Directed Search, Mismatch and Efficiency

Frederic Gavrel[*] [†]

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Abstract

This short paper provides a directed search model of the labor market in which the persistency of vacant jobs results from a mismatch problem, not from a pure coordination problem. Since firms cannot commit to an output cutoff lower than the posted wage, laissez-faire is inefficient. But, under a binding condition, public policy can restore market efficiency by associating a minimum wage with a layoff tax.

Key words : Directed search, Mismatch, Efficiency, Layoff tax.

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[*] CREM, CNRS (UMR 6211) and University of Caen Normandy, Esplanade de la Paix, 14000 Caen, France; frederic.gavrel@unicaen.fr

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1 Introduction

In the real world, firms obviously use the wage they post as a means to attract workers (i.e. a means to direct workers' search). To some extent, as in the housing market, the price of labor is likely to be renegotiated ex post, but assuming no renegotiation seems to be the right starting point for an acceptable theory of wages. Thus, it is not surprising that labor economists were not entirely satisfied with Pissarides (2000) wage bargaining assumption.

In the literature on directed search (or competitive search, see Wright et al. (2017) for a guided tour), where agents are usually homogenous, firms can commit to a posted wage and (persistent) vacancies result from a pure coordination problem in workers' search. Some firms have more applicants than needed whereas others have not any applicant at all. The present paper develops an alternative approach in which workers and jobs are heterogenous. We come back to the idea that the vacancy of jobs results from the mismatch between those jobs and the workers who apply to them as in Layard et al. (1991), thus not from a coordination problem which can be seen as a restriction on workers' mobility, hence as a short run phenomenon. In other words, our basic argument is that the reason why some jobs remain vacant is not that firms do not find any applicant. The reason rather is that the applicants they find are unsuited, meaning that their productivities are lower than the posted wages.

Like Marimon and Zilibotti (1999), workers and jobs are horizontally differentiated according to Salop circle. Firms post wages and workers then decide on the (single) firm they will join - not on the probability of going to each firm, as usual in the literature - depending on the wages and on the size of firms' applicant pools. Ex ante, workers do not know the quality of a match with the firm they choose. Next, firms rank their applicants, select the best, and hire her if her productivity is higher than their posted wage. If not, the job remains vacant. Thus, vacancies result from a mismatch problem. As usual with wage posting, the wages are set by maximizing the expected profits subject to the indifference constraint: in equilibrium, all applicant pools should generate the same expected income for workers. Thus, when deciding on its posted wage, a firm faces a tradeoff between its labor costs and its expected output which grows with the number of applicants, hence with the wage. The model is closed by the assumption of free-entry. Concerning free-entry, it is worth noting that in this context applicant ranking plays a crucial role as it creates a decreasing relationship between job creation and expected profits. An important feature of our model is that firms post a single wage (not as many wages as possible productivity levels) and that the hiring cutoff (i.e. the productivity level below which the best candidate is rejected) coincides with the wage. We believe that these assumptions are quite reasonable since the output of a match is not verifiable by a third party, meaning that firms can neither commit to a wage function (of productivity) nor to a mismatch trigger lower than the wage. See for instance Peters (2010) for similar incompleteness assumptions.

By contrast to usual directed search models, market equilibrium is (constrained) inefficient. The reason for this is that firms cannot commit to a hiring cutoff lower than the posted wage. As a consequence, in this credibility-constrained equilibrium, wages are too high in terms of job rejection but too low in terms of job creation. The reason is that a wage increase (hence, an increase in the hiring cutoff) lowers the probability of filling a vacancy as well as the probability of finding a job. This creates a wage moderation effect which explains why wages are too low as concerns job creation. To make this point crystal-clear, we consider an hypothetical "unconstrained equilibrium" in which firms actually can commit to the wage as a well as to the hiring cutoff (different from the wage). We find that this unrealistic equilibrium is a social optimum which satisfies two conditions. One the one hand, the hiring cutoff coincides with the utility of leisure and, on the other hand, job creation fulfills the so-called "generalized Hosios condition", as Mangin and Julien (2017) put it. Firms internalize the usual congestion effect. Overall, they also internalize that additional vacancies reduce the number of applicants per firm, thus lowering the expected productivity of the best applicant: the productivity effect. The social optimality of this hypothetical equilibrium indicates how public policy could remedy the inefficiency of *laissez-faire* (i.e. the credibility-constrained equilibrium). Public intervention should make the hiring trigger be equal to the value of leisure. This can be achieved through a self-financed tax/subsidy Pigovian system. We show that, under a binding condition, an appropriate layoff tax associated with a minimum wage - equal to the unconstrained-equilibrium wage - can decentralize the social optimum.[1]

Following Wright et al. (2017), one can distinguish two strands of papers in the literature on directed search. On the one hand, there are *market utility* models (as Wright et al. put it) like Moen (1997) or Shimer (1996). Our setup is close to these papers. We also examine the behavior of a deviant firm to deduce the wage setting equation. But, contrary to this approach, we do not introduce search frictions between a firm and its pool of applicants. In our setting, the reason why jobs can remain vacant is that applicants are unsuited to them and the probability of such a situation grows with an increase in market tightness. On the other hand, there are models in which the microeconomic foundations are more explicit like Peters (1984), Julien et al. (2000) or Burdett et al. (2001) and others. In these models, contrary to ours, workers' search strategies are mixed and the persistence of vacant jobs results from a pure coordination problem, not from a mismatch problem as in the present paper.[2] There are also some directed search papers with heterogenous agents and non contingent wages.[3] See for instance Lang et al. (1999) or, more recently, Peters (2010).[4] In general, these papers seek at accounting for the dispersion of wages. To that end, the agents are vertically differentiated and workers have some information about the types of the jobs they apply to and on the composition of the applicants pools of these jobs. In our setting, this information is only available *ex post* and all agents face the same situation, implying that there is a unique equilibrium wage. Our setup can also be related to the competing auction literature starting with work by Mc Afee (1993) and Peters (1997). In some of these papers like Albrecht et al. (2014), the type realization is an ex post shock like in ours. The difference is that in our setup worker heterogeneity is not private value, but affects the value of the firm.[5]

[1] To our knowledge, this rationale for layoff taxes is new. In general the motivation for firing taxes (or for experience rating) is that they make firms internalize the consequences of their dismissal behavior on unemployment costs. See for instance Blanchard and Tirole (2008), Cahuc and Malherbet (2004), Gavrel (2018) or L'Haridon and Malherbet (2009).

[2] With infinite numbers of agents, these mixed-strategy models provide the market utility approach with a microeconomic foundation. In the limit, search frictions are described with urn-ball matching.

[3] It is worth noting that in some of these papers, firms can commit to hire a worker whose productivity is lower than the posted wage. See for instance Michelacci and Suarez (2006).

[4] Peters (2010) provides a brief review of the literature with heterogenous agents.

[5] Regarding the modeling of heterogeneity, different papers use Salop circle with bargained wages.

There are two ways of understanding the presence of vacant jobs. The literature on directed search puts the emphasis on the coordination problem. Alternatively, the present paper develops a directed search model of the labor market in which persistent vacancies result from a mismatch problem. The implications are as follows. First, mixed strategies are not required, which sounds more realistic. Second, due to a credibility constraint, market equilibrium is unavoidably inefficient for crystal clear reasons. Third, in a public policy perspective, this inefficiency is a (new) rationale for the introduction of layoff taxes.

Section 2 describes our setup and defines an equilibrium of the labor market. Section 3 provides the welfare analysis and study how public policy can decentralize a social optimum.

2 Market structure and equilibrium

2.1 Market structure

In this market, there are "very large" numbers of workers and firms who are risk-neutral and heterogeneous. The number of workers is denoted by U. An important feature of this economy is that workers and jobs are horizontally differentiated according to Salop circular model.

The distance between the firm j and the worker i along the skills circle, denoted by x, measures the mismatch between the offered skills of worker i and the skill requirements of firm j. The output of a match (i, j) is then a decreasing (mismatch) function $y(x)$ of this distance. Assuming that the workers and the jobs are uniformly distributed on the circle, one can see that to any circular model, defined by the mismatch function $y(x)$, one can associate a model of match specific productivities whose cumulative distribution satisfies $F(y) = 1 - 2x(y)$, with $x(y)$ being the reciprocal of $y(x)$.[6] This isomorphism allows us to use match specific productivities which are simpler and more familiar than the circular model but, at this stage, we would like to stress that this simple tool of analysis can be derived from first principles. Thus,

See for instance Marimon and Zilibotti (1999)(without on-the-job search), Gautier et al.(2010)(with on-the-job search), or Gavrel (2012)(without on-the-job search but with recruitment selection).

[6]The circle length is normalized to one.

the productivity of a match is assumed to be a random variable whose cumulative distribution, $F(y)$, is strictly increasing in the interval $[0,1]$, and such that $F(0)=0$ and $F(1)=1$. Its density is denoted by $f(y)$ ($f(y)>0$).

Another peculiarity concerns the matching process. We retain very simple but understandable assumptions. As usual in wage posting models, workers know where to find the firms as well as the wages which they have posted. But they don't know the quality of a match with these jobs. Depending on the posted wages and on the number of applicants to each firm, a worker chooses which applicant pool to join. We then exclude multiple applications, implying that *ex post* firms' competition for workers is ruled out.[7] The same holds for *ex post* competition between the workers of the same applicant pool.

The timing of this game is the following. In the first stage, firms freely decide on entering the market or not. If they enter the market, they create a single vacancy, paying the cost c to that end. Next, firms post a wage knowing that an increase in this wage affects the size of their applicant pool. And, knowing those wages, the workers decide on the applicant pool they will join (the job they will apply to) by comparing the expected incomes associated with the different jobs. Finally, firms interview all their applicants and select the best. If the output of the best worker is higher than the wage, then they hire this worker. Notice that, as usual with wage posting, firms are able to commit to the wage that they announce. As a consequence, the wage plays two roles here. On the one hand, it is used as a means to compete for workers, and it determines the output cutoff below which firms decide not to hire their best applicant, on the other hand. This assumption sounds reasonable but it clearly raises an efficiency issue, as emphasized in the welfare and policy analysis.

Formally, let w denotes the wage posted by a firm and n the number of its applicants. The output of the best applicant is the maximum statistic of a sample of n draws of the distribution $F(.)$. Its density is

$$g(y,n)=nF(y)^{(n-1)}f(y),$$

and the expected output of this firm is given by

[7]See Albrecht et al. for *ex post* competition for workers in a directed search model.

$$\int_w^1 g(y,n)ydy.$$

Consequently, the expected profits of this firm (gross of the entry cost, c), denoted by π, satisfy

$$\pi = \tilde{\pi}(w,n) = n\int_w^1 F(y)^{n-1}f(y)(y-w)dy.$$

Notice that the probability, $\tilde{m}(w,n)$, of the firm filling its vacancy is given by

$$\tilde{m}(w,n) = \int_w^1 g(y,n)dy = 1 - F(w)^n.$$

Indeed, as $F(w)^n$ is the probability of all candidates having a productivity lower than the wage, $1 - F(w)^n$ is the probability of at least one applicant having a productivity higher than the wage, hence the probability of the firm hiring a worker.

As all applicants face the same situation, their probability of getting the job is

$$p = \tilde{p}(w,n) = \frac{\tilde{m}(w,n)}{n}.$$

A peculiarity of this model is that the wage, which acts as an output cutoff, directly (negatively) affects the probability of filling a job as well as the probability of finding a job. Regarding the size of the applicant pool, one can see that an increase in the number of applicants raises the probability of the firm hiring a worker as well as its expected output. By contrast, the probability of a candidate getting the job is reduced.

It results that the expected income of an applicant to this firm, denoted by W, has the following expression:

$$W = \tilde{W}(w,n) = d + \frac{\tilde{m}(w,n)}{n}(w-d),$$

with d being the utility of leisure.

As already mentioned, workers choose an applicant pool according to expected incomes. The stability of the applicant pools then requires that they all generate the same expected income.[8] For given wages, this indifference condition (which is a kind of migration equilibrium) determines the distribution of workers across the applicant pools (i.e. the firms with a vacancy). As the expected income is a decreasing function of the number of applicants, we can deduce that such a distribution of workers across firms is stable.

In what follows, $\bar{y}$ denotes the expected output of a filled job, that is

$$\bar{y} = \tilde{\bar{y}}(w, n) = \frac{n \int_w^1 F(y)^{n-1} f(y) y dy}{\tilde{m}(w, n)}.$$

Following the same line of reasoning as Gavrel (2012), one can show that, in conformity with intuition, the expected output, $\tilde{\bar{y}}(.)$, is an increasing function in the number of applicants, n.

2.2 Market equilibrium

All firms with a vacancy know the indifference condition. They then set their posted wage and the size of their applicant pool by maximizing their expected profit subject to the indifference condition.

Let j denote a firm with a vacant job, *ex ante*. A market equilibrium can be defined as a number of vacancies, V^*, a pair of functions $(w^*(j), n^*(j))$, and an expected income W^* such that

-i) $W(j) = W^*$, for all $j = 1, ..., V$ (indifference condition),

-ii) $(w^*(j), n^*(j))$ maximizes $\pi(j)$ subject to the indifference condition $W(j) = W^*$, for all $j = 1, ..., V$,

-iii) $\sum_1^V n(j) = U$,

-iv) $-c + \pi(j) = 0$ for all $j = 1, ..., V$ (free-entry).

In general competitive analysis, the agents are price takers. Here, the firms which are "very small" take the expected income W as given. Thus, this model describes a "competitive search" situation.

It is easy to see that this equilibrium is symmetric. As a consequence, in this symmetric equilibrium, workers are uniformly distributed among firms, implying that

[8] The same holds in *market utility* models like Moen (1997). Contrary to this literature, we do not introduce search frictions between a firm and its applicants.

all applicant pools have the same size, $n = U/V = 1/\theta$, with θ denoting market tightness.

In conformity with notational use, all function, $x = \tilde{x}(w, n)$, is now expressed in terms of market tightness, $\theta = V/U$, that is $x = x(w, \theta) = \tilde{x}(w, 1/\theta)$. We can then define an equilibrium in a (much) simpler way:

Definition 1. *A market equilibrium is a pair, (w^*, θ^*), which maximizes the expected profit, $\pi(w, \theta)$, subject to the indifference constraint, $W(w, \theta) = W(w^*, \theta^*)$, and which satisfies the free-entry equation, $-c + \pi(w, \theta) = 0$.*

As usual in directed search, the wage is determined by examining the behavior of a deviant firm who sets the wage w and (the inverse of) its applicant pool, θ, by maximizing its expected gross profit, $\pi(w, \theta)$, subject to the constraint that the expected income of its applicants, $W(w, \theta) = p(w, \theta)w + (1 - p(w, \theta))d$, be equal to the expected income of other workers, W^*.

Consider an interior private optimum ($w > d$). The indifference constraint implicitly determines the posted wage as a function of market tightness. Differentiating this equation gives the impact of an increase in market tightness on the wage. We obtain:

$$\frac{dw}{d\theta} = -\frac{\frac{\partial p(.)}{\partial \theta}(w - d)}{p(.) + \frac{\partial p(.)}{\partial w}(w - d)} \tag{1}$$

An increase in market tightness (a decrease in the number of applicants) lowers the expected incomes. Holding the cutoff as a constant, a wage increase raises the expected income as usual in market utility models (term $p = \theta m > 0$). But, here, the wage also exerts a negative effect on the probability of an applicant being hired by lowering the lower bound of the output (term $\partial p(.)/\partial w < 0$). As a consequence, a wage increase does not necessarily lead to an increase in the number of applicants (i.e. a decrease in market tightness). If this negative effect is sufficiently strong, part of the applicants will leave the deviant firm, implying that the firm should set a lower wage.

Let $\varepsilon(m(.), w)$ denote the elasticity of $m(.)$ to w in absolute value. Thus, an interior (private) optimum should satisfy the following condition

Condition (IC). $\varepsilon(m(.), w)\frac{w-d}{w} < 1$.

It results that the total derivative of expected profits with respect to market tightness has the expression below

$$\frac{d\pi(\theta)}{d\theta} = \frac{\partial(m(.)\bar{y}(.))}{\partial\theta} - \frac{\partial m(.)}{\partial\theta}w - m(.)\frac{dw}{d\theta}.$$

Consider the term

$$-m(.)\frac{dw}{d\theta} = \frac{m(.)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)}[\frac{m(.)}{\theta} + \frac{\partial m(.)}{\partial\theta}](w-d).$$

We have

$$\frac{m(.)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)} = 1 - \frac{\frac{\partial m(.)}{\partial w}(w-d)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)}.$$

Let $\Gamma(.)$ denote the expression

$$\Gamma(.) \equiv \frac{\frac{\partial m(.)}{\partial w}(w-d)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)} = -\frac{\varepsilon(m(.), w)\frac{w-d}{w}}{1 - \varepsilon(m(.), w)\frac{w-d}{w}} < 0 \quad (2)$$

Substitution into the previous expression for the derivative yields

$$\frac{d\pi(\theta)}{d\theta} = \frac{\partial(m(.)\bar{y}(.))}{\partial\theta} - \frac{\partial m(.)}{\partial\theta}d + \frac{m(.)}{\theta}(w-d) - \Gamma(.)\frac{m(.)}{\theta}(1-\eta(.))(w-d),$$

where $\eta(.)$ denotes the elasticity of $m(.)$ to θ in absolute value.

Since

$$\frac{\partial m(.)\bar{y}(.)}{\partial\theta} = -\frac{m(.)}{\theta}[\eta(.)\bar{y}(.) + \alpha(.)(\bar{y}(.) - d)],$$

with $\alpha(.)$ being the elasticity of $\bar{y} - d$ to θ in absolute value[9], it follows that

[9]Note that the elasticities of $m(.)$ and $\bar{y} - d$ to θ are negative.

$$\frac{\theta}{m(.)}\frac{d\pi(\theta)}{d\theta} = w - d - [\eta(.) + \alpha(.)](\bar{y}(.) - d) - \Gamma(.)(1 - \eta(.))(w - d),$$

implying that a private (interior) optimum should satisfy

$$w = d + \frac{[\eta(w,\theta) + \alpha(w,\theta)][\bar{y}(w,\theta) - d]}{1 - \Gamma(w,\theta)(1 - \eta(w,\theta))} \tag{3}$$

It is worth noting that, as the equilibrium value of workers' expected income is deduced from the equilibrium values of the wage and the market tightness, the previous equation only means that in a private optimum (of firms) firms and workers face the same marginal exchange rate between the wage and the number of applicants. Note also that, under condition (IC), the wage actually is higher than the value of leisure.

On the other hand, the free-entry equation can be written as

$$-c + m(w,\theta)(\bar{y}(w,\theta) - w) = 0 \tag{4}$$

To summarize:

Proposition 1. *A market equilibrium is a pair,* (w^*, θ^*)*, which jointly satisfies equations* (3)*, and* (4).

In the following, we also refer to this market equilibrium as the credibility-constrained equilibrium. Indeed, posting a wage higher than the output cutoff is not credible.

3 Efficiency and public policy

We now show that, despite wage posting, the market equilibrium (i.e. the credibility-constrained equilibrium) is inefficient. Then, to get a precise understanding of this result, we consider the hypothetical case in which firms can commit to an output cutoff lower than the posted wage. We refer to this equilibrium as the unconstrained equilibrium. On this basis, we determine how public policy can restore efficiency.

3.1 Social optimum and market efficiency

As usual, our welfare criterion is the (net) aggregate income. The (net) aggregate income *per* head, denoted by Σ, depends on two variables which are the output cutoff, $\hat{y}$, and the number of jobs per worker, $n = U/V = 1/\theta$, with θ being market tightness.

It can be written as follows

$$\Sigma = \Sigma(\hat{y}, \theta) = -\theta c + \theta m(\hat{y}, \theta)\bar{y}(\hat{y}, \theta) + [1 - \theta m(\hat{y}, \theta)]d \tag{5}$$

It is easy to see that, whatever market tightness might be, the optimum cutoff coincides with the utility of leisure, d. Thus, the aggregate income can be rewritten as

$$-\theta c + \theta m(d, \theta)\bar{y}(d, \theta) + [1 - \theta m(d, \theta)]d \tag{6}$$

Differentiating the social surplus *per* worker with respect to θ gives

$$-c + [m(.) + \theta \frac{\partial m(.)}{\partial \theta}](\bar{y}(.) - d) + \theta m(.) \frac{\partial \bar{y}(.)}{\partial \theta}$$

It follows that the first order condition for the maximization of the aggregate income can be written as

$$\frac{\partial \Sigma(d, \theta)}{\partial \theta} = -c + m(d, \theta)[1 - \eta(d, \theta) - \alpha(d, \theta)][\bar{y}(d, \theta) - d] = 0 \tag{7}$$

With $\alpha(.) = 0$, we would obtain the usual Hosios condition for the efficiency of job creation. With $\alpha(.) > 0$, we face the "generalized Hosios condition" as Mangin and Julien (2017) put it.

This optimality condition takes into account two externalities of an increase in the number of vacancies. The first is the usual congestion effect. The second effect results from the ranking of candidates. As already mentioned, it is easy to check that an increase in market tightness lowers the average output: the elasticity of $\bar{y} - d$ with respect to θ ($-\alpha(.)$) is negative. In conformity with intuition, more vacancies lower the number of applicants per firm, implying that the average of the maximum statistic is reduced. This negative productivity effect tends to reduce job creation in an optimum. In other words, if the basic Hosios condition holds, then job creation is excessive.

Let us now compare the credibility-constrained equilibrium (market equilibrium of Proposition 1) with the social optimum. Substitution of the wage equation (3) into the free-entry condition (4) yields

$$-c + m(.)[1 - \frac{\alpha(.) + \eta(.)}{1 - \Gamma(.)(1 - \eta(.))}](\bar{y} - d) = 0 \tag{8}$$

Knowing that the equilibrium wage is higher than the value of leisure, the comparison of equations (7) and (8) show that the laissez-faire equilibrium is inefficient in terms of the output cutoff as well as in terms of market tightness. For obvious reasons, lowering the cutoff for a given labor cost - meaning that only the probabilities $m(.)$ and $p(.)$ are (positively) affected - increases the aggregate income. This holds true for any cutoff higher than the value of leisure. Lowering the cutoff makes that some viable matches are no longer rejected. Now, since $\Gamma(.) < 0$, equation (8) implies that the derivative of the aggregate income with respect to market tightness,

$$-c + m(.)[1 - (\eta(.) + \alpha(.))](\bar{y} - d) < -c + m(.)[1 - \frac{\eta(.) + \alpha(.)}{1 - \Gamma(.)(1 - \eta(.))}](\bar{y} - d) = 0,$$

is strictly negative in the neighborhood of market equilibrium. This means that job creation is excessive. On the other hand, one can check that, according to the free-entry equation (4), market tightness is a decreasing (implicit) function of the wage. A small increase in the labor cost for a given rejection trigger - meaning that probabilities $m(.)$ and $p(.)$ are left unchanged - increases the aggregate income. To summarize, we can state the following

Proposition 2. *In market equilibrium, the wage is too high in terms of match rejection but, too low in terms of job creation*

The reason why the wage is too low regarding job creation is that its negative impact on the transition rates ($m(.)$ and $p(.)$) exerts a wage moderation effect.

3.2 Unconstrained equilibrium and public policy

3.2.1 Relaxing the credibility constraint makes equilibrium efficient

To reach a better understanding of the welfare results as well as to drive the analysis of public policy, we here consider the (unrealistic) case in which firms could commit to the wage as well as to the output cutoff. In other words, in this "unconstrained" equilibrium, firms are assumed to be able to disconnect the cutoff, $\hat{y}$, from the wage, w. Consequently, the transition probabilities $m(.)$ and $p(.)$ no longer depends on the wage but on the cutoff (as in the determination of a social optimum).

Let us first study how the output cutoff is determined in this situation. To that end, we show that for a given applicant pool - meaning that any variation in the cutoff should be associated with an appropriate variation in the wage which keeps workers' expected income constant - the optimum of expected profits is reached for $\hat{y} = d$.

The proof is as follows. First, from the indifference condition,

$$p(\hat{y}, \theta)w + (1 - p(\hat{y}, \theta))d = W,$$

we deduce workers' marginal rate of substitution between the wage and the cutoff

$$\frac{dw}{d\hat{y}} = -\frac{\frac{\partial m(\hat{y},\theta)}{\partial \hat{y}}}{m(\hat{y}, \theta)}(w - d) \tag{9}$$

For obvious reasons, this MRS is positive. Indeed, holding the expected income, W, constant, an increase in the cutoff leads to a decrease in the probability of getting the job. This is is neutralized by a wage increase.

In this unconstrained case, the expected profits (gross of entry costs) are written as

$$\pi = \pi(\hat{y}, \theta, w) = \frac{1}{\theta} \int_{\hat{y}}^{1} F(y)^{\frac{1-\theta}{\theta}} f(y)(y - w)dy.$$

Consequently the total derivative of expected profits with respect to the cutoff, for a given market tightness, is

$$\frac{d\pi}{d\hat{y}} = -\frac{1}{\theta} F(\hat{y})^{\frac{1-\theta}{\theta}} f(\hat{y})(\hat{y} - w) - m(\hat{y}, \theta)\frac{dw}{d\hat{y}} \tag{10}$$

Knowing that

$$\frac{\partial m(\hat{y},\theta)}{\partial \hat{y}} = -\frac{1}{\theta}F(\hat{y})^{\frac{1-\theta}{\theta}}f(\hat{y}),$$

substituting (9) into (10) yields

$$\frac{d\pi}{d\hat{y}} = \frac{\partial m(\hat{y},\theta)}{\partial \hat{y}}(\hat{y} - d) \tag{11}$$

Since $\partial m(\hat{y},\theta)/\partial\hat{y} < 0$, we deduce from the previous derivative that an increase in the cutoff raises (reduces) profits if and only if the cutoff is lower (higher) than d, whatever the number of applicants is. Unsurprisingly, this means that in an unconstrained equilibrium , the cutoff necessarily coincides with the domestic output. What implies that firms would be able to commit to a cutoff which can generate a loss.

Let us now turn to wage setting. The wage is derived from the maximization of profits subject to the indifference condition for $\hat{y} = d$. In the unconstrained situation, using the indifference constraint, the expected profit can be rewritten as a function of market tightness $\Pi(\theta)$:

$$\Pi(\theta) = m(d,\theta)[\bar{y}(d,\theta) - d] - \frac{W-d}{\theta}.$$

Thus, maximizing the expected profit with respect to θ gives

$$\frac{p(.)}{\theta^2}(w-d) = -\frac{\partial m(.)}{\partial\theta}(\bar{y}(.) - d)) - m(.)\frac{\partial(\bar{y}-d)}{\partial\theta}.$$

Or

$$w = d + [\eta(d,\theta) + \alpha(d,\theta)][\bar{y}(d,\theta) - d] \tag{12}$$

From the substitution of (12) into the free-entry condition

$$-c + m(d,\theta)[\bar{y}(d,\theta) - w] = 0,$$

we deduce the equation for the social optimality of market tightness (equation 7).

We can state

Proposition 3. *In the absence of the credibility constraint, equilibrium is efficient.*

We would like to stress that this unconstrained equilibrium is purely theoretical. Firms cannot commit to an output cutoff lower than the wage. Nonetheless, this possibility explains why market equilibrium (as defined in Proposition 1) is inefficient and how public policy could remedy this inefficiency.

3.2.2 Public policy. A rationale for layoff taxes

From the analysis above, we can deduce that if firms are given the incentive to decide on a trigger equal to the value of leisure, then market efficiency will improve. There exist different Pigovian tax/subsidy schemes whose introduction in the credibility-constrained equilibrium (defined in Proposition 1) are capable of creating the right incentive for firms, i.e. of "tying their hands". The simplest one consists of taxing *ex post* vacancies while subsidizing all jobs.

Suppose that tax authorities levy a tax f on (*ex post*) vacant jobs which is dedicated to the financing of a subsidy to job creation (i.e. a subsidy to all created jobs), h. In so doing, tax authorities collect the (total) tax amount, $(1 - m(.))Vf$, and pay the (total) subsidy amount, Vh. The balanced budget constraint then implies that $h = (1 - m(.))f$.

The Pigovian tax, f, should ensure that firms decide on the trigger $\hat{y} = d < w$. To that end, the *ex post* loss, $w - d$, should be equal to this "non-hiring" tax, f. If the maximum productivity (among the applicant pool) is higher (lower) than d, then the firm hires (rejects) its best applicant.

It is worth noting that this tax/subsidy scheme resembles a layoff tax. In general, the motivation for layoff taxes is that firms should perceive that their layoff behavior increases the costs of unemployment. These costs result from unemployment insurance. See Blanchard and Tirole (2008) for instance. Our analysis shows that introducing a layoff tax is desirable even in the absence of unemployment insurance (workers being risk-neutral). To our knowledge, this rationale for layoff taxes is novel.[10]

[10] Building on Shapiro and Stiglitz (1984), another strand of literature argues that the layoff tax allows firms to reduce their efficiency wage (or their monitoring costs) by committing them to a

However, introducing a layoff tax is not sufficient to restore efficiency. Public policy should also act on job creation. As an illustration, following Flinn (2006), we consider the introduction of a minimum wage, σ. Let θ_S denote the socially optimal value for market tightness (equation (7)). From the analysis above (equation (12)), we deduce that the minimum wage should be set to

$$\sigma = \sigma_S \equiv d + [\eta(d, \theta_S) + \alpha(d, \theta_S)][\bar{y}(d, \theta_S) - d].$$

We now show that this minimum wage, σ_S, associated with the layoff tax, $f_S = \sigma_S - d$, permits the decentralization of the social optimum if and only if the following (binding) condition,

Condition (BC). $\eta(d, \theta_S) < \varepsilon(m(d, \theta_S), w)\frac{\sigma_S - d}{\sigma_S}$,

is satisfied.

Due to the budget constraint, we can see that with the pair (σ_S, f_S) the equilibrium equation for job creation,

$$-c + m(d, \theta)(\bar{y}(d, \theta) - \sigma_S) = 0,$$

coincides with the (social) optimum equation. But this not sufficient. The minimum wage σ_S should be binding, meaning that firms should not be willing to set a higher wage than this minimum. We must show that, under condition (BC), the total derivative of expected profits with respect to the wage is negative. The calculus of the (total) derivative of the expected profits with respect to θ is very close to the proof of Proposition 1. The appendix shows that this derivative has the following expression:

$$\frac{\theta}{m(.)}\frac{d\pi(\theta)}{d\theta} = w - d - [\eta(.) + \alpha(.)](\bar{y}(.) - d) - [\eta(.)f + \Gamma(.)(1 - \eta(.))(w - d)] \quad (13)$$

For the minimum wage, σ_S, and the layoff tax, $f_S = \sigma_S - d$, the derivative reduces to

$$\frac{d\pi(\theta)}{d\theta} = -\frac{m(.)}{\theta}[\eta(.)f_S + \Gamma(.)(1 - \eta(.))f_S].$$

lower separation rate (which raises the utility of non-shirking workers). See Saint Paul (1995), Fella (2000), and Fath and Fuest (2005).

Thus, the derivative of the expected profits with respect to market tightness, $d\pi/d\theta$ has the same sign as

$$-\eta(d, \theta_S) - \Gamma(d, \theta_S)(1 - \eta(d, \theta_S)),$$

or[11]

$$-\eta(d, \theta_S) + \varepsilon(m(d, \theta_S), w)\frac{\sigma_S - d}{\sigma_S}.$$

The previous expression is positive under condition (BC). Since $dw/d\theta < 0$ under condition (IC) (see the appendix), this proves that the minimum wage, σ_S, is binding under condition (BC).

To summarize:

Proposition 4. *Under condition (BC), the minimum wage, σ_S, coupled with the layoff tax, $f_S = \sigma_S - d$, permits the decentralization of a social optimum.*

One could wonder why the optimum minimum wage is not necessarily binding. The reason is that the layoff tax can create an incentive to raise the wage. Indeed, a wage increase attracts more applicants to the job. This lowers the probability of firing (not hiring) the best applicant, then the probability of paying the layoff tax. This effect is unlikely to overcompensate the direct increase in labor costs, meaning that condition (BC) sounds reasonable.

4 Conclusion

This paper develops a directed search model with heterogeneous agents in which the vacancy of jobs results from a mismatch problem. Under the reasonable assumption that firms cannot commit to a mismatch cutoff which could generate losses, laissez-faire equilibrium is inefficient. But, under a binding condition, associating a minimum wage with a layoff tax permits the decentralization of a social optimum.

From a purely theoretical perspective, it is not surprising that labor theorists first soke at providing a model in which agents are homogenous. We subscribe to this

[11]Remember that the analysis restricts to the case in which $dw/d\theta < 0$, hence to the case in which $\varepsilon(m(.), w) < 1$.

criterion of good theory. But the application of this criterion revealed to be quite difficult, implying that one is led to introduce some *ad hoc* hypothesis like mixed strategies or search frictions between a firm and its applicants. We acknowledge that our model is a "lesser evil", but we believe that horizontal heterogeneities are a better solution than mixed strategies. Relaxing the assumption of applicant ranking would lead to a simple model better suited to applied theory. We conjecture that the same inefficiencies (leading to the same policy measures) would hold if it were assumed that firms interview their applicants one by one until they find one whose productivity is higher than their posted wage. An interesting area for further investigations would consist of introducing Bertrand competition for the job when all applicants have a productivity lower than the wage.

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Appendix. Derivative of expected profits in the presence of a layoff tax

In the presence of the layoff tax, the expected profits have the following expression:

$$\pi(w,\theta) = \int_{w-f}^{1} g(y,\theta)ydy - m(w-f,\theta)w - (1-m(w-f,\theta))f.$$

On the one hand, the derivative of profits with respect to the wage reduces to

$$\frac{\partial \pi(.)}{\partial w} = -m(.).$$

On the other hand, the derivative of expected profits with respect to market tightness is written as

$$\frac{\partial\pi(.)}{\partial\theta} = \frac{\partial(m(.)\bar{y}(.))}{\partial\theta} - \frac{\partial m(.)}{\partial\theta}(w-f).$$

Now, from the indifference constraint, we deduce

$$\frac{dw}{d\theta} = -\frac{\frac{\partial p(.)}{\partial\theta}(w-d)}{p(.) + \frac{\partial p(.)}{\partial w}(w-d)} < 0.$$

It follows that the total derivative of expected profits with respect to market tightness has the expression below.

$$\frac{d\pi(\theta)}{d\theta} = \frac{\partial(m(.)\bar{y}(.))}{\partial\theta} - \frac{\partial m(.)}{\partial\theta}(w-f) - m(.)\frac{dw}{d\theta}.$$

Consider the term

$$-m(.)\frac{dw}{d\theta} = \frac{m(.)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)}[\frac{m(.)}{\theta} + \frac{\partial m(.)}{\partial\theta}](w-d).$$

We have

$$\frac{m(.)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)} = 1 - \frac{\frac{\partial m(.)}{\partial w}(w-d)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)}.$$

Let $\Gamma(.)$ denote the expression

$$\Gamma(.) \equiv \frac{\frac{\partial m(.)}{\partial w}(w-d)}{m(.) + \frac{\partial m(.)}{\partial w}(w-d)} = -\frac{\varepsilon(m(.),w)\frac{w-d}{w}}{1-\varepsilon(m(.),w)\frac{w-d}{w}}.$$

Substitution into the previous expression for the derivative yields

$$\frac{d\pi(\theta)}{d\theta} = \frac{\partial(m(.)\bar{y}(.))}{\partial\theta} + \frac{\partial m(.)}{\partial\theta}(f-d) + \frac{m(.)}{\theta}(w-d) - \Gamma(.)\frac{m(.)}{\theta}(1-\eta(.))(w-d).$$

As

$$\frac{\partial m(.)\bar{y}(.)}{\partial\theta} = -\frac{m(.)}{\theta}[\eta(.)\bar{y}(.) + \alpha(.)(\bar{y}(.)-d)],$$

we obtain equation (13) of the text.