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Do positional preferences cause welfare gains?

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Abstract

We examine conditions for which positional preferences for voluntary contribution to a public good can be welfare enhancing in a one-shot public good game, where individuals may also enjoy a return from their contribution ranking. We show that positional preferences are welfare-increasing only under certain conditions. We find that when agents’ positional preferences are homogeneous, they overinvest in the public good compared to equilibrium with no positional preferences, resulting in a zero-sum positional race with a higher public good provision. When agents have heterogeneous positional preferences, the overall impact on social welfare is positive when endowments are homogeneous.
1. Introduction

It is widely recognized that people care for their relative position. But, the co-existence of positional goods\(^1\) with non-positional goods may generate a crucial social dilemma. Positional concerns are stronger in some domains than others so that resources are diverted from non-positional goods to positional goods and lead to a decrease in social welfare (Frank, 2005). Because of the very specific nature of positional goods (Hirsch, 1976), seeking a higher rank is necessarily gained at the expense of other agents, resulting in zero or negative sum games. Each step up the status ladder for one person logically requires a step down for another. Consequently, status seeking agents make expenditures on positional goods to get a higher relative position, but as all agents make the same efforts, all obtain an identical position. In an economy with private consumption goods, positional preferences lead to a welfare loss, which can be aggravated if negative externalities are considered (Ng and Wang, 1993; Brekke et al., 2003; Van Long and Wang, 2008; Frank, 2008).

Although consumer positional motivations were identified early (e.g., Veblen, 1899; Hirsch, 1976; Frank, 1985; Alpizar et al., 2005), theoretical and empirical investigations about their importance in relation to public goods are scarce (e.g., Hollländer, 1990; Solnick and Hemenway, 2005, Muñoz-Garcia, 2011). Contrasting with this literature gap, anecdotal evidence supports the fact that charitable contributions or contributions to public goods may be motivated by positional concerns (Wilkinson, 2006). For instance, Ted Turner inspired the successful idea of the Slate 60 list of the top American donors. Indeed, he argued that the Forbes 400 list of richest Americans was discouraging the wealthy from giving away their money for fear of slipping down the rankings (Dowd, 1996). School auctions are another example where status seeking concerns can lead to higher private provision of public goods (Kuczynski, 2003; Pertusini, 2009). Similarly, a recent marketing study showed that status competition can promote pro-environmental behavior, even at a private cost for individuals (Griskevicius et al., 2010). The authors argue that ‘visible’ ecofriendly purchases (by contrasting shopping alone online with shopping in public) are rooted in the idea of competitive altruism, that is people compete for status by trying to appear more altruistic. Gintis et al. (2003, p. 159) state that hyper-fair offers in ultimatum games can reflect Melanesian culture of status-seeking through gift giving.

In this paper, our contribution is to enlarge the analysis of the impact of positional preferences by taking into account, in a single framework, the heterogeneity of endowments, the full range of positional preferences (positive and negative) and the impact on social welfare. Two specific articles are closely related to our work. Hollländer (1990), who considers homogeneous agents (both in positional preferences and endowments), shows that, at the equilibrium, the only effect of status orientation is a decrease in social approval through self-assessment. Muñoz-Garcia (2011) compares simultaneous and sequential mechanisms in terms of overall public good contribution, and shows that, in case of heterogeneous positional preferences (supposed to be positive), the sequential mechanism should be preferred. The analytic framework considers a non-linear payoff function, putting higher weight on lower positional preferences. The study only focuses on the impact of positional preferences on public good provision and whether the decision making process (simultaneous or sequential) allows higher contributions. However, the author provides no social welfare analysis.

Adding to the literature, we analyze whether policy makers could promote status races on public goods. Simply put, we explore all the effects on social welfare of such positional concerns, not only in terms of provision of public goods but also in terms of positional

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\(^1\) We use the terms status and position interchangeably, status being the ultimate positional good.
externalities. To do so, we use a simple model of a one-shot voluntary contribution to a public good where besides a return to the public good, individuals may enjoy a return from the ranking of their public good contribution. We carry out a comparative statics analysis to determine how positional preferences drive public good provision and impact social welfare. We show that welfare is influenced by the level of two externalities; individual contributions affect the level of public goods (public good externality) but also influence the rank of individuals (positional externality) in the contribution hierarchy. When agents have homogeneous positional preferences over the public good, they overinvest in the positional public good compared to equilibrium with no positional preferences. There is no positional gain (everyone runs to keep the same place), but a higher provision of the public good. Indeed, positional preferences allow public good provision, prevent free rider behavior and thus increase social welfare. When agents differ in their positional preferences over the public good, the overall impact on social welfare is positive when endowments are homogeneous.

This paper proceeds as follows. Section 2 presents a basic framework of consumer behavior with positional preferences on public goods contributions and shows that the standard free riding equilibrium is not the only equilibrium. Then, in section 3, we analyze the impact of such positional preferences on social welfare when positional preferences are homogeneous (section 3.1) and when they are heterogeneous (section 3.2). Section 4 concludes.

2. A model of voluntary contribution with a positional public good

Consider a one-shot public good game where individual $i$ has positional preferences and chooses his public good contribution $x_i$ to maximize utility in equation (1). Utility depends on $i$'s own contribution but also on the contributions of others not only because of the nature of the public good but also because of relative standing issues.

$$U_i(x_i, x_j, \alpha_i) = \pi_i(x_i) + g_i(x_i, x_j) + R_i(x_i, x_j, \alpha_i)$$

(1)

The utility function is composed of three parts: utility from private consumption $\pi_i(x_i)$, utility from public good consumption $g_i(x_i, x_j)$, and utility from relative standing $(R_i(x_i, x_j, \alpha_i))$.

The first part of equation (1), $\pi_i(x_i) + g_i(x_i, x_j)$, constitutes the standard payoff function for a voluntary contribution mechanism. For the sake of simplicity and as a first step, we assume this payoff function to be linear both for private as well as public consumption (cf. equation (2)); Utility from private consumption, $\pi_i(x_i)$, equals $D_i$ (the monetary endowment that $i$ might allocate to public good provision, i.e. disposable income for charitable giving)\(^2\) minus $px_i$ which is the cost of individual $i$'s contribution to the public good. Individual $i$ may contribute any amount of $x$ to the public good such that $x_i \in \left[0, \frac{D_i}{p}\right]$. Public good consumption,

\(^2\) We suppose that an individual $i$ with revenue $B_i$, might allocate a maximum amount of money to charitable giving, $D_i$, such that $B_i = D_i + D_i$. This amount $D_i$ is exogenously determined and depends on revenue and life expenses (food, housing, etc.). Then, without loss of generality and for sake of simplicity, we set $D=0$.  


$g_i(x_i, x_j)$, depends on contributions of all $N$ group members $(x_i + \sum_{j\neq i} x_j)$ and group marginal payoff ($G$). Thus marginal per capita return of the public good equals $\frac{G}{N}$.

Thus, under the assumption of linear payoff functions, the marginal cost of public good consumption equals $p$, whereas the marginal benefit equals $\frac{G}{N}$. We assume that $p > \frac{G}{N}$ and that $p < G$, which corresponds to standard public good dilemma assumptions.

To summarize, the first part of equation (1) writes as in equation (2).

$$\pi_i(x_i) + g_i(x_i, x_j) = D_i - px_i + \frac{G}{N}(x_i + \sum_{j\neq i} x_j)$$

(2)

The last part of equation (1), $R_i(x_i, x_j, \alpha_i)$, represents the positional payoff (cf. equation (3))

where $\bar{x}_j = \frac{\sum x_j}{(N-1)}$ is defined as the average contribution of all other players $(j \neq i \in [1,N])$.

$$R_i(x_i, x_j, \alpha_i) = \alpha_i \times (x_i - \bar{x}_j)$$

(3)

The positional payoff function is composed of two terms:

- the difference between individual $i$’s contribution and the average contribution of all other players $(x_i - \bar{x}_j)$, that is $i$’s relative position (as in Brekke et al., 2003).

- the individual positional parameter $\alpha_i$ that measures the nature and strength of positional concerns. If $\alpha_i < 0$, individual $i$’s utility increases when $i$ contributes less than others, but if $\alpha_i > 0$, individual $i$’s utility increases when $i$ contributes more than others. If $\alpha_i = 0$, individual $i$ does not care about his relative standing and only enjoys the monetary payoff, which corresponds to standard homo economicus preferences.

We would like to stress the role played by the positional parameter. If someone contributes to the public good for the same amount as others on average (i.e., identical relative position), his contribution will not give him benefits other than the monetary payoff. Let us assume a higher relative position for individual $i$, i.e., $(x_i - \bar{x}_j) > 0$, that is individual $i$ contributes more than others, on average. Then, if individual $i$ has a positive positional parameter $\alpha_i > 0$ (resp. a negative positional parameter $\alpha_i < 0$), $i$’s positional payoff increases (resp. decreases). Conversely, assuming a lower relative position for $i$, i.e., $(x_i - \bar{x}_j) < 0$ only the individuals that have a negative positional parameter $(\alpha_i < 0)$ will increase their positional payoff. In this particular case, individual $i$ benefits from the public good with a personal contribution lower than the average. He enjoys the pleasure of “making a better deal than other players”.

The utility maximizing behavior leads to the following lemma.

**Lemma 1:** An individual with positive positional concerns contributes all his endowment (respectively nothing) to the public good, if the non-monetary value of his relative position, $\alpha_i$, is higher (respectively lower) than the monetary loss of
contributing to the public good, \( p - \frac{G}{N} \). An individual with negative positional concerns never contributes to the public good.

**Proof:** see Appendix

Linearity of payoffs implies equilibrium contributions to be boundary. Thus, there exists two equilibria in this game: player \( i \) either donates all his endowment to the public good or submits zero contributions. In an economy where individuals don’t possess any positional preferences or where contributions of others are not observable (i.e. individuals don’t benefit from any reputational value from public good consumption), and given our assumption \( p > \frac{G}{N} \), the unique equilibrium is to contribute nothing. In an economy where individuals possess positional preferences, voluntary contribution may become a dominant strategy, as long as \( \alpha > p - \frac{G}{N} \). Our results show that, under the assumption of linear utility functions, *status seeking may counter-balance free-riding incentives in a one shot public good game as long as positional preferences are sufficiently high.*

The assumption of linear payoff functions might be considered to be restrictive and lack generalization of our results. Especially when arguing that in real life often both "private and public goods are subject to diminishing marginal values" (Laury and Holt, 2008).

So for more generality, we will discuss the way equilibria are modified when linearity is either released at the private consumption level or released at the public good provision level. *We still find in both cases that public good contributions increase with positional preferences.*

In the case of diminishing returns in private consumption \( (\pi_i' > 0, \pi_i'' < 0 \ \forall i) \), the optimal decision is no longer a corner solution. Indeed, the marginal cost of contributing to the public good is now an increasing function with \( x_i \) because the amount of private good becomes rarer. As the optimal voluntary contribution is determined such that the marginal benefit of contributing (i.e. marginal benefit of public good consumption \( \frac{G}{N} \) and his/her relative position \( \alpha_i \)) equals the marginal cost, some individuals who value their relative position not necessarily contribute their entire endowment to the public good. We can thus distinguish three different types of behavior. Individuals with \( \alpha_i \leq \alpha_l \) don’t contribute, individuals with \( \alpha_i \geq \alpha_h \) donate their entire endowment, and individuals with \( \alpha_i \in ]\alpha_l, \alpha_h[ \) will contribute only a part of their endowment to the public good.

An illustration is given in Figure 1 supposing a quadratic function for private consumption: \( \pi_i(x_i) = D_i - p(x_i + \delta x_i^2) \) for any parameter \( \delta > 0 \) (Laury and Holt, 2008).
When considering diminishing marginal returns in public good consumption \((g_i' > 0; g_{ii}'' < 0\) and \(g_{ij}'' < 0\) \(\forall i, j\) there may exist multiple individual equilibria. Diminishing returns may be prevailing in different sectors such as health care, research and development, national defense. An extra contribution does not increase the marginal benefit to all users by the same amount. Under this assumption, private voluntary contribution by an individual will not only depend on his/her personal positional preference but also on the level of others' contributions. The marginal benefit of contribution for individual \(i\) decreases with his own contribution level, but also when others contribute more.

For an illustration, we use the quasi-linear function of Laury and Holt (2008), \(g_i(x_i, x_j) = \frac{g}{N}(x_i + \sum_{j \neq i} x_j) - \varepsilon(x_i + \sum_{j \neq i} x_j)^2\) for any parameter \(\varepsilon > 0\). Best response function for \(i\) can then be written: \(x_i^{br} = \frac{\alpha_i - (\frac{g}{N})}{2\varepsilon} - \sum_{j \neq i} x_j\), and represented in Figure 2. Contribution levels to the public good increase with \(\alpha_i\) (marginal benefit from individual \(i\)'s position relative to all the others), but are diminishing with the contribution level of other individuals within the economy.
3. Welfare analysis under positional preferences

According to Frank (2005, 2008), the consumption of positional goods necessarily gives rise to welfare losses. More precisely, the idea is that if one good is more positional than another good, the search for status leads to an increase in consumption of positional goods and a decrease of consumption of non-positional goods. However, as everyone consumes more of the positional good, nobody increases his relative position. Individuals do not get the expected positional benefit. This result is conditional on private positional goods. In the case of public goods, positional concerns may divert some resources from private consumption to private provision of public goods.

We consider now an economy composed of N individuals. Social welfare is the sum of individuals’ utility functions.

\[ W = \sum_{i=1}^{N} U_i(x_i, x_j, \alpha_i) = \sum_{i=1}^{N} \left( \pi_i(x_i) + g_i(x_i, x_j) + R_i(x_i, x_j, \alpha_i) \right) \]  (4)

Social welfare is thus the sum of the social welfare effects of private good and public good consumption, on one side, and of the social welfare effects of the positions on public good contribution.

\[ W = \sum_{i=1}^{N} \left( \pi_i(x_i) + g_i(x_i, x_j) \right) + \sum_{i=1}^{N} R_i(x_i, x_j, \alpha_i) \]  (5)

The aim of our paper is to determine the impact of positional preferences on social welfare gains/losses, that is, the impact of a change of the parameters \( \alpha_i \) on \( W \), and to determine under which conditions positional preferences can be welfare enhancing for the society.

For sake of simplicity, let us assume that all parameters \( \alpha_i \) increase by \( d\alpha \).

\[ \frac{\partial W}{\partial \alpha} = \sum_{i=1}^{N} \left( \frac{\partial \pi_i}{\partial x_i} + \sum_{j \neq i} \frac{\partial g_i}{\partial x_i} \frac{\partial x_i}{\partial \alpha_i} + \sum_{j \neq i} \frac{\partial R_i}{\partial \alpha_i} + \sum_{j \neq i, j \in [1,N]} \frac{\partial R_j}{\partial x_i} \frac{\partial x_i}{\partial \alpha_i} \right) \]  (6)
The first part of this equation represents the sum of individual marginal benefits from both private and public good consumption. It can be assumed without loss of generality, because of the characteristics of a public good, that:

\[
\frac{\partial \pi_i}{\partial x_i} + \frac{\partial g_i}{\partial x_i} + \sum_{j \neq i} \frac{\partial g_j}{\partial x_i} \geq 0 \quad \forall i
\]

Moreover, section 2 highlights that \(\frac{\partial x_i}{\partial \alpha_i} \geq 0\) whatever the assumptions regarding \(\pi_i\) and \(g_i\).

Consequently, the first part of (6) is positive. The effect of an increase of the parameters \(\alpha_i\) on the sum of the positional preferences is more ambiguous and depends on the assumptions on positional preferences.

We consider now positional preferences defined as in equation (3).

### 3.1. Homogeneous positional preferences

**Proposition 1**: If individuals have homogeneous positional preferences, i.e., \(\alpha_i = \alpha_j = \alpha \quad \forall i, j \in \{1, 2, \ldots, N\}\), social welfare increases or remains constant when positional preferences increase.

**Proof**: see Appendix

If individuals have homogeneous positional preferences, an increase of all positional parameters \(\alpha\) by \(d\alpha\) leaves the sum of positional benefits/losses unchanged.

\[
\sum_{i=1}^{N} \left( \frac{\partial R_i}{\partial \alpha_i} + \sum_{j \neq i, j \in [1,N]} \frac{\partial R_j}{\partial x_i} \frac{\partial x_i}{\partial \alpha_i} \right) = 0 \text{ if } \alpha_i = \alpha_j \quad \forall i, j
\]

From (6), it can be deduced that:

\[
\frac{\partial W}{\partial \alpha} = \sum_{i=1}^{N} \left( \frac{\partial \pi_i}{\partial x_i} + \frac{\partial g_i}{\partial x_i} + \sum_{j \neq i, j \in [1,N]} \frac{\partial g_j}{\partial x_i} \frac{\partial x_i}{\partial \alpha_i} \right) \geq 0
\]

With homogeneous positional preferences, there is no social welfare gain to status-seeking in public goods as there was none with private goods (Frank, 2005, 2008). Positional benefits for some will be counterbalanced by positional losses for others. This result comes from our assumption of homogeneous preferences (all individuals give the same value to their relative position) and our definition of relative position (comparison between individuals and average contribution of all other individuals). However, the public good is overprovided as compared with the benchmark provision level and social welfare increases. In other words, positional preferences can prevent free-rider behavior. Note that the gain in the public good level mainly depends on the group marginal payoff.

### 3.2. Heterogeneous positional preferences

It is clear from section 2 that the individual contribution depends on the individual positional preferences. Without loss of generality, it can be assumed that \(k\) individuals will contribute to the public good after an increase of their positional preferences, and that the other individuals will not contribute to the public good, even after an increase of their search for status.
Proposition 2: Assume that: individuals differ in their positional preferences, that \( \forall i \in [1, k], \forall j \in [k+1, N] \) \( \alpha_i > \alpha_j \), and that \( \forall j \in [k+1, N] \) \( x_j = 0 \) and \( \frac{\partial x_j}{\partial x_i} = 0 \)

a. An increase of positional preferences leads to an increase in social welfare if its effect on private contribution to the public good is the same for all individuals \( \frac{\partial x_i}{\partial x} > 0 \ \forall i \in [1, k] \).

b. If individuals contribute heterogeneous amounts to the public good, the effect of public goods positional preferences on social welfare is:
   - positive if contributors do have homogeneous positional preferences on one side, and non-contributors do also have homogeneous preferences on the other side
   - positive if \( \frac{\partial x_i}{\partial x} < \frac{\partial x_i}{\partial x} \ \forall i, l \) such that \( \alpha_i - \frac{\sum_{j \neq l} \alpha_j}{N-1} < 0 \) and \( \alpha_l - \frac{\sum_{j \neq l} \alpha_j}{N-1} > 0 \).
   - negative if \( \frac{\partial x_i}{\partial x} > \frac{\partial x_i}{\partial x} \ \forall i, l \) such that \( \alpha_i - \frac{\sum_{j \neq l} \alpha_j}{N-1} < 0 \) and \( \alpha_l - \frac{\sum_{j \neq l} \alpha_j}{N-1} > 0 \).

Proof: see Appendix

Given our assumption regarding positional preferences (3), the positional component of social welfare is equal to:

\[
\sum_{i=1}^{N} \left( \alpha_i - \frac{\sum_{j \neq i} \alpha_j}{N-1} \right) \frac{\partial x_i}{\partial x} = \sum_{i=1}^{N} \left( x_i - \frac{\sum_{j \neq i} x_j}{N-1} \right) + \sum_{i=1}^{k} \left( \alpha_i - \frac{\sum_{j \neq i} \alpha_j}{N-1} \right) \frac{\partial x_i}{\partial x} \tag{7}
\]

The first term of this equation represents the social welfare effect of an increase of all positional tastes by \( d\alpha \), letting all contributions to public good unchanged. It is always equal to 0, since what some people do gain in terms of positional welfare is equal to what others do lose when all positional tastes increase by \( d\alpha \).

The second term of this equation is the positional welfare effect of the change of individual contributions due to the change of positional tastes by \( d\alpha \). If the marginal effect of an increase of positional taste is the same for the \( k \) contributors \( \frac{\partial x_i}{\partial x} = \frac{\partial x}{\partial x} \ \forall i \in [1, k] \), then the second term of equation (7) is positive:

\[
\sum_{i=1}^{k} \left( \alpha_i - \frac{\sum_{j \neq i}^{[1,N]} \alpha_j}{N-1} \right) \frac{\partial x}{\partial x} = \left( N-k \sum_{i=1}^{k} \alpha_i - \frac{k}{N-1} \sum_{j=k+1}^{N} \alpha_j \right) \frac{\partial x}{\partial x} > 0 \tag{8}
\]

This proposition shows that when all contributors give the same amount to the public good, social welfare is increased by an increase of positional preferences, not only because of the positive externality on public good consumption, but also because the positional benefit of those who contribute is higher than the positional loss of non-contributors. Indeed those who contribute have a higher positional taste (\( \alpha_i \)) than those who do not contribute. Unlike the results of Frank (2005) on private goods, we show that heterogeneous positional preferences over the public goods with identical contributions can ultimately enhance social welfare.
Let us now consider that the marginal effect on contribution does differ between contributors (the assumption \( \frac{\partial x_i}{\partial \alpha} = \frac{\partial x}{\partial \alpha} \) is not verified) and that contributors do have homogeneous preferences (\( \forall i \in [1, k], \alpha_i = \alpha \)) as non-contributors (\( \forall j \in [k + 1, N], \alpha_j = \alpha \)). Then:

\[
\sum_{i=1}^{k} \left( \alpha_i - \frac{\sum_{j=[1,N]}^i \alpha_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha} = \frac{N-k}{N-1} (\overline{\alpha} - \alpha) \sum_{i=1}^{k} \frac{\partial x_i}{\partial \alpha} > 0 \tag{9}
\]

Intuitively, if all contributors have the same positional preferences, the social welfare effect of the comparison between contributors is equal to 0 (the increase of social status of someone is compensated by the decrease of social status by someone else). However, the increase of status of all contributors is higher than the loss of status of non-contributors, since contributors give a higher value to status than non-contributors.

This result can be reversed when contributors do not have the same positional preferences. In that case, the social welfare effect of an increase of positional preferences is a sum of the marginal effect on individual contributions \( \frac{\partial x_i}{\partial \alpha} \) weighted by individual relative positional preferences, which might be either positive or negative \( \left( \sum_{i=1}^{k} \left( \alpha_i - \frac{\sum_{j=[1,N]}^i \alpha_j}{N-1} \right) \right) \). When those who have negative relative positional preferences \( \left( \frac{\sum_{j=[1,N]}^i \alpha_j}{N-1} < 0 \right) \) have the lowest marginal increase of individual contribution \( \frac{\partial x_i}{\partial \alpha} \), the overall effect is positive.

Table 1 summarizes Propositions 1 and 2.

It should be noticed that, in case of linear payoff functions as assumed in (2), then individual contributions to public good are equal to \( \frac{D}{p} \); consequently the results depend then on the homogeneity of endowments.

Interestingly, if individuals have heterogeneous positional preferences (and different endowments) and if they compare their contribution to the average contribution of other persons having the same endowment, then the first part of proposition 2 applies and positional preferences in public good provision lead to an increase in social welfare. Indeed, it seems more realistic to assume that people do compare themselves with similar persons. Aristotle argued that ‘we envy those who are near us in time, place, age, or reputation’ (Rhetoric, 1338a). For instance, Ted Turner is engaged in a competition with the wealthiest Americans, and is careful to maintain his position in this group and far from poor people. In the same vein, Clark and Oswald (1996) showed that individuals compare themselves to reference groups including other persons similar to themselves in some dimensions.
**Table I. Overall effect of positional preferences for public goods contribution on social welfare**

<table>
<thead>
<tr>
<th>Individual contributions to public good</th>
<th>Positional preferences on public good contribution</th>
<th>Positional preferences on public good contribution</th>
<th>Positional preferences on public good contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td>Increase</td>
<td>Increase</td>
<td>Increase</td>
</tr>
<tr>
<td>Heterogeneous</td>
<td>Increase</td>
<td>Increase</td>
<td>Increase</td>
</tr>
</tbody>
</table>

Increase if:

\[
\frac{\partial x_i}{\partial \alpha} < \frac{\partial x_l}{\partial \alpha} \text{ for all } i, l \text{ such that } \alpha_i - \frac{\sum_{j \neq i} \alpha_j}{N-1} < 0 \text{ and } \alpha_l - \frac{\sum_{j \neq l} \alpha_j}{N-1} > 0
\]

Decrease if:

\[
\frac{\partial x_i}{\partial \alpha} > \frac{\partial x_l}{\partial \alpha} \text{ for all } i, l \text{ such that } \alpha_i - \frac{\sum_{j \neq i} \alpha_j}{N-1} < 0 \text{ and } \alpha_l - \frac{\sum_{j \neq l} \alpha_j}{N-1} > 0
\]
4. Concluding remarks

While positional races on private goods can be detrimental to social welfare, our results show that positional races on public goods can be conducive to Paretian improvements. Consequently, a major issue for policymakers is to limit positional races in the private goods domain while promoting them in the public goods domain. ‘Good’ institutions can help to channel status seeking energy in directions conducive to socially desirable outcomes. For instance, it is widely admitted that positional preferences are more likely to remain latent if there is no socially visible way to rank individuals on the considered dimension. From a practical viewpoint, ‘social visibility’ and the possibility to rank on this dimension can stimulate positional choices in the public goods realm.

A natural extension to our study would be to analyze the overall impact of different combinations of positional preferences regarding public and private goods on the whole economy. Extending the framework to endogenize the positional parameters of individuals as a function of their wealth or/and as a function of the size of their reference group may also provide fruitful insights for future research. Moreover, adding strategic concerns could enrich the analysis. For example, in the case of imperfect information on positional preferences, contribution levels become a strategic variable and constitute signals of individual ranking seeking. Or in the case where charitable contributions cannot be perfectly observed, then expected beliefs on others’ contributions will drive individual decision making.
References


Appendix

Proof of Lemma 1:
Maximizing earnings, i.e. $\max_{x_i} U_i(x_i, x_j, \alpha_i)$ results in an equilibrium condition in which the marginal cost of contributing, $p$, equals the marginal benefit in public good provision, $\frac{G}{N}$, and in social status, $\alpha_i$.

From which it can be deduced that:
\[
\frac{\partial U_i(x_i, x_j, \alpha_i)}{\partial x_i} = -p + \frac{G}{N} + \alpha_i > 0 \iff \alpha_i > p - \frac{G}{N}
\]

The optimal contribution to the public good is such that:
\[
x^*_i = 0 \text{ if } \alpha_i < p - \frac{G}{N} \text{ and } x^*_i = \frac{D_i}{p} \text{ if } \alpha_i < p - \frac{G}{N}.
\]

Proof of proposition 1:
We want to show that $\frac{\partial W}{\partial \alpha}$ (equation (6)) is always positive when individuals have homogeneous positional preferences. The first part of $\frac{\partial W}{\partial \alpha}$ (equation (6)) is always positive, since it is the sum of individual marginal benefits from both private and public good consumption. The second part of $\frac{\partial W}{\partial \alpha}$ is equal to
\[
\sum_{i=1}^{N} \left( \frac{\partial R_i}{\partial \alpha} + \sum_{j \neq i} \frac{\partial R_j}{\partial x_j} \frac{\partial x_j}{\partial \alpha} \right) = \sum_{i=1}^{N} \left( x_i - \frac{\sum_{j \neq i} x_j}{N-1} \right) + \sum_{i=1}^{N} \left( \alpha_i - \sum_{j \neq i} \frac{\alpha_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha}
\]

It is obvious that: $\sum_{i=1}^{N} \left( x_i - \frac{\sum_{j \neq i} x_j}{N-1} \right) = 0$

Under the assumption of homogeneous positional preferences, we have:
\[
\sum_{i=1}^{N} \left( \alpha_i - \sum_{j \neq i} \frac{\alpha_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha} = \sum_{i=1}^{N} \left( \alpha - \frac{(N-1)\alpha}{(N-1)} \right) \frac{\partial x_i}{\partial \alpha} = 0 \text{ if } \forall i,j, \alpha_i = \alpha_j = \alpha
\]

Proof of proposition 2:
Under our assumption of positional preferences (3) the positional component of social welfare variation (second part of equation (6)) is equal to :
\[
\sum_{i=1}^{N} \left( \frac{\partial R_i}{\partial \alpha_i} + \sum_{j \neq i} \frac{\partial R_j}{\partial \alpha_i} \frac{\partial x_j}{\partial \alpha_i} \right) = \sum_{i=1}^{N} \left( x_i - \frac{\sum_{j \neq i} x_j}{N-1} \right) + \sum_{i=1}^{N} \left( \alpha_i - \sum_{j \neq i} \frac{\alpha_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha}
\]

Simple arithmetic shows that the first term is equal to 0. Next parts analyze the second term.
Part (a)

In case where \( \frac{\partial x_i}{\partial \alpha} = \frac{\partial x}{\partial \alpha} \)

\[
\sum_{i=1}^{k} \left( a_i - \sum_{j \neq i, j \in [1,N]} \frac{a_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha} = \sum_{i=1}^{k} \left( \frac{\partial x_i}{\partial \alpha} - \sum_{j \neq i, j \in [1,N]} \frac{a_j}{N-1} \right) \frac{\partial x}{\partial \alpha} = \left( \sum_{i=1}^{k} a_i - \frac{\sum_{j \neq i}^{1,N} a_j}{N-1} - \frac{k}{N-1} \sum_{j=k+1}^{N} a_j \right) \frac{\partial x}{\partial \alpha}
\]

Which can be rearranged into:

\[
\sum_{i=1}^{k} \left( a_i - \sum_{j \neq i}^{1,N} \frac{a_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha} = \left( \frac{N-k}{N-1} \sum_{i=1}^{k} a_i - \frac{k}{N-1} \sum_{j=k+1}^{N} a_j \right) \frac{\partial x}{\partial \alpha}
\]

The hypothesis that \( a_i > a_j \forall i \in [1,k] \) and \( \forall j \in [k+1,N] \) can be rewrote into:

\( (N-k)k a_i > k(N-k) a_j \) \( \forall i \in [1,k] \) and \( \forall j \in [k+1,N] \)

Consequently \( (N-k) \sum_{i=1}^{k} a_i > k \sum_{j=k+1}^{N} a_j \), and \( \left( \frac{N-k}{N-1} \sum_{i=1}^{k} a_i - \frac{k}{N-1} \sum_{j=k+1}^{N} a_j \right) \frac{\partial x}{\partial \alpha} > 0 \) and \( \frac{\partial \mathcal{W}}{\partial \alpha} > 0 \)

Part (b)

Let us consider the case with heterogeneous marginal effects on individual contributions to public good \( (\forall i \neq j, \frac{\partial x_i}{\partial \alpha} \neq \frac{\partial x_j}{\partial \alpha}) \), and homogeneous positional preferences among contributors, \( (a_i = \bar{\alpha} \forall i \in [1,k]) \), and among non-contributors on the other side and \( (a_j = \underline{\alpha} (\forall j \in [k+1,N])). \) It is then easy to show that:

\[
\sum_{i=1}^{k} \left( a_i - \sum_{j \neq i}^{1,N} \frac{a_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha} = \sum_{i=1}^{k} \left( \bar{\alpha} - \sum_{j \neq i}^{1,k} \frac{a_j}{N-1} - \frac{\sum_{j=k+1}^{N} a_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha}
\]

\[
= \frac{N-k}{N-1} (\bar{\alpha} - \bar{\alpha}) \sum_{i=1}^{k} \frac{\partial x_i}{\partial \alpha}
\]

which is positive since \( \bar{\alpha} > \alpha \).

In case of heterogeneous positional preferences \( (a_i \neq a_j \forall i \neq j) \) and heterogeneous marginal effects on individual contributions to public good \( (\frac{\partial x_i}{\partial \alpha} \neq \frac{\partial x_j}{\partial \alpha} \forall i \neq j) \), the social welfare positional effect \( \sum_{i=1}^{k} \left( a_i - \sum_{j \neq i}^{1,N} \frac{a_j}{N-1} \right) \frac{\partial x_i}{\partial \alpha} \) cannot be simplified. However, knowing that \( \frac{\partial x_i}{\partial \alpha} > 0 \forall i \in [1,k] \), this sum is positive if:

\[
\frac{\partial x_i}{\partial \alpha} < \frac{\partial x_i}{\partial \alpha} \forall i, l \text{ such that } a_i - \frac{\sum_{j \neq i} a_j}{N-1} < 0 \text{ and } a_i - \frac{\sum_{j \neq i} a_j}{N-1} > 0
\]