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Production Network and International Fiscal Spillovers

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Abstract

This paper analyzes the impact of fiscal spending shocks in a multi-country model with international production networks. In contrast to standard results suggesting that production network linkages are unimportant for the aggregate response to macro shocks in a closed economy, we show that network structures may place a central role in the international propagation of fiscal shocks, particularly when wages are slow to adjust. The paper first develops a simple general equilibrium multi-country model and derives some analytical results on the response to fiscal spending shocks. We then apply the model to an analysis of fiscal spillovers in the Eurozone, using the calibrated sectoral network structure from the World Input Output Database (WIOD). In a version of the model with sticky wages, we find that fiscal spillovers from Germany and some other large Eurozone countries may be large, and within the range of empirical estimates. More importantly, we find that the Eurozone production network is very important for the international spillovers. In the absence of international production network linkages, spillovers would be only a third as large as predicted by the baseline model. Finally, we explore the diffusion of identified German government spending at the sectoral level, both within and across countries. We find that government expenditures have both significant upstream and downstream effects when these links are measured by the direction of sectoral production linkages.

Keywords: Production Network, Fiscal Policy, Spillovers, Eurozone, Terms of Trade, Nominal Rigidities

JEL Codes: E23 E62 F20 F42 H50

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1 Introduction

During the global financial crisis, many countries initiated massive fiscal stimulus to boost their domestic economies. These fiscal stimuli may have had various impacts on other countries, through international trade and capital flows. Empirical evidence shows that fiscal spillovers across borders can be large, depending on how fiscal shocks are identified (e.g., [Auerbach and Gorodnichenko, 2012a](#)). Nevertheless, most model based analyses suggest that fiscal spillovers across countries are negligible, given the size of trade openness at the aggregate level. ¹

Recent research argues that production linkages between countries can have important implications for aggregate co-movements, for instance see [Burstein, Kurz and Tesar \(2008\)](#), [Giovanni and Levchenko \(2010\)](#), and [Giovanni and Levchenko \(2012\)](#). Motivated by this, our paper investigates the importance of production networks in accounting for fiscal spillovers across countries. It has been well documented that international trade is substantially dominated by trade in intermediate products. Firms in many countries intensively use intermediate inputs produced by their own industry and also by other home and foreign industries. These input-output linkages not only link production lines globally, but also serve as important channels for shock propagation within a country and across borders. How do these input-output linkages affect domestic and foreign fiscal multipliers? Can production networks account for the discrepancy between fiscal spillovers in the data and models?

Our baseline model is a multi-country version of [Bigio and La'O \(2016\)](#), which is a general equilibrium multi-sector model incorporating production distortions at the sectoral level. The main question addressed is how the production network connections across sectors and countries affect the response to fiscal policy shocks, both at the aggregate country-level as well as at the level of individual sectoral responses.

We employ two versions of the model. In one version, all prices and wages are flexible, and sectoral shocks work through both supply and demand channels. In particular, in a multi-country setting, shocks in one country will impact on relative factor prices across countries, and in turn these price changes cause adjustments along the production network. In a second version of the model, we assume that nominal

¹See for instance, [Cwik and Wieland \(2011\)](#), [Goujard \(2016\)](#), and [Ivanova and Weber \(2011\)](#).

wages are sticky, and cannot immediately adjust to fiscal policy shocks. In this version, fiscal shocks are diffused through production networks solely through demand channels. Our results, described further below, suggests that the distinction between the two models has major implications for the importance of production networks in aggregate co-movements. We also examine how the responses to fiscal shocks depend on the presence of production distortions (or 'wedges', in the language of [Bigio and La'O \(2016\)](#)), and the sectoral composition of fiscal spending.

The first section of the paper develops some simple theoretical results on the responses to fiscal policy shocks in closed and open economies with production networks. In a closed economy, absent production distortions, we show that the production network structure is irrelevant for the aggregate government spending multiplier. Moreover, the spending multiplier is independent of the sectoral composition of fiscal spending. This result mirrors well known conclusions from the literature on production networks in closed economies (e.g. [Hulten 1978](#)). By contrast, in a multi-country model, the production network is always relevant for the response to fiscal spending. In the flexible price and wage version of the model, the response to fiscal shocks depends crucially on the movements in the terms of trade across countries. With sticky wages, the production network itself matters directly for the way in which shocks are channeled across sectors and countries.

We then apply the model to the data. We use the World Input Output Database (WIOD) for the Eurozone countries over the period of 2000-2014 and consider German government expenditure shocks. The model is simple enough that all the share parameters can be directly calibrated from the WIOD. The focus on the Eurozone subsample is motivated by a number of factors. First, it is likely that production networks within the European Union are more important than those among other less well connected countries within the WIOD. Secondly, a large policy literature has developed on the role of fiscal policy within the EU and the Eurozone, and there is quite a lot of empirical evidence on the impact of fiscal shocks within this group of countries. Finally, and relatedly, the Eurozone has a common monetary policy, so looking at the diffusion of fiscal shocks along the Eurozone production network does not require us to take a stand on national monetary policies for each country.

We first note some features of production networks in the Eurozone. Although the Eurozone has more production linkages than many other regions, we find that cross-country production network connections are much more sparse than those in countries within the Eurozone. Network connectivity (defined in various ways) has increased over the sample period, but production networks are still highly 'home-

biased'. As is common in many countries, there is also very substantial home bias in private and especially public spending.

As noted, our main focus of interest is the impact of shocks to German fiscal expenditures. We first explore this at the aggregate level, looking at the responses to spending shocks on aggregate GDP in the calibrated model, both in the home economy and through cross-country spillovers. We find that there is a major difference between the two versions of the model. In the model with fully flexible wages and prices, supply and demand channels tend to work against one another in so far as cross-country spillovers are concerned. The aggregate spillovers in the model are very small, and almost all the aggregate response to fiscal shocks is felt within the domestic economy. We also find that the production network connections across countries in the Eurozone has little effect on the responses to fiscal spending shocks. Recalibrating the model to shut down international network has negligible effects on the predicted response to shocks.

By contrast, when nominal wages are sticky, we find that spillovers working solely through the demand channel can be large, and certainly within the range of empirical estimates of fiscal spillovers in the Eurozone. Moreover, in this case the network linkages among countries has first order effects on the response to fiscal shocks. When we solve a recalibrated version of the model shutting down the international production network, the implied fiscal spillovers are reduced by more than two-thirds.

A second empirical exercise explores the diffusion of fiscal shocks at the sectoral level, both within and across countries. In this exercise, we ask how the production network determines the magnitude and direction of shocks. We first measure German fiscal policy shocks using the SVAR identification procedure of [Blanchard and Perotti \(2002\)](#). Following this, using the WIOD fiscal spending shares at the country-sectoral level, we construct a measure of sectoral demand shocks using the approach of [Acemoglu, Akcigit and Kerr \(2015b\)](#). Using a simple regression approach to both the model simulations and sectoral data, we find strong support for the implications of the model at the level of sectoral responses to demand shocks. A key aspect of this empirical approach is to distinguish between 'downstream' and 'upstream' effects of demand shocks. As shown by [Acemoglu, Akcigit and Kerr \(2015b\)](#), demand shocks should work upstream in benchmark production networks. Our regression analysis strongly confirms this prediction.

This paper links to three branches of literature. First, our paper contributes to the literature on production networks and macroeconomic impact of shocks. Following the pioneering work of [Gabaix \(2011\)](#) and [Acemoglu, Carvalho, Ozdaglar and TahbazSalehi \(2012\)](#), [Acemoglu, Ozdaglar and Tahbaz-Salehi \(2015a\)](#) investigate the propagation of microeconomic shocks through input-output linkages and

find that when input-output linkages are unbalanced, micro shocks can lead to sizable macroeconomic tail risks. [Baqaee and Farhi \(2017b\)](#) provide conditions that production networks can substantially amplify sectoral productivity shocks in dynamic stochastic general equilibrium models. They suggest to use higher-order approximation techniques to solve dynamics models in order to capture the effect of production networks.² In a companion research, [Baqaee and Farhi \(2017a\)](#) provide a general non-parametric formula for aggregating microeconomic shocks in general equilibrium economies with distortions. In most cases the production network literature has focused on productivity shocks (see [Bigio and La’O, 2016](#); [Baqaee and Farhi, 2017b](#)), with substantial attention to the idea that the networks may allow sectoral productivity shocks to propagate to the aggregate economy.³ Nevertheless, our focus is on demand shocks, and specifically fiscal spending shocks.

The most related papers are [Acemoglu, Akcigit and Kerr \(2015b\)](#) and [Bigio and La’O \(2016\)](#). [Bigio and La’O \(2016\)](#) explore how production networks and distortions affect the propagation of sectoral productivity shocks in a closed economy multi-sector real business cycle framework and apply the framework to analyze shock propagation through the USA input-output linkages. Our paper takes use of a similar framework as [Bigio and La’O \(2016\)](#), but there are several important differences between our paper and theirs. First, we focus on the role of both domestic and international production networks in a multi-country world economy. Second, we explore how the demand side shocks–fiscal spending shocks–are propagated across borders through international input-output linkages. Third, our model features nominal wage rigidities as in the literature (see [Schmitt-Grohé and Uribe \(2016\)](#)). We find that international production networks substantially increase fiscal policy spillovers across borders in a model with nominal wage rigidities, while the spillovers are quite small in a model with flexible prices and wages. [Acemoglu, Akcigit and Kerr \(2015b\)](#) explore the propagation of macroeconomic shocks through input-output and geographic networks and they find that demand-side shocks propagate upstream and supply-side shocks propagate downstream in a closed economy. Our paper features labor markets segmentation across countries and we find that government expenditures have both significant upstream and downstream effects when these links are measured by the direction of sectoral production linkages.

²[Pasten, Schoenle and Weber \(2016\)](#) study a multi-sector Calvo model with intermediate inputs and explore the real effects of monetary policy shocks. Based on first-order approximations, they find heterogeneity in input-output linkages contributes only marginally to the real effects of monetary policy shocks. Following a similar approach, [Bouakez, Rachedi and Emiliano \(n.d.\)](#) explore fiscal multiplier in a multi-sectoral closed-economy monetary model.

³More recent works include [Jones \(2011\)](#); [Atalay \(2017\)](#); [Luo \(2016\)](#); [Acemoglu and Azar \(2017\)](#); [Lim \(2018\)](#); [Dong and Wen \(2018\)](#) and [Miranda-Pinto and Young \(2018\)](#).

Second, our paper is related to studies on intermediate inputs and business cycles. [Basu \(1995\)](#) and [Dupor \(1999\)](#) incorporate multi-sector production structures into closed economy business cycle models. [Bouakez, Cardia and Ruge-Murcia \(2009\)](#) explore the idiosyncratic sectoral dynamics in an economy with heterogeneous production sectors in response to monetary policy shocks. Along international dimension, [Giovanni and Levchenko \(2012\)](#) investigate how country size and international trade affect macroeconomic volatility. Based on a cross-country, industry-level panel dataset of manufacturing production and trade, [Giovanni and Levchenko \(2010\)](#) find that sector pairs that experience more bilateral trade exhibit stronger comovement, particularly for cross-border industry pairs that use each other as intermediate inputs. The input-output linkages in our model are similar to those of the above papers, but we follow the recent literature that deviates from Hulten’s theorem ([Hulten, 1978](#); [Baqae and Farhi, 2017b](#)) via introducing production wedges and especially nominal rigidities into a multi-country world economy.

Third, this study also provides a micro-foundation of fiscal spillovers. Theoretically, our results indicate that the fiscal spillovers could be large or small, depending on which sectors a government chooses to spend, how sectors are linked via various production networks, and how factor prices adjust. This may provide a new angle to reconcile various empirical studies on fiscal spillovers, i.e., [Beetsma, Giuliodori and Klaassen \(2008b\)](#), [Auerbach and Gorodnichenko \(2012b\)](#) and [Corsetti and Muller \(2013\)](#).

4

The structure of paper is as follows. Section 2 lays out the baseline model. Analytical results for various versions of the model are presented in Section 3. Section 4 reviews the evidence on fiscal spillovers in the Eurozone. The calibration of the model to the WIOD is discussed in section 5 and the quantitative results of the model for the aggregate response to fiscal policy shocks follow next. Section 6 looks more closely at the sectoral responses to fiscal shocks.

2 The model

We consider a variant of [Bigio and La’O \(2016\)](#) economy with (a) constant return to scale production technologies, (b) production wedges which distort firms’ input choices, and (c) constant elasticity of

⁴[Cacciatore and Traum \(2018\)](#) study the role of international trade following unanticipated government spending and income tax changes in a two-country environment. Section 5.5 below provides a review of the empirical literature on fiscal spillovers within the Eurozone.

substitution among firms' intermediate inputs. Firms/sectors may face differences in (a) intermediate input expenditure shares (production networks), (b) production wedges (or distortions) and (c) final absorption by households and the government. This general model has the advantage that it can be directly incorporated into the structure of production linkages within and between countries defined in the World Input Output Database.

There are K countries in the world economy. Country k has N_k sectors, each of which can be represented by a competitive firm with constant elasticity of substitution production. Countries differ in both sectoral production efficiency and size (output and employment). Labour is mobile across sectors within each country but immobile across countries.⁵ Normalizing the total world population to unity, we let country k have population n_k , so that $\sum_{k=1}^K n_k = 1$.

2.1 Production

Following the international trade literature (i.e., [Anderson and Van Wincoop, 2003](#); [Allen, Arkolakis and Takahashi, 2014](#); [Caliendo, Parro and Tsyvinski, 2017a](#)), we assume constant returns to scale in production, so that for a competitive firm in sector i of country k , the technology is

$$X_{ki} = \exp(\alpha_{ki}\epsilon_{ki})\ell_{ki}^{(1-\alpha_{ki})}M_{ki}^{\alpha_{ki}} \quad (1)$$

X_{ki} denotes the gross output produced by the firm. ϵ_{ki} is a productivity term, and ℓ_{ki} is employment. α_{ki} captures the intermediate input cost share in production. M_{ki} is the composite of intermediate inputs,

$$M_{ki} = \prod_{m=1}^K \prod_{j=1}^{N_m} X_{kimj}^{\omega_{kimj}}$$

In the baseline model we assume an elasticity of substitution of unity across intermediate inputs, as in [Acemoglu, Akcigit and Kerr \(2015b\)](#), and [Bigio and La'O \(2016\)](#).⁶ In particular, X_{kimj} represents the use of country m sector j product by sector i in country k . The input-output matrix is denoted by \mathbf{W} with entry ω_{kimj} , where $\sum_{m=1}^K \sum_{j=1}^{N_m} \omega_{kimj} = 1$, with $\omega_{kimj} \geq 0$.

⁵Although goods and labor markets are highly integrated in the Eurozone ([Caliendo, Opromolla, Parro and Sforza, 2017b](#)), spatial wage gaps still persist even within a country ([Heise and Porzio, 2018](#)).

⁶In Appendix C, we explore the results under a more general CES technology, estimating elasticities from the data. We find that the results are quite similar to those of the baseline case.

A representative firm in country k sector i faces the following problem

$$\max_{\{\ell_{ki}, X_{ki}\}} \pi_{ki} = (1 - \tau_{ki})P_{ki}X_{ki} - \widetilde{w}_k \ell_{ki} - \sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj} X_{kimj} \quad (2)$$

subject to the technology constraint (1). P_{ki} is the product price, \widetilde{w}_k is the wage rate. Here, τ_{ki} is equivalent to a tax on the firm's revenue which plays the role of a production wedge, or distortion. Let sales in sector ki be $S_{ki} = P_{ki}X_{ki}$ and total expenditures on inputs $U_{ki} = \widetilde{w}_k \ell_{ki} + \sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj} X_{kimj}$.

The optimality conditions are

$$\widetilde{w}_k \ell_{ki} = (1 - \alpha_{ki})U_{ki} \quad (3)$$

$$P_{mj} X_{kimj} = \alpha_{ki} \omega_{kimj} U_{ki} \quad (4)$$

$$U_{ki} = \phi_{ki} S_{ki} \quad (5)$$

with $\phi_{ki} \equiv 1 - \tau_{ki}$. Profits can be written as $\pi_{ki} = (1 - \tau_{ki})S_{ki} - U_{ki} = 0$, and with competitive markets, profits are zero.

2.2 Households

A representative household's preference in each country k has the form

$$u(c_k, \widetilde{L}_k) = \frac{(c_k(1 - \widetilde{L}_k)^{\lambda_k})^{1-\sigma}}{1 - \sigma}$$

where \widetilde{L}_k is total labour supply of the household and labour services are homogeneous within a country, with total available labour supply normalized to unity. c_k is a consumption aggregator, which has the following Cobb-Douglas aggregation for the consumption basket,

$$c_k = \prod_{m=1}^K \prod_{j=1}^{N_m} c_{kmj}^{\beta_{kmj}}$$

where $\beta_{kmj} \geq 0$, $\sum_{m=1}^K \sum_{j=1}^{N_m} \beta_{kmj} = 1$.⁷

We abstract from all asset trade across countries, and assume that each country satisfies its within-

⁷Appendix (C) generalizes the consumption aggregator to allow for non-unitary elasticity of substitution across goods.

period budget constraint. In country k , the aggregate households' budget constraint is given by

$$P_k C_k = \sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj} C_{kmj} = \widetilde{w}_k L_k + \sum_{i=1}^{N_k} \pi_{ki} + T_k \quad (6)$$

where T_k represents lump-sum transfers from the government, and we have used the aggregate consumption $C_{kmj} = n_k c_{kmj}$ and labour supply $L_k = n_k \widetilde{L}_k$. A representative household chooses consumption c_{kmj} and labor supply \widetilde{L}_k to maximize the lifetime utility. The optimality conditions for households in country k become

$$P_{mj} c_{kmj} = \beta_{kmj} C_k P_k \quad (7)$$

where P_k is defined as

$$P_k = \prod_{m=1}^K \prod_{j=1}^{N_m} \left(\frac{P_{mj}}{\beta_{kmj}} \right)^{\beta_{kmj}}$$

Optimal labour supply is characterized by the condition

$$\widetilde{w}_k (1 - \widetilde{L}_k) = \lambda_k P_k C_k$$

Multiplying country size n_k , yields,

$$\widetilde{w}_k (n_k - L_k) = w_k - \widetilde{w}_k L_k = \lambda_k P_k C_k$$

where we define $w_k = \widetilde{w}_k n_k$ as the country-size weighted wage rate in country k .

2.3 Government Policy

Governments spend on goods in many sectors and many countries. Assume that governments have access to lump sum taxation. The government budget constraint is then

$$T_k = \sum_{i=1}^{N_k} (1 - \phi_{ki}) S_{ki} - \sum_{m=1}^K \sum_{j=1}^{N_m} G_{kmj} \quad (8)$$

The first term on the right hand side is the government revenue raised from the tax distortion, rebated in lump sum back to households. The second term is the lump sum tax revenue raised to finance government spending.

2.4 Monetary Policy

In the model with fully flexible wages, monetary policy plays no role in determining real allocations. But when wages are sticky, the stance of monetary policy makes a difference. We introduce money into the model by assuming a simple quantity theory relationship between nominal consumption and the money supply.⁸ In addition, because our application below focuses on the fiscal spillovers within the Eurozone, we assume that there is a monetary union among countries which can be described by one monetary policy rule. But we also incorporate the ‘rest of the world’ as the residual country, it is necessary to allow for an independent monetary policy for the non-Eurozone residual country. To describe this, assume that the Eurozone consists of the first $K - 1$ countries, while the K th country is the rest of the world. For the Eurozone countries and the rest of the world, respectively, then we have

$$M^{ez} = \sum_{k=1}^{K-1} P_k C_k, \quad M^{row} = P_K^{row} C_K \quad (9)$$

where M represents money supply.

Where P_K^{row} indicates the country K price in terms of country K currency. In what follows, we will mostly express prices in terms of the currency of the first $K - 1$ countries. Assuming the law of one price holds at the sectoral level across all countries, then we can define prices in the rest of the world currency implicitly by the condition

$$\xi P_{mj}^{row} = P_{mj}$$

where ξ is the nominal exchange rate (Eurozone price of ROW currency). The assumptions governing monetary policy will be specified more clearly below.

2.5 Market clearing conditions

The labour market clearing condition reads

$$L_k = \sum_{i=1}^{N_k} \ell_{ki}$$

⁸This can be given a more microeconomic foundation using a simple money in the utility function specification applied to a static setting - e.g. [Obstfeld and Rogoff \(2002\)](#).

and market clearing condition for goods produced by sector ki reads,

$$S_{ki} = \sum_{m=1}^K \frac{\beta_{mki}}{\lambda_m} \left[w_m - \sum_{j=1}^{N_m} \phi_{mj} (1 - \alpha_{mj}) S_{mj} \right] + \sum_{m=1}^K \sum_{j=1}^{N_m} \phi_{mj} \omega_{mjk} \alpha_{mj} S_{mj} + \sum_{m=1}^K G_{mki} \quad (10)$$

2.6 The competitive equilibrium

Substituting labour market clearing conditions, government budget constraint, equation (3) and households optimal labour supply into households' aggregate budget constraints, yields

$$\lambda_k \left(\sum_{i=1}^{N_k} S_{ki} (1 - \alpha_{ki} \phi_{ki}) - \sum_{m=1}^K \sum_{j=1}^{N_m} G_{kmj} \right) = w_k - \sum_{i=1}^{N_k} \phi_{ki} (1 - \alpha_{ki}) S_{ki} \quad (11)$$

Substituting demand for factor inputs into the production function, yields the total sales for sector ki ,

$$P_{ki} X_{ki} = P_{ki} \exp(\alpha_{ki} \epsilon_{ki}) \left(\frac{(1 - \alpha_{ki}) \phi_{ki} S_{ki}}{\tilde{w}_k} \right)^{(1 - \alpha_{ki})} M_{ki}^{\alpha_{ki}}$$

$$M_{ki} = \prod_{m=1}^K \prod_{j=1}^{N_m} \left(\frac{\alpha_{ki} \omega_{kimj} \phi_{ki} S_{ki}}{P_{mj}} \right)^{\omega_{kimj}}$$

which can be simplified as

$$P_{ki} = \exp(-\alpha_{ki} \epsilon_{ki}) (1 - \alpha_{ki})^{\alpha_{ki} - 1} \alpha_{ki}^{-\alpha_{ki}} \phi_{ki}^{-1} \tilde{w}_k^{1 - \alpha_{ki}} (P_{ki}^M)^{\alpha_{ki}} \quad (12)$$

with the price for intermediate inputs used by sector ki

$$P_{ki}^M = \prod_{m=1}^K \prod_{j=1}^{N_m} \left(\frac{P_{mj}}{\omega_{kimj}} \right)^{\omega_{kimj}}$$

In the competitive equilibrium, we have $2N + K + 1$ unknowns S_{ki} , P_{ki} , and w_k , for $k = 1, \dots, K$, $i = 1, \dots, N_k$, and the exchange rate between the $K - 1$ countries of the monetary union and the rest of the world K , ξ_t . There are $2N + K + 2$ equations (10), (11) and (12) and (9). One equation is redundant by Walras' Law (we drop the last equation in (11)).

2.7 Sticky Nominal Wages

The model above assumes that all prices and wages are flexible. In the application to the Eurozone, this assumption is questionable. Substantial evidence from the Eurozone crisis and before attests to the slow adjustment of wages to macro shocks (Schmitt-Grohé and Uribe, 2016). We will explore a version of the model in which wages are pre-set in each period, and cannot adjust to unexpected fiscal shocks. A full micro-foundation for this assumption can be developed based on a model in which labour is differentiated and firms demand labour from each of a continuum of workers with elasticity of demand $\zeta > 1$ (Obstfeld and Rogoff, 2002). In equilibrium each worker within a country sets the same wage and has the same employment level. Here, we can state that the equilibrium wage setting equation is characterized by

$$Eu_c(c_k, \widetilde{L}_k) \widetilde{L}_k \left[\frac{\widetilde{w}_k}{\widetilde{P}_k} - \hat{\zeta} \frac{c_k \lambda_k}{1 - \widetilde{L}_k} \right] = 0 \quad (13)$$

where $\hat{\zeta} = \frac{\zeta}{\zeta - 1}$.

Given a fixed nominal wage in each country, we assume that employment is determined by labour demand. The set of equilibrium prices then may be recovered using (12). This requires knowledge not just of the wages (which are ex-post sticky) but also exchange rates. Following the discussion of monetary policy above, we will assume that the first $K - 1$ countries consists of a monetary union, and the K th country follows an independent monetary policy.

The competitive equilibrium with sticky wages is described in a different way. Conditional upon pre-set nominal wages (in domestic currency), we use the following three conditions,

$$S_{ki} = \sum_{m=1}^K \beta_{mki} P_m C_m + \sum_{m=1}^K \sum_{j=1}^{N_m} \phi_{mj} \omega_{mjk} \alpha_{mj} S_{mj} + \sum_{m=1}^K G_{mki} \quad (14)$$

$$P_k C_k = \sum_{i=1}^{N_k} S_{ki} (1 - \alpha_{ki} \phi_{ki}) - \sum_{m=1}^K \sum_{j=1}^{N_m} G_{kmj} \quad (15)$$

$$P_{ki} = \exp(-\alpha_{ki} \epsilon_{ki}) (1 - \alpha_{ki})^{\alpha_{ki} - 1} \alpha_{ki}^{-\alpha_{ki}} \phi_{ki}^{-1} \widetilde{w}_k^{1 - \alpha_{ki}} (P_{ki}^M)^{\alpha_{ki}} \quad (16)$$

Along with (9), and the law of one price condition $\xi P_{ki}^{row} = P_{ki}$ can be solved for S_{ki} , C_k , P_{ki} P_{ki}^{row} ,

and ξ , for $k = 1..K$ and $i = 1..N_k$.⁹

Before moving to the quantitative results of the model, we first explore analytically how production networks affect the fiscal multiplier and international spillover of fiscal shocks. In the following section 3.1, we start with a closed economy with production network and investigate the domestic own fiscal multiplier. In section 3.2, we extend tractable analysis to a multi-country environment.

3 The Fiscal Multiplier in Production Networks

3.1 A Closed Economy

We set $K = 1$ in the above model. In the closed economy, there is a single wage, given labour market mobility within the country. Assume that monetary rule (9) targets the CPI. Then it can easily be shown from (12) that the nominal wage is independent of fiscal spending shocks. Moreover, this implies that nominal prices for each sector don't respond to shocks and therefore are constant. As a result, we can compute the real effects of fiscal shocks from the equilibrium of the N sales equations, repeated here as (we drop country index k or m):

$$S_i = \sum_{j=1}^N \alpha_j \phi_j \omega_{ji} S_j + \frac{\beta_i}{\lambda} \left(w - \sum_{j=1}^n (1 - \alpha_j) \phi_j S_j \right) + G_i, \quad i = 1..N \quad (17)$$

We use variables without subscripts to denote the corresponding vectors. Define the effective Leontief matrix as

$$\mathbf{B} \equiv \left(\mathbf{I} - (\mathbf{e}_N \alpha') \circ \mathbf{W}' \circ (\mathbf{e}_N \phi') + \frac{\beta}{\lambda} (\phi' \circ ((1 - \alpha)')) \right)^{-1}$$

with column vector \mathbf{e}_N whose entries are one, α represents the column vector with entry α_i , ϕ for the column vector with entry ϕ_i , and \circ denoting the Hadamard product (entrywise product). This effective Leontief matrix is adjusted by heterogeneities in wedges across sectors and the indirect effect of labour market clearing. Then (17) may be rewritten as

$$S = \mathbf{B} \left(\beta \frac{w}{\lambda} + G \right) \quad (18)$$

For the special case with homogeneous wedges, so that $\phi_i = \phi$, and equal labour shares $\alpha_i = \alpha$ we

⁹Note that in (14)-(16), all nominal values are expressed in Eurozone currency, so the country K wage is defined as $\xi \tilde{w}_K^{r^ow} = \tilde{w}_K$, where $\tilde{w}_K^{r^ow}$ represents the preset sticky wage in currency K .

can verify that the sum of entries in each column of matrix \mathbf{B} equals $\frac{\lambda}{\lambda(1-\phi\alpha)+\phi(1-\alpha)}$.¹⁰

First, focus on a special case where the labour share and the production wedge is equal across all sectors, so that $\alpha_i = \alpha$ and $\phi_i = \phi$ for all i . Then we can sum the columns in the sales equation (18), to express total sales as

$$\sum_{j=1}^N S_j = \frac{w + \lambda \sum_{j=1}^n G_j}{\lambda(1 - \alpha\phi) + (1 - \alpha)\phi} \quad (19)$$

Since the nominal wage is preset and prices are independent of spending shocks, we can equate GDP to total nominal value added, or total sales less payments to intermediate inputs. In this case of uniform α and ϕ , GDP is then

$$GDP = (1 - \alpha\phi) \sum_{j=1}^N S_j = \frac{(1 - \alpha\phi)(w + \lambda \sum_{j=1}^n G_j)}{\lambda(1 - \alpha\phi) + (1 - \alpha)\phi} \quad (20)$$

Now, using (3) and (20), in combination with the labour market equilibrium, and normalizing $w = 1$, we can solve for equilibrium employment as

$$L = \frac{(1 - \alpha)\phi(1 + \lambda \sum_{j=1}^n G_j)}{\lambda(1 - \alpha\phi) + (1 - \alpha)\phi} \quad (21)$$

Given (20) and (21), we can state

Result 1. *Given equal labour share and production wedges across all sectors, the government spending multiplier in a network economy is independent of the network composition. This holds for both aggregate government spending as well as any individual sector-specific spending.*

The aggregate impact of a sectoral government spending shock, which represents a demand shock in our model, is the same, no matter how connected is the sector. With less connected sectors the aggregate impact will be felt through the sector, while with more connected sectors the impact will be diffused more generally, but in aggregate, there is no difference in terms of the full effect on GDP.

While the aggregate multiplier in the uniform economy is independent of the network, it is clear from (20) that the multiplier does depend on the production wedge. It is easy to see that the government spending multiplier will be increasing, the higher is the common production wedge (the lower is ϕ).

¹⁰But even in this special case, the sum of entries in each row of \mathbf{B} is not constant unless W is a regular matrix (i.e., $\sum_{i=1}^n \omega_{ij} = 1$ for all j), and households spend their income equally on all goods $\beta_i = \frac{1}{n}$.

Intuitively, this arises due to the fact the production wedge lowers the equilibrium employment level, as we see from (21).¹¹

Result 1 relies both on identical labour shares and identical production wedges across sectors. With differential labour shares, the network composition will affect the fiscal multiplier even in the presence of a uniform production wedge. But in the special case of zero production wedges, i.e. $\phi_i = 1$ for all i , we can establish

Result 2. *With zero production wedges in all sectors, the aggregate and sectoral government spending multipliers are equal to $\frac{\lambda}{\lambda+1}$.*

Thus, without production wedges, the government spending multiplier is identical to that in an economy without production networks, even though labour shares may differ across sectors.

In the more general case with non-zero and differential production wedges across sectors, then both the aggregate and sectoral multiplier will depend on the network structure. To illustrate how the production network makes a difference, we take two examples.

Example 1: A closed economy

Let $n = 5$, $\beta_i = \frac{1}{5}$, $\alpha_i = \alpha$ for $i = 1, \dots, 5$. In addition, assume the production network is characterized by a symmetric regular matrix, given by $\omega_{ii} = \frac{1}{\delta}$, $\omega_{ij} = \frac{1}{4}(1 - \frac{1}{\delta})$, where $\delta = 1, \dots, 5$. Thus, δ measures the degree of connectedness of the production network, with $\delta = 1$ representing zero connectedness, and $\delta = 5$ representing a symmetric network fully connected. In addition, assume that $\phi_i = 1$ for $i = 1, \dots, 4$, but $\phi_5 < 1$. Hence, only sector 5 suffers from a production wedge.

We can then establish two properties of this example.

1. The multiplier is higher in the distorted sector than in the undistorted sector.

$$\frac{dGDP}{dG_5} - \frac{dGDP}{dG_1} = -\frac{20\delta(1-\alpha)(1-\phi_5)}{\Delta} > 0$$

where $\Delta < 0$.¹²

2. A more connected network increases the government spending multiplier in the undistorted sector

¹¹Note that while nominal GDP given by (20) is increasing in the production wedge, real GDP, given by $\frac{GDP}{P}$, is decreasing in the wedge, since it is easy to show that the CPI will be related to the wedge in proportion to $\phi^{-\frac{1}{1-\alpha}}$. The multiplier result described above pertains both to nominal and real GDP.

¹²In particular $\Delta = 5\delta\alpha^2\lambda\phi_5 + 5\delta\alpha^2\phi_5 - 25\alpha^2\lambda\phi_5 + 15\delta\alpha\lambda - \delta\alpha\phi_5 - 25\alpha^2\phi_5 + 20\alpha\lambda\phi_5 + 16\delta\alpha - 20\delta\lambda - 4\delta\phi_5 + 5\alpha\lambda + 25\alpha\phi_5 - 16\delta$.

but reduces the multiplier in the distorted sector.

$$\frac{d^2GDP}{dG_i d\delta} = \frac{100\alpha(1-\alpha)(1-\phi_5)[\lambda(1-\alpha\phi_5) + \phi_5(1-\alpha)]}{\Delta^2} > 0, \text{ with } i = 1, \dots, 4$$

$$\frac{d^2GDP}{dG_5 d\delta} = \frac{-400\alpha\phi_5(1-\alpha)^2(1-\phi_5)(1+\lambda)}{\Delta^2} < 0, \text{ with } i = 5$$

The first point follows intuitively from the results above - since production wedges raise the aggregate government spending multiplier, a distortion in one sector will raise the multiplier relative to the undistorted case, and spending in the more distorted sector will have a larger aggregate effect. Point 2 reveals that the production network plays an important role with heterogenous production wedges. A more connected network leads spending in non-distorted sectors to have a greater influence on the distorted sector, and thereby raising the aggregate spending multiplier.

How do these results differ in the case of sticky wages? In fact, in the closed economy model where the money rule targets the CPI, sticky wages do not matter for the responses to fiscal spending shocks. This is clear from the above discussion. The equilibrium nominal wage is independent of government spending shocks in the closed economy model. But this is not true with multiple countries, as we see below.

3.2 The Multi-Country Model

Now we move to a multi-country world economy and assume first that prices and wages are flexible. With many countries, there are $N = \sum_{k=1}^K N_k$ conditions determining the equilibrium sales of each sector in each country (equation (10)). In the one country case, given a monetary rule that targeted the CPI, we saw that the nominal wage was independent of fiscal spending shocks. Therefore conditions (17) alone determined the equilibrium vector of sales. But with K countries, there are $K - 1$ relative wage rates which are determined in equilibrium given that labour markets must clear separately in each country. Country level resource constraints give additional K conditions (equation (11)). In addition, there must be a monetary rule which pins down nominal values. It is convenient in this case to assume that the monetary rule targets the sum of nominal wages (weighted by country size).¹³ Thus, we

¹³This normalization makes it easier to illustrate the results of fiscal spending shocks in the multi-country case algebraically. With flexible prices and wages, the impact of spending on real magnitudes is independent of the monetary rule in any case.

assume that $\sum_{k=1}^K w_k = 1$. For simplicity but without loss of generality, we assume that $\lambda_k = \lambda$ hold for all countries.

The equilibrium conditions can be written as

$$S = \mathbf{B} \left(\beta \frac{w}{\lambda} + G_M e_K \right) \quad (22)$$

Matrix \mathbf{B} has a similar definition to above except that it is defined across all K countries now and w is the corresponding column vector for w_k .

$$\mathbf{B} \equiv \left(\mathbf{I}_N - (\mathbf{e}_N \alpha') \circ \mathbf{W}' \circ (\mathbf{e}_N \phi') + \frac{\beta}{\lambda} (\Phi \circ \Gamma) \right)^{-1}$$

where \mathbf{I}_N is the identity matrix with dimension N , α and ϕ are corresponding vectors for α_{ki} , ϕ_{ki} , and G_M represents a matrix of government spending by country and sector.¹⁴

Now however, there are $K - 1$ endogenous relative wage rates, satisfying (11). We can rewrite the budget constraints as:

$$w = (\lambda \mathbf{I} \mathbf{K} - (1 + \lambda)(\Phi \circ (\mathbf{I} \mathbf{K} - \Gamma)) + \Phi) S - \lambda (G'_M e_N) \quad (23)$$

We drop the last equation of the constraints above due to Walras' law. Combining with the normalization

¹⁴In particular,

$$\Phi = \begin{bmatrix} \phi_{1,1}, \dots, \phi_{1,N_1}, & 0 & \dots & 0 \\ 0 & \phi_{2,1}, \dots, \phi_{2,N_2}, & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \phi_{K,1}, \dots, \phi_{K,N_K} \end{bmatrix}, \quad G_M = \begin{bmatrix} G_{11} & G_{21} & \dots & G_{K1} \\ G_{12} & G_{22} & \dots & G_{K2} \\ \vdots & \vdots & \vdots & \vdots \\ G_{1N} & G_{2N} & \dots & G_{KN} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} (1 - \alpha_{1,1}), \dots, (1 - \alpha_{1,N_1}), & 0 & \dots & 0 \\ 0 & (1 - \alpha_{2,1}), \dots, (1 - \alpha_{2,N_2}), & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & (1 - \alpha_{K,1}), \dots, (1 - \alpha_{K,N_K}) \end{bmatrix}$$

$$\beta = \begin{bmatrix} \beta_{1,1} & \beta_{2,1} & \dots & \beta_{K,1} \\ \beta_{1,2} & \beta_{2,2} & \dots & \beta_{K,2} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{1,N} & \beta_{2,N} & \dots & \beta_{K,N} \end{bmatrix}, \quad \mathbf{I} \mathbf{K} = \begin{bmatrix} \underbrace{1, \dots, 1}_{N_1}, & 0 & \dots & 0 \\ 0 & \underbrace{1, \dots, 1}_{N_2}, & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \underbrace{1, \dots, 1}_{N_K} \end{bmatrix}$$

Here we have assumed that government in any country may spend in any other country or sector.

$\sum_{k=1}^K w_k = 1$, we have $N + K$ equations solve for $N + K$ variables S_{ki} and w_k .

Equations (22) and (23) determine the nominal sale values for each sector and each country, as well as the wage rates w_k , $k = 1, \dots, K$. But unlike the closed economy case, now the sectoral and country prices are not determined by production efficiency alone, because wage rates are endogenous. As a result, relative prices across sectors and countries are generally affected by government spending. This is equivalent to saying that the terms of trade is endogenously determined. The implication of this as we see below is that the structure of the production network will generically affect the fiscal spending multiplier in the open economy model.

Following the approach of the previous section, the vector of log product prices across all sectors and countries may be written as:

$$p = -\mathbf{D}(\alpha \circ \epsilon) - \Omega + \mathbf{D}((1 - \alpha) \circ \hat{w}_N) \quad (24)$$

where $\mathbf{D} = (\mathbf{I} - (\alpha e'_N) \circ \mathbf{W})^{-1}$.¹⁵

From (22), (23) and (24) we can compute real GDP for country k as

$$GDP_k = \sum_{i=1}^{N_k} \frac{S_{ki}(1 - \alpha_{ki}\phi_{ki})}{P_k^{GDP}} \quad (25)$$

with GDP deflator P_k^{GDP} (Comparing purchasing power if deflated by CPI P_k). We can establish some general results from the multi-country case. Note that as before in the case where $\phi_{ki} = \phi$ and $\alpha_{ki} = \alpha$ (as well as λ) is common across all sectors and countries, the columns of \mathbf{B} each sum to $\frac{\lambda}{(1-\phi\alpha)\lambda + \phi(1-\alpha)}$. Using this, along with the normalization $\sum_{k=1}^K w_k = 1$ we can state the following result,

Result 3. *Given equal labour shares and production wedges across all sectors countries, the impact of government spending on world nominal GDP is independent of the network composition. This holds for both aggregate government spending as well as any individual sector-specific spending.*

¹⁵ We define $\Omega = \mathbf{D}\tilde{\Omega}$, with $\tilde{\Omega}_{ki} = (1 - \alpha_{ki}) \ln(1 - \alpha_{ki}) + \alpha_{ki} \ln(\alpha_{ki}) + \ln(\phi_{ki}) + \alpha_{ki} \sum_{m=1}^K \sum_{j=1}^{N_m} \omega_{kimj} \ln(\omega_{kimj})$, and $\hat{w}_N \equiv \underbrace{[\ln \tilde{w}_1, \dots, \ln \tilde{w}_1]}_{N_1} \underbrace{[\ln \tilde{w}_2, \dots, \ln \tilde{w}_2]}_{N_2} \dots \underbrace{[\ln \tilde{w}_N, \dots, \ln \tilde{w}_N]}_{N_K}$.

To show this, use the two properties of \mathbf{B} and the wage rates just noted. Then we can establish that

$$\sum_{k=1}^K \sum_{i=1}^{N_k} S_{ki}(1 - \phi_{ki}\alpha_{ki}) = \frac{(1 - \alpha\phi)(1 + \lambda \sum_{m=1}^K \sum_{j=1}^N G_{mj})}{(1 - \phi\alpha)\lambda + \phi(1 - \alpha)}$$

Result 3 establishes that the irrelevance of the production network for aggregate world nominal fiscal multipliers extends to the case where labour is immobile across countries. Even though individual wages may be affected by country specific shocks, in the aggregate the production links across sectors and countries have no implication for the world response.

In the case of zero production wedges, the extension of Result 2 applies, whereby the government spending multiplier for aggregate nominal GDP is equal to $\frac{\lambda}{1+\lambda}$, even if factor shares differ across countries and sectors. But now it is important to distinguish nominal from real GDP, because in general movements in government spending will affect the sectoral prices and therefore the CPI.

Aside from this, Result 3 is relevant only for world aggregates. When we focus on individual country GDP as defined by (25) the production network will generically affect the impact of government spending on GDP, irrespective of the presence or absence of production wedges or the composition of factor shares. The reason is clear; from (22) and (23), government spending shocks will generically impact on country specific wage rates, which from (24) determine individual countries terms of trade. But as we show below the response of wages and the terms of trade, and hence GDP, will be impacted by the structure of domestic and international production networks.

Example 2: A two-country economy with flexible wages.

Let $K = 2$, $N_k = 1$ for $k = 1, 2$, $\beta_k = \frac{1}{2}$, $\alpha_i = \alpha$ for $i = 1, \dots, 2$. In addition, assume the production network is characterized by a symmetric regular matrix and define ω as the common share of non-domestic inputs in production for each country with $\omega \leq 1/2$. Hence ω represents both the production network and the extent of cross-country production linkages. In addition, let the production distortion ϕ be common across the two countries. Government expends on domestic goods only.

We can establish the following three results.

1. The domestic government expenditure multiplier is positive but falling in the size of the cross-country production linkages. Define nominal sectoral value-added as $Y_1 \equiv S_1(1 - \alpha\phi)$,

$$\frac{dY_1}{dG_1} = \frac{1}{2} \frac{(1 - \phi\alpha)(\phi(1 - \alpha) + 2\lambda(1 - \alpha\phi(1 - \omega)))}{\Delta_1\Delta_2} > 0$$

$$\frac{d^2 Y_1}{dG_1 d\omega} = -\frac{\alpha\phi(1-\alpha\phi)}{\Delta_2^2} < 0$$

where $\Delta_1 = (\phi(1-\alpha) + \lambda(1-\alpha\phi))$, $\Delta_2 = 1 - \alpha\phi(1-2\omega)$.

Note that nominal prices in a multi-country model may vary with government expenditure shocks. Fortunately, we can verify in this example that $\frac{dP_k}{dG_1}|_{G_1=0} = 0$ with $k = 1, 2$. That is, consumer prices don't respond to government expenditure shocks when evaluating at $G_1 = 0$. Therefore, nominal GDP is identical to real GDP when deflated by CPI.

2. The fiscal multiplier on foreign GDP is negative (positive) if $\omega < \frac{1-\alpha}{2\lambda\alpha}$ ($\omega > \frac{1-\alpha}{2\lambda\alpha}$). The foreign multiplier is increasing in the size of cross-country production linkages.

$$\frac{dY_2}{dG_1} = -\frac{1}{2} \frac{\phi(1-\alpha\phi)(1-\alpha-2\lambda\alpha\omega)}{\Delta_1\Delta_2} \leq 0$$

$$\frac{d^2 Y_2}{dG_1 d\omega} = \frac{\alpha\phi(1-\alpha\phi)}{\Delta_2^2} > 0$$

3. The fiscal multiplier on foreign GDP is negative (positive) as the domestic terms of trade appreciates (deteriorates).

To see this, note that in this example the terms of trade is just $\frac{P_1}{P_2}$. But from the pricing functions, this must be proportional to $\frac{w_1}{w_2}$. Then, given the normalization $w_1 + w_2 = 1$, the terms of trade improves (deteriorates) as w_1 rises (falls). In this example we can show that

$$\frac{dw_1}{dG_1} = \frac{1}{2} \frac{\phi(1-\alpha-2\lambda\alpha\omega)}{\Delta_2} \leq 0$$

This example establishes that the sign of the cross-country government spending multiplier coincides with the sign of the terms of trade response. Moreover, the production network is a critical feature of the size of the domestic spending multiplier and the sign and size of the foreign multiplier.

Example 3: A two-country economy with sticky wages.

Now take an example where wages are sticky, and the two countries are in a monetary union. In particular, assume that $K = 2$, $N_k = 1$, $\beta_k = \frac{1}{2}$, $\alpha_i = \alpha$ for $i = 1, \dots, 2$. Let ω represent the same parameter as in Example 2. Assume also that the home country fiscal shock falls on home (foreign) goods in proportion $g \leq 1$ (or $(1-g)$). In addition, since the monetary rule of the previous example would be

redundant with sticky nominal wages, we assume that monetary policy stabilizes nominal consumption. Thus, we assume that $\sum_{k=1}^K P_k C_k$ is constant.

Note that from equation (16), sticky wages in a monetary union imply sticky prices. Therefore, nominal GDP equals real GDP when deflated by CPI in this example, and we can establish the following results:

1. The own-country and foreign country multipliers for this example are

$$\frac{dY_1}{dG_1} = \frac{(g(1 - \alpha\phi) + \omega\alpha\phi)}{\Delta_2} > 0$$

$$\frac{dY_1}{dG_2} = \frac{((1 - g)(1 - \alpha\phi) + \omega\alpha\phi)}{\Delta_2} > 0$$

Both the home and foreign country multipliers are strictly positive, so long as part of the home spending falls on foreign goods ($g < 1$), and/or there are production linkages between countries ($\omega > 0$). By contrast, when $\omega = 0$ and $g = 1$, the own (foreign) multiplier is unity (zero).

The simple economics of this example captures the intuition that demand side factors are the critical features of the fiscal spillover when wages, and hence relative prices do not respond to fiscal spending shock.

2. The own (foreign) multiplier is increasing (decreasing) in production distortions when $g > \frac{1}{2}$.

$$\frac{d^2Y_1}{dG_1d\phi} = \frac{\alpha\omega(1 - 2g)}{\Delta_2^2} < 0, \quad \frac{d^2Y_2}{dG_1d\phi} = -\frac{\alpha\omega(1 - 2g)}{\Delta_2^2} > 0,$$

The key distinction between Example 2 and Example 3 is the absence of endogenous supply responses in the spillover from fiscal spending across countries. We see that this difference represents the major factor in computing quantitative estimates of fiscal spillovers in the Eurozone in the next section.

4 Evidence on international fiscal spillovers

We first review some recent empirical and quantitative studies of fiscal spillovers. One result that emerges from this literature is a discrepancy between some of the empirical studies, which find significant spillovers, and quantitative DSGE models based mostly on trade linkages, which tend to find very small spillovers.

A considerable empirical literature has estimated fiscal policy spillovers with the European Union and the Eurozone. [Alcidi, Määttänen and Thirion \(2016\)](#), define a fiscal spillover as the influence of fiscal policy measures (such as taxes or government spending) in one country (source country) on another (recipient country) country. [Beetsma, Giuiodori and Klaassen \(2006\)](#) employ a two-block approach to investigate the trade spillovers of fiscal policy in EU. In the first step, they estimate the response of output to fiscal shocks in a panel VAR model (fiscal block). In the second step, they consider a dynamic version of gravity model to estimate the effects of domestic output on bilateral exports (trade block). Combining the estimates from both blocks, they are able to calculate the effect of fiscal shocks on bilateral exports and thereby on foreign output. Their results indicate that fiscal spillovers are economically significant. But the magnitude of spillovers varies with the size of the source country in which the fiscal shock originates and with the intensity of trade between the source country and recipient countries. A fiscal stimulus of 1% GDP in Germany leads to an average increase of 0.15% of foreign GDP for a spending increase, and 0.05% for a tax cut. In contrast, a fiscal stimulus in Greece increases foreign GDP by an average of 0.01% (0.005%) for a spending increase (a tax cut).

Empirically, there is little consensus on how the real exchange rate responds to domestic fiscal shocks. [Kim and Roubini \(2008\)](#) and [Monacelli and Perotti \(2010\)](#) find that the real exchange rate depreciates, while [Beetsma, Giuiodori and Klaassen \(2008a\)](#) report real exchange rate appreciation, though with a delay. More recently, [Auerbach and Gorodnichenko \(2016\)](#), using high frequency data of U.S defense spending, find that the exchange rate appreciates in response to spending announcements, but not to actual fiscal outlays.

[Bénassy-Quéré and Cimadomo \(2006\)](#) examine the fiscal spillovers from Germany to the seven largest European Union economies over the period 1971-2004, using a Factor Augmented VAR model. They find that expansionary fiscal policies in Germany, especially tax cuts, have a positive effect on neighbouring countries, but less so for remote ones.

Using narrative data on fiscal adjustments identified by [Pescatori, Leigh, Guajardo and Devries \(2011\)](#), [Hebous and Zimmermann \(2013a\)](#) find that the international transmission of fiscal consolidations works mainly through trade linkages, rather than financial linkages. The reaction of the interest rate to fiscal consolidations is insignificant, whereas the response of exports to foreign fiscal consolidations is negative, particularly for European countries, although the U.S seems to be insulated from foreign fiscal consolidations. Despite these empirical findings, model-based estimations tend to suggest a limited

scope of fiscal spillovers. Simulating four major macroeconomic models, [Gros and Hobza \(2001\)](#) find that effect of fiscal spillovers is quite small. A government spending shock of 1% of GDP in Germany has an average spillover effect in the whole euro area between -0.03% and +0.04% on impact. Similarly, [Attinasi, Lalik, Vetlov et al. \(2017\)](#), simulating the consolidation episodes of 2010-2013 in euro area countries in the New Multi-Country Model, find that the overall fiscal spillovers are limited.

Using a multi-country model to evaluate the spillover effect of German fiscal stimulus plan for 2009 and 2010 on France and Italy, [Cwik and Wieland \(2011\)](#) find that spillover effects are negligible. For Italy, the effect actually turns negative after one year. They argue that the negative effect is due to the appreciation of the common currency euro following the fiscal expansion in Germany, which reduces the competitiveness of member countries with respect to the rest of the world.

[Goujard \(2016\)](#) find that the short-term effect of fiscal consolidation within a currency union is more negative than between countries with flexible exchange rates. However, in the medium term, negative spillovers become larger between countries with flexible exchange rates.

Taking a simple analytical framework, [Ivanova and Weber \(2011\)](#) simulate the effect of actual fiscal consolidation plans in 20 countries for years 2011 and 2012, using estimates of fiscal multipliers and import elasticities from other studies. They find that aggregate spillovers through trade channels from synchronized consolidation plans are quite small compared to the impact of domestic fiscal policy. However, for small open economies such as Ireland, Belgium, Netherlands and Austria, spillovers are rather substantial.

[Favero, Giavazzi and Perego \(2011\)](#) examine the effects of fiscal policy among OECD countries controlling for country heterogeneity in the styles of fiscal policy (tax-based v.s. expenditure-based), fiscal reaction functions (implied debt-deficit dynamics) and degrees of openness. Using a Global VAR model, they find that international fiscal spillovers are important but the output response to fiscal consolidations differ significantly across countries, ranging from expansionary to contractionary. Similarly, [Hebous and Zimmermann \(2013a\)](#) also report notable heterogeneity in output dynamics across member countries in the euro area following a fiscal shock.

Another way to deal with heterogeneity in fiscal multipliers is to follow the approach of [Dabla-Norris, Dallari and Poghosyan \(2017\)](#). They estimate a GVAR model using quarterly data on 10 Euro countries: Germany, France, The Netherlands, Belgium Austria, Italy, Spain, Ireland, Finland and Portugal over the period 1999-2016. With this approach, they can deal with country heterogeneity in the response to

the fiscal shock. They show that larger economies generate larger spillovers. Closer trade links between countries (country of origin of the shock- country receiving the shock) imply as well higher spillovers. Regressing the size of spillovers on pair of country’s links, they show that trade links seems to matter more than financial links to explain the size of spillovers. Fiscal spillovers are the highest when Germany or France are the country originating the fiscal shock (cumulative spillovers higher than 1.4). Finally, fiscal multipliers are positive and higher than 0.5 in Germany, France, the Netherlands, Italy and Ireland.

In the subsequent analysis, we will compare our theoretical results on multipliers and spillovers to some of these empirical estimates.

5 Application to World Input Output Database

To the best of our knowledge, none of the above studies of fiscal spillovers have used detailed measures of European production networks to investigate the size and direction of fiscal spillovers. In this section, we pursue this question using an application of the World Input Output Database (WIOD) to the Eurozone. In particular, we explore the quantitative predictions of Eurozone fiscal spillovers by implementing a mapping between the above multi-country model and the WIOD. The system (22) and (23), along with (24), gives an integrated world economy consisting of K countries and N_k sectors per country, with an equilibrium value of sectoral output levels, sectoral prices, and country specific wage rates when prices and wages are flexible (similarly, the system (14)-(16) for sticky wages). This system can be directly applied to the WIOD in order to calibrate the parameters of the production network, private and government spending shares, labour shares, and relative country size. Given this calibration we can explore the domestic and spillover effects of government spending shocks both in the aggregate and the sectoral level.

5.1 WIOD description

The WIOD is a time-series of world input output tables (WIOT) and Socio-Economic Accounts which covers 43 countries over the period 2000-2014. The WIOT is an integrated system of input-output tables for 56 sectors identically defined for all countries, expressed in current value US dollars. We focus on the first 54 sectors only.¹⁶ The coverage is for 28 EU countries and 15 other major economies.

¹⁶The last two sectors in the WIOT are sector 55 “Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use” and sector 56 “Activities of extraterritorial organizations and

The WIOD Socio Economic Accounts give sectoral gross output, value added, employment, labour compensation and prices. The sectoral breakdown in the Socio Economic Accounts is identical to that in the WIOT, and the two data bases can be combined to identify key parameters of the model.

We focus on a data set of 11 Euro area countries, with the rest of the world (ROW) as a residual. The focus on the Eurozone allows us to focus on the role of production networks in the transmission of fiscal policy without having to take a stand on heterogeneity in monetary policy responses to fiscal shocks. In addition, the Eurozone represents an integrated economic area within which intermediate trading linkages are likely to be substantially greater than among other economies. Moreover, by using the time series of input output tables over the 2000-2014 period, we can investigate how the evolution of production networks within the Eurozone has changed the pattern of fiscal spillovers implied by the model.

5.2 Calibration

To implement the equilibrium model defined by the system of equations given by (22)-(24) (or (14)-(16)), we need to calibrate the intermediate input coefficients ω_{kimj} , the labour share parameters $1 - \alpha_{ki}$, the spending parameters from preferences, β_{kmj} , the government spending shares, g_{kmj} , the production wedges ϕ_{ki} , and the labour supply parameters λ_k . Given the assumption of Cobb-Douglas production and unit elasticity of substitution among consumption goods, all parameters can be directly measured from the WIOD except for λ_k when prices and wages are flexible.¹⁷

Column ki , row mj in the WIOT Tables gives expenditure by sector i in country k on intermediate inputs from sector j in country m . From the assumption of Cobb-Douglas production, the demand for intermediate input mj by sector ki is $P_{mj}X_{kimj} = \alpha_{ki}\omega_{kimj}P_{ki}X_{ki} = \alpha_{ki}\omega_{kimj}S_{ki}$. Dividing by the total demand for intermediate inputs (using homogeneity), we can write $\omega_{kimj} = \frac{P_{mj}X_{kimj}}{\sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj}X_{kimj}}$. Doing this for all countries and sectors gives us the transpose of the matrix \mathbf{W} , following the notation from section 2 above. Note that rows of \mathbf{W} sum to unity.¹⁸

The WIOT also gives private consumption expenditure for each sector-country by each country

bodies", both of which account for a tiny fraction of gross output. We omit these two sectors due to data availability.

¹⁷In the Appendix, we depart from the assumption of Cobb-Douglas production and unit elasticity of substitution in consumption, estimating elasticity parameters directly from the data for each country. The results are similar to those below.

¹⁸The WIOT tables include a measure for inventory adjustment by each country for each sector-country. To incorporate this in our model, which lacks inventories, we add inventory adjustment to intermediate goods demand for each sector mj in proportion to the share of demand coming from each country-sector ki .

$P_{mj}C_{kimj}$. Our static model does not contain investment. To account for this, we add to private consumption expenditure a share of gross fixed capital formation. To determine the private sector share of gross fixed capital formation we use World Bank data, which gives for each country a decomposition of gross fixed capital formation into private and public sectors. Again using the Cobb-Douglas specification, we obtain $\beta_{kmj} = \frac{P_{mj}C_{kmj}}{\sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj}C_{kmj}}$, where consumption is now measured including a share gross fixed capital formation at the sectoral level.

To compute government spending we use the column in the WIOT tables reporting final government consumption expenditure by each country on each country-sector, and add to that the share of gross fixed capital formation attributable to government. Let G_k represent the total nominal government spending of country k , including government investment. Then from the WIOT matrix we can construct the shares $g_{kmj} = \frac{G_{kmj}}{G_k}$, which describes the share of country k government expenditures going to sector j in country m .

Each parameter above can be calibrated for each year from 2000 to 2014 from the WIOD tables. The composition of the input-output tables did change over this period, although, as shown below, the changes did not fundamentally alter the predictions of the model for fiscal spillovers. Hence, we first report the results using an average of all shares over the 2000-2014 period. In the discussion below, we report how the predicted evolution of the effects of fiscal shocks changes year by year over the sample.

Combining labour compensation from WIOD Socio-Economic Accounts $\widetilde{w}_k l_{ki}$ with intermediate consumption $X_{kimj}P_{mj}$, we can compute labour income shares and production wedges. Total expenditure by firms in sector ki is

$$U_{ki} \equiv \sum_{m=1}^K \sum_{j=1}^{N_m} X_{kimj}P_{mj} + \widetilde{w}_k l_{ki}$$

and total sales is (column ‘total output’ at WIOT)

$$S_{ki} \equiv P_{ki}X_{ki} \equiv \sum_{m=1}^K \sum_{j=1}^{N_m} P_{ki}X_{mjki} + \sum_{m=1}^K P_{ki}C_{mki} + \sum_{m=1}^K G_{mki}$$

The sectoral labour income share is then defined as $\alpha_{ki} = 1 - \frac{\widetilde{w}_k l_{ki}}{U_{ki}}$. We measure the production wedge following the strategy of [Bigio and La’O \(2016\)](#). In particular, if the firm is profit maximizing and competitive, in the absence of wedges, then sales should equal costs. Given this, the production wedge can be inferred by the implied markup of sales over costs. Under this interpretation, the production

wedge implied by the model is defined as, $\phi_{ki} = \frac{U_{ki}}{S_{ki}}$. With a zero wedge in country-sector ki , we have $\phi_{ki} = 1$. As we have shown in previous sections, parameter λ can completely pin down the fiscal multiplier in some special circumstances in section 3. We take $\lambda = 7$ in the baseline model, implying the own fiscal multiplier would be around $\lambda/(1 + \lambda)$ when prices and wages are flexible.

The effects of fiscal spending shocks at the aggregate level is measured by the response of real GDP. Real GDP is measured as nominal GDP divided by the GDP deflator, using value added per sector. In particular, define $RGDP_{k,t}$ as real GDP for country k , and $RGDPD_{k,t}$ as the GDP deflator. In particular, we define

$$RGDP_{k,t} = \frac{GDP_{k,t}}{GDPD_{k,t}}$$

where

$$GDP_{k,t} = \sum_{i=1}^{N_k} S_{ki,t}(1 - \phi_{ki}\alpha_{ki}) = \sum_{i=1}^{N_k} P_{ki,t}X_{ki,t}(1 - \phi_{ki}\alpha_{ki}),$$

and

$$GDPD_{k,t} = \frac{\sum_{i=1}^{N_k} P_{ki,t}X_{ki,t}(1 - \phi_{ki}\alpha_{ki})}{\sum_{i=1}^{N_k} P_{ki,0}X_{ki,t}(1 - \phi_{ki}\alpha_{ki})}$$

where $t = 0$ represents the ‘base year’ for evaluation. In practice, we will measure changes in GDP using the pre-shock prices as the base year prices.

We can also measure GDP using a CPI based measure. This is equivalent to a PPP adjusted measure of GDP. In particular, we define $GDPCPI_{k,t}$ as follows

$$GDPCPI_{k,t} = \frac{\sum_{i=1}^{N_k} P_{ki,t}X_{ki,t}(1 - \phi_{ki}\alpha_{ki})}{P_{k,t}}, \text{ with } P_{k,t} = \prod_{m=1}^K \prod_{j=1}^{N_m} \left(\frac{P_{mj,t}}{\beta_{kmj}} \right)^{\beta_{kmj}}$$

One clear feature of the input output tables is the degree of home bias in spending patterns. This can be seen clearly in the country-specific shares in Figure 1. The Figure shows, for each country, the share of total government and private sector spending falling on domestic sectors, and in addition, the share of total intermediate input demand falling on domestic sectors.¹⁹ For all countries, government spending is heavily biased towards the domestic sector, with a share in excess of 90 percent. Private sector spending is less home biased than government spending, but the share in all countries exceeds 70 percent. Intermediate input usage is less domestically biased, but for the large European economies

¹⁹The latter share is computed, for each country k , as $\omega_k = \text{mean}_i(\sum_{j=1}^{N_k} \omega_{kikj})$.

intermediate inputs still displays over 70 percent home bias.

Figure 2 shows how the home bias in government spending and intermediate production network has evolved over the sample. Overall, across countries there is very little change in the strong domestic bias of government spending. By contrast, the production network has become more international, while still remaining heavily domestically biased.

An alternative perspective on the degree of production linkages across countries can be seen from network measures of centrality. Figure 3 presents a network centrality measure based on the number of incoming edges to each node, the in-degree centrality, for the year of 2000, 2007 and 2014. This in-degree centrality measure lies between 0 and 1. A zero means that the node is not connected to any other, while a 1 means that the node is connected to all the other nodes of the networks. Figure 3 shows that the 54 sectors of the 11 countries of the Eurozone become increasingly connected between 2000 and 2014. The 2014 yellow curve depicts a double peak compared to 2000 and 2007, suggesting that the number of sectors with a high in-degree centrality has largely increased between 2007 and 2014, with an important number of sectors between 0.96 to 1. However, the home bias measure above indicates that this large number of very “central” sectors hides low weights of input-output matrix coefficients at the cross country level. In Appendix A, Figure A-2 illustrates this more clearly in the form of a ‘heatmap’ of sectoral linkages across sectors and countries.

Figure 4 shows the model equilibrium levels of GDP by country for all 11 countries, plus the rest of the world. Note that GDP levels are not directly calibrated, but are determined by the equilibrium of the model given government and private sector spending patterns, as well as the production matrix \mathbf{W} . In this case, we see that the model does a good job in accounting for relative GDP levels among countries.

5.3 Fiscal Spillovers in the Baseline Model with Flexible Wages

Using the flexible price and wage version of the model, Table 1 shows the response of the two alternative measures of real GDP growth to a shock to German government spending approximately equal to one percent of German GDP. The Table reports in each case the responses using the baseline model, as well as a re-calibrated version of the model where all cross-country production linkages are

removed. ²⁰

The response of German GDP indicates that the own multiplier is approximately 0.9 for GDP. This falls slightly in the case of zero cross country production linkages. But the Table also indicates that in the baseline model, the spillover effects to GDP are completely negligible. The highest spillover is to Austria, where a one percent of GDP spending increase in Germany increases Austrian GDP growth by only .05 of one percent. In the absence of cross-country production linkages, the fiscal spillover is effectively zero. These results indicate that the cross-country production network does increase the fiscal spillover across countries, but quantitatively the impact of the production network is negligible.

When GDP is measured in terms of the CPI, the domestic response is somewhat smaller, and spillover effects larger. When measured in terms of CPI, each country gains from a lower price of German goods (see below). But again, the overall effects are still very small. The final column in the Table 1 shows the eliminating production distortions (setting $\phi_{ki} = 1$ for all ki) has little impact on the model prediction for the impact of fiscal spending shocks. Appendix B shows that the results are very similar when we simulate a spending shock in France or Italy; the own effects are substantial, but the spillover effects are very small, whether or not there are cross-country production linkages.

Table 2 gives some further insight into the results of Table 1. We show the response of national PPI's and employment to the German government spending shock. Prices in Germany fall sharply, and employment rises. By contrast, prices in other countries rise, and employment falls slightly. The key factor in this response is the income effect on labour supply - German consumers have a strong negative income effect as taxes rise to finance the spending increase. This increases labour supply, driving down the wage and German producer prices, while increasing employment. In the rest of Europe prices increase slightly, and employment falls by a small amount.

5.4 Fiscal Spillovers in the Sticky Wage Model

We have seen that in the baseline model, firstly, fiscal spillovers are small, and secondly, cross-country production networks have almost no relevance for fiscal spillovers. Is this result robust to alternative formulations of the model? We saw also that a key mechanism governing the response to fiscal shocks in the baseline model is the response of labour supply. But evidence from the Eurozone suggests that

²⁰In this case, we rescale the within-country input-output production function for all countries, so that the production function is constant returns to scale in within-country intermediates and labour.

Country	g_Y	$g_{Y,nn}$	g_{Ycpi}	$g_{Ycpi,nn}$	$g_{Y,\phi=1}$
Aut	0.05	0.01	0.07	0.08	0.05
Bel	0.01	0.00	0.03	0.03	0.01
Ger	0.83	0.79	0.88	0.83	0.90
Esp	0.01	0.00	0.01	0.01	0.01
Fin	0.00	-0.01	0.02	0.02	0.00
Fra	0.00	-0.01	0.01	0.02	0.00
Grc	0.01	0.00	0.01	0.01	0.00
Irl	-0.01	-0.03	0.03	0.03	-0.02
Ita	0.01	0.00	0.01	0.01	0.01
Nld	0.01	0.01	0.03	0.03	0.01
Prt	0.01	0.00	0.01	0.01	0.01

Table 1: The table shows the response of real GDP growth to a one-percent of GDP increase in German Government spending. The first and second column shows real GDP measured using the GDP deflator, respectively with and without international production connections. The third and fourth columns show the same responses, but using the CPI deflated measure of real GDP. The final column shows the response of GDP (using the GDP deflator), in the absence of production distortions.

Country	g_{RER}	$g_{RER,nn}$	g_L	$g_{L,nn}$
Aut	0.03	0.02	-0.01	-0.01
Bel	0.04	0.03	-0.01	-0.01
Ger	-0.60	-0.22	0.99	0.90
Esp	0.03	0.02	-0.01	0.00
Fin	0.09	0.07	-0.02	-0.01
Fra	0.04	0.03	-0.01	-0.01
Grc	0.06	0.05	-0.01	-0.01
Irl	0.13	0.12	-0.03	-0.02
Ita	0.02	0.02	0.00	0.00
Nld	0.02	-0.01	0.00	0.00
Prt	-0.01	-0.01	0.00	0.00

Table 2: The table shows the response of the real exchange rate and employment to a one-percent of GDP increase in German Government spending. The first and second column shows the growth in the real exchange rate, respectively with and without international production connections. The third and fourth columns show the same responses for employment growth.

wages are slow to respond to shocks. With sticky wages, labour supply plays no short run role in the response to a fiscal shock. To see how this alternative specification affects the results, we now analyze the fiscal shock in the sticky wage version of the model.

Now, Table 3 illustrates the results of a fiscal shock in Germany, measured in the same manner as before, but assuming nominal wages are sticky, and monetary policy targets the value of nominal consumption in the Eurozone, as described in section 2 above.²¹ The Table shows both the response in the model using the calibrated international production network, as well as the case where there are no international production linkages, recalibrating the model as described in above. We also show the implied results when government spending in Germany falls only on domestic sectors, recalibrating the model in a similar way.

The most striking feature of Table 3 is the very large cross-country spillovers of fiscal policy shocks, while at the same time the substantial fall in the own country multiplier. For a one percent of GDP fiscal expansion in Germany, German GDP grows by 0.53 percent (indicating a multiplier of about 0.5). But the response of all other country's GDP is substantial, particularly relative to the baseline case with flexible wages. The spillover effects range from 0.18 percent in Italy to 0.27 percent in Ireland. By contrast with the flexible price and wage version of the model, these spillovers are in the range estimated by [Beetsma, Giuliadori and Klaassen \(2006\)](#).

The key difference from the baseline case is the predominance of the demand channel of fiscal spending. In the baseline case, the main response to fiscal shocks is supply driven - through movements in labour supply. But with sticky wages, and profit maximizing firms, prices are not directly affected by fiscal shocks - thus the main response is through demand effects.²² In combination with the monetary rule which targets nominal consumption, this implies that a fiscal shock in Germany diffuses throughout the Eurozone, raising output in all countries.

A second result from Table 3 concerns the importance of the cross-country production network. The second column shows the response to the same sized shock, but assuming the absence of international production networks. In this case, the own country multiplier is increased by fifty percent, but the cross

²¹ It is assumed that the rest of the world follows a similar monetary policy, targeting rest of the world consumption, and the exchange rate between the Eurozone and the rest of the world is flexible. Different assumptions about the rest of the world monetary policy make little differences to the results of the Table

²²It is not quite true that prices are constant. Because the model includes the 'rest of the world' (ROW), and assumes that monetary policy in ROW targets nominal consumption, in fact the nominal exchange rate between the Eurozone and the ROW will depreciate. This raises prices of ROW goods in the Eurozone. This adjustment is incorporated in the responses described in Table 3. But in fact these adjustments are very minor.

country demand effects are much less important, and the spillover effects from the German fiscal shock are substantially lower. Hence, the production network plays a key role in the cross country propagation of fiscal shocks in the environment where the demand channel is predominant, i.e. in the case with sticky wages.

But the table also shows that the pattern of government spending is important. Holding the international production network constant, a redirection of government spending towards only domestic sectors raises the domestic multiplier almost as much as does the eliminating of international production networks, and at the same time reduces international spillovers in the same dimension. Interestingly, even though government spending is substantially biased toward domestic goods in the data, the combination of government spending spillovers to foreign goods and an international production network lead to significant spillovers in the case of sticky wages, where demand shocks play a predominant role.

These results are based on a model calibrated to the average WIOT and Socioeconomic Accounts over the 2000-2014 period. But there have been some changes in the distribution of the shares over this time. The overall pattern of international production network connections increased, as shown in Figure 3 above. To explore how these changes affect the quantitative predictions of the model for spillovers, we show the evolution of fiscal responses when the model is calibrated separately for each year in the 2000-2014 sample.

Figure 5 shows the time variation in model predictions for the response to a fiscal spending shock in Germany when the model is calibrated separately year by year to the corresponding WIOD. The domestic spending multiplier and the predicted spillovers vary somewhat over the sample, but for the most part remain at the same orders of magnitude as reported in Table 3. The Figure also reports the implied responses if the cross-country production network was held constant at the 2000 value. The striking feature of this is that it makes very little difference to the quantitative predictions for fiscal spillovers. While the production network is quantitatively very important for the magnitude of spillovers (as shown in Table 3), changes in the network over the sample period make little difference to the quantitative size of spillovers.

Figure 6 presents a similar comparison, but now exploring how changes in the composition of government spending make a difference to the predictions of the model. In this case, we see that the domestic spending multiplier would have remained larger, and the international spillover effects would have been smaller, had the composition of German government spending been frozen at the 2000 value. This is

Country	g_Y	$g_{Y,nn}$	$g_{Y,G}$	$g_{Y,\phi=1}$
Aut	0.20	0.09	0.11	0.23
Bel	0.20	0.09	0.11	0.23
Ger	0.53	0.80	0.74	0.45
Esp	0.19	0.08	0.09	0.22
Fin	0.23	0.13	0.13	0.26
Fra	0.20	0.09	0.09	0.23
Grc	0.21	0.11	0.11	0.24
Irl	0.27	0.17	0.09	0.30
Ita	0.18	0.08	0.10	0.21
Nld	0.17	0.05	0.10	0.20
Prt	0.16	0.06	0.09	0.18

Table 3: The table shows the response of real GDP growth to a one-percent of GDP increase in German Government spending when nominal wages are sticky in all countries. The first and second column shows real GDP measured using the GDP deflator, respectively with and without international production connections. The third column shows the same responses, but assuming that all government spending in Germany is targeted towards domestic sectors only. The final column shows the response in the absence of production distortions.

particularly true for the middle years of the sample. Figure 7 helps to explain this. The home bias in German government spending fell significantly towards the middle of the sample, and then rose again over the end of the sample.

5.5 Comparing the model with empirical estimates

How do the spillovers from the sticky wage model compare with those of empirical estimates described in section above? We focus on estimates from [Beetsma, Giuliadori and Klaassen \(2006\)](#) and [Dabla-Norris, Dallari and Poghosyan \(2017\)](#), who provide empirical measures of spillovers for the countries used in our calibrated model. In each case, the comparison is one where there is a one percent of GDP fiscal shock in the origin country - we use three countries as originators - Germany, France, and Italy. The spillover measures are equivalent to the percent of GDP responses in the neighbouring countries.

Figure 8 shows that for a shock to German government spending both the own multipliers and the spillover multipliers are in the range of empirical estimates, and mostly in between the estimates of [Beetsma, Giuliadori and Klaassen \(2006\)](#) and [Dabla-Norris, Dallari and Poghosyan \(2017\)](#)²³. Some of the spillovers from the model are somewhat larger than estimated from the data (e.g. Spain, France and Finland), while others are smaller (Ireland, Netherlands), but on average the predictions of the model

²³ [Beetsma, Giuliadori and Klaassen \(2006\)](#) do not provide an estimate of the own multiplier for Germany, so we use the multiplier from [Blanchard and Perotti \(2002\)](#), which uses a similar identification approach.

are quite close to those of the empirical estimates for Germany.

Figure 9 shows the same comparison for a French government spending shock of 1 percent of GDP. The two key features of this Figure are a) the spillovers from the data are substantially smaller than those from the German shock (except for Ireland), and b), the model matches very well the size of the spillovers, since the spillovers from France in the model are smaller than those from Germany.

Finally, Figure 10 shows the effect of a model generated shock to Italian government spending and compares this with the empirical estimates. Both the model and the data predict substantially smaller spillover effects than those of Germany or France, but again, the model predictions are very close to those of the data.

6 Government expenditures and input linkages with other sectors

So far we have only looked at aggregate responses to fiscal spending shocks. We now turn to more detailed sectoral responses, both domestic and cross-country. This will clearly depend on a) the sector which receives most of the government spending, and b) the linkages, both domestic and international, between these sectors and other sectors.

In this vein, we first explore which sectors a government spends on most and how these sectors are linked with other sectors in the rest of the economy. Figure 12 shows that the governments in the Eurozone have a similar expenditure pattern. Overall government spending is highly concentrated in a small number of sectors. Three sectors constitute the bulk of government spending: sector 51 “Public administration and defence, compulsory social security”, sector 52 “Education” and sector 53 “Human health and social work activities”. Other sectors which receive significant but smaller shares are: sector 30 “Retail trade, except of motor vehicles and motorcycles”, sector 47 “Scientific research and development” and sector 54 “Other service activities”. These sectors in total account for a very large fraction of total government expenditures.

Figure 13 looks at the correspondence between German government spending in the Eurozone, and the sectoral output responses in each sector, country by country. Although the sectoral spending shares of German government spending outside Germany are very small, there is a close correspondence between spending shares and output responses, as would be expected. In particular, sector 12 (“Manufacture of basic pharmaceutical products and pharmaceutical preparations”) in many Eurozone countries receives

a significant share of government spending from Germany.

6.1 Sectoral linkages of fiscal responses

We now develop an empirical strategy to explore the impacts of German fiscal shocks at the sectoral level, and ask how this depends on the production linkages across sectors. Our approach follows closely the method of [Acemoglu, Akcigit and Kerr \(2015b\)](#).

Following the definitions of [Acemoglu, Akcigit and Kerr \(2015b\)](#), we define two ratios

$$a_{kimj} \equiv \frac{\text{Expenditure}_{mj \rightarrow ki}}{\text{Expenditure}_{ki}} = \alpha_{ki} \omega_{kimj}, \quad \widehat{a_{kimj}} \equiv \frac{\text{Expenditure}_{ki \rightarrow mj}}{\text{Expenditure}_{ki}} = a_{mjk_i} \frac{U_{mj}}{U_{ki}}$$

where a_{kimj} reflects the propagation of shocks from sector mj to sector ki , while $\widehat{a_{kimj}}$ captures the propagation of shocks from sector ki to sector mj . We write the overall *downstream* effect of shock propagation as $\mathbf{D} = (\mathbf{I} - \mathbf{A})^{-1}$, with $\mathbf{A} \equiv (\alpha e'_N) \circ \mathbf{W}$. The overall *upstream* effect of shock propagation as $\widehat{\mathbf{D}} = (\mathbf{I} - \widehat{\mathbf{A}})^{-1}$, with $\widehat{\mathbf{A}} \equiv \mathbf{A}' \circ (e_N U') \circ (U e'_N)$, where U is a column vector for sectoral total expenditures.

We define two measures for downstream and upstream propagation of shocks:

$$\text{Downstream}_{ki,t} = \sum_{m,j} (\text{Input}\%_{ki,mj,t-1} - 1_{mj=ki}) G_{c,mjt}$$

where $\text{Input}\%_{ki,mj,t-1} \equiv d_{kimj,t-1}$, d is an element of matrix \mathbf{D} and $1_{mj=ki}$ is an indicator function which equals one if $mj = ki$.

$$\text{Upstream}_{ki,t} = \sum_{m,j} (\text{Output}\%_{ki,mj,t-1} - 1_{mj=ki}) G_{c,mjt}$$

where $\text{Output}\%_{ki,mj,t-1} \equiv \widehat{d_{kimj,t-1}}$ and \widehat{d} is an element of matrix $\widehat{\mathbf{D}}$.

In order to look at the sectoral responses to fiscal shocks, we first need to identify exogenous fiscal spending. We follow the approach of [Blanchard and Perotti \(2002\)](#) and use quarterly data to estimate government expenditure shocks in Germany. Figure 11 shows the time line of estimated government expenditure shocks.

We then aggregate the shocks to the annual frequency, since the WIOD data is annual. Using

the estimated fiscal shocks, the matrix of government spending weights and upstream and downstream measures country by country from WIOD, we construct a country-sector-time panel of government spending shocks from the source countries and take use of the following specification,

$$\Delta Y_{ki,t} = \beta_0 G_{c,ki,t} + \beta_1 \text{Upstream}_{ki,t} + \beta_2 \text{Downstream}_{ki,t} + \gamma Z_t + u_{ki,t} \quad (26)$$

where $\Delta Y_{ki,t}$ can be sectoral real value added and $G_{c,ki,t}$ is the constructed spending shock to country k , sector i , at time t from the center country c . The $G_{c,ki,t}$ variable is constructed from the final public absorptions from the WIOD, which gives the share of spending of c country aggregate government spending that is spent on sector i , country k good in time t . Call this share $g_{c,ki,t}$. Then

$$G_{c,ki,t} = g_{c,ki,t-1} G_{c,t}$$

where $G_{c,t}$ is the aggregate spending shock in the centre country. β_0 is a measure of the spillover multiplier at the sectoral level. β_1 captures the upstream effects from all other sectors (and countries), while β_2 captures the downstream effects. Z_t includes controls and country, sector and time fixed effects.

Note that we only have demand side shocks in our model. In the closed economy with Cobb-Douglas specifications, we show in section 3.1 that product prices don't respond to government expenditure shocks. The downstream effect will disappear $\beta_2 = 0$ since the government expenditure doesn't change product price and input costs for the downstream sector but changes demand for intermediate inputs in that sector. Consequently, government expenditure shocks work upstream only, captured by β_1 . Following the closed economy model, we still take use of $\text{Downstream}_{ki,t}$ and $\text{Upstream}_{ki,t}$ in a multi-country environment. When labour is immobile across borders, government expenditures are able to change the labor costs $w_{k,t}$, which in turn change country-sectoral product prices and terms of trade across countries. Therefore, government expenditures have both upstream and downstream effects.

We do the following two exercises. First, we construct variables on both sides of specification (26) based on the WIOD directly. We use labour income distribution across country sectors, the input-output table, sectoral expenses on intermediate inputs, and government expenditure distribution in year $t - 1$ to construct upstream and downstream measures on the RHS of specification (26). The LHS of (26) uses sectoral real value-added from the WIOD. Second, we construct the model counterparts of specification

(26). Given the calibration and government expenditure shocks, we solve the model and compute the change of value-added for each country sector, $\Delta Y_{ki,t}$ and the RHS variables of specification (26).

Table 4: Production network and government expenditures

Variable	Data		Flexible Wages		Sticky Wages	
	(1)	(2)	(3)	(4)	(5)	(6)
Gov-shock	0.201*** (3.962)	0.211** (2.183)	0.409*** (8.767)	1.259*** (19.680)	0.240*** (7.053)	1.256*** (19.668)
Upstream	0.691*** (3.663)	0.608* (1.653)	0.094*** (5.951)	0.460** (2.246)	0.066*** (4.474)	0.381* (1.869)
Downstream	-2.294*** (-3.196)	-4.085** (-2.228)	0.304*** (11.164)	-0.743 (-1.406)	-0.532*** (-19.445)	-0.684 (-1.304)
Constant	0.111*** (13.192)	0.048 (1.345)	0.000 (0.791)	-0.008 (-0.811)	-0.006*** (-19.255)	-0.001 (-0.113)
Country FE	YES		YES		YES	
Year FE	YES	YES	YES	YES	YES	YES
Sector FE	YES	YES	YES	YES	YES	YES
Adjusted R-squared	0.531	0.456	0.219	0.582	0.588	0.546
Observations	6,402	591	6,404	583	6,403	583

Notes: t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The odd column (1), (3) and (5) are for the whole sample of Euro-11 (the rest of world is excluded from the regression analysis). The even column (2), (4) and (6) are for Germany only. The first two columns report regression results based on the data, the following two columns (3) and (4) are for the model with flexible wages and the last two columns are for the model with sticky wages. Gov-shock is for government expenditure shocks and FE denotes the fixed effect. Both the model and data are censored at the top and bottom 1% of total observations for sectoral real value-added. All of regressions are weighted by sectoral sales.

Table 4 reports three sets of regression results for specification (26). The results in columns labeled 'Data' are obtain directly from the WIOD, while results from 'Flexible Wages' and 'Sticky Wages' are obtained from the corresponding model simulations. Results from both the model and data show that German government expenditure shocks can directly boost not only domestic sectoral value-added but also foreign sectoral value-added on average. The effect of government expenditure on domestic economy is stronger than its spillover to other countries. The production network plays an important role in affecting the direction of shock propagation. The results state that government expenditure shocks work upstream positively but work downstream negatively. Higher government expenditure increases demand for goods produced by sector ki , for instance. Firms in sector ki expand their output by hiring a higher amount of intermediate inputs. Consequently, gross output and value-added rise in the upstream sectors along the production network. On the other hand, higher demand for goods in sector ki may drive up the equilibrium good price P_{ki} and labour cost w_k . The rise of input costs faced by the downstream

sectors would lead to lower employment and production, and consequently lower value-added in the downstream sectors.

7 Conclusion

This paper analyzes the impact of fiscal shocks on domestic and foreign productions in a model with production network. Using a simple theoretical setting, we show that, in a closed economy, neither the production network structure nor the sectoral composition of fiscal spending are relevant for the aggregate government spending multiplier. By contrast, in a multi-country model, the production network is always relevant for the response to fiscal spending. In the flexible price and wage version of the model, the response to fiscal shocks depends crucially on the movements in the terms of trade across countries. With sticky wages, the production network itself matters directly for the way in which shocks are channeled across sectors and countries.

We calibrate the model using the World Input Output Database (WIOD) for the Eurozone countries over the period of 2000-2014 and consider German government expenditure shocks. In the model with fully flexible wages and prices, supply and demand channels in response to fiscal shocks tend to work against one another making aggregate spillovers very small. By contrast, when nominal wages are sticky, spillovers working solely through the demand channel can be large, and certainly within the range of empirical estimates of fiscal spillovers in the Eurozone. Moreover, in this case the network linkages among countries have first order effects on the response to fiscal shocks. When we solve a recalibrated version of the model shutting down the international production network, the implied fiscal spillovers are reduced by more than two-thirds.

We then explore the diffusion of fiscal shocks at the sectoral level, both within and across countries. We measure German fiscal policy shocks using the SVAR identification procedure of [Blanchard and Perotti \(2002\)](#), and sectoral demand shocks using the approach of [Acemoglu, Akcigit and Kerr \(2015b\)](#). We find that government expenditures have both significant upstream and downstream effects when these links are measured by the direction of sectoral production linkages.

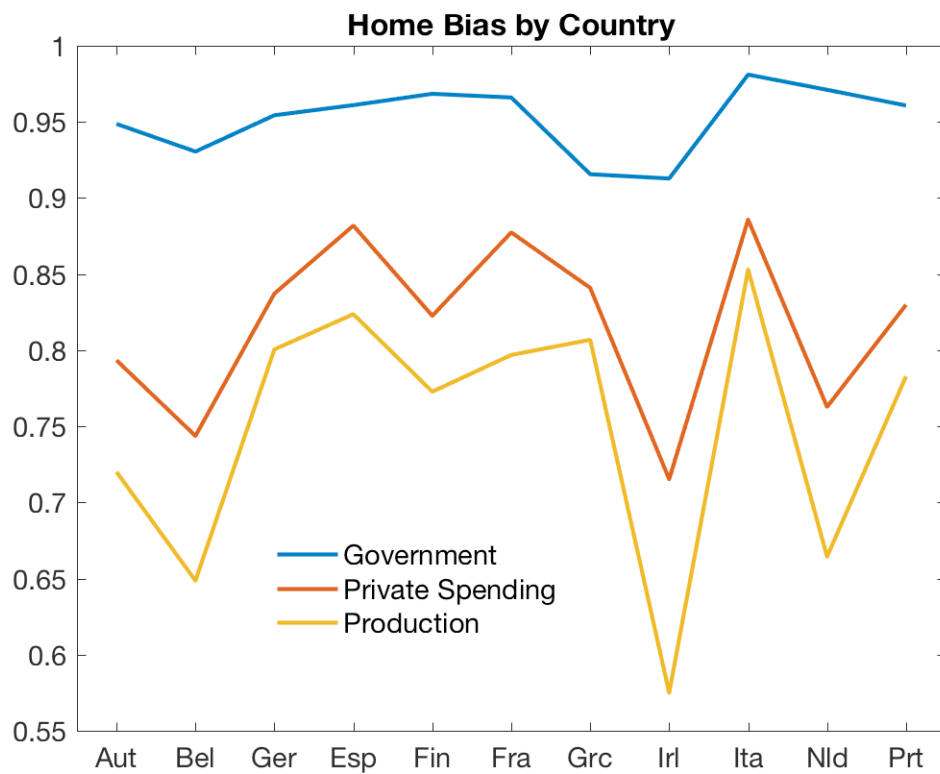


Figure 1: Shares of government spending, private sector spending, intermediate input spending falling on domestic sectors, listed by country.

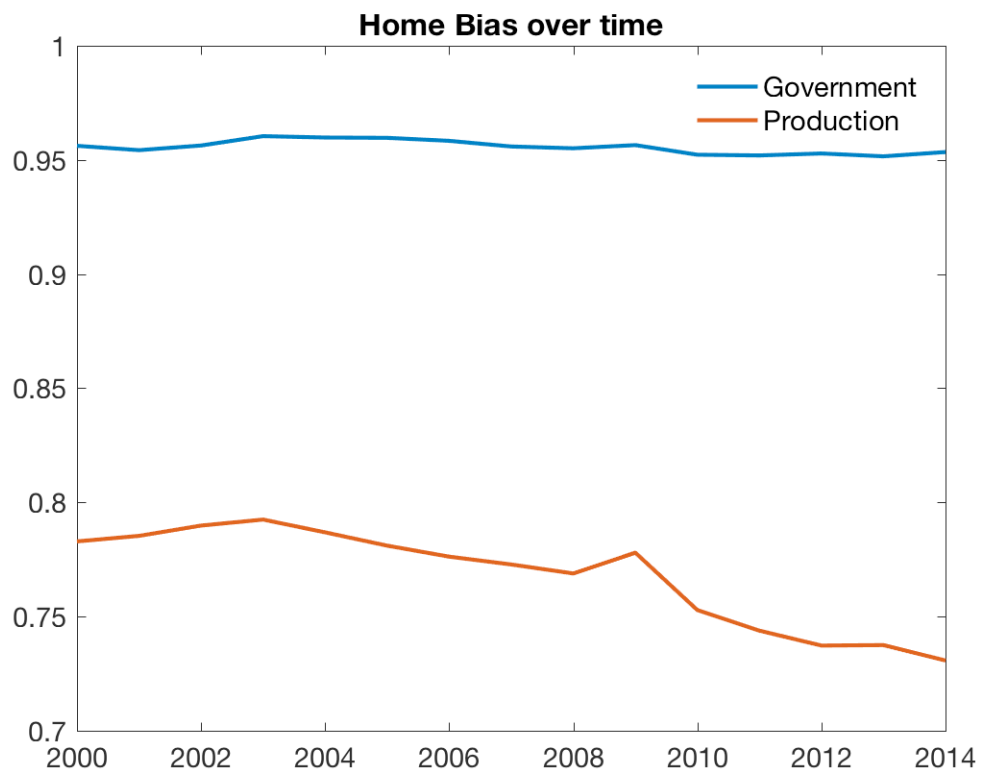


Figure 2: Shares of government spending and intermediate input spending falling on domestic sectors, by year, averaged over countries.

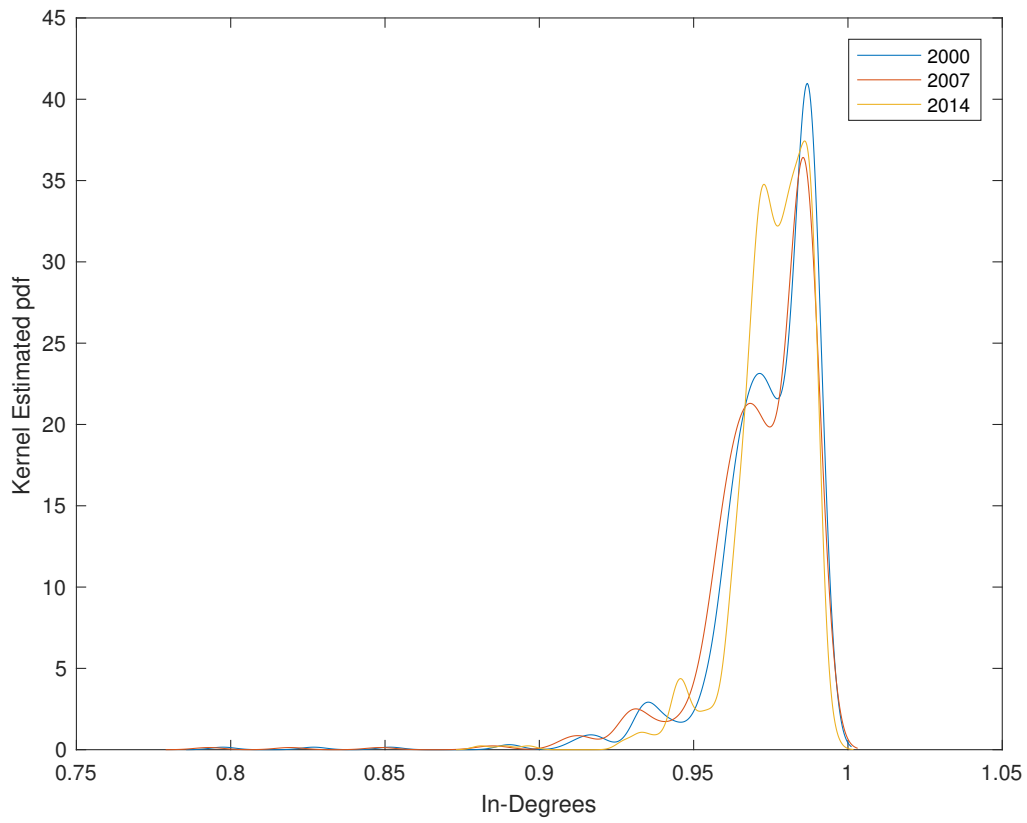


Figure 3: A centrality measure of production network for the Euro zone over time.

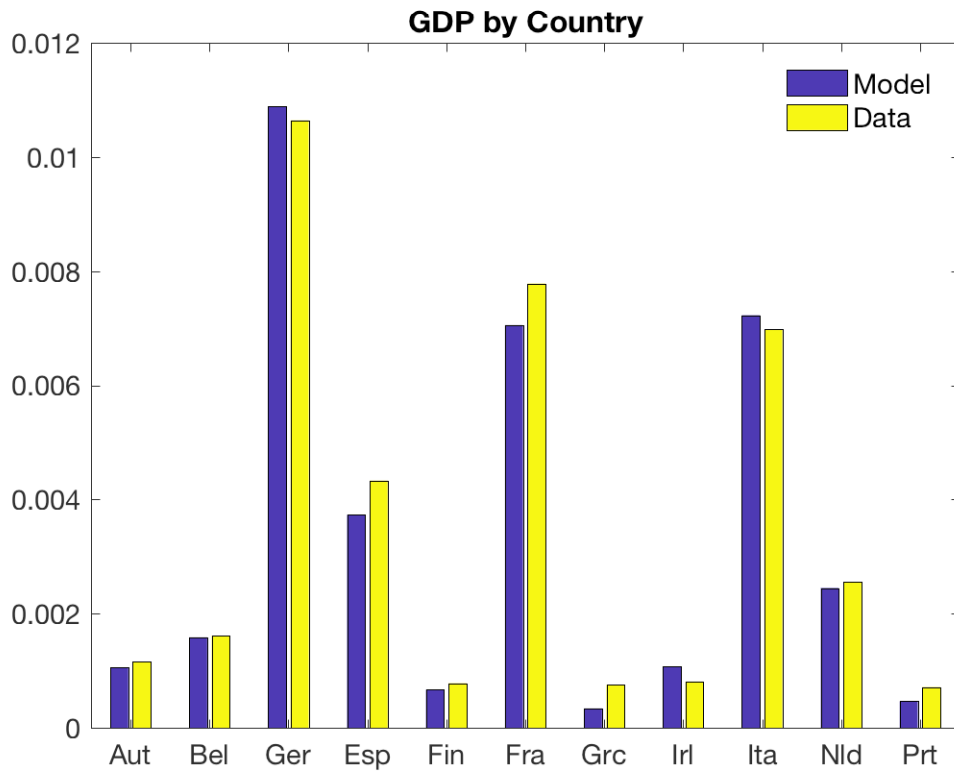


Figure 4: Model predictions for GDP levels, compared with measured relative GDP.

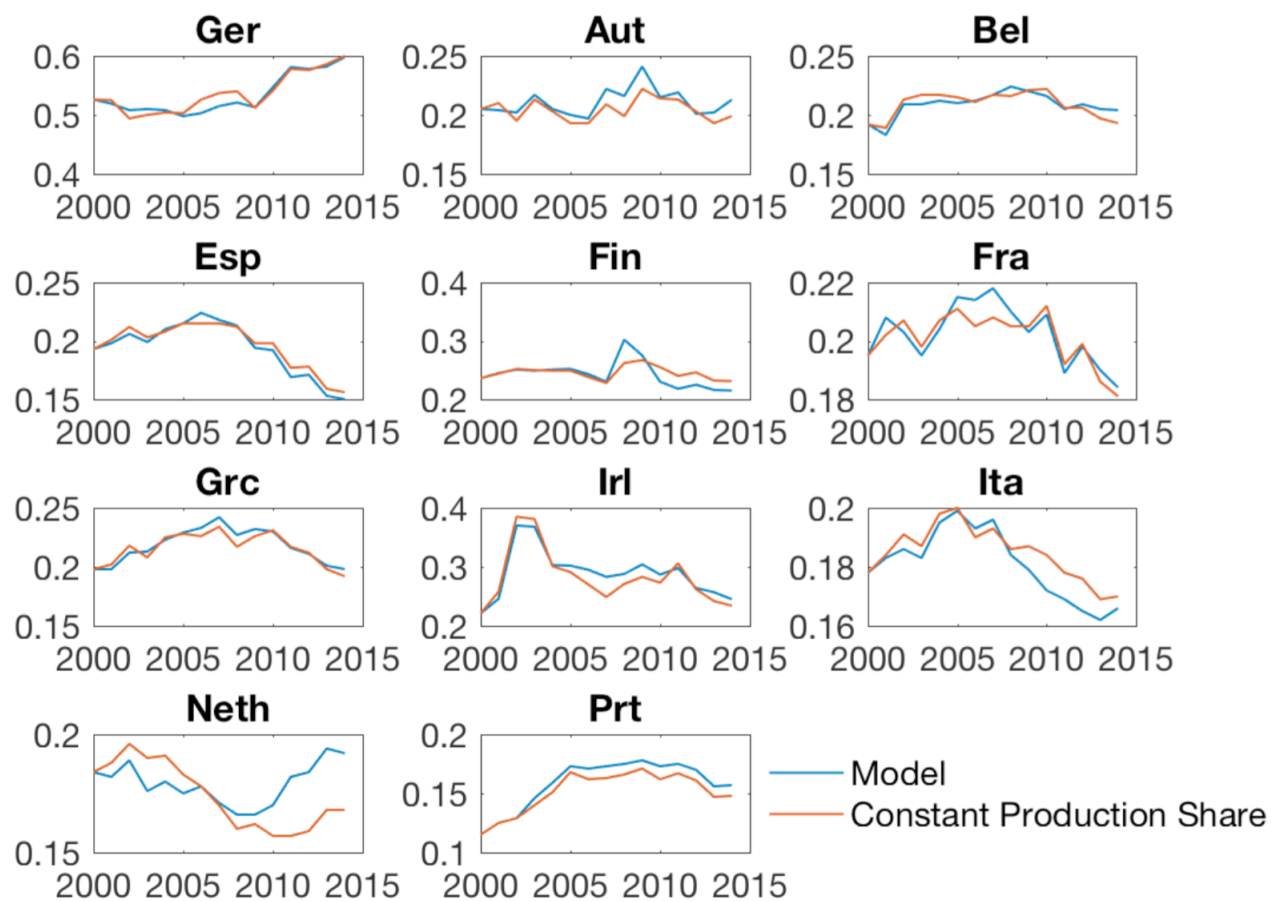


Figure 5: Model predictions for GDP responses to a German government spending shock equal to 1 percent of GDP, evaluated year by year based on WIOD data year calibrations. The blue line uses the WIOD directly. The Red line shows the predictions when the international production network is held constant at 2000 values.

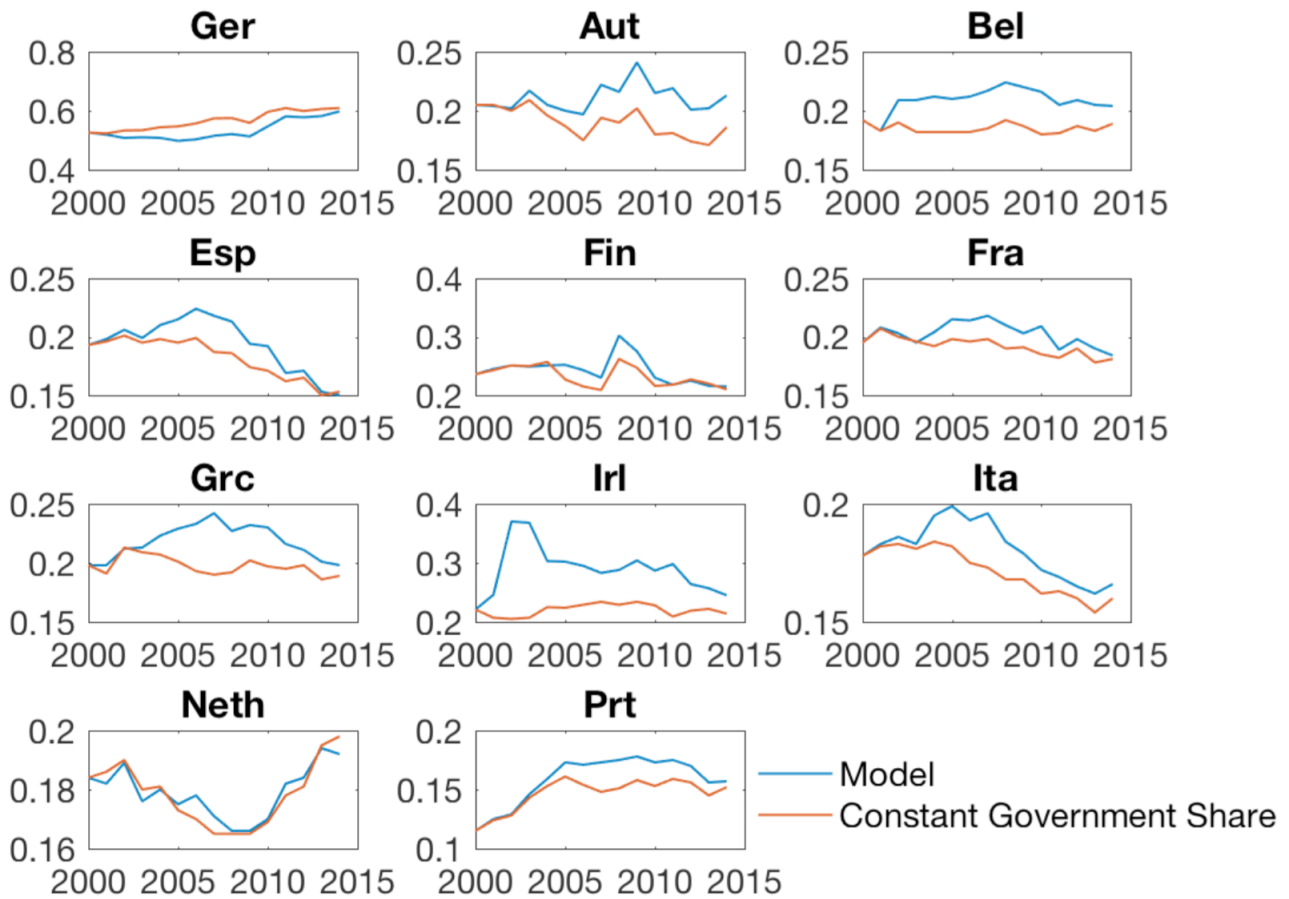


Figure 6: Model predictions for GDP responses to a German government spending shock equal to 1 percent of GDP, evaluated year by year based on WIOD data year calibrations. The blue line uses the WIOD directly. The Red line shows the predictions when the composition of German government spending is held constant at 2000 values.



Figure 7: The evolution of home bias in German government spending over the 2000-2014 period.

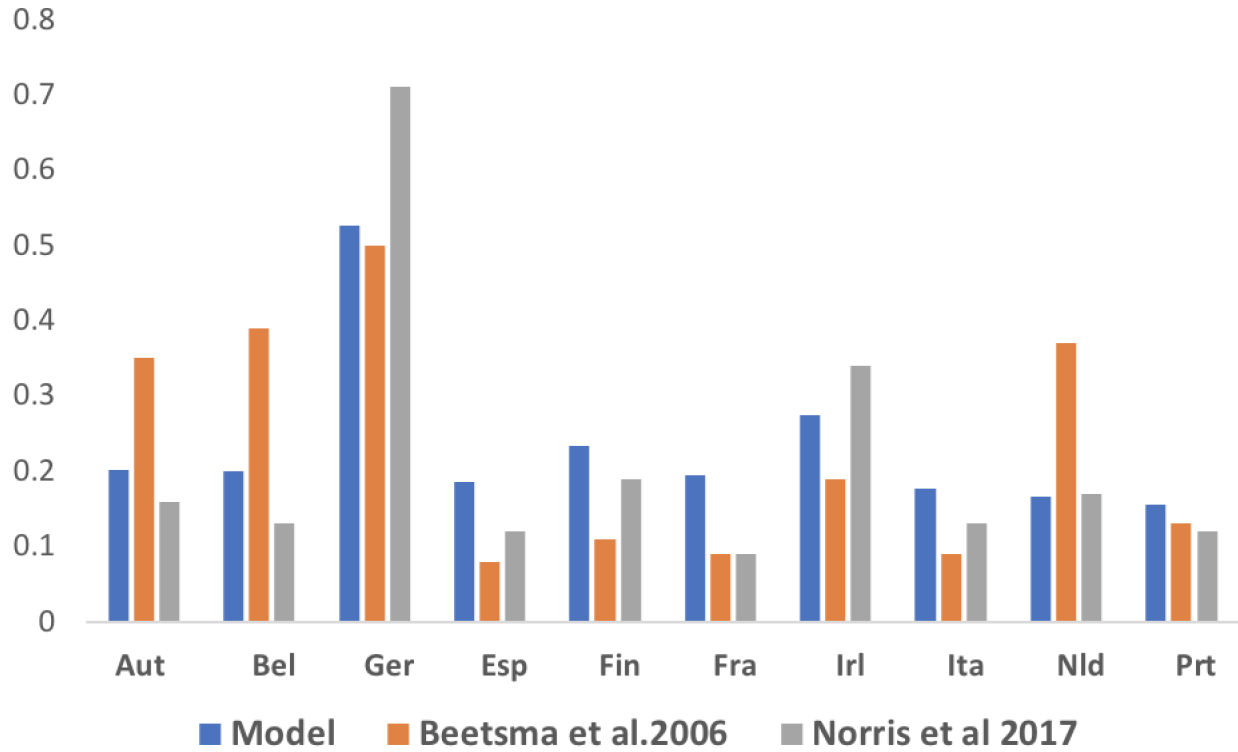


Figure 8: Comparing spillovers in the model versus empirical estimates for Germany.

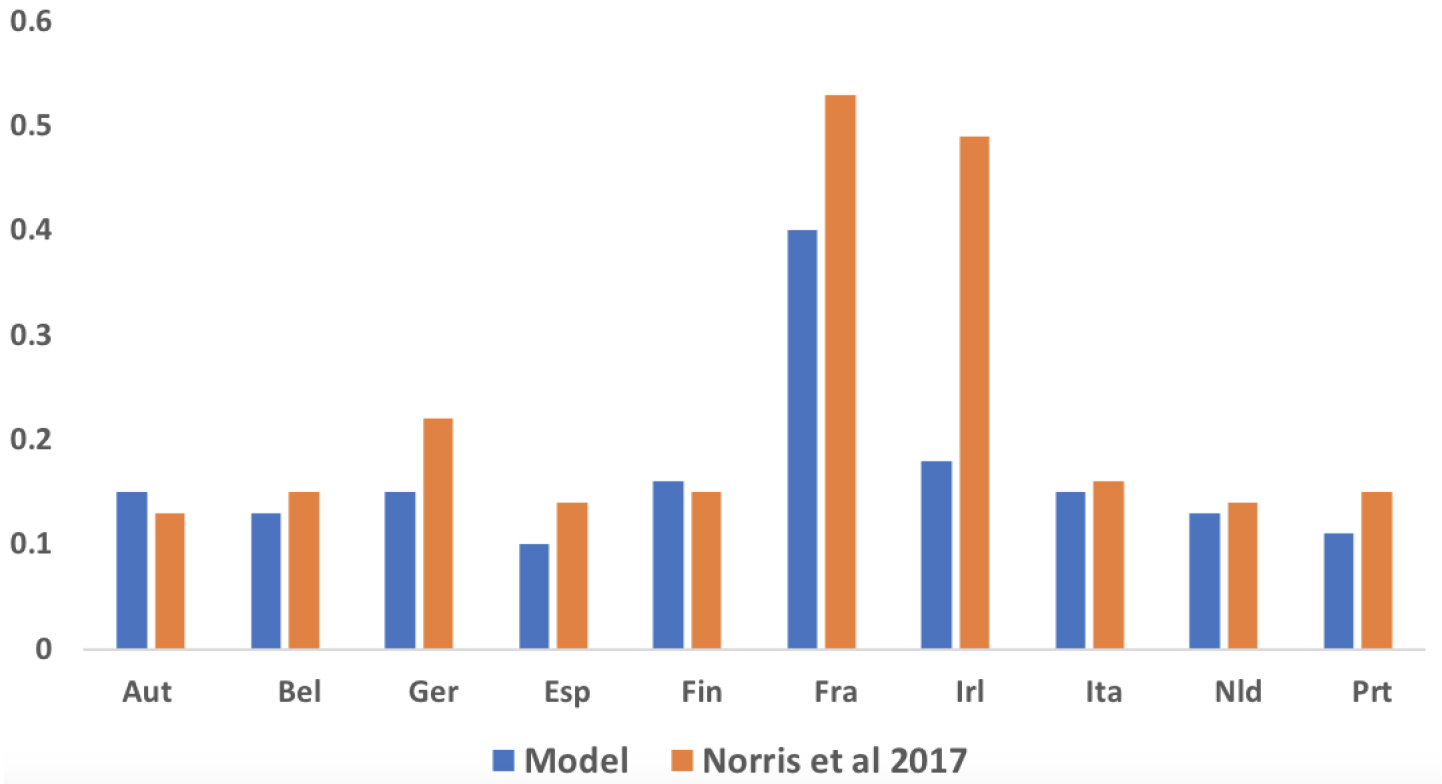


Figure 9: Comparing spillovers in the model versus empirical estimates for France.

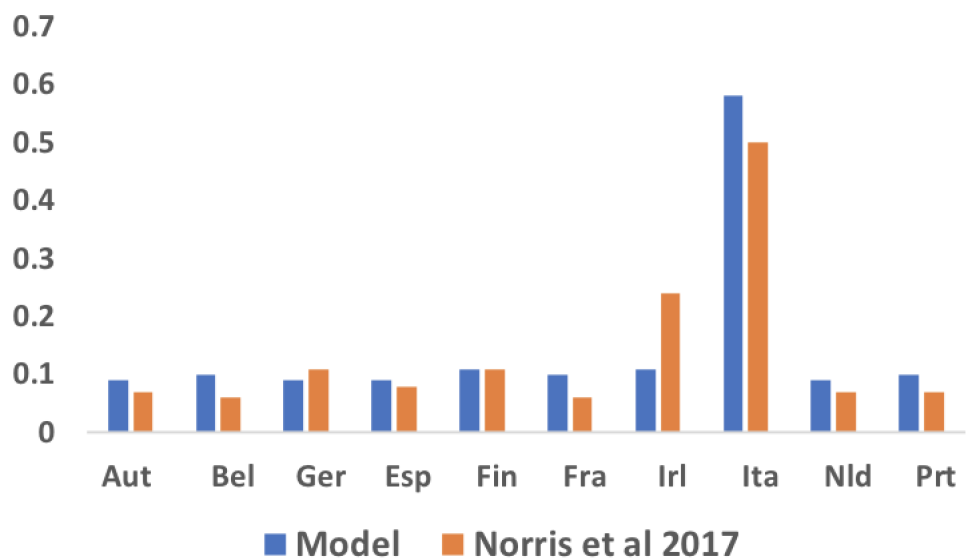


Figure 10: Comparing Spillovers in the model versus empirical estimates for Italy.

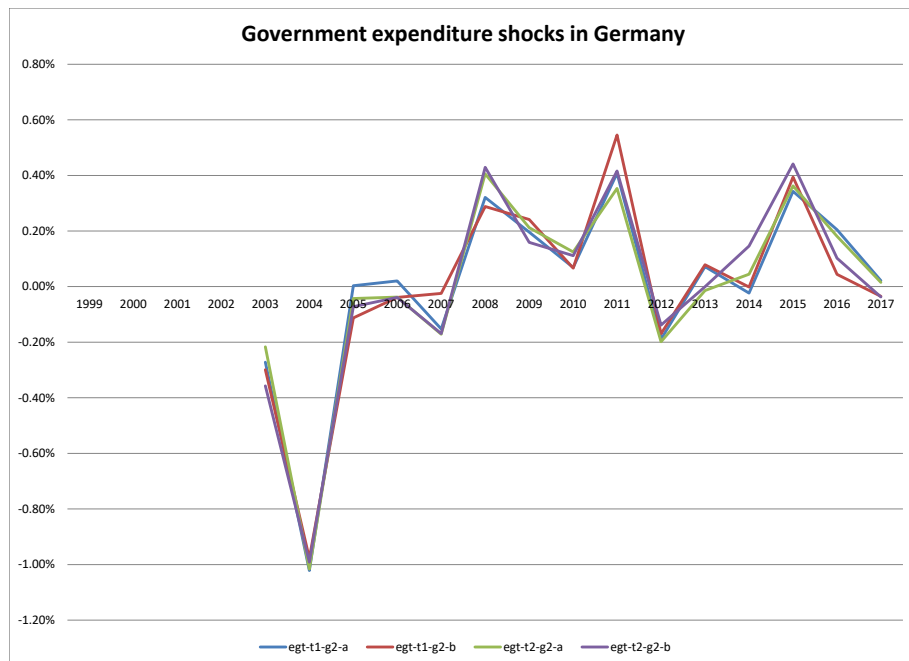


Figure 11: The estimated German fiscal expenditure shocks.

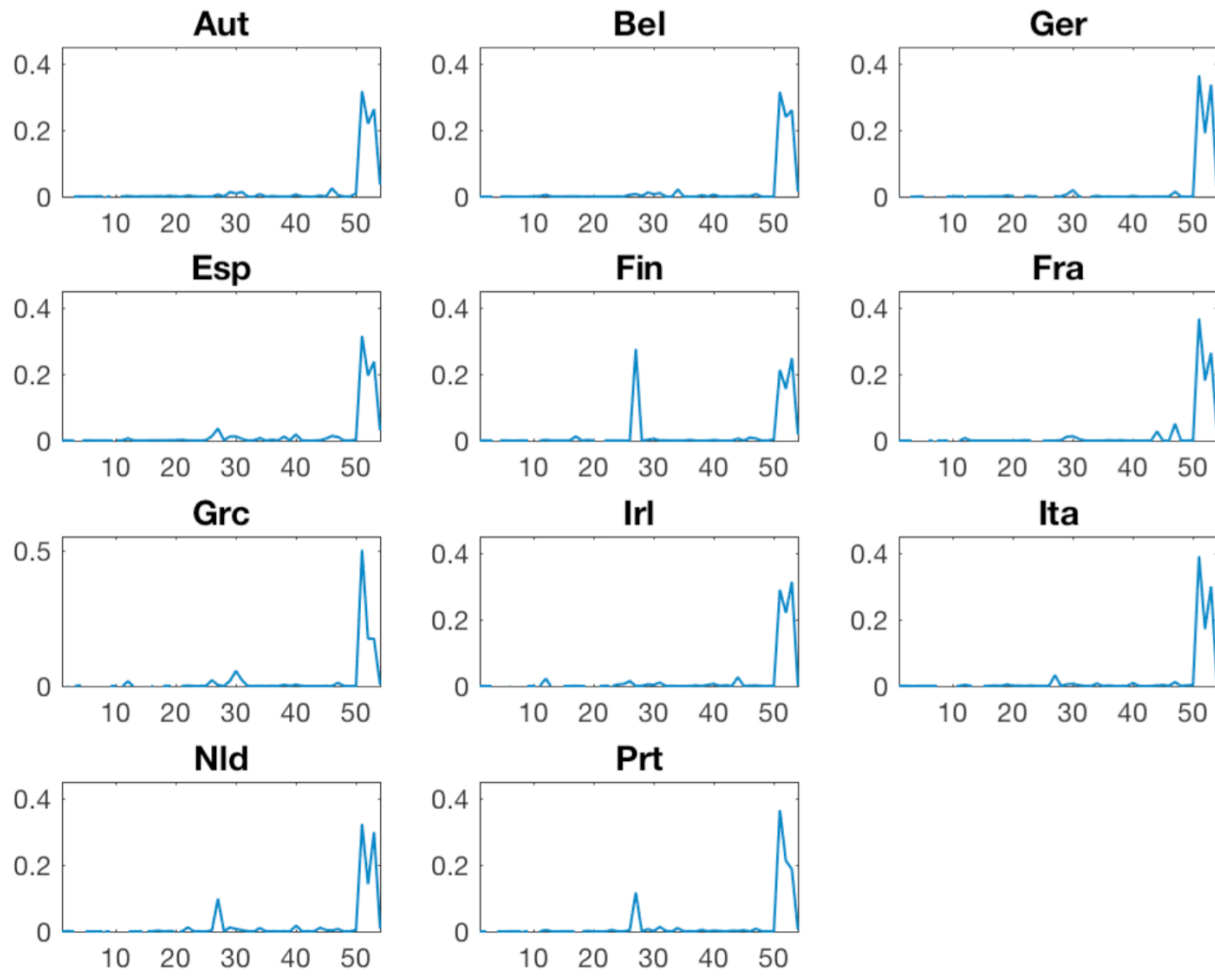


Figure 12: Government expenditure shares in domestic sectors in the Euro 11 countries. Sector code can be found in the WIOD.

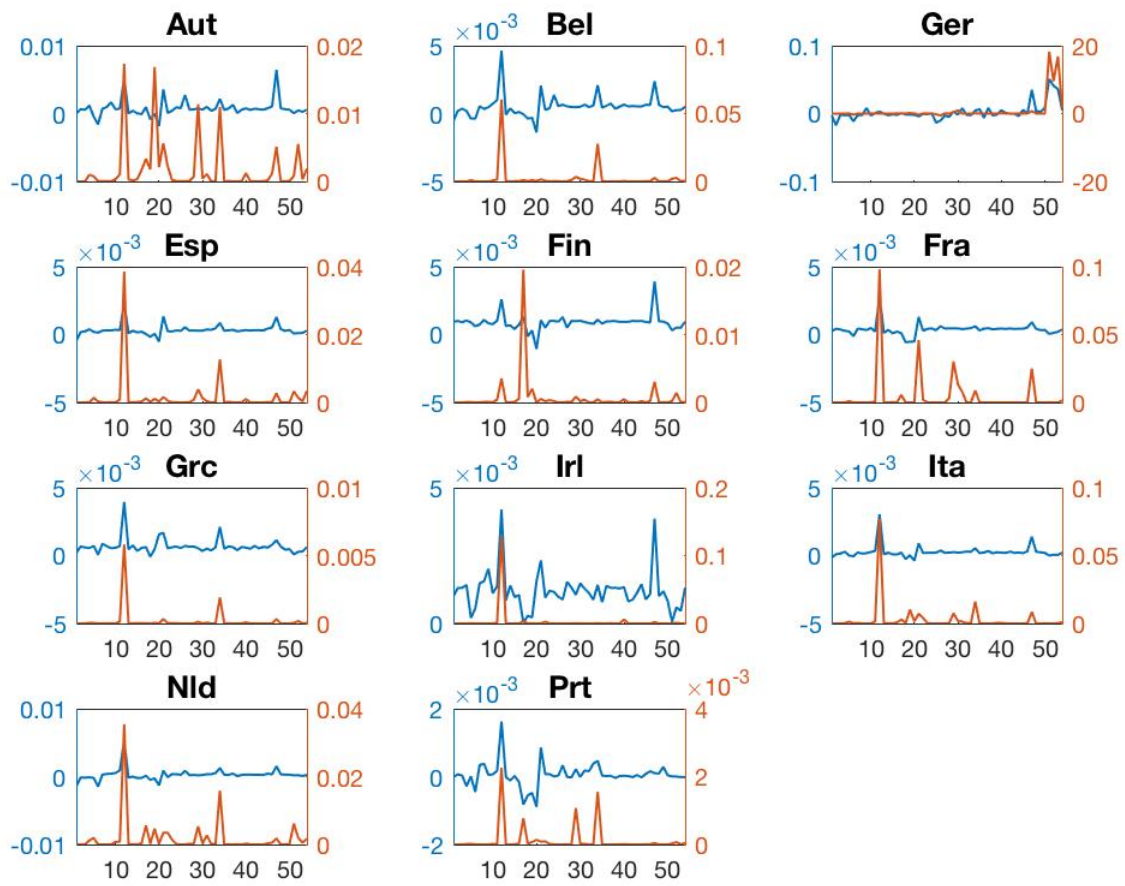


Figure 13: German government expenditure shares by sector in each country (right axis %) and sectoral growth rate in response to a German government spending shock (left axis %)

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Technical Appendix

A Measures of production network

The WIOD network is a weighted directed network. To determine whether we really need to conduct an analysis of the directed network we follow FRS(2007) testing for symmetry measuring the number of bilateral links in the network, and comparing the adjacency matrix and its transpose. The degree of symmetry of the matrix is the percentage of reciprocal links we have. We observe that 95.05% of the links are reciprocal and that the degree of symmetry of the matrix is 48% (comparing each element of the adjacency matrix with its transpose, this number would be 100% in the case of a symmetric matrix)). This degree of symmetry indicates that the direction of the network is quite important.

We can use different measures of centrality: pagerank, in-degree or out-degree (number of outgoing edges to each node). Since we deal with government shocks, we consider in-degree rather than out-degree. Figure A-1 shows that the evolution of the network across time through the kernel density of the pagerank centrality. Similar to Figure 3 in the text, the 2014 yellow curve depicts a double peak compared to 2000 and 2007, suggesting that the number of sectors with a high pagerank centrality has largely increased between 2007 and 2014. However, this large number of very "central" sectors hides low weights of input-output matrix coefficients. These low coefficients are depicted in Figure A-2, a heatmap of the input-output matrix for 11 countries of the Eurozone in 2014, with 54 sectors in each country.

The colors indicate the logarithms of the input-output linkage weights. Lighter colors reflect more intensive input usage while darker colors reflect less intensive input usage. We observe light colors on the diagonal which means that many intermediate trade take place within the intermediate sector itself. We find as well a high concentration of light colors on the last row, which represents the Rest of the World.

Another measure of centrality that can be considered is the "country centrality", given by figure A-3 below. This depicts the average "country centrality" based on the average of in-degree centrality over the 54 sectors for each country and across time. Alternatively, figures A-4 and A-5 depict the "country centrality" based respectively on pagerank and betweenness to indicate how important the "average" sector by country is in terms of connecting other sectors. For example, France is the more central country in terms of betweenness which means that French sectors are more often than other countries on the path relating one foreign sector to another foreign sector.

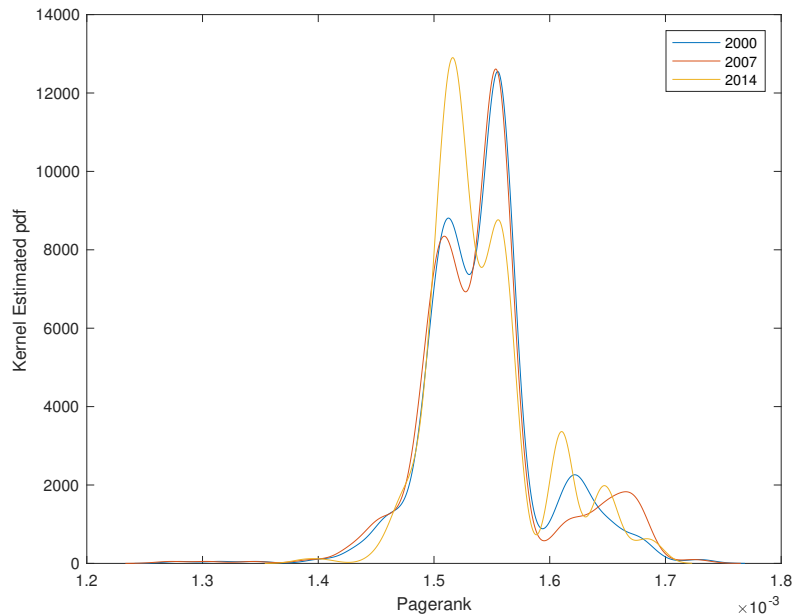


Figure A-1: Kernel density of pagerank centrality across time.

B Shocks in other Eurozone Countries

Here we show that the results of Table 3 for German government spending shocks are quite robust to different assumptions about the source of the spending shock. Table A-1 shows that spillovers are significant for French government spending shocks in the sticky wage model (although not as large as spillovers from German shocks shown in Table 3.). Spillovers from Italian government spending shocks in Table A-2 are smaller. In both cases, spillovers are much smaller in the absence of cross-country production networks, or when government spending is fully concentrated in domestic sectors.

Figure A-6 represents aggregate spillovers divided by country size following a symmetric shock in country i (all sectors of country i) against average weighted pagerank of country i . This positive relationship shows that spillovers divided by country size are lower when the country originating the shock has a lower weighted pagerank. The weighted pagerank measure of centrality does not only consider the average position of the country's different nodes in the network but also the weight on these nodes. A-24. Notice that there is no clear relationship between average pagerank and aggregate spillovers which suggests that both the direction of the network as well as the intensity of the relationships matter.

^{A-24}The Weighted PageRank algorithm (WPR) takes into account the importance of both the inlinks and the outlinks of the nodes and distributes rank scores based on the utilization of the nodes (See Xing and Ghorbani (2004)). It consists in multiplying each edge's probability by its weight vector.)

Country	g_Y	$g_{Y,nn}$	$g_{Y,G}$	$g_{Y,\phi=1}$
Aut	0.15	0.07	0.07	0.17
Bel	0.13	0.05	0.08	0.15
Ger	0.14	0.06	0.07	0.16
Esp	0.10	0.03	0.06	0.12
Fin	0.16	0.08	0.08	0.18
Fra	0.39	0.79	0.66	0.29
Grc	0.16	0.07	0.08	0.18
Irl	0.18	0.11	0.06	0.19
Ita	0.14	0.06	0.08	0.16
Nld	0.13	0.05	0.07	0.14
Prt	0.11	0.03	0.07	0.13

Table A-1: The table shows the response of real GDP growth to a one-percent of GDP increase in French Government spending when nominal wages are sticky in all countries. The first and second column shows real GDP measured using the GDP deflator, respectively with and without international production connections. The third column shows the same responses, but assuming that all government spending in France is targeted towards domestic sectors only. The final column shows the response in the absence of production distortions.

Country	g_Y	$g_{Y,nn}$	$g_{Y,G}$	$g_{Y,\phi=1}$
Aut	0.09	0.02	0.06	0.11
Bel	0.10	0.03	0.05	0.12
Ger	0.09	0.03	0.05	0.11
Esp	0.09	0.03	0.05	0.12
Fin	0.11	0.05	0.05	0.13
Fra	0.10	0.03	0.05	0.12
Grc	0.10	0.03	0.06	0.13
Irl	0.11	0.05	0.05	0.14
Ita	0.58	0.86	0.76	0.49
Nld	0.09	0.03	0.05	0.11
Prt	0.10	0.02	0.06	0.12

Table A-2: The table shows the response of real GDP growth to a one-percent of GDP increase in Italian Government spending when nominal wages are sticky in all countries. The first and second column shows real GDP measured using the GDP deflator, respectively with and without international production connections. The third column shows the same responses, but assuming that all government spending in Italy is targeted towards domestic sectors only. The final column shows the response in the absence of production distortions.

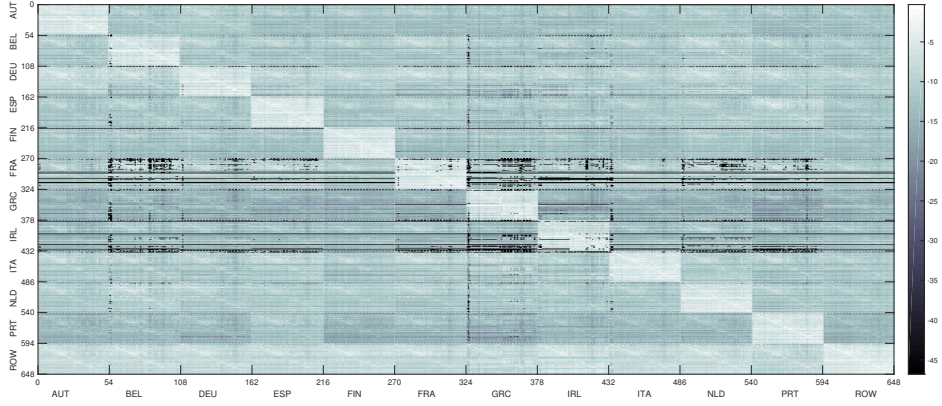


Figure A-2: Production network (logarithm) in the Euro zone in 2014.

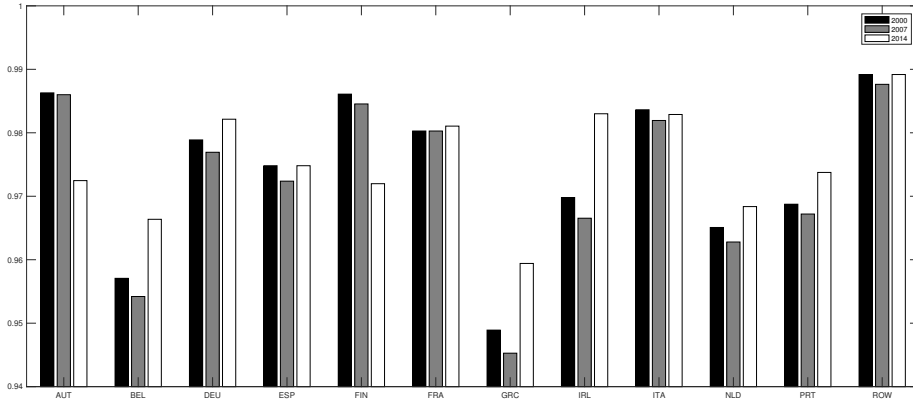


Figure A-3: Average country centrality based on indegree.

C General CES specifications

This appendix extends the analysis in the main text to a general specification for production and consumption. We assume that the composite of intermediate inputs M_{ki} has a following CES form,

$$M_{ki} = \left[\sum_{m=1}^K \sum_{j=1}^{N_m} \omega_{kimj}^{\frac{1}{\theta}} X_{kimj}^{1-\frac{1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution among intermediate inputs.

The optimality conditions for production read

$$\tilde{w}_k \ell_{ki} = (1 - \alpha_{ki}) U_{ki} \tag{C.1}$$

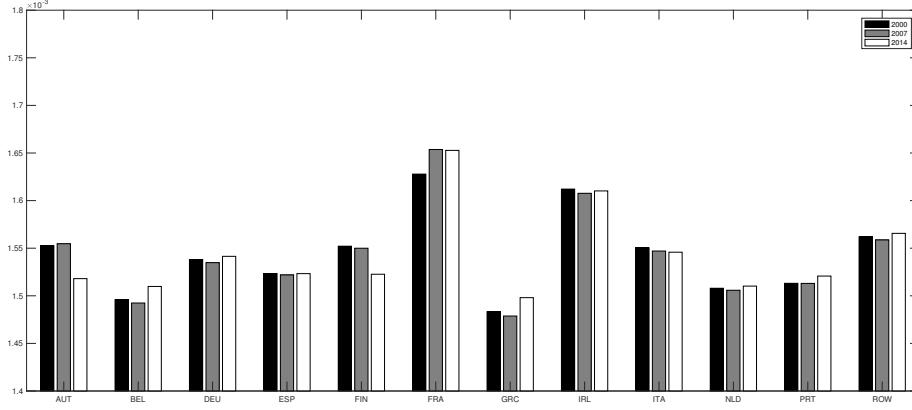


Figure A-4: Average country centrality based on pagerank.

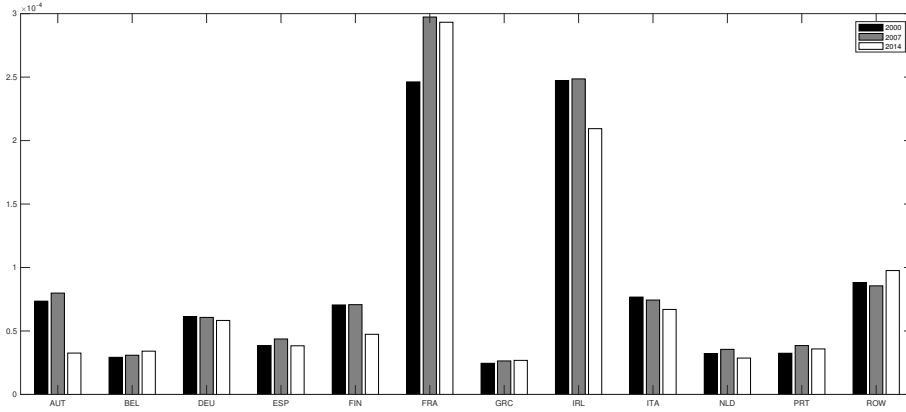


Figure A-5: Average country centrality based on betweenness.

$$P_{ki}^M M_{ki} = \alpha_{ki} U_{ki} \quad (\text{C.2})$$

$$P_{mj} X_{kimj} = \left(\frac{P_{mj}}{P_{ki}^M} \right)^{1-\theta} \alpha_{ki} \omega_{kimj} U_{ki} \quad (\text{C.3})$$

$$U_{ki} \equiv \widetilde{w}_k \ell_{ki} + \sum_{m=1}^K \sum_{j=1}^{N_k} P_{mj} X_{kimj} = \phi_{ki} S_{ki} \quad (\text{C.4})$$

The price for intermediate input composite M_{ki} is linked to prices of intermediate inputs as

$$P_{ki}^M = \left[\sum_{m=1}^K \sum_{j=1}^{N_m} \omega_{kimj} (P_{mj})^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

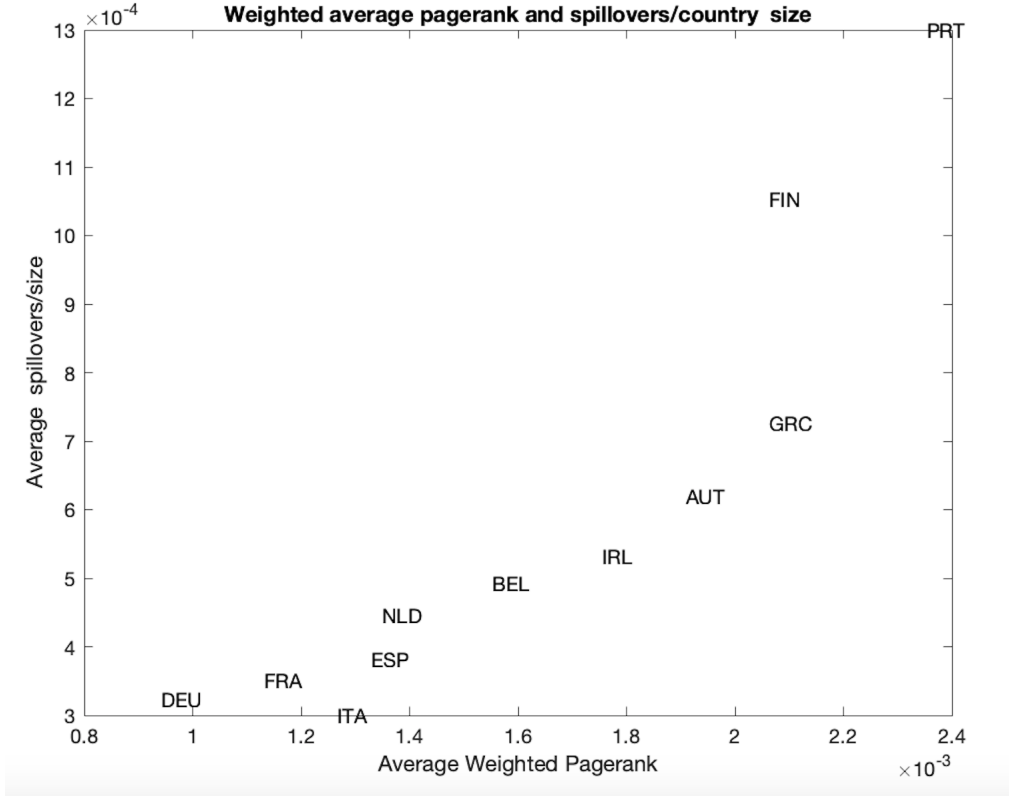


Figure A-6: Weighted pagerank against average spillovers by country

Similarly, the consumption basket for a representative household has the following CES aggregation,

$$c_k = \left[\sum_{m=1}^K \sum_{j=1}^{N_m} \beta_{kmj}^{\frac{1}{\gamma}} c_{kmj}^{1-\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

γ is the elasticity of substitution among varieties in the consumption basket.

The optimality conditions for households in country k become

$$P_{mj} c_{kmj} = \left(\frac{P_{mj}}{P_k} \right)^{1-\gamma} \beta_{kmj} c_k P_k \quad (\text{C.5})$$

with consumer price index P_k

$$P_k = \left[\sum_{m=1}^K \sum_{j=1}^{N_m} \beta_{kmj} (P_{mj})^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

The market clearing condition for goods produced by sector ki now becomes,

$$S_{ki} = \sum_{m=1}^K \left(\frac{P_{ki}}{P_m} \right)^{1-\gamma} \frac{\beta_{mki}}{\lambda_m} \left[w_m - \sum_{j=1}^{N_m} \phi_{mj} (1 - \alpha_{mj}) S_{mj} \right] + \sum_{m=1}^K \sum_{j=1}^{N_m} \left(\frac{P_{ki}}{P_{mj}^M} \right)^{1-\theta} \phi_{mj} \omega_{mjki} \alpha_{mj} S_{mj} + \sum_{m=1}^K G_{mki} \quad (\text{C.6})$$

C.1 Estimation of elasticities

We use price information, input output table and private consumption expenditure distribution from WIOD to estimate elasticity of substitution among intermediate inputs and elasticity of substitution in a consumption basket. Data construction is delegated to the appendix.

The WIOD provides useful information to recover the elasticity parameters θ and γ . First we define the intermediate input share of sector mj in sector ki in year t

$$IIshare_{ki,mj,t} \equiv \frac{P_{mj,t} X_{ki,mj,t}}{\sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj,t} X_{ki,mj,t}} = \omega_{ki,mj} \left(\frac{P_{mj,t}}{P_{ki,t}^M} \right)^{1-\theta}$$

which can be calculated directly from the WIOT and the second equation comes from (C.3).

We define the relative intermediate input share across periods as

$$RII_{ki,mj,ts} \equiv \frac{IIshare_{ki,mj,t}}{IIshare_{ki,mj,s}} = \left(\frac{P_{mj,t}/P_{mj,s}}{P_{ki,t}^M/P_{ki,s}^M} \right)^{1-\theta}$$

Taking logs on both sides, yields

$$\ln RII_{ki,mj,ts} = (1 - \theta) \left[\ln \left(\frac{P_{mj,t}}{P_{mj,s}} \right) - \ln \left(\frac{P_{ki,t}^M}{P_{ki,s}^M} \right) \right] \quad (\text{C.7})$$

We take a similar approach to pin down the elasticity in consumption γ . First, we define the share of consumption expenditure on each variety, which can be obtained from WIOT

$$CShare_{k,mj,t} \equiv \frac{P_{mj,t} C_{k,mj,t}}{\sum_{m=1}^K \sum_{j=1}^{N_m} P_{mj,t} C_{k,mj,t}} = \left(\frac{P_{mj,t}}{P_{k,t}} \right)^{1-\gamma} \beta_{k,mj}$$

The relative consumption expenditure share

$$RCE_{ki,mj,ts} \equiv \frac{CEshare_{ki,mj,t}}{CEshare_{ki,mj,s}} = \left(\frac{P_{mj,t}/P_{mj,s}}{P_{k,t}/P_{k,s}} \right)^{1-\gamma}$$

Taking logs on both sides, yields

$$\ln RCE_{ki,mj,ts} = (1 - \gamma) \left[\ln \left(\frac{P_{mj,t}}{P_{mj,s}} \right) - \ln \left(\frac{P_{k,t}}{P_{k,s}} \right) \right] \quad (\text{C.8})$$

The socio-economic database of WIOD provides constant sectoral prices for intermediate inputs and gross outputs. We use “price levels gross output (2010=100)”, GO-PI, to measure $P_{mj,t}/P_{mj,s}$, and “price levels of intermediate inputs (2010=100)”, II-PI, to measure $P_{ki,t}^M/P_{ki,s}^M$, with $m, k = 1, \dots, K$, $i, j = 1, \dots, N_k$, $t = 2, \dots, T$. Regressing the LHS on the RHS of the above equation (C.7) yields the estimate $\hat{\theta}$. Taking the country level CPI data to measure $P_{k,t}/P_{k,s}$ (2010=100) from eurostat or OECD database, regressing the LHS on the RHS of the above equation (C.8) yields the estimate $\hat{\gamma}$. Figure A-7 shows the estimated $\hat{\theta}$ and $\hat{\gamma}$ based on year over year regressions. We take the average value in the data sample $\hat{\theta} = 0.87$ and $\hat{\gamma} = 0.77$ in the following analysis.

Note that the constant input-output matrix \mathbf{W} and consumption expenditure shares β in the model are not directly observed from WIOT under general production and preference specifications. But we know that without any shocks, ω_{kimj} captures the steady state input-output linkages and β_{kmj} for the steady state consumption expenditure shares. We use the following information to pin down ω_{kimj} and β_{kmj}

$$\omega_{kimj} = \frac{\sum_{t=1}^T IIshare_{ki,mj,t}}{T}$$

$$\beta_{kmj} = \frac{\sum_{t=1}^T CEsare_{k,mj,t}}{T}$$

Similarly we can calibrate other parameters including intermediate income share α_{ki} , government expenditure share g_{kmj} and production wedge ϕ_{ki} .

C.2 Fiscal multipliers and spillovers

Table A-3 presents fiscal multipliers and spill overs for the Euro countries facing German government increases fiscal expenditures when prices and wages are flexible in all countries. Similar to the Cobb-

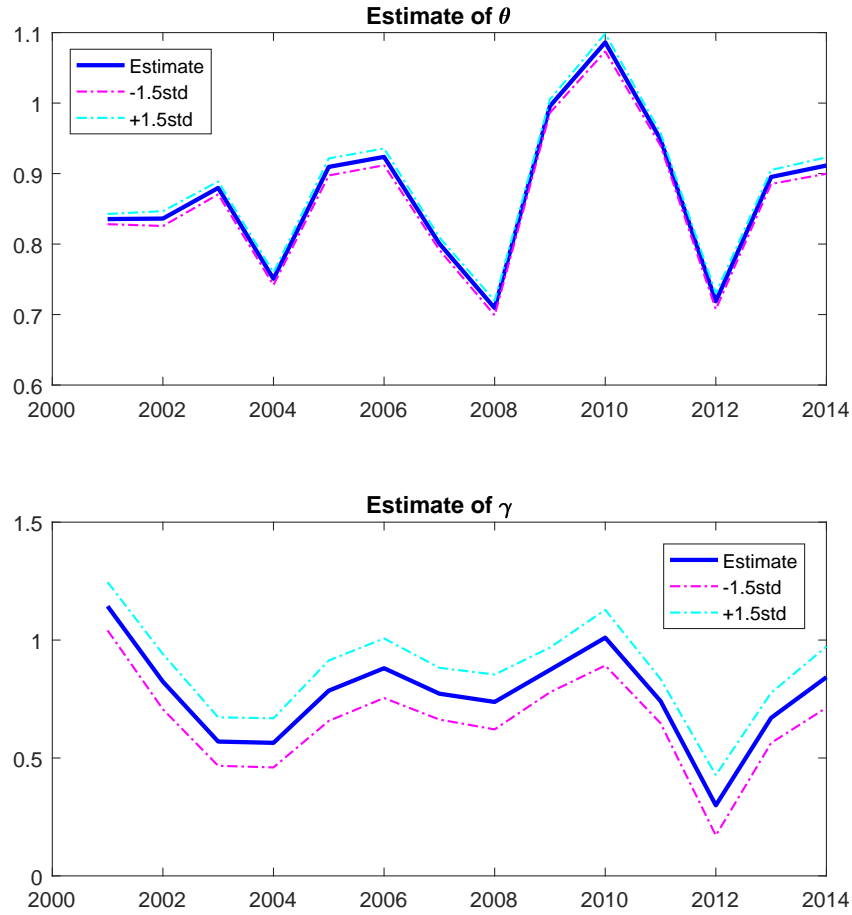


Figure A-7: Estimation of elasticities

Douglas specification in the main text, fiscal spillover under flexible prices and wages is negligible. Shutting down international production network could significantly reduces spillovers when real GDP is deflated by GDP deflator. But real GDP deflated by CPI doesn't change much with international production network. Production wedge again doesn't play a significant role as in the Cobb-Douglas specification.

Country	g_Y	$g_{Y,nn}$	$g_{Y,nsym}$	$g_{Y,\phi=1}$	g_{cpi}	$g_{cpi,nn}$	$g_{cpi,nsym}$	$g_{cpi,\phi=1}$
Aut	0.01	-0.01	-0.01	0.02	0.03	0.00	0.01	0.03
Bel	0.00	-0.01	-0.01	0.01	0.01	0.00	0.00	0.02
Deu	0.85	0.91	0.91	0.82	0.81	0.88	0.88	0.78
Esp	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	0.01
Fin	0.00	-0.02	-0.02	0.01	0.01	-0.01	0.00	0.02
Fra	-0.01	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.01
Grc	-0.01	-0.02	-0.02	0.00	0.00	-0.01	-0.01	0.01
Irl	0.01	-0.02	-0.01	0.01	0.02	0.02	0.02	0.01
Ita	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	0.00
Nld	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	0.01
Prt	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	0.00

Table A-3: Fiscal multipliers (CES). The table shows the response of real GDP growth to a one-percent of GDP increase in German Government spending with flexible prices and wages in all countries. The first to fourth columns show real GDP measured using the GDP deflator, respectively with and without international production connections. ‘nn’ denotes no international trade in intermediate inputs. ‘nsym’ denotes no trade in intermediate inputs and a symmetric input structure. Column with $\phi = 1$ is for the case without production wedges. The fifth to eighth columns show the same responses, but GDP is deflated by CPI.