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1 Introduction

The Great Recession and the subsequent zero lower bound episode led the Federal Reserve to engage in QE programs. Those consisted in buying several types of assets to restore the functioning of credit markets and foster the real economy. It is recognized in the literature that QE pressured long-term yields and successfully improved financial conditions for banks and firms.¹ In addition, in the below-mentioned studies, the macroeconomic effects of QE are found to be large. However, these results emerge from vector autoregression (VAR) models. These dynamic systems are, in our view, subject to various potential issues, that lead us to raise the following question: to what extent are these estimates reliable?

VAR models are routinely used rather than justified upon the underlying generating process of data. As such, the approach is susceptible to lead to important misspecification errors, especially when looking at the way in which QE programs were conducted in details. Indeed, the Federal Reserve had quite different objectives for each of the QE programs implemented.

In a nutshell, the QE1 program was an emergency plan to revitalize frozen private credit markets. The QE2 program, along with the Operation Twist, was employed to foster economic conditions and combat deflationary pressures. The QE2 program differed from its predecessor, as it only focused on purchasing long-term Treasury bonds rather than toxic assets. The QE3 program was state-contingent: intended to continue indefinitely until the economic fundamentals improve.

Consequently one can only expect differences in the macroeconomic effects, and in the underlying transmission channels (TC hereafter) of QE programs. Hence, we argue that models with constant coefficients might not be well suited to capture what we consider as *structural changes* in the macroeconomic impacts of QE programs. Along the same line, it is also worth testing for stochastic volatility.

VAR models allow for many degrees of freedom thanks to a relatively low number of variables. However, the use of sparse information might induce large overestimation of the identified effects. Furthermore, standard estimation procedures usually ignore model uncertainty. Beyond the fact that many econometricians arbitrarily select a set of variables, the habit is to add or switch several variables as robustness checks instead of discriminating over all variable combinations. These approaches are questionable in light of the results of Hoeting et al. (1999), who demonstrate that considering model uncertainty greatly improves out-of-sample forecasting.

Therefore, there are good reasons to think that many empirical studies on QE programs

¹See for example Gagnon et al. (2011), Baumeister & Benati (2012) and Christensen & Rudebusch (2012), among others.

are potentially affected by one or all of these problems. In this paper, we evaluate these issues using a large set of model specifications with different degrees of time-variation in parameters. Models are also estimated along with a QE factor that combines different variables with time-varying factor weights, capturing the different TCs of QE. The term *specification* refers to the modeling assumptions that allow to capture structural changes and stochastic volatility in the systems. As such, the usual VAR specification with constant parameters and homoskedasticity is a particular case where additional restrictions are imposed. Overall, our evaluation comprises three parts.

First, we use a grid-search algorithm to select the optimal specification across all the possible variants in the set of models. The ability to forecast macroeconomic and QE variables determines the optimal specification.²

Second, for each of the model specification, we account for model uncertainty in the time-varying relationship between QE, the real economy, and the associated TC of QE. More precisely, the uncertainty arises from variable switching in the estimation of the TC factor.

Third, we discuss the implications of QE: *(i)* We gauge the time-varying forecasting importance of the portfolio balance, signaling, balance sheet and risk-taking channels of QE and *(ii)* we quantify the macroeconomic effects of QE programs based on the identification of structural shocks and the subsequent impulse responses. Identification is achieved by means of Cholesky decompositions and a mix of zero and sign restrictions.

Our forecasting exercise suggests that the time-variation in parameters is of great importance. However, stochastic volatility does not help improve the fit with the data. Furthermore, we find that the portfolio balance and signaling channels gain forecasting power over time. Conversely, the balance sheet channel has a steady implication, while the risk-taking channel is marginal. Further, despite the increasing forecasting importance of the two channels, the impulse responses show that the macroeconomic effects of QE are not linearly and monotonically increasing with the magnitude of QE shocks. Indeed, we find that the QE1 program has relatively more effects on real variables than QE2 and QE3 programs, suggesting that QE programs might have asymmetric macroeconomic effects as discussed below.

The transmission channels of QE are relatively well identified in the theoretical literature, but there are discussions about their importance and in the size of the associated macroeconomic

²Given the large set of models to estimate, we resort to a very flexible estimation technique using the algorithm of Koop & Korobilis (2014), the task being computationally overwhelming otherwise. This method allows estimations to take a couple of days instead of several months, and consists of a two-step dual linear Kalman filter and smoother. See the online appendix for further details.

effects.³ On the one hand, Curdia & Woodford (2011) claim that QE should be neutral and have no macroeconomic effects. On the other hand, studies using a mechanism à la Gertler & Karadi (2011) find that QE programs should have large effects. From an empirical perspective, QE programs are found to have strong effects on the real allocations; as in Gambacorta et al. (2014) and Weale & Wieladek (2016) (WW). We believe that our results – QE programs had significant macroeconomic effects but decreasing over time – may reconcile theoretical with empirical results. Indeed, QE had significant effects during the period of economic distress during which QE1 was implemented. At that time, financial frictions were exacerbated, easing the transmission of the QE1 program. Conversely, QE has smaller macroeconomic effects during the recovery period, when QE2 and QE3 programs were implemented, because financial constraints were slack (or slacker) at that time.

The paper proceeds as follows. Section 2 presents the general framework, the estimation and selection methods. Section 3 describes the data, and reports the results of the forecasting exercise. Section 4 offers a discussion of the policy analysis. Section 5 concludes.

2 Framework, Estimation and Selection methods

2.1 Framework

In the literature, researchers have frequently neglected to check for the adequacy between model specification and the generating processes of data, using mostly low-dimensional VAR systems. However, the scarceness of information is generally little advised in econometrics. The short sample of QE programs that we situate between 2008M12 and 2014M11 is clearly characterized by different asset targets but also a mixture of unconventional monetary tools such as “forward guidance” and rescue plans. One can therefore consider an evolution in the transmission channels of QE, as well as the presence of heteroskedasticity. Our goal is gauge the marginal contributions of the addition of structural changes through time-varying parameters and stochastic volatility.

A model misspecification can lead to substantial biases; therefore, misleading inference about forecasting and the magnitude of impulse responses. We thus first generalize the standard VAR process to include time-varying parameters. Second, we augment each model specification with a time-varying unobserved factor; that is, variables entering the estimate can switch over time.

Hereafter, we perform a model *specification* and a model *selection*. The model specification

³Woodford (2012) provides a long discussion to feed the debate.

consists of testing different modeling assumptions across a set of variants. The changes of specifications will depend on the value of a bunch of hyperparameters, as later explained.

The model selection periodically picks out a combination, or averages on all the possible combinations of variables that compose the factor. Precisely, we perform Bayesian/Dynamical Model Averaging (BMA, DMA) and Bayesian/Dynamical Model Selection (BMS, DMS) using the algorithm of Koop & Korobilis (2014) (KK). The discriminating criterion is the forecasting power of each of the variable entering the estimation of the TC factor, as detailed later.

The general model specification over the variants is an TVP-FAVAR with stochastic volatility. For $t = 1, \dots, T$, let y_t be an $s \times 1$ vector of macroeconomic and QE variables. Let f_t be the unobserved TC factor. The observables used for the estimation of f_t are comprised in x_t , an $n \times 1$ vector. The model selection is performed across the $2^n - 1$ possible combinations of x_t .⁴ Consequently, the M_i for $i = 1, \dots, I$ models of this general specification with p -lags, are of the form of:

$$x_{i,t} = \lambda_{i,t}^f f_{i,t} + \lambda_{i,t}^y y_t + \mu_{i,t} \quad (1)$$

where $\lambda_{i,t}^f$ are $n \times 1$ vectors of factor loadings and $\lambda_{i,t}^y$ are $n \times s$ matrices of regression coefficients. The Gaussian errors $\mu_{i,t} \sim \mathcal{N}(0, V_{i,t})$ are time-varying. Notice that equations (1) are linear space equations, the systems being thus completed by i state equations, with:

$$\begin{pmatrix} f_{i,t} \\ y_t \end{pmatrix} = B_{i,t,1} \begin{pmatrix} f_{i,t-1} \\ y_{t-1} \end{pmatrix} + \dots + B_{i,t,p} \begin{pmatrix} f_{i,t-p} \\ y_{t-p} \end{pmatrix} + \varepsilon_{i,t} \quad (2)$$

where $(B_{i,t,1}, \dots, B_{i,t,p})$ are time-varying VAR coefficients and $\varepsilon_{i,t} \sim \mathcal{N}(0, Q_{i,t})$ are Gaussian errors. The VAR coefficients and factor loadings evolve according to random walk equations:

$$\lambda_{i,t} = \lambda_{i,t-1} + \nu_{i,t} \quad (3)$$

$$\beta_{i,t} = \beta_{i,t-1} + \eta_{i,t} \quad (4)$$

where $\lambda_{i,t} = ((\lambda_{i,t}^f)^T, (\lambda_{i,t}^y)^T)^T$, $\beta_{i,t} = (\text{vec}(B_{i,t,1})^T, \dots, \text{vec}(B_{i,t,p})^T)^T$, and $\nu_{i,t} \sim \mathcal{N}(0, W_{i,t})$, $\eta_{i,t} \sim \mathcal{N}(0, R_{i,t})$ are also Gaussian distributed. All of the above errors are by assumption, uncorrelated over time. This concludes the description of the i benchmark TVP-FAVAR models with stochastic volatility.

⁴We obviously preclude the empty set of variables.

2.2 Estimation

We estimate a *great* number of different model specifications. For each of the model specification, the procedure also involves estimating all the i combinations of x_t . Thus, the entire estimation procedure is computationally overburdening, even inconceivable with simulation methods such as MCMC algorithms. Consequently, the simulation-free algorithm of KK is a doable solution among the alternatives, and consist of a dual linear Kalman filter and smoother (KFS). The algorithm proceeds as follows:⁵ The first KFS update the parameters $\theta_t = (\lambda_t, \beta_t)$ given the principal component estimate of the factor $\hat{f}_t$. Afterward, the second KFS update f_t given the estimates of θ_t .

The error covariance matrices (V_t, Q_t, W_t, R_t) are also estimated using simulation-free methods. The covariances V_t and Q_t are computed with an exponentially weighted moving average method. In this estimator, two decay hyperparameters κ_1 and κ_2 manage the degree of stochastic volatility in, respectively, the space (1) and state (2) equations. In turn, W_t and Q_t are estimated using a variance discounting method à la Koop & Korobilis (2013), in which two forgetting hyperparameters κ_3 and κ_4 control the time-variation of, respectively, the parameters λ_t and β_t . The special case where $\kappa_1 = \kappa_2 = 1$ imposes V_t and Q_t to be time-invariant. In that situation, the model specifications are thus homoskedastic. Similarly, λ_t and β_t are constant when $\kappa_3 = \kappa_4 = 1$. In that case, the model specification is a FAVAR. The optimal values of the κ_i are determined by a grid-search algorithm, as explained below.

This Bayesian filtering and smoothing algorithm requires prior conditions. We choose uninformative priors for two reasons: the estimates of KK and the stationarity of data. In addition, a training sample would be misleading because QE was exclusively implemented during our sample. The initial conditions are given by:

$$\begin{aligned} f_0 &\sim \mathcal{N}(0, 4), & \beta_0 &\sim \mathcal{N}(0, R_\beta), & R_\beta &= \frac{\gamma}{r^2}, \\ \lambda_0 &\sim \mathcal{N}(0, 4 \times \begin{pmatrix} I \\ (s+1) \times (s+1) \end{pmatrix}) & V_0 &\equiv \begin{pmatrix} I \\ n \times n \end{pmatrix}, & Q_0 &\equiv \begin{pmatrix} I \\ (s+1) \times (s+1) \end{pmatrix}. \end{aligned} \quad (5)$$

where $r = 1, \dots, p$. R_β is a diagonal covariance matrix in the flavor of Minnesota priors, penalizing distant lags. In addition, V_t is diagonal so that $\mu_{i,t}$ errors are idiosyncratic; therefore, the estimation of the factor uses TC variables only.

To sum up, the algorithm is common to all of the variants of the model specifications. Each specification differs in the values of the κ_i . For instance, a model is a TVP-FAVAR when either κ_3 or κ_4 is lower than 1, or a specification is heteroskedastic if either κ_1 or κ_2 is lower than 1.

⁵A complete description of the algorithm is provided in the online appendix.

In the grid-search algorithm, we restrict the κ_i to three values $\{0.96, 0.98, 1.00\}$, in line with the recommendations of KK, and the three implied distinct cases: The parameters are static for $\kappa_i = 1$, slowly varying for $\kappa_i = 0.98$ and fairly moving $\kappa_i = 0.96$. As seen in Figure 1, a $\kappa_1 = 0.96$ induces that the data point from a year ago weighs 60% as much as the $t - 1$ point in the estimation of covariance V_t , causing a high stochastic volatility in the equation (1).

Figure 1: Decaying importance in the estimation of covariances

These lines display the decaying importance of data from x months ago in the estimation of each covariance, for each of the κ_i cases.

We use Bayesian/dynamical selection/averaging techniques (BMA, BMS, DMA, DMS) to shed light on the time-varying forecasting importance of each TC variable. For each t , DMS dynamically selects the x_t combination with the highest forecasting performance. Conversely, DMA is a time-varying weighted average of all the combinations, where the weights evolve according to the forecasting power of each combination. Hence, let be $\pi_{t|t-1,i}$ the predicted and $\pi_{i,t|t}$ the updated weights determined by the following forms:⁶

$$\pi_{i,t|t-1} = \frac{\pi_{i,t-1|t-1}^\alpha}{\sum_{j=1}^I \pi_{j,t-1|t-1}^\alpha} \quad (6)$$

$$\pi_{i,t|t} = \frac{\pi_{i,t|t-1} p_i(y_t | Data_{1:t-1})}{\sum_{j=1}^I \pi_{j,t|t-1} p_j(y_t | Data_{1:t-1})} \quad (7)$$

where $\alpha \in [0, 1]$ is a forgetting factor that controls the degree of switching between the com-

⁶The method to predict, and update the time-varying weight is derived from Raftery et al. (2010).

binations.⁷ As for the κ_i , a low value of α leads to a high degree of time-variation. The case where $\alpha = 1$ induces the recursive BMA or BMS. The difference with DMA/DMS being that BMA/BMS are recursive windowing methods, while DMA/DMS are dynamic updating methods. In other words, DMA/DMS apply decaying weights to old observations.

In a nutshell, the updated probabilities $\pi_{i,t|t}$ are time-varying according to the predictive likelihood $p_i(y_t|Data_{1:t-1})$, and can exponentially decay if $0 < \alpha < 1$.

3 Data and Forecasting Exercise

3.1 Data

We collect US monthly data from the Federal Reserve Economic Database.⁸ The period of the estimation spans the QE programs of the Federal Reserve, from 2008M12 to 2014M11. The three variables in y_t are the Consumer Price Index (CPI), the real Gross Domestic Product (GDP), and the total assets of all Federal Reserve banks (proxy for QE). The monthly frequency of the real GDP is approximated using a piecewise cubic hermite interpolator.⁹ These series are transformed in first log-difference.¹⁰

The TC factor is extracted using eight time series related to four TC of QE; the portfolio balance channel, the signaling channel, the balance sheet channel and the risk-taking channel. The portfolio balance channel is captured by ten-year Treasury term premium estimates of Adrian et al. (2013), and the Moody's BAA corporate bond yield relative to the ten-year Treasury constant maturity rate. The signaling channel is proxied with the estimates of the ten-year expected average short-term rates also by Adrian et al. (2013) and the ten-year breakeven inflation rate. The balance sheet channel consists of households and nonfinancial corporate business net worth series, both taken in first log-difference and interpolated from quarterly frequencies. The risk-taking channel is captured by the growth rates of St. Louis stress and the CBOE volatility indexes.

All model variants have two lags and the total number of different estimated models is $3^5 \times (2^8 - 1) = 61965$, given the five hyperparameters and the eight x_t variables.¹¹

⁷The weights are equally initialized such that $\pi_{i,0|0} = \frac{1}{I}$.

⁸Ref: <https://fred.stlouisfed.org/>, more details about data are given in the appendix.

⁹We believe that such interpolation is more reliable than any use of monthly proxies such as industrial or activity indexes.

¹⁰The series are also standardized in the forecasting exercise.

¹¹We consider two lags to preserve enough degrees of freedom. Results are robust to higher lag lengths.

3.2 Preliminary discussion

The macroeconomic implications of QE programs have been studied at length in the literature. However, the most intriguing is that empirical studies find strong and robust macroeconomic effects of QE (Baumeister & Benati (2012), WW, while some theoretical studies support weak or neutral impacts of QE (Curdia & Woodford (2011), Jouvanceau (2019)). In that respect, Curdia & Woodford (2011) argue that QE is *irrelevant* absent of financial frictions, but potentially effective through a signaling channel. In other words, QE is effective if it is able to change expectations about the future stance of monetary policy. Empirically, Krishnamurthy & Vissing-Jorgensen (2011) or Christensen & Rudebusch (2012) find supportive evidence about these signaling components. Hence, we test for the relevance of this channel by including measures of market expectations such as the ten-year expected average short-term rates and the ten-year breakeven inflation rate.

According to Bernanke (2012), QE transmits through the portfolio balance channel. The latter roots in the presence of financial frictions. For Vayanos & Vila (2009) financial frictions emerge in investors' preferred-habitats. In this theory markets are segmented because of the preferences of investors regarding the maturities of assets. For investors to invest in another segment, the expected return must offset the incurred risk; therefore assets are not perfect substitutes. In such environment, QE shifts the yields of close substitutes due to the change in the supply of a targeted asset. In particular, QE exerts a downward pressure on the duration risk of assets; hence, reducing long-term rates. WW provide empirical evidence of the portfolio balance of QE. In this work, we assume that the ten-year Treasury term premium and the ten-year corporate bond spread are thus conceivable variables to evaluate the potential contribution of this channel.

More generally, the seminal theories of Kiyotaki & Moore (1997) and Bernanke et al. (1999) are of importance for the macroeconomic effects of QE programs. As a quick reminder, Kiyotaki & Moore (1997) highlight the role of collateral constraints, while Bernanke et al. (1999) design a mechanism of financial accelerator to reconcile the financial and macroeconomic fluctuations. In such theories, asymmetric information and the net worth situations entail excessive premia, affecting the real allocations. These premia are interest spreads between risk-free rates and the cost of capital.

Recently, Gertler & Karadi (2011) (GK) adopt a financial accelerator mechanism in a new-Keynesian model to capture the squeeze in the credit supply in the aftermath of a financial crisis. In this framework, the supply of credit is a function of the size of bankers' balance sheets,

causing an excess premium. Jouvanceau (2019) generalizes the model of GK with, *inter alia* a collateral constraint in a stylized housing market, and refine the theoretical implications of QE. Indeed, Gertler & Karadi (2011) argue that QE is largely effective by loosening balance sheet constraints, hence pressuring excess premia. However, Jouvanceau (2019) nuances these findings and asserts that most of the macroeconomic effects of QE are weaker than found by GK, and are mainly transmitted through income effects – asset prices. Accordingly, we believe that the fluctuations in net worth of private agents are crucial indicators of, both, the upcoming feedback and income effects. For these reasons, we use series of households and nonfinancial corporate business net worth to proxy more generally the balance sheet channel of QE.

One last transmission channel of QE programs is the risk-taking channel. In theory, risk-taking behavior is negatively correlated with the level of short-term rates. Dell’Ariccia et al. (2017) provide empirical evidence of this phenomenon. From that, one can infer that a downward pressure on long-term rates has opposite effects on the attitude toward risk, characterized by search for yields. In addition, the excess of liquidity leaves room for the funding of uncreditworthy agents. We thus control for the risk components by including the St. Louis stress and the CBOE volatility indexes.

Knowing the outlines for each of these channels, we gauge for their relative importance in the following forecasting exercise.

3.3 Forecasting exercise

The forecasting exercise determines the optimal model specification in the set of the possible variants. The hyperparameter optimization is conducted using a grid-search algorithm. The grid comprises 3^5 combinations of the five hyperparameters $(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \alpha)$ for the values $\{0.96, 0.98, 1.00\}$. As a reminder, the hyperparameters define the model specification and the degree of time-variation in the coefficients and between the combinations. In particular, the decay factors κ_1 and κ_2 handle the stochastic volatility in, respectively, the space (1) and the state (2) equations. The forgetting factors κ_3 and κ_4 control the time-variation of, respectively, the VAR coefficients and the factor loadings in the random walk equations (3) and (4). The forgetting factor α shapes the rate of switching between the combinations of x_t variables which is represented by the probabilities $\pi_{i,t|t}$ (see equation (7)).

For instance, the case where $(\kappa_i = \alpha = 0.96)$ defines an TVP-FAVAR model with high stochastic volatility and fast DMA/DMS. The situation where $(\kappa_i = \alpha = 0.98)$ is also a TVP-FAVAR, but with slower time-variation in the coefficients, stochastic volatility and in

DMA/DMS. Last, the case where ($\kappa_i = \alpha = 1.00$) induces an homoskedastic FAVAR with BMA/BMS methods. As such, the pattern to build all the model variants is straightforward. The recursive out-of-sample forecasting exercise is applied to the $3^5 \times (2^8 - 1) = 61965$ estimated models.¹²

The two metrics of forecasting evaluation are the mean squared forecast errors (MSFE) and the one-step ahead predictive likelihoods (PL). The forecasting period spans from 2009M3 to 2014M11-h for $h = 1, 2, 3, 4$ months ahead.¹³ The ranking procedure proceeds as follows. In a first step, we compute the arithmetic means of MSFE and PL over the h horizons and the dimension s of y_t .¹⁴ In a second step, the means of the step one are ranked in an ascending/descending order for PL/MSFE and BMA/DMA or BMS/DMS. Finally, we compute the average of the rankings of the previous step. Consequently, the top 1 specification is the one with the lowest average ranking across the metrics of evaluation, and over DMA/DMS or BMA/BMS.

Table 1 presents the top 10 of model specifications. The first line shows the averaged metrics of the top 1 specification. Other specifications are normalized by the results of the top 1 specification, so that any entry lower than 1 indicates a worse forecasting performance. The top 1 specification is a TVP-FAVAR with a weak stochastic volatility in the space equation (1), homoskedasticity in the state equation (2) and a fast time-variation in the VAR and the loading coefficients. In addition, the BMA/BMS methods are favored. In particular, BMA should not be misunderstood. BMA does not imply that the x_t combinations are always equally weighted in the estimation of the factor.¹⁵ BMA is a windowing recursive model averaging method.¹⁶ To give the reader an overview, the worst specification is an heteroskedastic FAVAR, that performs roughly 15% worst than the top 1 specification. We believe that such underperformance is rather considerable given the size of the sample. This worst specification is characterized by a fast decaying volatility in the space equation and a weak volatility in the state equation. Moreover, it demands constant VAR and loading coefficients. The corresponding hyperparameters are $\alpha = 1.00$, $\kappa_1 = 0.96$, $\kappa_2 = 0.98$, $\kappa_3 = 1.00$, $\kappa_4 = 1.00$.

Conversely, the differences between the top 10 specifications are clearly insignificant and close specifications improve the fit. These models share a fast time-variation in the β_t coefficients

¹²This task is computationally heavy. The use of leaps-and-bounds algorithms or Occam's windowing are highly recommended for higher dimensions.

¹³Forecasts are iterated for $h > 1$.

¹⁴The PL are one-step ahead predictions, therefore are not averaged.

¹⁵This is the case if $\alpha = 0$.

¹⁶The interested reader would find further explanations about BMA/BMS/DMA/DMS in the paper of Raftery et al. (2010).

and a fair change in the loadings λ_t . In addition, six out of the ten specifications highlight the benefit of the absence of stochastic volatility, while the remaining four require a weak stochastic volatility, only in the space equation. From these findings, we assert that QE had asymmetric transmission to the real economy. In addition, the updated time-varying probabilities $\pi_{i,t|t}$ in the Figure 2 shed light in the evolution of the transmission of QE.¹⁷ The portfolio balance and the signaling channels have increasing forecasting power over time. Conversely, the risk-taking channel is completely muted while the balance sheet channel has a relatively steady weight.

The importance of the portfolio balance and signaling channels is shared with the above-cited literature. However, the asymmetric transmission of QE, is a new contribution. Overall, this exercise shows the importance of testing for various model specification and uncertainty so that to avoid misleading inference. In that direction, the next section highlights the consequence of misspecification in the estimation of macroeconomic effects of QE derived from an analysis of impulse responses to structural QE shocks.

Table 1: Top 10 of model specifications

TOP	PL DMA	PL DMS	MSFE DMA	MSFE DMS	SPECIFICATIONS	α	κ_1	κ_2	κ_3	κ_4
1	0.4929	0.4924	0.6416	0.6190	TVP-FAVAR	1.00[†]	0.98	1.00	0.96	0.96
2	0.9997	0.9998	1.0009	1.0029	TVP-FAVAR*	1.00	1.00	1.00	0.96	0.96
3	0.9999	0.9995	0.9978	0.9968	TVP-FAVAR*	1.00	1.00	1.00	0.98	0.96
4	0.9995	1.0014	0.9966	0.9851	TVP-FAVAR	0.98	0.98	1.00	0.96	0.96
5	1.0001	0.9965	0.9969	0.9901	TVP-FAVAR	1.00	0.98	1.00	0.98	0.96
6	0.9993	1.0006	0.9973	0.9898	TVP-FAVAR*	0.98	1.00	1.00	0.96	0.96
7	0.9996	1.0003	0.9951	0.9847	TVP-FAVAR*	0.98	1.00	1.00	0.98	0.96
8	0.9997	1.0009	0.9943	0.9796	TVP-FAVAR	0.98	0.98	1.00	0.98	0.96
9	0.9993	1.0006	0.9960	0.9821	TVP-FAVAR*	0.96	1.00	1.00	0.96	0.96
10	1.0000	0.9985	0.9939	0.9868	TVP-FAVAR*	1.00	1.00	1.00	1.00	0.96

[†] When $\alpha = 1$, BMA and BMS are conducted.

*Are fully homoskedastic model specifications.

MSFE are the mean squared forecast errors. PL are the predictive likelihoods. The first line displays the mean of the MSFE over the 4 h -steps and the one-step ahead PL for the top 1 model specification. The reminder are normalized by the corresponding values of the top 1 specification. Hence, any entry lower than 1 indicate a worse forecasting performance.

¹⁷Each x_t variable enters the estimation of the factor with a probability of $\frac{\sum_{k=1}^n \binom{n-1}{k-1}}{\sum_{k=1}^n \binom{n}{k}} = 50.20\%$, as $n = 8$.

Figure 2: Time-varying probabilities $\pi_{i,t|t}$

The red solid lines show the time-varying probabilities $\pi_{i,t|t}$ in the top 1 specification. The black solid lines are the theoretical probabilities of equally included x_t variables. Details about data and acronyms are given in the appendix.

4 Policy analysis

4.1 Identification of structural shocks

The above analysis makes it clear that simple VARs are most likely misspecified, and that TVP-FAVAR models provide a better fit with the data. As such, one can arguably question the results from the literature that stem from simple VARs regarding the macroeconomic effects of QE programs. In this section, we compute the structural QE shocks in the top 1 specification and compare it to the literature. One of the i combination of the top 1 TVP-FAVAR can be written in a VMA form as:¹⁸

$$\begin{aligned} \begin{pmatrix} x_t \\ y_t \end{pmatrix} &= \tilde{\lambda}_t B_t(L)^{-1} \varepsilon_t + \tilde{\mu}_t \\ &= \tilde{\Phi}_t(L) \eta_t \end{aligned} \quad (8)$$

where $\tilde{\mu}_t = (\mu'_t, 0)'$, $\tilde{\lambda}_t = \begin{pmatrix} \lambda_t^f & \lambda_t^y \\ 0_{1 \times k} & 1 \end{pmatrix}$, $B_t(L) = I - \sum_{l=1}^p B_{t,l} L^l$ and L is the lag operator. For the sake of parsimony, we limit the analysis to the structural shocks related to the y_t variables.¹⁹

¹⁸The generalization to the i combinations are straightforward, see the online appendix for further details.

¹⁹The refinement to the entire set of x_t combinations would imply $(2^8 - 1)$ identification restrictions. The latter is feasible for simple triangular zero restrictions but computationally huge for a mix of zero and sign

Consequently, the VMA form of the top 1 TVP-VAR is sufficient to pin down the following desired structural shocks:

$$z_t = \begin{pmatrix} f_t \\ y_t \end{pmatrix} = B_t(L)^{-1} \varepsilon_t \quad (9)$$

where ε_t are the reduced-form shocks. The uncovered structural shocks can be defined as $\vartheta_t = \mathcal{S}_{0,t} \varepsilon_t$ in which $\mathcal{S}_{0,t}$ are the structural contemporaneous parameters so that the state covariance writes $Q_t = (\mathcal{S}_{0,t} \mathcal{S}_{0,t}^T)^{-1}$.²⁰ In line with the theoretical concepts of Rubio-Ramirez et al. (2010) (RMZ), we propose two sorts of linear restrictions on the space of structural parameters. The first restriction scheme considers zero upper triangular restrictions to the short-run impulse responses. Fortunately, the properties of variance-covariance matrices ensure that Q_t has a unique Cholesky factorization. Thus, the system is globally identified regarding the necessary and sufficient rank conditions.²¹ However, despite its tractability, such recursive identification is at odds with theoretical predictions. Indeed, time ordering assumptions are bold. A second identifying scheme of restrictions is thus considered. We remain agnostic and use a mix of zero and sign restrictions. In the presence of sign restrictions, an TVP-VAR is not identified. To tackle this issue, the algorithm of Binning (2013) allows to compute a set of admissible impulse responses that satisfy the chosen zero and sign restrictions. The flexibility of sign restrictions allows for shocks to be unrestricted. Consequently, we do not impose restrictions to the structural impacts of QE. That way, we assert that the outcomes are robust elements of evidence about the macroeconomic effects of QE programs. The restriction schemes are summarized in the Table 2.

Table 2: Identification scheme: lower triangular zero restrictions

	AS	AD	QE	TC
CPI	1	0	0	0
GDP	x	1	0	0
AST	x	x	1	0
FAC	x	x	x	1

A “x” indicates that a structural shock is left unrestricted. A “0” is a zero restrictions. AS = aggregate supply, AD = aggregate demand, TC = transmission channels.

In the first identification scheme, the restrictions have the following interpretation. The structural shocks of the transmission channels of QE have no contemporaneous effects on inflation, GDP and the total of assets of all Federal Reserve banks.²² The structural QE shocks

restrictions. This would require further technical progress that are beyond the scope of this paper.

²⁰ Assuming that $\mathbb{E}_t[\vartheta_t \vartheta_t^T] = I$.

²¹ Proofs and algorithms are detailed in the online appendix

²² The reader is reminded that GDP and the total assets held by Federal Reserve banks are expressed in log-difference.

Table 3: Identification scheme: mixture of zero and sign restrictions

	AS	AD	QE	TC
CPI	+	+	x	x
GDP	−	+	x	x
AST	0	0	+	x
FAC	x	x	x	x

The “+”, “−” and “0” are the signs and zero restrictions to the structural shocks.

have no short-run effects on inflation and GDP. The structural aggregate demand shocks have no contemporaneous impacts on inflation.

In the second identification scheme, the restrictions has the following interpretation. The structural aggregate supply and demand shocks have no contemporaneous effects on the total assets held by the Federal Reserve banks. In addition, we impose that the structural aggregate supply shocks have a positive impact on inflation and a negative effect on GDP. The aggregate demand shocks increase inflation and GDP. Last, structural QE shocks are characterized by an increase in the the total of assets of all Federal Reserve banks. The effects of shocks to the factor are left unrestricted.

4.2 Impulse responses analysis

The structural shocks are computed over the period 2009M3-2014M11 by using the smoothed estimates of the parameters $\beta_{i,t}$ and $Q_{i,t}$ of the top 1 model specification. The time-varying structural IRF are first presented in three-dimensional surface plots. Second, we dissociate the responses by averaging the IRF for each of the QE programs.²³ In details, the QE1 was in place between 2009M3 to 2010M6 while the QE2 was spreading over 2010M10 to 2012M12. Finally, the QE3 lasted between 2013M1 and 2014M11.

Figures 3 to 14 display the 3D IRFs to unitary QE structural shocks. Remember that in the lower triangular scheme, inflation and GDP do not contemporaneously respond to QE shocks while in the mix of zero and sign restrictions, inflation and GDP are left unrestricted. Moreover, the 68% and 90% percentiles of draws are computed as confidence intervals. In the Cholesky scheme, the draws are obtained with a bootstrap algorithm. In the mixture of zero and sign restrictions, the draws are given by the algorithm of Binning (2013).²⁴

For the ease of visibility, we show the median responses in the surface plots, the confidence

²³The corresponding matrices of IRFs are huge due to the number of dimensions (sample size, horizons, number of variables, number of shocks, draws, number of combinations). Hence, we restrict the number of draws to 100 in both bootstrap and sign restriction algorithms to avoid memory issues. A higher number of draws marginally refines the IRFs.

²⁴A sketch of this algorithm is detailed in the online appendix.

intervals are displayed in the 2D plots only. A first inspection of the results highlights that the IRFs are qualitatively and quantitatively evolving over time. In particular, the effects on inflation are high and persistent in the QE1 period. Conversely, the effects are sharp, weaker and less persistent during QE2 and QE3. The effects on the growth rate of GDP seem qualitatively similar to those of inflation. However, the magnitudes are lower on average. In addition, the peak effects are delayed by a semester.

Figures 15 to 20 display the averaged IRFs for each of the QE programs. In QE1, we thus learn that the statistical significance of the responses of inflation occurs after a semester, especially in the mixture of restrictions. The average responses of GDP shows a similar pattern. In other words, QE shocks have delayed impacts on inflation and GDP. This pattern is reinforced in the QE2 period; GDP and inflation being entirely in phase. However, in that period, the persistence in the effects is distinctively less important than during QE1. The IRF of QE3 resemble to those of QE2. Overall, these findings are robust to the use of BMA or BMS methods.

Looking at these IRFs, one notices that the magnitude of the effects of QE shocks on GDP and inflation decline over time. However, one should remain cautious, since the total amounts of purchased assets differed in each of the QE programs. Consequently, we gauge the total effects of QE shocks by computing accumulated IRFs. Figures 21 to 26 display the corresponding results. In both schemes, the IRFs are significant in the width of two standard-deviations around the medians. Hence, we compute the total effects of QE shocks by proceeding to back of envelope calculations. First, we average the accumulated IRFs across schemes and BMA/BMS. Second, we multiply the long-term steady values of CPI and GDP by the total change in the assets for each of QE programs. In particular, the total assets held by the Federal Reserves changed by 17.76% in the QE1 program, by 25.88% in the QE2 program and by 51.33% in the QE3 program.

The calculations then suggest that QE1 roughly lead to a total rise of 1.30% in CPI and 1.65% in GDP. If we relate these findings to the multipliers of previous studies reported in WW, our results are close. For example, Baumeister & Benati (2012) and WW respectively estimate that QE1 increased CPI by 0.90% and 1.50%, and GDP by 1.08% and 1.40%. However, being computed with BVAR models with data from 2009M3 to 2014M5, the estimates of WW (among others) are somehow problematic for further inference. In addition, they assert that: “asset purchases did not become less effective over time”. In particular, they find that the maximum impacts of a unitary structural QE shock induces an increase of 0.58% in GDP

and 0.62% in CPI.²⁵ Hence, by linearity, their multipliers should be 2.10% and 3.92% for CPI and 1.97% and 3.67% for GDP in the QE2 and QE3 programs, respectively.²⁶ In our top 1 specification the corresponding multipliers are up to {1.57%, 2.03%} (CPI, GDP) for the QE2 program, and {2.16%, 2.95%} for the QE3 program, which is overall significantly lower.

We proceed to further comparison by also estimating VAR models augmented with the estimated factors of the top 1 specification. The IRFs of these VAR models are depicted in Figures 27 to 50. In those specifications, the multipliers are of {1.45%, 2.11%, 3.94%} for CPI and {1.95%, 2.79%, 5.28%} for GDP, in QE1, QE2 and QE3, respectively. In other words, having constant effects over time; the impact of QE are linearly increasing with the magnitude of the amounts purchased. However, in our top 1 specification with time-varying parameters, the multipliers for CPI {1.29%, 1.57%, 2.16%} and GDP {1.65%, 2.03%, 2.95%} are clearly not monotonic.

The lesson from the above analysis is that the effects of QE shocks estimated by VAR specifications are certainly overestimated. In addition, QE had asymmetric effects on the real economy. In particular, QE was more effective when the markets were disrupted as when the QE1 program was implemented; that is, when financial frictions were exacerbated. Conversely, QE had marginally lower macroeconomic effects when the QE2 and QE3 programs were executed; even though the portfolio balance and the signaling channels gained forecasting importance over time.

5 Conclusion

In the empirical literature regarding the macroeconomic effects of QE, studies rely on VAR specification and sparse information. In this paper, we expose the potential issues of such modeling assumptions. We find that they can induce misleading insights about the transmission of QE to the real economy, and may overestimate the macroeconomic effects of QE programs.

These results emerge from an optimal selection among a large set of model specifications that allows for structural changes in parameters and stochastic volatility. The optimal specification is found using a grid-search algorithm over hyperparameters that rule the variants. Specifications are ranked according to their ability to predict macroeconomic and QE variables. All models are augmented with an unobserved factor that represents the different theoretical TC of QE. In addition, we take uncertainty into account by using all possible combinations of the TC variables

²⁵In the study of WW, QE structural shocks are expressed in percent of GDP (2009Q1).

²⁶These estimates are computed by considering that the total assets of QE purchases was worth 3.40% and 6.32% of 2009Q1 GDP in QE2 and QE3, respectively.

which serve in the estimation of the factor. Afterward, dynamic selection and averaging methods inform about the time-varying forecasting importance of each TC.

On the one hand, our results suggest that the time-variation in parameters improves the fit of the estimation. On the other hand, stochastic volatility is not critical. Given the importance of time-variation in parameters, we find that QE was asymmetrically transmitted to the real economy across the different QE programs. This assertion is further supported by a policy analysis of structural QE shocks. Indeed, we find that the macroeconomic effects of QE do not increase monotonically in the amount of purchases. In particular, QE was more impacting during QE1, when financial markets were severely impaired, from 2009M3 to 2010M6. On the contrary, despite the increasing forecasting importance of the portfolio balance and signaling channels between 2010M10 to 2014M11 (QE2 and QE3 periods), the estimated macroeconomic impacts of QE are lower. We argue that the slackness of financial constraints, in boom cycles, potentially accounts for the reduced macroeconomic effects of QE. These asymmetries pave the way for additional and interesting theoretical studies, that could look at the effects of QE depending on the slackness of financial constraints.

Appendix A - Figures

Figure 3: TOP 1: Median IRF of CPI to QE shocks: Cholesky and BMA

Figure 4: TOP 1: Median IRF of GDP to QE shocks: Cholesky and BMA

Figure 5: TOP 1: Median IRF of total assets to QE shocks: Cholesky and BMA

Figure 6: TOP 1: Median IRF of CPI to QE shocks: Cholesky and BMS

Figure 7: TOP 1: Median IRF of GDP to QE shocks: Cholesky and BMS

Figure 8: TOP 1: Median IRF of total assets to QE shocks: Cholesky and BMS

Figure 9: TOP 1: Median IRF of CPI to QE shocks: Zero and Sign restrictions and BMA

Figure 10: TOP 1: Median IRF of GDP to QE shocks: Zero and Sign restrictions and BMA

Figure 11: TOP 1: Median IRF of total assets to QE shocks: Zero and Sign restrictions and BMA

Figure 12: TOP 1: Median IRF of CPI to QE shocks: Zero and Sign restrictions and BMS

Figure 13: TOP 1: Median IRF of GDP to QE shocks: Zero and Sign restrictions and BMS

Figure 14: TOP 1: Median IRF of total assets to QE shocks: Zero and Sign restrictions and BMS

Figure 15: TOP 1: Average IRF of QE structural shocks during QE1 with BMA

Figure 16: TOP 1: Average IRF of QE structural shocks during QE1 with BMS

Figure 17: TOP 1: Average IRF of QE structural shocks during QE2 with BMA

Figure 18: TOP 1: Average IRF of QE structural shocks during QE2 with BMS

Figure 19: TOP 1: Average IRF of QE structural shocks during QE3 with BMA

Figure 20: TOP 1: Average IRF of QE structural shocks during QE3 with BMS

Figure 21: TOP 1: Accumulated IRF of QE structural shocks during QE1 with BMA

Figure 22: TOP 1: Accumulated IRF of QE structural shocks during QE1 with BMS

Figure 23: TOP 1: Accumulated IRF of QE structural shocks during QE2 with BMA

Figure 24: TOP 1: Accumulated IRF of QE structural shocks during QE2 with BMS

Figure 25: TOP 1: Accumulated IRF of QE structural shocks during QE3 with BMA

Figure 26: TOP 1: Accumulated IRF of QE structural shocks during QE3 with BMS

Figure 27: VAR: Median IRF of CPI to QE shocks: Cholesky and BMA

Figure 28: VAR: Median IRF of GDP to QE shocks: Cholesky and BMA

Figure 29: VAR: Median IRF of total assets to QE shocks: Cholesky and BMA

Figure 30: VAR: Median IRF of CPI to QE shocks: Cholesky and BMS

Figure 31: VAR: Median IRF of GDP to QE shocks: Cholesky and BMS

Figure 32: VAR: Median IRF of total assets to QE shocks: Cholesky and BMS

Figure 33: VAR: Median IRF of CPI to QE shocks: Zero and Sign restrictions and BMA

Figure 34: VAR: Median IRF of GDP to QE shocks: Zero and Sign restrictions and BMA

Figure 35: VAR: Median IRF of total assets to QE shocks: Zero and Sign restrictions and BMA

Figure 36: VAR: Median IRF of CPI to QE shocks: Zero and Sign restrictions and BMS

Figure 37: VAR: Median IRF of GDP to QE shocks: Zero and Sign restrictions and BMS

Figure 38: VAR: Median IRF of total assets to QE shocks: Zero and Sign restrictions and BMS

Figure 39: VAR: Average IRF of QE structural shocks during QE1 with BMA

Figure 40: VAR: Average IRF of QE structural shocks during QE1 with BMS

Figure 41: VAR: Average IRF of QE structural shocks during QE2 with BMA

Figure 42: VAR: Average IRF of QE structural shocks during QE2 with BMS

Figure 43: VAR: Average IRF of QE structural shocks during QE3 with BMA

Figure 44: VAR: Average IRF of QE structural shocks during QE3 with BMS

Figure 45: VAR: Accumulated IRF of QE structural shocks during QE1 with BMA

Figure 46: VAR: Accumulated IRF of QE structural shocks during QE1 with BMS

Figure 47: VAR: Accumulated IRF of QE structural shocks during QE2 with BMA

Figure 48: VAR: Accumulated IRF of QE structural shocks during QE2 with BMS

Figure 49: VAR: Accumulated IRF of QE structural shocks during QE3 with BMA

Figure 50: VAR: Accumulated IRF of QE structural shocks during QE3 with BMS

Appendix B - Data

Description & Acronyms	Units	Sources
Real Gross Domestic Product (GDP)	Billions	US B.E.A.
All Federal Reserve Banks: Total Assets (AST)	Millions	H.4.1 F.A.R.B.
Consumer Price Index for All Urban Consumers (CPI)	Index	US B.E.A.
Ten-year Treasury Term Premium (10Y TP)	Rates	Adrian et al. (2013)
Moody's Seasoned Baa Corporate Bond Yield (BAA)	Rates	Moody's
Ten-Year Treasury Constant Maturity Rate (10Y)	Rates	H.15 S.I.R.
Ten-year Expected Average Short-Term Rates (10Y exp. ST rates)	Rates	Adrian et al. (2013)
10-Year Breakeven Inflation Rate (10Y infl)	Rates	Federal Reserve St. Louis
Households and Nonprofit Organizations; Net Worth (HH NW)	Billions	Z.1 F.A.U.S.
Nonfinancial Corporate Business; Net Worth (Corp. NW)	Billions	Z.1 F.A.U.S.
St. Louis Fed Financial Stress Index (STL stress)	Index	Federal Reserve St. Louis
CBOE Volatility Index: VIX (CBOE VIX)	Index	CBOE Market Statistics
Initialisms: B.E.A. = Bureau of Economic Analysis, F.A.R.B. = Factors Affecting Reserve Balances, S.I.R. = Selected Interest Rates, F.A.U.S. = Financial Accounts of the United States.		

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