resourcism or Lifetimism makes such a large difference here? To have a clue, Figure 9 reproduces the indifference map in the (income, lifetime) space, under the low VSL estimate, as well as the equivalent income index and the equivalent lifetime index. Figure 9 makes appear that the reason why the equivalent income and the equivalent lifetime indexes lead to different pictures lies in the curvature of indifference curves in the area of the indifference map between the initial point (2010) and the War point (2016).

Consider first the equivalent income index. The high slope of indifference curves for income levels lower than the War level explains why a small movement along the indifference curve - and thus a small income reduction - suffices to compensate for the 5-year improvement in life expectancy when the reference (pre-conflict) survival conditions are imposed. This low WTP for coming back to pre-conflict survival conditions can be explained by the extreme poverty due to the War. This low WTP, in income terms, for an increase in lifetime, explains why the equivalent income is very close to the standard income in 2016.

Consider now the equivalent lifetime index. The high slope of the indifference curve around the War point explains that a large lifetime reduction is needed to compensate the substantial loss in income (from \$2805 to \$1215). Thus the high WTP, in life-year terms, for an increase in income explains why the equivalent lifetime index is much lower than (unadjusted) lifetime in 2016. Note that this high WTP (in life-year terms) for coming back to the pre-War income is also explained by the extreme poverty due to the War. Extreme poverty explains why, although individuals would be willing to give up little income to turn back to pre-conflict survival conditions, they would be willing to give up a large number of life-years to turn back to pre-War material standards of living.

[31] The size of the differential between the standard income and the equivalent income is quite small. The gap, for 2016, equals only $\$1215 - \$1140 = \$75$ under the lower bound of the VSL, and $\$1215 - \$1071 = \$144$ under the higher bound of the VSL.

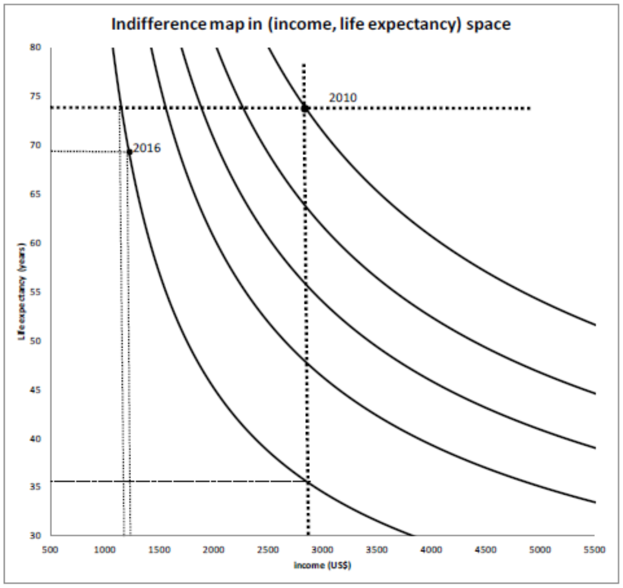

Figure 9. Construction of equivalent income index and equivalent lifetime index for 2016.

All in all, the measurement of the (average) well-being loss due to the War illustrates that relying on Resourcism or on Lifetimism leads to different pictures of the deprivation caused by the War. The reason why the pictures provided by the two indexes are so different lies in the fact that the War bundle lies in an area of the indifference map where life-years have a low value with respect to income (or, alternatively, income has a high value with respect to life-years). Hence, relying on the income metrics or on the lifetime metrics makes a substantial difference when describing the overall deprivation due to the War.

9 Conclusions

In this paper, we proposed to examine the role of the metric in the measurement of well-being by means of equivalent indexes, by comparing, in the (income, lifetime) space, the equivalent income index with the equivalent lifetime index. At first glance, one may believe that relying on the money metric or on the life-year metric does not make a difference for well-being measurement. However, our analysis revealed that relying on a particular metric makes a substantial difference, at various levels of analysis (Table 2).

	equivalent income	equivalent lifetime	alternative equivalent lifetime
Existence conditions	weak	strong	weak
Respect for Preferences	yes	no	yes
Same Preference	yes	yes	no
Resourcism	yes	no	no
Lifetimism	no	yes	no
Alternative Lifetimism	no	no	yes
Respect for Value of Life	no	no	yes
|Absolute WB variations|	larger	smaller	it depends
|Relative WB variations|	larger	smaller	it depends

Table 2: Summary of our results.

A first important difference lies in the fact that existence conditions are weaker for the equivalent income index than for the equivalent lifetime index, but the alternative equivalent lifetime index allows to solve, to some extent, the existence problems faced by the latter. Table 2 also shows that the three indexes under comparison rely on quite different approaches for the interpersonal comparison of well-being: Resourcism and (Alternative) Lifetimism, which, under Respect for Preferences, lead to contradictory rankings. Quite interestingly, Table 2 points also to some form of dilemma: the alternative equivalent lifetime index fails to satisfy Same Preference, but, at the same time, it is the only index under comparison to satisfy Respect for Value of Life. Thus, from a qualitative perspective, the postulated metric definitely affects well-being comparisons.

From a quantitative perspective, relying on a particular metric also matters. Under a unique indifference map, absolute and relative well-being variations are larger under the equivalent income index than under the equivalent lifetime index. That point is illustrated by the measurement of the (average) well-being loss due to the Syrian War. Our calculations also show that, whereas the equivalent income index is, in War times, very close to the standard income, this is not the case for the equivalent lifetime index, which is, in War times, much lower than the standard lifetime. Therefore, the two well-being indexes provide different pictures of the well-being loss due to the War.

In sum, our comparison of the equivalent income index and the equivalent lifetime index shows that the choice of the metric matters for well-being measurement. This is true when considering the comparison of well-being across individuals having distinct indifference maps. But even if one assumes a unique indifference map, the chosen metric still matters, not from a qualitative perspective (since rankings are here preserved), but from a quantitative perspective.

To conclude, it should be stressed that this paper focused only on the issue of the metric for well-being measurement, while relying on equivalent indexes, constructed by fixing (constant) reference levels for some dimensions of well-being. Alternatively, one may consider other well-being indexes relying not on a fixed reference level, but, instead, on a reference ray increasing in both arguments, as in Fleurbaey and Maniquet (2017, 2018, 2019). Relying on such a reference ray is a way to escape from other criticisms against the standard equivalent income index, which point to the arbitrariness of the (fixed) reference level (see Fleurbaey 2016). The present paper did not consider that issue, and focused instead on a more particular problem, i.e. the comparison of the income and the lifetime metrics for well-being measurement. However, a more comprehensive study of well-being measurement should include all those aspects of the construction of well-being indexes. Much work remains to be done, in the future, on the construction of appealing well-being indexes.

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11 Appendix

11.1 Proof of Proposition 4

Regarding the equivalent income index, we have, for two bundles (y_i, L_i) and (y_i', L_i'), equivalent income levels $\hat{y}_i$ and $\hat{y}_i'$ satisfying:

$$U_i\left(\hat{y}_i, \bar{L}\right) = U_i(y_i, L_i) \text{ and } U_i\left(\hat{y}_i', \bar{L}\right) = U_i(y_i', L_i')$$

Given the monotonicity of $U_i(\cdot)$ in y_i, if $U_i(y_i, L_i) > U_i(y_i', L_i')$, then $\hat{y}_i > \hat{y}_i'$. Moreover, if $U_i(y_i, L_i) < U_i(y_i', L_i')$, then $\hat{y}_i < \hat{y}_i'$. Finally, if $U_i(y_i, L_i) = U_i(y_i', L_i')$, then $\hat{y}_i = \hat{y}_i'$. We thus have: $\hat{y}_i' \gtreqless \hat{y}_i \iff U_i(y_i', L_i') \gtreqless U_i(y_i, L_i)$, that is, Respect for Preferences is satisfied.

Consider now the equivalent lifetime index. Assume $y_i > \tilde{y}_i$ and $\bar{y} > \tilde{y}_i$. For two bundles (y_i, L_i) and (y_i', L_i'), equivalent lifetime $\hat{L}_i$ and $\hat{L}_i'$ satisfy:

$$U_i\left(\bar{y}, \hat{L}_i\right) = U_i(y_i, L_i) \text{ and } U_i\left(\bar{y}, \hat{L}_i'\right) = U_i(y_i', L_i')$$

If $\bar{y} > \tilde{y}_i$, it is easy to see that if $U_i(y_i, L_i) > U_i(y_i', L_i')$, then it has to be the case, by monotonicity of $U_i(y_i, L_i)$ in L_i, that $\hat{L}_i > \hat{L}_i'$. Moreover, if $U_i(y_i, L_i) < U_i(y_i', L_i')$, then $\hat{L}_i < \hat{L}_i'$. Finally, if $U_i(y_i, L_i) = U_i(y_i', L_i')$, then $\hat{L}_i = \hat{L}_i'$. Thus Respect for Preferences is satisfied when $y_i > \tilde{y}_i$ and $\bar{y} > \tilde{y}_i$.

Assume now $y_i < \tilde{y}_i$ and $\bar{y} < \tilde{y}_i$. If $U_i(y_i, L_i) > U_i(y_i', L_i')$, then we need $U_i\left(\bar{y}, \hat{L}_i\right) > U_i\left(\bar{y}, \hat{L}_i'\right)$, which implies $\hat{L}_i < \hat{L}_i'$. Thus Respect for Preferences is not satisfied in that case.

Concerning the alternative equivalent lifetime index, three cases can arise.

If $y_i > y_i' > \tilde{y}_i$, Respect for Preferences is satisfied, and the proof is similar to the one for the standard equivalent lifetime index (since in that case $\breve{L}_i = \hat{L}_i$), except that the reference income is now $\bar{y}_{i2}$.

If $y_i < y_i' < \tilde{y}_i$, we have, for two bundles (y_i, L_i) and (y_i', L_i'), alternative equivalent lifetime levels $\breve{L}_i = -\hat{L}_i$ and $\breve{L}_i' = -\hat{L}_i'$ where $\hat{L}_i$ and $\hat{L}_i'$ satisfy:

$$U_i\left(\bar{y}_{i1}, \hat{L}_i\right) = U_i(y_i, L_i) \text{ and } U_i\left(\bar{y}_{i1}, \hat{L}_i'\right) = U_i(y_i', L_i')$$

Given that $y_i < y_i' < \tilde{y}_i$, we have that if $U_i(y_i, L_i) > U_i(y_i', L_i')$, then it has to be the case, by monotonicity of $U_i(y_i, L_i)$ in L_i, that $\hat{L}_i < \hat{L}_i'$, leading to $\breve{L}_i > \breve{L}_i'$. Moreover, if $U_i(y_i, L_i) < U_i(y_i', L_i')$, then $\hat{L}_i > \hat{L}_i'$, leading to $\breve{L}_i < \breve{L}_i'$. Finally, if $U_i(y_i, L_i) = U_i(y_i', L_i')$, then $\hat{L}_i = \hat{L}_i'$, leading to $\breve{L}_i = \breve{L}_i'$. Thus Respect for Preferences is satisfied.

If $y_i < y_i' = \tilde{y}_i$ or $y_i > \tilde{y}_i = y_i'$, Respect for Preferences also holds, since in the former case we have $\breve{L}_i = -\hat{L}_i < 0 = \breve{L}_i'$, whereas in the latter case $\breve{L}_i = \hat{L}_i > \breve{L}_i' = 0$.

11.2 Proof of Proposition 5

Regarding the equivalent income index, we have, for two bundles (y_i, L_i) and (y_j, L_j), equivalent income levels $\hat{y}_i$ and $\hat{y}_j$ satisfying:

$$U\left(\hat{y}_i, \bar{L}\right) = U(y_i, L_i) \text{ and } U\left(\hat{y}_j, \bar{L}\right) = U(y_j, L_j)$$

Hence, we have that if $U(y_i, L_i) \gtreqless U(y_j, L_j)$, it follows that $\hat{y}_i \gtreqless \hat{y}_j$. Alternatively, if $\hat{y}_i \gtreqless \hat{y}_j$, then $U(y_i, L_i) \gtreqless U(y_j, L_j)$. Thus the Same Preference principle is satisfied by the equivalent income index.

Regarding the equivalent lifetime index, two cases must be distinguished.

If $y_i, y_j > \tilde{y}_i, \tilde{y}_j$ and $\bar{y} > \tilde{y}_i, \tilde{y}_j$, equivalent lifetime $\hat{L}_i$ and $\hat{L}_j$ satisfy:

$$U\left(\bar{y}, \hat{L}_i\right) = U(y_i, L_i) \text{ and } U\left(\bar{y}, \hat{L}_j\right) = U(y_j, L_j)$$

Hence, we have that if $U(y_i, L_i) \gtreqless U(y_j, L_j)$, it follows that $\hat{L}_i \gtreqless \hat{L}_j$. Alternatively, if $\hat{L}_i \gtreqless \hat{L}_j$, then $U(y_i, L_i) \gtreqless U(y_j, L_j)$. Thus the Same Preference principle is satisfied by the equivalent lifetime index in that case.

However, this is not true when $y_i, y_j < \tilde{y}_i, \tilde{y}_j$ and $\bar{y} < \tilde{y}_i, \tilde{y}_j$. Indeed, in that case, we have that if $U(y_i, L_i) \gtreqless U(y_j, L_j)$, it follows that $\hat{L}_i \lesseqgtr \hat{L}_j$. And if $\hat{L}_i \gtreqless \hat{L}_j$, then $U(y_i, L_i) \lesseqgtr U(y_j, L_j)$. Thus the Same Preference principle is not satisfied by the equivalent lifetime index in that case.

Concerning the alternative equivalent lifetime index, it is easy to show that it does not satisfy the Same Preference principle. To see this, take two individuals with same preferences. Suppose that $y_i, y_j > \tilde{y}_i, \tilde{y}_j$ with $\bar{y}_{j2} < y_j < y_i < \bar{y}_{i2}$. Suppose also $L_i = L_j = L$. The alternative equivalent lifetime indexes satisfy:

$$U\left(\bar{y}_{i2}, \breve{L}_i\right) = U(y_i, L) \text{ and } U\left(\bar{y}_{j2}, \breve{L}_j\right) = U(y_j, L)$$

We have $U(y_i, L) > U(y_j, L)$. The Same Preference principle requires that $\breve{L}_i > \breve{L}_j$. However, since $\bar{y}_{j2} < y_j$, we have $\breve{L}_j > L$. But since $y_i < \bar{y}_{i2}$, we have also $\breve{L}_i < L$. Thus we have $\breve{L}_j > L > \breve{L}_i$, contradicting $\breve{L}_i > \breve{L}_j$. The alternative equivalent lifetime index does not satisfy the Same Preference principle.

11.3 Proof of Proposition 7

Take the equivalent income index. When $L_i = L_j = \bar{L}$, we have:

$$\begin{aligned} U_i\left(\hat{y}_i, \bar{L}\right) &= U_i(y_i, \bar{L}) \iff \hat{y}_i = y_i \\ U_j\left(\hat{y}_j, \bar{L}\right) &= U_j(y_j, \bar{L}) \iff \hat{y}_j = y_j \end{aligned}$$

Hence it follows that: $\hat{y}_i \gtreqless \hat{y}_j \iff y_i \gtreqless y_j$, that is, that Resourcism is satisfied.

Take the equivalent lifetime index. When $y_i = y_j = \bar{y}$, we have

$$\begin{aligned} U_i\left(\bar{y}, L_i\right) &= U_i(\bar{y}, \hat{L}_i) \iff \hat{L}_i = L_i \\ U_j(\bar{y}, L_j) &= U_j(\bar{y}, \hat{L}_j) \iff \hat{L}_j = L_j \end{aligned}$$

Hence it follows that: $\hat{L}_i \gtreqless \hat{L}_j \iff L_i \gtreqless L_j$, i.e., that Lifetimism is satisfied.

Take the alternative equivalent lifetime index. Suppose that $\bar{y}_{i1} < \tilde{y}_i < y_i < y_j < \bar{y}_{j1} < \tilde{y}_j < \bar{y}_{i2} < \bar{y}_{j2}$. When $L_i = L_j = \bar{L}$, we have:

$$\begin{aligned} \breve{L}_i &= \hat{L}_i \text{ where } U_i\left(\bar{y}_{i2}, \hat{L}_i\right) = U_i(y_i, \bar{L}) \\ \breve{L}_j &= -\hat{L}_j \text{ where } U_j\left(\bar{y}_{j1}, \hat{L}_j\right) = U_j(y_j, \bar{L}) \end{aligned}$$

Given that $L_i = L_j = \bar{L}$, Resourcism would require that j is regarded as better off than i, since $y_i < y_j$. But we have $\breve{L}_i > \breve{L}_j$. Thus Resourcism is not satisfied.

Regarding Lifetimism, suppose now that $y_i = y_j = \bar{y}$ and that $L_j > L_i$. Suppose also that $\bar{y}_{i1} < \tilde{y}_i < y_i = y_j < \bar{y}_{j1} < \tilde{y}_j < \bar{y}_{i2} < \bar{y}_{j2}$. We have:

$$\begin{aligned} \breve{L}_i &= \hat{L}_i \text{ where } U_i\left(\bar{y}_{i2}, \hat{L}_i\right) = U_i(\bar{y}, L_i) \\ \breve{L}_j &= -\hat{L}_j \text{ where } U_j\left(\bar{y}_{j1}, \hat{L}_j\right) = U_j(\bar{y}, L_j) \end{aligned}$$

Given that $y_i = y_j = \bar{y}$, Lifetimism would require that j is regarded as better off than i, since $L_i < L_j$. But we have $\breve{L}_i > \breve{L}_j$.

11.4 Proof of Proposition 10

Note first that, from the definitions of the equivalent income and the equivalent lifetime indexes, we have, by transitivity of equality:

$$\bar{L}u\left(\hat{y}\left(y'', L''\right)\right) = L''u(y'') = \hat{L}\left(y'', L''\right)u(\bar{y}) \implies \hat{y}\left(y'', L''\right) = u^{-1}\left(\frac{\hat{L}\left(y'', L''\right)}{\bar{L}}u(\bar{y})\right)$$

Consider the measurement of absolute variations in well-being. We have:

$$\begin{aligned} \Delta\hat{y} \gtreqless \Delta\hat{L} &\iff u^{-1}\left(\frac{\hat{L}\left(y'', L''\right)}{\bar{L}}u(\bar{y})\right) - \bar{y} \gtreqless \hat{L}\left(y'', L''\right) - \bar{L} \\ &\iff u^{-1}\left(Xu(\bar{y})\right) \gtreqless X\bar{L} - \bar{L} + \bar{y} \end{aligned}$$

where we denote $X \equiv \frac{\hat{L}\left(y'', L''\right)}{\bar{L}}$. Since $u'(\cdot) > 0$, we have:

$$\begin{aligned} \Delta\hat{y} \gtreqless \Delta\hat{L} &\iff u \circ \left[u^{-1}\left(Xu(\bar{y})\right)\right] \gtreqless u(\bar{L}(X-1) + \bar{y}) \\ &\iff Xu(\bar{y}) \gtreqless u(\bar{L}(X-1) + \bar{y}) \end{aligned}$$

In case of well-being gain, we have $X > 1$, so that, by concavity of $u(\cdot)$, we have $Xu(\bar{y}) > u(X\bar{y})$. Moreover, from $\bar{y} > \bar{L}$, we have $X\bar{y} > \bar{L}(X-1) + \bar{y}$, so that $u(X\bar{y}) > u(\bar{L}(X-1) + \bar{y})$. Hence, by transitivity, it follows that: $Xu(\bar{y}) > u(\bar{L}(X-1) + \bar{y})$. Hence $\Delta\hat{y} > \Delta\hat{L}$. In case of a well-being loss (i.e. $X < 1$), we have, by concavity of $u(\cdot)$, $Xu(\bar{y}) < u(X\bar{y})$. Moreover, from $\bar{y} > \bar{L}$, we have also, under $X < 1$, that $X\bar{y} < \bar{L}(X-1) + \bar{y}$, so that $u(X\bar{y}) < u(\bar{L}(X-1) + \bar{y})$.

Hence, by transitivity, it follows that: $Xu(\bar{y}) < u(\bar{L}(X-1)+\bar{y})$. Thus we have $\Delta\hat{y} < \Delta\hat{L}$. But $\Delta\hat{y}$ and $\Delta\hat{L}$ are both negative, so that, in absolute value, we have: $|\Delta\hat{y}| > \left|\Delta\hat{L}\right|$.

Consider now relative variations in well-being. We have:

$$\begin{aligned} \frac{\Delta\hat{y}}{\hat{y}\left(y',L'\right)} \gtrless \frac{\Delta\hat{L}}{\hat{L}\left(y',L'\right)} \quad &\Longleftrightarrow \quad \frac{\hat{y}\left(y'',L''\right)-\bar{y}}{\bar{y}} \gtrless \frac{\hat{L}\left(y'',L''\right)-\bar{L}}{\bar{L}} \\ &\Longleftrightarrow \quad u^{-1}\left(Xu(\bar{y})\right) \gtrless X\bar{y} \\ &\Longleftrightarrow \quad Xu(\bar{y}) \gtrless u\left(X\bar{y}\right) \end{aligned}$$

When there is a well-being gain ($X > 1$), we have $Xu(\bar{y}) > u\left(X\bar{y}\right)$, implying that $\frac{\Delta\hat{y}}{\hat{y}} > \frac{\Delta\hat{L}}{\hat{L}}$. On the contrary, when there is a loss in well-being, $X < 1$, we have $Xu(\bar{y}) < u\left(X\bar{y}\right)$, so that $\frac{\Delta\hat{y}}{\hat{y}} < \frac{\Delta\hat{L}}{\hat{L}}$. However, $\frac{\Delta\hat{y}}{\hat{y}}$ and $\frac{\Delta\hat{L}}{\hat{L}}$ are both negative, so that, in absolute value, we obtain $\left|\frac{\Delta\hat{y}}{\hat{y}}\right| > \left|\frac{\Delta\hat{L}}{\hat{L}}\right|$.

11.5 Proof of Proposition 11

Let us first define:

$$\begin{aligned} \Lambda\left(y'',L''\right) \quad &\equiv \quad \hat{y}\left(y'',L''\right)-y'' \text{ and } \frac{\Lambda\left(y'',L''\right)}{y''} = \frac{\hat{y}\left(y'',L''\right)-y''}{y''} \\ \Omega\left(y'',L''\right) \quad &\equiv \quad \hat{L}\left(y'',L''\right)-L'' \text{ and } \frac{\Omega\left(y'',L''\right)}{L''} = \frac{\hat{L}\left(y'',L''\right)-L''}{L''} \end{aligned}$$

Consider first the comparison of $\Lambda\left(y'',L''\right)$ with $\Omega\left(y'',L''\right)$. We have:

$$\begin{aligned} \hat{y}\left(y'',L''\right)-y'' \gtrless \hat{L}\left(y'',L''\right)-L'' \quad &\Longleftrightarrow \quad u^{-1}\left(Xu(\bar{y})\right)-y'' \gtrless \bar{L}X-L'' \\ &\Longleftrightarrow \quad Xu(\bar{y}) \gtrless u\left(\bar{L}X-L''+y''\right) \end{aligned}$$

By concavity of $u(\cdot)$, we have, in case of well-being gain ($X > 1$), that $Xu(\bar{y}) > u(X\bar{y})$. Let us now compare $X\bar{y}$ with $\bar{L}X-L''+y''$. Here we have $X\left(y'-L'\right) \gtrless y''-L''$. If $y'-L' > y''-L'' > 0$ or $L''-L' > y''-y' > 0$, then $u(X\bar{y}) > u\left(\bar{L}X-L''+y''\right)$, which implies that $\Lambda\left(y'',L''\right) > \Omega\left(y'',L''\right)$. On the contrary, if $0 < L''-L' < y''-y'$, then $u(X\bar{y}) < u\left(\bar{L}X-L''+y''\right)$, so that $\Lambda\left(y'',L''\right) \gtrless \Omega\left(y'',L''\right)$. Consider now a loss of well-being ($X < 1$). By concavity of $u(\cdot)$, we have that $Xu(\bar{y}) < u(X\bar{y})$. Hence if $L''-L' > y''-y'$, then $u(X\bar{y}) > u\left(\bar{L}X-L''+y''\right)$, so that $\Lambda\left(y'',L''\right) \gtrless \Omega\left(y'',L''\right)$. However, if $L''-L' < y''-y'$, then $u(X\bar{y}) < u\left(\bar{L}X-L''+y''\right)$, so that $\Lambda\left(y'',L''\right) < \Omega\left(y'',L''\right)$. But given that $\Lambda\left(y'',L''\right)$ and $\Omega\left(y'',L''\right)$ are negative, we have, in absolute value, $\left|\Lambda\left(y'',L''\right)\right| > \left|\Omega\left(y'',L''\right)\right|$.

Consider now the comparison of $\frac{\Lambda\left(y'',L''\right)}{y''}$ and $\frac{\Omega\left(y'',L''\right)}{L''}$. We have:

$$\begin{aligned} \frac{\Lambda\left(y'',L''\right)}{y''} \gtrless \frac{\Omega\left(y'',L''\right)}{L''} \quad &\Longleftrightarrow \quad u^{-1}\left(Xu(\bar{y})\right) \gtrless X\bar{y}\frac{L'y''}{L''y'} \\ &\Longleftrightarrow \quad Xu(\bar{y}) \gtrless u\left(X\bar{y}\frac{L'y''}{L''y'}\right) \end{aligned}$$

Consider a well-being gain ($X > 1$), where income rise dominates, i.e. $\frac{L'y''}{L''y'} > 1$. By concavity, we have $Xu(\bar{y}) > u(X\bar{y}) < u\left(X\bar{y}\frac{L'y''}{L''y'}\right)$, so no conclusion can be drawn in that case. Consider now a well-being gain, where lifetime rise dominates, i.e. $\frac{L'y''}{L''y'} < 1$. By concavity, we have $Xu(\bar{y}) > u(X\bar{y}) > u\left(X\bar{y}\frac{L'y''}{L''y'}\right)$. so that $\frac{\Lambda\left(y'',L''\right)}{y''} > \frac{\Omega\left(y'',L''\right)}{L''}$. Consider now a well-being loss ($X < 1$), where income loss dominates, i.e. $\frac{L'y''}{L''y'} < 1$. By concavity $Xu(\bar{y}) < u(X\bar{y}) > u\left(X\bar{y}\frac{L'y''}{L''y'}\right)$, so no conclusion can be drawn in that case. Consider now well-being loss, where lifetime loss dominates, i.e. $\frac{L'y''}{L''y'} > 1$. By concavity $Xu(\bar{y}) < u(X\bar{y}) < u\left(X\bar{y}\frac{L'y''}{L''y'}\right)$, so $\frac{\Lambda\left(y'',L''\right)}{y''} < \frac{\Omega\left(y'',L''\right)}{L''}$. But both are negative, so we have in absolute value $\left|\frac{\Lambda\left(y'',L''\right)}{y''}\right| > \left|\frac{\Omega\left(y'',L''\right)}{L''}\right|$.