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Bundling, Belief Dispersion, and Mispricing in Financial Markets

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Bundling, Belief Dispersion, and Mispricing in Financial Markets*

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Abstract

Bundling assets of heterogeneous quality results in dispersed valuations when these are based on investor-specific samples from the pool. A monopolistic bank has the incentive to create heterogeneous bundles only when investors have enough money as in that case prices are driven by more optimistic valuations. When the number of banks is sufficiently large, oligopolistic banks choose extremely heterogeneous bundles even when investors have little money and even if this turns out to be collectively detrimental to the banks, which we refer to as a Bundler's Dilemma.

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1 Introduction

Many financial products such as mutual fund shares or asset-backed securities consist of claims on composite pools of assets. Pooling assets has obvious advantages, for example in terms of improved diversification, but it may sometimes make it harder for investors to evaluate the resulting financial products. Due to time or other constraints, investors may only be able to assess limited samples of assets in the underlying pool. At the same time, as implied by many behavioral studies, investors may tend to rely too much on their own sample, trading as if this sample were fully representative of the underlying pool.[1]

If investors overweight their own limited sample when evaluating pools of assets, bundling assets of heterogeneous quality may induce dispersion in investors' valuations and this may in turn affect asset prices. We wish to study in such an environment the incentives for banks or other financial institutions to offer financial products backed by pools of assets of heterogeneous quality. In particular, we wish to investigate how these incentives change depending on whether potential investors have more or less money in their hands and whether there is more or less market competition in the banking system. Addressing such questions is essential -we believe- for the large debate on the increasing complexity of financial products.[2]

We develop a simple and deliberately stylized model to address our research question. Specifically, we consider several banks holding assets (say, loan contracts) of different quality (say, probability of default). Banks are able to package their assets into pools as they wish and sell claims backed by these pools. We abstract from the design of possibly complex security structures and assume that banks can only sell pass-through securities. Each investor randomly samples one asset from each pool and assumes that the average value of the assets in the pool coincides with this draw considered as representative. In our baseline specification, we consider an extreme version of excessive reliance on the sample and assume that no other information is used for assessing the value of a pool. In particular, investors do not consider how banks may strategically allocate assets into pools,[3] nor do they draw any inference from market prices. We discuss below how

[1] This can be derived from forms of representativeness heuristic, extrapolation, overconfidence, or cursedness. We discuss these models in more details below.

[2] Krugman (2007) and Soros (2009) are prominent actors of such a debate.

[3] Through the choice of how heterogeneous the assets are, the bank affects whether small samples are more likely to be representative of the entire pool. If banks were to pool homogenous assets, one draw would be highly representative of the assets in the pool. If banks instead tend to pool assets of heterogeneous quality (as we show they do) this is no longer the case.

alternative specifications can be accommodated without affecting our main logic.

We further assume that the draws determining the representative samples are made independently across investors. This implies that if the underlying assets of a given package are heterogeneous, the evaluations of the package are dispersed across investors. This captures the view that more complex or innovative financial products, interpreted in our framework as products backed by assets of more heterogeneous quality, are harder to evaluate.[4] Hence, even starting with the same objective information, investors may end up with very different assessments. This heterogeneity of valuations was documented for example in the context of asset-backed securities, where even highly sophisticated investors used different valuation methods (Bernardo and Cornell (1997); Carlin, Longstaff and Matoba (2014)).[5] But, note that irrespective of banks' strategies valuations across investors are on average correct so that banks cannot induce any systematic bias in investors' evaluations. Market clearing prices, however, need not reflect average valuations and, under conditions we will describe, each bank may find it optimal to induce a mean preserving spread in the distribution of valuations.

Investors are assumed to be risk neutral to emphasize that our mechanism is unrelated to risk aversion. They are also wealth-constrained and cannot short-sell. Thus, each investor allocates his whole wealth to the securities perceived as most underpriced. Pooling heterogeneous assets excludes from trading those investors who end up with low valuations, and at the same time it extracts more wealth from those investors who end up with good valuations. The larger the wealth, the more optimistic the marginal investor who determines the market clearing price, which in turns increases the incentive for banks to induce disagreement by creating heterogeneous pools. As it turns out, the market structure of the banking system is also a key determinant of whether banks find it good to create heterogeneous pools. The main message of our paper is that more wealth and/or more competition can explain the emergence of such heterogeneous pools.

We first consider a monopolistic setting. We characterize conditions on in-

[4]Such a view differs from one according to which bundles may be easier to assess due to the cancelling out of the idiosyncratic noise attached to the evaluation of the individual assets. This alternative view requires that the distribution from which individual values are drawn is known and stable across assets, while our model is best suited for unfamiliar assets for which such a common distribution is yet to be discovered.

[5]Mark Adelson (S&P chief credit officer): *"It [Complexity] is above the level at which the creation of the methodology can rely solely on mathematical manipulations. Despite the outward simplicity of credit-ratings, the inherent complexity of credit risk in many securitizations means that reasonable professionals starting with the same facts can reasonably reach different conclusions."* Testimony before the Committee on Financial Services, U.S. House of Representatives, September 27, 2007. Quoted in Skreta and Veldkamp (2009).

vestors' wealth under which the monopolistic bank prefers to pool all assets into a single bundle, thereby creating the largest dispersion in investors' evaluations. We also define a threshold on investors' wealth such that when investors' wealth exceeds the threshold, the bank prefers to sell its loans with some non-trivial packaging, while when wealth falls short of this threshold, disagreement decreases asset prices, and so selling the loans as separate assets is optimal for the bank.

Our next central question is whether increasing competition between banks affects their incentives to induce belief dispersion by pooling assets of heterogeneous quality. Our main result is that these incentives are increased when several banks compete to attract investors' capital. A key observation is that, in a market with many banks, investors who happen to sample the best asset from some bundles must be indifferent between buying any of those, as otherwise the market would not clear. This implies that, irrespective of investors' wealth, the ratio between the price of a bundle and the value of its best asset must be the same across all bundles.

Each bank has then an incentive to maximize the most valued asset in a bundle, which can be achieved by pooling all its assets into a single bundle. We show that such a full bundling is the only equilibrium when the number of banks is sufficiently large, irrespective of investors' wealth. This should be contrasted with the monopolistic case, in which the bank has no incentive to bundle at low levels of wealth.

In other words, we show that more wealth in the hands of investors and/or more competition between banks to attract investors increase the incentives for banks to increase belief dispersion by proposing more complex financial products; that is, products backed by assets of more heterogeneous quality. In a monopolistic market with very wealthy investors, inducing belief dispersion is profitable since those who end up with less optimistic views prefer to stay out from the market. In a market with many banks, and even if investors' wealth is low, inducing belief dispersion is the best strategy as doing otherwise would be beneficial to other banks (due to investors' comparisons of assets) and in turn attract a lower fraction of investors' wealth.

The implications of bundling in terms of asset prices, and so in terms of banks' and investors' payoffs, are however quite different in monopoly and oligopoly. In fact, we show that even though full bundling is the only equilibrium in the highly competitive case, banks would be in some cases better off by jointly opting for a finer bundling strategy. We refer to such a situation as a Bundler's Dilemma. We show that Bundler's Dilemmas are driven by the fact that any bank is worse

off when the other banks offer larger bundles, so that bundling creates a negative externality on the other banks. When offering larger bundles, each bank is not only "stealing" investors' wealth from its competitors, but it is also decreasing the total amount of wealth attracted in the market, thereby making banks collectively worse off. We also discuss cases in which Bundler's Dilemmas would prevent banks and investors from fully exploiting potential gains from trade.

While obviously stylized, our analysis suggests several insights that could be brought to the data. Our framework can serve as a building block for a systematic investigation of the incentives to issue asset-backed securities along the business cycle. We suggest that pool heterogeneity tends to be larger in good times, which is consistent with Downing, Jaffee and Wallace (2009) and Gorton and Metrick (2012) in relation to the 2008 crisis. In terms of asset prices, existing evidence suggests that overpricing tends to be associated with low breadth of ownership (Chen, Hong and Stein (2002)), higher investors' disagreement (Diether, Malloy and Scherbina (2002)), and higher asset complexity (Henderson and Pearson (2011), Célérier and Vallée (2017), and Ghent, Torous and Valkanov (2017)). Our model suggests how to think in a unified way about these findings and it proposes a precise link between complexity, disagreement, and overpricing, which should be the subject of future tests.

Literature

The heuristic followed by our investors builds on several closely-related behavioral aspects previously discussed in the literature. Our investors extrapolate from small samples as modelled by Osborne and Rubinstein (1998). The corresponding valuation method can be related to the representativeness heuristic (in particular, to the law of small numbers) as well as to the extrapolative heuristic, which have been widely discussed in psychology as well as in the context of financial markets.[6] Our formalization is most similar to Spiegler (2006) and Bianchi and Jehiel (2015), but the literature offers several other models of extrapolative investors.[7]

The excessive reliance on the sample used by our investors can also be related to

[6] Tversky and Kahneman (1975) discuss the representativeness heuristic and Tversky and Kahneman (1971) introduce the "law of small numbers" whereby "*people regard a sample randomly drawn from a population as highly representative, that is, similar to the population in all essential characteristics.*" In financial markets, evidence on extrapolation comes from surveys on investors' expectations (Shiller (2000); Dominitz and Manski (2011); Greenwood and Shleifer (2014)) as well as from actual investment decisions (Benartzi (2001); Greenwood and Nagel (2009); Baquero and Verbeek (2008)).

[7] These include De Long, Shleifer, Summers and Waldmann (1990), Barberis, Shleifer and Vishny (1998), Rabin (2002), and Rabin and Vayanos (2010).

a form of base rate neglect (they insufficiently rely on outside information such as the prior) or to a form of overconfidence (leading investors to perceive their signals as much more informative than everything else, in a similar vein as in Scheinkman and Xiong (2003)). This also makes investors exposed to the winner's curse, as they do not take sufficiently into account the information that other investors may have and that may be revealed by the prices.[8] Compared to the previous behavioral models in financial economics, our focus on the bundling strategies of banks has no counterpart. As already highlighted, its key and novel aspect is that it structures the distribution of signals that investors receive.

Our model is also related to the literature on financial markets with heterogeneous beliefs and short-selling constraints as in Harrison and Kreps (1978).[9] Part of this literature has also studied how financial institutions can exploit investors' heterogeneity by offering securities catered to different investors (see e.g. Allen and Gale (1988) for an early study and Broer (2018) and Ellis, Piccione and Zhang (2017) for recent models). Unlike in that literature, the heterogeneity of beliefs in our setting is not a primitive of the model (in fact, we do not need any ex-ante heterogeneity across investors), but it is endogenously determined by the bundling decisions of banks. Relative to security design, our focus on banks' bundling decision is complementary, and it shows that inducing dispersed valuations may be profitable even if banks cannot offer differentiated securities.

Finally, the potential benefits of bundling have been studied in several other streams of literature, from IO to auctions.[10] In particular, a recent literature on obfuscation in IO studies how firms can exploit consumers' naïveté by hiding product attributes or by hindering comparisons across products.[11] Our banks can be viewed as using bundling to make it harder to evaluate their assets, but unlike in models à la Gabaix and Laibson (2006) they cannot make assets more or less visible to investors.

[8] Previous theoretical approaches to the winner's curse include the cursed equilibrium (Eyster and Rabin (2005)) or the analogy-based expectation equilibrium (Jehiel (2005) and Jehiel and Koessler (2008)) that have been applied to financial markets by Eyster and Piccione (2013), Steiner and Stewart (2015), Kondor and Koszegi (2017), or Eyster, Rabin and Vayanos (2017). See also Gul, Pesendorfer and Strzalecki (2017) for an alternative modelling of coarseness in financial markets.

[9] See Xiong (2013) for a recent review and Simsek (2013) for a model of financial innovation in such markets.

[10] In the context of a monopolist producing multiple goods, see e.g. Adams and Yellen (1976) and McAfee, McMillan and Whinston (1989). For models of auctions, see e.g. Palfrey (1983) and Jehiel, Meyer-Ter-Vehn and Moldovanu (2007).

[11] See Spiegler (2016) for a recent review of these models, and Carlin (2009) for an application of obfuscation to financial products.

2 Model

There are N risk-neutral banks. Each bank $i = 1, ..., N$ possesses several assets. We denote asset j of bank i by X_j^i and its expected payoff by x_j^i. For concreteness, asset X_j^i may be thought of as a loan contract with face value normalized to 1, probability of default $1 - x_j^i \in [0, 1]$, and zero payoff upon default. We order assets in terms of increasing expected payoff. That is, we have $x_j^i \leq x_{j+1}^i$ for each i and j.

Each bank may pool some of its assets and create securities backed by these pools. Each bank can package its assets into pools as it wishes. We represent the selling strategy of bank i as a partition of $X^i = \{X_j^i, j = 1, ..., J\}$, denoted by $\alpha^i = \{\alpha_r^i\}_r$, in which the set of bundles are indexed by $r = 1, 2, ...$ We focus on complexity considerations that arise merely from banks' bundling strategies. That is, we do not consider the use of possibly complex contracts that would map the value of the underlying pool to the payoff of the securities, and we assume that each bank i simply creates pass-through securities backed by the pool α_r^i for each r. Accordingly, an investor who buys a fraction ω of the securities backed by α_r^i is entitled to a fraction ω of the payoffs generated by all the assets in α_r^i. The expected payoff of bank i choosing α^i is defined as

$$\pi^i = \sum\nolimits_r \left|\alpha_r^i\right| p(\alpha_r^i), \tag{1}$$

where $|\alpha_r^i|$ is the number of assets contained in α_r^i and $p(\alpha_r^i)$ is the price of the security backed by α_r^i. We denote the set of bundles sold by all banks by $A = \{\{\alpha_r^i\}_r\}_{i=1}^N$.

There are K risk-neutral investors, indexed by k. As investors are risk neutral and they buy claims on the total payoff generated by bundle α_r^i, they care about the average expected value of the assets in α_r^i.[12] For each bundle α_r^i, investor k samples one basic asset from α_r^i at random (uniformly over all assets in α_r^i) and assumes that the average expected value of the assets in α_r^i coincides with this draw. In our baseline model, no other information is used for assessing the value of a pool. As it will be clear, our logic would not be affected if investors were allowed to draw larger but finite samples from each bundle. We also discuss later on how the valuation method could be modified to allow that investors' samples may depend on banks' bundling strategies (in particular, fixing the number of

[12] The model can be naturally extended, without affecting its main logic, to introduce risk aversion as well as more complicated security structures, which would induce investors to focus also on other characteristics of the underlying assets.

draws independently of the number of bundles available in the market) as well as to accommodate possibly less extreme forms of heuristics for example including the price or some prior about the value of bundles into the subjective valuations.

Specifically, denote by $\widetilde{x}_k(\alpha_r^i)$ the evaluation that investor k attaches to the average asset in α_r^i: $\widetilde{x}_k(\alpha_r^i)$ takes value x_j^i with probability $1/\left|\alpha_r^i\right|$ for every $X_j^i \in \alpha_r^i$. We assume that the draws are independent across investors.[13] It follows that if $|\alpha_r^i| = 1$, investors share the same correct assessment of bundle α_r^i. But, if $|\alpha_r^i| > 1$, investors may attach different values to α_r^i depending on their draws. As already mentioned, however, bundling heterogenous assets only induces belief dispersion and no systematic bias in the average valuation across investors.

Prices are determined by market clearing, assuming that each investor k has a fixed budget denoted by w_k and cannot borrow nor short-sell (an assumption we discuss in Section 5).[14] The supply and demand of the securities backed by α_r^i are defined as follows. If α_r^i consists of $|\alpha_r^i|$ assets, the supply of α_r^i is

$$S(\alpha_r^i) = \left|\alpha_r^i\right|. \tag{2}$$

The demand for α_r^i is defined as

$$D(\alpha_r^i) = \frac{1}{p(\alpha_r^i)} \sum\nolimits_k w_k \lambda_k(\alpha_r^i), \tag{3}$$

where $\lambda_k(\alpha_r^i) \in [0,1]$ is the fraction of the budget w_k allocated to bundle α_r^i. Given the risk-neutrality assumption, each investor allocates his entire budget to the securities perceived as most profitable. That is,

$$\lambda_k(\hat{\alpha}) > 0 \text{ iff } \hat{\alpha} \in \arg\max_{\alpha_r^i \in A} \frac{\widetilde{x}_k(\alpha_r^i)}{p(\alpha_r^i)} \text{ and } \widetilde{x}_k(\hat{\alpha}) - p(\hat{\alpha}) \geq 0,$$

and

$$\Sigma_{\alpha_r^i} \lambda_k(\alpha_r^i) = 1 \text{ if } \max_{\alpha_r^i \in A} (\widetilde{x}_k(\alpha_r^i) - p(\alpha_r^i)) > 0.$$

With the exception of Section 3.1, we take the number of investors K to be infinite. This can be interpreted as the limiting case as $K \to \infty$ of a setting in which, from the law of large numbers, each asset in bundle α_r^i is sampled by a fraction $1/\left|\alpha_r^i\right|$ of investors. Considering such a limiting case simplifies our analysis as it removes

[13] More generally, the insights developed below would carry over, as long as there is no perfect correlation of the draws across investors.

[14] Investors' wealth is taken as given. An interesting next step would be to endogenize this wealth, possibly as a function of banks' strategies and of market prices, as well as to analyze its dynamics.

the randomness of prices (which would otherwise vary stochastically as a function of the profile of realizations of the assessments of the various investors), and it allows us to focus on the effect of the aggregate budget across all investors, which we denote as

$$W = \sum\nolimits_k w_k,$$

as opposed to its exact distribution across investors.

The timing is as follows. Banks simultaneously decide their selling strategies so as to maximize the expected payoff as described in (1); investors assess the value of each security according to the above described procedure and form their demand as in (3); a competitive equilibrium emerges, which determines the price for each security so as to clear the markets for all securities.

3 Monopoly

We start by analyzing a monopolistic setting with $N = 1$ (we omit the superscript i for convenience). This is the simplest setting to highlight some basic insights, in particular the effect of investors' wealth on the incentives for the bank to bundle its assets. The larger the wealth, the more optimistic are the investors who fix the market clearing price, and so the bigger the incentive for the bank to create heterogeneous bundles.

3.1 Bundling is Optimal only with Disagreement

A preliminary observation is that the presence of several investors -and so the possibility of disagreement- is needed to make bundling profitable to the bank. To see this, let there be a single investor, $K = 1$, and assume that the bank sells its assets separately (we refer to this case as full separation). Each asset X_j is correctly perceived as having value x_j, so the payoff derived by the bank is:

$$\min(\sum\nolimits_j x_j, W). \tag{4}$$

Assume by contrast that the bank pools all its assets into a single bundle α (we refer to this case as full bundling). A generic loan in the bundle is perceived to have value x_j with probability $1/|\alpha|$ for each j and thus the payoff of the bank is:

$$\sum\nolimits_j \frac{1}{|\alpha|} \min(|\alpha| \, x_j, W).$$

Such a payoff cannot strictly exceed the payoff in (4) due to the concavity of $\min(.,w)$ and Jensen's inequality. The argument extends to any other partition, as reported in the following proposition.

Proposition 1 *Suppose $K = 1$. Irrespective of W, the monopolistic bank prefers full separation.*

3.2 Bundling is Optimal only with Enough Wealth

Turning, from now on, to the case of infinite number of investors, we note that bundling is profitable to the extent that only the investors who overvalue the bundle (as compared with the fundamental value) are willing to buy. The question is whether the wealth possessed by those investors is sufficient to satisfy the corresponding market clearing conditions at such high prices. An immediate observation is that bundling cannot be profitable if the aggregate wealth W falls short of the fundamental value of the assets which are sold in the market, since selling assets separately exhausts the entire wealth and the payoff from any bundling cannot exceed W (while it can sometimes fall short of W due to the possibly pessimistic assessment of the bundle).

Another simple observation is that when investors are very wealthy ($W/J > Jx_J$ where x_J is the best asset), the price of any bundle is determined by the most optimistic evaluation of the bundle -that is, by the maximum of the draws across investors- irrespective of the bank's bundling strategy. In this case, it is optimal for the bank to create as much disagreement as possible, so full bundling strictly dominates any other strategy.

More generally, the larger the aggregate wealth W, the more profitable it is to create bundles with several assets of heterogeneous value. While full bundling is optimal when W is large enough, some non-trivial but partial bundling is optimal at intermediate levels of wealth whereas at sufficiently low levels of wealth, the bank finds it optimal to sell its assets separately.[15] More precisely, if wealth is so low that pooling $\{X_1, X_2\}$ and offering the other assets separately is dominated by offering all assets separately, then no other bundling can be profitable, which in turn yields:

Proposition 2 *Suppose $K \to \infty$. Some bundling dominates full separation if and only if $W > 2(x_2 + x_1)$.*

[15] See the Online Appendix for elaborations.

4 Oligopoly

We now consider multiple banks, and observe that the incentives to offer assets in bundles are larger in markets with sufficiently many banks. As it turns out, when N is large, full bundling is the only equilibrium, even at levels of wealth at which a monopolistic bank would sell its assets separately. We then show that bundling creates a negative externality on the other banks, which can lead banks to situations similar to a Prisoner's Dilemma.

4.1 Full Bundling is the Only Equilibrium

We assume that all banks have the same set of assets $X = \{X_j, j = 1, ..., J\}$ with $J \geq 2$ and $0 < x_1 < ... < x_J$. Consider some partition of assets across banks. Let α_r be a generic bundle (the identity of the selling bank is not important), $J_r \geq 1$ the number of elements in α_r, x_r^* the highest value of the assets in bundle α_r, p_r the market clearing price of a security backed by α_r and define

$$\mu_r \equiv \frac{p_r}{x_r^*}.$$

We first show that, when N is large, market clearing requires that the ratio μ_r is constant across all bundles sold by all banks.

Lemma 1 *Suppose $K \to \infty$. There exist $\mu_0 \in (0, 1]$ and N_0 such that if $N \geq N_0$ then market clearing requires*

$$p_r = \mu_0 x_r^* \text{ for all } \alpha_r \in A. \tag{5}$$

Moreover, N_0 can be chosen irrespective of the partition of assets into bundles.

To have an intuition for Lemma 1, observe that the ratio μ_r determines the attractiveness of bundle r for investors who happen to sample the best asset x_r^* in that bundle. Suppose a bundle r_1 had a strictly lower ratio than all other bundles, it would attract at least those investors who sample its best asset, which is a fixed proportion of investors irrespective of N. All other bundles would receive at most the remaining wealth, which would be split among a larger number of bundles as N increases. As these bundles would then become cheaper as N increases, the condition that bundle r_1 had a strictly lower ratio would be violated for sufficiently large N. Similarly, if a bundle r_2 had a strictly higher ratio than all other bundles, it would attract at most those investors who sample no best asset from any of the

other bundles, thereby corresponding to a proportion of investors that decreases exponentially fast as N increases. As a result, bundle r_2 would become comparatively cheaper as N increases and, for sufficiently large N, that would contradict the premise that r_2 had a strictly higher ratio. The proof extends this intuition, showing that the markets would not clear unless the ratios μ_r are equated across the various bundles. That N_0 can be set independently of the partitions of assets into bundles follows because there are only finitely many possible partitions of the assets for any bank.

Condition (5) implies that, when N is large, the price of each bundle is driven by its highest valued asset. This suggests that each bank has an incentive to maximize the most valued asset in a bundle, which can be achieved by pooling all assets into a single bundle. Of course, this loose intuition does not take into account that the constant of proportionality μ_0 depends itself on the bundling strategies of the banks. But, as it turns out, full bundling is the only equilibrium when N is large given that in this case a single bank cannot affect much μ_0.

Proposition 3 *Suppose $K \to \infty$. Irrespective of W, there exists N^* such that if $N \geq N^*$ then full bundling is the only equilibrium.*

To have a finer intuition as to why full bundling is an equilibrium, suppose all banks propose the full bundle and bank j deviates to another partition. From Lemma 1, the fraction of wealth allocated to each bundle depends on the value of its best asset. Full bundling gives a price proportional to x_J for all assets, while the deviating bank would at best sell $J-1$ assets at a price proportional to x_J and one asset at a price proportional to its second best asset x_{J-1}. Relative to the other banks, the deviating bank would experience a loss proportional to $(x_J - x_{J-1})$, and this remains positive irrespective of N. At the same time, all banks could benefit from the deviation if the total amount of wealth invested were to increase. Such an increase is bounded by the fraction of wealth not invested before the deviation, which corresponds at most to the mass of those investors who sample no best asset from any of the bundles. When N is large, these investors are not many, and so the increase in wealth is small, which makes the deviation not profitable.

The proposition also rules out any other possibly asymmetric equilibrium. Starting from an arbitrary profile of (possibly asymmetric) bundles, we show that the bank receiving the lowest payoff would be better off by deviating to full bundling.

4.2 The Bundler's Dilemma

Another implication of Lemma 1 is that each bank is better off when the other banks choose finer partitions than when they offer coarser partitions of their assets. Let us introduce the following definition.

Definition 1 *Consider two partitions $\tilde{\alpha}^i$ and α^i of X^i. We say that $\tilde{\alpha}^i$ is coarser than α^i (or, equivalently, that α^i is finer than $\tilde{\alpha}^i$) if $\tilde{\alpha}^i$ can be obtained from the union of some elements of α^i.*

We can show that, irrespective of its strategy, each bank receives lower payoffs when the other banks offer coarser partitions than when they offer finer partitions. When the other banks offer coarser partitions, the total amount of wealth invested is lower since the probability of sampling an asset whose value is lower than the price from all bundles is larger. At the same time, from Lemma 1, banks offering coarser partitions receive a larger fraction of this wealth as some of their best assets would be included in larger bundles. We then have the following proposition.

Proposition 4 *Suppose $K \to \infty$ and consider partitions $\tilde{\alpha}$ and α, where $\tilde{\alpha}$ is coarser than α. If $N \geq N_0$, irrespective of its strategy, each bank is better off when all other banks offer partition α than when they offer partition $\tilde{\alpha}$.*

Proposition 4 implies in particular that each bank is better off when the other banks sell their assets separately than when they offer them in bundles. In this sense, we say that bundling creates a negative externality on the other banks.

This externality leads to a new phenomenon, which we call Bundler's Dilemma (with obvious reference to the classic Prisoner's Dilemma).[16] Full bundling can be the only equilibrium and at the same time be collectively bad for banks, in the sense that if banks could make a joint decision they would rather choose a finer bundling strategy.

Definition 2 *We have a Bundler's Dilemma when i) Full bundling is the only equilibrium, and ii) Banks would be better off by jointly choosing a finer bundling strategy.*

A special (extreme) case of the Bundler's Dilemma arises when banks would be collectively better off by selling their assets separately, while in equilibrium they are induced to offer the full bundle. This occurs under the following conditions.

[16] We thank Laura Veldkamp for suggesting this terminology.

Corollary 1 *Suppose* $K \to \infty$, $N \geq N^*$ *and*

$$\frac{W}{N} \in (Jx_1, \frac{\sum_j x_j}{1-(\frac{1}{J})^N}). \tag{6}$$

We have a Bundler's Dilemma in which full bundling is the only equilibrium while banks would collectively prefer full separation.

Equation (6) follows from simple algebra. When $W/N > Jx_1$, the price of each bundle is strictly greater than x_1. Otherwise, all investors would be willing to buy irrespective of their draw, all wealth would be extracted, and the price of each bundle would exceed x_1, leading to a contradiction. It follows that investors who draw X_1 from all bundles, that is a fraction $(1/J)^N$ of investors, do not participate and each bundle gets at most

$$\frac{W}{N}(1-(\frac{1}{J})^N). \tag{7}$$

The upper bound in (6) is derived by imposing that (7) does not exceed $\sum_j x_j$ so that each bank would be better off if all assets were sold separately.

In a Bundler's Dilemma as defined in (6), the mere option of banks to offer assets in bundles, together with investors' inability to correctly assess the values of the bundles, makes investors better off. There are levels of W such that prices would be equal to fundamentals if banks offered assets separately and turn out to be below fundamentals only due to bundling. Moreover, as we discuss in the next section, a Bundler's Dilemma may occur even if banks can withhold some of their assets. In this case, a Bundler's Dilemma would prevent banks and investors from fully exploiting potential gains from trade.

5 Extensions

Our model while obviously stylized is open to many extensions. In this section, we suggest that our main insights are robust to several modifications. We refer to the Online Appendix for more formal results as well as for additional extensions.

Capital Constraints and Short Selling

In our model, the valuation of a given pool across investors is on average correct. The price of the pool however need not reflect such average valuation: optimistic

investors cannot invest more than their wealth, pessimistic investors cannot short-sell. Moreover, when facing several bundles, investors only trade those bundles perceived as most profitable. In this way, some beliefs do not influence prices. Capital constraints, and not the absence of short-selling per se, are the key reasons why heterogeneous valuations, even if correct on average, may lead to mispricing.

In fact, as we show, introducing short-selling may even increase the incentives for banks to create disagreement in a competitive setting. Suppose that short-selling too is limited by capital constraints (say due to collateral requirements) and investors allocate their wealth between buying and short-selling. While short-selling decreases the payoff from bundling (investors with low evaluations can drive the price down), it also decreases the payoff from deviations. With no short-selling, deviations from bundling can attract those investors who refrain from investing as they end up with bad evaluations. If these investors could instead short-sell, deviations would attract a lower amount of wealth and so become less profitable. We show in a simple example that introducing short-selling can make both bundling and the Bundler's Dilemma more likely to occur.

Varying the Sophistication of Investors

Observe that in the baseline model investors could make one draw for each bundle independently of the number of bundles in the market. It may be desirable to disentangle the number of draws that an investor can make from the total number of bundles put in the market. To this end, assume that investors never buy bundles for which they have no signal, and let M denote the number of draws each investor can make (irrespective of the number of banks). Proposition 3 can be reformulated as saying that for some M^* and all $M > M^*$, there exists N^* such that for all $N > N^*$ full bundling is the only equilibrium. The subsequent considerations on the Bundler's Dilemma remain valid.[17] So our insights carry over to the extent that investors can make sufficiently many draws across different bundles.

As discussed earlier, natural extensions of the above valuation model would allow investors to consider that their sample need not be fully representative of the underlying pool, thereby including other aspects in their assessments. Many variants can be considered. In addition to their sample, investors could put weight on some prior (if they have one), on the average value of the assets, or on the value

[17] In the Online Appendix, we consider an example with $N = M = 2$ in which investors can draw more assets from the same bundle, with no replacement. We observe that in this case, full bundling is more likely to occur than in the baseline model

of the worst asset (as an extreme form of ambiguity aversion), or on the price (so as to partly correct for the winner's curse). As long as the weight attached to the sample is the same across bundles, no qualitative property in our analysis would be affected.

A richer set of insights could be derived by allowing the weight on the sample to depend more finely on banks' bundling strategies. One may for example let investors know the size of the bundles and apply some discount to any bundle which contains more than one asset (say, they put more weight on some worst-case scenario when evaluating those bundles). In a setting with N large, each bank would issue a bundle containing its best asset (which drives the price, due to Lemma 1) together with all assets with value below some threshold (so as to benefit from the overpricing). Differently from the baseline model, assets of intermediate value would be sold as separate assets.[18]

One may also consider a setting in which investors are heterogeneous in their sophistication, possibly with a fraction of them being referred to as omniscient knowing the fundamental values (they can be thought of as making infinitely many draws). We would have a Bundler's Dilemma exactly under the same conditions as in Corollary 1 if omniscient investors are not too many. These investors would still find it optimal to buy the bundle as its price is below the fundamental.[19] A more systematic analysis of a market in which investors would use heterogeneous valuation methods would be an extremely interesting next step opening the door to whether banks specialize in attracting certain types of investors and how that could be achieved through well-chosen bundling strategies.

When Banks Decide what to Sell

The previous analysis has focused on banks' bundling strategy taking as given the set of assets sold in the market. Suppose at the other extreme that banks have no fundamental reason to sell the assets (assets have the same value for banks as for investors) and they can decide which assets to sell in addition to their bundling. This modification does not substantially affect our main insights. In particular, banks may still be led for strategic reasons into a Bundler's Dilemma. If banks were to coordinate, they would limit the supply of low quality assets up to a point where the benefit of adding an extra asset to their bundles would be offset by the induced reduction in the price of the bundles. A single bank however does not

[18] Creating securities backed by barbell-shaped loan pools has been viewed as a common way to deceive investors in the recent financial crisis (Lewis (2010)).

[19] The share of rational investors has to be small enough to make the deviations from full bundling undesirable.

internalize this effect. This leads to an excess supply of low quality assets and so to lower prices than what banks could obtain if they could act cooperatively. We illustrate this idea in a simple example with three assets in which in equilibrium banks are induced to offer the full bundle while they would be collectively better off if they were committed to keeping one bad asset and selling the rest as a bundle.

It is also clear that, in these more general settings, the Bundler's Dilemma affects not only prices but also which assets are sold in the market. Banks may prefer keeping their assets rather than selling them at low prices, even when those assets are more valuable to investors than to banks. The Bundler's Dilemma may then prevent banks and investors from fully exploiting possible gains from trade, thereby resulting also in welfare losses.

6 Conclusion

We have studied banks' incentives to package assets into composite pools when investors base their assessments on a limited sample of the assets in the pool. While we have focused on a specific heuristic of investors and a specific financial instrument for banks, we believe our approach can be viewed as representative of a more general theme in which investors use simple valuation models -for example, models that worked well for similar yet more familiar products- and product complexity is endogenous.

Future research could explore the incentives for financial institutions to create complexity when investors use other heuristics as well as to investigate other forms of complexity. The evaluation of some financial products could be difficult not only because of the heterogeneity of the underlying assets (which was our focus) but also because of the complex mapping between the value of the underlying and the payoff to investors.[20] Extending our model so as to allow banks to offer more general securities could provide novel perspectives to the standard security design literature.[21]

[20] Think for example at the various tranching structures of mortgage-backed securities or at the various ways in which the returns of a structured financial product can be defined in relation to a benchmark index.

[21] See for example DeMarzo and Duffie (1999) and Biais and Mariotti (2005) for classic contributions and Arora, Barak, Brunnermeier and Ge (2011) for a discussion of complexity in security design.

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7 Proofs

Proof of Proposition 1

Denote with $\alpha = \{\alpha_r\}_r$ with $r = 1, 2, ..., R$ an arbitrary partition of the $\hat{J}$ assets. We denote with x^r a generic element of bundle α_r and with $y = (x^1, x^2, .., x^R)$ a generic vector in which one asset of bundle α_1 is associated with one asset from each of the bundles $\alpha_2, \alpha_3, .., \alpha_R$. We denote with Y the set of all possible vectors y. The payoff from selling assets with partition α is $\pi(\alpha) = \sum_{y \in Y} \eta(\alpha) \min(W, \pi_0(y))$, where $\eta(\alpha) = \prod_r \frac{1}{|\alpha_r|}$, and $\pi_0(y) = \sum_{x^r \in y} |\alpha_r| x^r$. Notice that by definition $\sum_{y \in Y} \eta(\alpha) = 1$ and so $\pi(\alpha) \leq W$. Notice also that $\pi(\alpha) \leq \sum_{j \in \hat{J}} x_j$ since by definition $\sum_{y \in Y} \eta(\alpha) \pi_0(y) = \sum_{j \in \hat{J}} x_j$. Hence, $\pi(\alpha)$ cannot strictly exceed the payoff from selling assets separately, as defined in (4).

Proof of Proposition 2

Suppose $W > \max(2(x_2 + x_1), \sum_j x_j)$, full separation gives $\sum_j x_j$. Suppose the bank bundles assets $\{X_1, X_2\}$ and sells the other assets separately. Consider first a candidate equilibrium in which investors who sample x_2 from the bundle are indifferent between trading the single asset x_j and the bundle. That requires $2x_2/p_2 = x_j/p_j$ for all $j > 2$, where p_2 is the price of the bundle and p_j is the price of the asset x_j. In addition, we need that $p_2 + \sum_{j>2} p_j \leq W$, so aggregate wealth is enough to buy at prices p_2 and p_j. The above conditions give $p_2 \leq 2x_2 W/(\sum_{j>2} x_j + 2x_2)$, and $p_j \leq x_j W/(\sum_{j>2} x_j + 2x_2)$. In addition, we need that $p_2 \leq W/2$ so those investors who have valuation x_2 for the (x_2, x_1) bundle can indeed drive the price to p_2. Suppose $2x_2 < \sum_{j>2} x_j$, we have $\frac{2x_2}{\sum_{j>2} x_j + 2x_2} < \frac{W}{2}$ and so $p_2 = \min(2x_2, \frac{2x_2}{\sum_{j>2} x_j + 2x_2} W)$ and $p_j = \min(x_j, \frac{x_j}{\sum_{j>2} x_j + 2x_2} W)$ for $j > 2$.So the payoff of the bank is

$$\min(W, 2x_2 + \sum_{j>2} x_j),$$

which exceeds $\sum_j x_j$. Suppose $2x_2 \geq \sum_{j>2} x_j$, which can only occur if $J = 3$ and $2x_2 \geq x_3$. Then we must have $p_2 = W/2$, and $p_3 = x_3 W/4x_2$. That cannot be in equilibrium since investors who sample x_l still have money and would like to drive the price p_3 up. So if $2x_2 > x_3$ investors are indifferent only if $p_2 = 2x_2$ and $p_3 = x_3$. That requires $W > 4x_2$. If $W < 4x_2$, then we must have $p_2 < 2x_2 \frac{p_3}{x_3}$. If $W \in (2x_3, 4x_2)$, we have $p_2 = \frac{W}{2}$ and $p_3 = x_3$. If $W < 2x_3$, we have $p_2 = p_3 = \frac{W}{2}$. The payoff of the bank is

$$\min(W/2, 2x_2) + \min(W/2, x_3),$$

which also exceeds $\sum_j x_j$. Suppose $W \leq \max(2(x_2 + x_1), \sum_j x_j)$. If $W \leq \sum_j x_j$, then no bundling strictly dominates full separation. If $W \in (\sum_j x_j, 2(x_2 + x_1)]$, we must have $\sum_j x_j < 2(x_2 + x_1)$, that cannot be for $J > 3$. For $J = 3$ and $W \leq 2(x_2 + x_1)$, no bundling strictly dominates full separation.

Proof of Lemma 1

Denote with $\mathcal{H}$ the set of (possibly identical) bundles $r \in \arg\min_r \mu_r$ and with $\mathcal{L}$ the set of (possibly identical) bundles $r \notin \arg\min_r \mu_r$, with $|\mathcal{H}| = H$ and $|\mathcal{L}| = L$. Suppose by contradiction equation (5) is violated, then $H \geq 1$ and $L \geq 1$ and

$$\mu_r < \mu_{\tilde{r}} \text{ for all } r \in \mathcal{H} \text{ and all } \tilde{r} \in \mathcal{L}. \tag{8}$$

Given (8), the H bundles would attract at least all those investors who sample x_r^* from at least one bundle $r \in \mathcal{H}$, and so at least

$$W_{\hat{r}} = (1 - \prod\nolimits_{r \in \mathcal{H}} (\frac{J_r - 1}{J_r}))W.$$

The other bundles would attract at most the remaining wealth $W - W_{\hat{r}}$. Denote with $\hat{r} \in \mathcal{H}$ the bundle which receives the largest fraction of $W_{\hat{r}}$, it would attract at least $1/H$ of it. Similarly, denote with $\bar{r} \in \mathcal{L}$ the bundle which receives the lowest fraction of $W - W_{\hat{r}}$, it would attract at most $1/L$ of it. This implies that

$$p_{\hat{r}} \geq \min(x_{\hat{r}}^*, \frac{1}{H}\frac{W_{\hat{r}}}{J_{\hat{r}}}),$$

and

$$p_{\bar{r}} \leq \frac{1}{L}\frac{W - W_{\hat{r}}}{J_{\bar{r}}}.$$

Notice that if

$$x_{\hat{r}}^* \leq \frac{1}{H}\frac{W_{\hat{r}}}{J_{\hat{r}}},$$

then $\mu_{\hat{r}} = 1$ and so $\mu_{\hat{r}} < \mu_{\bar{r}}$ would imply $p_{\bar{r}} > x_{\bar{r}}^*$, which violates market clearing. Hence, $\mu_{\hat{r}} < \mu_{\bar{r}}$ requires

$$\frac{1}{H}\frac{W_{\hat{r}}}{J_{\hat{r}}}\frac{1}{x_{\hat{r}}^*} < \frac{1}{L}\frac{W - W_{\hat{r}}}{J_{\bar{r}}}\frac{1}{x_{\bar{r}}^*}, \tag{9}$$

which gives

$$L < \frac{J_{\hat{r}}}{J_{\bar{r}}}\frac{x_{\hat{r}}^*}{x_{\bar{r}}^*}\frac{W - W_{\hat{r}}}{W_{\hat{r}}}H. \tag{10}$$

Notice that the r.h.s. of equation (10) decreases in H and tends to zero as $H \to \infty$.

In fact we can write

$$\prod_{r\in\mathcal{H}}(\frac{J_r-1}{J_r})=z^H,$$

for some $z\in(0,1)$ and so

$$\frac{W-W_{\hat{r}}}{W_{\hat{r}}}H=\frac{z^H}{1-z^H}H. \tag{11}$$

We notice that the r.h.s. of equation (11) decreases in H iff $H\ln z-z^H+1<0$, that $H\ln z-z^H+1$ decreases in H and that $\ln z-z+1<0$ for all $z\in(0,1)$. Hence, condition (10) is violated if either H or L (or both) grow sufficiently large, which must be the case when $N\to\infty$ since $H+L\geq N$ (the total number of bundles cannot fall short of the number of banks). Hence, there exists a N_0 such that equation (5) must hold for $N\geq N_0$, which proves our result.

Proof of Proposition 3

We first show that full bundling is an equilibrium when N is large. We then show that no other bundling can be an equilibrium when N is large.

Part 1. If N is sufficiently large, full bundling in an equilibrium for all W.

Suppose all banks offer the full bundling and denote with π^F the payoff for each bank. If $(1-(\frac{J-1}{J})^N)\frac{W}{N}\geq Jx_J$ then $\pi^F=Jx_J$ and no other bundling can increase banks' payoffs. Suppose then

$$(1-(\frac{J-1}{J})^N)\frac{W}{N}<Jx_J. \tag{12}$$

We have

$$\pi^F\geq(1-(\frac{J-1}{J})^N)\frac{W}{N}. \tag{13}$$

The condition is derived by noticing that each bundle attracts at least those who sample the maximal asset in the bundle x_J. These investors are willing to invest all their wealth since the price of the bundle is strictly lower than their evaluation. From (12), the price of the bundle is lower than x_J.

Suppose bank j deviates and offers at least two bundles, indexed by r. The payoff of the deviating bank is $\pi^j=\sum_{r\geq1}J_rp_r$.From (5) we have

$$\pi^j=\mu_0\sum_{r\geq1}J_rx_r^*, \tag{14}$$

and so

$$\pi^j\leq\mu_0(J-1)x_J+\mu_0x_{J-1}. \tag{15}$$

Notice also that $\pi^j + (N-1)J\mu_0 x_J \leq W$ and since from (14) $\pi^j \geq \mu_0 \sum_j x_j$, we have

$$\mu_0 \leq \frac{W}{\sum_j x_j + (N-1)Jx_J}.$$

Together with (15), that gives

$$\pi^j \leq \frac{((J-1)x_J + x_{J-1})W}{\sum_j x_j + (N-1)Jx_J}.$$

In order to show that the deviation is not profitable, given (13), it is enough to show that

$$\frac{((J-1)x_J + x_{J-1})W}{\sum_j x_j + (N-1)Jx_J} < (1 - (\frac{J-1}{J})^N)\frac{W}{N}. \tag{16}$$

Equation (16) can be written as

$$(\frac{J-1}{J})^N < \frac{\sum_j x_j - Jx_J + N(x_J - x_{J-1})}{\sum_j x_j + (N-1)Jx_J}. \tag{17}$$

Notice that the l.h.s. of equation (17) decreases monotonically in N and tends to zero as $N \to \infty$, while the r.h.s. of equation (17) increases monotonically in N and tends to

$$\frac{x_J - x_{J-1}}{Jx_J} > 0,$$

as $N \to \infty$. Hence, $\pi^j < \pi^F$ and so full bundling is an equilibrium for N sufficiently large.

Part 2. If N is sufficiently large and irrespective of W, no alternative bundling is an equilibrium.

Suppose there is one bank, say bank j, which offers at least two bundles and it deviates by offering the full bundle. From (5), the payoff of the deviating bank can be written as $\mu_0 Jx_J$. If $\mu_0 = 1$, then the deviation is profitable since any other bundling would give strictly less than Jx_J. Suppose then $\mu_0 < 1$. As the price of each bundle is strictly lower than the best asset from the bundle, those who sample x_r^* from at least one bundle r will invest all their wealth. The total amount

of wealth invested is then at least

$$\hat{W} = 1 - \prod\nolimits_r (\frac{J_r - 1}{J_r})^M,$$

where M is the number of bundles offered after the deviation. Since $M \geq N$ and $J_r \leq J$ for all r, we have

$$\hat{W} \geq 1 - (\frac{J-1}{J})^N. \tag{18}$$

Consider first a candidate symmetric equilibria in which the payoff of the non-deviating banks is the same and it is denoted by π^{-j}. By definition we have $\pi^j + (N-1)\pi^{-j} \geq \hat{W}$ and $\pi^{-j} \leq \mu_0 x_{J-1} + (J-1)\mu_0 x_J$, which gives $\mu_0 J x_J + (N-1)\mu_0 x_{J-1} + (N-1)(J-1)\mu_0 x_J \geq \hat{W}$. Hence,

$$\mu_0 \geq \frac{\hat{W}}{J x_J + (N-1)x_{J-1} + (N-1)(J-1)x_J},$$

and so

$$\pi^j \geq \frac{\hat{W} J x_J}{J x_J + (N-1)x_{J-1} + (N-1)(J-1)x_J}.$$

Since the payoff before deviation was at most W/N, the deviation is profitable if

$$\frac{J x_J (1 - (\frac{J-1}{J})^N) W}{J x_J + (N-1)x_{J-1} + (N-1)(J-1)x_J} > \frac{W}{N},$$

which writes

$$x_J - x_{J-1} > \frac{N}{N-1}(\frac{J-1}{J})^N J x_J,$$

and that shows that $\pi^j > \pi^F$ and so the alternative bundling is not an equilibrium for for N sufficiently large.

Suppose there exists an asymmetric equilibrium. Denote with π^j the payoff of a generic bank j in such equilibrium, we must have $\min_j \pi^j < \max_j \pi^j$. Consider bank $\tilde{j} \in \arg\min_j \pi^j$. Suppose bank $\tilde{j}$ deviates and offers the full bundle. From the above argument, its payoff after deviation would be at least $\hat{W}/N$ and from (18) $\hat{W} \to W$ as $N \to \infty$. Since $\tilde{j} \in \arg\min_j \pi^j$, we have $\pi^{\tilde{j}} < W/N$. Hence, $\pi^{\tilde{j}} < \hat{W}/N$ for N sufficiently large, which rules out the possibility of asymmetric equilibria when N is large.

Proof of Proposition 4

Denote with r the bundles offered by a generic bank j. Its payoff can be written

as $\pi^j = \sum_{r\geq 1} J_r p_r$ and from (5) we have

$$\pi^j = \mu_0 \sum_{r\geq 1} J_r x_r^*, \tag{19}$$

for some μ_0. If all other banks offer a partition $\alpha = \{\alpha_f\}_f$, we have

$$\mu_0^F \left(\sum_{r\geq 1} J_r x_r^* + (N-1)\sum_{f\geq 1} J_f x_f^*\right) = W^F, \tag{20}$$

for some μ_0^F, and where W^F is the total amount of wealth invested. Suppose instead that the other banks offer a partition $\tilde{\alpha} = \{\alpha_c\}_c$, which is coarser than α, we have

$$\mu_0^C \left(\sum_{r\geq 1} J_r x_r^* + (N-1)\sum_{c\geq 1} J_c x_c^*\right) = W^C, \tag{21}$$

for some μ_0^C and W^C. Suppose that $\mu_0^C \geq \mu_0^F$, then we must have $W^C \leq W^F$. The fraction of wealth which is not invested corresponds to the probability that an investor samples an asset with value lower than the price from all bundles. This probability cannot be larger in α than in $\tilde{\alpha}$. By definition, there exists at least one element $\tilde{\alpha}_c \in \tilde{\alpha}$ which is obtained by the union of at least two elements $\tilde{\alpha}_f, \hat{\alpha}_f \in \alpha$. Hence, if $\mu_0^C \geq \mu_0^F$, the probability to sample an asset whose value is lower than the price in $\tilde{\alpha}_c$ cannot be lower than the probability to sample such an asset both in $\tilde{\alpha}_f$ and in $\hat{\alpha}_f$. Notice that from (20) and (21) $\mu_0^C \geq \mu_0^F$ contradicts $W^C \leq W^F$ since by definition $\sum_{c\geq 1} J_c x_c^* > \sum_{f\geq 1} J_f x_f^*$. Hence, we must have $\mu_0^C < \mu_0^F$ and from (19) this shows that bank j receives an higher payoff when the other banks offer a finer partition.

8 Online Appendix

8.1 Extensions

8.1.1 Short Selling

Suppose that an investor with wealth w can short-sell $\theta w/p$ units of an asset of price p (the baseline model corresponds to $\theta = 0$). Consider a setting with $N = 2$ banks each with two assets with value $x_1 = 0$ and $x_2 > 0$, and a continuum of investors. Suppose both banks bundle and denote by p^B the price of the corresponding security. An investor drawing x_1 from one bundle and x_2 from another bundle prefers to buy rather than short-selling if

$$p^B \leq \frac{x_2}{1+\theta}.$$

As we show, bundling can be sustained in equilibrium when W is sufficiently large, and the required lower bound on wealth is defined by

$$\frac{3-\theta}{8}\bar{W} = \frac{x_2}{1+\theta}. \tag{22}$$

In equation (22), the l.h.s. is the payoff from bundling and the r.h.s. is the payoff from deviation. An interesting observation is that both payoffs decrease with θ, but the payoff from deviation decreases more. This implies that $\bar{W}$ decreases in θ for $\theta < 1$. Hence, bundling is more likely to occur when $\theta > 0$ than when no short-selling is allowed.

A corollary is that short-selling creates the possibility of a Bundler's Dilemma, which is not possible with $N = 2$ and $\theta = 0$. The payoff from bundling is lower than what banks would get by jointly deciding not to bundle whenever

$$\frac{3-\theta}{8}W < x_2.$$

That is, when $W < \bar{W}(1+\theta)$. We then have the following.

Proposition 5 *Suppose that $N = 2$.*
i) Bundling is an equilibrium iff $W \geq \bar{W}$.
ii) A Bundler's Dilemma occurs for $W \in (\bar{W}, \bar{W}(1+\theta))$.

Proof. Point i). Suppose all banks bundle. Consider an investor drawing x_1 from first bundle and x_2 from the second bundle. When the price p of each asset

in each bundle is p, buying is preferred to short-selling if

$$p < \frac{x_2}{1+\theta}. \tag{23}$$

Suppose that is the case, the price of each asset in each bundle is given by $\frac{3}{8}\frac{W}{p} = 2 + \frac{1}{8}\frac{\theta w}{p}$, in which the demand for the bundle comes from investors who have sampled x_2 from at least one bundle (equally shared among the two banks). That gives

$$p = \frac{3-\theta}{16}W. \tag{24}$$

Conditions (23) and (24) give $W < 16x_2/(1+\theta)(3-\theta)$.Collecting (23) and (24) we have that the payoff of each bank is $2p = \min(\frac{3-\theta}{8}W, \frac{2x_2}{1+\theta})$ for $W < 16x_2/(1+\theta)(3-\theta)$. Suppose now condition (23) is violated. The price of each asset in each bundle is given by $\frac{1}{8}\frac{W}{p} = 2 + \frac{3}{8}\frac{\theta w}{p}$, in which the demand for the bundle comes from investors who have sampled x_2 from all bundles (equally shared among the two banks). That gives $p = (1-3\theta)W/16$, and so the payoff of each bank is $2p = \min(\frac{1-3\theta}{8}W, 2x_2)$ for $W \geq 16x_2/(1+\theta)(1-3\theta)$. That defines π^θ as

$$\pi^\theta = \begin{cases} \min(\frac{3-\theta}{8}W, \frac{2x_2}{1+\theta}) & \text{for} \quad W \leq \frac{16x_2}{(1+\theta)(1-3\theta)} \\ \min(\frac{1-3\theta}{8}W, 2x_2) & \text{for} \quad W \geq \frac{16x_2}{(1+\theta)(1-3\theta)}. \end{cases} \tag{25}$$

Suppose one bank deviates. If the price of the single asset x_2 is p, an investor drawing x_1 from the bundle prefers buying the single asset x_2 rather than short-selling the bundle if condition (23) holds. In that case, both the bundle and the single asset attract half of the aggregate wealth. The payoff of the deviating bank is $\min(\frac{1}{2}W, \frac{x_2}{1+\theta})$. If condition (23) is violated, an investor drawing x_1 from the bundle prefers short-selling the bundle rather than buying the single asset x_2. Only those who sample x_2 from the bundle are willing to buy the single asset x_2. If p_1 is the price of the single asset x_2 and p_2 is the price of each asset in the bundle, we need

$$p_1 = p_2. \tag{26}$$

Denote with λ the fraction of investors who draw x_2 from the bundle and buy the bundle. For the bundle, we have $\lambda\frac{W}{2p_2} = 2 + \frac{\theta W}{2p_2}$. For the single asset, we have $(1-\lambda)W = 2p_1$. Together with (26), these conditions give $\lambda = (2+\theta)/3$, and so $p_1 = p_2 = W(1-\theta)/6$. As we need $p > x_2/(1+\theta)$, we need $W > 6x_2/((1-\theta)(1+\theta))$. Hence, the payoff of the deviating bank is $\min(\frac{1-\theta}{6}W, x_2)$ for

$W > 6x_2/((1-\theta)(1+\theta))$. That defines $\pi^{-\theta}$ as

$$\pi^{-\theta} = \begin{cases} \min(\frac{1}{2}W, \frac{x_2}{1+\theta}) & \text{for} \quad W \leq \frac{6x_2}{(1-\theta)(1+\theta)} \\ \min(\frac{1-\theta}{6}W, x_2) & \text{for} \quad W \geq \frac{6x_2}{(1-\theta)(1+\theta)}. \end{cases} \tag{27}$$

Finally, notice that $\pi^{\theta} \geq \pi^{-\theta}$ when $W \geq \bar{W}$, where $\frac{3-\theta}{8}\bar{W} = \frac{x_2}{1+\theta}$. In fact, the only intersection between π^{θ} and $\pi^{-\theta}$ is at $\bar{W}$.

Point ii). We need to show that if bundling is an equilibrium then no bundling is not an equilibrium. Suppose no bank bundles and one bank deviates and offers the bundle. Its payoff are defined in the proof defining $\pi^{-\theta}$ (since we only have 2 banks) and they write as

$$\pi^{-U} = \begin{cases} \min(\frac{1}{2}W, \frac{2x_2}{1+\theta}) & \text{for} \quad W \leq \frac{6x_2}{(1-\theta)(1+\theta)} \\ \min(\frac{1-\theta}{3}W, 2x_2) & \text{for} \quad W > \frac{6x_2}{(1-\theta)(1+\theta)}. \end{cases}$$

We have $\pi^{-U} > \pi^{U}$ for $W > 2x_2$. Since $2x_2 < 8x_2/(3-\theta)(1+\theta)$,which is the minimal wealth required to have bundling in equilibrium, we have that if bundling is an equilibrium then no bundling is not an equilibrium. ■

8.1.2 Varying the sophistication of investors

Consider the simplest setting with $J = 2$, $x_1 = 0$ and $x_2 > 0$, and $K \to \infty$. Suppose that each investor has a fixed number of signals M he can draw and assume $N = M = 2$. Suppose first that investors sample each bundle at most once (possibly due to the fact that they do not observe the size of each bundle). One can show that our main analysis would not be affected. Suppose instead that investors observe the size of the bundle and that they can sample several assets from the same bundle (they randomly draw across all assets with no replacement). Relative to the baseline model, bundling is now more likely to occur. This is shown in the next proposition (which should be compared to Proposition 5 with $\theta = 0$).

Proposition 6 *Suppose that $N = M = 2$. Bundling is an equilibrium for all W.*

Proof. Suppose $N = M = 2$. As there are 4 assets and 2 signals to be drawn, there are 6 possible realizations of the draws, each with equal probability. Suppose all banks bundle. A fraction 1/6 of investors sample two signals from the first bundle, 1/6 of investors sample two signals from the second bundle and the remaining investors sample one signal from each bundle. Hence, for each bundle, 1/3 of investors have valuation $2x_2$, 1/6 of investors have valuation $x_1 + x_2$, 1/3 of

investors have valuation $2x_1$ and 1/6 of investors have no valuation (and do not buy that bundle).

Each bundle can be sold at price $2x_2$ when the fraction of investors who draw x_2 from at least one bundle (that is, half of the investors) have enough wealth. That requires $W/2 > 4x_2$. When $W/2 < 2(x_1 + x_2)$, investors with valuation $x_1 + x_2$ start buying and so each bundle attracts a fraction 5/12 of wealth. The payoff for each bank is

$$\pi_2^B = \begin{cases} \min(\frac{1}{4}W, 2x_2) & \text{for} \quad W \geq 4x_2, \\ \min(\frac{5}{12}W, x_2) & \text{for} \quad W < 4x_2. \end{cases}.$$

Suppose bank j deviates and sell the assets separately, denote its payoff as π_2^{-B}. It can be easily shown that in equilibrium investors who sample x_2 from the bundle and a single asset x_2 always prefer to buy the bundle. Hence, bank j attracts only those investors who sample x_2 from j and do not sample x_2 from the bundle (that is, 1/3 of investors). We have $\pi_2^{-B} = \min(W/3, x_2)$. The result follows from noticing that $\pi_2^B \geq \pi_2^{-B}$ for all W. ■

8.1.3 When banks decide what to sell

The selling strategy of bank i can be represented as a partition of X^i which specifies which assets are put in the market and how those assets are bundled. Following the previous notation, such partition can be written as $\alpha^i = \{\alpha_0^i, \{\alpha_r^i\}_r\}$, in which the set of assets which are kept in the bank is α_0^i. The expected payoff of bank i choosing α^i is now defined as

$$\pi^i = \sum_r \left|\alpha_r^i\right| p(\alpha_r^i) + \sum_{x_j^i \in \alpha_0^i} x_j^i, \tag{28}$$

where x_j^i is the value of keeping asset X_j^i in the bank. The simplest setting to illustrate the possibility of a Bundler's Dilemma is one with $N \geq 3$ banks, each of them with two assets valued $x_1 = 0$ and one asset valued $x_2 > 0$. Full bundling occurs when each bank pools all its assets in a single bundle. We say there is partial bundling when each bank keeps one asset with value x_1 and pools the other assets (with value x_1 and x_2) in a single bundle. If all banks offer the full bundle, the payoff of each bank is

$$\pi^F = \min(3x_2, \frac{1}{N}(1 - (\frac{2}{3})^N)W), \tag{29}$$

where the term $\frac{1}{N}(1-(\frac{2}{3})^N)W$ accounts for the total wealth of the share of investors who make a positive draw X_2 for at least one of the N bundles. Full bundling can be sustained in equilibrium only if $\pi^F \geq x_2$, since each bank can get x_2 by withholding its assets. That requires $W \geq \tilde{W}$, where

$$\tilde{W} = \frac{N}{1-(\frac{2}{3})^N} x_2. \tag{30}$$

As we show in the next proposition full bundling is indeed an equilibrium for $W \geq \tilde{W}$. If banks could jointly follow the partial bundling strategy, their payoff would be

$$\pi^P = \min(2x_2, \frac{1}{N}(1-(\frac{1}{2})^N)W). \tag{31}$$

Full bundling is dominated by partial bundling when $\pi^F < 2x_2$, that is for $W < 2\tilde{W}$. We can then show the following:

Proposition 7 *There is a Bundler's Dilemma for* $W \in (\tilde{W}, 2\tilde{W})$.

Proof. Step 1. We show that full bundling is an equilibrium for $W \geq \tilde{W}$ when $N \geq 3$. To see this, we first show that if bank j deviates and withdraws one x_1 asset and offers the bundle (x_2, x_1) its payoff is $\pi^{-F} = x_1 + \min(\pi_0^{-F}, 2x_2)$, where

$$\pi_0^{-F} = \max(\frac{1}{2}(\frac{2}{3})^{N-1}W, \frac{2}{3N-1}(1-\frac{1}{2}(\frac{2}{3})^{N-1})W).$$

In π_0^{-F}, the first term corresponds to the case in which the only potential buyers of the deviating bank are those investors who sample x_2 from the deviating bank and x_1 from all the other bundles and the second term corresponds to the case in which investors who sample x_2 from the (x_2, x_1) bundle and x_2 from at least one (x_2, x_1, x_1) bundle are indifferent between trading any of those bundles. One can easily show that the first term applies for $N \leq 3$ and the second term for $N > 3$. To see when full bundling is an equilibrium, notice that $\frac{2}{3N-1}(1-\frac{1}{2}(\frac{2}{3})^{N-1}) < \frac{1}{N}(1-(\frac{2}{3})^N)$ for all N. Notice also that $\frac{1}{2}(\frac{2}{3})^{N-1} < \frac{1}{N}(1-(\frac{2}{3})^N)$ for $N = 3$.

Step 2. We show that full bundling is dominated by partial bundling, that is $\pi^F < \pi^P$, when $W < 2\tilde{W}$. The payoff π^F is increasing linearly in W up to $W^F = 3Nx_2(1-(\frac{2}{3})^N)^{-1}$, and it is equal to $3x_2$ afterwards. The payoff π^P is increasing linearly in W up to $W^P = 2Nx_2(1-(\frac{1}{2})^N)^{-1}$, and it is equal to $2x_2$ afterwards. Since $\pi^F = \pi^P = 0$ at $W = 0$, $W^F > W^P$, and $\frac{d\pi^F}{dW} < \frac{d\pi^P}{dW}$ for $W < W^P$, it follows that $\pi^F < \pi^P$ when $W < 2\tilde{W}$.

Step 3. We show that if full bundling is an equilibrium then partial bundling is not an equilibrium. We first show that if all banks offer the bundle (x_2, x_1) and

bank j deviates by offering (x_2, x_1, x_1) its payoff is $\pi^{-P} = \min(3x_2, \pi_0^{-P})$, where

$$\pi_0^{-P} = \max(\frac{1}{3}W, \frac{3}{2N+1}(1 - \frac{2}{3}(\frac{1}{2})^{N-1})W). \tag{32}$$

As in π_0^{-F}, the first term corresponds to the case in which investors who sample x_2 from a (x_2, x_1) bundle and x_2 from the (x_2, x_1, x_1) bundle prefer buying the latter while the second term corresponds to the case in which these investors are indifferent between the two. To see that if full bundling is an equilibrium then partial bundling is not an equilibrium, notice that $\frac{3}{2N+1}(1-\frac{2}{3}(\frac{1}{2})^{N-1}) > \frac{1}{N}(1-(\frac{1}{2})^N)$ for all N. Notice that $\frac{1}{3} > \frac{1}{N}(1-(\frac{1}{2})^N)$ for $N = 3$. Hence, we have $\pi^{-P} > \pi^P$ for all W when $N \geq 3$. ■

8.2 Additional Results in Monopoly

8.2.1 Small Number of Investors

We consider a setting with a finite number of investors K. We establish that if there are at least two investors, one sufficiently rich and another not too poor, then full separation is strictly dominated by full bundling.

Proposition 8 *Suppose $K > 1$ and there exist two investors $k_1 \neq k_2$ such that $w_{k_1} > Jx_J$ and $w_{k_2} > Jx_1$. Then full bundling strictly dominates full separation.*

To show the above result, recall that by definition $x_J = \max_j x_j$ and $x_1 = \min_j x_j$. The condition $w_{k_1} > Jx_J$ ensures that full bundling delivers at least the same payoff as full separation. For each x_j, investor k_1 can pay Jx_j when sampling an asset with value x_j. Hence, irrespective of other investors' wealth, the expected payoff to the bank is at least $\sum_j \frac{1}{J} Jx_j$. If in addition we have $w_{k_2} > Jx_1$, then full bundling strictly dominates full separation. When investor k_1 draws x_1 and investor k_2 draws $x_j \neq x_1$ (which occurs with strictly positive probability) investor k_2 drives the price of the bundle strictly above Jx_1. Hence, the expected payoff from full bundling exceeds $\sum_j x_j$, that is, the payoff from full separation.

Following the same logic, we show that if investors are sufficiently wealthy, bundling all assets into one package is optimal.

Proposition 9 *Suppose $K > 1$ and $w_k > Jx_J$ for all k. Then full bundling strictly dominates any other strategy.*

Proof. The condition on w_k ensures that whatever the bundling, the price of α is the maximum of the draws of the various investors. If α and X are separate,

the issuer gets $|\alpha|\, E[\max_k \widetilde{X}_k(\alpha)] + X$. If α and X are bundled then the issuer gets $(|\alpha|+1)E[\max_k \widetilde{X}_k(\alpha \cup X)]$. Note that $|\alpha|\, E[\max_k \widetilde{X}_k(\alpha)] + X$ is the same as $(|\alpha|+1)E[\widetilde{\widetilde{X}}(\alpha)]$ where $\widetilde{\widetilde{X}}(\alpha) = \max_k[\frac{|\alpha|}{|\alpha|+1}\widetilde{X}_k(\alpha) + \frac{1}{|\alpha|+1}X]$ where $+$ denotes here the classic addition. When α and X are bundled, $\widetilde{X}_k(\alpha \cup X)$ is the lottery $\frac{|\alpha|}{|\alpha|+1}\widetilde{X}_k(\alpha) \oplus \frac{1}{|\alpha|+1}X$. Thus, it is a mean preserving spread of $\frac{|\alpha|}{|\alpha|+1}\widetilde{X}_k(\alpha) + \frac{1}{|\alpha|+1}X$. Since for any three independent random variables, $\widetilde{Y}_1, \widetilde{Y}_1', \widetilde{Y}_2$ such that $\widetilde{Y}_1'$ is a mean preserving spread of $\widetilde{Y}_1$, we have that $E[\max(\widetilde{Y}_1', \widetilde{Y}_2)] > E[\max(\widetilde{Y}_1, \widetilde{Y}_2)]$ (this can be verified noting that $y_1 \to E_{y_2}[\max(y_1, y2)]$ is a convex function of y_1), we can conclude that $(|\alpha|+1)E[\max_k \widetilde{X}_k(\alpha \cup X)] > |\alpha|\, E[\max_k \widetilde{X}_k(\alpha)] + X$ and thus full bundling is optimal. ■

More generally, in the limit of a very large number of investors and for a given bundling strategy, the belief of the marginal investor is deterministic and so the bank does not face any uncertainty in its payoff. In a setting with a finite number of investors, or in which investors have stochastic wealth, the marginal belief becomes stochastic. A (mean-preserving) spread of this belief may increase or decrease the payoff associated to a given bundling strategy, and this depends on the level of aggregate wealth. When wealth is low, the marginal belief is the lowest evaluation in the population, so there is nothing to lose but possibly something to gain from a spread in beliefs. In this case, randomness increases the incentives to bundle. The opposite occurs when wealth is large. Moreover, in these settings, realizations (of wealth or of evaluations) may be worse than expected and so bundling may be profitable ex-ante but detrimental to the bank ex-post.

8.2.2 Intermediate Levels of Wealth

We consider intermediate levels of wealth so as to highlight more generally how investors' wealth affects the incentives to increase the belief dispersion, which in turns determines the optimal form of securitization. We consider the simplest setting for this purpose, one with three assets and a continuum of investors. To illustrate how market clearing prices are set, suppose the bank creates a single bundle consisting of all three assets $\{X_1, X_2, X_3\}$. A fraction 1/3 of investors samples X_1 and assesses that on average assets in the bundle have value x_1; 1/3 of investors assesses the average asset as x_2, and 1/3 of investors assesses the average asset as x_3. If $W/3$ exceeds $3x_2$, the most optimistic investors drive the price to a level at which no other investor is willing to buy. In that case, the payoff of the bank is $\min(W/3, 3x_3)$. When $W/3$ is slightly lower than $3x_2$, prices are such that also investors who draw x_2 are willing to buy. Hence, the bank gets

$\min(2W/3, 3x_2)$. When $W/3$ is slightly lower than $3x_1$, also investors who draw x_1 are willing to buy and the bank gets $\min(W, 3x_1)$.

It is clear that when W is sufficiently large, full bundling dominates any other strategy as it allows to sell assets as if they all had value x_3. We now characterize more precisely the optimal selling strategy -referred to as partition α^*- as a function of W. We show that the larger the aggregate wealth W, the more profitable it is to create bundles with several assets of heterogeneous value. As W decreases, the bank prefers to bundle fewer assets and assets of more similar value. When wealth is sufficiently large, full bundling is optimal. The next optimal bundling is $\{X_1, X_3\}$ followed then by $\{X_1, X_2\}$ up to the point where it is best to sell assets separately.

Proposition 10 *Suppose $K \to \infty$ and $J = 3$. Then*

$$\alpha^* = \begin{cases} \alpha_1 = \{X_1, X_2, X_3\} & \textit{for} \quad W \geq 6x_3 + 3x_2 \\ \alpha_1 = \{X_1, X_3\},\ \alpha_2 = \{X_2\} & \textit{for} \quad W \in [2x_3 + 2x_2, 6x_3 + 3x_2) \\ \alpha_1 = \{X_1, X_2\},\ \alpha_2 = \{X_3\} & \textit{for} \quad W \in [2x_1 + 2x_2, 2x_3 + 2x_2) \\ \alpha_1 = \{X_1\},\ \alpha_2 = \{X_2\},\ \alpha_3 = \{X_3\} & \textit{for} \quad W < 2x_1 + 2x_2. \end{cases}$$

Proof. The payoff from offering the full bundle $\{X_1, X_2, X_3\}$ is

$$\begin{cases} \min(3x_3, W/3) & \text{for} \quad W \geq 9x_2 \\ \min(3x_2, 2W/3) & \text{for} \quad W \in [9x_2/2, 9x_1/2) \\ \min(3x_1, W) & \text{for} \quad W < 9x_1/2 \end{cases}$$

Suppose instead the bank offers the bundle $\{X_1, X_2\}$ and $\{X_3\}$ as separate asset. We show that the payoff for the bank is

$$\begin{cases} \min(W, 2x_2 + x_3) & \text{for} \quad 2x_2 < x_3 \\ \min(W/2, 2x_2) + \min(W/2, x_3) & \text{for} \quad 2x_2 > x_3. \end{cases} \tag{33}$$

In these computations we never consider the possibility that the price of the bundle is driven by its lowest evaluation, since in that case it is clear that bundling cannot strictly dominate full separation. Consider first a candidate equilibrium in which investors who sample x_2 from the (x_2, x_1) bundle are indifferent between trading the single asset x_3 and the bundle. That requires $2x_2/p_2 = x_3/p_3$, where p_2 is the price of the bundle and p_3 is the price of the asset x_3. In addition, we need that $p_2 + p_3 \leq W$, so aggregate wealth is enough to buy prices p_2 and p_3. The above conditions give $p_2 \leq 2Wx_2/(x_3 + 2x_2)$ and $p_3 \leq Wx_3/(x_3 + 2x_2)$. In addition,

we need that $p_2 \leq W/2$,so those investors who have valuation x_2 for the (x_2, x_l) bundle can indeed drive the price to p_2. Suppose $\frac{2x_2}{x_3+2x_2} < \frac{1}{2}$ that is $2x_2 < x_3$. Then we must have $p_2 = 2Wx_2/(x_3+2x_2)$, and $p_3 = Wx_3/(x_3+2x_2)$.So the payoff of the bank is $\min(W, 2x_2 + x_3)$ when $2x_2 < x_3$. Suppose $2x_2 > x_3$. Then we must have $p_2 = W/2$, and $p_3 = x_3W/4x_2$. That cannot be in equilibrium since investors who sample x_l still have money and would like to drive the price p_3 up. So if $2x_2 > x_3$ investors are indifferent only if $p_2 = 2x_2$ and $p_3 = x_3$. That requires $W > 4x_2$. If $W < 4x_2$, then we must have $p_2 < 2x_2\frac{p_3}{x_3}$. If $W \in (2x_3, 4x_2)$, we have $p_2 = \frac{W}{2}$ and $p_3 = x_3$. If $W < 2x_3$, we have $p_2 = p_3 = \frac{W}{2}$.

Consider the other possible partitions. Since $2x_3 > x_j$ for $j = 1, 2$, the payoff follows the second case on the payoff in (33). Hence, for each $j = 1, 2$, keeping X_j and offering the bundle $\{X_l, X_3\}$ where $x_j \neq x_l$ gives payoff $\min(W/2, 2x_3) + \min(W/2, x_j)$.Comparing the various payoffs, one can see that when $W < 2(x_1 + x_2)$, no bundling can strictly dominate full separation. For $W > 2(x_1 + x_2)$, the bundling $\{X_1, X_2\}$ and $\{X_3\}$ is optimal until $x_3 + 2x_2 = x_2 + W/2$, that is for $W \in [2x_1 + 2x_2, 2x_3 + 2x_2)$, the bundling $\{X_1, X_3\}$ and $\{X_2\}$ is optimal until $3x_3 + x_2 = W/3$. For $W/3 > 3x_3 + x_2$, the full bundle is optimal. Notice also that the bundling $\{X_2, X_3\}$ and $\{X_1\}$ is dominated by $\{X_1, X_3\}$ and $\{X_2\}$ for $W > 2(x_1 + x_2)$. ■

8.2.3 Heterogeneity and Bundle Composition

In the following proposition, we consider the case in which several assets have the same value, and we observe that within homogeneous compositions, it is best to create bundles which are as small as possible.

Proposition 11 *Suppose $K \to \infty$ and the bank has $2\chi_1$ assets Y with value 0 and $2\chi_2$ assets Z with value $z > 0$, where χ_1 and χ_2 are positive integers. Then creating a single bundle $\{2\chi_1 Y, 2\chi_2 Z\}$ is dominated by creating two bundles, each with $\{\chi_1 Y, \chi_2 Z\}$.*

Proof. The payoff from the single bundle is $\pi^1 = \min(2(\chi_1 + \chi_2)z, \frac{\chi_2}{\chi_1+\chi_2}W)$. The payoff from the two identical bundles is $\pi^2 = \min(2(\chi_1+\chi_2)z, (1-(\frac{\chi_1}{\chi_1+\chi_2})^2)W)$. Notice that $\pi^2 \geq \pi^1$ since $1 - (\frac{\chi_1}{\chi_1+\chi_2})^2 > 1 - (\frac{\chi_1}{\chi_1+\chi_2}) = \frac{\chi_2}{\chi_1+\chi_2}$. ■

The intuition behind the result is simple, and can be illustrated when $\chi_1 = \chi_2 = 1$. If the monopolist pools all its assets in the bundle $\{Y, Y, Z, Z\}$, its payoff is $\min(4z, W/2)$ since the maximal wealth that can be extracted comes from investors making a Z draw, i.e., half the population of investors. By creating two

bundles $\{Y, Z\}, \{Y, Z\}$, its payoff is $\min(4z, 3W/4)$ where the term $3W/4$ accounts for the fact that an investor making a good draw from either bundle (there are 3/4 of them) is potentially willing to put his wealth in the market. By disaggregating, the monopolist does not affect the probability of inducing over evaluations of the bundles since the composition of each bundle remains the same. But disaggregating allows the monopolist to extract more wealth since it reduces the fraction of investors who end up with bad draws from all bundles.

8.2.4 Risk Aversion

Our main analysis assumes that investors are risk neutral so as to abstract from risk sharing considerations that could motivate bundling. Allowing for risk aversion in our baseline model may actually reinforce the incentives for bundling, as we now explain. Suppose our basic assets are loans with face value equal to 1 and probability of default equal to $1-x_j$. Suppose defaults are (perceived as) independent across loans. Suppose also that investors observe the size of each bundle.[22] If two assets with respective values x_2 and x_1 are offered separately, the investor perceives an expected value of $x_2 + x_1$ and a variance of $x_1(1-x_1) + x_2(1-x_2)$. If the assets are bundled, an investor drawing x_2 believes that the bundle has expected value $2x_2$ and variance $2x_2(1-x_2)$. If the investor buys $(x_2+x_1)/2x_2$ units of the bundle, he perceives the same expected value x_2+x_1, but a variance of $(1-x_2)(x_2+x_1)^2/2x_2$, which is lower than $x_1(1-x_1) + x_2(1-x_2)$. If investors dislike payoffs with larger variance, the bank has an extra incentive to bundle assets x_1 and x_2. Bundling may lead investors not only to overestimate the expected value but also to underestimate the variance of returns.

8.2.5 Tranching

An additional motive for bundling assets is to create different tranches which are then sold to investors with different risk appetites. In our setting, tranching can be profitable even if investors are risk neutral. Tranching may be a way to exploit belief heterogeneity and, relative to selling pass-through securities, it may allow the bank to extract a larger share of wealth. We also show this is the case even if the bank were required to keep the most junior tranche.

To see this most simply, consider a bank ($N = 1$) with two assets with $0 < x_1 < x_2$. The bank can offer a pass-through security or slice the bundle into a

[22] This is not needed as the argument would hold irrespective of the size of the bundle, but it simplifies the exposition.

junior and a senior tranche. The senior tranche pays 1 if at least one loan is repaid, the junior tranche pays 1 if both loans are repaid. Assume $K \to \infty$, investors are risk neutral and they observe the size of each bundle. Investors who sample x_1 value the senior tranche as $s_1 = 1-(1-x_1)^2 = 2x_1 - x_1^2$ and the junior tranche as $j_1 = x_1^2$. Similarly, investors who sample x_2 value the senior tranche as $s_2 = 2x_2 - x_2^2$, and the junior tranche as $j_2 = x_2^2$.

We provide some intuition about how the equilibrium works. Suppose the monopolist sells the two tranches and denote its payoff as π^T. At low levels of wealth, everyone buys both tranches and the payoff is W, which coincides with the payoff from offering a pass-through security π^B when $W \leq 2x_1$. When W is large, those sampling x_2 drive the prices so high that investors sampling x_1 prefer not to buy any tranche. In particular, this occurs when $W > 2W_4$, where

$$W_4 = \frac{s_1}{s_2}(j_2 + s_2).$$

In that case, we have again $\pi^T = \pi^B$. Tranching is however strictly preferred to the pass-through security for intermediate levels of wealth, for which investors sampling x_2 buy both tranches while those sampling x_1 only buy the senior tranche. Tranching is profitable to the bank as it allows to price discriminate between those having optimistic views and those having pessimistic views about the bundle. This is shown in the next proposition.[23]

Proposition 12 *We have $\pi^T \geq \pi^B$ for all W and $\pi^T > \pi^B$ for $W \in (2x_1, 2W_4)$.*

Proof. We first show that the payoff from tranching writes as

$$\pi^T = \begin{cases} \min(\frac{W}{2}, 2(x_1+x_2)) & \text{for} \quad W \geq 2W_4. \\ \min(W, W_4) & \text{for} \quad W < 2W_4. \end{cases} \tag{34}$$

To see this, notice first that investor drawing signal x_z for $z = \{1, 2\}$ prefers to buy the junior tranche as opposed to the senior tranche iff $p_s/p_j \geq s_z/j_z$.Notice also that $s_2/j_2 < s_1/j_1$,since $x_1 < x_2$. This implies that in equilibrium it must be that

$$\frac{p_s}{p_j} = \frac{s_2}{j_2}. \tag{35}$$

[23]It should be noted though that $W_4 < (x_1 + x_2)$, so that the monopolist stills prefer selling its assets separately rather than as a bundles with two tranches when $W < 2(x_1 + x_2)$. Hence, in this example, allowing the monopolist to sell assets in tranches does not strictly improve its payoffs. We suspect it could be otherwise in more elaborated situations.

Suppose by contradiction that $p_s/p_j < s_2/j_2$. Then everyone would strictly prefer buying the senior tranche and no one would buy the junior tranche, which cannot happen in equilibrium. Suppose instead

$$\frac{p_s}{p_j} > \frac{s_2}{j_2}. \tag{36}$$

Those who sample x_2 only buy the junior tranche while those who sample x_1 must buy the senior tranche. Notice first that we must have $W/2 \leq j_2$, or those who sample x_2 would still have money left and they would be willing to buy the senior tranche given $p_s \leq s_1 < s_2$. Suppose $W/2 \leq s_1$. Then those who sample x_1 only buy the senior tranche and $p_s = W/2$. We would have $p_s = p_j = W/2$ and would contradict (36) since $s_2 > j_2$. Suppose $W/2 > s_1$, which is consistent with $W/2 \leq j_2$ only if $s_1 < j_2$. We would have $p_s = s_1$ and $p_j = W/2$, which contradicts (36) since $s_2 > j_2$ and $W/2 > s_1$. Hence, it cannot be that those who sample x_2 only buy either the junior tranche or the senior tranche.

Hence, for $W > 2W_4$, only those who sample x_2 buy both tranches, in which case $\pi^T = \min(\frac{W}{2}, 2(x_1 + x_2))$. If $W < 2W_4$, investors sampling x_1 are attracted in the market and they only buy the senior tranche. Investors sampling x_2 buy both tranches and we have $p_s = s_1$ and $p_j = \frac{j_2}{s_2} s_1$ due to (35). That gives $\pi^T = s_1(1 + \frac{j_2}{s_2}) = W_4$. That occurs until $W \geq W_4$. If $W < W_4$, we have $\pi^T = W$. The payoff from offering the bundle writes as

$$\pi^B = \begin{cases} \min(\frac{W}{2}, 2(x_1 + x_2)) & \text{for} \quad W \geq 4x_1. \\ \min(W, 2x_1) & \text{for} \quad W < 4x_1. \end{cases}$$

Hence, given (34) and noticing that $W_4 > 2x_1$, we have $\pi^T \geq \pi^B$ for all W and $\pi^T > \pi^B$ for $W \in (2x_1, 2W_4)$. Finally, it should be noted that in our setting the bank would still have an incentive to securitize even if it were required to keep the most junior tranche. Indeed, if investors' wealth is large enough, the bank would still benefit from selling the senior tranche to the most optimistic investors. ■