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Abstract

This study contributes to the existing literature on the empirical characteristics of virtual currency allowing for dynamic transition between different economic regimes and considering various crashes and rallies over the business cycle, that are captured by jumps. We combine Markov-switching models with Levy jump-diffusion offer a new model that captures the different sub-period of crises over the business cycle, that are captured by jumps. This method also enables to test the relevance of dynamic measures of regime switching with respect to independent pure-jump process, which are not frequently used in the literature. Bitcoin offer something different than a traditional currency; there is potential value of having a network that helps as a secure repository for the common knowledge of all transactions. In addition, value of bitcoin fluctuates so wildly that it may be too risky to serve as a credible store of value.

Keywords: Bitcoin; Jump process; Markov-switching model.

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1 Introduction

This study contributes to the existing literature on the empirical characteristics of virtual currency allowing for dynamic transition between different economic regimes and considering various crashes and rallies over the business cycle, that are captured by jumps. In our model, we combine Markov-switching models with Levy jump-diffusion offer a new model that captures the different sub-period of crises over the business cycle, that are captured by jumps, whereas the trend is simply modelled under a Gaussian process. This method also enables to test the relevance of dynamic measures of regime switching with respect to independent pure-jump process, which are not frequently used in the literature.

Our study differs from past ones in that we investigate the Bitcoin price fluctuations. Bitcoin offer something different than a traditional currency; there is potential value of having a network that helps as a secure repository for the common knowledge of all transactions. In addition, value of bitcoin fluctuates so wildly that it may be too risky to serve as a credible store of value. In fact, Bitcoin hogged the limelight in the cryptocurrency markets in the first week of May as it fell to one-month lows below \$8,000. Nevertheless, while the world's largest cryptocurrency by market capitalization suffered a 3.6 percent week-on-week drop in prices, it still outclassed other major names like bitcoin cash (BCH) and EOS, which both reported double-digit losses. Furthermore, the US CFTC rules that bitcoin and other crypto-currencies are commodities subject to the CFTC's jurisdiction. Thus, bitcoin falls within the definition of "commodity" under the Commodity Exchange Act such that bitcoins derivatives contracts are regulated by the CFTC. Consequently, bitcoin is officially a commodity, according to U.S regulator. Dyhrberg (2015) claim that bitcoin has liquidity limitations like other commodities. In fact, bitcoin and gold derive most of their value from the fact that they are scarce and costly to extract.

2 Stochastic model

In this preliminary section, we review the basic intuitions behind our modeling strategy. Lévy processes have many appealing properties in financial economics, and constitute the first building block of our model. Second, we recall the very intuitive interpretation of the aperiodic, irreducible and ergodic Markov chain. Third, we set the objective of the newly proposed regime-switching Lévy model.

2.1 Lévy jumps

Lévy processes can be thought of as a combination of a diffusion process and a jump process. Both Brownian motion (i.e. a pure diffusion process) and Poisson processes (i.e. pure jump processes) are Lévy processes. As such, Lévy processes represent a tractable extension of Brownian motion to infinitely divisible distributions. In addition, Lévy processes allow the modeling of discontinuous sample paths, whose properties match those of empirical phenomena such as financial time series. There have been many efforts to apply Lévy processes, such as the variance gamma (VG) model (Carr and Madan (1999)), and the Normal Inverse Gaussian (NIG) model (Rydberg (1997)). Kijima (2002), Cont and Tankov (2004) and Schoutens (2003) are general books which discuss the use of Lévy processes in finance.

Jumps are discontinuous variations in assets' prices. By nature, jumps consist of rare and dramatic events that dominate the trading days during which they occur. In financial economics, jumps are expected to appear due to dividend payments, microcrashes due to short-term liquidity challenges or news, such as macroeconomic announcements. Such events have been made partly accountable for the non-Gaussian feature of financial returns, as they can only be captured by fat-tailed distributions. By definition, jumps generate returns that lie outside their usual scale of value. Hence, the higher the jump activity, the higher the uncertainty for market participants. This is why measuring jumps matters.

Jumps are an essential building block of the underlying data-generating process in financial and commodity markets, both in the returns and volatility dynamics. The frequency of occurrence and the size of the jumps are found to be very different from one market to another. Kaeck (2013) studies several jump-diffusion processes (including Lévy processes) in the S&P 500 index. and evaluates their performance in terms of option pricing and jump risk premia. Deaton and Laroque (1992) find empirical evidence that agricultural prices are agitated by jumps, which led to numerous theoretical contributions (see Casassus and Collin-Dufresne (2005), Liu and Tang (2011)). Recently, Brooks and Prokopczuk (2013) have documented that a large negative return in the crude oil price should trigger a jump in its volatility (alongside other examples for soybean, etc.).

2.2 Markov-switching

The normal behavior of economies is occasionally disrupted by dramatic events that seem to produce quite different dynamics for the variables that economists study. Chief among these is the business cycle, in which economies depart from their normal growth behavior and a variety of indicators go into decline (Hamilton and Raj (2002)).

Following Hamilton (1989a,b), time series may be modeled by following different processes at different points in time, with the shifts between processes determined by the outcome of an unobserved Markov chain. In this framework, the parameters and the variance of an autoregressive process depend upon an unobservable regime variable, which represents the probability of being in a particular state of the world. As explained by Engle and Hamilton (1990), the basic idea is to decompose time series into a sequence of stochastic, segmented time trends. A complete description of the Markov-switching model requires the formulation of a mechanism that governs the evolution of the stochastic and unobservable regimes on which the parameters of the autoregression depend. Once a law has been specified for the states, the evolution of regimes can be inferred from the data. Typically, the regime-generating process is an ergodic Markov chain with a finite number of states defined by the transition probabilities, which determine the probability that volatility will switch to another regime, and thus the expected duration of each regime.[1]

The regime at any given date is presumed to be the outcome of a Markov chain whose realizations are unobserved to the econometrician. The task facing the econometrician is to characterize the regimes and the law that governs the transitions between them. These parameters estimates can then be used to infer which regime the process was in at any historical date. Although the state of the business cycle is not observed directly by the econometrician, the statistical model implies an optimal way to form an inference about the unobserved variable and to evaluate the likelihood function of the observed data. The techniques developed in Hamilton (1996) rely on the EM algorithm. The standard errors are calculated considering the covariance matrix of the estimators (Bollerslev and Wooldridge (1992)). The residuals are calculated as a weighted sum of the residuals in the four states (for a two-regime model), with weights given by the filtered probabilities.

In terms of recent applications of this technique, we may cite Li et al. (2013), who consider a continuous-time regime-switching term structure model applied to monetary policy, and underline the relevance of switching regimes for term structure modeling. Besides, Tu (2010) shows that stock market displays regime switching between upturns and downturns. Therefore, the author recommends the use of regime switching models in portfolio decisions.

2.3 Regime-switching Lévy

In this paper, we choose to combine Markov-switching models with Lévy jump-diffusion to match the empirical characteristics of financial and commodity markets. From a statistical point of view, it makes sense to introduce a Markov chain with the existence of a Lévy jump in order to disentangle potentially normal economic regimes (e.g. with a Gaussian distribution) versus agitated economic regimes (e.g. crises periods with stochastic jumps). By combining these two features, we offer a new model that captures well the various crashes and rallyes over the business cycle, that are captured by jumps, whereas the trend is simply modelled under a Gaussian framework. On the one hand, the benefits of resorting to regime-switching dynamics lie in

[1]The estimation routine generates two by-products. First, the regime probability at time t is the probability that state t will operate at t, conditional on information available up to $t-1$. The second by-product is the smooth probability, which is the probability of a particular state in operation at time t conditional on all information in the sample.

disentangling different market regimes that do not have the same parameters of modelling. These separate dynamics are endogenous to the asset price, that need to be uncovered by the Markov chain. On the other hand, Lévy processes represent a very flexible class of stochastic processes, since they allow for the presence of a diffusion (e.g. a scaled Brownian motion) and/or the presence of an independent pure-jump process. Consequently, the regime-switching Lévy model allows identifying the presence of discontinuities for each market regime. This feature constitutes the objective of the proposed model.

Lévy processes are key to this study, since they enable us to measure the intensity of jumps. The higher the jump intensity during one market regime, the higher the need to include a pure-jump process. Conversely, in the absence of jumps during another market regime, then a continuous diffusion with a Brownian motion is indicated. The use of Lévy processes in finance was pioneered by Madan and Seneta (1990) and Madan, Carr and Chang (1998) who paid particular attention to their use in option pricing. Recent extensions include Carr, Geman, Madan and Yor (2003) who called these models CGMY after their own initials. Barndorff-Nielsen and Shephard (2012) further discuss the empirical fit of Lévy processes. Computationally, an elegant discussion of the EM-algorithm is given in Tanner (1996). In the next section, we formally introduce the notations for Lévy processes modulated by a Markov chain.

3 Evidence of jump, crisis and economic states

Let $(\omega, \mathcal{F}, P)$ be a filtered probability space and T be a fixed terminal time horizon. We propose in this paper to model the dynamic of a sequence of historical values of price using a regime-switching stochastic jump-diffusion. This model is defined using the class of Lévy processes.

3.1 Lévy Process

Definition 1 *A Lévy process L_t is a stochastic process such that*

1. $L_0 = 0$.

2. *For all $s > 0$ and $t > 0$, we have that the property of stationary increments is satisfied. i.e. $L_{t+s} - L_t$ as the same distribution as L_s.*

3. *The property of independent increments is satisfied. i.e. for all $0 \leq t_0 < t_1 < \cdots < t_n$, we have that $L_{t_i} - L_{t_{i-1}}$ are independent for all $i = 1, \ldots, n$.*

4. *L has a Cadlag paths. This means that the sample paths of a Lévy process are right continuous and admit a left limits.*

Remark 1 *In a Lévy process, the discontinuities occur at random times.*

3.2 Markov-switching

Definition 2 *Let $(Z_t)_{t\in[0,T]}$ be a continuous time Markov chain on finite space $\mathcal{S} := \{1, 2, \ldots, K\}$. Denote $\mathcal{F}_t^Z := \{\sigma(Z_s); 0 \leq s \leq t\}$, the natural filtration generated by the continuous time Markov chain Z. The generator matrix of Z, denoted by Π^Z, is given by*

$$\Pi_{ij}^Z \geq 0 \quad \text{if } i \neq j \text{ for all } i, j \in \mathcal{S} \quad \text{and} \quad \Pi_{ii}^Z = -\sum_{j\neq i} \Pi_{ij}^Z \quad \text{otherwise.} \tag{1}$$

Remark 2 *The quantity Π_{ij}^Z represents the switch from state i to state j.*

3.3 Regime-switching Lévy

Let us define the regime-switching Lévy Model:

Definition 3 *For all $t \in [0, T]$, let Z_t be a continuous time Markov chain on finite space $\mathcal{S} := \{1, \ldots, K\}$ defined as in Definition 2. A regime-switching model is a stochastic process (X_t) which is solution of the stochastic differential equation given by*

$$dX_t = \kappa(Z_t)\left(\theta(Z_t) - X_t\right) dt + \sigma(Z_t) dY_t \tag{2}$$

where $\kappa(Z_t)$, $\theta(Z_t)$ and $\sigma(Z_t)$ are functions of the Markov chain Z. Hence, they are constants which take values in $\kappa(\mathcal{S})$, $\theta(\mathcal{S})$ and $\sigma(\mathcal{S})$

$$\kappa(\mathcal{S}) := \{\kappa(1), \ldots, \kappa(K)\} \in \mathbb{R}^{K^*}, \quad \theta(\mathcal{S}) := \{\theta(1), \ldots, \theta(K)\}, \quad \sigma(\mathcal{S}) := \{\sigma(1), \ldots, \sigma(K)\} \in \mathbb{R}^{K^+}.$$

where Y is a stochastic process which could be a Brownian motion or a Lévy process.

Remark 3 *The following classic notations apply:*

- *κ denotes the mean-reverting rate;*
- *θ denotes the long-run mean;*
- *$\sigma_.$ denotes the volatility of X.*

Remark 4

- *In this model, there are two sources of randomness: the stochastic process Y appearing in the dynamics of X, and the Markov chain Z. There exists one randomness due to the market information which is the initial continuous filtration $\mathcal{F}$ generated by the stochastic process Y; and another randomness due to the Markov chain Z, $\mathcal{F}^Z$.*

- *In our model, the Markov chain Z infers the unobservable state of the economy, i.e. expansion or recession. The processes Y^i estimated in each state, where $i \in \mathcal{S}$, capture: a different level of volatility in the case of Brownian motion (i.e. $Y^i \equiv W^i$), or a different jump intensity level of the distribution (and a possible skewness) in the case of Lévy process (i.e. $Y^i \equiv L^i$).*

Remark 5 *One could propose to use a regime-switching stochastic volatility model, à la Heston, to better capture the flexibility of the volatility changes and levels. Nevertheless, this kind of model increases dramatically the computational burden during the simulations without improving greatly the empirical fit.*

3.4 NIG distribution

We recall the main properties of the Normal Inverse Gaussian (NIG) distribution. Indeed, we assume that a Lévy process L follows a Normal Inverse Gaussian (NIG) distribution. Note the Variance-Gamma could have been an alternative at this stage (Kaishev and Dimitrova, 2009). The NIG family of distribution was introduced by Barndorff-Nielsen and Halgreen (1977). The NIG density belongs to the family of normal variance-mean mixtures, i.e. one of the most commonly used parametric densities in financial economics.

Taking $\delta > 0$, $\alpha \geq 0$, then the density function of a NIG variable $NIG(\alpha, \beta, \delta, \mu)$ is given by

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha}{\pi} \exp\big(\delta\sqrt{\alpha^2 - \beta^2} + \beta(x-\mu)\big) \frac{K_1\big(\alpha\delta\sqrt{1+(x-\mu)^2/\delta^2}\big)}{\sqrt{1+(x-\mu)^2/\delta^2}} \ . \tag{3}$$

where K_ν is the third Bessel kind fonction with index ν. It can be represented with the following integral

$$K_\nu(z) = \frac{1}{2}\int_0^\infty y^{\nu-1} \exp\big(-\frac{1}{2}z(y+y^{-1})\big)dy \ .$$

For a given real ν, the function K_ν satisfies the differential equation given by

$$x^2 y^{''} + xy^{'} - (x^2 + \nu^2)y = 0 \ .$$

This class of distribution is stable by convolution as the classic normal distribution. i.e.

$$NIG(\alpha, \beta, \delta_1, \mu_1) * NIG(\alpha, \beta, \delta_2, \mu_2) = NIG(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2) \ .$$

Lemma 1 *If $X \sim NIG(\alpha, \beta, \delta, \mu)$ then for any $a \in \mathbb{R}^+$ and $b \in \mathbb{R}$, we have that*

$$Y = aX + b \sim \left(\frac{\alpha}{a}, \frac{\beta}{a}, a\delta, a\mu + b\right) .$$

The log cumulative function of a NIG variable is given by

$$\phi^{NIG}(z) = \mu z + \delta \left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + z)^2} \right), \quad \text{for all } |\beta + z| < \alpha \ , \tag{4}$$

The first moments are given by

$$\mathbb{E}[X] = \mu + \frac{\delta\beta}{\gamma} \ , \quad \mathrm{Var}[X] = \frac{\delta\alpha^2}{\gamma^3} \ . \tag{5}$$

with $\gamma = \sqrt{\alpha^2 - \beta^2}$. And finally the Lévy measure of a NIG$(\alpha, \beta, \delta, \mu)$ law is

$$F_{NIG}(dx) = e^{\beta x} \frac{\delta\alpha}{\pi|x|} K_1(\alpha|x|)\, dx \ . \tag{6}$$

Remark 6 *Each parameter in $NIG(\alpha, \beta, \delta, \mu)$ distributions can be interpreted as having a different effect on the shape of the distribution:*

- α - *tail heaviness of steepness.*
- β - *skewness.*
- δ - *scale.*
- μ - *location.*

4 Application on Bitcoin

In this paper we apply the estimation process of Chevallier and Goutte (2017). This methodology is a two-step approach by estimating in (2) *(i)* model parameters in a regime-switching Brownian process, and *(ii)* the distribution parameters. We fit a regime-switching Lévy model such as (2) where the stochastic process Y is a Lévy process that follows a Normal Inverse Gaussian (NIG) distribution. Thus the optimal set of parameters to estimate is $\hat{\Theta} := \left(\hat{\kappa}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{\mu}_i, \hat{\Pi} \right)$, for $i \in \mathcal{S}$.

We have the three parameters of the dynamics of X, the four parameters of the density of the Lévy process L, and the transition matrix of the Markov chain Z.

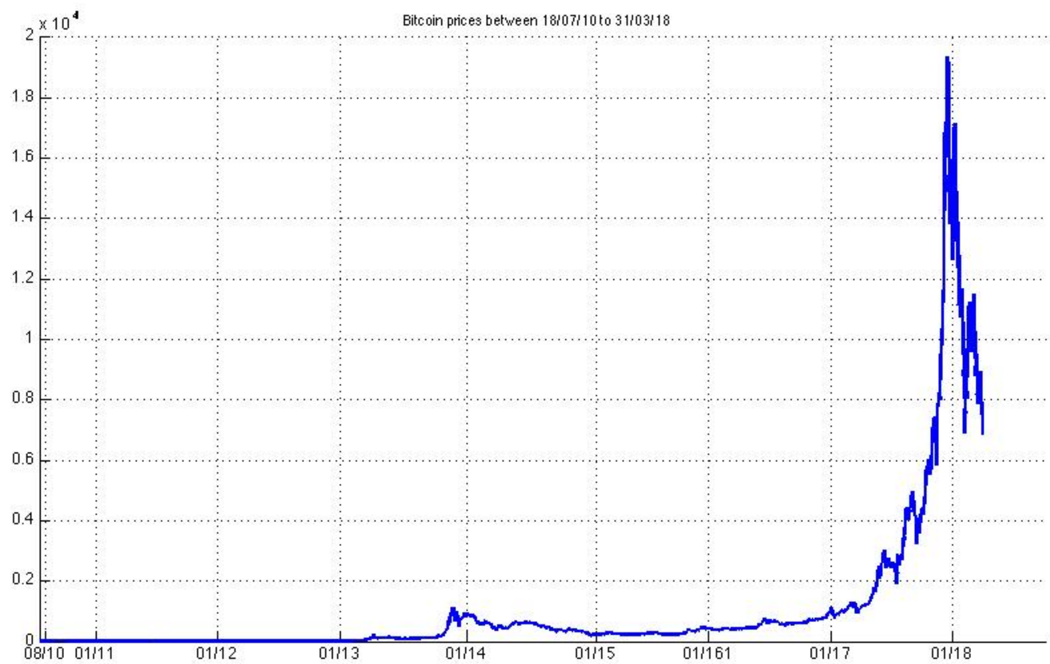
The data is retrieved from XXX over the period going from July, 10th 2010 to March, 31th 2018 with a daily frequency, totaling 2814 observations.

Figure 1 displays the data. The table 1 reports the results of: (i) the set of diffusion parameters, and (ii) the NIG density parameters of the Lévy jump process fitted to each regime. The remaining problem in this work is to specify the number of regimes in the Markov chain.

For simplicity, we proceed with two regimes that relate to the 'boom' and 'bust' phases of the business cyle.

We also report a plot where each regime is reported with a different color (e.g. blue (red) corresponds to regime 1 (regime 2)). To provide the reader with a clearer picture, we have chosen to plug the regimes identified back into the raw (non-stationary) data. Of course, all the estimates were performed on returns, e.g. stationary data. Below this first plot, the filtered and smoothed probabilities are displayed. They reflect the regime switches at stake.

Figure 1: Bitcoin price



4.1 MS-Lévy model

Table 1: Estimated parameters

Process	κ	θ	σ	P_{ii}^{Z}
State 1	1.0299	0.3082	23.8652	0.9552
	(0.1055)	(0.0074)	(0.7274)	
State 2	0.9130	9.9467	1367.4328	0.8550
	(14.4028)	(0.0389)	(7529.9928)	

NIG	α	β	δ	μ
State 1	0.0801	0.0238	0.1603	-0.0499
	(0.0672)	(0.0023)	(0.7534)	(10.0061)
State 2	0.0012	0.0001	161.0635	-14.5843
	(0.0883)	(0.0001)	(0.5497)	(0.0001)

Standard errors are given in parentheses below parameter estimates.

In Table 1, the parameter α represents the jump intensity. The lower the alpha, the higher the jump intensity in a given regime. We observe that $\alpha = 0.0012$ during regime 2, which indicates a high jump intensity. On the contrary, $\alpha = 0.0801$ during regime 1, which points out a low jump intensity.

Inspecting the Figures 1 and 3, we observe that the regime 1 is characterized by a rather stable price path for the Bitcoin crypto-money (before April 2013 and between all crisis and bublbes whose appear in 2013 and 2014 (i.e. The Meltdown of April 2013, Pops goes the 2013 Buble and The Mt. Gox Calamity of 2014). Visually, the model captures well these three main events (by switching in regime two) whose appear in standard economic state (i.e. regime one) and then switch definitively in regime two of high volatility and high frequency of jump after 2017. So beyond that date, the data enters the regime two which is characterized by ups and downs, and thus a higher jump activity.

β is the skewness parameter: $\beta < 0$ ($\beta > 0$) implies a density skewed to the left (right). The skewness of the density increases as β increases. In the case where β is equal to 0, the density is symmetric around μ. In both regime the values of β is close to 0, these values reflect a balance between positive and negative jumps, as illustrated in Figure 3. δ is the scale parameter representing a measure of the spread of the returns. We clearly see that the regime two with high frequency and intensity of jumps exhibits a huge value of delta $\delta = 161.0635$ against a vanishing value for regime one. This fact demonstrates the crisis or bubble situation capture by the regime two of our regime-switching Lévy model.

In Table 1, of particular interest is the volatility parameter σ: it is more than sixty times higher during regime 2 than during regime 1. The relatively higher jump intensity during regime

2 therefore translates into higher volatility levels. The mean-reverting parameters κ are close to zero in both regimes.

When inspecting the filtered and smoothed probabilities in the middle and bottom panels of Figure 2, we notice that the probability to stay in the current regime is very high (e.g. close to unity). This information is also visible in the last column of Table 1: the probability to stay in regime 1 (in regime 2) is equal to 95.52% (85.50%). If there is some general form of persistence in the chain (e.g. high probability of staying in a given regime), then this could have important implications for the computation of the Value-at-Risk and dynamic portfolio allocation, because the benefits of portfolio diversification would be less volatile.

What we learn mainly from these probability graphs is that the stochastic process fitted to each regime does not have the same jumps characteristics during the sample period. Indeed, there are periods of time with an obvious presence of jumps (recorded during regime 2) in the Bitcoin price, and others without. Hence, this first set of results illustrates the interest of resorting to the regime-switching Lévy model to model the Bicoin dynamic.

Figure 2: Probabilities

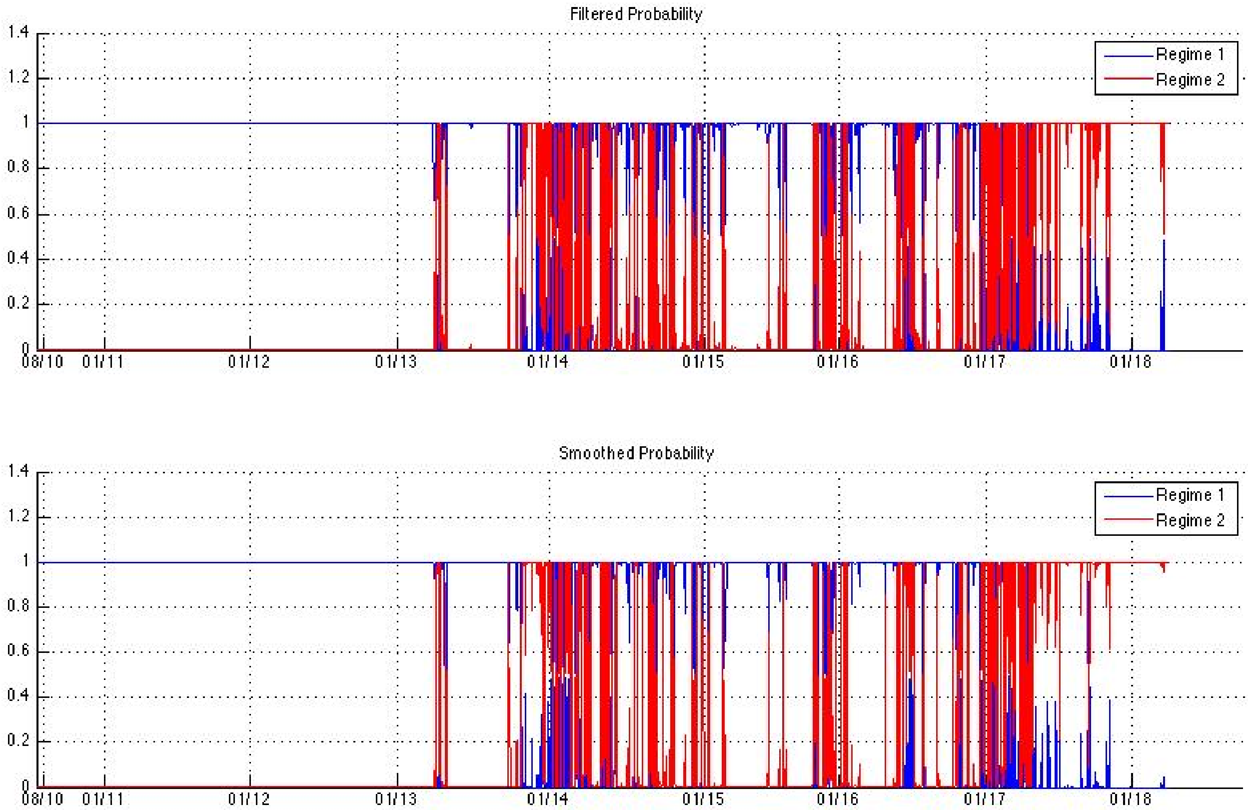
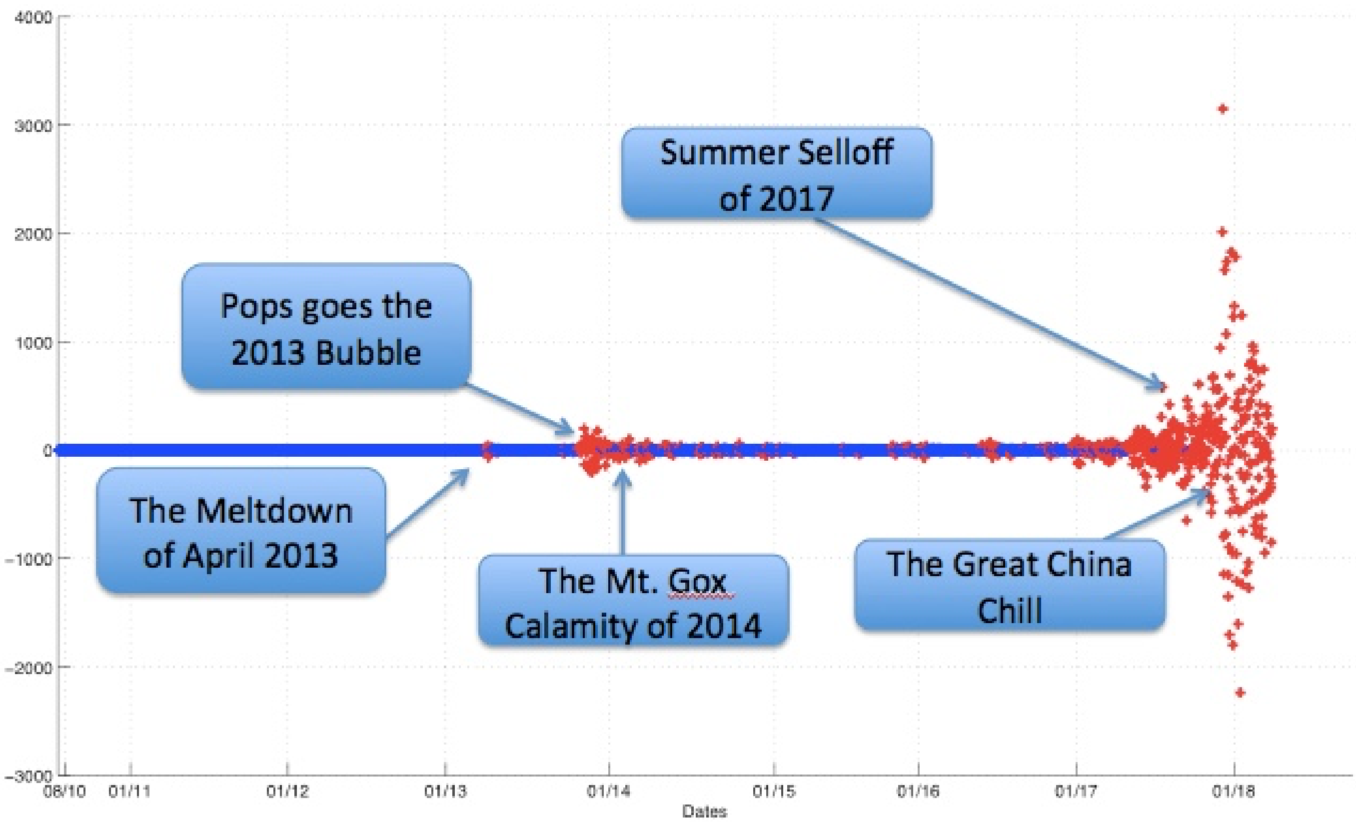


Figure 3:   Important Dates



4.2 Regime-switching classification

An ideal model is one that classifies the regimes sharply, with smoothed probabilities close to either zero or one. In order to measure the quality of regime classification, we propose the following two measures:

(1) The **regime classification measure (RCM)** introduced by Ang and Bekaert (2002) and generalized for multiple states by Baele (2005).

$$\text{RCM}(K) = 100. \left(1 - \frac{K}{K-1}\frac{1}{T}\sum_{k=1}^{N}\sum_{Z_{t_k}}\left(P\left(Z_{t_k} = i|\mathcal{F}_{t_M}^X;\hat{\Theta}_1^{(n)}\right) - \frac{1}{K}\right)^2\right), \tag{7}$$

where the quantity $P\left(Z_{t_k} = i|\mathcal{F}_{t_M}^X;\hat{\Theta}_1^{(n)}\right)$ is the smoothed probability and $\hat{\Theta}_1^{(n)}$ is the vector of parameters estimated. The constant serves to normalize the statistic to be between 0 and 100. Good regime classification is associated with low RCM statistic value: a value of 0 means perfect regime classification, and a value of 100 implies that no information about the regimes is revealed. Consequently, even if a model has the highest log-likelihood value, its RCM needs to be close to zero.

(2) The **smoothed probability indicator**, introduced by Goutte and Zou (2013) and developed by Goutte (2014). With this indicator, a good classification for the data can be achieved when the smoothed probability is less than 0.1 or greater than 0.9. This means that the data at time $t \in [0, T]$ is in one of the regimes at the 10% error level.

Table 2: Regime Classification Measure (RCM) and Smoothed Probability Indicator

	RCM	$P^{10\%}$
Financial markets		
Bitcoin	4.96	94.88%

For the Bitcoin, we observe in Table 2 that the RCM statistic is close to zero (4.96). In that case, the regime-switching Lévy model is able to discriminate perfectly between the two regimes. In the second column of 2, we notice that the smoothed probability indicator is equal to 94.88%. Again, we conclude that the discrimination between regime is accurately performed by the model, as it is very close to the upper bound of 100%. With this battery of diagnostic tests, we have established the robustness of the results obtained with the Lévy regime-switching model.

5 Conclusion

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