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Abstract

In this paper, we show that *(i)* the volatility of worker flows increases with age in US CPS data, and *(ii)* a search and matching model with life-cycle features, endogenous separation and search effort, is well suited to explain this fact. With a shorter horizon on the labor market, older workers' outside options become less responsive to new employment opportunities, thereby making their wages less sensitive to the business cycle. Their job finding and separation rates are then more volatile along the business cycle. The horizon effect cannot explain the significant differences between prime-age and young workers as both age groups are far away from retirement. A lower bargaining power on the youth labor market brings the model closer to the data.

JEL Classification: E32, J11, J23

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1 Introduction

US labor market fluctuations actually hide a great deal of heterogeneity across age groups. This paper first aims at providing new empirical evidence on the business cycle behavior of unemployment and worker flows across age groups. We then propose a search and matching model with life-cycle features, endogenous search effort and separation to shed light on our empirical findings.

Average labor market flows differ by age, with a decreasing pattern in job separation and job finding rates (Elsby et al. (2010) & (2011), Gervais et al. (2012), Choi et al. (2015), Menzio et al. (2016)). However, little is known about the *cyclical behavior* of these transition rates across age groups. This study addresses this issue by documenting the patterns of volatility in job separation and job finding rates by age using monthly CPS data. Even though individuals at the beginning and at the end of the working life are fewer than prime-age workers, their age-specific business cycle behaviors are interesting to further understand the life-cycle dimensions of the labor market. The volatilities of separation and finding rates as well as unemployment display a significant age-increasing pattern: the older the worker, the more volatile the labor flows. We perform several checks to ensure that the stylized fact is robust.[1] We find that worker flows' volatility differs across age groups, which calls for further analysis of life-cycle features. In addition, we also show that the cyclicality of job separation accounts for 30 (45%) of youth (prime age and old workers') unemployment fluctuations, thereby suggesting that a relevant life-cycle model shall include endogenous separations.

To explain these facts, we consider the standard Mortensen - Pissarides (hereafter MP) model[2], augmented with life-cycle features along the lines of Cheron et al. (2013). We then face two interrelated challenges: the model must match aggregate *and* age-related

[1]As in Gomme et al. (2005) and Jaimovich & Siu (2009), our empirical study suggests that old workers' labor market fluctuations are more volatile than prime-age workers'. Gomme et al. (2005) and Jaimovich & Siu (2009) report a U-shaped pattern of fluctuations across age groups (fluctuations are highest for younger and older workers, and are lowest for middle aged workers). Our empirical evidence differs from theirs only on the youth labor market. This might be due to the difference in the data. While Gomme et al. (2005) and Jaimovich & Siu (2009) focus on employment and hours, we examine unemployment, job finding and separation. At the aggregate level, it is well known that employment is nearly as volatile as output while unemployment is several times more volatile than output. Our stylized facts underline that the discrepancy between the business cycle behavior of employment and unemployment also holds at the disaggregate level, across age groups.

[2]The MP model is the textbook model of labor economics. It was widely used in applied macroeconomics: it was first integrated into RBC models (see Merz (1994), Langot (1995), or Andolfatto (1996)) and later in New-Keynesian DSGE models (see Cheron & Langot (2000), Walsh (2005), Blanchard & Gali (2010), and Christiano et al. (2016)).

labor market volatility in a fully consistent way.[3] We first show that the interactions between endogenous search effort and endogenous separations allow the MP model to reach the aggregate objective. We then show that these amplifying mechanisms are also key to explaining the age-increasing volatility found in the data. In that sense, the aggregate and the life-cycle approaches are strongly interrelated, though the life-cycle features contribute marginally to the explanation of aggregate volatility.

Regarding the aggregate volatility issue, our results underline the complementary of two approaches already discussed separately in the literature: the ability of the endogenous search effort to magnify labor market fluctuation, as discussed in Gomme & Lkhagvasuren (2015),[4] and the necessity to introduce endogenous separation, as stressed by Fujita & Ramey (2012).[5] Moreover, we show that the complementary between search strategies of firms and workers restores the Beveridge curve in the MP model with endogenous separations.

Regarding the age-volatility issue, we focus on in this paper, any life-cycle quantitative model faces a daunting quantitative challenge as the theoretical predictions must match two types of targets: *(i)* age-specific average values of job finding, job separation and unemployment (targets based on first-order moments) and *(ii)* volatility of labor market fluctuations across age groups (targets based on second-order moments). First (*(i)*), the levels of the unemployment, job finding and job separation generated by the model must decrease with age as found in US data.[6] Second, given these parameter restrictions allowing to match the first-order moments, we show that volatilities of labor market stocks and flows are age-increasing, as in the data. Moreover, the model is able to capture the size of the cyclical fluctuations by age groups observed in the US (*(ii)*). These results rely on life-cycle mechanisms, which effects turn out to be amplified in the presence of

[3]The MP model fails to explain the high responsiveness of job finding rate to the business cycle (Shimer (2005)), volatile job separations (Fujita & Ramey (2009)) and the Beveridge curve (Fujita & Ramey (2012), Krause & Lubik (2007)).

[4]Gomme & Lkhagvasuren (2015) show that endogenous search effort exacerbates the complementarity between workers' and firms' investments in the search process. The intuitive view of a pro-cyclical search effort was first disputed by Shimer (2004), who uses an indirect measure of the search effort based on CPS data. Nevertheless, using a direct measure of the search effort based on ATUS data, Gomme & Lkhagvasuren (2015) find that search effort is strongly pro-cyclical. See Appendix C in which we argue that empirical evidence on search effort seems consistent with search effort dynamics predicted by the model.

[5]Fujita & Ramey (2009) show that changes in separations are sizeable and accounts for at least one third of unemployment fluctuations. Fujita & Ramey (2012) also stress that the Beveridge curve is lost with endogenous separation in the MP model.

[6]By matching these first order moments, our paper contributes to the literature that studies the pattern of worker flows over the life cycle: Ljungqvist & Sargent (2008), Cheron et al. (2013), Menzio et al. (2016), Kitao et al. (2016).

endogenous search effort and separation. Due to retirement, old workers expect to remain on the labor market for a very short time (the *"horizon effect"*). As a result, their current labor market status seems almost permanent: there is little room for future outside options as retirement gets closer (older workers' wage are less responsive to the business cycle), which tends to make old workers' flows very responsive to current productivity changes. In contrast, for younger workers, with a long expected working life, many future search opportunities can be seized, which tends to dampen their business cycle response to current productivity shocks. This *horizon effect* cannot explain the significant differences between volatilities of prime-age and young workers because both age groups are far away from retirement. We show that a lower bargaining power for young workers, consistent with their weaker union affiliation in US data, allows our model to replicate the volatility of their transition rates.[7] In addition to these mechanisms intrinsic to the life-cycle framework, we show that the interaction between search effort and endogenous separations help the model fit the age-pattern volatilities. In booms, younger workers are less willing to remain within the firm as outside opportunities are expanding, all the more so when workers' search effort is endogenous. This widens the volatility gap between old workers' worker flows and their younger counterparts'.

The paper is organized as follows. Section 2 documents workers' transitions rates by age group and the age-related pattern in their responsiveness to business cycles using US data. Section 3 presents the MP model with life cycle features, and then examines the theoretical age-pattern of labor market flows at the steady state and in response to productivity shocks. Section 4 applies the model to the data after calibrating its key parameters to match the level of transition rates by age. We also investigate wage fluctuations by age in Section 4.3. Section 5 concludes.

2 Labor market fluctuations by age

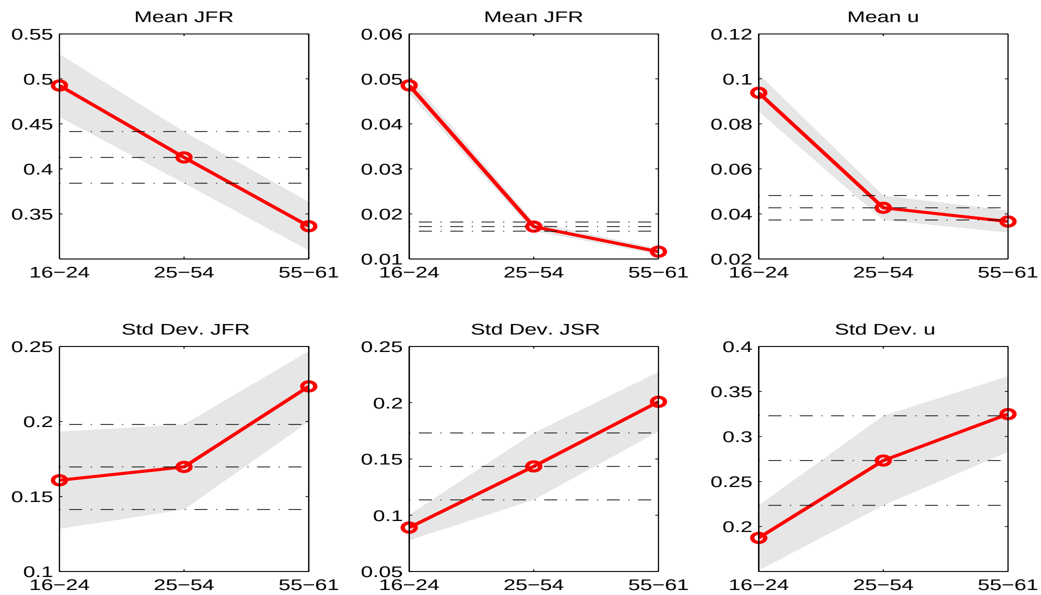
In this section, we use CPS data for the male population[8] to study the age profile of transitions for 2 states in the labor market: from employment to unemployment (job separation) and from unemployment to employment (job finding). Using monthly CPS data between January 1976 and March 2013, we follow all steps described in Shimer (2012).

[7] Hence, unlike Hall (2005) or Hall & Milgrom (2008), but in line with Pissarides (2009), our approach favors a Nash-bargained wage rather than an exogenously rigid wage that would apply in a similar fashion to all age groups.

[8] Female transitions are also linked to fertility and child rearing, which we do not model here. We check in Appendix A.4 the relevance of our stylized facts on data with male and female workers.

We compute sample-weighted gross flows between labor market states and seasonally adjusted time series using the same ratio-to-moving average technique as in Shimer (2012). We correct these for time aggregation to account for the transitions that occur within the month. We then average the time series of these instantaneous transition rates for each age group on a quarterly basis to reduce noise, which gives quarterly data about workers' instantaneous transition rates (job separation rates JSR_t and job finding rates JFR_t), and the corresponding unemployment conditional steady state $\left(u_t = \frac{JSR_t}{JSR_t+JFR_t}\right)$. In order to measure the volatility of these time series, we consider cyclical component of logged-data extracted by the HP filter with a smoothing parameter $\lambda_{HP} = 10^5$. In doing so, we follow the literature (Shimer (2005, 2012), Lise & Robin (2017)).

Figure 1: Job Separation Rate JSR, Job Finding Rate JFR, and Unemployment u by age group: Levels fall with age, volatilities increase with age



CPS quarterly averages of monthly data, Men, 1976 Q1 - 2013Q2. "JFR" Job Finding Rate. "JSR" Job Separation Rate. "u" unemployment rate. "Std Dev": Standard deviation of logged HP-filtered data with smoothing parameter 10^5. 3 age-groups (16-24, 25-54, 55-61). All moments are estimated using GMM with a weighting matrix corrected for heteroskedasticity and serial correlation using Newey & West (1987)'s method. 95% confidence band in shaded area. Horizontal lines are 95% band for prime-age workers. The comparison between the shaded area and the 2 horizontal lines allows to compare prime-age workers to younger or older workers. Authors' calculations.

We consider 3 age groups: 16-24, 25-54, and, 55-61. Since we do not consider retirement choices in the model, we discard individuals aged 62 and above.[9]

[9]In this model, we want to illustrate the effect of short distance to retirement on old workers' business cycle response. We then define old workers such that the short distance to retirement is likely to be relevant. As male retirement age peaks at age 62 in the US (Gruber & Wise (1999), Hairault & Langot (2016)), we consider male individuals aged 55-61.

Levels of inflow and outflow rates of unemployment fall with age. Figure 1 reports the mean of the time series. Like Elsby et al. (2010), we find large differences in the levels of separation rates by age group. Young workers have a separation rate 2.82 times higher than that of prime-age workers. The average job tenure during youth amounts to 21 months ($\frac{1}{1-e^{-0.049}}$) versus 91.4 months during older age ($\frac{1}{1-e^{-0.011}}$). The differences in job finding rates are less striking but significant: the length of an unemployment spell is 2.6 months ($\frac{1}{1-e^{-0.49}}$) for young workers versus 3.6 months ($\frac{1}{1-e^{-0.33}}$) for older workers.

The levels of inflow and outflow rates of unemployment fall with age. As Elsby et al. (2011) stress for UK data, with faster exits from employment and shorter unemployment spells, youth face a more fluid labor market than their older counterparts. This faster exit from employment and unemployment does not appear when one simply looks at unemployment rates across age groups. In addition, the differences in job finding rates would actually predict an age increasing profile for unemployment. Thus, Figure 1 suggests that the high level of unemployment rates for youth is actually driven by their high rates of exit from employment.[10] This is consistent with Elsby et al. (2011) and Gervais et al. (2012).[11]

Business cycle volatility increases with age. Figure 1 provides the standard deviations of logged de-trended time data. Older workers' transition rates are highly responsive to the business cycle, much more so than young and prime-age individuals. The increase in volatility in the job finding rate is weaker for young to prime-age individuals (the volatility increases from 0.16 to 0.17) than for prime-age workers to older individuals (the volatility goes up from 0.17 to 0.22). This gap is statistically significant at the 5% level only between prime-age and older workers. Regarding the job separation rates, the volatility gap between young and prime-age workers, as well as that between older and prime-age workers, is significant. The cyclical behavior of the unemployment rate is also consistently age-increasing.

Robustness. The age-increasing pattern of labor market volatility is robust when considering alternative age-groups (Appendix A.1), alternative smoothing parameter for the

[10] The unemployment steady state is consistent with the BLS unemployment rate across age groups: 11.07% for 20-24, 5.17% for 25-54, 4.12% for 55+, and 6.46% for 16+. Source: BLS monthly SA data, 1976 Jan-2013 March, Men

[11] The estimated means are consistent with the decreasing transitions with age found in the male population in Choi et al. (2015), Menzio et al. (2016), and Gervais et al. (2012). The transition rates in our data show higher levels than in their calculations because we discard labor market transitions considered in these studies (namely inactivity for Choi et al. (2015) and job-to-job transitions for Menzio et al. (2016)). Our results are not comparable to Menzio et al. (2016) because they restrict their sample to individuals with a high school degree.

HP filter (Appendix A.2). Moreover, they are also robust to the number of labor market states (Inactivity-Unemployment-Employment, Appendix A.3), gender (transitions for men and women, Appendix A.4), or method for computing transition rates (we use Elsby et al. (2010)'s publicly available data, they use the macroeconomic formula as in Shimer (2005) to compute transition rates, see Appendix A.5). We also check that our age effect is not a skill composition effect due to the higher proportion of low-skilled individuals in the population of older workers: the level and volatilities have the same age profiles within each sub-group, "High school degree and below" and "College and above" (Appendix A.6). We also check that the stylized fact holds when we look at age-specific labor market responsiveness to the business cycle using a structural VAR (Appendix A.7). We finally look at age-specific volatility of unemployment duration along the business cycle (Appendix A.8). The stylized facts remain robust: the older the worker, the more volatile the worker flows.

The contributions of age-specific fluctuations to aggregate cyclicality. In this section, we quantify the contributions of age-specific fluctuations to aggregate cyclicality using β decompositions as in Shimer (2012). In doing so, we consider the economy at the conditional steady state: unemployment inflows equal unemployment outflows, such that $\left(u_t = \frac{JSR_t}{JSR_t+JFR_t}\right)$.[12]

Transitions rates : We first consider the aggregate job finding rate and analyze whether aggregates changes are due to changes in the age-composition of the economy, or to changes in the propensity to find a job conditional on each age group. Using the log-deviation with respect to the mean, we decompose the change in the job finding probability due to changes in the age composition of unemployment β_i^u and changes due to changes in the job finding probability for each age-group β_i^{JFR}. We repeat the exercise for the aggregate separation rate, with β_i^n and β_i^{JSR}. Table 1 displays the results. As in Shimer (2012), we find that observable changes in workers' age composition explain little of the overall fluctuations in the job finding probability (β_i^u and β_i^n are not significantly different from zero). Virtually all of the change in the job finding probability is driven by the cyclicality of age-specific unemployment outflows (β_i^{JFR}). The same comment applies to job separation rate β_i^{JSR}. This calls for a further understanding of age-specific cyclicality of ins and outs of unemployment. This paper aims at filling this gap.

Table 1 suggests that the cyclicality of ins and outs of unemployment from prime-age

[12]See Appendix A.9 for further details.

Table 1: Contribution of each age group to fluctuations in aggregate job flows

	Changes in transitions rates				Changes in age composition		
Age group i	16-24	25-54	55-61		16-24	25-54	55-61
β_i^{JFR}	0.3575^{*} (a)	0.5263^{*}	0.0775^{**}	β_i^{u}	$0.1441^{\diamond}$	$-0.0957^{\diamond}$	$-0.0196^{\diamond}$
β_i^{JSR}	0.2472^{*}	0.6737^{**}	0.0935^{**}	β_i^{n}	$-0.0203^{\diamond}$	$0.0036^{\diamond}$	$0.0023^{\diamond}$
p_i	0.3918	0.5312	0.0770	p_i	0.3918	0.5312	0.0770

CPS quarterly averages of monthly logged data, Men. 1976Q1 - 2013Q1. Authors' calculations. See Appendix A.9.
β_i^{JFR} contribution of changes in Job Finding Rate of age group i in aggregate fluctuations of Job Finding Rate. Similarly for the Job Separation Rate, β_i^{JSR}.
$p_i = \frac{JFR_i(u_i/u)}{JFR}$ share of JFR of age group i in total JFR. As we consider the economy at the conditional steady state (unemployment ins equal unemployment outs), $p_i = \frac{JSR_i(n_i/n)}{JSR}$ is also the share of of JSR of age group i in total JSR.
**: Statistically larger than p_i (with a 95% confidence level). *: Statistically lower than p_i. $^{\diamond}$: Statistically equal to zero.
(a). Changes in young workers' JFR explains 35.75% of changes in aggregate JFR. This is significantly lower than p_i their relative weight in worker flows (39.18%).

workers drive more than half of aggregate fluctuations of JFR and JSR. Fluctuations on the youth labor market account for approximately a third (a fourth) of aggregate changes in JFR (in JSR, respectively). The cyclicality of old workers account for a little bit less than 10% of aggregate fluctuations of labor flows. Finally, we show that the contribution of each age group to the aggregate labor flows is not a simple reflection of their relative weight in the labour force. Young workers' weight in worker flows is p_i =39% but account for less than 39% of changes in the aggregate job finding rate. In contrast, old workers' contribution to aggregate volatility is larger than their relative weight in worker flows.

Unemployment : We then look at the contribution of age-specific transitions to aggregate unemployment fluctuations. Using a log-linear approximation of steady state unemployment for each age group, we first compute the contribution of ins and outs of unemployment of each age group to age-specific unemployment changes. We report the results in Table 2, rows 1-3. On the youth labor market, changes in job separation accounts for approximately a 29% of young workers' unemployment fluctuations. The contribution of employment exits rises to 45% for prime age and old workers. This suggests that an empirically relevant life-cycle model should include endogenous separations. As the contribution of unemployment inflows is 45% for old workers, this suggests that omitting the dynamics of job separations in a life-cycle model would provide bad predictions on the cyclicality of old workers' unemployment. Row 4 of Table 2 provides the contribution of changes in age-specific transition rates to the volatility of aggregate unemployment. The cyclicality of prime-age workers' JFR alone accounts for 36.54% of fluctuations of aggregate unemployment. Old workers' contribution to aggregate unemployment volatility is approximately 11% (5.58% from changes in JSR and 5.97% from JFR), while youth worker flows account for 27% of changes in aggregate unemployment.

Our empirical exercise shows that life cycle features are worth investigating as young and

Table 2: Variance decomposition of unemployment fluctuations

	β_Y^{JSR}	β_Y^{JFR}	β_A^{JSR}	β_A^{JFR}	β_O^{JSR}	β_O^{JFR}
1. u_Y	0.2943 (a)	0.7057				
2. u_A			0.4497	0.5503		
3. u_O					0.4580	0.5420
4. u	0.0731 (b)	0.1970	0.2442	0.3654	0.0558	0.0597

Subscript Y,A,O relates to young, prime-age and old workers, respectively. For instance, u_Y denotes youth unemployment rate, u_A prime-age workers' unemployment rate, u_O old workers' unemployment rate. u is the aggregate unemployment rate "JFR" Job Finding Rate. "JSR" Job Separation Rate. β relates to variance decomposition. Example : β_Y^{JSR} is the contribution of changes in young workers' Job Separation Rate to labor market fluctuations.
(a) Changes in young workers' JFR accounts for 29.43% of youth unemployment fluctuations.
(b) Changes in young workers' JFR accounts for 7.31% of aggregate unemployment fluctuations.

old workers' labor market fluctuations differ from prime age workers'. In addition, a model focusing on prime age workers' fluctuations would capture approximately 60% of aggregate fluctuations. We show that the contribution of youth labor market to aggregate changes hovers around 30%, with 10% for old workers' contribution. Old workers' contribution to aggregate fluctuations seem quantitatively small. However, we argue that old workers' fluctuations are still worth investigating as they provide an interesting opportunity to assess the relevance of the short distance to retirement, thereby providing an additional opportunity to further test the DMP model.

3 A life-cycle matching model with aggregate uncertainty

In this section, we extend Mortensen & Pissarides (1994)'s model that introduces endogenous search effort and separation with aggregate shocks in a framework with life cycle features. We model the life cycle as stochastic aging, as in Castañeda et al. (2003), Ljungqvist & Sargent (2008), and Hairault et al. (2010). Unlike Cheron et al. (2013), we consider *(i)* aggregate shocks, as in Fujita & Ramey (2012), and *(ii)* age-directed search, as in Menzio et al. (2016).[13] With age-directed search, there is no externality due to workers' heterogeneity in the matching function. Following Bagger et al. (2014) and Menzio et al. (2016), we also consider human capital accumulation through a deterministic exogenous process. This allows the model to capture the evolution of wage over the life-cycle.

[13]For simplicity, we discard job-to-job transitions (as in Fujita & Ramey (2012)) and savings (as in Lise (2013)). These extensions are left for future research.

3.1 Demographic setting and aggregate shock

The Life Cycle. Consistent with our empirical results, we consider three age groups, i, which is enough to describe the working-life cycle: young Y, prime-age A, and older workers O. All young workers Y enter the labor market as unemployed workers. We assume stochastic aging. The probability of remaining a prime-age (young) worker in the next period is π_A (π_Y). Conversely, the probability of becoming older (prime-age) is $1-\pi_A$ ($1-\pi_Y$). To account for the non-linearity in the horizon effect, the period as older workers is divided into N years: $O = \{O_i\}_{i=T-N}^{T}$, with π_{O_i} the probability of remaining in age group O_i next period.[14] With probability $1-\pi_{O_T}$, older workers reach the exogenous retirement age of $T+1$. To maintain a constant population size, we assume that the number of exiting workers is replaced by an equal number of young workers.

More formally, we assume for simplicity that the matrix Π governing the age Markov-process is:

$$\Pi = \begin{bmatrix} \pi_Y & 1-\pi_Y & 0 & 0 & \cdots & 0 \\ 0 & \pi_A & 1-\pi_A & 0 & \cdots & 0 \\ 0 & 0 & \pi_{O_{T-N}} & 1-\pi_{O_{T-N}} & \cdots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ 1-\pi_{O_T} & 0 & 0 & 0 & \cdots & \pi_{O_T} \end{bmatrix}$$

We deduct the size of each group from Π_∞, the matrix of the unconditional probabilities, given that the total size of the population is normalized to unity. We divide the population of each group into two types of agents: unemployed u_i and employed n_i, such that $m_i = u_i + n_i$, with $1 = \sum_i m_i$. We thereby discard the participation margin. In our view, this is not a very restrictive assumption because we introduce an age-specific search effort that can converge towards zero at the end of working life (before retirement). These older unemployed workers with a zero-search can thus be considered as non-participants. Their number is endogenously determined at equilibrium.

Shocks. A worker-firm match is characterized by the aggregate z and the match-specific ϵ productivity factors. We assume that the aggregate productivity component follows the

[14] Our paper analyzes the effect of the short-distance to retirement on labor market fluctuations. The question is: at which age does the old workers' expected surplus start falling due to the short distance to retirement? At which age is an old worker considered as "close to retirement"? This is an endogenous outcome in the model. In addition, as discounting is exponential, the effect of the short distance to retirement appears in a non-linear fashion in our model. By considering this demographic structure for old workers, we let the model endogenously respond in a non-linear fashion at the end of the working life. We then aggregate all old workers to fit the age group of 55-61 as in the data.

exogenous process:

$$\log(z') = \rho \log(z) + \nu' \quad (1)$$

where ν' is an i.i.d. normal disturbance with mean zero and standard deviation σ_ν.

For a common aggregate component of productivity z, idiosyncratic productivity shocks hit jobs at random. At the end of each period t, there is a new productivity level for period $t+1$ drawn with probability $\lambda_i \leq 1$ in the distribution $G(\epsilon)$, with $\epsilon \in [0,1]$. The higher λ_i, the lower the persistence of the current productivity draw. The probability of drawing a new match-specific productivity may be specific to age i. Firms decide to discard any job whose productivity is below an idiosyncratic productivity threshold (the reservation productivity) denoted by $R_i(z)$. Unlike in MP, new jobs are not opened at the highest productivity: their productivity level is also drawn in the distribution $G(\epsilon)$.[15] Age-$(i-1)$ workers become age-i workers (with probability $1-\pi_{i-1}$), and, if contacted at the age $i-1$, will be hired if and only if their productivity is above the threshold $R_i(z)$, i.e., the reservation productivity of an age-i worker, because their productivity value is revealed after the firm has met the worker.

Finally, we account for human capital accumulation to mimic the observed pattern of individual wage earnings over the life cycle. Human capital can be considered general (related to experience) or specific (related to tenure).[16] In the following, we assume that h_i denoting the human capital at age i, is a general human capital that is transferable (it can be used in all jobs and in home production): individuals accumulate this capital inside and outside firms. We assume that every age-group is associated with a particular level of human capital h_i, with $h_i < h_{i+1}$. The productivity of the job is then $z\epsilon h_i$, and the instantaneous opportunity cost of employment $b_i = bh_i$. Hence, this age-increasing component of the individual productivity allow older workers to accept lower job-specific productivity ϵ.[17]

[15] This assumption is a particular case of Nagypal & Mortensen (2007*b*)'s framework, where the initial value of the idiosyncratic productivity is drawn from a distribution $\widetilde{G}(\epsilon)$ with $G \neq \widetilde{G}$. With our assumption, whether the worker is in the firm or in the pool of job seekers, their future opportunities are easily comparable, either through unemployment search $\gamma e_i p(\theta_i)$ or through labor hoarding $(1-s_e)\lambda_i$ (see Section 4.1.). Note that adopting this assumption in the MP model would give a comparative advantage to job seekers relative to workers within the firms, which is not realistic.

[16] In the *Mincerian* wage equations, these two components explain the increase in wage earnings during the workers' life cycle.

[17] These simplifying assumptions are made for model tractability. Indeed, there is no depreciation in worker's human capital during unemployment spells, as in Ljungqvist & Sargent (2008) or stagnation of this accumulation process during unemployment spells, as in Bagger et al. (2014). Nevertheless, given the short-time duration of an unemployment spell in the US, this seems to be a reasonable approximation.

Matching with directed search. We consider an economy in which labor market frictions imply a costly delay in the process of filling vacancies. Age is perfectly observed and a worker who applies to a job not matching the age-characteristic will have a nil production, and thus a nil surplus. Firms choose how many and what type of vacancies to open. The type of vacancy is simply defined by a worker's age. Since search is directed, the probability that a worker meets a firm depends on her age.

Since firms can *ex-ante* age-direct their search, there is one matching function by age. Let $v_i(z)$ be the number of vacancies, $u_i(z)$ the number of unemployed workers, and $e_i(z)$ the endogenous search effort for a worker of age i. The matching function gives the number of contacts, $M(v_i(z), e_i(z)u_i(z))$, where M is increasing and concave in both arguments and with constant returns-to-scale. From the firm's perspective, the contact probability is $q(\theta_i(z)) \equiv \frac{M(v_i(z),e_i(z)u_i(z))}{v_i(z)} = M(1, \theta_i^{-1}(z))$ with $\theta_i(z) = \frac{v_i(z)}{e_i(z)u_i(z)}$ as the corresponding labor market tightness. The probability for unemployed workers of age i to be employed is then defined by $e_i(z)p(\theta_i(z))[1 - G(R_i(z))]$ with $p(\theta_i(z)) \equiv \frac{M(v_i(z),e_i(z)u_i(z))}{e_i(z)u_i(z)} = M(\theta_i(z), 1)$ as the contact probability of the effective unemployed worker. Note that the hiring process is then age-differentiated via a firms' age-specific search intensity ($v_i(z)$), an age-specific reservation productivity ($R_i(z)$), and an age-specific search effort from unemployed workers ($e_i(z)$).

3.2 Firms' and workers' intertemporal values

Firms' problem. Any firm is free to open a job vacancy directed at an age-specific labor market and engage in hiring. c denotes the flow cost of hiring a worker and $\beta \in [0, 1]$ the discount factor. Let $V_i(z)$ be the expected value of a vacant position in the age-i labor market, given the aggregate state of the economy z at time t, and $J_i(z, \epsilon)$ the value of a job filled by a worker of age i with productivity ϵ in aggregate state z. The firm's search value is given by:

$$V_i(z) = -c + q(\theta_i(z))\beta E_z \left[\pi_i \int_0^1 J_i(z', x)dG(x) + (1 - \pi_i)V_i(z')\right] + (1 - q(\theta_i(z)))\beta E_z V_i(z')$$

where the operator E_z denotes the expectation with respect to aggregate productivity z. Given that search is directed, if the worker ages between the meeting and the production processes (with a probability $1 - \pi_i$), the job is not filled. We will assume hereafter the standard free-entry condition, i.e., $V_i(z) = 0$, $\forall i, z$, which leads to:

$$\frac{c}{q(\theta_i(z))} = \beta \pi_i E_z \int_0^1 J_i(z', x) dG(x)$$

Vacancies are determined according to the expected value of a contact with an age-i unemployed worker, which depends on the uncertainty in the hiring process arising from the two components of productivity, z and ϵ.

Given a state vector (z, ϵ) and for a bargained wage $w_i(z, \epsilon)$, the expected value $J_i(z, \epsilon)$ of a filled job by a worker of age i, $\forall i \in \{Y, ..., O_{T-1}\}$, is defined by:

$$J_i(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} z\epsilon h_i - w_i(z,\epsilon) \\ +\beta\pi_i(1-s_e) \left(\begin{array}{c} \lambda_i E_z \int_0^1 J_i(z',x)dG(x) \\ +(1-\lambda_i)E_z J_i(z',\epsilon) \end{array} \right) \\ +\beta(1-\pi_i)(1-s_e) \left(\begin{array}{c} \lambda_{i+1} E_z \int_0^1 J_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z J_{i+1}(z',\epsilon) \end{array} \right) \end{array} ; 0 \right\}$$

where s_e is the exogenous separation rate. Notice that, for $i = O_T$, aging implies retirement. The value function becomes:

$$J_{O_T}(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} z\epsilon h_{O_T} - w_{O_T}(z,\epsilon) \\ +\beta\pi_{O_T}(1-s_e) \left(\begin{array}{c} \lambda_{O_T} E_z \int_0^1 J_{O_T}(z',x)dG(x) \\ +(1-\lambda_{O_T})E_z J_{O_T}(z',\epsilon) \end{array} \right) \end{array} ; 0 \right\}$$

The short horizon reduces the value of a filled job for a given wage.

Workers' problem. Values for employed (on a match of productivity ϵ) and unemployed workers of any age $i \neq O_T$, are respectively given by:

$$\mathcal{W}_i(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} w_i(z,\epsilon) \\ +\beta\pi_i \left[\begin{array}{l} (1-s_e) \left(\begin{array}{c} \lambda_i E_z \int_0^1 \mathcal{W}_i(z',x)dG(x) \\ +(1-\lambda_i)E_z \mathcal{W}_i(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_i(z') \end{array} \right] \\ +\beta(1-\pi_i) \left[\begin{array}{l} (1-s_e) \left(\begin{array}{c} \lambda_{i+1} E_z \int_0^1 \mathcal{W}_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z \mathcal{W}_{i+1}(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_{i+1}(z') \end{array} \right] \end{array} ; \mathcal{U}_i(z) \right\}$$

$$
\mathcal{U}_i(z) = \max_{e_i(z)} \left\{ \begin{array}{l} bh_i - \phi(e_i(z)) \\ +\beta\pi_i \left(\begin{array}{l} e_i(z)p(\theta_i(z))E_z \int_0^1 \mathcal{W}_i(z', x)dG(x) \\ +(1 - e_i(z)p(\theta_i(z)))E_z\mathcal{U}_i(z') \end{array} \right) \\ +\beta(1-\pi_i) \left(\begin{array}{l} e_{i+1}(z)p(\theta_{i+1}(z))E_z \int_0^1 \mathcal{W}_{i+1}(z', x)dG(x) \\ +(1 - e_{i+1}(z)p(\theta_{i+1}(z)))E_z\mathcal{U}_{i+1}(z') \end{array} \right) \end{array} \right\}
$$

with $bh_i \geq 0$ denoting the instantaneous opportunity cost of employment indexed on human capital h_i and $\phi(.)$ The convex function capturing the disutility of search effort is e_i. For $i = O_T$, these values are simply given by:

$$
\mathcal{W}_{O_T}(z, \epsilon) = \max \left\{ \begin{array}{l} w_{O_T}(z, \epsilon) \\ +\beta\pi_{O_T} \left[\begin{array}{l} (1 - s_e) \left(\begin{array}{l} \lambda_{O_T} E_z \int_0^1 \mathcal{W}_{O_T}(z', x)dG(x) \\ +(1 - \lambda_{O_T})E_z\mathcal{W}_{O_T}(z', \epsilon) \end{array} \right) \\ +s_e E_z\mathcal{U}_{O_T}(z') \end{array} \right] \end{array} ; \mathcal{U}_{O_T}(z) \right\}
$$

$$
\mathcal{U}_{O_T}(z) = \max_{e_{O_T}(z)} \left\{ \begin{array}{l} bh_{O_T} - \phi(e_{O_T}(z)) \\ +\beta\pi_{O_T} \left(\begin{array}{l} e_{O_T}(z)p(\theta_{O_T}(z))E_z \int_0^1 \mathcal{W}_{O_T}(z', x)dG(x) \\ +(1 - e_{O_T}(z)p(\theta_{O_T}(z)))E_z\mathcal{U}_{O_T}(z') \end{array} \right) \end{array} \right\}
$$

The worker's optimal search effort decision then satisfies the following condition:

$$
\phi'(e_i(z)) = \beta\pi_i p(\theta_i(z))E_z \left[\int_0^1 \mathcal{W}_i(z', x)dG(x) - \mathcal{U}_i(z') \right]
$$

The marginal cost of the search effort at age i is equal to its expected marginal return.

3.3 Job surplus, Nash sharing rule, and reservation productivity

The surplus $S_i(z, \epsilon)$ generated by a job of productivity $z\epsilon$ is the sum of the worker's and the firm's surplus $S_i(z, \epsilon) \equiv \mathcal{W}_i(z, \epsilon) - \mathcal{U}_i(z) + J_i(z, \epsilon)$ given that $V_i(z) = 0$ at equilibrium. Thus, using the definitions of $J_i(z, \epsilon)$, $\mathcal{W}_i(z, \epsilon)$, and $\mathcal{U}_i(z)$, the surplus is given by:

$$
S_i(z, \epsilon) = \max \left\{ \begin{array}{l} z\epsilon h_i - bh_i + \phi(e_i(z)) \\ +\beta\pi_i(1 - s_e) \left(\begin{array}{l} \left[\lambda_i - \frac{\gamma e_i(z)p(\theta_i(z))}{1-s_e}\right] E_z \int_0^1 S_i(z', x)dG(x) \\ +(1 - \lambda_i)E_z S_i(z', \epsilon) \end{array} \right) \\ +\beta(1-\pi_i)(1 - s_e) \left(\begin{array}{l} \left[\lambda_{i+1} - \frac{\gamma e_{i+1}(z)p(\theta_{i+1}(z))}{1-s_e}\right] E_z \int_0^1 S_{i+1}(z', x)dG(x) \\ +(1 - \lambda_{i+1})E_z S_{i+1}(z', \epsilon) \end{array} \right) \end{array} ; 0 \right\}
$$

Reservation productivity $R_i(z)$ such that $S_i(z, R_i(z)) = 0$. As in MP, a crucial implication of this rule is that job destruction is mutually optimal for the firm and the worker. $S_i(z, R_i(z)) = 0$ indeed entails $J_i(z, R_i(z)) = 0$ and $\mathcal{W}_i(z, R_i(z)) = \mathcal{U}_i(z)$. Note that the lower bound of any integral over $S_i(z, \epsilon)$ is actually the reservation productivity because any productivity level below $R_i(z)$ yields a negative job surplus. Given $S_i(z, \epsilon)$, the Nash bargaining leads to $\mathcal{W}_i(z, \epsilon) - \mathcal{U}_i(z) = \gamma_i S_i(z, \epsilon)$ and $J_i(z, \epsilon) = (1 - \gamma_i) S_i(z, \epsilon)$, where worker's bargaining power γ_i may be specific to age i. Using this sharing and value functions, the wage rule is:

$$w_i(z, \epsilon) = \gamma_i \left(z\epsilon h_i + ce_i(z)\theta_i(z) + \frac{1 - \pi_i}{\pi_{i+1}} \frac{\gamma_{i+1}}{\gamma_i} ce_{i+1}(z)\theta_{i+1}(z) \right) + (1 - \gamma_i)\left(bh_i - \phi(e_i(z))\right)$$

Because workers age, the returns on search activity is an average between age i and $i + 1$.

3.4 Equilibrium

Definition 1. *The labor market equilibrium with directed search in a finite-horizon environment is defined by labor market tightness, search effort and separation rule (reservation productivity), respectively,* $\theta_i(z)$, $e_i(z)$, *and* $R_i(z)$*:*

$$\frac{c}{q(\theta_i(z))} = (1 - \gamma_i)\beta\pi_i E_z \overline{S}_i(z') \tag{2}$$

$$\phi'(e_i(z)) = \gamma_i p(\theta_i(z))\beta\pi_i E_z \overline{S}_i(z') \tag{3}$$

$$zh_i R_i(z) = bh_i + \Sigma_i(z) - \Lambda_i(z) - \Gamma_i(z) \tag{4}$$

given average and individual surpluses:

$$\overline{S}_i(z') \equiv \int_{R_i(z')}^{1} S_i(z', x) dG(x) \tag{5}$$

$$S_i(z, \epsilon) = \max \left\{ \begin{array}{l} zh_i(\epsilon - R_i(z)) \\ +\beta\pi_i(1 - \lambda_i)(1 - s_e)E_z[S_i(z', \epsilon) - S_i(z', R_i(z))] \\ +\beta(1 - \pi_i)(1 - \lambda_i)(1 - s_e)E_z[S_{i+1}(z', \epsilon) - S_{i+1}(z', R_i(z))] \end{array} ; 0 \right\} \tag{6}$$

where "search value", "labor hoarding value" and "continuation value" are respectively defined as follows:

$$\Sigma_i(z) = -\phi(e_i(z)) + \beta \left[\begin{array}{l} \pi_i \gamma_i e_i(z) p(\theta_i(z)) E_z \overline{S}_i(z') \\ +(1-\pi_i)\gamma_{i+1} e_{i+1}(z) p(\theta_{i+1}(z)) E_z \overline{S}_{i+1}(z') \end{array} \right] \tag{7}$$

$$\Lambda_i(z) = \beta(1-s_e)\left[\pi_i \lambda_i E_z \overline{S}_i(z') + (1-\pi_i)\lambda_{i+1} E_z \overline{S}_{i+1}(z')\right] \tag{8}$$

$$\Gamma_i(z) = \beta(1-s_e)\left[\begin{array}{l} \pi_i(1-\lambda_i) E_z S_i(z', R_i(z)) \\ +(1-\pi_i)(1-\lambda_{i+1}) E_z S_{i+1}(z', R_i(z)) \end{array} \right] \tag{9}$$

The stock-flow dynamics on the labor market are described by equations (27), (28), (29), and (30) in Appendix E.4, whereas the law of motion of aggregate shock z is driven by equation (1).

As in Menzio & Shi (2010), directed search implies that the problem is block-recursive[18]: after solving the dynamics of the forward variables, we deduce the dynamics of the backward variables ((un)-employment rates) as well as the dynamics of the job distribution.

3.5 Identification of model parameters using age heterogeneity

We derive the restrictions allowing the model to fit the first-order moments of the data by age (the levels, in section 3.5.1), and deduce the implications of these restrictions on the second-order moments by age (volatilities, in section 3.5.2). In Appendix D, we propose a stylized version of our model, in continuous time, which allows to recovers familiar expressions of the labor market elasticities based on comparative statics of the steady state.[19]

[18]See proposition 8 in Appendix E.2.

[19]In Appendix D, we first show that the interaction between search effort and endogenous separations can solve the Shimer (2005) puzzle. In particular, the amplifying mechanism through the endogenous search effort, discussed in Gomme & Lkhagvasuren (2015), is presented as a special case of our model without age structure and without endogenous separations. We then mimic Nagypal & Mortensen (2007*a*)'s analysis (another special case of our model without age heterogeneity and without endogenous search effort) in order to show that the combination of endogenous separations and search effort allows the MP model to generate a Beveridge curve. When the age structure is introduced, we show that a short horizon distance to retirement affects the labor market elasticities with respect to aggregate productivity, as if the interest rate were augmented with the risk of workers' retirement. Notice that the finite horizon on the labor market implies that this risk becomes infinite when the worker reaches the retirement age (increasing risk with the worker's age). For the youth labor market, we discuss the effects of changes in the worker's bargaining power on the labor market elasticities, in the spirit of Hagedorn & Manovskii (2008)'s work.

3.5.1 Steady state properties: why do levels of labor flows fall with age?

At the steady state,[20] the model must generate an age-pattern of transition rates such that, for age group i:

$$JSR_i \approx s_e + (1 - s_e)\lambda_i G(R_i) > JSR_{i+1} \tag{10}$$

$$JFR_i \approx e_i p(\theta_i)[1 - G(R_i)] > JFR_{i+1} \tag{11}$$

These age pattern in the finding and separation rates is consistent with the evidence found in US data if $JFR_i > JFR_{i+1}$ and $JSR_i > JSR_{i+1}$ (see Figure 1, top panels).[21]

Job separation rate falls with age. Equation (10) suggests that the job separation rate is driven by the reservation productivity by age (R_i). Reservation productivity differs across age groups because workers differ in terms of expected time on the labor market. Intuitively, the economic mechanism is the following: with $R_i > R_{i+1}$, older workers are less selective than younger workers when new opportunities are available. A shorter horizon leads old workers to accept lower and lower job opportunities because they know that the number of draws before retirement is falling. We refer to this effect as a "selection effect". This can be seen in the model equations. For workers close to retirement, only current surplus matters. Equation (4) shows that reservation productivity converges to the unemployed worker's current surplus bh_i as the worker ages, i.e. when the expected gains on labor market, contingent to their future status (Σ_i, Λ_i and Γ_i) tend towards zero[22]. In contrast, prime-age workers have a long work-life expectancy: the expected gains are larger than zero. Equation (4) shows that, if the return on search is larger than the one on labor hoarding, i.e. if $\Sigma_i > \Lambda_i$,[23] then $R_i > h_i b$. For prime-age workers, the larger value of their unemployment option raises their wages and thus the reservation productivity of their jobs. This leads to an age-decreasing pattern of job separation.[24]

[20] For the sake of brevity, we consider $z = 1$ in the steady state analysis.

[21] Cheron et al. (2013) analyze all the other cases: the age-increasing reservation productivity case and the U-shaped pattern of the reservation productivity. Given that US data are not in line with these two last cases, we restrict our analysis to the case where the steady state of the model matches the long run values of JFR and JSR.

[22] The retirement age is a terminal condition and acts as if the discount factor β were tending towards 0 when worker gets closer to retirement.

[23] We show in proposition 9 in Appendix E.3 that this restriction is equivalent in our case to $\gamma_i e_i p(\theta_i) > (1 - s_e)\lambda_i$

[24] The separation rate also depends on the exogenous probability λ_i. Hereafter, we restrict our analysis to a sufficiently flat age profile for this exogenous variable to ensure that our results are not exogenously determined by the calibration of λ at the end of the working life. Were the growth of this probability highly tilted over the life cycle, the age-pattern of R could have been dominant in shaping the separation

Both Σ_i and Λ_i depend on the age-specific average surplus ($\overline{S}_i$), they are actually different as the value of search Σ_i can be manipulated by agents through their choices for $\{e_i, \theta_i\}$, whereas labor hoarding Λ_i depends on an exogenous probability λ_i: in our paper, the younger the worker, the lower λ_i, the larger the incentive to invest in search on the labor market because a longer horizon allows them to recoup search costs, and thus the larger the gap between the search value and the value of labor hoarding.

The job finding rate falls with age. Equation (11) shows that the job finding rate depends on search efforts by unemployed workers e_i and firms θ_i, but also on the reservation productivity R_i. We have just discussed the "selection effect": a shorter horizon leads old workers to accept lower and lower job opportunities because they know that the number of draws before retirement is falling. This effect tends to increase the job finding rate as the worker ages, which is counterfactual. In order for the model to be consistent with the age-decreasing pattern of the job finding rate, it must be the case that the age profile of $\{e_i, \theta_i\}$ is prevalent and leads to a decline in the job finding rate with age. The decline of $\{e_i, \theta_i\}$ with age must be induced by the fall in the average surplus with age ($\overline{S}_i$).[25] Hence, the crucial point for the age profile of $\{e_i, \theta_i\}$ is to generate an age-decreasing pattern in $\overline{S}_i$. This is the case for the "horizon effect" because a shorter horizon prior to retirement causes the match-specific surplus to decline with worker's age: the gains from the job are capitalized on a duration that falls when worker ages. The falling average expected surplus ($\overline{S}_i > \overline{S}_{i+1}$) implies that the "horizon effect" dominates the "selection effect," leading search effort (e_i and θ_i) to decline with age. Note that the horizon effect can be offset by a large increase in human capital at the end of the working life. Its growth rate, $\delta_i = (h_{i+1} - h_i)/h_i$, is calibrated to replicate the observed wage age-profile.[26]

3.5.2 Cyclical properties: why do volatilities of labor flows increase with age?

Are the conditions that imply an age-decreasing pattern for the levels of the transition rates compatible with the fact that older workers' flows are more responsive to the business

rate. We exclude this case *a priori*, and we check that our calibration is consistent with this restriction.

[25] Equation (2) provides the link between firms' search effort θ_i and $E_z\overline{S}_i(z')$, whereas the combination of equations (2) and (3) shows that unemployed workers' search effort can be expressed as a function of only θ_i, and thus depends only on $E_z\overline{S}_i(z')$.

[26] In proposition 10 of Appendix E.3, we derive the conditions under which the horizon effect dominates the selection effect and show analytically that the horizon effect dominates the selection effect if δ_i is below a threshold value. Our quantitative results show that, with the calibrated δ_i, using US data, this condition holds.

cycle than for their younger counterparts? It can be shown that the restrictions for which the model can reproduce the age-profile of worker transitions by age group *at the steady state* ensure that the age-pattern of their *volatility* is also matched.[27]

Why a more volatile JFR for older workers? In the MP model, in order to understand the response of labor market tightness with respect to aggregate productivity, one needs to look at fluctuations in the value of a filled vacancy (as the free entry condition provides a direct link between labor market tightness θ and job value J.[28]). Log-linear approximation of the job creation condition leads to

$$\widehat{J}_i = \frac{zh_iX(R_i)}{zh_iX(R_i) - (bh_i + \Sigma_i)(1 - G(R_i))}\widehat{z} - \frac{\Sigma_i(1 - G(R_i))}{zh_iX(R_i) - (bh_i + \Sigma_i)(1 - G(R_i))}\widehat{\Sigma}_i \quad (12)$$

$$\underset{\Sigma_i \to 0}{\longrightarrow} \widehat{J}_O = \frac{zh_OX(R_O)}{zh_OX(R_O) - bh_O(1 - G(R_O))}\widehat{z} \quad (13)$$

where hat variables denote log-deviation from the steady state, $J_i = \int_{R_i}^1 J_i(x)dG(x)$ and $X(R_i) = \int_{R_i}^1 xdG(x)$ are respectively the average job values for age-i worker and their average productivity. Equation (12) illustrates previous results found in the literature. Shimer (2005) finds a large wage elasticity to the aggregate shock z. Following a positive productivity shock ($\widehat{z} > 0$), the increase in wages leaves profits nearly unchanged. As a result, firms' hiring incentives, captured by $\widehat{J}$, does not respond much to the business cycle. This can be seen in equation (12), the pro-cyclical wage response is due to improved labor market opportunities for unemployed workers ($\widehat{\Sigma}_i > 0$), which tends to dampen the response of firms' hiring incentives ($\widehat{J} > 0$ but small). Hagedorn & Manovskii (2008) match the volatility of market tightness after focusing on the size of the percentage changes of profits in response to changes in productivity. They argue that these percentage changes are large if the size of profits is small and the increase in productivity is not fully absorbed by an increase in wages. This leads them to consider a large firm's bargaining power (small γ) and high unemployment benefit b. Indeed, in equation (12), a high unemployment benefit b tends to increase the response of firms' hiring incentives to the aggregate shock (the coefficient in front of $\widehat{z}$ goes up). In addition, with Nash bargaining, search opportunities on the labor market expand in booms, which drives wages

[27] In Appendix E.5, we analytically derive the business cycle elasticity by age and show that older worker's responsiveness to the aggregate shock is larger than their younger counterparts' if the restrictions at the steady state (Propositions 9 and 10) are satisfied.

[28] J refers to the relevant hiring incentive upon entry (the firm still does not know the match-productivity draw). J is the expected value of a filled vacancy, with expectations with respect to micro-economic match-productivity draw.

upward. Under Hagedorn & Manovskii (2008)'s calibration, with workers' low bargaining power, $\widehat{\Sigma}_i$ disappears from equation (12),[29] thereby making hirings more responsive to the business cycle.

In our paper, we consider business cycle response by age, without using Hagedorn & Manovskii (2008)'s calibration. Old workers' search value Σ converges to zero ("horizon effect" implies $\Sigma_i \to 0$). This leads the fluctuations of job value to depend only on productivity shock (equation (13)). In contrast, for young and prime-age workers, the search value is positive in economic boom and thus the impact of the productivity shock is dampened by the pro-cyclical response of the search value to the business cycle shocks. With Nash bargaining, wages react not only to productivity changes, but also to fluctuations of outside opportunities. The higher the wage adjustment, the lower the volatility in labor market. As generations of workers differ according to their sensitivity to future opportunities, they differ in terms of wage adjustments. Specifically, this implies that older workers have less pro-cyclical individual wages[30], and thus higher volatility in labor market tightness, which in turn also makes the search effort more responsive to the business cycle. In contrast, as younger workers are more responsive to outside options, their wages are more pro-cyclical. This dampens the firm's incentives to post vacancies directed to these younger workers. The age-varying influence of the outside options in the Nash wage bargaining is then key to explain the age-pattern of the transition rates' volatility.

Why a more volatile JSR for older workers? Log-linear approximation of the job destruction condition leads to

$$\widehat{R}_i = -\frac{bh_i + \Sigma_i}{bh_i + \Sigma_i + \Gamma_i}\widehat{z} + \frac{\Sigma_i}{bh_i + \Sigma_i + \Gamma_i}\widehat{\Sigma}_i - \frac{\Lambda_i}{bh_i + \Sigma_i + \Gamma_i}\widehat{\Lambda}_i \quad \underset{\Sigma_i,\Lambda_i \to 0}{\longrightarrow} \widehat{R}_O = -\widehat{z} \tag{14}$$

Equation (14) summarizes the business cycle response of job destruction. As in Mortensen & Pissarides (1994), following an increase in aggregate productivity ($\widehat{z} > 0$), reservation productivity falls ($\widehat{R}_i < 0$): firms want to keep more workers. $\widehat{\Sigma}_i$ captures changes in opportunity cost of employment for the worker. In booms, the increase in expected gains from search on the labor market ($\widehat{\Sigma}_i > 0$) makes workers less willing to remain within

[29] In Appendix E.5., equation (31) shows that $\Sigma_i = 0$ when worker's bargaining power γ_i tends to zero in our model.

[30] This low elasticity of individual wages when workers age is not easy to measure in the data. Indeed, data provides information about aggregate wages for each age group. However, theory predicts that, *(i)* within each age class, there is substantial productivity heterogeneity and *(ii)* productivity distribution changes along the business cycle. Hence, the dynamics of aggregate wage per age do not provide sufficient information about the dynamics of individual wages. See Section 4.3 and Appendix B.1.

the firm. This tends to raise the reservation productivity in booms ($\widehat{R}_i > 0$). The third term in equation (14) relates to changes in labor hoarding. $\widehat{\Lambda}_i$ measures the extent to which the employer is willing to incur a loss now in anticipation of a future improvement in the value of the match's product. It is the option value of retaining an existing match. In boom, firms keep more workers, rather than waiting for new workers to arrive from the matching market. The value of labor hoarding increases in booms ($\widehat{\Lambda}_i > 0$). Given that the steady state restrictions allowing to match the age pattern of transition rates are such that the search value dominates the labor hoarding value ($\Sigma_i > \Lambda_i$), we also have $\widehat{\Sigma}_i > \widehat{\Lambda}_i$. Therefore, the impact of an aggregate shock on the reservation productivity is dampened by pro-cyclical changes in the search value for young and prime age workers. In contrast, for older workers, with a shorter horizon on the labor market, the "search value" and "labor hoarding" are close to zero: this leads to the high response of reservation productivity to current aggregate shocks.

4 Quantitative Analysis

In this section, we apply the model to the data. The model is calibrated to match the first-order moments found in the data (section 4.1). Under this calibration, we assess the model's ability to generate second-order moments consistent with aggregate data and stylized facts by age. This quantitative analysis aims to demonstrate that *(i)* the parameter restrictions imposed to match the first-order moments are sufficient to generate the age-increasing volatilities observed in the data (Sections 4.2.1 and 4.2.2), *(ii)* the size of volatilities of labor market aggregates (flows and stocks) are well matched (Section 4.2.3) and *(iii)* the interaction between search effort and endogenous separations is key for the replication of volatility across age groups as well as the size of aggregate labor market volatility (Section 4.2.4). In Section 4.3, we assess the model's fit with respect to wage fluctuations by age.

4.1 Calibration

The vector of parameters is $\Phi = \{\Phi_1, \Phi_2\}$ with $dim(\Phi) = 48$. Functional forms for the matching process and search costs are respectively

$$M(v_i, e_i u_i) \quad = \quad H v_i^{1-\eta}(e_i u_i)^{\eta} \quad \text{and} \quad \phi(e_i) = \chi \frac{e_i^{1+\phi}}{1+\phi}$$

with $0 < \eta < 1$ and $\phi > 0$. All parameters calibrated using external information are:

$$\Phi_1 = \left\{\beta, \{\pi_i\}_{i=Y}^{O_T}, \{s_{e,i}\}_{i=Y}^{O}, c, \{\gamma_{e,i}\}_{i=Y}^{O}, \eta, \phi, b, \rho, \sigma_\nu\right\} \quad dim(\Phi_1) = 22$$

The discount factor β is calibrated to match a weekly discount factor consistent with an annual interest rate of 4%. We set π_i such that an age class corresponds to the same age groups as in the data: $i = Y, A$ are $16 - 24$ and $25 - 54$ year-old workers, and $i = O_j$ for $j = 1, ..., 7$ ($T = 7$) are the $55, ..., 61$ year-old workers. The parameters for the aggregate productivity process ρ and σ_ν are set to the values proposed by Shimer (2005). For exogenous job separation rates by age, $s_{e,i}$, for $i = Y, A, O$, we follow Fujita & Ramey (2012): at each age, the exogenous job separations represent 34 percent of total separations. The calibration of the cost of vacancy posting c is based on Barron et al. (1997) and Barron & Bishop (1985) who suggest an amount equal to 17 percent of a 40-hour workweek (nine applicants for each vacancy filled, with two hours of work time required to process each application). The elasticity parameter of the matching function η is arbitrary set at its standard value of 0.5 (Petrongolo & Pissarides (2001)), because there is no information about the matching function by age. Moreover, the endogeneity of search effort implies that the elasticity of the matching function also depends on the elasticity of the cost function for search effort (Gomme & Lkhagvasuren (2015)). Finally, we set $\phi = 0.45$, which is an intermediate value of estimates of Lise (2013) and Christensen et al. (2005).

For the other parameters, we need some restrictions in order to identify these parameters using our first-order moments on labor market flows. Thus, we assume that *(i)* older workers share the same level of human capital, leading to $\{h_i\}_{i=Y}^{O_7} = \{1, h_A, h_O\}$,*(ii)* older workers share the same λ_i, leading to $\{\lambda_i\}_{i=Y}^{O_7} = \lambda_O$.. We are left with 3 parameters: $\{\lambda_Y, \lambda_A, \lambda_O\}$, and *(iii)* younger workers have a specific bargaining power $\gamma_Y \neq \gamma$, where $\gamma = \gamma_A = \gamma_O$. The bargaining power of age-i workers for $i = A, O$ are such that $\gamma_i = \eta$. This last restriction is also used by Shimer (2005). Hence, 10 parameters are estimated:

$$\Phi_2 = \{H, \chi, h_A, h_O, b, \gamma_Y, \lambda_Y, \lambda_A, \lambda_O, \sigma_\epsilon\} \quad dim(\Phi_2) = 10$$

The calibrated parameters are the solution to $\min_{\Phi_2} ||\Psi^{theo}(\Phi_2) - \Psi||$, where the numerical solution for $\Psi^{theo}(\cdot)$ is provided by the algorithm described in Appendix F.[31] The 10 free parameters are the elements of Φ_2, whereas the 10 first-order moments provided by the

[31] We use a global solution method to avoid the lack of accuracy when solving DMP model (see Petrosky-Nadeau & Zhang (2013)).

data are:

$$\Psi = \{\widehat{b}, \overline{w}, JFR_Y, JFR_A, JFR_O, JSR_Y, JSR_A, JSR_O, w_A/w_Y, w_O/w_Y\}$$

with $dim(\Psi) = 10$. We denote $X_O = \frac{\sum_{i=O_1}^{O_7} m_i n_i X_i}{\sum_{i=O_1}^{O_7} m_i n_i}$ for $X = JSR, w$ and $JFR_O = \frac{\sum_{i=O_1}^{O_7} m_i u_i JFR_i}{\sum_{i=O_1}^{O_7} m_i u_i}$. We choose as an additional target the value of the opportunity cost of employment measured by Hall & Milgrom (2008), which is $\widehat{b} = 0.7$.[32] Note that, in our model with endogenous search effort, this instantaneous value of leisure is actually $b_i - \phi_i(e_i(z))$, not b_i where $b_i = bh_i$ for age group i. We report the empirical targets in Table 3, as well as the model fit: the estimation results show that the steady state of the model is very close to empirical targets.

Table 3: First order moments: Ψ

	JFR_Y	JFR_A	JFR_O	JSR_Y	JSR_A	JSR_O
Data	0.44	0.41	0.33	0.047	0.017	0.011
Model fit	0.45	0.42	0.30	0.041	0.020	0.013
	$\widehat{b}$	$\overline{w}$	w_A/w_Y	w_O/w_Y		
Data	0.7	1.37	1.47	1.45		
Model fit	0.6786	1.39	1.50	1.48		

Overbar refers to aggregate average. Data source in Appendix B.1.

Table 4: Benchmark calibration

	External information Φ_1					
	β	$\eta = \gamma$	c	$s_{e,i}$	ϕ	
	$r = 4\%$	0.5	0.17	34% JSR	0.45	
	ρ	σ_ν	π_A	$\{\pi_i\}_{i=Y}^{O_7}$		
	0.9895	0.0034	25-54	55-56...60-61		
	Calibration Φ_2					
	H	b	σ_ϵ	χ		
HLS	0.125	0.92	0.15	0.75		
FR	0.061	0.934	0.124	-		
	λ_Y	λ_A	λ_O	h_A	h_O	γ_Y
HLS	0.0175	0.045	0.055	1.58	1.57	0.4
FR	-	0.085	-	-	-	-

HLS = Our weekly calibration

FR = Fujita and Ramey's (2011) weekly calibration

Table 4 summarizes the calibration. Our calibration strategy matches the observed labor flow rates per age at the steady state (Table 3). This imposes particular restrictions on the model. Indeed, matching the gaps between flow rates across ages at the steady state (i.e., the elasticities to some profitability differentials) is likely to discipline parameter calibration. Especially that related to the age-pattern of search effort, and then the

[32]This value is also used by Menzio et al. (2016).

elasticity of search cost function, which is key in the ability of the model to match these gaps.

We must stress three other points about the calibrated parameters. First, younger workers are less likely to benefit from changes in productivity. A smaller λ for young workers indicates a lower ability to move within the firm, whereas the higher value of λ for prime-age and older workers may reflect their ability to adapt to new tasks within the firm as they have accumulated higher levels of specific human capital.

Secondly, to account for the differences between young and prime-age workers with respect to the job finding rate JFR, at the steady state, bargaining power must be youth-specific: a value equal to 0.4 for the younger workers is then able to match the relatively high value of their job finding rate. This lower value is consistent with BLS statistics[33], which provides indirect evidence of an age-specific bargaining power. Indeed, for men aged 16 to 24 years old, the percentage of workers with a union affiliation equals 4.9%, versus 13% for those aged 25 to 64 years.

Table 5: Implied values of the outside option

$\widehat{b} \approx \sum_{i=Y}^{O_7} m_i \frac{bh_i - \phi(e_i^{ss})}{h_i}$	$\frac{bh_Y - \phi(e_Y^{ss})}{h_Y}$	$\frac{bh_A - \phi(e_A^{ss})}{h_A}$	$\sum_{i=O_1}^{O_7} m_i \frac{bh_O - \phi(e_i^{ss})}{h_O}$			
0.6786	0.675	0.668	0.727			
		$\frac{bh_O - \phi(e_{O_i}^{ss})}{h_O}$ for $i = 1, ..., 7$				
0.69	0.69	0.69	0.69	0.70	0.737	0.891

The third comment deals with the value of b. The results reported in Table 4 show that our calibrated value for b is lower than that used by Fujita & Ramey (2012) in their calibration à la Hagedorn & Manovskii (2008). The net value of home production, shown in Table 5, is closer to Hall & Milgrom (2008)'s estimated value for outside opportunities. Nevertheless, the calibration leads to higher values for the outside option than that used by Shimer (2005). This can help the model generate large responses to productivity shocks.

4.2 Worker flows and unemployment fluctuations

Table 6 reports labor market volatility across age groups. Comparing row 1 (Model) to row 5 (Data), the model can generate the observed age pattern in the volatility of labor flows and unemployment. This suggests that, given the parameter restrictions found at the steady state to match age-patterns of transition rates by age (section 3.5.1) hold, such that the age-pattern of volatility is matched by the model.

[33]See http://www.bls.gov/news.release/union2.t01.htm

Table 6: Model Predictions by Age Group i: 2nd Order Moments

X_i		U_Y	JFR_Y	JSR_Y	U_A	JFR_A	JSR_A	U_O	JFR_O	JSR_O
		$i = Y$ (16-24)			$i = A$ (25-54)			$i = O$ (55-61)		
$\sigma(X_i)$	1. Model	0.11	0.06	0.07	0.20	0.13	0.11	0.29	0.19	0.15
	2. No Hete.	0.19	0.12	0.11	0.19	0.12	0.11	0.25	0.17	0.13
	3. No HC	0.12	0.07	0.08	0.21	0.14	0.12	0.30	0.20	0.15
	4. Low γ_Y	0.17	0.11	0.10	0.20	0.13	0.11	0.25	0.17	0.13
	5. Data[a]	0.18	0.16	0.08	0.27	0.17	0.14	0.32	0.22	0.20
Corr(X_i, U_i)	6. Model	1	-0.81	0.88	1	-0.88	0.86	1	-0.93	0.89
	7. No Hete.	1	-0.89	0.85	1	-0.89	0.86	1	-0.92	0.87
	8. No HC	1	-0.83	0.87	1	-0.89	0.86	1	-0.94	0.90
	9. Low γ_Y	1	-0.90	0.88	1	-0.89	0.86	1	-0.92	0.87
	10. Data[a]	1	-0.91	0.68	1	-0.92	0.89	1	-0.81	0.76

[a]: CPS quarterly averages of monthly data, Men, 1976Q1 - 2013Q1, HP filtering of logged data with $\lambda_{HP} = 10^5$.

"Model": benchmark model, heterogeneous h and λ, low γ_Y

"No HC": benchmark model with homogenous h: $h_i = 1$

"No Hete": model with homogenous h & λ & γ: $h_i = 1$, $\lambda = \lambda_A$, $\gamma_i = \eta$

"Low γ_Y": idem "No Hete" except $\gamma_Y = 0.4$ and $\gamma_i = \eta$ for $i \geq A$.

4.2.1 Higher volatility for older workers on the labor market: the horizon effect

The model slightly overestimates the volatility gaps across age of the job finding rate (old workers' JFR is $0.19/0.13 = 1.46$ times higher than prime age workers', versus $0.22/0.17 = 1.29$ in the data; for young workers, the volatility gap is $0.06/0.13 = 0.46$ versus $0.16/0.17 = 0.9$ in the data), and matches those of the job separation rates (old workers' JSR is $0.15/0.11 = 1.36$ times higher than prime age workers', versus $0.2/0.14 = 1.42$ in the data; for young workers, the volatility gap is $0.07/0.11 = 0.63$ versus $0.08/0.14 = 0.57$ in the data). The volatility levels of job finding, separation and unemployment rates of the labor market of the prime-age worker are well reproduced, compared to Shimer (2005).[34]

In order to understand these results, we explore the quantitative predictions of several versions of the model.

The effect of the short distance to retirement. We consider the life-cycle model, in which we remove any exogenous age-heterogeneity in the calibrated parameters ($h_i = h$, $\gamma_i = \gamma$, and $\lambda_i = \lambda$, $\forall i$). When all exogenous sources of heterogeneity by age are removed, heterogeneity across age groups only comes from the horizon effect: workers are heterogeneous only with respect to their working-life expectancy, prior to retirement. Table 6 (row 2, "No Hete") reports simulation results. Without heterogeneity, young workers obviously

[34]Search effort is also more volatile for older workers: we obtain $\sigma(e_O) = 0.18$, versus $\sigma(e_A) = 0.12$ for the prime-age workers and $\sigma(e_Y) = 0.08$ for young workers. The volatility of aggregate search effort is $\sigma(e) = 0.13$, which is consistent with business moments found by Gomme & Lkhagvasuren (2015).

display the same business cycle fluctuations as prime-age workers'. In order to have a sense of the impact of the short distance to retirement, let us have a look at old versus prime age workers. Older vs. prime-age workers display an age-increasing pattern in volatilities (old workers' JFR is $0.17/0.12 = 1.4$ times larger than prime age workers' versus $.19/.13{=}1.46$ in the benchmark model; the volatility increase for JSR is $0.13/0.11 = 1.18$ versus $0.15/0.11{=}1.36$ in the benchmark model), when considering no other heterogeneity than that created by the varying distance to retirement. This shows that the horizon effect is key to generating the higher cyclicality of older workers' labor market flows.

The impact of human capital. Human capital was introduced to match the life-cycle wage profile, at the steady state. We simulate the model after removing only the increase in human capital as the worker ages. The simulation results are displayed in row 3 of Table 6. Comparing rows 1 and 3, older workers' volatilities slightly rise. Indeed, human capital makes older workers more profitable, which increases older workers' surplus while the horizon effect lowers older workers' surplus. By removing human capital, the model is left with only the horizon effect, which lowers older workers' surplus and makes them more responsive to aggregate shocks. However, comparing rows 1 and 3 for older workers, the volatility increase is small. This suggests that the life-cycle profile of human capital that is necessary to reproduce the observed life-cycle wage is not large enough to dampen the horizon effect in a sizable way.[35]

The impact of λ_i the age-specific probability of match-productivity draw. The impact of the age-specific λ can be assessed by comparing the results of the model for old workers with "No Hete" (row 2, where $\lambda_O = \lambda_A$) with those of the model with "No HC" (row 3, where $\lambda_O \neq \lambda_A$). The main impact of a reduction in λ is to reduce all volatilities. When λ are age-specific, older worker have the highest λ, leading all their job rates to be more sensitive to the fluctuations of reservation productivity. As changes in reservation productivity affects job separation but also job finding (by defining the range of acceptable jobs), this exogenous heterogeneity in λ matters for the goodness of the fit. Notice that quantitative results underline that the horizon effect is the main force at work for generating the large volatility gaps across ages because a large fraction of the volatility gaps across age is driven by the distance to retirement only, not by exogenous heterogeneity.

[35] Theoretical restrictions in proposition 10 of Appendix E.3 turn out to hold.

4.2.2 Low volatilities for younger workers on the labor market: a market for outsiders

As both young and prime-age workers are far away from retirement, specificities on the youth labor market cannot be explained by the distance to retirement. We stress that adding a lower bargaining power for younger workers is necessary to replicate their lower volatility on the labor market. At the steady state, a low bargaining power for young workers is essential to match their high level of job finding rate. In Table 6, we consider the "Low γ_Y" case, where the only exogenous heterogeneity comes from a lower bargaining power for young workers. Notice that, in the "Low γ_Y" case (row 4), fluctuations in the youth labor market are less volatile than in the case where all workers have the same bargaining power (row 2 "No Hete"). Indeed, with low bargaining power, two opposing forces are at work: on the one hand, as in Hagedorn & Manovskii (2008), a lower workers' bargaining power tend to increase volatility; on the other hand, with low workers' bargaining power, the steady value of youth labor market tightness increases: young workers get a lower share of the surplus, which makes firms more willing to hire them. In a market with high tensions on the labor market, the wage is more responsive to changes in job opportunities. This last effect dominates. In a boom, young workers' wage respond more to aggregate shocks than their older counterparts', which tends to dampen the volatility of hiring incentives, hence JFR. In addition, as young workers are responsive to changes in search value, they are less willing to stay within the firm, which tends to dampen the fall in the reservation productivity in economic booms. Hence, youth fluctuations in JSR are less volatile than their older counterparts'.[36]

Calibrating the model to replicate the labor market age-pattern at the steady state delivers a rather good match for business cycle features.

4.2.3 Aggregate variables

This good fit of the age heterogeneity is convincing if the model can also match the dynamics of US aggregate labor market variables. Table 7 reports the second-order moment of aggregate labor market variables.[37] Notice that (i) the model can predict the magnitude of aggregate labor market volatilities. Concerning vacancies, their volatility is slightly underestimated, and *(ii)* their correlation with unemployment (the Beveridge curve) is also well reproduced. Hence, in spite of endogenous separation, our model captures the dynam-

[36] We also derive this result analytically in Appendix D.2.3.

[37] The Barnichon's (2010) data are updated (see https://sites.google.com/site/regisbarnichon/data) and rescaled as in Adjemian et al. (2017).

Table 7: Model Predictions on Aggregate variables: 2nd Order Moments

		u	v^b	JFR	JSR
Std. Dev.	Data^a	0.24	0.09	0.16	0.11
	Model	0.18	0.05	0.12	0.09
	RA Model	0.17	0.06	0.12	0.07
	Model 25-54	0.20	—	0.13	0.11
Corr. with u	Data^a	1	-0.87	-0.95	0.88
	Model	1	-0.55	-0.90	0.85
	RA Model	1	-0.43	-0.98	0.93
	Model 25-54	1	—	-0.88	0.86

"u" aggregate unemployment, "v" vacancies, "JFR" Job Finding Rate, "JSR" Job Separation Rate
"Std. Dev." Standard Deviation. "Corr. with u" Correlation with u
"RA" Representative Agent model
^{a}: CPS quarterly averages of monthly data, Men, 1976Q1 - 2013Q1, HP-filtered logged data, with $\lambda_{HP} = 10^5$.
^{b}: Barnichon (2010)'s logged data.

ics around the aggregate Beveridge curve via a pro-cyclical search effort. Indeed, search effort increases the elasticity of the vacancies-unemployment ratio with respect to productivity at all ages.[38] There are two channels leading to higher vacancies-unemployment elasticity when search effort is endogenous: *(i)* the elasticity of vacancies-unemployment ratio with respect to "net profits" is always larger when search effort is endogenous, because search effort is complement to investment in vacancies, *(ii)* the elasticity of "net profits" with respect to productivity is always larger when search effort is endogenous because the share of productivity in profits is larger than in an economy where search effort is constant. Finally, while the correlation between unemployment and vacancies is ambiguous without endogenous search effort[39], it becomes negative for sufficiently high values for the elasticity of search effort. Indeed, in the case without search effort, counter-cyclical separations can amplify the counter-cyclical responses of the unemployment rate to productivity shocks, and the small response of the vacancy-unemployment ratio can lead to a counter-cyclical response in the vacancy rate. In contrast, with endogenous search effort, a sufficiently high elasticity of search effort ensures that the vacancy rate is pro-cyclical, leading to a negative correlation between vacancies and unemployment.[40] Pro-cyclical response of the search effort eliminates incentives for firms to use recessions to change the composition of their workforce, and preserves the Beveridge curve.

Finally, we also report in Table 7 simulations from a model with a "Representative Agent" (RA model, endogenous separation and search effort), and for prime-age workers (25-54) of

[38]This result is consistent with findings in Gomme & Lkhagvasuren (2015) in a model with infinitely-lived agents and exogenous separation and in the case of a wage posting equilibrium. See proposition 2 and corollary 1 in Appendix D for analytical results in the case of the MP model.

[39]Krause & Lubik (2007) show that a New Keynesian model with search and matching frictions in the labor market fails to generate the negative correlation between vacancies and unemployment in the data.

[40]See proposition 3 in Appendix D for analytical results.

our life-cycle model. As far as job finding rate is concerned, the RA model, the model with life-cycle features or results for prime-age workers deliver similar business cycle statistics. However, differences in the volatility of job separation rates are more sizeable. This comes from the high non-linerarities, increasing with age, of decision rules on separations: the RA model, or the group prime-age workers cannot account for the specificities of old workers' labor market.

4.2.4 The contributions of search effort and endogenous separations

In this section, we provide an evaluation of the interactions between search effort and endogenous separations in our life cycle model of equilibrium unemployment. We want to show that the interaction between the endogenous search effort and endogenous separations not only magnifies the volatilities of aggregate variables,[41] but it also contributes to magnify the differences across age, and thus allowing the model to be close to the data.[42]

We thus propose two simulations. First, we freeze the response of workers' search effort to the business cycle by imposing a low elasticity for search effort ("$\varepsilon_{e|z}$ low"). Secondly, we restrict separations to be exogenous (“Exo. Sep.").

Table 8: Model Predictions by Age Group i: 2nd Order Moments

		$i = Y$ (16-24)			$i = A$ (25-54)			$i = O$ (55-61)		
X_i		U_Y	JFR_Y	JSR_Y	U_A	JFR_A	JSR_A	U_O	JFR_O	JSR_O
$\sigma(X_i)$	Bench	0.11	0.06	0.07	0.20	0.13	0.11	0.29	0.19	0.15
	$\varepsilon_{e\|z}$ low	0.088	0.022	0.083	0.101	0.033	0.089	0.12	0.045	0.103
	Exo. Sep.	0.157	0.144	0	0.163	0.151	0	0.18	0.16	0
$\text{Corr}(X_i, U_i)$	Bench	1	-0.81	0.88	1	-0.88	0.86	1	-0.93	0.89
	$\varepsilon_{e\|z}$ low	1	-0.54	0.97	1	-0.60	0.95	1	-0.77	0.97
	Exo. Sep.	1	-0.99	0	1	-0.99	0	1	-0.99	0

"Bench": benchmark calibration

"$\varepsilon_{e|z}$ low": $\phi = 2$ and $b = 0.8$ to match $u = 4.86$. Exogenous search effort, endogenous separations.

"Exo. Sep.": $\lambda_i = \sigma_\epsilon = 0$ and $s_{e,i} = JSR_i$, with $b = 0.8$ to match $u = 4.86$. Endogenous search effort. Exogenous separations.

We change the opportunity cost of employment in order to maintain the same level of aggregate unemployment in each model.

[41] In Appendix D, proposition 2 shows that the elasticity of labor market tightness is magnified by endogenous search effort, leading JFR (JSR) to be more (less) volatile (see corollary 1). We also prove that endogenous search effort allows the MP model with endogenous separation to generate a negative correlation between u and v (see proposition 3).

[42] The Log-linear approximations of the transition are $\widehat{JFR}_i = \widehat{e}_i + (1-\eta)\widehat{\theta}_i - \frac{G(R_i)}{1-G(R_i)}\varepsilon_{G|R}\widehat{R}_i$ and $\widehat{JSR}_i = \frac{(1-s_e)\lambda_i G(R_i)}{s_e+(1-s_e)\lambda_i G(R_i)}\varepsilon_{G|R}\widehat{R}_i$. Given that $\widehat{\theta}|_{\text{variable } e} > \widehat{\theta}|_{\text{constant } e}$ (proposition 2 in Appendix D), obviously $\widehat{e}|_{\text{variable } e} > \widehat{e}|_{\text{constant } e} = 0$, and $\widehat{R}|_{\text{variable } e} < \widehat{R}|_{\text{constant } e}$ (see the proof of proposition 11 in Appendix E.5), the interaction between endogenous search effort and separations magnifies the volatilities of transition rates. Moreover, the age-increasing pattern of the volatilities is magnified by the endogenous search effort: $\widehat{JFR}_{i+1} - \widehat{JFR}_i$ declines when $\phi \to \infty$ (see the proof of proposition 11 in Appendix E.5) as well $\widehat{JSR}_{i+1} - \widehat{JSR}_i$ as $\widehat{R}|_{\text{variable } e} < \widehat{R}|_{\text{constant } e}$.

In Table 8, the model with a constant search effort can generate age-increasing volatilities. This suggests that the horizon effect still prevails when distance to retirement affects volatility across age-groups only through firms' search effort θ_i. However, the magnitude of volatility gaps across age groups can be matched only with endogenous search effort. With constant search effort, the volatility gaps are reduced by 20pp on average, thereby moving away from the stylized facts. Hence, the search effort channel is key to understand the observed age heterogeneity over the business cycle.

Exogenous separations also lead to small increases in volatility as the worker ages. Hence, it is not only the endogenous search effort that allows the model to generate significant volatility gaps across age groups, but also the endogenous job separation rates. This is consistent with CPS data: the contribution of fluctuations in the job separation rate to age-specific unemployment fluctuations is sizable and increases as the worker ages (see Table 2, rows 1-3). Hence, by omitting this age specificity in the job separation rates, the model with exogenous separation rates also move the theory away from the data.

Hence, both experiments underline the key role of the interaction between endogenous search effort and endogenous separations in order to account for the volatility gaps across ages.

Table 9: Model Predictions on Aggregate variables: 2nd Order Moments

		u	v	JFR	JSR
Std. Dev.	Bench	0.18	0.05	0.12	0.09
	$\varepsilon_{e\|z}$ low	0.08	0.05	0.03	0.07
	Exo. Sep.	0.16	0.10	0.14	0
Corr. with u	Bench	1	-0.55	-0.90	0.85
	$\varepsilon_{e\|z}$ low	1	0.53	-0.66	0.95
	Exo. Sep.	1	-0.76	-0.99	0

"Bench": benchmark calibration. "u" aggregate unemployment rate. "v" aggregate vacancies.
"$\varepsilon_{e|z}$ low": $\phi = 2$ and $b = 0.8$ to match $u = 4.86$. Exogenous search effort. Endogenous separation.
"Exo. Sep.": $\lambda_i = \sigma_\epsilon = 0$, $s_{e,i} = JSR_i$ and $b = 0.8$ to match $u = 4.86$. Exogenous separation, endogenous search effort.
We change the opportunity cost of employment in order to maintain the same level of aggregate unemployment in each model.

The results for aggregate variables are reported in Table 9. The magnitudes of volatilities can only be matched by the "complete" model. In the model with exogenous separation rates ($\lambda_i = \sigma_\epsilon = 0$ & $s_{e,i} = JSR_i$), search effort is variable: as in Gomme & Lkhagvasuren (2015), our result on aggregate data suggest that search effort can be a sufficient mechanism for solving the volatility puzzle.[43] Hence, this simplified model can be considered as a good approximation of the data for prime age workers, but it is not able to fully account for life-cycle features. Results in Table 9 also show that a model with constant search ef-

[43]These quantitative results clearly support analytical results in Appendix D.

fort fails to generate the magnitude of fluctuations observed on the aggregate of the labor market. As in Fujita & Ramey (2012), this model with constant search effort generate a positive correlation between unemployment and vacancies, which is counterfactual.

Our simulation results underline the interactions between search effort and endogenous separations: these mechanisms allow a better match with the data than the sum of these two channels, taken independently. This comes from the fact that, with an endogenous search effort, reservation productivity is more sensitive to aggregate shocks, leading older workers JSR and JFR to be more volatile. Indeed, we stressed in section 3.5.2 that, in Equation (14), the response of reservation productivity to the business cycle is mainly driven by current changes in aggregate productivity and fluctuations in the expected gains from search on the labor market Σ, given the restriction parameters at the steady state. The response of expected gains from search is magnified when search effort is endogenous. There lies the interaction between endogenous separation and endogenous workers' search effort. When considering response of reservation productivity by age, we underlined in section 3.5.2 that expected gains from search differ by age. We argue here that it is all the more the case when search effort is endogenous. In booms, younger workers are less willing to remain within the firm as outside opportunities are expanding, all the more so when workers' search effort is endogenous. This widens the volatility gap between old workers' worker flows and their younger counterparts'.

4.3 Wage cyclicality

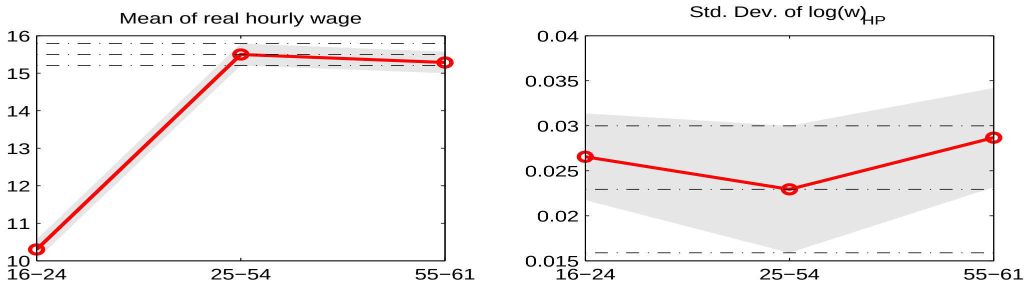
We first present stylized fact based on monthly CPS data, and secondly discuss the implications of our model in terms of wage age-dynamics. We then compare the model's cyclical properties with the observed volatility of real hourly wage.

Real wage in the data. Figure 2 reports the descriptive statistics for male real hourly wages and weekly earnings.[44] The level of wages increases between younger and prime-age workers, and declines at the end of the life-cycle, which is consistent with the view that experience makes workers more productive until the age when the depreciation of the human capital becomes faster.

Cyclical wage volatility is U-shaped over ages, though the gaps between age volatilities are not significant. It seems to be inconsistent with our findings on worker flows and unemployment stock, which would imply an age-decreasing profile for wages, as, in a

[44]See appendix B.1.1 for a description of the data.

Figure 2: Real wage by age



3 age-groups, CPS quarterly averages of monthly data, Men, 1979 Q1 - 2013Q2. "Std Dev" : Standard deviation of logged HP-filtered data (with smoothing parameter 10^5). All moments are estimated using GMM, with a weighting matrix corrected for heteroskedasticity and serial correlation using Newey & West (1987)'s method. 95% confidence band. Authors' calculations.

market with more rigid prices, the large part of adjustments would fall on quantities. However, the decline in the wage volatility over ages would be relevant in a model with a representative worker by age class, ie. a model with an exogenous separation rate. In this case, the age-increase of the volatilities of labor market flows would imply a more rigid wage for older workers. As this property of the wage dynamics per age is not observed, this provides some support to a model with endogenous separations where the average wage dynamics are driven by fluctuations in individual wages combined with changes in the productivity distribution of job-worker pairs, as it is the case in our model. Indeed, in our model, individual wages differ from the average wage, as the latter takes into account changes in employment composition. For older workers, the horizon effect generates wage rigidity in individual wages. However, the composition of older workers' employment also responds to the business cycle.[45]

Real wage in the model. Table 10 compares the model prediction with the data. Only the model with search effort and endogenous separation generates a U-shape pattern of the wage volatilities per age.[46] As suggested above, fluctuations in average productivity seem to reduce the (average) wage fluctuations. This is why the benchmark model generates smaller volatilities than the model with exogenous separations. Consistently, the volatility of older workers' average wage is larger than prime-age workers', since this dampening effect becomes smaller at the end of the life-cycle.

Table 10 shows that *incomplete* models (with low elasticity of search effort, or with exogenous separation rate) perform poorly: the aggregate wage dynamics is driven by the

[45]See the appendix B.1.2 for a analytical discussion.

[46]Notice that we obtain promising results with respect to the age-pattern of the wage cyclicalities, but the model slightly under-predicts the level of these wage volatilities.

Table 10: Second order moments: data versus theory

		All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
Std. Dev	US Data	0.021	0.026	0.023	0.028
	Model	0.0124	0.0127	0.0122	0.0123
	$\varepsilon_{e\|z}$ low	0.0131	0.0126	0.0131	0.0133
	Exo. Sep.	0.0128	0.0130	0.0128	0.0125

US data: Standard deviation of CPS Monthly HP-filtered logged data, Men, 1979 Q1 - 2013Q2
"$\varepsilon_{e|z}$ low": $\phi = 2$ and $b = 0.8$ to match $u = 4.86$. Exogenous search effort. Endogenous separation.
"Exo. Sep.": $\lambda_i = \sigma_\epsilon = 0$, $s_{e,i} = JSR_i$ and $b = 0.8$ to match $u = 4.86$. Exogenous separation, endogenous search effort.
We change the opportunity cost of employment in order to maintain the same level of aggregate unemployment in each model.

ones of individual wages. Hence, these findings on the wage volatilities across age groups can be viewed as additional supports to our findings on worker flows and unemployment stock: in the absence of interactions between endogenous search effort (highly dependent on workers' horizon) and the job separation rate (highly sensitive to workers' horizon), it seems more difficult to reproduce the age-pattern of wage volatilities.[47]

5 Conclusion

We document business cycle fluctuations in worker flows by age group for the US economy. We extend the current literature by looking at the age profile of both *average* and *volatilities* in workers' transition rates on CPS data. We then develop a life cycle Mortensen & Pissarides (1994) model with age-directed search, endogenous search effort and separations. Older workers' shorter horizon endogenously reduces the cyclicality of their outside options, thereby making their wages less sensitive to the business cycle. Thus, in a market where wage adjustments are small, quantities vary considerably. This is the case for older workers, whereas their younger counterparts behave like infinitively lived agents. Furthermore, the horizon effect cannot explain the significant volatility differences between prime-age workers and young workers because both age groups are far away from retirement. The lower bargaining power for young workers consistent with their weaker union affiliation allows us to replicate the volatility of their transition rates.

We also show that search effort and endogenous separations provide useful mechanisms to magnify the volatility difference across ages as well as to understand the dynamics of the aggregate labor market: the complementarity between firms' and workers' search strategies generates a significant amplification mechanism. Endogenous separations also

[47]In Appendix B.2, we also discuss the model predictions on wage distribution conditional on separation and non-separation. We show that, in CPS data, young and prime-age workers exhibit the same compositional effect as the one analyzed in Mueller (2017) for the whole population: in recessions, the pool of unemployed shifts toward workers with higher wages in the previous job. Nevertheless, this is not the case for older workers. We leave this issue for future research.

help the model match aggregate volatilities. Moreover, workers' pro-cyclical search efforts reduce firms' incentives to use recessions to change the composition of their workforce, leading the model to generate a Beveridge curve.

We subscribe to the view that the understanding the age-differences of unemployment are important because one needs to discipline any employment policy issue using data coming from different age groups. We propose in this paper a model that could be used to start thinking about the business cycle effects of policy reforms across age-groups. Our work also suggests that future research should investigate endogenous retirement choices in order to account for pathways to retirement, with transitions periods of bridge jobs, as well as changes in worker heterogeneity in order to explain the age-specific changes in the composition of unemployment over the business cycle.

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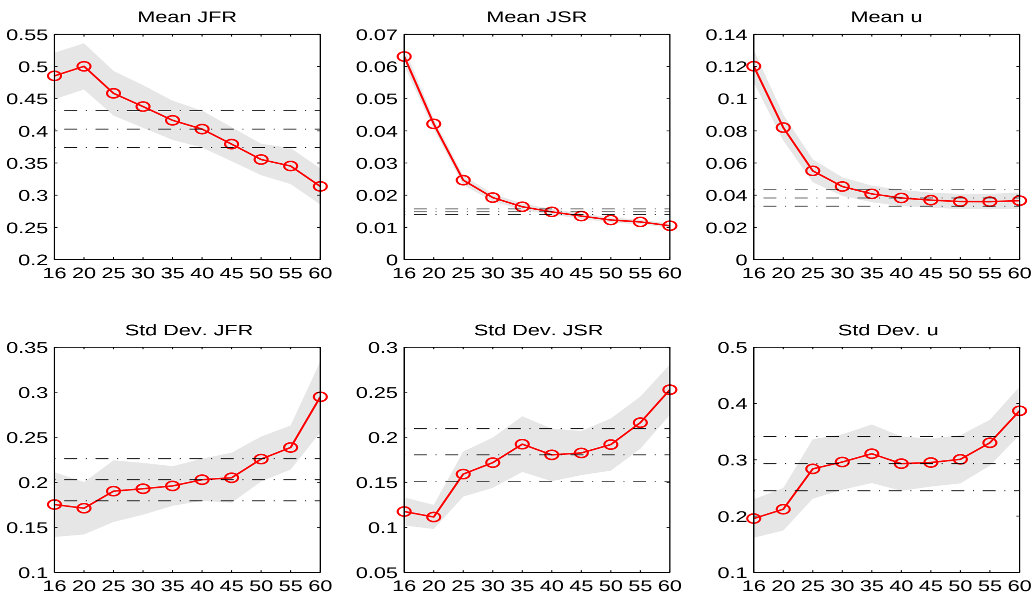
Online Appendix

A Stylized facts on worker flows

A.1 Alternative age-groups

We divide the working life into 10 age groups: 16-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, and 60-64. Each age group includes individuals with a maximum age difference of 5 years, except for younger workers, where the minimum working age is the lower bound. Figure 3 reports the mean of the time series. It appears that JFR and JSR are age-decreasing. Compared to the 40-44 age group (the reference group), JFR is significantly larger for workers below 30 years of age, and significantly lower for workers aged 55+. Given the highest accuracy of the estimates for JSR, these gaps become significant for workers less than 34 years old, and those older than 50.

Figure 3: Job Separation Rate JSR, Job Finding Rate JFR, and Unemployment u by age



10 age-groups. CPS quarterly averages of monthly data, Men, 1976 Q1 - 2013Q2. "Std Dev": Standard Deviation of logged HP-filtered data (with smoothing parameter 10^5). All moments are estimated using GMM with a weighting matrix corrected for heteroskedasticity and serial correlation using Newey & West (1987)'s method. 95% confidence band in shaded area. Horizontal lines are 95% band for prime-age workers. The comparison between the shaded area and the 2 horizontal lines allows to compare prime-age workers to younger or older workers. Authors' calculations.

The cyclical behavior of logged transition rates are obtained using HP filter with a smoothing parameter of 10^5. Volatility of fluctuations in logged transition rates display a significant age-increasing pattern. Figure 3 shows that older workers have significantly larger standard deviations than prime-age workers do. Indeed, workers older than 54 have a significantly larger standard deviation than those in the 40-44 age group. For young workers, significant differences appear between those younger than 24 and prime-age workers. The cyclical behavior of the unemployment rate also contains age-increasing volatility. Nevertheless, this feature is less pronounced than for worker flows because the variance of the unemployment rate, given by $E[\widehat{u}^2] = (1-u)^2\{E[\widehat{JSR}^2]+E[\widehat{JFR}^2]-2E[\widehat{JSR}\widehat{JFR}]\}$, is dampened at the end of the life cycle by covariances between JSR and JFR ($E[\widehat{JSR}\widehat{JFR}]$), which are less negative than for prime-age workers. Overall, we synthesize the age-pattern of labor market stock and flows based on these first results by considering only three age groups: 16-24, 25-54, and, 55-61.

A.2 HP-filtering with $\lambda = 1600$

Table 11 reports business cycle facts when the smoothing parameter is 1600 rather than 10^5. The age-increasing pattern in volatility is robust.

Table 11: Standard deviation. CPS data, quarterly averages of monthly instantaneous transition rates,1976Q1-2013Q1, Men, logged HP-filtered data with smoothing parameter 1600. Authors' calculations.

	All: 16-61	16-24	25-54	55-61
JSR	0.084492	0.077361	0.10856	0.17571
		0.7126	1	1.6186 (a)
JFR	0.11897	0.1219	0.12439	0.18647
		0.98004	1	1.4991
u	0.16976	0.13661	0.19427	0.25264
		0.7032	1	1.3004

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate.
(a) Old workers' JSR volatility equals 1.6186 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

A.3 Accounting for inactivity

Using Shimer (2012)'s methodology for 3 employment states (Employment, Unemployment and Inactivity), on CPS data for Men, we obtain the results reported in Tables 12 and 13. With 3 employment states, steady state unemployment includes all transitions rates, including those involving inactivity. Tables 12 and 13 suggest that our business cycle facts across age groups remain robust when separations and findings are purged from the transition to and from inactivity. Exit from employment as well as the job finding rate fall with age while their volatility increases with age.

When we decompose unemployment fluctuations using β decomposition as in Shimer (2012), based on hypothetical unemployment rates, we find that the transitions between unemployment and unemployment account for 76% of unemployment fluctuations.[48]

[48] We compute counter-factual steady states predicted by time varying finding and separation rates, while other transition rates are set at their historical mean. We log and HP-filter the time series using a smoothing parameter of 10^5 and compute the variance decomposition of the cyclical component of steady state unemployment based on βs. We find that $\beta^{EU} + \beta^{UE} = 0.7575$.

Table 12: Mean. Quarterly averages of monthly CPS data, 3 states (Employment, Unemployment, Inactivity), 1976Q1 - 2013Q1, Men. Authors' calculations.

	All: 16-61	16-24	25-54	55-61
JSR (eu)	0.022289	0.049672	0.017704	0.012269
		2.8057	1	0.69298 (a)
JFR (ue)	0.38394	0.41585	0.37888	0.30254
		1.0976	1	0.79851
u	0.062021	0.12627	0.049089	0.045553
		2.5723	1	0.92796
ei	0.017989	0.060717	0.009086	0.01725
ui	0.25019	0.38769	0.173	0.2178
ie	0.087675	0.10631	0.089063	0.039792
iu	0.094369	0.12409	0.097061	0.026687

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate. "ei" transition rate from employment to inactivity. "ui" transition rate from unemployment to inactivity. "ie" transition rate from inactivity to employment. "iu" transition rate from inactivity to unemployment.

[(a)] Old workers' JSR equals 0.69298 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

Table 13: Standard deviation. Quarterly averages of monthly CPS logged data, 3 states (Employment, Unemployment, Inactivity), 1976Q1 - 2013Q1, Men. Logged HP-filtered data with smoothing parameter 10^5. Authors' calculations.

	All: 16-61	16-24	25-54	55-61
JSR	0.10936	0.095019	0.14095	0.19785
		0.67413	1	1.4037 (a)
JFR	0.15626	0.15902	0.1637	0.21629
		0.97144	1	1.3213
u	0.21511	0.15902	0.25506	0.29212
		0.62344	1	1.1453
ei	0.066495	0.073621	0.074905	0.10924
ui	0.13181	0.098655	0.15932	0.23929
ie	0.094361	0.11697	0.10092	0.12961
iu	0.09026	0.09714	0.12326	0.23116

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate. "ei" transition rate from employment to inactivity. "ui" transition rate from unemployment to inactivity. "ie" transition rate from inactivity to employment. "iu" transition rate from inactivity to unemployment.

[(a)] Old workers' JSR equals 1.4037 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

A.4 Employment and Unemployment for all workers

Using Shimer (2012)'s methodology for 2 employment states (Employment, Unemployment), on CPS data for Men and women, we get results reported in Tables 14 and 15. The main stylized facts remain relevant : the mean transition rates fall with age while their volatility increases with age.

Table 14: Mean. Quarterly averages of monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men and Women. Authors' calculations.

	All: 16-61	16-24	25-54	55-61
JSR	0.018792	0.042009	0.015466	0.010627
		2.7162	1	0.68713 (a)
JFR	0.42404	0.49514	0.39915	0.33679
		1.2405	1	0.84376
u	0.044798	0.081417	0.039634	0.03305
		2.0542	1	0.83387

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate.
[a] Old workers' average JSR equals 0.68713 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

Table 15: Standard deviation. Quarterly averages of monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men and Women, logged HP-filtered data with smoothing parameter 10^5. . Authors' calculations.

	All: 16-61	16-24	25-54	55-61
JSR	0.086295	0.069806	0.11134	0.16103
		0.62694	1	1.4463 (a)
JFR	0.15671	0.14693	0.16161	0.20154
		0.90912	1	1.247
u	0.21221	0.16513	0.2388	0.28343
		0.6915	1	1.1869

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate.
[a] Old workers' JSR volatility equals 1.4463 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

A.5 Robustness check using Elsby et al. (2010)'s data

We consider an alternative method of computing transition rates. Elsby et al. (2010) use Shimer (2012)'s formula based on quarterly stocks of unemployed and employed workers (in which separations are proxied by short-term unemployment) rather than disaggregated data as we do. Their approach yields higher levels of transition rates. In order to test the sensitivity of our results to the transition rate calculation method, we re-compute the same business cycle statistics on their database using their methodology. Since the time series are now quarterly, the smoothing parameter on the HP filter is equal to 1600. The results are reported in Tables 16 and 17. We do find that the levels of transition rates fall with age while the standard deviations increase with age.

Table 16: Mean. Elsby et al. (2010) data, 1977Q2 - 2009Q4, Quarterly data, Men and Women.

	All: 16+	Young: 16-24	Prime-age: 25-54	Old: 55+
JSR	0.03527	0.10047	0.023898	0.015542
		4.2043	1	0.65034 (a)
JFR	0.54514	0.7111	0.46652	0.43876
		1.5243	1	0.94049

"JSR" Job Separation Rate. "JFR" Job Finding Rate.
(a) Old workers' average JSR equals 0.65034 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

Table 17: Standard deviation. Logged HP-filtered data, smoothing parameter 1600, 1977Q2 - 2009Q4, Quarterly data, Men and Women. Elsby et al. (2010) data.

	All: 16+	Young: 16-24	Prime-age: 25-54	Old: 55+
JSR	0.044485	0.046366	0.060651	0.092973
		0.76447	1	1.5329 (a)
JFR	0.10627	0.095022	0.113	0.15079
		0.8409	1	1.3344

"JSR" Job Separation Rate. "JFR" Job Finding Rate.
(a) Old workers' JSR volatility equals 1.5329 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

A.6 Employment and unemployment per skill

The data per age can mix an age effect and a skill effect. In order to deal with this identification problem, we propose to distinguish two skill groups (high school diploma and less, and college or more).

Table 18: Mean. Monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men. Authors' calculations.

	High school and less			College and more		
	16-24	25-54	55-61	16-24	25-54	55-61
JSR	0.065079	0.027775	0.016455	0.034743	0.012331	0.0085209
	2.343	1	0.59242 (a)	2.8176	1	0.69104
JFR	0.45922	0.42343	0.35042	0.56922	0.39952	0.30959
	1.0845	1	0.82757	1.4248	1	0.77491
u	0.12986	0.06568	0.050573	0.062228	0.032214	0.029889
	1.9772	1	0.76999	1.9317	1	0.92782

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate.
(a) Old workers' average JSR equals 0.59242 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

After controlling for educational attainment, the levels are age-decreasing and the volatilities are age-increasing. Thus, our stylized facts account for phenomena linked to worker age, and are not the result of a composition effect.

Table 19: Standard deviation. Employment and Unemployment, Monthly CPS data, 1976Q1 - 2013Q1, Men, logged HP-filtered data, smoothing parameter 10^5, Authors' calculations.

	High school and less			College and more		
	16-24	25-54	55-61	16-24	25-54	55-61
JSR	0.11934	0.16678	0.24422	0.15384	0.1669	0.27525
	0.71555	1	1.4643 (a)	0.92172	1	1.6492
JFR	0.1803	0.17097	0.27954	0.19894	0.18454	0.28459
	1.0546	1	1.635	1.078	1	1.5421
u	0.20022	0.27448	0.36584	0.24774	0.30167	0.38679
	0.72945	1	1.3328	0.82122	1	1.2822

"JSR" Job Separation Rate. "JFR" Job Finding Rate. "u" unemployment rate.
[(a)] Old workers' JSR volatility equals 1.4643 times prime-age workers'. Small numbers refer to values relative to prime-age workers'.

A.7 Conditional variance: a structural VAR

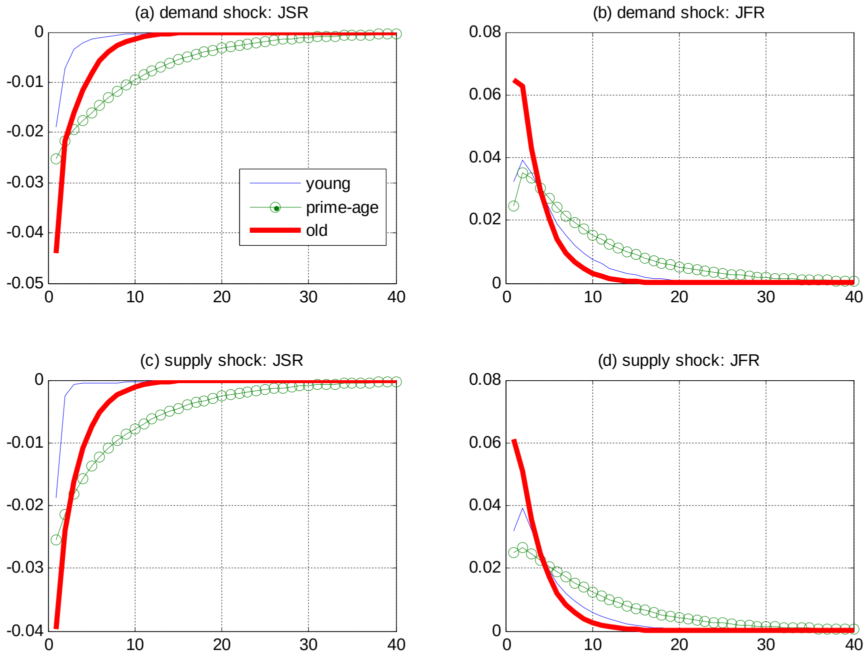
In this section, we provide evidence that our stylized facts (based on unconditional standard deviation of labor flows by age group) are robust when considering responses to structural shocks in a structural VAR. Old workers' response of job finding and job separation to aggregate shocks remain larger than their younger counterparts. With a smaller sample size each month than for the other age groups, the time series of the workers aged 55-61 may include a noise component. To deal with this problem, we estimate a structural VAR where IRFs cannot then be driven by noise, that is uncorrelated with structural shocks, by definition. Moreover, we restrict all the standard deviations of the structural shocks to be equal to unity. Hence, the size of Impulse Response Function are comparable across age-groups.

We follow Fujita (2011) who uses structural VARs with sign restrictions in order to study US worker reallocation along the business cycle. We estimate a structural VAR for each age group (1976Q1-2013Q2). A first trivariate VAR includes US CPI inflation, job separation and finding for male young workers. The second (and third) VAR includes US CPI inflation, job separation and finding for male prime-age (old) workers. As in Fujita (2011), we detrend the data using deterministic quadratic trends. Lag length is set to 1 based on the AIC criteria. We impose sign restrictions to identify two aggregate shocks (supply and demand) and a reallocation shock. All sign restrictions are imposed for 1 quarter.

- *Restriction 1: the supply shock.* In response to a positive aggregate supply shock, inflation and job separation respond negatively, while job finding increases
- *Restriction 2: the demand shock.* In response to a positive aggregate demand shock, job separation responds negatively, while inflation and job finding increase
- *Restriction 3: the reallocation shock.* In response to a positive reallocation shock, job separation and job finding both increase

Impulse Response Functions (IRF) to a one-standard-deviation shock are reported in Figure 4. In response to an aggregate shock (whether supply or demand shock), older workers' job finding and separation respond more than their younger counterparts'.

Figure 4: IRFs to aggregate shocks: Job Separation Rate JSR, Job Finding Rate JFR by age group



Median IRFs to aggregate structural shocks identified using a VAR on US data (1976Q1-2013Q2). Authors' calculations.

In order to gauge the significance of older workers' larger response to aggregate shock, we compute the probability that older workers' IRF lies above the younger counterpart's median IRF. Results are displayed in Table 20. The results show that response of the job finding and separation rates to

Table 20: Probability that older workers' IRF lies above younger counterparts' median IRF

	Supply shock		Demand shock	
	Young	Prime age	Young	Prime age
JFR	0.67	0.76^a	0.68	0.73
JSR	0.72	0.65	0.77	0.68

JFR Job Finding Rate. JSR Job Separation Rate.

^{a}: Following a supply shock, 76% of old workers' IRF of JFR lie above median JFR IRF of prime age workers, 1 quarter after the shock.

demand and supply shocks are significantly larger for older workers. Hence, beyond unconditional moments, conditional moments by age group suggest that older workers' labor market fluctuations are more volatile than their younger counterparts'.

A.8 Volatility of unemployment duration across age groups

In this section, we check the robustness of our stylized fact by looking at fluctuations of unemployment duration across age groups. In the CPS survey, an unemployed person is not asked about her job separation rate or job finding rate. She is asked about how long she has been looking for work. The answer to this question on unemployment duration might be seen as providing more direct evidence on age-specific labor market responsiveness along the business cycle.

Unemployment duration is related to the job finding rate: by the law of large number, unemployment duration equals the inverse of job finding rate. Intuitively, this corresponds to the idea that, if unemployed workers face a 10% probability of getting a job each week, the average unemployment duration is 10 weeks. Our focus on unemployment duration allows to assess the robustness of the stylized fact on the job finding rate.

A.8.1 Measuring unemployment duration: conceptual and practical questions

Measuring unemployment duration is actually challenging due to the combination of several elements.

Censored data. CPS provides limited *longitudinal* data on unemployment episodes as each household entering the CPS is administered 4 monthly interviews, then ignored for 8 months, then interviewed again for 4 more months. It is then difficult to follow each unemployed workers from the beginning to the end of the unemployment spell. In the CPS, many unemployment spells are censored (Horrigan (1987)) as there is no continuous monitoring of spells of unemployment (from their inception to their completion). In addition, respondents often round their amounts of time spent unemployed and the amounts are sometimes inconsistent with the actual time between surveys (Bowers & Horvath (2016)). In a nutshell, based on longitudinal data, the CPS cannot provide an accurate measure of *completed* spells of unemployment.

Definition: In-progress spells of currently unemployed workers versus expected completed spells of newly unemployed. If we use the *cross-sectional* nature of CPS data, then the replies to the question on how long the person is looking for work can only provide information on the *average age of unemployment spells among the currently unemployed.* Indeed, BLS computes measure of average unemployment duration based on the *in-progress* spells of unemployment at the time of the survey. However, such a measure does not capture the average length of *completed* spells of unemployment, which is also the relevant measure in our model. The BLS measure is actually likely to overestimate the average length of completed unemployment spells as, when computing length of existing unemployment spells, the pool of currently unemployed workers also include long-term unemployed who are more likely to leave the labor force.

CPS re-designs. Two CPS re-designs affected the measure of unemployment duration. First re-design took place in 1994. This major CPS re-design actually introduced a trend break in the measured duration of unemployment, increasing it relative to its measurement using the earlier survey design (Polivka & Miller (1988)). This phenomenon is documented by Abraham & Shimer (2001), Elsby et al. (2009) and Busch (2012), among others. The second re-design took place in 2011. The CPS was modified to allow respondents to report longer durations of unemployment. Prior to that time, the CPS accepted unemployment durations of up to 2 years; any response of unemployment duration greater than this was entered as 2 years. Starting with data for January 2011, respondents were able to report unemployment durations of up to 5 years. This change affected estimates of average (mean) duration of unemployment. This generated an unprecedented rise in the number of persons with very long durations of unemployment.[49]

A.8.2 Our measure of unemployment duration across age-groups using matched CPS Basic monthly data

In spite of the difficulties mentioned above, we propose here to measure unemployment duration. To abstract from the 2 CPS re-design (as there is little consensus on the appropriate way of correcting the survey data), we focus on data between 1995 and 2010. We show that business cycle volatility of unemployment increases with age, which is consistent with our stylized fact on the job finding rate.

Empirical strategy Using CPS Monthly data, we compute age-specific unemployment duration using the answers to the question on how long the person is looking for work.

- We want to measure *completed* spells of unemployment.
- As a result, we look at month-to-month matched CPS data. We focus on the unemployed in a given month who subsequently report that they are employed in the next month's survey
- We look at their answer to the question on unemployment duration.
- Unemployment duration UD is the number of weeks a person has been looking for a job, when this person subsequently found a job in the next month's survey.
- We compute average UD for men, aged 16-24.
- We repeat the process for men, aged 25-54, then 55-61 (same age-groups as in Section 2)[50]

Result: the volatility of unemployment duration increases with age We seasonally adjust the data using x12, then proceed with the quarterly average of monthly data. The standard deviation of HP-filtered logged data with smoothing parameter 10^5 is displayed in Table 21. Our stylized fact holds: the volatility of unemployment duration increases with age.

[49]https://www.bls.gov/cps/duration.htm. U.S. Bureau of Labor Statistics. 2011. "Changes to Data Collected on Unemployment Duration." Labor Force Statistics from the Current Population Survey.

[50]In order to check the empirical relevance of this measure of unemployment duration, we compute this measure for the whole working-age population, we compare it with alternative measures, namely: BLS average duration of unemployment, Kaitz (1970)'s measure of unemployment duration as the ratio of unemployment level to the level of inflows, $1/JFR$ where the job finding rate JFR is based either on Shimer (2005)'s formula or our JFR CPS from section 2. The volatility of HP-filtered logged data on unemployment duration is similar across measures. Results are available upon request.

Table 21: Business cycle volatility of unemployment duration by age group

Age	std
16-24	0.171
25-54	0.180
55-61	0.259

CPS data, Men only, quarterly average of monthly data. "std": standard deviation of HP-filtered logged data with smoothing parameter 10^5. 1995Q1-2010Q4.

A.9 Contributions of age-specific transition rates to aggregate fluctuations

A.9.1 The contribution of age-specific transition rates to aggregate transition rates

Let us consider the economy at the conditional steady state in which unemployment ins equal unemployment outs. We use the following decomposition to measure the contribution of each age group in the fluctuations of the aggregate job flows:

$$x_t \;=\; \sum_i x_{i,t}\varpi_{i,t} \Rightarrow \widehat{x}_t \approx \sum_i p_i \widehat{x}_{i,t} + \sum_i p_i \widehat{\varpi}_{i,t}$$

with $x = JSR$ ($x = JFR$) then $\varpi_i = n_i/n$ ($\varpi_i = u_i/u$), knowing that $p_i = \frac{JSR_i(n_i/n)}{JSR} = \frac{JFR_i(u_i/u)}{JFR}$. $\widehat{x}$ denotes log deviation with respect to the mean. We denote β_i^x as the contribution of age-i worker flow x_i to the variance of the corresponding aggregate job flow x. Following Shimer (2012), we have:

$$E[(\widehat{x}_t)^2] \;\approx\; \sum_i p_i E[\widehat{x}_t \widehat{x}_{i,t}] + \sum_i p_i E[\widehat{x}_t \widehat{\varpi}_{i,t}] \quad \Rightarrow 1 \approx \sum_i \beta_i^x + \sum_i \beta_i^{\varpi}$$

We can then decompose the contribution of each age group to the total volatility into two elements: the first one captures the volatility of age-specific transition rate and the second is associated with changes in the age composition of labor flows. To compute these statistics, we use the employment and unemployment conditional steady states, and the time series of worker flows.

A.9.2 The contribution of age-specific transition rates to aggregate unemployment fluctuations

In this section, to simplify the notation, job separation rate is denoted s and job finding is denoted f.

The unemployment volatility for age-i workers. For each state z, the unemployment rate for the age-i worker is given by

$$u_{i,z} = \frac{s_{i,z}}{s_{i,z} + f_{i,z}} = \widetilde{u}(\overline{s}_i(1+\varepsilon_{i,s}), \overline{f}_i(1+\varepsilon_{i,f})) \quad \text{with} \quad \left\{ \begin{array}{lcl} f_{i,z} & = & \overline{f}_i(1+\varepsilon_{i,f}) \\ s_{i,z} & = & \overline{s}_i(1+\varepsilon_{i,s}) \end{array} \right.$$

implying that $\varepsilon_{i,x} = \frac{x-\overline{x}}{\overline{x}} \approx \log(x/\overline{x})$ for $x = s, f$. Within each age-group, the volatility of the unemployment rate is given by

$$
\begin{aligned}
Var(\log(u_i)) &= \sum_t \pi(z_t)(\log(u_{i,t}) - \log(\overline{u}_i))^2 = \sum_t \pi(z_t)(\log(\widetilde{u}(\overline{s}_i(1+\varepsilon_{s,i,t}), \overline{f}_a(1+\varepsilon_{f,i,t}))) - \log(\overline{u}_i))^2 \\
&\approx (1-\overline{u}_i)^2 \left[\sigma^2_{i,s} + \sigma^2_{i,f} - 2cov(s_i, f_i)\right]
\end{aligned}
$$

Using the approximation of the steady state unemployment rate by age ($\widehat{u}_i = \log(u_i/\overline{u}_i) = (1-\overline{u}_i)(\widehat{s}_i - \widehat{f}_i)$), we deduce the contributions of each transition rate in the unemployment fluctuations:

$$
\begin{aligned}
E(\widehat{u}_i^2) &= E[(1-\overline{u}_i)\widehat{s}_i\widehat{u}_i] + E[(1-\overline{u}_i)(-\widehat{f}_i)\widehat{u}_i] \\
\Rightarrow 1 &= \frac{(1-\overline{u}_i)^2(\sigma^2_{s,i} - cov(s_i, f_i))}{\sigma^2_{u,i}} + \frac{(1-\overline{u}_i)^2(\sigma^2_{f,i} - cov(s_i, f_i))}{\sigma^2_{u,i}} \quad \Leftrightarrow \quad 1 = \beta_i^{u,s} + \beta_i^{u,f}
\end{aligned}
$$

Aggregate unemployment. At the conditional steady state, the impact of the age specific volatilities of JFR_i and JSR_i on the fluctuations of the aggregate unemployment is deduced from

$$
\begin{aligned}
Var(\log(u)) &= \sum_t \pi(z_t)\left(\log\left(\sum_i \varsigma_i u_{i,t}\right) - \log\left(\sum_i \varsigma_i \overline{u}_i\right)\right)^2 \\
&\approx \sum_t \pi(z_t)\left(\frac{1}{\sum_i \varsigma_i u_i}\sum_i \varsigma_i\left[\varepsilon_{s,i,t}\overline{s}_i\widetilde{u}'_1(\overline{s}_i, \overline{f}_i) + \varepsilon_{f,i,t}\overline{f}_i\widetilde{u}'_2(\overline{s}_i, \overline{f}_i)\right]\right)^2 \\
&\approx \varsigma_Y^2\left(\frac{\overline{u}_Y}{\overline{u}}\right)^2 Var(\log(u_Y)) + \varsigma_A^2\left(\frac{\overline{u}_A}{\overline{u}}\right)^2 Var(\log(u_A)) + \varsigma_O^2\left(\frac{\overline{u}_O}{\overline{u}}\right)^2 Var(\log(u_O)) \\
&\quad +2\left(\frac{1}{\overline{u}}\right)^2\left[\begin{array}{l}
\varsigma_Y\varsigma_A u_Y(1-u_Y)u_A(1-u_A)\left(cov(s_Y, s_A) - cov(s_Y, f_A) - cov(f_Y, s_A) + cov(f_Y, f_A)\right) \\
+\varsigma_Y\varsigma_O u_Y(1-u_Y)u_O(1-u_O)\left(cov(s_Y, s_O) - cov(s_Y, f_O) - cov(f_Y, s_O) + cov(f_Y, f_O)\right) \\
+\varsigma_A\varsigma_O u_A(1-u_A)u_O(1-u_O)\left(cov(s_A, s_O) - cov(s_A, f_O) - cov(f_A, s_O) + cov(f_A, f_O)\right)
\end{array}\right]
\end{aligned}
$$

where the weight of each age-class in working age population is ς_a. These weights are those used in the calibrated model. By rearranging this formula, we can obtain the contribution of each age-specific transition rates in the volatility of aggregate unemployment:

$$
\begin{aligned}
\beta^u_{f_i} &= \frac{\varsigma_i^2\left(\frac{\overline{u}_i}{\overline{u}}\right)^2(1-\overline{u}_i)^2\left[\sigma^2_{i,f} - cov(s_i, f_i)\right] + \left(\frac{\varsigma_i u_i(1-u_i)}{\overline{u}^2}\right)\sum_{j\neq i}\varsigma_j u_j(1-u_j)\left[cov(f_i, f_j) - cov(f_i, s_j)\right]}{Var(\log(u))} \\
\beta^u_{s_i} &= \frac{\varsigma_i^2\left(\frac{\overline{u}_i}{\overline{u}}\right)^2(1-\overline{u}_i)^2\left[\sigma^2_{i,s} - cov(s_i, f_i)\right] + \left(\frac{\varsigma_i u_i(1-u_i)}{\overline{u}^2}\right)\sum_{j\neq i}\varsigma_j u_j(1-u_j)\left[cov(s_i, s_j) - cov(f_i, s_j)\right]}{Var(\log(u))}
\end{aligned}
$$

which satisfies $1 = \sum_i\left(\beta^u_{f_i} + \beta^u_{s_i}\right)$.

B Wage

B.1 Cyclicality of real hourly wage across age groups

B.1.1 Computing average real hourly wage by age in CPS data

We compute the average real hourly wage by age in order to compute the empirical targets w_A/w_Y and w_O/w_Y in Table 3. Using monthly CEPR MORG data between January 1979 and June 2013, we document the business cycle response of male real hourly wages (w). Hourly wage is usual weekly earnings divided by usual weekly hours. Data represent earnings before taxes and other deductions and include any overtime pay, commissions, or tips usually received. The data excludes all self-employed persons, regardless of whether their businesses are incorporated. After dealing with outliers[51], we divide the time series of nominal hourly wages by a trend derived from the aggregate wage time-series. This trend captures long-term increases in inflation and technology. After correcting for seasonal movements using x12, we consider the quarterly averages of monthly observations and then look at logged-HP filtered real hourly wages using 10^5 as a smoothing parameter. We check that levels of real hourly wages are consistent with findings in Heathcote et al. (2010), as well as BLS data on weekly earnings by age. We also check that business cycle features are close to Jaimovich & Siu (2009)'s statistics on annual wage data. We get age-increasing levels of real hourly wages, which is consistent with the view that experience makes workers more productive. We then compute w_A/w_Y and w_O/w_Y, values are reported in Table 3.

B.1.2 Wage cyclicality: From theory to data

One might wonder what are the model's predictions with respect to wage volatility by age, and compare the model's predictions with the corresponding data. In this section, we argue that answering this question would not be very informative.

In the data. Using CEPR-MORG data, we can compute only average real hourly wage by age group, not individual wages.

In the model. Wages are heterogeneous within each age group because each job has a match-specific productivity. Thus, the model generates a wage distribution by age. If we want to compare the model predictions with wages in the data, it is then necessary to compute an *average wage* by age group, defined as follows:

$$\mathcal{W}_i(z) \quad = \quad \gamma_i z \mathcal{G}(R_i(z)) + (1-\gamma_i)\left(b + \Sigma_i(z)\right) \tag{15}$$

where $\mathcal{G}(R_i(z)) = \frac{1}{n_i(z)} \int_{R_i(z)}^{1} x dn_i(z,x)$ denotes the average productivity of age-i workers, $dn_i(z,x)$ is the number of age-i employees on a x-productivity job. This distribution is endogenous and subject to cyclical changes, and depends on the cyclical changes in separation, finding, and productivity

[51] Hourly wages higher than 250 US dollars, wages less than half the net minimum wage, and young workers working more than 45 hours per week

changes: $dn_i(z,\epsilon)$ changes with the business cycle. Given that the average wage depends on both individual wages and the wage distribution, a rigid individual wage and a volatile average wage could be mutually consistent. The log-linear approximation of equation (15) leads to

$$\widehat{\mathcal{W}_i} = \gamma_i \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i}\left(\widehat{z}+\widehat{\mathcal{G}_i}\right) + (1-\gamma_i)\frac{\Sigma_i}{\mathcal{W}_i}\widehat{\Sigma}_i \quad \text{with } \widehat{\mathcal{G}_i} = \Gamma_i^z\widehat{z} + \Gamma_i^r\widehat{R}_i + \Gamma_i^s\widehat{\Sigma}_i$$

where the term $\widehat{\mathcal{G}_i}$ accounts for the changes in the job composition on wage fluctuations.[52] The parameters Γ_i^z, Γ_i^r and $\Gamma_i^s \equiv \gamma_i^s\Sigma_i$ are the elasticities of the average productivity with respect to z, R_i, and Σ_i, respectively[53]. This approximation underlines the channels through which average productivity $\mathcal{G}_i$ depends on the business cycle. The signs of these elasticities Γ_i^x, for $x = z, r, s$, are ambiguous. Aggregate productivity (z) has a positive effect on aggregate employment, and thus lowers average productivity, but also raises employment at each level of the productivity distribution through its impact on search efforts ($e_i(z)$ and $\theta_i(z)$). In boom, the fall in R_i increases the set of jobs (the integral has a larger span) but lowers their quality (more jobs are concentrated at the bottom), Moreover, when R_i declines, employment increases, thus average productivity falls. Finally, a rise in the search value Σ_i reduces incentives to post new vacancies. This has a negative effect on the two dimensions of employment: by reducing its aggregate level, it raises average productivity, whereas its negative effect on each point lowers average productivity. Finally, we deduce that the average wage dynamics is given by:

$$\widehat{\mathcal{W}_i} = \gamma_i \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i}\left((1+\Gamma_i^z)\widehat{z} + \Gamma_i^r\widehat{R}_i + \Gamma_i^s\widehat{\Sigma}_i\right) + (1-\gamma_i)\frac{\Sigma_i}{\mathcal{W}_i}\widehat{\Sigma}_i$$

Thus, for older workers, i.e., when $\Sigma_O \to 0$, the average wage can be proxied by:

$$\widehat{\mathcal{W}_O} = \gamma_i \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i}\left((1+\Gamma_O^z)\widehat{z} + \Gamma_O^r\widehat{R}_O\right)$$

This shows that this average wage can be highly pro-cyclical if $\Gamma_i^r > 0$ is large enough. Moreover, given that the volatility of the JSR is age-increasing, implying $\widehat{R}_i < \widehat{R}_{i+1}$, the impact of $\widehat{R}_O$ on $\widehat{\mathcal{W}_O}$ can be reinforced by the large volatility in the reservation productivity.

In a nutshell, in our model, individual wages differ from the average wage, as the latter takes into account changes in employment composition. For older workers, the horizon effect generates wage rigidity in individual wages. However, the composition of older workers' employment also responds to the business cycle. The final effect of workers' age on the volatility of average wage is therefore theoretically ambiguous.

B.2 Wage conditional on separation and non-separation

We use CPS data from Mueller (2017) in order to investigate the wage cyclicality. Mueller (2017) compares the wage of continuing jobs (wage conditional on non-separation) with pre-displacement

[52] See Appendix E.5 for the derivation of the formula for $\widehat{\mathcal{G}_i}$

[53] The notation $\Gamma_i^s \equiv \gamma_i^s\Sigma_i$ allows us to use the property that $\Gamma_O^s \to 0$ for older workers, simply because $\Sigma_O \to 0$, and γ_O^s is bounded.

wage of unemployed workers (wage conditional on separation). In his paper, using CPS data, Mueller (2017) shows that, in recessions, the pool of unemployed shifts towards high-wage workers. Mueller (2017) stresses that the standard MP model with endogenous separation fails to account for this empirical feature. He then proposes two extensions to the model to bring the theory closer to the data: the first one is based on an ad-hoc calibration of the variance of match-specific productivity among high-ability workers, the second relies on credit constraints. We extend his empirical analysis by looking at age-groups.

Data. We use Mueller (2017)'s data available of the *American Economic Review* website. We first replicate his results on the whole population using CPS March supplement. This annual survey collects information on wages in the prior year, which allows to compute the average wage from the previous year by current labor market status (employed versus unemployed). We also consider his data based on matched CPS ORG. Using the rotating panel structure of CPS basic monthly surveys, Mueller (2017) looks at the wage of those who lose their job and become unemployed. He focuses on wage data at the 4th interview and analyzes the employment status in subsequent months. We then extend Mueller (2017)'s work by focusing on age groups.

Results. Results are reported in Table 22.

Table 22: Changes in the unemployment composition: wages conditional on separation and non-separation by age

		Raw wage	Decomposition of predicted wage Total	Exp.	Educ. attain.	Gender	Marital status	Race	State	Indus.	Occ.	Residual wage Total
	Results in Mueller's paper (Table 1 in his paper, Panel B, March CPS, page 2092)											
1	Cyclicality	2.59	1.64	0.23	0.10	0.12	0.23	0.02	0.05	0.70	0.19	0.95
		(0.28)***	(0.19)***	(0.04)***	(0.06)*	(0.02)***	(0.02)***	(0.01)**	(0.05)	(0.08)***	(0.08)**	(0.13)***
	Young (16-24)											
2	Cyclicality	1.92	1.32	0.12	0.11	0.16	0.21	0.03	0.05	0.43	0.20	0.61
		(0.52)***	(0.29)***	(0.03)***	(0.06)*	(0.03)***	(0.07)***	(0.02)**	(0.08)	(0.15)***	(0.13)	(0.32)*
	Prime age (25-54)											
3	Cyclicality	2.50	1.44	0.03	0.09	0.13	0.20	0.03	0.04	0.73	0.21	1.06
		(0.30)***	(0.17)***	(0.03)	(0.07)	(0.03)***	(0.02)***	(0.01)**	(0.06)	(0.08)***	(0.08)**	(0.18)***
	Old (55-61)											
4	Cyclicality	0.67	0.42	-0.03	-0.03	0.02	0.04	-0.05	0.06	0.60	-0.19	0.25
		(0.72)	(0.48)	(0.03)	(0.17)	(0.08)	(0.08)	(0.03)	(0.07)	(0.20)***	(0.13)	(0.52)
	Results in Mueller's paper (Table 1 in his paper, Panel A, Matched CPS ORG, page 2092)											
5	Cyclicality	2.77	2.01	0.32	0.17	0.14	0.24	-0.03	0.12	0.69	0.37	0.75
		(0.51)***	(0.38)***	(0.08)***	(0.10)	(0.03)***	(0.04)***	(0.04)	(0.10)	(0.10)***	(0.10)***	(0.20)***
	Young (16-24)											
6	Cyclicality	1.89	0.98	0.03	0.09	0.12	0.04	-0.13	0.11	0.50	0.22	0.92
		(0.39)***	(0.25)***	(0.07)	(0.05)*	(0.05)**	(0.03)	(0.07)*	(0.09)	(0.16)***	(0.12)*	(0.27)***
	Prime age (25-54)											
7	Cyclicality	2.47	1.63	0.06	0.13	0.14	0.24	-0.01	0.09	0.66	0.31	0.84
		(0.50)***	(0.30)***	(0.06)	(0.11)	(0.03)***	(0.04)***	(0.05)	(0.10)	(0.11)***	(0.11)***	(0.30)***
	Old (55-61)											
8	Cyclicality	0.44	0.54	-0.07	-0.31	0.21	0.11	-0.11	0.47	0.53	-0.29	-0.11
		(0.91)	(0.51)	(0.03)**	(0.22)	(0.08)***	(0.11)	(0.06)*	(0.17)**	(0.29)*	(0.24)	(0.76)

Source: Mueller (2017). Merged CPS Outgoing Rotation Group sample for the years 1980 to 2012, the CPS March supplement for the years 1968 to 2012 (the years 1962-1967 were not included as no information was available on industry in previous year).
Notes: Newey-West corrected standard errors in parentheses. * p<0.1 ** p<0.05 *** p<0.01. All series are yearly averages, HP-filtered with a smoothing parameter of 100. Note that the coefficients on the predicted and residual wage add up to the coefficient on the raw wage.
Rows 1 and 5 : same estimates as in Mueller (2017). Line 1: from March CPS. Line 5: from Matched CPS ORG.

We measure the cyclicality of the compositional changes in the unemployment pool as the coefficient β in the regression $log(w_t^u) - log(w_t) = \alpha + \beta U_t + \epsilon_t$, where w_t^u is the average wage from the previous year for those unemployed at time t, w_t is the average wage from the previous year for the full sample, and U_t is the official unemployment rate from the Bureau of Labor Statistics. As in Mueller

(2017), rows 1 and 5 in Table 22 show that the unemployed in recessions are more experienced, more educated, more likely to be male, more likely to be married, more likely to be white, and more likely to come from industries and occupations that pay high wages, compared to periods of low unemployment. These results are based on data with all workers of all ages. We find similar results when we restrict the sample to young or prime-age workers (rows 2-3 and 6-7 in Table 22). Interestingly, when we look at older workers, the results are different (rows 4 and 8 in Table 22). In recession, for older workers only, the pool of unemployed does not shift towards high-wage workers.

New challenges. The empirical investigation of wages conditional on separation and non-separation over the life cycle shows that old workers' labor market is not affected by the same compositional effect as the labor market of young and prime age workers. This non-homogeneity of the empirical results conditional on workers' age suggest that the extensions proposed by Mueller (2017) are not well suited for an analysis that accounts for life cycle features.

Obviously, for young and prime-age workers, it would be possible to assume a low variance of the match-specific productivity for high-ability workers, as suggested by Mueller (2017) (in the context of a model without life-cycle). However, this assumption would need to be less relevant for old workers, as Mueller (2017)'s stylized fact is not observed on older workers. It is difficult to argue that such an assumption holds for all workers, except at the end of the working life. It would be interesting to find an endogenous mechanism, related to workers' age, to account for the age-specific changes in the composition of unemployed workers along the business cycle. Therefore, the inspection of the data raises new questions, which paves the way for future developments that lie beyond the scope of our current paper.

C Empirical evidence on search effort

We present *(i)* the methods allowing to measure the search effort, *(ii)* the debate on its cyclicality, and *(iii)* its age pattern.

C.1 Measuring search effort

There are two main data sets used to measure search effort by unemployed workers: the first one, the American Time Use Survey (ATUS), provides a measure of the time spent on job-searching activities per day, whereas the second, the Current Population Survey (CPS), gives an indirect measure via the types and number of search methods used. If the first measure is the most natural quantitative proxy for job search effort, ATUS suffers from its small sample size and its short sample period (starting in 2003), unlike the CPS.

C.2 Pro-cyclicality of aggregate search effort

Shimer (2004) uses CPS data to build an indirect measure of search effort (the number of methods used). His main result is that this search effort measure is counter-cyclical. These first investigations are supported by the more recent work of Mukoyama et al. (2014) who use CPS data to infer the average time used to search for job prior to the ATUS sample. Nevertheless, the results reported by Mukoyama et al. (2014) cast some doubt on the quality of the econometric method allowing to compute this "imputed" time used to search for a job. In particular, if one looks at unemployed workers, the "actual" and the "imputed" time series are consistent only during the periods 2007-2008, displaying counter-cyclical movements during the periods 2003-2007 and 2009-2011. Moreover, the use of this measure of search effort proposed in Shimer's pioneer work has been criticized by Tumen (2014) who shows that the probability of exiting unemployment is a decreasing function of the number of search methods, whereas, if he uses the number of search methods per week unemployed as an alternative measure of search effort, search intensity becomes strongly pro-cyclical. This result confirms the one in DeLoach & Kurt (2013). After controlling for composition bias, Gomme & Lkhagvasuren (2015) also find that search effort (measured by time spent on this activity) is strongly pro-cyclical. Moreover, Gomme & Lkhagvasuren (2015) underline that *(i)* the "OLS regressions of search time on the number of search methods delivers a very low R^2, well below 10%, even after controlling for the individual level characteristics"[54] and *(ii)* "despite the positive individual-level link between the two variables, they do not move in the same direction over the business cycle". These empirical findings favor the view that the supply side of labor market adjustments (via search effort) can complement the demand driven adjustments, usually analyzed in the simplified version of the DMP model. Given that the measure of search effort is highly debated in the literature, one can favor the empirical approaches that use worker flows to estimate the dynamics of search effort. This is the path followed by Hornstein & Kudlyak (2016) who show that structural estimation of worker flows cannot reject the scenario of a pro-cyclical job search effort, if the weight of vacancies in the matching function is small enough. These results echo the ones in Elsby et al. (2015) who note that a counter-cyclical search effort is difficult to reconcile with movements in the Beveridge curve during and after the Great Recession.

C.3 Levels of search effort by age

C.3.1 Levels of search effort falls with age

Regarding search effort by age based on the ATUS data (the direct measure of the search effort), Aguiar et al. (2013) find that search effort has an inverted U-shaped profile with age.[55] Nevertheless, using time use surveys in the UK, Germany, France, Italy, and Spain (MTUS), they also show that search effort is an age-decreasing in all other countries. Hence, the US appear as an outliers with respect to this behavior. Aguiar et al. (2013) do not provide any hint about the reason why the US seems an outlier. We suggest that this surprising result for the US comes from the use of the raw ATUS data, whereas some other components of the time use are age specific. With respect to Aguiar et al. (2013), we measure search effort as search activities divided by total time available during the day, which differs across age groups. This point is very important: the available time appears shorter

[54] This regression shows that a unit increase in search methods is correlated with a 10 minute increase in search time. This positive correlation between the two measures is also underlined by Mukoyama et al. (2014)

[55] These results based on the direct measure of search intensity are in accordance with those provided by Mukoyama et al. (2014).

Table 23: Search effort by age group: ATUS data

	22-34	35-54	55+	Total
Median	0.161	0.143	0.138	0.146
♯ Obs	302	544	171	1061

for young people than for their older counterpart (11.90 hours a day for respondents younger than 34, 13.6 hours a day for prime age respondents of 35-54 years old, 13.3 hours for 55-70 years-old), as they spend more time on education, sleeping, eating and drinking. Moreover, we consider respondents with positive search effort, in accordance with the definition of unemployment status. Search effort is then computed as the number of minutes of job search activities divided by the total available time each day (see the presentation of our sample in section C.3.2). Table 23 shows that the older the respondent, the lower the search effort in the US, which is consistent with the empirical pattern found for all countries in Aguiar et al. (2013).

C.3.2 Search effort data in ATUS

In order to estimate search effort by age, we use 2003-2015 waves of the ATUS (American Time Use Survey). The BLS conducts the ATUS based on a sample drawn from the outgoing panel of the CPS (Current Population Survey). Each respondent reports activities from the previous day.

- **Total length of each day by age.** In the model, search effort is measured as time spent on activities undertaken to find a job *relative to* the total length of the period, that is normalized to 1. In the data, the total length of each day can differ across age as time spent on activities that are not modeled in the paper (such as sleep, eating and drinking, education) can differ across age. For each respondent, we compute the sum of activities that individuals do not choose in our model (personal care, eating and drinking, education, respectively mnemonics t01, t11 and t06, with the associated travel time t1801, t1811 and t1806). We then infer the time available for activities that are chosen by economic agents in our model. This available time appears shorter for young people than for their older counterpart, as they spend more time on education, eating and drinking, or personal care.
- **Positive search effort.** Job search activities are recorded under mnemonics 0504 (t050403, t050404, t050405, t050481 and t050499). They include time spent on filling out job application, sending resumes, interviewing, etc... We consider respondents with positive search effort, in accordance with the definition of unemployment status (not employed *and* looking for a job).

D Analytical analysis

In order to provide analytical results, we use simplifying assumptions with respect to the model presented in the main text: Time is continuous; we also characterize the steady-state and derive comparative statics results that describe how the vacancy-unemployment ratio changes with aggregate productivity across steady states. We also adopt a gradual approach.

- *Representative-agent model:* In section D.1, we first look at the elasticity of the vacancy-unemployment ratio with respect to productivity in a representative agent model with constant versus variable search effort. We then show that variable search effort magnifies the responsiveness of the vacancy-unemployment ratio to aggregate productivity. This first step allows the reader to recover Nagypal & Mortensen (2007*a*)'s results as a special case of constant search effort (and endogenous separation) and Gomme & Lkhagvasuren (2015)'s conclusions as a special case of constant separation (and variable search effort). In addition, as Gomme & Lkhagvasuren (2015) consider only exogenous separation, they do not assess the role of variable search effort on the elasticity of separations to aggregate productivity. This section fills this gap. In particular, we show how endogenous search effort helps the model match the slope of the Beveridge curve, which is missed by the standard DMP model with no search effort and endogenous separation (Krause & Lubik (2007), Fujita & Ramey (2012)).

- *With life-cycle features:* In section D.2, we then look at a simplified version of our model with variable search effort, endogenous separations and life-cycle features. We analyze how life-cycle alters the discount rate. To illustrate how the short distance to retirement amplifies the responsiveness to aggregate shock, we will focus on older workers' elasticity to aggregate shock. Using the results from section D.1, we illustrate the interaction between life-cycle features on the one hand, and endogenous search effort and separations on the other hand. Endogenous search effort and separations magnify the effect of the short distance to retirement on the elasticity of vacancy-employment ratio and reservation productivity to aggregate shocks. Finally, in section D.2.3, using the simplified version of our full model, we analytically derive the impact of workers' bargaining power on the elasticity to aggregate shock. This section relates to our quantitative results on the effect of young workers' bargaining power on their labor market volatility (Section 4.2.2).

For expositional simplicity, the disutility of search effort is given by $\phi(e) = \frac{e^{1+\phi}}{1+\phi}$ and the matching function is given by $M(v, eu) = v^{1-\eta}(eu)^{\eta}$, with $\phi > 0$ and $\eta \in (0; 1)$. γ refers to workers' bargaining power. Labor market tightness is defined as $\theta \equiv \frac{v}{eu}$, whereas the vacancy-unemployment ratio is denoted $\vartheta \equiv \frac{v}{u}$. As usual in the literature, we assume that time is continuous (the discount rate is r), and we focus on the steady state.

D.1 Infinite lifetime horizon and endogenous search effort

Proposition 1. *In the infinitely-lived agent model, the elasticities of the vacancy-unemployment ratio ($\vartheta \equiv \frac{v}{u}$) with respect to aggregate productivity z are:*

$$\begin{array}{ll} \textit{Endogenous Search Effort} & \textit{Constant Search Effort} \\ \widehat{\vartheta} = \frac{(r+JSR)\frac{1+\phi}{\phi}+\gamma JFR}{(r+\lambda)\eta+\gamma JFR}\frac{1}{1-\widetilde{b}}\widehat{z} & \widehat{\vartheta} = \frac{r+JSR+\gamma JFR}{(r+\lambda)\eta+\gamma JFR}\frac{1}{1-\widetilde{b}+\widetilde{\phi}(\overline{e})}\widehat{z} \end{array} \tag{16}$$

Proof. **With exogenous search effort.** In an infinite lifetime horizon model, with exogenous search effort $\overline{e}$, equilibrium of the labor market is defined by the intersection of job destruction (JD) and job creation (JC):

$$
\begin{aligned}
z\left(R+\frac{\lambda}{r+\lambda}\int_R^1[1-G(x)]dx\right) &= b-\phi(\overline{e})+\frac{\gamma}{1-\gamma}c\vartheta && (JD)\\
c\vartheta^{\eta} &= (1-\gamma)\frac{z}{r+\lambda}\int_R^1[1-G(x)]dx && (JC)
\end{aligned}
$$

(JD) and (JC) are equations (7) and (8) in Nagypal & Mortensen (2007a)'s paper. These equations determine the equilibrium values of the vacancy-unemployment ratio, ϑ, and reservation productivity, R. Log-linear approximation of this system leads to:

$$
\begin{aligned}
\left(R(r+\lambda)+\lambda\int_R^1[1-G(x)]dx\right)\widehat{z}+(r+JSR)R\widehat{R} &= \frac{\gamma}{1-\gamma}\frac{c\vartheta}{z}(r+\lambda)\widehat{\vartheta}\\
\eta\widehat{\vartheta} &= \widehat{z}-\frac{1-G(R)}{\int_R^1[1-G(x)]dx}R\widehat{R}
\end{aligned}
$$

Using (JC) and (JD) conditions at the steady state,

$$
\left.\begin{aligned}
(r+\lambda)\tfrac{c\vartheta}{(1-\gamma)JFR} &= z\tfrac{\int_R^1[1-G(x)]dx}{1-G(R)}\\
(r+\lambda)\tfrac{zR-(b-\phi(\overline{e}))}{\gamma JFR+JSR-\lambda} &= z\tfrac{\int_R^1[1-G(x)]dx}{1-G(R)}
\end{aligned}\right\}\Rightarrow\left\{\begin{aligned}
zR &= \tfrac{(\gamma JFR+JSR-\lambda)c\vartheta+(1-\gamma)JFR(b-\phi(\overline{e}))}{(1-\gamma)JFR}\\
c\vartheta &= \tfrac{(1-\gamma)JFR(zR-(b-\phi(\overline{e})))}{\gamma JFR+JSR-\lambda}
\end{aligned}\right.
$$

and their log-linear approximations, we deduce :

$$
\left(\frac{(b-\phi(\overline{e}))(1-\gamma)JFR}{c\vartheta}+\gamma JFR+JSR+r\right)\widehat{z} = ((r+JSR)\eta+\gamma JFR)\,\widehat{\vartheta}
$$

Given the definition of aggregate productivity $\overline{z}=z\left(R+\frac{\int_R^1[1-G(x)]dx}{1-G(R)}\right)$, and the definition of the replacement rate

$$
\begin{aligned}
\widetilde{b}_{\overline{e}}\equiv\frac{b-\phi(\overline{e})}{\overline{z}} &= \frac{b-\phi(\overline{e})}{\frac{(\gamma JFR+JSR-\lambda)c\vartheta+(1-\gamma)JFR(b-\phi(\overline{e}))}{(1-\gamma)JFR}+(r+\lambda)\frac{c\vartheta}{(1-\gamma)JFR}}\\
&= \frac{\left(\frac{b-\phi(\overline{e})}{c}\right)(1-\gamma)JFR\lambda}{(\gamma JFR+JSR)(r+\lambda)\vartheta+(1-\gamma)JFR\lambda\frac{b-\phi(\overline{e})}{c}-(1-\gamma)JFRr\frac{zR-(b-\phi(\overline{e}))}{c}}
\end{aligned}
$$

we obtain:

$$
\frac{(b-\phi(\overline{e}))(1-\gamma)JFR}{c\vartheta}=\frac{\widetilde{b}_{\overline{e}}(r+\lambda)(JSR+\gamma JFR)-\widetilde{b}_{\overline{e}}r\Gamma}{(1-\widetilde{b}_{\overline{e}})\lambda}
$$

where $\Gamma=(1-\gamma)JFRr\frac{zR-(b-\phi(\overline{e}))}{c\vartheta}=\gamma JFR+JSR-\lambda$. We then get the equation on the right-hand side of (16) where $\widetilde{b}_{\overline{e}}=\widetilde{b}-\widetilde{\phi}(\overline{e})$. The expression of $\widetilde{b}_{\overline{e}}$ is similar to equation (15) in Nagypal & Mortensen (2007a)'s paper.

With endogenous search effort. After using the FOC on search effort in both the JC and the

JD curves, the equilibrium is defined by:

$$z\left(R+\frac{\lambda}{r+\lambda}\int_R^1[1-G(x)]dx\right) = b+\frac{\phi}{1+\phi}\frac{\gamma}{1-\gamma}c\vartheta \qquad (JD)$$

$$c\left(\frac{1-\gamma}{\gamma c}\right)^{\frac{\eta}{1+\phi}}\vartheta^{\frac{\eta\phi}{1+\phi}} = (1-\gamma)\frac{z}{r+\lambda}\int_R^1[1-G(x)]dx \qquad (JC)$$

where $\vartheta=\frac{v}{u}$ and thus $\vartheta=e\theta$. Log-linear approximations of these two equations lead to:

$$\left(R(r+\lambda)+\lambda\int_R^1[1-G(x)]dx\right)\widehat{z}+(r+JSR)R\widehat{R} = \frac{\phi}{1+\phi}\frac{\gamma}{1-\gamma}\frac{c\vartheta}{z}(r+\lambda)\widehat{\vartheta}$$

$$\eta\frac{\phi}{1+\phi}\widehat{\vartheta} = \widehat{z}-\frac{1-G(R)}{\int_R^1[1-G(x)]dx}R\widehat{R}$$

Using (JC) and (JD) conditions at the steady state,

$$\left.\begin{array}{rcl}(r+\lambda)\frac{c\vartheta}{(1-\gamma)JFR} &=& z\frac{\int_R^1[1-G(x)]dx}{1-G(R)}\\ (r+\lambda)\frac{zR-b}{\gamma\frac{\phi}{1+\phi}JFR+JSR-\lambda} &=& z\frac{\int_R^1[1-G(x)]dx}{1-G(R)}\end{array}\right\}\Rightarrow\left\{\begin{array}{rcl}zR &=& \frac{(\gamma\frac{\phi}{1+\phi}JFR+JSR-\lambda)c\vartheta+(1-\gamma)JFRb}{(1-\gamma)JFR}\\ c\vartheta &=& \frac{(1-\gamma)JFR(zR-b)}{\gamma\frac{\phi}{1+\phi}JFR+JSR-\lambda}\end{array}\right.$$

and their log-linear approximations, we deduce :

$$\left(\frac{b(1-\gamma)JFR}{c\vartheta}+\gamma\frac{\phi}{1+\phi}JFR+JSR+r\right)\widehat{z} = \frac{\phi}{1+\phi}\left((r+JSR)\eta+\gamma JFR\right)\widehat{\vartheta}$$

Given the definitions of aggregate productivity and replacement rate

$$\widetilde{b}\equiv\frac{b}{\overline{z}} = \frac{\left(\frac{b}{c}\right)(1-\gamma)JFR\lambda}{\left(\gamma\frac{\phi}{1+\phi}JFR+JSR\right)(r+\lambda)\vartheta+(1-\gamma)JFR\lambda\frac{b}{c}-(1-\gamma)JFRr\frac{zR-b}{c}}$$

we get:

$$\frac{b(1-\gamma)JFR}{c\vartheta}=\frac{\widetilde{b}(r+\lambda)\left(JSR+\gamma\frac{\phi}{1+\phi}JFR\right)-\widetilde{b}r\Gamma}{(1-\widetilde{b})\lambda}$$

where $\Gamma=(1-\gamma)JFRr\frac{zR-b}{c\vartheta}=\gamma\frac{\phi}{1+\phi}JFR+JSR-\lambda$. We then deduce the equation on the left-hand side of (16) where $\widetilde{b}=b/\overline{z}$. □

Proposition 2. *The elasticity of vacancy-to-unemployment ratio with respect to aggregate productivity is higher with endogenous search effort than with constant search effort, given the same calibration targets.*

Proof. For given values of observed data (r, JFR, and JSR), and parameters (γ, η, and b), we deduce from (16) that $\frac{(r+JSR)\frac{1+\phi}{\phi}+\gamma JFR}{(r+\lambda)\eta+\gamma JFR} > \frac{(r+JSR)+\gamma JFR}{(r+\lambda)\eta+\gamma JFR}$ and $\frac{1}{1-\widetilde{b}} > \frac{1}{1-\widetilde{b}+\widetilde{\phi}(\overline{e})}$, leading unambiguously to $\widehat{\vartheta}\Big|_{\text{variable } e} > \widehat{\vartheta}\Big|_{\text{constant } e}$. $\square$

Even if the calibrations of $\widetilde{b}\Big|_{\text{variable } e}$ and $\widetilde{b}\Big|_{\text{constant } e}$ are different in order to ensure that $\widetilde{b}\Big|_{\text{variable } e} = \widetilde{b} - \widetilde{\phi}(\overline{e})\Big|_{\text{constant } e}$, Proposition 2 holds.

Proposition 2 generalizes the result found by Gomme & Lkhagvasuren (2015) where separations are exogenous. The intuition behind this result is the following: in booms, an increase in productivity increases the value of a match. As a consequence, firms post more vacancies, which boosts workers' job finding rate, thereby raising workers' outside option (the value of being unemployed). With constant search effort, as the outside option increases, wages increase, thereby eating up the gains received by firms (due to increased productivity), thereby lowering the response of vacancy. With variable search effort, in booms, unemployed search more, which dampens the rise of the value of being unemployed, and so the wage increase. The business cycle response of the flow value of unemployment is a key determinant of the success of the DMP model.

Using these first results, we then analyze the volatility of worker flows with and without variable search effort.

Corollary 1. *There exists a set of realistic parameters such that the volatilities of the job separation and the job finding rates are larger in the model with endogenous search effort than in the model with constant search effort.*

Proof. Let us define the log deviations around the steady state values of the job finding rates in economies with variable and constant search effort, as respectively $\widehat{JFR}\Big|_{\text{var e}} = \widehat{e}+(1-\eta)\widehat{\theta}-\frac{G'(R)R}{1-G(R)}\widehat{R}$ and $\widehat{JFR}\Big|_{\text{const e}} = (1-\eta)\widehat{\vartheta} - \frac{G'(R)R}{1-G(R)}\widehat{R}$. Job separation rates have the same expression for the two economies $\widehat{JSR} = \varepsilon_{G|R}\widehat{R}$. Using the free entry conditions, $\widehat{\theta} = \frac{1}{\eta}(\widehat{z} - \varepsilon_{I|R}\widehat{R})$ and $\widehat{\vartheta} = \frac{1}{\eta}(\widehat{z} - \varepsilon_{I|R}\widehat{R})$, we deduce:

$$\begin{aligned}
\Delta\widehat{JSR} &= \frac{\varepsilon_{G|R}}{\varepsilon_{I|R}}\eta\left(-\frac{\phi}{1+\phi}\,\widehat{\vartheta}\Big|_{\text{var e}} + \widehat{\vartheta}\Big|_{\text{const e}}\right) \\
\Delta\widehat{JFR} &= \frac{\eta}{1+\phi}\widehat{\vartheta} + (1-\eta)\Delta\widehat{\vartheta} - \frac{G(R)}{1-G(R)}\varepsilon_{G|R}\Delta\widehat{R}
\end{aligned}$$

given that $\widehat{\theta} = \widehat{\vartheta} - \widehat{e}$ and $\widehat{e} = \frac{1}{\phi}\widehat{\theta}$.

Let us define $\varpi \in (0;1)$ such that $\varpi\frac{1}{1-b} = \frac{1}{1-b+\phi(\overline{e})}$. Using (16), we deduce that $\text{Sign}\left(\Delta\widehat{JSR}\right) = \text{Sign}\left((\varpi-1)(r+JSR) + \left(\varpi - \frac{\phi}{1+\phi}\right)\gamma JFR\right)$, and it is negative if $\frac{\varpi}{1-\varpi} < \phi$. Under this condition, the job separation rate is more countercyclical in a model with endogenous job search. This is then true for R, ie. $\Delta\widehat{R} < 0$. As a result, $\Delta\widehat{JFR}$ is unambiguously positive because Proposition 2 implies $\Delta\widehat{\vartheta} \geq 0$. Finally, the restrictions on the parameters allowing to obtain this equilibrium are realistic. Indeed, if we take a value for the same home production as Hall & Milgrom (2008), approximatively 0.7, and an elasticity of the job search effort close to one estimated by Christensen et al. (2005), approximatively 0.5, we deduce that $b = 0.9$ and $\phi(\overline{e}) = 0.2$ are admissible, by using $\frac{\varpi}{1-\varpi} < \phi$ and given that $b = 0.9$ is lower than the upper bound of the value in Hagedorn & Manovskii (2008). $\square$

Proposition 2 and Corollary 1 show that search effort allows the MP model to generate larger volatilities of worker flows than the models with constant search effort.[56]

Proposition 3. *While the correlation between unemployment and vacancies is ambiguous with constant search effort, it becomes negative for sufficiently high values of the elasticity of search effort.*

Proof. Given $\theta = \frac{1}{e}\vartheta \Rightarrow \widehat{\theta} = -\widehat{e} + \widehat{\vartheta}$ and using the log-linear approximation of the FOC on e (leading to $\widehat{e} = \frac{1}{\phi}\widehat{\theta}$), we deduce $\widehat{\theta} = \frac{\phi}{1+\phi}\widehat{\vartheta} = \frac{(r+JSR)+\gamma\frac{\phi}{1+\phi}JFR}{(r+\lambda)\eta+\gamma JFR}\frac{1}{1-\widehat{b}}\widehat{z}$, showing that the "efficient" labor market tightness is less volatile that the vacancies-unemployment ratio and is bounded $\widehat{\theta} \in \left[\frac{(r+JSR)}{(r+\lambda)\eta+\gamma JFR}\frac{1}{1-\widehat{b}}\widehat{z}; \frac{(r+JSR)+\gamma JFR}{(r+\lambda)\eta+\gamma JFR}\frac{1}{1-\widehat{b}}\widehat{z}\right]$ for respectively $\phi \to 0$ and $\phi \to \infty$. Hence, the dynamics of the Beveridge curve is determined by:

Variable search effort	Constant search effort
$\widehat{u} = (1-u)\left(\frac{\varepsilon_{G\|R}}{1-G(R)}\widehat{R} - (1-\eta)\widehat{\theta} - \frac{1}{\phi}\widehat{\theta}\right)$	$\widehat{u} = (1-u)\left(\frac{\varepsilon_{G\|R}}{1-G(R)}\widehat{R} - (1-\eta)\widehat{\vartheta}\right)$
$\widehat{v} = (1-(1-\eta)(1-u))\widehat{\theta} + u\frac{1}{\phi}\widehat{\theta} + \frac{(1-u)\varepsilon_{G\|R}}{1-G(R)}\widehat{R}$	$\widehat{v} = (1-(1-\eta)(1-u))\widehat{\vartheta} + \frac{(1-u)\varepsilon_{G\|R}}{1-G(R)}\widehat{R}$

given that $\widehat{u} = (1-u)\left(\widehat{JSR} - \widehat{JFR}\right)$. With constant search effort, if the volatility of the vacancy-unemployment ratio is small, $\widehat{v}$ can be countercyclical and driven by the dynamics of $\widehat{R}$ allowing to match the volatility of the JSR. With endogenous search effort, it exists ϕ such that v becomes procyclical for bounded values of $\widehat{\vartheta}$. $\square$

In the case with constant search effort, counter-cyclical separations can amplify the counter-cyclical responses of unemployment to aggregate productivity shocks, and the small response of the vacancy-unemployment ratio can lead to a counter-cyclical response in the vacancy rate ($\widehat{v} = \widehat{\vartheta} + \widehat{u}$). In contrast, with endogenous search effort, a sufficiently small value for ϕ (a high elasticity of search effort) ensures that the vacancy rate is pro-cyclical ($\widehat{v} = \widehat{\theta} + \widehat{e} + \widehat{u}$ with $\widehat{e} = \frac{1}{\phi}\widehat{\theta}$), leading then to a negative correlation between v and u.

Gomme & Lkhagvasuren (2015) consider only exogenous separation. They do not assess the role of variable search effort on other dimensions of the DMP model. This section fills this gap. In particular, Fujita & Ramey (2012) show that the MP model without search effort generates a counterfactual positive slope of the Beveridge curve, as it the case in Krause & Lubik (2007) in a New Keynesian DMP model with endogenous separations but without search effort. Beyond its amplifying properties,

[56]Note that the case with endogenous search effort converges to the case with constant e when $\phi \to \infty$.

we show how endogenous search effort helps the model with endogenous separation match the negative slope of the Beveridge curve.

D.2 Labor market elasticities in a life-cycle model

For simplicity, we assume that there are 3 age groups $i \in \{Y, A, O\}$ on the labor market. When an older worker exits the labor market, she is replaced by a young worker. Transition rates between ages are denoted by π_i.

D.2.1 Elasticity of vacancy-unemployment ratio with respect to aggregate productivity

Proposition 4. *In a life-cycle model, the elasticities of vacancy-unemployment ratio ($\vartheta_i \equiv \frac{v_i}{u_i}$) with respect to aggregate productivity z are:*

$$\begin{array}{ccc} & \textit{Variable Search Effort} & & \textit{Constant Search Effort} \\ \widehat{\vartheta}_i = & \frac{(\widetilde{r}_i+\lambda+\Omega_i(JSR_i-\lambda))\frac{1+\phi}{\phi}+\gamma_i JFR_i}{(\widetilde{r}_i+\lambda)\eta+\gamma_i JFR_i}\frac{1}{1-\widetilde{b}_i}\widehat{z} & \widehat{\vartheta}_i = & \frac{\widetilde{r}_i+\lambda+\Omega_i(JSR_i-\lambda)+\gamma_i JFR_i}{(\widetilde{r}_i+\lambda)\eta+\gamma_i JFR_i}\frac{1}{1-\widetilde{b}_i+\widetilde{\phi}_i(\overline{e})}\widehat{z} \end{array} \tag{17}$$

where $\widetilde{r}_i = r + \pi_i$, $\Omega_O = 1$ and $\Omega_i = 1 + \frac{\pi_i}{\widetilde{r}_{i+1}+\lambda}\Omega_{i+1}$ for $i < O$.

Proof. The surplus from a match at age i is given by $S_i(\epsilon) = \Omega_i\frac{z(\epsilon-R_i)}{\widetilde{r}_i+\lambda}$. These expressions for the surplus implies that $\overline{S}_i = \frac{z\Omega_i}{\widetilde{r}_i+\lambda}\int_{R_i}^1[1-G(z)]dz$ where R_i is the reservation productivity at age i.

With constant search effort, job destruction (JD) and job creation (JC) determine the equilibrium at age i:

$$z\left(R_i + \frac{\lambda\Omega_i}{\widetilde{r}_i+\lambda}\int_{R_i}^1[1-G(z)]dz\right) = b - \phi(\overline{e}) + \frac{\gamma}{1-\gamma}c\vartheta_i \tag{JD}$$

$$c\vartheta_i^\eta = (1-\gamma)\frac{z\Omega_i}{\widetilde{r}_i+\lambda}\int_{R_i}^1[1-G(z)]dz \tag{JC}$$

With endogenous search effort. The equilibrium is defined by

$$z\left(R_i + \frac{\lambda\Omega_i}{\widetilde{r}_i+\lambda}\int_{R_i}^1[1-G(x)]dx\right) = b + \frac{\phi}{1+\phi}\frac{\gamma}{1-\gamma}c\vartheta_i \tag{JD}$$

$$c\left(\frac{1-\gamma}{\gamma c}\right)^{\frac{\eta}{1+\phi}}\vartheta_i^{\frac{\eta\phi}{1+\phi}} = (1-\gamma)\frac{z\Omega_i}{\widetilde{r}_i+\lambda}\int_{R_i}^1[1-G(x)]dx \tag{JC}$$

Using exactly the same algebra than for the proof of proposition 1, we get (17), where $\widetilde{b}_i = b/\overline{z}_i$ and $\widetilde{\phi}_i(\overline{e}) = \phi(\overline{e})/\overline{z}_i$, given that aggregate productivity is $\overline{z}_i = z\left(R_i + \frac{\int_{R_i}^1[1-G(x)]dx}{1-G(R_i)}\right)$. □

For simplicity, we now focus on older workers' labor market and analyze the impact of their short horizon prior to retirement on the elasticity of this labor market segment.

Proposition 5. *Propositions 2 and 3 as well as Corollary 1 apply to older workers' labor market.*

Proof. Obviously, because $\Omega_O = 1$ and thus only the definition of r changes in (JC) and (JD). □

The properties of the infinitely-live agent model carry through the life-cycle model.

Proposition 6. *With the range of parameter values used in the calibrated model,*
(i) the elasticity of vacancy-employment ratio increases as the worker gets closer to retirement.
(ii) The impact of the shorter distance to retirement is larger in the model with endogenous search effort.

Proof. **With constant search effort.** Let us define $F(\widetilde{r}_O, JFR_O, JSR_O) \equiv \frac{\widetilde{r}_O+JSR_O+\gamma JFR_O}{(\widetilde{r}_O+\lambda)\eta+\gamma JFR_O}$ and $C \equiv \frac{1}{1-\widetilde{b}_O+\widetilde{\phi}_O(\overline{e})}$, leading to

$$\widehat{\vartheta}_O = F(\widetilde{r}_O, JFR_O, JSR_O) \times C \times \widehat{z} \tag{18}$$

C is considered as a constant because, even if $\overline{z}_O$ can change when parameters change, C keeps its initial value because parameters b and $\phi(\overline{e})$ must adjust in order to match the calibrated value of outside opportunities.

Distance to retirement, captured by π_O, has 2 effects on the multiplier $F(\widetilde{r}_O, JFR_O, JSR_O)$

- A direct effect captured by the first derivative of F with respect to $F(\widetilde{r}_O, JFR_O, JSR_O)$, denoted F_1'
- An indirect effect: π_O affects steady state values of JFR and JSR. Indeed, when π_O changes, the definition of older workers changes and thus the steady-state flows of these populations.

Therefore, we have

$$\frac{dF}{d\widetilde{r}_O} = F_1' + F_2' \frac{\partial JFR_O}{\partial \vartheta_O} \frac{d\vartheta}{d\widetilde{r}_O} + \left(F_2' \frac{\partial JFR_O}{\partial R_O} + F_3' \frac{\partial JSR_O}{\partial R_O} \right) \frac{dR_O}{d\widetilde{r}_O}$$

The direct effect is given by

$$F_1' = \frac{\partial F}{\partial \widetilde{r}_O} = 2 \frac{\lambda + (1/2)JFR_O - JSR_O}{[\widetilde{r} + \lambda + JFR_O]^2}$$

and is positive for our calibrated parameters (see Figure 5).

The indirect effect goes through steady state values of worker flows. Indeed, at the steady state, we have $JFR_O = p(\vartheta_O)[1 - G(R_O)]$ and $JSR_O = \lambda G(R_O)$, which depend on $\widetilde{r}_O$ though ϑ_O and R_O.

After differentiating (JC) and (JD), we get

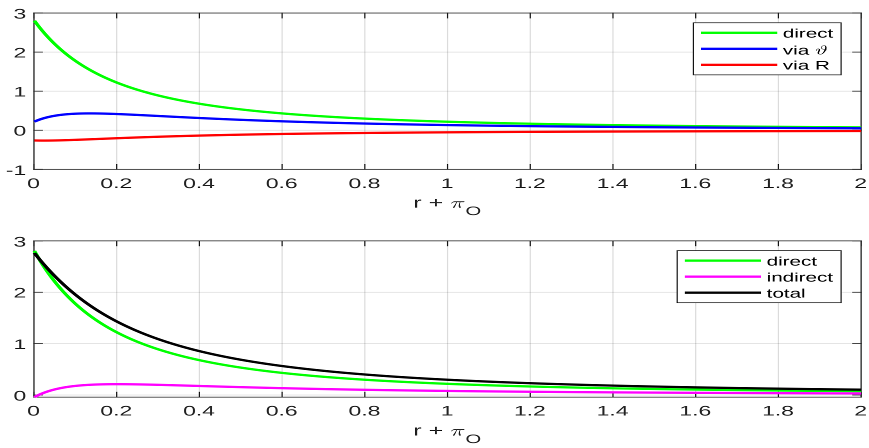
$$\begin{aligned} dR_O &= \frac{c}{JFR_O}\left(JFR_O + JSR_O - \lambda\right) d\vartheta_O \\ d\widetilde{r}_O &= -\frac{1}{\vartheta_O}\left[\eta\left(\widetilde{r}_O + JSR_O\right) + \gamma JFR_O\right] d\vartheta_O \end{aligned}$$

Normalizing $z = 1$ and assuming that $\gamma = \eta = 1/2$ as in the calibration (Table 4), these expressions lead to

$$\begin{aligned} F_2' \frac{\partial JFR_O}{\partial \vartheta_O} \frac{d\vartheta}{d\widetilde{r}_O} &= 2\frac{(1/2)(\widetilde{r}_O + \lambda) - JSR_O}{[\widetilde{r}_O + \lambda + JFR_O]^2} \frac{JFR_O}{\widetilde{r}_O + JSR_O + JFR_O} > 0 \\ \left(F_2' \frac{\partial JFR_O}{\partial R_O} + F_3' \frac{\partial JSR_O}{\partial R_O}\right) \frac{dR_O}{d\widetilde{r}_O} &= -2\frac{\frac{\widetilde{r}_O + \lambda}{1 - G_O}(JFR + 2\lambda - 2JSR) + 2\frac{JFR_O}{1 - G_O}(\lambda - 2JSR_O)}{[\widetilde{r}_O + \lambda + JFR_O]^2} \\ &\quad \times \frac{\frac{c}{q(\vartheta_O)} \frac{G_O'}{1 - G_O}\left(JFR + JSR - \lambda\right)}{\widetilde{r}_O + JSR_O + JFR_O} < 0 \end{aligned}$$

where the signs are deduced from numerical analysis of these functions with our calibrated parameters (see Figure 5).

Figure 5: Elasticity of ϑ to z as function of the worker's horizon: exogenous e



All the parameters and job flows are those presented in the calibration (Table 4).

Figure 5 shows that, for the calibrated values of the model (Table 4), we always have $\frac{\partial F}{\partial \widetilde{r}_O} > 0$: a shorter horizon (a rise of $\widetilde{r}_O$), increases the labor market sensitivity to the business cycle.

With variable search effort. Let us define $\mathcal{G}(\widetilde{r}_O, JFR_O, JSR_O) \equiv \frac{(\widetilde{r}_O + JSR_O)\frac{1+\phi}{\phi} + \gamma JFR_O}{(\widetilde{r}_O + \lambda)\eta + \gamma JFR_O}$ and $C' \equiv \frac{1}{1 - \widetilde{b}_O}$, leading to

$$\widehat{\vartheta}_O = \mathcal{G}(\widetilde{r}_O, JFR_O, JSR_O) \times C' \times \widehat{z} \tag{19}$$

As previously, C' is kept constant in the calibration procedure and does not change when the worker's horizon shortens.

The function $\mathcal{G}$ can be decomposed as follows:

$$\mathcal{G} = F + \underbrace{\frac{1}{\phi}\frac{\widetilde{r}_O + JFR_O}{(\widetilde{r}_O + \lambda)\eta + \gamma JFR_O}}_{=g(\widetilde{r}_O, JFR_O)}$$

Distance to retirement, captured by π_O, has 2 effects on the multiplier $\mathcal{G}$

- A direct effect captured by $\mathcal{G}_1'$
- An indirect effect as π_O affects steady state values of JFR and JSR.

Assuming that $\gamma = \eta$ (as in the calibration, Table 4), the direct impact of the worker's horizon on the elasticity is given by

$$\mathcal{G}_1' = F_1' + g_1' = 2\frac{\lambda + (1/2)JFR_O - JSR_O}{[\widetilde{r} + \lambda + JFR_O]^2} + \frac{2}{\phi}\frac{\lambda}{r_O + \lambda + JFR_O} > F_1' > 0$$

Therefore, endogenous search effort magnifies the impact of the worker's horizon changes on the elasticity of the labor market tightness. This comes from the complementarity between search on both sides of the market: if firms invest less in vacancies because the horizon is too short to recoup hiring costs, then unemployed workers face a lower incentive to search for a job.

As for the indirect effect, any change in $\widetilde{r}_O$ modifies the steady state values of JFR_O and JSR_O, given that $JFR_O = \left(c\frac{\gamma}{1-\gamma}\right)^{\frac{\eta}{1+\phi}} \vartheta_O^{\frac{1+\phi(1-\eta)}{1+\phi}} [1 - G(R_O)]$ when the search effort is endogenous, implying $\frac{\partial JFR_O}{\partial \vartheta_O} = \frac{1+\phi(1-\eta)}{1+\phi}\frac{JFR_O}{\vartheta_O}$ and $\frac{\partial JFR_O}{\partial \vartheta_O} = -JFR_O\frac{G_O}{1-G_O}\frac{G_O'}{G_O}$. By differentiating the equations (JC) and (JD), we get

$$\begin{aligned}
\frac{dR_O}{d\vartheta_O} &= \frac{\phi}{1+\phi}\frac{c}{JFR_O}(JFR_O + JSR_O - \lambda) \\
\frac{d\vartheta_O}{d\widetilde{r}_O} &= -\frac{1+\phi}{\phi}2\vartheta_O\frac{1}{\widetilde{r}_O + JSR_O + JFR_O}
\end{aligned}$$

Using these expressions, we deduce that the impact of changes in ϑ_O is given

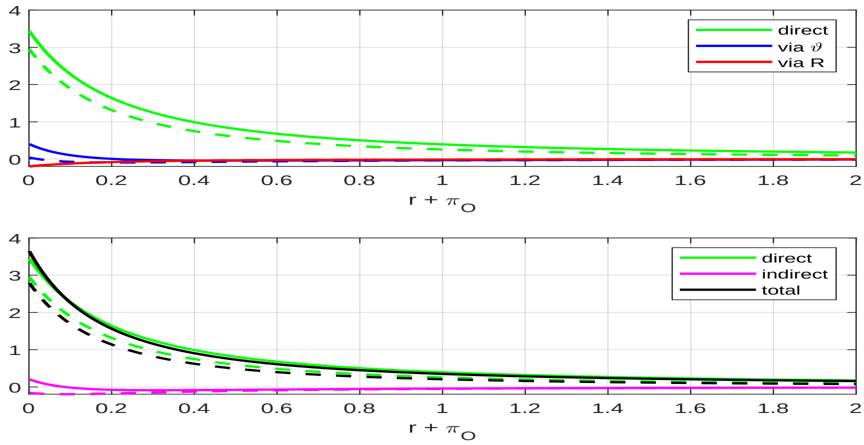
$$\mathcal{G}_2'\frac{\partial JFR_O}{\partial \vartheta_O}\frac{d\vartheta_O}{d\widetilde{r}_O} = 2\frac{(1/2)\widetilde{r}_O + \lambda((1/2) - (1/\phi)) - JSR_O}{(\widetilde{r}_O + \lambda + JFR_O)^2}\frac{(2+\phi)JFR_O}{\phi(\widetilde{r}_O + JSR_O + JFR_O)}$$

The impact of the changes in R_O is given by the sum of two terms:

$$\begin{aligned}
(F_2' + g_2')\frac{\partial JFR_O}{\partial R_O}\frac{dR_O}{d\widetilde{r}_O} &= -4\frac{JFR_O}{1-G_O}\left(\frac{(1/2)\widetilde{r}_O + \lambda((1/2) - (1/\phi)) - JSR_O}{(\widetilde{r}_O + \lambda + JFR_O)^2}\right)\left(\frac{c}{q_O}\frac{G_O'}{1-G_O}\right)\left(\frac{JFR_O + JSR_O - \lambda}{\widetilde{r}_O + JSR_O + JFR_O}\right) \\
F_3'\frac{\partial JSR_O}{\partial R_O}\frac{dR_O}{d\widetilde{r}_O} &= -4\lambda\left(\frac{1}{\widetilde{r} + \lambda + JFR_O}\right)\left(\frac{c}{q_O}\frac{G_O'}{1-G_O}\right)\left(\frac{JFR_O + JSR_O - \lambda}{\widetilde{r}_O + JSR_O + JFR_O}\right)
\end{aligned}$$

Figure 6 shows that the indirect effect is dominated by the direct effect: a shorter horizon increases the sensitivity of the labor market tightness to the business cycle. Figure 6 also shows that the interaction between the incentives for firms to invest in vacancies and the incentives for workers to search for a job magnifies the impact of the worker's horizon on the elasticity of the labor market

Figure 6: Elasticity of ϑ to z as function of the worker's horizon: endogenous e



All the parameters and job flows are those presented in the calibration (Table 4). The dotted lines correspond to a calibration with low elasticity of the search effort $\phi = 2$.

tightness.

□

D.2.2 Elasticity of reservation productivity with respect to aggregate productivity

Corollary 2. *With the range of parameter values used in the calibrated model, a fall in the worker's horizon magnifies the sensitivity of reservation productivity R_O to the business cycle.*

Proof. The elasticity of reservation productivity with respect to aggregate productivity is given by

$$\widehat{R}_O = \underbrace{-\frac{I(R_O)}{[1-G(R_O)]R_O}\left(\frac{\eta\phi}{1+\phi}\mathcal{G}\times C' - 1\right)}_{=\Psi}\widehat{z}$$

which, using (19), can be written as

$$\widehat{R}_O = \underbrace{-\frac{I(R_O)}{[1-G(R_O)]R_O}\left(\frac{\eta\phi}{1+\phi}\frac{\widehat{\vartheta}}{\widehat{z}} - 1\right)}_{=\Psi}\widehat{z} \tag{20}$$

As in Nagypal & Mortensen (2007a)'s paper, we can ensure that the elasticity of the reservation productivity to the business cycle is negative (ie. $\mathcal{G}\times C' > \frac{1+\phi}{\eta\phi}$). It is the case with our calibration. In addition, in equation (20), as in Nagypal & Mortensen (2007a)'s paper, the elasticity of reservation productivity with respect to aggregate productivity also depends on the elasticity of vacancy-unemployment ratio $\frac{\widehat{\vartheta}}{\widehat{z}}$, which is larger with variable search effort and magnifies the effects of the distance to retirement on worker's elasticity (Proposition 6). This illustrates the intuition behind the idea that the model correctly predicts the labor market volatility by age group because of the interaction between endogenous search effort and separation on the one hand, and life-cycle features on the other hand.

Let us now demonstrate formally this point. For simplicity, assuming $G(x) = x$ and setting $\gamma = \eta$ (as in the calibration), we obtain $\widehat{R}_O = -\Psi\widehat{z}$, with $\Psi = \frac{1-R_O}{2R_O}\left(\frac{\phi}{2(1+\phi)}\mathcal{G} \times C' - 1\right)$, it is easy to deduce that $\frac{\partial\Psi}{\partial R_O} = -\frac{2}{4R_O^2} < 0$, $\frac{\partial R_O}{\partial \widetilde{r}_0} = -\frac{2c\vartheta_O}{JFR_O}\left(\frac{JFR_O+JSR_O-\lambda}{\widetilde{r}_O+JSR_O+JFR_O}\right) < 0$ and $\frac{\partial\Psi}{\partial\Theta} = \frac{1-R_O}{2R_O}\frac{\phi}{2(1+\phi)} > 0$. Using the previous analysis of $\mathcal{G} \times C'$, we get $\frac{d\Psi}{d\widetilde{r}_O} = \frac{\partial\Psi}{\partial R_O}\frac{\partial R_O}{\partial\widetilde{r}_0} + \frac{\partial\Psi}{\partial\Theta}\frac{\partial\Theta}{\partial\widetilde{r}_0} > 0$. This elasticity (in absolute value) increases when the worker's horizon is shorter. □

D.2.3 Bargaining power and elasticity of the labor market

In our calibrated model (section 4.2.2.), we introduce a lower bargaining power for young workers. We derive here, in our simplified model, the impact of changes in young workers' bargaining power on labor market elasticity. Our analytical results are consistent with the simulation results found in section 4.2.2.

Proposition 7. *For our range of calibration, a fall in worker's bargaining power reduces the responsiveness of the vacancy-employment ratio to aggregate shocks.*

Proof. Let us define $H(\gamma_i, JFR_i, JSR_i) \equiv \frac{(\widetilde{r}_i+\lambda+\Omega_i(JSR_i-\lambda))\frac{1+\phi}{\phi}+\gamma_i JFR_i}{(\widetilde{r}_i+\lambda)\eta+\gamma_i JFR_i}$ and $C'' \equiv \frac{1}{1-\widetilde{b}_i}$, leading to $\widehat{\vartheta}_i = H(\gamma_i, JFR_i, JSR_i) \times C'' \times \widehat{z}$ where C'' is kept constant during the calibration procedure. For $i = Y$, the signs of H_1' and H_2' are ambiguous, whereas $H_3' > 0$. However, using our calibration (Table 4), we have $H_1' < 0$ and $H_2' < 0$. Therefore, two opposite forces are at work:

- $H_1' < 0$ refers to Hagedorn & Manovskii (2008)'s effect, ie. a low bargaining power increases the elasticity,
- whereas $H_2' < 0$ and $H_3' > 0$ refer to the indirect effect of a change in γ_Y through the change in the steady state values of JFR_Y and JSR_Y.

The total effect is given by

$$\frac{\partial H}{\partial\gamma_Y} = H_1' + H_2'\frac{\partial JRF_Y}{\partial\vartheta_Y}\frac{\partial\vartheta_Y}{\partial\gamma_Y} + \left(H_2'\frac{\partial JRF_Y}{\partial R_Y} + H_3'\frac{\partial JSF_Y}{\partial R_Y}\right)\frac{\partial R_Y}{\partial\gamma_Y}$$

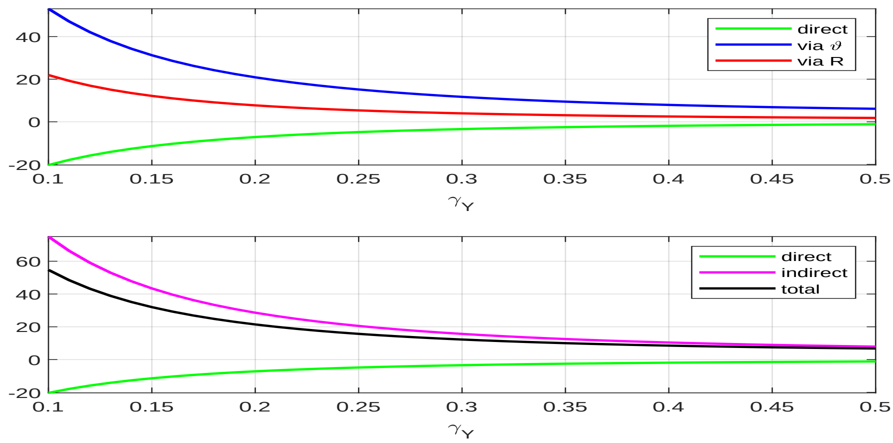
The impact of a change in γ_Y on the steady state is obtained by differentiating the system (JC)-(JD).

For simplicity, we assume that G is an uniform distribution. $\frac{\partial \vartheta_Y}{\partial \gamma_Y}$ and $\frac{\partial R_Y}{\partial \gamma_Y}$ are deduced from

$$\begin{aligned}
\frac{\phi}{1+\phi}\frac{1}{1-\gamma_Y}c\left(\frac{\vartheta_Y}{1-\gamma_Y}d\gamma_Y+\gamma_Y d\vartheta_Y\right) &= \left(1-\lambda\frac{\Omega_Y}{\widetilde{r}_Y+\lambda}(1-R_Y)\right)dR_Y \\
\frac{\eta}{1+\phi}c\left(\frac{1-\gamma_Y}{\gamma_Y c}\right)^{\frac{\eta}{1+\phi}}\vartheta_Y^{\frac{\eta\phi}{1+\phi}}\left(\frac{1}{\gamma_Y(1-\gamma_Y)}d\gamma_Y+\frac{\phi}{\vartheta_Y}d\vartheta_Y\right) &= -\frac{\Omega_Y}{\widetilde{r}_Y+\lambda}\frac{(1-R_Y)^2}{2}d\gamma_Y \\
&\quad -(1-\gamma_Y)\frac{\Omega_Y}{\widetilde{r}_Y+\lambda}(1-R_Y)dR_Y
\end{aligned}$$

Therefore, using the definitions of worker flows, we obtain the numerical results reported in Figure 7, showing that $\frac{\partial H}{\partial \gamma_Y} > 0$ for our range of calibration. Figure 7 shows that this results is robust to the choice of the value of γ_Y. □

Figure 7: Sensitivity of the elasticity of ϑ to z to worker's bargaining power ($\frac{\partial H}{\partial \gamma_Y}$) as a function of the worker's bargaining power (γ_Y): endogenous e



All the parameters and job flows are those presented in the calibration (Table 4). Worker's bargaining power (γ_Y) varies in the interval $[0.1; 0.5]$.

First, Hagedorn & Manovskii (2008)'s effect is always negative: this direct effect implies that a fall in worker's bargaining power increases the elasticity of ϑ to z. However, the indirect effect (via the changes in steady state values of ϑ_Y and R_Y) overcompensates the direct effect. Therefore, a fall in worker's bargaining power reduces the responsiveness of the vacancy-employment ratio ϑ to aggregate shocks z. This analytical result is consistent with the simulation results found in section 4.2.2.

E The Model with Life Cycle Features

This section provides additional material on the model described in section 3 of the paper. We assume again that $\phi(e) = \frac{e^{1+\phi}}{1+\phi}$ and $M(v, eu) = v^{1-\eta}(eu)^\eta$, with $\phi > 0$ and $\eta \in (0; 1)$.

E.1 Steady state surplus

The surplus function is defined by:

$$S_i(z,\epsilon) = \max\left\{ \begin{array}{l} zh_i(\epsilon - R_i(z)) + \beta\pi_i(1-\lambda_i)(1-s_e)E_z[S_i(z',\epsilon) - S_i(z',R_i(z))] \\ +\beta(1-\pi_i)(1-\lambda_{i+1})(1-s_e)E_z[S_{i+1}(z',\epsilon) - S_{i+1}(z',R_i(z))] \end{array} ;0 \right\}$$

Thus, at age $i+1$ and for $\epsilon = R_i(z)$, we have, at the conditional steady state:

$$S_{i+1}(z,R_i(z)) = \max\left\{ \begin{array}{l} zh_{i+1}(R_i(z) - R_{i+1}(z)) + \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)S_{i+1}(z,R_i(z)) \\ +\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)[S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))] \end{array} ;0 \right\}$$

Assuming that $S_{i+1}(z,R_i(z)) > 0$, we obtain:

$$\begin{aligned} S_{i+1}(z,R_i(z)) &= \frac{zh_{i+1}(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)}[S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))] \end{aligned}$$

For age $i+2$, we then have:

$$\begin{aligned} S_{i+2}(z,R_i(z)) &= \frac{zh_{i+2}(R_i(z) - R_{i+2}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+2}(z))] \end{aligned}$$

We deduce the value for $S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))$, which is:

$$\begin{aligned} S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z)) &= \frac{zh_{i+2}(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+1}(z))] \end{aligned}$$

Introducing this result in the expression of $S_{i+1}(z,R_i(z))$, we obtain:

$$\begin{aligned} S_{i+1}(z,R_i(z)) &= \frac{zh_{i+1}(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \left[\frac{zh_{i+2}(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \right. \\ &\left. + \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+1}(z))] \right] \end{aligned}$$

This leads to $S_{i+1}(z,R_i(z)) = \Omega_{i+1}z(R_i(z) - R_{i+1}(z))$. More generally, the surplus is given by $S_i(z,\epsilon) = \Omega_i z(\epsilon - R_i(z))$, $\forall \epsilon \geq R_i(z)$, where $\Omega_i = a_i\left\{h_i + a_{i+1}b_{i+1}\left[h_{i+1} + a_{i+2}b_{i+2}(\ldots)\right]\right\}$ with $a_i = \frac{1}{1-\beta\pi_i(1-\lambda_i)(1-s_e)}$ and $b_i = \beta(1-\pi_i)(1-\lambda_{i+1})(1-s_e)$ and until $i+n \leq O_7$. Thus, we have, e.g., $\Omega_{O_T} = \frac{h_{O_T}}{1-\beta\pi_{O_T}(1-\lambda_{O_T})(1-s_e)}$ and $\Omega_{O_{T-1}} = \frac{1}{1-\beta\pi_{O_{T-1}}(1-\lambda_{O_{T-1}})(1-s_e)}\left[h_{O_{T-1}} + h_{O_T}\frac{\beta(1-\pi_{O_{T-1}})(1-\lambda_{O_T})(1-s_e)}{1-\beta\pi_{O_T}(1-\lambda_{O_T})(1-s_e)}\right]\ldots$

E.2 Model solution: a block recursive equilibrium

Proposition 8. *The equilibrium is block recursive.*

Proof. As in Menzio & Shi (2010), if we find a fixed point for $S_{O_T}(z,\epsilon)$, which is a function of choices at age O_T (the terminal age) only, we then obtain $S_{O_T}(z,\epsilon)$, $\theta_{O_T}(z)$, $R_{O_T}(z)$, and $e_{O_T}(z)$ $\forall z,\epsilon$ using equations (2), (3), and (4). Given these solutions for the labor market for age-O_T workers, we can solve for the age-O_{T-1} workers using the equation system given in definition 1 until age $i = Y$. $\square$

E.3 Steady-state properties

At the steady-state, for age i, the model must generate an age-pattern of transition rates such that

$$\begin{aligned} JSR_i &\approx s_e + (1-s_e)\lambda_i G(R_i) > JSR_{i+1} && (21)\\ JFR_i &\approx e_i p(\theta_i)[1-G(R_i)] > JFR_{i+1} && (22)\end{aligned}$$

At the conditional steady state, we have (we omit z for the sake of simplifying the notations):

$$\begin{aligned} \frac{c}{q(\theta_i)} &= (1-\gamma_i)\beta\pi_i\overline{S}_i \quad (JC) && (23)\\ R_i &= b + \Sigma_i - \Lambda_i - \Gamma_i(R_i) \quad (JD) && (24)\end{aligned}$$

where Σ_i, Λ_i and $\Gamma_i(R_i)$ are given by (7), (8) and (9).

Proposition 9. *In the data, we observe* **(i)** *by $JSR_i > JSR_{i+1}$ and* **(ii)** *$JFR_i > JFR_{i+1}$.* **(i)** *is compatible with the steady state equilibrium of the model if $\frac{\lambda_{i+1}}{\lambda_i} < \frac{G(R_i)}{G(R_{i+1})}$, i.e. for a sufficiently flat age dynamic of λ_i, allowing to rewrite $JSR_i > JSR_{i+1}$ as $R_i > R_{i+1}$.* **(ii)** *is compatible with the steady state equilibrium of the model if $\{e_i,\theta_i\} > \{e_{i+1},\theta_{i+1}\}$ and if the age pattern of search effort e and labor tightness θ dominates the age profile of reservation productivity R.*

Proof. Assume for simplicity that $\lambda_i = \lambda_{i+1}$, an extreme case where the age dynamic of λ_i is sufficiently flat. Straightforward from Equation (10) for $R_i > R_{i+1}$. For $JFR_i > JFR_{i+1}$, as $R_i > R_{i+1}$, it is also straightforward from Equation (11) that $e_i p(\theta_i) > e_{i+1}p(\theta_{i+1})$ is needed. Given that $\phi'(e_i(z)) = \frac{\gamma_i}{1-\gamma_i}c\theta_i(z)$, we deduce that e_i and θ_i share the same dynamics. Hence $\{e_i,\theta_i\} > \{e_{i+1},\theta_{i+1}\}$ is needed. From Equation (11), it is obvious that the $\{e_i,\theta_i\} > \{e_{i+1},\theta_{i+1}\}$ may be not sufficient to compensate for $R_i > R_{i+1}$. The job finding rate (Equation (22)) declines with worker age if the fall in e_i and θ_i dominates the decline in R_i; i.e. if $[e_i p(\theta_i) - e_{i+1}p(\theta_{i+1})]\int_{R_{i+1}}^{1} dG(x) > e_i p(\theta_i)\int_{R_{i+1}}^{R_i} dG(x)$. $\square$

As the value of a match is determined by a single variable, its surplus, the observed age-decreasing pattern of the worker flows may be puzzling. The following proposition decomposes the main forces at work in the agent behaviors.

Proposition 10. *If the "search value" is larger than the "labor hoarding value", i.e. if $\gamma_i e_i p(\theta_i) > (1-s_e)\lambda_i$ $\forall i$, then $R_i > R_{i+1}$. If the "horizon effect" dominates the "selection effect", i.e. if $\overline{S}_i > \overline{S}_{i+1}$, and if human capital accumulation is moderate, then $\{e_i,\theta_i\} > \{e_{i+1},\theta_{i+1}\}$.*

Proof. Under the restriction on λ_i given by Proposition 9, $R_i > R_{i+1}$ ensures that $JSR_i > JSR_{i+1}$. Using (24), we deduce that we have $R_i > b$ as long as $\Sigma_i > \Lambda_i + \Gamma_i(R_i)$. If we consider the marginal job, we have $\Gamma_i(R_i) \to 0$. Thus, a sufficient condition for $R_i > R_{i+1}$ is $\Sigma_i > \Lambda_i$. Using (7) and (8), this last condition becomes $\gamma_i e_i p(\theta_i) > (1 - s_e)\lambda_i$, $\forall i$.

From Equation (22), we deduce that the decline in the reservation productivity must be dominated by the a large decline in search efforts ($\{e_i; \theta_i\}$) in order to generate $JFR_i > JFR_{i+1}$ (See Proposition 9). $\{e_i, \theta_i\}$ are positively linked to the expected surplus $\overline{S}_i$. If $R_i > R_{i+1}$, then $S_i(\epsilon) = \Omega_i(\epsilon - R_i)$ (see appendix E.1). We have $\overline{S}_i = \int_{R_i}^1 S_i(x)dG(x) = \Omega_i \int_{R_i}^1 (x - R_i)dG(x) = \Omega_i \int_{R_i}^1 [1 - G(x)]dx$. Finally, the age profile of the expected surplus can be defined as follows:

$$\overline{S}_i - \overline{S}_{i+1} = \underbrace{(\Omega_i - \Omega_{i+1}) \int_{R_i}^1 [1 - G(x)]dx}_{\text{“horizon effect”}} - \underbrace{\Omega_{i+1} \int_{R_{i+1}}^{R_i} [1 - G(x)]dx}_{\text{“selection effect”}}$$

The expected surplus is age-decreasing when the horizon effect dominates the selection effect. A necessary condition is that the human capital accumulation is not too strong. Indeed, $\Omega_i > \Omega_{i+1}$ iff

$$R1: \quad \frac{h_{i+1} - h_i}{h_i} = \delta_i < \widetilde{\delta}_i \equiv \frac{\beta(1 - \pi_i)(1 - \lambda_{i+1})(1 - s_e)}{1 - \beta(1 - \pi_i)(1 - \lambda_{i+1})(1 - s_e)}$$

The restriction $R1$ is necessary to get a decreasing age-pattern for the expected surplus. In this case ($\overline{S}_i > \overline{S}_{i+1}$), we have $\{e_i; \theta_i\} > \{e_{i+1}; \theta_{i+1}\}$. □

E.4 Stock-flow dynamics

E.4.1 Levels of employment and unemployment

The number of age-i workers employed during period t in a firm such that $\tau \in [R_{i,t}, x]$, is $n_i(z, x) = \int_{R_i(z)}^x \mu(\tau)d\tau$, where $\mu(\tau)$ the number of firms with a productivity z. This stock of jobs evolves as follows:

$$\begin{array}{rcl} & & \text{If } i = Y \\ n_Y(z', x) & = & \pi_Y \left[\begin{array}{l} [(1 - s_e)\lambda_Y(m_Y - u_Y(z)) + e_Y(z)p(\theta_Y(z))u_Y(z)][G(x) - G(R_Y(z'))] \\ +(1 - s_e)(1 - \lambda_Y)[n_Y(z, x) - n_Y(z, R_Y(z'))] \end{array} \right] \end{array} \tag{25}$$

$$\begin{array}{rcl} & & \text{If } i \neq Y \\ n_i(z', x) & = & \pi_i \left[\begin{array}{l} [(1 - s_e)\lambda_i(m_i - u_i(z)) + e_i(z)p(\theta_i(z))u_i(z)][G(x) - G(R_i(z'))] \\ +(1 - s_e)(1 - \lambda_i)[n_i(z, x) - n_i(z, R_i(z'))] \end{array} \right] \\ & & +(1 - \pi_{i-1}) \left[\begin{array}{l} [(1 - s_e)\lambda_{i+1}(m_{i-1} - u_{i-1}(z)) + e_i(z)p(\theta_i(z))u_{i-1}(z)][G(x) - G(R_i(z'))] \\ +(1 - s_e)(1 - \lambda_{i+1})[n_{i-1}(z, x) - n_{i-1}(z, R_i(z'))] \end{array} \right] \end{array} \tag{26}$$

where, as in Hairault et al. (2010), we assume that when worker ages (from $i-1$ to i), his job contact probability ($e_i(z)p(\theta_i(z))$), and his reservation productivity $R_i(z)$ are those of a worker of age i.

E.4.2 Unemployment and employment rates

The dynamics of unemployment rates by age are given by $u_i(z) = m_i - n_i(z,1) \Leftrightarrow u_i^r(z) \equiv \frac{u_i(z)}{m_i} = 1 - \frac{n_i(z,1)}{m_i} \equiv 1 - n_i^r(z,1)$, $\forall i, z$. The dynamics of employment rates are given by

$$\begin{aligned}
& \text{If } i = Y \\
n_Y^r(z',x) &= \pi_Y \left[\begin{array}{l} [(1-s_e)\lambda_Y(1-u_Y^r(z)) + e_Y(z)p(\theta_Y(z))u_Y^r(z)][G(x) - G(R_Y(z'))] \\ +(1-s_e)(1-\lambda_Y)[n_Y^r(z,x) - n_Y^r(z,R_Y(z'))] \end{array} \right] \qquad (27) \\
& \text{If } i \neq Y \\
n_i^r(z',x) &= \pi_i \left[\begin{array}{l} [(1-s_e)\lambda_i(1-u_i^r(z)) + e_i(z)p(\theta_i(z))u_i^r(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda_i)[n_i^r(z,x) - n_i^r(z,R_i(z'))] \end{array} \right] \\
&\quad +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [(1-s_e)\lambda_{i+1}(1-u_{i-1}^r(z)) + e_i(z)p(\theta_i(z))u_{i-1}^r(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda_{i+1})[n_{i-1}^r(z,x) - n_{i-1}^r(z,R_i(z'))] \end{array} \right] \qquad (28)
\end{aligned}$$

Given equations (27) and (28), $G(1) = 1$ and $u_i^r(z) = 1 - n_i^r(z,1)$, we obtain for $i = Y$ and $i \neq Y$

$$\begin{aligned}
u_Y^r(z') &= \pi_Y \left[\begin{array}{l} [1 - e_Y(z)p(\theta_Y(z))(1 - G(R_Y(z')))]u_Y^r(z) \\ +(1-s_e)(1-\lambda_Y)n_Y^r(z,R_1(z')) \\ +[s_e + (1-s_e)\lambda_Y G(R_Y(z'))](1 - u_Y(z)) \end{array} \right] + (1-\pi_{O_T})\frac{m_{O_T}}{m_Y} \qquad (29) \\
u_i^r(z') &= \pi_i \left[\begin{array}{l} [1 - e_i(z)p(\theta_i(z))(1 - G(R_i(z')))]u_i^r(z) \\ +(1-s_e)(1-\lambda_i)n_i^r(z,R_i(z')) \\ +[s_e + (1-s_e)\lambda_i G(R_i(z'))](1 - u_i^r(z)) \end{array} \right] \\
&\quad +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [1 - e_i(z)p(\theta_i(z))(1 - G(R_i(z')))]u_{i-1}^r(z) \\ +(1-s_e)(1-\lambda_{i+1})n_{i-1}^r(z,R_i(z')) \\ +[s_e + (1-s_e)\lambda_{i+1} G(R_i(z))](1 - u_{i-1}^r(z)) \end{array} \right] \qquad (30)
\end{aligned}$$

Unemployed workers of age i in period $t+1$ are those of age i in period t who do not age, and

- who do not find a job (first term of the first line of the right-hand side of equations (29) and (30)),
- employed workers of age i who lose their job in period $t+1$ due to a change in aggregate productivity leading to a change in the reservation productivity. When $R_i(z) < R_i(z')$, the number of obsolete jobs depends on job creations over the past. Obviously, if $R_i(z) > R_i(z')$, these jobs do not exist. (second term of the first line),
- the age-i employed workers who lose their jobs due to a separation, which can result from an exogenous reason with a probability s_e and from endogenous decisions with a probability $(1-s_e)\lambda_i G(R_i(z'))$ (first term of the second line),
- and new participants (last term of the second line).

Due to aging for those unemployed of age-i, there is a number of unemployed aged age-$i-1$ who age without finding a job (the last lines of (30)). Note that unemployment dynamics are a function of $n_i(z, R_i(z'))$ and $n_{i-1}(z, R_i(z'))$, which are themselves function of past values of unemployment. This underlines the interdependence between age-i unemployment stock and unemployment level at previous age $i-1$. Average unemployment rate is: $u_t^r = \sum_{i=1}^{T} u_{i,t}$.

E.4.3 Transition rates

The job finding rate (JFR) and the job separation rate (JSR) are respectively:

$$\begin{aligned}
JFR_i(z) &= \frac{e_i(z)p(\theta_i(z))(1-G(R_i(z'))\left[\pi_i u_i^r(z) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}u_{i-1}^r(z)\right]}{u_i^r(z)} \\
JSR_i(z) &= \frac{(1-s_e)(1-\lambda_i)\left[\pi_i n_i^r(z, R_i(z')) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}n_{i-1}^r(z, R_i(z'))\right]}{n_i^r(z,1)} \\
&\quad + \frac{[s_e + (1-s_e)\lambda_i G(R_i(z'))]\left[\pi_i(1-u_i^r(z)) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}(1-u_{i-1}^r(z))\right]}{n_i^r(z,1)}
\end{aligned}$$

In the basic infinite horizon model, we have $\pi_i = 1, \forall i$, $m_i = 1$, and $n_i(z, R_i(z')) = n_{i-1}(z, R_i(z')) = 0$ leading to $JFR(z) = e(z)p(\theta(z))(1-G(R(z'))$ and $JSR(z) = s_e + (1-s_e)\lambda_i G(R(z'))$. These definitions of worker flows have an empirical counterpart and are used by Fujita & Ramey (2012) to test the ability of the MP model to match labor market features. In the data, it is only possible to detect the worker's age before a labor market transition. Thus, we compute the transition rate conditionally on being of a given age prior to the labor market transition. In this case, all workers have "the same" age in our measure of the transition rates by age. The counterparts in the model are:

$$\begin{aligned}
JFR_i(z) &= e_i(z)p(\theta_i(z))[1-G(R_i(z'))] \\
JSR_i(z) &= \frac{(1-s_e)(1-\lambda_i)n_i^r(z, R_i(z')) + [s_e + (1-s_e)\lambda_i G(R_i(z'))]n_i^r(z,1)}{n_i^r(z,1)}
\end{aligned}$$

where $n_i^r(z,1) = 1 - u_i^r(z)$. We use this usual approximation of the worker flows per age in order to measure the ability of the theory to explain the observed data, computed using the same formula.

E.5 The derivation of the model elasticity to the business cycle

In order to decompose the impact of the aggregate productivity shock on $\{\theta_i, e_i, R_i\}$, we consider the following system:

$$(JC) \quad \left\{ \begin{aligned}
\frac{c}{q(\theta_i(z))} &= \beta\pi_i J_i(z) \\
J_i(z) &= zh_i X(R_i(z)) - w_i(z) + (1-\gamma_i)[(1-G(R_i(z)))\Lambda_i(z) + \Gamma_i(z)] \\
w_i(z) &= \gamma_i z h_i X(R_i(z)) + (1-\gamma_i)\left(bh_i + \Sigma_i(z)\right)(1-G(R_i(z)))
\end{aligned} \right.$$

$$(JD) \quad \left\{ \begin{aligned}
zh_i R_i(z) &= w_i(z, R_i(z)) - (1-\gamma_i)[\Lambda_i(z) + \Gamma_i(z, R_i(z))] \\
w_i(z, R_i(z)) &= \gamma_i z h_i R_i(z) + (1-\gamma_i)\left(bh_i + \Sigma_i(z)\right)
\end{aligned} \right.$$

where $J_i(z) = \int_{R_i(z)}^1 J_i(z,x)dG(x)$, $w_i(z) = \int_{R_i(z)}^1 w_i(z,x)dG(x)$, $X(R_i(z)) = \int_{R_i(z)}^1 x dG(x)$ and $\Gamma_i(z) = \int_{R_i(z)}^1 \Gamma_i(z,x)dG(x)$. The decision rule on θ leads to $p(\theta_i(z))\int_{R_i(z)}^1 S_i(z,x)dG(x) = \frac{1}{(1-\gamma_i)\beta\pi_i}c\theta_i(z)$.

The decision rule on e leads to $\phi'(e_i(z)) = \frac{\gamma_i}{1-\gamma_i} c\theta_i(z)$. Using the functional form, we obtain

$$\frac{e_i(z)^{1+\phi}}{1+\phi} = \frac{1}{1+\phi}\frac{\gamma_i}{1-\gamma_i} c e_i(z)\theta_i(z) \Rightarrow \widehat{e}_i(z) = \frac{1}{\phi}\widehat{\theta}_i(z)$$

Given the solution for the surplus (see appendix E.1), we have $S_i(z,\epsilon) = \Omega_i z(\epsilon - R_i(z))$, implying $\overline{S}_i(z) = \Omega_i z I(R_i(z))$, where $I(R_i(z)) = \int_{R_i(z)}^1 (1-G(x))dx$, and thus $\widehat{\overline{S}}_i(z) = \widehat{z} - \varepsilon_{I|R}\widehat{R}_i(z)$, where $\varepsilon_{I|R} = \left|\frac{I'R}{I}\right|$. Finally, given the free entry condition, the FOC with respect to e and the solution for the surplus, the implied solution for $\Sigma_i(z)$, $\Lambda_i(z)$, and $\Gamma_i^r(z)$, are

$$\Sigma_i(z) = c\left[\frac{\gamma_i}{1-\gamma_i}\frac{\phi}{1+\phi} e_i(z)\theta_i(z) + \frac{1-\pi_i}{\pi_{i+1}}\frac{\gamma_{i+1}}{1-\gamma_{i+1}} e_{i+1}(z)\theta_{i+1}(z)\right] \tag{31}$$

$$\Lambda_i(z) = (1-s_e)c\left[\frac{\lambda_i}{1-\gamma_i}\theta_i(z)^\eta + \frac{\lambda_{i+1}}{1-\gamma_{i+1}}\frac{1-\pi_i}{\pi_{i+1}}\theta_{i+1}(z)^\eta\right] \tag{32}$$

$$\Gamma_i^r(z) = \beta(1-s_e)(1-\pi_i)(1-\lambda_{i+1})\Omega_{i+1} z(R_i(z) - R_{i+1}(z)) \tag{33}$$

Hence, log-linear approximation of equilibrium consists of the approximation of $(JC)-(JD)$ systems. For the (JC) curve, we have:

$$(JC) \quad \begin{cases} \widehat{\theta}_i = \frac{1}{1-\eta}\widehat{J}_i \\ \widehat{J}_i = \frac{zh_iX(R_i)}{zh_iX(R_i(z))-w_i(z)}\widehat{z} - \frac{w_i}{zh_iX(R_i(z))-w_i(z)}\widehat{w}_i - R_iG'(R_i)\frac{zh_iR_i+(1-\gamma_i)\Lambda_i}{zh_iX(R_i(z))-w_i(z)}\widehat{R}_i \\ \widehat{w}_i = \gamma_i\frac{zh_iX(R_i)}{w_i}\widehat{z} + (1-\gamma_i)\frac{\Sigma_i(1-G(R_i))}{w_i}\widehat{\Sigma}_i - R_iG'(R_i)\frac{w(R_i)}{w_i}\widehat{R}_i \end{cases}$$

Using the approximation $\frac{1-\pi_i}{\pi_{i+1}} \to 0$ that allows us to obtain $\Lambda_i(z) = \beta(1-s_e)\pi_i\lambda_i\frac{J_i(z)}{1-\gamma_i}$, leading to $J_i(z) = \frac{zh_ie(R_i(z))-w_i(z)}{1-\beta(1-s_e)\pi_i\lambda_i\frac{1-(1-\lambda_i)G(R_i(z))}{1-\lambda_i}}$ and $\widehat{\Lambda}_i = \widehat{J}_i$, we deduce:

$$\widehat{J}_i = \frac{zh_iX(R_i(z))}{zh_iX(R_i(z)) - (bh_i+\Sigma_i)(1-G(R_i(z)))}\widehat{z} - \frac{\Sigma_i(1-G(R_i(z)))}{zh_iX(R_i(z)) - (bh_i+\Sigma_i)(1-G(R_i(z)))}\widehat{\Sigma}_i \tag{34}$$

Hence, Log-linear approximation of the (JD) system is given by:

$$(JD) \quad \begin{cases} \widehat{R}_i = -\widehat{z} + \frac{w_i(R_i)}{bh_i+\Sigma_i}\widehat{w}_i^r - \frac{(1-\gamma_i)\Lambda_i}{bh_i+\Sigma_i}\widehat{\Lambda}_i - \frac{(1-\gamma_i)\Gamma_i}{bh_i+\Sigma_i}\widehat{\Gamma}_i^r \\ \widehat{w}_i^r = \gamma\frac{zh_iR_i}{w_i(R_i)}(\widehat{z}+\widehat{R}_i) + (1-\gamma_i)\frac{\Sigma_i}{w_i(R_i)}\widehat{\Sigma}_i \end{cases}$$

where $\Gamma_i^r(z,R_i(z)) = \beta(1-s_e)(1-\pi_i)(1-\lambda_{i+1})S_{i+1}(z,R_i(z))$, leading to $\widehat{\Gamma}_i^r \approx \widehat{R}_i$, given the equation of the surplus. The (JD) system then leads to

$$\widehat{R}_i = -\frac{bh_i+\Sigma_i}{bh_i+\Sigma_i+\Gamma_i}\widehat{z} + \frac{\Sigma_i}{bh_i+\Sigma_i+\Gamma_i}\widehat{\Sigma}_i - \frac{\Lambda_i}{bh_i+\Sigma_i+\Gamma_i}\widehat{\Lambda}_i$$

Given that Log-linear approximations of the free entry condition and FOC with respect to e lead to $\widehat{\Sigma}_i \approx \frac{1+\phi}{\phi}\widehat{\theta}_i$ and $\widehat{\Lambda}_i \approx \eta\widehat{\theta}_i$ respectively, we deduce that $\widehat{\Sigma}_i > \widehat{\Lambda}_i$.

Using the free-entry condition $\frac{c}{q(\theta_i)} = (1-\gamma_i)\beta\pi_i\widehat{\overline{S}}_i$, which leads to $\eta\widehat{\theta}_i \approx \widehat{\overline{S}}_i \Leftrightarrow \widehat{\theta}_i \approx \frac{1}{\eta}\left[\widehat{z} - \varepsilon_{I|R}\widehat{R}_i(z)\right]$, the FOC on the search effort, which leads to $\widehat{e}_i(z) = \frac{1}{\phi}\widehat{\theta}_i(z)$, the Log-approximation of the "search value" $\widehat{\Sigma}_i \approx \frac{1+\phi}{\phi}\widehat{\theta}_i$ and the one of the "labor hoarding value $\widehat{\Lambda}_i \approx \eta\widehat{\theta}_i$, we deduce from the JD system $\widehat{R}_i \approx -\frac{b+\Sigma_i-\Lambda_i-\Gamma_i}{b+\Sigma_i-\Lambda_i}\widehat{z} + \frac{\Sigma_i\frac{1+\phi}{\phi}-\Lambda_i\eta}{b+\Sigma_i-\Lambda_i}\widehat{\theta}_i$, under the assumption that when $\frac{1-\pi_i}{\pi_{i+1}} \to 0$, we also have $\Gamma_i \to 0$. Hence, we find the system of equations (35), (36) and (37).

The log-linear approximation of the equilibrium (Definition 1) is given by:

$$\begin{aligned}
\widehat{R}_i &\approx -\mathcal{M}_i\widehat{z} && (35)\\
\widehat{\theta}_i &\approx \frac{1}{\eta}\left[1+\varepsilon_{I|R}\mathcal{M}_i\right]\widehat{z} && (36)\\
\widehat{e}_i &\approx \frac{1}{\phi}\frac{1}{\eta}\left[1+\varepsilon_{I|R}\mathcal{M}_i\right]\widehat{z} && (37)
\end{aligned}$$

where $\varepsilon_{I|R} = \left|\frac{I'R}{I}\right|$, with $I(R) = \int_R^1(1-G(x))dx$ and $\mathcal{M}_i = \frac{bh_i+\Sigma_i\left(1-\frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{bh_i+\Sigma_i\left(1+\frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right)-\Lambda_i(1+\varepsilon_{I|R})}$.

Proposition 11. *If the restrictions in Propositions 9 and 10 are satisfied, volatilities of transition rates are age-increasing, which is consistent with the data.*

Proof. If the restrictions in Proposition 10 is satisfied, implying that $\Sigma_i > \Sigma_{i+1}$ and $\Sigma_i > \Lambda_i$, $\forall i$, and given that $\frac{\partial\mathcal{M}_i}{\partial\Sigma_i} < 0$ for any $x_i \in (0;1)$ such that $\Lambda_i = x_i\Sigma_i$, we conclude that $\widehat{R}_{i+1} < \widehat{R}_i$. The same arguments apply for $\widehat{\theta}_{i+1} > \widehat{\theta}_i$ and $\widehat{e}_{i+1} > \widehat{e}_i$ using equations (36) and (37). The age profile of worker flows are given by $\widehat{JFR}_i \approx \widehat{e}_i + (1-\eta)\widehat{\theta}_i - \frac{G(R_i)}{1-G(R_i)}\varepsilon_{G|R}\widehat{R}_i$ and $\widehat{JSR}_i \approx \frac{(1-s_e)\lambda_i G(R_i)}{s_e+(1-s_e)\lambda_i G(R_i)}\varepsilon_{G|R}\widehat{R}_i$, where $\varepsilon_{G|R}$ denotes the elasticity of function G with respect to R. Equations (35), (36), and (37) lead to $\widehat{JFR}_i \approx \left[\frac{1+\phi(1-\eta)}{\phi\eta}\left(1+\varepsilon_{I|R}\mathcal{M}_i\right) + \frac{G(R_i)}{1-G(R_i)}\varepsilon_{G|R}\mathcal{M}_i\right]\widehat{z}$ and $\widehat{JSR}_i \approx -\frac{(1-s_e)\lambda_i G(R_i)}{s_e+(1-s_e)\lambda_i G(R_i)}\varepsilon_{G|R}\mathcal{M}_i\widehat{z}$. Given that the restrictions in Proposition 10 implies that $\mathcal{M}_i$ increases with worker age because $\Sigma_i > \Sigma_{i+1}$, $\Sigma_i > \Lambda_i$, and $\frac{\partial\mathcal{M}_i}{\partial\Sigma_i} < 0$ $\forall i$, we deduce that $\widehat{JSR}_i < \widehat{JSR}_{i+1}$. For the dynamics of the finding rates, we have:

$$\widehat{JFR}_{i+1} - \widehat{JFR}_i = \left[\begin{array}{l} \left(\frac{1+\phi(1-\eta)}{\phi\eta}\varepsilon_{I|R} + \frac{G(R_i)}{1-G(R_i)}\varepsilon_{G|R}\right)(\mathcal{M}_{i+1}-\mathcal{M}_i) \\ -\frac{\int_{R_{i+1}}^{R_i}dG(x)}{[1-G(R_{i+1})][1-G(R_i)]}\varepsilon_{G|R}\mathcal{M}_{i+1} \end{array}\right]\widehat{z}$$

Using the restrictions in Proposition 9, $\mathcal{M}_{i+1} - \mathcal{M}_i > 0$ dominates $\int_{R_{i+1}}^{R_i} dG(x) > 0$. □

Proposition 11 shows that the restrictions for which the model can reproduce the shape of worker transitions per age at the steady state ensure that the age-pattern of their volatility is also matched.

F Numerical algorithm

In order to solve the model, we extend Fujita & Ramey (2012)'s algorithm along 2 dimensions. First, we take into account endogenous search effort, which was not included in Fujita & Ramey

(2012)'s paper. Secondly, we include life-cycle features (while Fujita & Ramey (2012) look at an infinitely-lived representative agent).

The model has three exogenous state variables: worker's age i, match-specific productivity ϵ and aggregate productivity z. For the grid of the match-specific productivity ϵ, we do not follow Fujita & Ramey (2012): its highest value x^h is set to sufficient large value to generate mean match productivity of 1, given that $G(\epsilon)$ is approximated by a discrete distribution with support $X = \{x_1, ..., x_M\}$, satisfying $x_1 = 1/M$, $x_m - x_{m-1} = x_M/M$. The associated probabilities $\{g_1, ..., g_M\}$ are $g_m = g(x_m)/M$ for $m = 1, ..., M-1$, where $g(x)$ is the Log-normal density, and $g_M = 1 - \sum_{i=1}^{M-1} g_i$. For the aggregate shock, we also follow Fujita & Ramey (2012) in order to represent the process z_t as a Markov chain with a state space $Z = \{z_1, ..., z_I\}$. The transition matrix of this process is $\Pi_z = [\pi_{ij}^z]$, where $\pi_{ij}^z = Pr(z_{t+1} = z_j | z_t = z_i)$. We then form two transition matrices: first, the matrix $\Pi_{z,\epsilon} = [\pi_{ij}^{z,\epsilon}]$ where $\pi_{ij}^{z,m} = Pr(z_{t+1} = z_j | z_t = z_i) g_m$, which gives the joint probability when both aggregate and match-specific shocks can change simultaneously, and second, the matrix $\overline{\Pi}_z = [\overline{\pi}_{ij}^z]$, where $\overline{\pi}_{ij}^z = Pr(z_{t+1} = z_j | z_t = z_i)\mathbb{I}_m$, which gives the probability when only aggregate shock can change, for each level of match-specific productivity.

Solving recursively, starting from the oldest worker O_T. Let $\mathcal{S}_{O_T}$ the vector $[S(x_1, z_1), \cdots, S(x_M, z_1), \cdots, S(x_1, z_I), \cdots, S(x_M, z_I)]$, and $\mathcal{R}$ be the vector $Z \bigotimes X$. Then, for an initial guess for $e_{O_T}(z)$ and $\theta_{O_T}(z)$, we find the fixed point of

$$\mathcal{S}_{OT} = \max \left\{ \mathcal{R} - z + \pi_{O_T}\beta \left[\lambda_{O_T}\Pi_{z,\epsilon}\mathcal{S}_{O_T} + (1-\lambda_{O_T})\Pi_z\mathcal{S}_{O_T} - \Pi_{z,\epsilon}^{e,\theta,O_T}\mathcal{S}_{O_T} \right]; 0 \right\}$$

where $\Pi_{z,\epsilon}^{e,\theta,O_T}\mathcal{S}_{O_T}$ is deduced from the definition of the opposite of the search value, which is $\phi(e_{O_T}) - \gamma_{O_T} e_{O_T} p(\theta_{O_T})\pi_{O_T}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_T}$. At each iteration, we use the FOC with respect to e to substitute $\phi(e_{O_T})$ by $\frac{1}{1+\phi}\gamma_{O_T} e_{O_T} p(\theta_{O_T})\pi_{O_T}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_T} \equiv \Pi_{z,\epsilon}^{e,\theta,O_T}\mathcal{S}_{O_T}$. When convergence criteria are satisfied, we obtain the decision rules $\theta_{O_T}^\star(z)$, $e_{O_T}^\star(z)$ and $R_{O_T}^\star(z)$, and the optimal value for the surplus $S_{O_T}^\star(x, z)$, $\forall z$ and $\forall x$.

Working backward, by looking at worker aged O_{T-1}. For $i = O_{T-1}$, we solve the same problem, except that we integrate the solution for the age $i = O_T$ in the agents' expectations. Then, we find the fixed point of

$$\mathcal{S}_{O_{T-1}} = \max \left\{ \begin{array}{l} \mathcal{R} - z + \pi_{O_{T-1}}\beta \left[\lambda_{O_{T-1}}\Pi_{z,\epsilon}\mathcal{S}_{O_{T-1}} + (1-\lambda_{O_{T-1}})\Pi_z\mathcal{S}_{O_{T-1}} - \Pi_{z,\epsilon}^{e,\theta,O_{T-1}}\mathcal{S}_{O_{T-1}} \right] \\ (1-\pi_{O_{T-1}})\beta \left[\lambda_{O_T}\Pi_{z,\epsilon}\mathcal{S}_{O_T}^\star + (1-\lambda_{O_T})\Pi_z\mathcal{S}_{O_T}^\star - \Pi_{z,\epsilon}^{e^\star,\theta^\star,O_T}\mathcal{S}_{O_T}^\star \right] \end{array} ; 0 \right\}$$

which gives, when the convergence criteria are satisfied, $\{\theta_{O_{T-1}}^\star(z), e_{O_{T-1}}^\star(z), R_{O_{T-1}}^\star(z)\}$, and $S_{O_{T-1}}^\star(x, z)$, $\forall z$ and $\forall x$.

Working backward. We repeat the procedure until $i = Y$. Given this complete set of decision rules, we can simulate the Markov chain for JFR and JSR and get the theoretical distribution of the employment per age, using equations (2), (4) and (3).