Skewed information transmission: the effect of complementarities in a multi-dimensional cheap talk game

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Abstract

We analyze a cheap talk game in a two-dimensional framework, with complementarities between the dimensions. A receiver chooses a two-dimensional costly effort in a productive activity. The Receiver's effort profile is determined by his individual ability profile, of which he is unaware. He is advised by an informed Sender, who makes one of two recommendations to maximize the Receiver's output. Output is a Constant Elasticity of Substitution function of the Receiver's two one-dimensional contributions. The credibility constraint on the Sender's recommendations requires her to truthfully contrast the Receiver's abilities. As a result, the Receiver associates greater (resp. less) effort with greater (resp. less) perceived ability in one (resp. the other) dimension. However, when there are strong complementarities, the Sender becomes more interested in the mismatch between effort and ability. Thus, strong complementarities preclude the Sender from making credible recommendations. By contrast, weaker complementarities allow the Sender to make credible recommendations, using either: (i) a symmetric comparison of abilities; or (ii) an asymmetric comparison, in which one ability could be perceived as greater, even if it is not. Sufficient complementarities make the asymmetric information revelation more stable and more productive.

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1 Introduction

In many important situations, agents determine their actions based on information revealed by another, better informed agent. For instance, an employee does this when considering her supervisor's recommendations about how to do her job; a student does so when acknowledging her teacher's expertise concerning which field of study she should pursue; and a child does so when conforming to parental rules. However, managers, teachers and parents have strong *biases* concerning the contribution of their "mentee". They do not incur the mentee's cost of effort, and prefer greater level of effort relative to the mentee's preferred level. According to the literature on strategic information transmission, this bias on the part of better-informed agents can preclude them from making influential recommendations (Crawford and Sobel, 1982), unless the parties take advantage of multiple dimensions for communication (Chakraborty and Harbaugh, 2007).

This paper addresses how communication is impacted when complementarities between the dimensions of the production are considered. Complementarities play a crucial role in production. For instance, at school or at home, human capital production involves complementarities between the child's skill dimensions (Cunha et al., 2010).[1] In the workplace, a bricklayer needs force *and* meticulousness to build a wall. When complementarities are considered, providing information to an uninformed agent might cause the agent to detrimentally allocate her efforts. This is due to the credibility constraint: when the informed agent prefers greater efforts, whatever her private information, she will always make a recommendation that will induce greater efforts along all dimensions (relative to an alternative recommendation). As a result, such a recommendation cannot be informative. Therefore, if providing information can increase output by driving efforts in more productive dimensions, it is at the cost of inducing lower levels of effort along other dimensions. This opportunity cost increases with more complementarities between the dimensions of production. Thus, strong complementarities can preclude communication. With limited complementarities, the trade-off can be solved through skewed information provision in favor of a single dimension production. The consequence is a potential mismatch between the actual information and its perception by the uninformed agent.

Our setting follows the standard Sender-Receiver setting in the cheap talk literature, with the following specifications. A two-dimensional state is realized from the uniform distribution on $[0,1]^2$. The state is the Sender's (she) information, known only to her, and represents her

[1] And teachers do no not consider the dimensions of knowledge as perfect substitutes.

evaluation of the Receiver's (he) abilities concerning a two-dimensional production.[2] Based on the Sender's private assessment, she recommends (or assigns) one of two activities to the Receiver. In doing this, she issues one of two messages to the Receiver about his ability profile. Given the message received, the Receiver chooses a two-dimensional effort profile.[3] We assume that in each dimension, the Receiver's contribution is the product of effort and ability, and that the Receiver's total production is a constant elasticity of substitution (CES) function of his two one-dimensional contributions. The CES specification permits investigation of the equilibrium conditions as a function of the degree of complementarity between the two one-dimensional contributions. The Sender maximizes the CES output. In particular she prefers greater efforts, whatever the Receiver's actual abilities. Given the cost of his efforts, in each dimension, the Receiver prefers to adjust his effort to his ability.

Strategically, the Sender chooses the recommendation that maximizes the CES output, given the Receiver's ability profile and anticipated effort profile. Reciprocally, the Receiver contributes according to his own perceived abilities, which he derives from the message he receives and his beliefs about the way the Sender associates abilities with messages. An influential equilibrium is sustained if, for each message, the Sender correctly anticipates the Receiver's subsequent contributions, and, reciprocally, the Receiver correctly chooses his level of efforts according to his actual abilities, and given the information received.

We first describe two necessary conditions for such an equilibrium to exist.

The first condition rules out the existence of a recommendation that induces greater efforts along both dimensions relative to the efforts induced by the alternative recommendation. Indeed, if such a recommendation exists, the Sender will always prefer to deviate to it, irrespective of her private information on abilities. As a result, it cannot be informative. Consequently, an influential recommendation inevitably induces a greater effort in one dimension and a lesser effort in the other dimension. Such efforts are induced by a Sender's comparison of abilities.

The second condition rules out an overly high degree of complementarity. Suppose, for instance, that the Sender wants to maximize the minimum of the Receiver's two one-dimensional contributions. She thus prefers to match the lowest effort with the highest ability and the highest effort with the lowest ability. However, the Receiver adjusts his efforts to his perceived abilities. Therefore, in equilibrium, the highest effort must be derived from a perceived

[2]For instance, in line with Cunha et al. (2010), the two dimensions might represent the cognitive and non-cognitive dimensions of the Receiver's skill profile.

[3]Note that even a child would proceed to Bayesian inferences to model the world around (Gopnik, 2012).

greater ability. This conflicts with the Sender's incentives.

Next, we show that when complementarities are limited, the Sender has multiple ways to compare the Receiver's abilities in equilibrium.[4] The Sender can symmetrically compare the Receiver's abilities. Then the Receiver exerts symmetrically balanced efforts across dimensions and recommendations (which, in turn, determines the symmetric comparison). The Sender can also asymmetrically compare the Receiver's ability. An asymmetric comparison biases the recommendations and the subsequent effort investments in favor of one of the dimensions.[5] One issue of the comparison is *ex-ante* more likely to be disclosed than the other; accordingly, one dimension is *ex-ante* more likely to be the most invested in by the Receiver. Moreover, the degree of the asymmetry of such an asymmetric comparison is exacerbated with a greater degree of complementarity between the Receiver's contributions.[6] The reason is that more complementarity makes the more expected recommendation even more profitable relative to alternative recommendation.

Finally, we show that relative to the symmetric equilibrium, the asymmetric equilibrium tends to be *ex ante* more productive with more complementarities.[7] The intuition is that more complementarities increase the Sender's interest in preventing the Receiver from neglecting one of the dimensions of his abilities. This is precisely the effect of the *ex-ante* lower informativeness of the asymmetric equilibrium. More precisely, in the asymmetric equilibrium the message most likely to be issued is poorly informative. With more complementarities, it is even less informative and more likely to be issued. This makes the Receiver more likely to exert more homogeneous efforts across the two dimensions. Relative to the information provided from the symmetric equilibrium, such homogeneous efforts becomes more productive

[4]For instance, concerning parental influence, Tenenbaum (2009) shows that when parents make recommendations concerning their children's course choices, the language they use to speak to their daughters tends to be more discouraging than the language they employ to talk to their sons, whatever the study domain. This suggests that parents and children play a different equilibrium conditional on the child's gender.

[5]Given the symmetry of the game, there are two asymmetric equilibria, one derived from the other by switching the dimension labels.

[6]The degree of complementarity has no effect on the symmetric equilibrium.

[7]In particular, there is generically an equilibrium which is more productive than the other. This illustrates the possibility of detrimental influence by an informed agent concerning, e.g., the ability profile of a pupil. As Heckman and Mosso (2014) argue, it is not budget constraints but parental attitudes, determined by their objective function, which are the major cause of observed low educational attainment children from low income families (Hackman et al., 2010; Knudsen et al., 2006; Heckman, 2008). Along the same lines, the parents' degree of risk aversion has been shown to be inversely related to the child's education attainment (Wölfel and Heineck, 2012; Brown et al., 2012; Checchi et al., 2014).

with strong complementarities.[8]

Note that from the Receiver's perspective, the asymmetric equilibrium is always detrimental relative to the symmetric equilibrium. It provides less information. Moreover, in the asymmetric equilibrium, the Receiver's perception of his highest ability may not correspond to its actual realization. Hence, the Sender could potentially misinform the Receiver about his "comparative advantage".

An extensive literature on strategic information transmission has emerged, triggered by the seminal paper by Crawford and Sobel (1982) (see Sobel (2013) for a review). Crawford and Sobel (1982) examine the case in which the Sender's private information and the Receiver's action are one-dimensional. Few theoretical papers consider the case in which players can communicate on multiple dimensions.

Chakraborty and Harbaugh (2007) consider a two-dimensional state of the world and a two-dimensional action. They show that the symmetric comparison of the one-dimensional state defines a Sender's equilibrium strategy if the state distribution is symmetric with respect to the two dimensions, and if the players' utility functions are additively separable with respect to the two dimensions and super-modular with respect to type and action in each dimension.[9] Chakraborty and Harbaugh's (2007) result illustrates that whatever the extent of their conflict, players might reach agreement by revealing and processing information orthogonally to their conflict. In our setting, the Sender always prefers a greater contribution from the Receiver along each dimension. Hence, conflict is high. Our result illustrates that in that case, the comparative method of transmitting credible information is also necessary. However, we show that multiple comparisons are possible in equilibrium. Along similar lines, Sémirat (forthcoming) considers a uniform state space, a binary message set, and separable and quadratic preferences for both players. The author characterizes equilibria of the game for any direction and any extent of the Sender's bias. He derives the same multiplicity of equilibria for any extent and direction of the players' conflict.

The present paper relaxes the separable assumption previously assumed in the literature,[10]

[8] According to Wang and Degol (2017), women are more likely to have an homogeneous ability profile, which would give them more possibilities concerning their choice of occupation. By contrast, men are more specialized toward math-intensive fields, and thus are more restricted in their occupational choice. This also suggests that a different equilibrium is played out during the skill formation process, depending on the gender of the child.

[9] In contrast, Levy and Razin (2007) show that the possibility of influence is precluded if the setting includes slight asymmetries and an extreme conflict of interest between players.

[10] An exception is Chakraborty and Harbaugh (2010). However Chakraborty and Harbaugh (2010) assume

and exploits the CES functional form to investigate the impact of a degree of complementarity on the revealed information. From the production perspective, the symmetric equilibrium is more efficient *iff* the degree of complementarity is low. Hence, complementarity fosters information withholding. However, whatever the degree complementarity, we also show that the uninformative babbling equilibrium is never *ex ante* more productive.[11] In particular, the Sender always benefit from some information provision for adequately allocating the Receiver's effort.

The economic motivation for our study originates from a recent strand of work on confidence management. This work investigates the informational content of firm decisions, and how they might affect the employees' efforts.

Crutzen et al. (2013) study the impact of the revelation or withholding of firms' rankings of employees. The authors focus on symmetric equilibria and show that in general, the firm has an interest in withholding such information. Asymmetric equilibria are investigated in Kamphorst and Swank (2016), who consider the promotion decision of a manager, derived from her private information concerning the two one-dimensional abilities of two employees. Like us, the authors obtain a symmetric equilibrium and an asymmetric equilibria (up to relabeling of the dimensions). In the asymmetric equilibrium, the manager discriminates in favor of the employee the players expect to be promoted. While Kamphorst and Swank (2016) consider two Receivers, each of whom contributes along a unique dimension, we consider a unique Receiver who contributes along two dimensions. Kamphorst and Swank's (2016) model is restricted to the case in which employees' contributions are perfect substitutes. Formally, our model extends Kamphorst and Swank's (2016) model by including the possibility of complementarities between employees contributions. Our characterization of the multiple equilibria according to the degree of complementarity includes Kamphorst and Swank's (2016) main result as a polar case. Our results highlight some important issues related to the asymmetric equilibria in cases where complementarities exist. First, we show that a high degree of complementarity induces the Sender to *ex-ante* prefer asymmetric over symmetric treatment. This is not true in the case of perfect substitution. Hence, the players' asymmetric treatment might be more likely with complementarities. Second, we illustrate that asymmetric treatment results from a qualitative effect. In particular, we show that the effect is robust to changes in the prior distribution that would preclude any asymmetric treatment in case of

state independent preferences for the Sender.

[11] By contrast, Chakraborty and Harbaugh (2010) note that a Sender with quasi-convex preferences relative to the multi-dimensional Receiver's action *ex-ante* prefers to be uninformative.

perfect substitution. Finally, we show that introducing slight exogenous asymmetries between the dimensions precludes the asymmetric equilibrium when there is perfect substitution; however, the asymmetric equilibrium is resistant to large exogenous asymmetries if the involved degree of complementarity is sufficiently high.

The paper is organized as follows. Section 2 describes the model set up and Section 3 presents the results. Section 3.1 and 3.2 discuss the strategies and necessary equilibrium conditions. Section 3.3 derives the existence of the symmetric and asymmetric equilibria for each degree of complementarity, and associates the degree of complementarity with the asymmetry of the asymmetric equilibrium. In Section 4, we provide arguments related to the robustness of the effect of complementarity on the asymmetric equilibrium, and Section 5 investigates selection criteria (with regard to stability and efficiency). Section 6 concludes. Proofs are provided in the appendix.

2 Model setup

A Receiver produces an output within a working environment. Production is decomposed along two dimensions $i = 1$ and $i = 2$. The Receiver contributes in each dimension $i \in \{1, 2\}$ at level y_i. Each contribution y_i relies on ability level a_i, and a chosen effort level e_i through

$$y_i = a_i e_i. \tag{1}$$

The Receiver derives utility from his contributions, but suffers a quadratic effort cost along each dimension.[12]

His utility is given by

$$U(e_1, e_2) = \sum_{i=1,2} \left(y_i - \frac{1}{2} e_i^2 \right). \tag{2}$$

The Receiver is not aware of his ability profile (a_1, a_2). He has a uniform prior on $[0, 1]$ in each dimension.

A Sender (she) is aware of (a_1, a_2). Given her observation, she recommends one of two activities to the Receiver. The recommendation takes the form of a message $m \in \{m_1, m_2\}$.

[12] We do not integrate spillover effects between effort costs, and assume a quadratic and additively separable utility function for the Receiver. This assumption is usual in the literature, and allows us to focus on the informational aspect of the equilibrium conditions (so that actions are identified with the expected states in equilibrium). It represents situations where a change of effort in one dimension has no effect on the effort in the other dimension. This might be due to independence of the dimensions considered (*e.g.* cognitive and non-cognitive efforts), or to efforts exerted at distant date.

The Sender's utility derived from Receiver's contributions is given by the CES function

$$Y_r = \left(\frac{y_1^r + y_2^r}{2}\right)^{\frac{1}{r}} \tag{3}$$

where $r \in (-\infty, 1]$ represents the degree of complementarity between the Receiver's contributions.[13]

The Sender makes her recommendation in order to maximize (3).

Given the Sender's recommendation $m \in \{m_1, m_2\}$, the Receiver chooses his effort levels to maximize his expected utility

$$\mathbb{E}[U|m].$$

The timing of the game is as follows:

1. Nature draws abilities a_1 and a_2, and reveals them to the Sender, but not to the Receiver, who has a uniform prior;
2. The Sender sends message $m \in \{m_1, m_2\}$ to Receiver;
3. The Receiver observes the Sender's message m, and updates his beliefs about his abilities;
4. The Receiver chooses his effort level $e_i(m)$, $i \in \{1, 2\}$, according to his posterior beliefs;
5. payoffs are realized.

We look for perfect Bayesian equilibria of the game, *i.e.*: (i) the Receiver's effort strategy is optimal given his beliefs about his abilities, (ii) whenever possible, beliefs are updated according to Bayes's rule, (iii) the Sender's disclosure strategy is optimal, given the Receiver's effort strategy and beliefs.

[13] Recall that for any $r \leq 1$, the elasticity of substitution between the contributions is given by $\rho = \frac{1}{1-r}$. In particular, at $r = 1$, we have $Y_r = \frac{y_1+y_2}{2}$ and perfect substitution between the contributions, at $r \to 0$, Y_r tends to the constant return to scale symmetric Cobb-Douglas function $Y_r = \sqrt{y_1 y_2}$, and at $r \to -\infty$, Y_r tends to the Leontief function $Y_r = \min\{y_1, y_2\}$.

3 Analysis

3.1 Strategies

Given the message $m \in \{m_1, m_2\}$ that he observes, the Receiver exerts his efforts to maximize his expected utility at

$$\begin{aligned}(e_1(m), e_2(m)) &= \underset{(e_1,e_2)\in\mathbb{R}^2}{\arg\max}\ \mathbb{E}[U(e_1,e_2)|m] \\ &= \underset{(e_1,e_2)\in\mathbb{R}^2}{\arg\max}\ \mathbb{E}\left[a_1e_2 - \frac{1}{2}e_1^2 + a_2e_2 - \frac{1}{2}e_2^2 \middle| m\right] \\ &= \underset{(e_1,e_2)\in\mathbb{R}^2}{\arg\max}\ e_2\mathbb{E}[a_1|m] - \frac{1}{2}e_1^2 + e_2\mathbb{E}[a_2|m] - \frac{1}{2}e_2^2 \\ &= (\mathbb{E}[a_1|m], \mathbb{E}[a_2|m]). \end{aligned} \tag{4}$$

Reciprocally, given the Receiver's effort strategy $m \mapsto (e_1(m), e_2(m))$, the Sender's utility derived from the recommendation m is given by

$$Y_r(m) = \left(\frac{a_1^r e_1^r(m) + a_2^r e_2^r(m)}{2}\right)^{\frac{1}{r}}.$$

Then the Sender chooses m_1 or m_2 conditional on $Y_r(m_1) \geq Y_r(m_2)$ or $Y_r(m_1) \leq Y_r(m_2)$ respectively.

Notice that if $e_1(m_1) = e_1(m_2)$ and $e_2(m_1) = e_2(m_2)$ then she is indifferent between m_1 and m_2, whatever (a_1, a_2). Reciprocally, in the case in which her choice does not depend on (a_1, a_2), then the Receiver's posterior beliefs (4) are equal to his prior beliefs, and $e_1(m_1) = e_1(m_2)$ and $e_2(m_1) = e_2(m_2)$. These strategies define a *babbling equilibrium*.

Henceforth, we focus on *influential equilibria*, *i.e.* equilibria in which $e_1(m_1) \neq e_1(m_2)$ or $e_2(m_1) \neq e_2(m_2)$.

3.2 Influential equilibrium conditions

Let us distinguish the cases $r < 0$ and $r \in (0, 1]$.

Case $r \in (0, 1]$**.** If $r \in (0, 1]$, then we obtain (up to a null measure set of abilities) the following Sender's strategy:

$m(a_1, a_2) = m_1$ *iff*

$$\begin{aligned} &\left(\frac{a_1^r e_1^r(m_1) + a_2^r e_2^r(m_1)}{2}\right)^{\frac{1}{r}} \geq \left(\frac{a_1^r e_1^r(m_2) + a_2^r e_2^r(m_2)}{2}\right)^{\frac{1}{r}} \\ \iff\ & a_1^r e_1^r(m_1) + a_2^r e_2^r(m_1) \geq a_1^r e_1^r(m_2) + a_2^r e_2^r(m_2) \\ \iff\ & a_1^r \left(e_1^r(m_1) - e_1^r(m_2)\right) \geq a_2^r \left(e_2^r(m_2) - e_2^r(m_1)\right). \end{aligned}$$

In equilibrium, we must have $e_1^r(m_1) - e_1^r(m_2) \neq 0$ and $e_2^r(m_2) - e_2^r(m_1) \neq 0$, with equal signs, since otherwise the same recommendation would always be issued. We derive the following conditional strategies:
if $e_1^r(m_1) > e_1^r(m_2)$ and $e_2^r(m_2) > e_2^r(m_1)$, that is $e_1(m_1) > e_1(m_2)$ and $e_2(m_2) > e_2(m_1)$, then

$$m(a_1, a_2) = m_1 \textit{ iff } a_1 \geq \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)} \right)^{\frac{1}{r}} \times a_2,$$

or, if $e_1(m_1) < e_1(m_2)$ and $e_2(m_2) < e_2(m_1)$, then

$$m(a_1, a_2) = m_1 \textit{ iff } a_1 \leq \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)} \right)^{\frac{1}{r}} \times a_2.$$

Note that the above Sender's strategies and the corresponding conditions are equivalent up to a relabeling of the messages m_1 and m_2.

Case $r < 0$**.** If $r < 0$, then we obtain the Sender's strategy:
$m(a_1, a_2) = m_1$ *iff*

$$\left(\frac{a_1^r e_1^r(m_1) + a_2^r e_2^r(m_1)}{2} \right)^{\frac{1}{r}} \geq \left(\frac{a_1^r e_1^r(m_2) + a_2^r e_2^r(m_2)}{2} \right)^{\frac{1}{r}}$$

$$\iff a_1^r e_1^r(m_1) + a_2^r e_2^r(m_1) \leq a_1^r e_1^r(m_2) + a_2^r e_2^r(m_2)$$

$$\iff a_1^r \left(e_1^r(m_1) - e_1^r(m_2) \right) \leq a_2^r \left(e_2^r(m_2) - e_2^r(m_1) \right).$$

Then we have: if $e_1^r(m_1) > e_1^r(m_2)$ and $e_2^r(m_2) > e_2^r(m_1)$, that is $e_1(m_1) < e_1(m_2)$ and $e_2(m_2) < e_2(m_1)$, then

$$m(a_1, a_2) = m_1 \textit{ iff } a_1 \geq \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)} \right)^{\frac{1}{r}} \times a_2,$$

or, if $e_1(m_1) > e_1(m_2)$ and $e_2(m_2) > e_2(m_1)$, then

$$m(a_1, a_2) = m_1 \textit{ iff } a_1 \leq \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)} \right)^{\frac{1}{r}} \times a_2.$$

Again, the two strategies and the corresponding conditions are equivalent up to a relabeling of the messages. However, note for instance that if $m(a_1, a_2) = m_1$ *iff* $a_1 \geq t a_2$ for some $t > 0$, then we have in equilibrium

$$e_1(m_1) = \mathbb{E}[a_1 | m_1] = \mathbb{E}[a_1 | a_1 \geq t a_2] > \mathbb{E}[a_1 | a_1 < t a_2] = \mathbb{E}[a_1 | m_2] = e_1(m_2),$$

and similarly, $e_2(m_2) < e_2(m_1)$. Therefore in the case that $r < 0$, the above strategies and the corresponding conditions are not compatible. In other words, if $r < 0$ there is no influential equilibrium. Thus we obtain the following necessary condition for an influential equilibrium to occur.

Lemma 1. *If $m \mapsto (e_1(m), e_2(m))$ and $(a_1, a_2) \mapsto m(a_1, a_2)$ define an influential strategy profile, then $r \in (0, 1]$, and up to a relabeling of the messages,*

$$e_1(m_1) > e_2(m_1), \text{ and } e_2(m_2) > e_1(m_2), \tag{5}$$

and

$$m(a_1, a_2) = \begin{cases} m_1, \text{ if } a_1 \geq t a_2, \\ m_2, \text{ if } a_1 < t a_2, \end{cases} \tag{6}$$

where

$$t = \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)} \right)^{\frac{1}{r}}. \tag{7}$$

Note that according to (4), effort represents the Receiver's perception of his ability. Hence, the sign condition (5) implies that the Sender's strategy must contrast the Receiver's perception of his abilities. For instance, if m_1 induces a greater perception concerning the first dimension ($e_1(m_1) > e_1(m_2)$) then it necessarily induces a lesser perception concerning the second dimension ($e_2(m_1) < e_2(m_2)$), and *vice versa*. Clearly, strategy (6) satisfies this requirement. Therefore, Conditions (4), (6) and (7) also are sufficient conditions for the existence of an equilibrium.

Finally, note that the game is fully symmetric with respect to the dimensions of abilities and efforts. This symmetry implies that any existing equilibrium strategy profile induces another equilibrium strategy profile as stated in the following lemma.

Lemma 2. *If for some $t \in (0, +\infty)$,*

$$m \mapsto \begin{cases} (e_1(m_1), e_2(m_1)) \text{ if } m = m_1, \\ (e_1(m_2), e_2(m_2)) \text{ if } m = m_2, \end{cases} \quad \text{and } (a_1, a_2) \mapsto \begin{cases} m_1, \text{ if } a_1 \geq t a_2, \\ m_2, \text{ if } a_1 < t a_2, \end{cases}$$

define an influential equilibrium strategy profile, then

$$m \mapsto \begin{cases} (e_2(m_2), e_1(m_2)) \text{ if } m = m_1, \\ (e_2(m_1), e_1(m_1)) \text{ if } m = m_2, \end{cases} \quad \text{and } (a_1, a_2) \mapsto \begin{cases} m_1, \text{ if } a_1 \geq \frac{1}{t} a_2, \\ m_2, \text{ if } a_1 < \frac{1}{t} a_2, \end{cases}$$

define an influential equilibrium strategy profile too.

In particular, Lemma 2 permits us to w.l.o.g. restrict to $t \in (0, 1]$ for the possible Sender's strategies (6) in equilibrium. Given the uniform prior distribution of abilities, this allows explicit expressions for the Receiver's derived efforts. According to (4), given (6), we obtain

$$\begin{aligned}
e_1(m_1) &= \mathbb{E}[a_1 | a_1 \geq t a_2] = \frac{\int_{a_2=0}^{1} \int_{a_1=ta_2}^{1} a_1 \, \mathrm{d}a_1 \, \mathrm{d}a_2}{\int_{a_2=0}^{1} \int_{a_1=ta_2}^{1} \mathrm{d}a_1 \, \mathrm{d}a_2} = \frac{1}{3}\frac{3-t^2}{2-t}, \\
e_2(m_1) &= \mathbb{E}[a_2 | a_1 \geq t a_2] = \frac{\int_{a_2=0}^{1} \int_{a_1=ta_2}^{1} a_2 \, \mathrm{d}a_1 \, \mathrm{d}a_2}{\int_{a_2=0}^{1} \int_{a_1=ta_2}^{1} \mathrm{d}a_1 \, \mathrm{d}a_2} = \frac{1}{3}\frac{3-2t}{2-t}, \\
e_1(m_2) &= \mathbb{E}[a_1 | a_1 < t a_2] = \frac{\int_{a_2=0}^{1} \int_{a_1=0}^{ta_2} a_1 \, \mathrm{d}a_1 \, \mathrm{d}a_2}{\int_{a_2=0}^{1} \int_{a_1=0}^{ta_2} \mathrm{d}a_1 \, \mathrm{d}a_2} = \frac{t}{3}, \\
e_2(m_2) &= E[a_2 | a_1 < t a_2] = \eta \frac{\int_{a_2=0}^{1} \int_{a_1=0}^{ta_2} a_2 \, \mathrm{d}a_1 \, \mathrm{d}a_2}{\int_{a_2=0}^{1} \int_{a_1=0}^{ta_2} \mathrm{d}a_1 \, \mathrm{d}a_2} = \frac{2}{3}.
\end{aligned} \tag{8}$$

Then, according to (7), equilibria of the game associated with $r \in (0,1]$, if any, are determined by the solutions $t \in (0,1]$ of

$$t = \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}. \tag{9}$$

Next we characterize these solutions and how they depend on r.

Figure 1 represents a Sender's strategy (6) associated with some $t \in (0,1)$, and the corresponding induced effort levels $e_i(m_j)$.

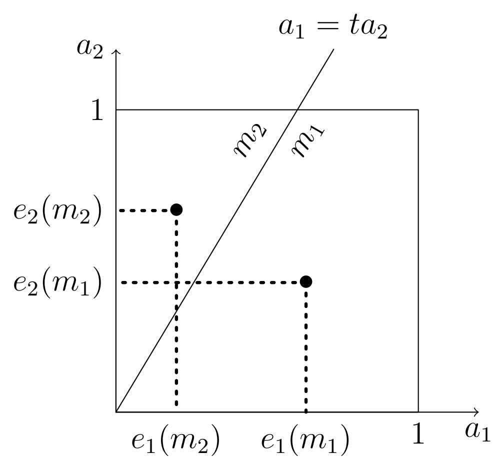

Figure 1: A Sender's disclosure rule, and the subsequent Receiver's optimal efforts

3.3 Influential equilibria

Notice first that for any $r \in (0,1]$, $t = 1$ is a solution of (9). The corresponding equilibrium strategies are given by

$$m(a_1, a_2) = \begin{cases} m_1 \text{ if } a_1 \geq a_2, \\ m_2 \text{ if } a_1 < a_2, \end{cases} \quad \text{and } (e_1(m), e_2(m)) = \begin{cases} \left(\frac{2}{3}, \frac{1}{3}\right) \text{ if } m = m_1, \\ \left(\frac{1}{3}, \frac{2}{3}\right) \text{ if } m = m_2. \end{cases}$$

They define a *symmetric equilibrium* in which the Sender symmetrically reveals whether one ability is greater than the other. Consequently, the Receiver symmetrically exerts efforts with respect to the two messages, with a greater exerted effort in the dimension corresponding to his greatest ability, and a lesser effort along the other dimension.

An intuition is as follows. Effort and ability are complementary factor of each one-dimensional contribution $y_i = a_i e_i$, $i \in \{1,2\}$. Hence, conditional on a limited degree of complementarity between y_1 and y_2, the CES output is maximized if effort is greater in the dimension where ability is greater.[14] Reciprocally, from the Receiver's perspective, a greater effort is derived from a greater inferred ability. This provides the Sender with the incentives to truthfully reveal the highest ability, and makes her recommendation credible in equilibrium.

The next proposition asserts that information is also potentially revealed and processed asymmetrically in equilibrium.

Proposition 1. *For each $r \in (0,1]$, there is a unique $t_r \in (0,1)$ such that*

$$m(a_1,a_2) = \begin{cases} m_1 \text{ if } a_1 \geq t_r a_2, \\ m_2 \text{ if } a_1 < t_r a_2, \end{cases} \quad \text{and } (e_1(m), e_2(m)) = \begin{cases} \left(\frac{1}{3}\frac{3-t_r^2}{2-t_r}, \frac{1}{3}\frac{3-2t_r}{2-t_r}\right) \text{ if } m = m_1, \\ \left(\frac{t_r}{3}, \frac{2}{3}\right) \text{ if } m = m_2. \end{cases}$$

define an asymmetric equilibrium strategy profile.

Equilibria described in Proposition 1 and Lemma 2 fully characterize the game's asymmetric equilibria. In an asymmetric equilibrium, the Sender is more likely to induce a greater effort in one of the two dimensions. In particular, she potentially induces a high effort in one dimension despite a greater ability in the other dimension. In that case, she induces a mismatch between abilities and efforts that is not in the Receiver's interest. For an intuition, consider the case depicted in Figure 1. Sender truthfully reveals whether m_1:"$\theta_1 \geq t_r\theta_2$" or m_2:"$\theta_1 < t_r\theta_2$" for some $0 < t_r < 1$. Given such an information transmission, the Receiver exerts either medium efforts $e_1(m_1)$ and $e_2(m_1)$ in the two dimensions, or a very low effort $e_1(m_2)$ and a medium effort $e_2(m_2)$ in dimensions 1 and 2 respectively. In particular, efforts are similar on receiving m_1, and very different on receiving m_2. By complementarity of effort and ability, the low effort $e_1(m_2)$ is more detrimental when a_1 is high than when a_1 is low. This precisely provides the Sender with the incentives to issue m_2 only when a_1 is much lower

[14] More precisely, it occurs whenever $\frac{\partial U^S((a_1,a_2),(e_1,e_2))}{\partial a_i \partial e_i} > 0$ for each $i \in \{1,2\}$, *i.e.* whenever $\frac{1}{2^{1/r}}((a_ie_i)^{r-1}((a_ie_i)^r + (a_{-i}e_{-i})^r)^{1/r-2}((a_ie_i)^r + r(a_{-i}e_{-i})^r) > 0$. In particular it is obtained when $r \in (0,1]$.

than a_2 in equilibrium.[15]

The above argument also illustrates that the greater the degree of complementary, the more asymmetric the Sender's information revelation in equilibrium. Indeed, a greater degree of complementarity exacerbates the detrimental effect on the CES output of the low effort $e_1(m_2)$. So message m_1 is even more likely with more complementarity.

Proposition 2. *Asymmetry of the asymmetric equilibrium is exacerbated with a greater degree of complementarity of the Receiver's contributions. In particular, when r tends to* 0 *(so that the CES output tends to $\sqrt{y_1 y_2}$), in the asymmetric equilibrium Sender tends to babble.*

Figure 2 depicts the two equilibria occurring at $t = 1$ (corresponding to the symmetric equilibrium) and at $t_r \in (0, 1)$ (corresponding to the asymmetric equilibrium) with respect to the degree of complementary $r \in (0, 1]$. At $r = 1$ (when contributions are perfect substitutes), we have $t_r = \frac{1}{2}$, and as r decreases to 0, so does t_r. Note that when t_r tends to 0, Sender tends to always issue m_1:"$\theta_1 \geq t_r\theta_2 \simeq 0$".

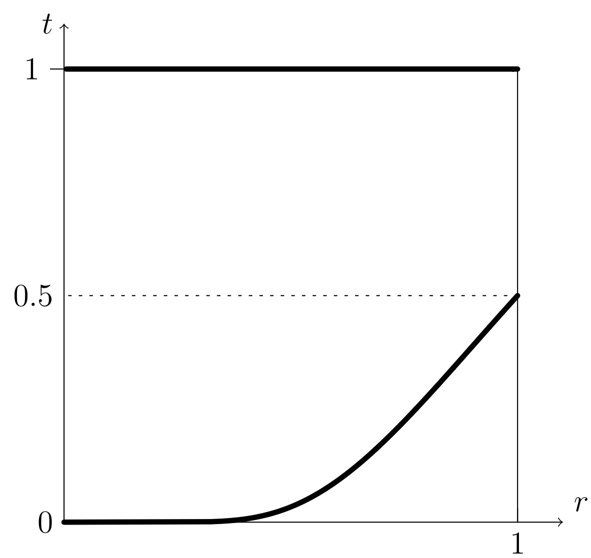

Figure 2: $t = 1$, and $t = t_r$ as a function of r

[15]The case of perfect substitution ($r = 1$) is the one investigated by Kamphorst and Swank (2016). In Section 4.1 we show that in that case, an asymmetric equilibrium is obtained from a cardinal argument, rather than from the similarity and differences in the effort profiles. More precisely, we argue that the existence of an asymmetric equilibrium is closely related to the uniform prior in case of perfect substitution, and we provide a different prior that rules out the asymmetric equilibrium when the degree of complementarity is low.

4 Robustness

4.1 A qualitative effect

In case of perfect substitution ($r = 1$), the Sender compares $a_1(e_1(m_1) - e_1(m_2))$ and $a_2(e_2(m_2) - e_2(m_1))$ and issues m_1 when the former expression is greater than the latter. In the corresponding asymmetric equilibrium (associated with $t_1 = 1/2$), m_2 is issued whenever a_1 is lower than half of a_2 (we have $e_1(m_1) - e_1(m_2) = 2(e_2(m_2) - e_2(m_1))$). In particular, m_1 is issued because relative to the efforts $e_1(m_1)$ and $e_2(m_1)$ derived from m_1, effort $e_2(m_2) = \mathbb{E}[a_2|a_2 > 2a_1]$ in dimension 2 does not compensate the very low effort $e_1(m_2) = \mathbb{E}[a_1|a_1 < a_2/2]$. From the Receiver's perspective, the "good news" m_2 concerning his ability in dimension 2 is not as good as how bad is the news concerning his ability in dimension 1. In this section, we show that this effect relies on the uniform distribution of abilities that we have specified. In particular, we specify a prior distribution such that the asymmetric equilibrium is precluded in case of perfect substitution, but persists in the presence of complementarity.

Consider the set of (a_1, a_2) such that

$$a_1 + a_2 \leq 1,\ a_1 \geq 0,\ a_2 \geq 0,$$

and assume that abilities are uniformly distributed on this set, as illustrated in Figure 3.

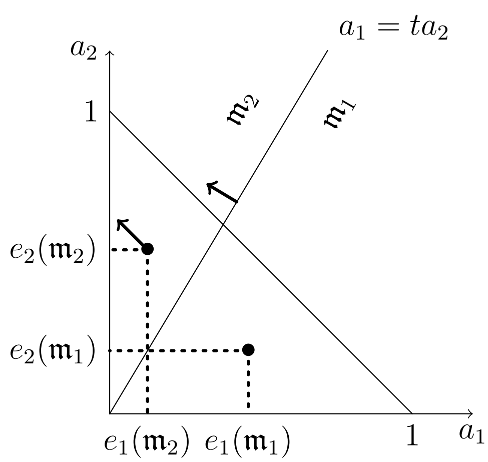

Figure 3: The decision-making rule and subsequent effort strategy with dependent prior abilities

Given such a prior, a Receiver who is informed to be worse in the first dimension does infer that he is much better in the second one. More precisely, we obtain, for any $t \geq 0$, as in

(8):

$$e_1(m_1) = \mathbb{E}[a_1|a_1 \geq ta_2] = \frac{1}{3}\frac{2t+1}{1+t}, \quad e_2(m_1) = \mathbb{E}[a_2|a_1 \geq ta_2] = \frac{1}{3}\frac{1}{1+t}, \quad e_1(m_2) = \mathbb{E}[a_1|a_1 < ta_2] = \frac{1}{3}\frac{t}{1+t},$$

In particular we have

$$e_1(m_1) - e_1(m_2) = e_2(m_2) - e_2(m_1) \iff \frac{e_2(m_2) - e_2(m_1)}{e_1(m_1) - e_1(m_2)} = 1$$

independently of t. This implies that when the Receiver's one-dimensional contributions are perfect substitutes, the symmetric equilibrium ($t = 1$) is the unique equilibrium of the game.[16] However, one can easily verify that given any $t \geq 0$, the equilibrium condition

$$t = \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)}\right)^{\frac{1}{r}}$$

is satisfied for a sufficiently low r. This implies the existence of an asymmetric equilibrium for sufficiently small rs. Figure 4 depicts solutions $t = 1$ and $t = t_r$ as a function of r in this setting. The symmetric equilibrium is the only equilibrium with low complementarity ($r \cong 1$) but for any sufficiently low r, the asymmetric equilibrium still occurs.

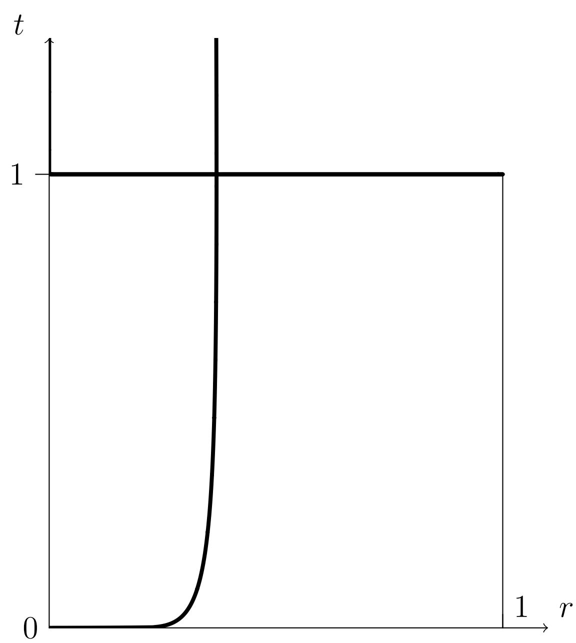

Figure 4: $t = 1$, and $t = t_r$ as a function of r in case of dependent abilities

[16]Moreover, any Sender's asymmetric comparison (corresponding to a $t \neq 1$) induces effort levels such that their Sender's best response is the symmetric comparison (corresponding to $t = 1$).

4.2 Exogenously asymmetric contributions

In this section, we assume that each recommendation represents an assignment to an activity, and that activities differ according to an exogenous relative importance (*i.e.* productivity) of the Receiver's contributions in each dimension. Thus Activities 1 and 2 correspond to $m = m_1$ and $m = m_2$ respectively, with respective production functions

$$Y_r(m_1) = (\eta(a_1 e_1(m_1))^r + (a_2 e_2(m_1))^r)^{\frac{1}{r}} ,$$

and

$$Y_r(m_2) = ((a_1 e_1(m_2))^r + \eta(a_2 e_2(m_2))^r)^{\frac{1}{r}} ,$$

for some $\eta > 1$ that represents the relative importance of the most important dimension of the activity.

The differentiation among the dimensions puts increasing importance on the matching of ability and effort in the most important dimension of an activity. The greater the relative importance of the contribution (a greater η), the more it prevents the Sender's mismatching of the greatest ability with the greatest effort in equilibrium. In particular, the Sender's incentive to sustain the asymmetric equilibrium vanishes with a sufficiently high η. However, complementarity severely mitigates the counter-effect of η on the existence of the asymmetric equilibrium. A high degree of complementarity requires an extreme differentiation of the dimensions in order to rule out the asymmetric equilibrium.

Proposition 3. *For any level of differentiation of the contributions η, the asymmetric equilibrium occurring at t_r persists provided there is sufficient complementarities between the Receiver's contributions.*

Figure 5 depicts the corresponding values of t_r according to different values of η.

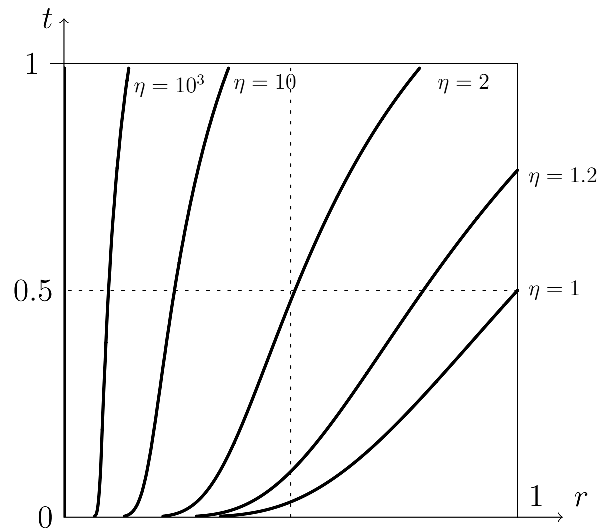

Figure 5: Equilibrium t_r as a function of r and η

If the degree of complementarity is fixed, the more differentiated the dimensions, the less asymmetric the asymmetric equilibrium (if any). However, more complementarity requires a greater level of differentiation η for the asymmetric equilibrium to be ruled out. Suppose for instance that $\eta = 10^3$ so that the important dimension of an activity is a thousand times more productive than its least important dimension. Then according to Figure 5, given sufficient complementarities, the asymmetric equilibria in which the Sender potentially mismatches the Receiver's highest ability to his highest effort still occurs. In addition, the extent of the asymmetry is still potentially very high ($t_r \to 0$ as $r \to 0$) in this setting.

By contrast, if the Receiver's contributions are perfect substitutes ($r = 1$), then a slight differentiation ($\eta \geq 1.5$) rules out the asymmetric equilibrium.[17]

[17] In a meta analysis of 172 studies, Lytton and Romney (1991) show that a gender bias does not emerge clearly within parent-child interactions except in the case of "encouragement of gendered activities". According to our model, such a gendered differentiation, irrespective of the observed abilities, is nevertheless a parental rational best-response to a child's asymmetric beliefs concerning the assignment. It is obtained conditional on a high degree of complementarity of the corresponding skill dimensions of the activities. An asymmetric gendered assignment to activities is less informative on the observed abilities than the ability-driven symmetric assignment. In the next section, we show that it is also an efficient assignment rule.

5 Selection

In this section, we investigate two selection arguments concerning the symmetric and the asymmetric equilibria. First, stability points to the asymmetric equilibrium. Second, the Receiver is *ex-ante* more productive in the asymmetric equilibrium when his two one-dimensional contributions have a high degree of complementarity.

5.1 Stability

Players' strategies are driven by the degree of asymmetry t of the comparison of the Receiver's abilities a_1 and a_2. We say that an equilibrium is stable if, given a player's small deviation from an equilibrium strategy associated with $t^* \in \{1, t_r\}$, the other player's best response to this deviation leads to a degree of asymmetry which is closer to t^* than the initial deviation. It is said to be unstable otherwise.

Proposition 4. *For each $r \in (0, 1]$, the symmetric equilibrium associated with $t^* = 1$ is unstable and the asymmetric equilibrium associated with $t^* = t_r \neq 1$ is stable. Moreover, in the symmetric equilibrium, the greater the degree of complementary (the lower r), the more important the degree of asymmetry of an agent's best response to a slight deviation of the other agent.*

Let us give a formal description of the result in Proposition 4. Set $t \in (0, 1)$ some degree of asymmetry of the Receiver's efforts $e_i(m_1) = \mathbb{E}[a_i|a_1 \geq ta_2]$ and $e_i(m_2) = \mathbb{E}[a_i|a_1 < ta_2]$, $i \in \{1, 2\}$, and let

$$\hat{t}(t) = \left(\frac{e_2^r(m_2) - e_2^r(m_1)}{e_1^r(m_1) - e_1^r(m_2)}\right)^{\frac{1}{r}}$$

be Sender's best response to the Receiver's profiles of efforts associated with t. We find that if $t_r < t < 1$, then $\hat{t}$ satisfies $t_r < \hat{t}(t) < t < 1$, so that the Sender confirms and amplifies any anticipated asymmetry that is lower than the equilibrium asymmetry associated with t_r. Also, if $0 < t < t_r$ then $0 < t < \hat{t}(t) < t_r$ so that the Sender's best response decreases any extra level of asymmetry in Receiver's effort profile and relative to the equilibrium asymmetry associated with t_r.

Moreover, for instance if $r = \frac{3}{4}$, then we find $\frac{\partial \hat{t}}{\partial t}(1) \cong 3$ in the symmetric equilibrium associated with $t = 1$. This implies $\left|\hat{t} - 1\right| \cong 3\left|t - 1\right|$, so that the Sender's best response to a Receiver's slight deviation from the symmetric equilibrium exacerbates the deviation up to factor 3. In contrast, in the asymmetric equilibrium associated with $t_r = t_{3/4} \cong \frac{1}{4}$, we find

$\frac{\partial \hat{t}}{\partial t}(t_r) \cong 0.35$. Hence $\left|\hat{t} - t_r\right| \cong 0.35\,|t - t_r|$ so that Sender's best response $\hat{t}$ is almost three times closer to the equilibrium value t_r relative to any Receiver's slight deviation from the degree of asymmetry $t \neq t_r$ associated with the asymmetric equilibrium.

5.2 Efficiency

In this section, we investigate the relative efficiency of the equilibria. Efficiency of an equilibrium is computed from Sender's perspective, as the expected output $\mathbb{E}[Y_r]$ obtained before the abilities are observed. In particular, if the Sender had the ability to commit to one of the equilibrium, then she would chose the most efficient equilibrium.

Proposition 5. *From the Sender's perspective, with more complementarities, the asymmetric equilibrium becomes more productive relative to the symmetric equilibrium. It is the most efficient equilibrium as $r \to 0$.*

The intuition of Proposition 5 relies on the relative informativeness of the equilibria, and its impact on the CES output according to the involved degree of complementarity.

The symmetric equilibrium is informative concerning abilities. It induces a match of high ability with high effort, and low ability with low effort. Therefore, it is highly productive conditional on a low degree of complementarity (r close to 1) and less productive conditional on a high degree of complementarity (r close to 0). The effect is reversed in an asymmetric equilibrium. More precisely, the informativeness of an asymmetric equilibrium decreases with its asymmetry, and hence, according to Proposition 2, it decreases with the degree of complementarity (as r goes to 0). Low informativeness induces efforts which are more likely close to medium efforts obtained out of any information transmitted (*i.e.* close to the prior expectations $\frac{1}{2}$). Such efforts are *ex ante* more productive with more complementarities. Therefore, with sufficient complementarities, the asymmetric equilibrium is *ex ante* more productive than the symmetric equilibrium.

As an illustration, let us compute the optimal level of asymmetry from the productive perspective of the CES output. Given $t \leq 1$ and $r = \frac{1}{n}$, $n \in \mathbb{N}^*$, the binomial expansion formula gives

$$\begin{aligned}\mathbb{E}[Y_t] =& \Pr(m_1) \iint\limits_{a_1 \geq t a_2} \left(\frac{(a_1 e_1(m_1))^{\frac{1}{n}} + (a_1 e_2(m_1))^{\frac{1}{n}}}{2} \right)^n \mathrm{d}a_1 \, \mathrm{d}a_2 \\ &+ \Pr(m_2) \iint\limits_{a_1 < t a_2} \left(\frac{(a_1 e_1(m_2))^{\frac{1}{n}} + (a_1 e_2(m_2))^{\frac{1}{n}}}{2} \right)^n \mathrm{d}a_1 \, \mathrm{d}a_2 \\ =& \frac{1}{2^n} \sum_{k=0}^{k=n} \left(\frac{n!}{k!(n-k)!} \frac{1}{3(1+\frac{k}{n})} \times \right. \\ & \left. \left(e_1^{k/n}(m_1) e_2^{1-k/n}(m_1) \frac{3 - t^{1+\frac{k}{n}}(2-\frac{k}{n})}{2-\frac{k}{n}} + e_1^{k/n}(m_2) e_2^{1-k/n}(m_2) t^{1+\frac{k}{n}} \right) \right)\end{aligned}$$

Figure 6 shows that the most productive degree of asymmetry corresponds to the symmetric equilibrium strategy ($t = 1$) only in the case in which the Receiver's contributions are perfect substitute. With complementarity, if the Sender commits to a t-comparison of the Receiver's abilities, she always prefers an out-of-equilibrium degree of asymmetry t which does involves asymmetry ($t \neq 1$). In particular, there is a threshold $\tilde{r}$ for the degree of complementarity ($\tilde{r} \cong \frac{1}{3}$) such that the asymmetric equilibrium is *ex-ante* more productive relative to the symmetric equilibrium *iff* $r < \tilde{r}$.

Note that the value $t = 0$ corresponds to a babbling equilibrium, which is non-informative, and is preferred to the symmetric equilibrium with stronger complementarities. However, the optimal t, as $r \to 0$, does not tend to $t = 0$. In particular, the Sender always prefers to deliver some information on the abilities.

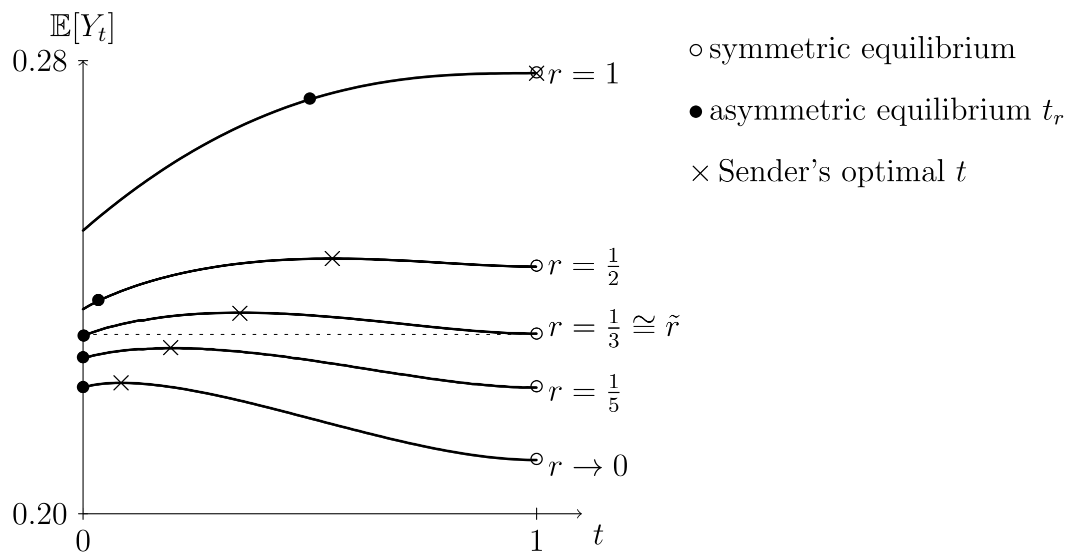

Figure 6: Sender's *ex-ante* utility, with respect to t and r

6 Conclusion

We examined a cheap talk game concerning a production with multiple dimensions and complementarities between the dimensions. We showed that multiple treatments can arise in equilibrium when an agent exerts specific efforts based on credible information about his abilities. The treatments differ in their informativeness and their efficiency. When weak complementarities exist between the production dimensions, a symmetric comparison of abilities is the most informative and most productive equilibrium strategy. When stronger complementarities exist, an asymmetric comparison becomes more productive. Nevertheless, it is poorly informative and could result in a mismatch between perceived abilities and actual abilities. An even stronger degree of complementarities precludes communication.

References

Brown, Sarah, Aurora Ortiz-Nuñez, and Karl Taylor, "Parental risk attitudes and children's academic test scores: Evidence from the US panel study of income dynamics," *Scottish Journal of Political Economy*, 2012, *59* (1), 47–70.

Chakraborty, Archishman and Rick Harbaugh, "Comparative cheap talk," *Journal of Economic Theory*, 2007, *132* (1), 70–94.

_ **and** _ , "Persuasion by Cheap Talk," *The American Economic Review*, 2010, *100* (5), 2361–2382.

Checchi, Daniele, Carlo V Fiorio, and Marco Leonardi, "Parents' risk aversion and children's educational attainment," *Labour Economics*, 2014, *30*, 164–175.

Crawford, Vincent P. and Joel Sobel, "Strategic information transmission," *Econometrica*, 1982, pp. 1431–1451.

Crutzen, Benoît S.Y., Otto H. Swank, and Bauke Visser, "Confidence management: on interpersonal comparisons in teams," *Journal of Economics & Management Strategy*, 2013, *22* (4), 744–767.

Cunha, Flavio, James J Heckman, and Susanne M. Schennach, "Estimating the technology of cognitive and noncognitive skill formation," *Econometrica*, 2010, *78* (3), 883–931.

Gopnik, Alison, "Scientific thinking in young children: Theoretical advances, empirical research, and policy implications," *Science*, 2012, *337* (6102), 1623–1627.

Hackman, Daniel A, Martha J Farah, and Michael J Meaney, "Socioeconomic status and the brain: mechanistic insights from human and animal research," *Nature Reviews Neuroscience*, 2010, *11* (9), 651–659.

Heckman, James J, "Schools, Skills, and synapses," *Economic Inquiry*, 2008, *46* (3), 289–324.

_ **and Stefano Mosso**, "The Economics of Human Development and Social Mobility," *Annual Review of Economics*, 2014, *6*, 689–733.

Kamphorst, Jurjen J.A. and Otto H. Swank, "Don't demotivate, discriminate," *American Economic Journal: Microeconomics*, 2016, *8* (1), 1–27.

Knudsen, Eric I, James J Heckman, Judy L Cameron, and Jack P Shonkoff, "Economic, neurobiological, and behavioral perspectives on building America's future workforce," *Proceedings of the National Academy of Sciences*, 2006, *103* (27), 10155–10162.

Levy, Gilat and Ronny Razin, "On the limits of communication in multidimensional cheap talk: a comment," *Econometrica*, 2007, *75* (3), 885–893.

Lytton, Hugh and David M Romney, "Parents' differential socialization of boys and girls: A meta-analysis.," *Psychological bulletin*, 1991, *109* (2).

Sémirat, Stéphan, "Information transmission despite conflict," *International Journal of Game Theory*, forthcoming.

Sobel, Joel, "Giving and receiving advice," in Daron Acemoglu, Manuel Arellano, and Eddie Dekel, eds., *Advances in Economics and Econometrics: Volume 1, Economic Theory: Tenth World Congress*, Vol. 49, Cambridge University Press, 2013.

Tenenbaum, Harriet R, ""You'd Be Good at That": Gender Patterns in Parent-Child Talk about Courses," *Social Development*, 2009, *18* (2), 447–463.

Wang, Ming-Te and Jessica L Degol, "Gender gap in science, technology, engineering, and mathematics (STEM): current knowledge, implications for practice, policy, and future directions," *Educational psychology review*, 2017, *29* (1), 119–140.

Wölfel, Oliver and Guido Heineck, "Parental risk attitudes and children's secondary school track choice," *Economics of Education Review*, 2012, *31* (5), 727–743.

A Proof of Proposition 1

Proof. We first establish the existence of the equilibria, and then their uniqueness.

Existence. Let $r \in (0,1]$. According to Equation (9) in the main text, we need to prove that there exists $t_r \in (0,1)$ such that

$$t_r = f_r(t_r),$$

where f_r is given by

$$f_r(t) = \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}.$$

Since f_r is continuous, and $[0, \frac{1}{2}]$ is compact and convex, we derive the result from Brouwer's fixed point theorem, if we show that the range $[0, \frac{1}{2}]$ is stable under f_r, and if we ensure that $t = 0$ is not a solution of $t = f_r(t)$ (so that $0 < t_r \leq \frac{1}{2} < 1$). For any $r \in (0,1]$ and any $t \in [0, \frac{1}{2}]$, we have $f_r(t) > 0$. So we need: for any $0 \leq t \leq \frac{1}{2}$,

$$f_r(t) \leq \frac{1}{2}. \tag{10}$$

To obtain (10), we first show

$$f_r(t) \leq f_r\left(\frac{1}{2}\right), \tag{11}$$

and then we show

$$f_r\left(\frac{1}{2}\right) \leq \frac{1}{2}, \tag{12}$$

for any $t \in [0, \frac{1}{2}]$ and $r \in (0,1]$.

Proof of (11). We show that f_r is increasing. Since $x \mapsto x^{\frac{1}{r}}$ is increasing, it is sufficient to show that $t \mapsto \frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}$ increases. We show that for any $r \in (0,1]$,

$$t \mapsto \left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \text{ increases,} \tag{11a}$$

and

$$t \mapsto \left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r \text{ decreases.} \tag{11b}$$

Proof of (11a). The derivative of $t \mapsto \left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r$ is $t \mapsto r\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}$, which is positive for $t \in (0, \frac{1}{2}]$.

Proof of (11b). The derivative of $t \mapsto \left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r$ is $t \mapsto r\left(\frac{1}{3}\right)^r \left(\left(\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \frac{1}{t^{1-r}} \right)$. It is negative if and only if

$$\left(\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} \leq \frac{1}{t^{1-r}} \iff \left(\frac{3-t^2}{t(2-t)}\right)^r \leq \frac{(3-t^2)(2-t)}{t(1-t)(3-t)}. \tag{13}$$

For $t \in (0, \frac{1}{2}]$, we have $\frac{3-t^2}{t(2-t)} > 1$, hence $\left(\frac{3-t^2}{t(2-t)}\right)^r \leq \frac{3-t^2}{t(2-t)}$. Then (13) derives from $\frac{3-t^2}{t(2-t)} \leq \frac{(3-t^2)(2-t)}{t(1-t)(3-t)}$. This completes the proof of (11).

Proof of (12). We show that for any $r \in (0, 1]$, $\left(\frac{(\frac{2}{3})^r - (\frac{4}{9})^r}{(\frac{11}{18})^r - (\frac{1}{6})^r}\right)^{\frac{1}{r}} \leq \frac{1}{2}$, *i.e.*

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \leq \left(\frac{11}{36}\right)^r - \left(\frac{1}{12}\right)^r. \tag{14}$$

The Mean Value Theorem gives

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r = \frac{r}{a^{1-r}}\left(\frac{2}{3} - \frac{4}{9}\right) = \frac{2r}{9}\frac{1}{a^{1-r}}$$

for some $a \in [\frac{4}{9}, \frac{2}{3}]$, and

$$\left(\frac{11}{36}\right)^r - \left(\frac{1}{12}\right)^r = \frac{r}{b^{1-r}}\left(\frac{11}{36} - \frac{1}{12}\right) = \frac{2r}{9}\frac{1}{b^{1-r}}$$

for some $b \in [\frac{1}{12}, \frac{11}{36}]$. Since $\frac{4}{9} > \frac{11}{36}$, we obtain $a > b$, and thus $\frac{2r}{9}\frac{1}{a^{1-r}} \leq \frac{2r}{9}\frac{1}{b^{1-r}}$, which gives (14).

Uniqueness. To show the uniqueness of an asymmetric equilibrium we need a unique solution to $t = f_r(t)$ on $(0, 1)$. This derives from the following claim (shown below):

$$\text{For each } r \in (0, 1], \text{ if } t \text{ solves } t = f_r(t) \text{ then } f_r'(t) < 1. \tag{15}$$

Indeed, since $f_r \neq \mathrm{Id}$, then as illustrated in Figure 7, if there were multiple solutions $t \in (0, 1)$ to $t = f_r(t)$, then at least one of them would satisfy $f_r'(t) \geq 1$.[18]

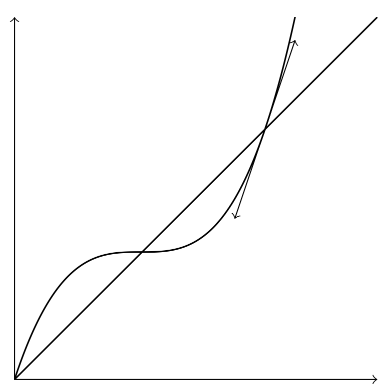

Figure 7: $f_r'(t) \geq 1$ at some solution of $t = f_r(t)$

[18] A formal proof is as follows. Let t_1 and t_2, $t_1 < t_2$, be two solutions to $t = f_r(t)$ such that $t \mapsto f_r(t) - t$ does not vanish on (t_1, t_2). Suppose that $f_r'(t_1) < 1$ and $f_r'(t_2) < 1$. Then $t \mapsto f_r(t) - t$ decreases at t_1 and there exists $t_1^+ > t_1$ such that $f_r(t_1^+) - t_1^+ < 0$. It also decreases at t_2 and there exists $t_2^- < t_2$ such that $f_r(t_2^-) - t_2^- > 0$. Then by continuity $f_r(t) - t$ vanishes on $(t_1^+, t_2^-) \subset (t_1, t_2)$, which is a contradiction.

We relegate the proof of (15) to a standalone proof below. □

Proof of (15). The proof is as follows. First, in Sept 1 we compute an upper bound $f_r'(t) \leq A_r(t) + B_r(t)$. In Step 2A and 2B respectively, we derive upper bounds $A_r(t) \leq A(r)$ and $B_r(t) \leq B(r)$ (independent of t). Finally, in Steps 3A and *3B* respectively, we derive upper bounds $A(r) \leq A$ and $B(r) \leq B$ such that $A + B < 1$ (independant of r).

Step 1. The derivative of $t \mapsto f_r(t)$ is

$$\begin{aligned} f_r'(t) &= \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}} \left(\frac{\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} - \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \frac{1}{3}\left(\frac{t}{3}\right)^{r-1}}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right) \\ &= f_r(t) \left(\frac{\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} - \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)}}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} + \frac{\frac{1}{t}\left(\frac{t}{3}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right), \end{aligned}$$

and since

$$\frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)}}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \geq 0,$$

we have

$$f_r'(t) \leq f_r(t) \left(\frac{\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} + \frac{\frac{1}{t}\left(\frac{t}{3}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right).$$

Thus we have

$$f_r'(t) \leq A_r(t) + B_r(t), \tag{16}$$

with

$$A_r(t) = \frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)},$$

and

$$B_r(t) = \frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \frac{1}{t} \left(\frac{t}{3}\right)^r.$$

Step 2A. From (11a) and (11b), we have, for any $0 < t \leq \frac{1}{2}$,

$$\frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} = \frac{\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right)^{\frac{1}{r}-1}}{\left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r\right)^{\frac{1}{r}}} \leq \frac{\left(\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r\right)^{\frac{1}{r}-1}}{\left(\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r\right)^{\frac{1}{r}}}.$$

Similarly, $t \mapsto \frac{1}{3}\frac{3-2t}{2-t}$ and $t \mapsto \frac{1}{(3-2t)(2-t)}$ decrease, and thus for any $0 < t \leq \frac{1}{2}$, $\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)} \leq \left(\frac{1}{2}\right)^r \frac{1}{3}$. Then we obtain for any $0 < t \leq \frac{1}{2}$,

$$A_r(t) \leq A(r), \tag{17}$$

with

$$A(r) = \frac{1}{3}\left(\frac{1}{2}\right)^r \frac{\left(\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r\right)^{\frac{1}{r}-1}}{\left(\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r\right)^{\frac{1}{r}}}.$$

Step 2B. Recall that t satisfies

$$\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}\right)^{\frac{1}{r}} = t.$$

Then we have

$$\begin{aligned} B_r(t) &= \frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}\right)^{\frac{1}{r}}}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \frac{1}{t}\left(\frac{t}{3}\right)^r = \frac{\left(\frac{t}{3}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} = \frac{1}{\left(\frac{3-t^2}{2-t}\right)^r \left(\frac{1}{t}\right)^r - 1} \\ &= \frac{1}{\left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} - 1}. \end{aligned} \tag{18}$$

Next we need a lower bound for $\left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}$. To this end, we show that it is decreasing with t. Its derivative with respect to t is

$$\begin{aligned} &\frac{r\left(\frac{3-t^2}{2-t}\right)^r}{\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right)^2} \Bigg(\frac{(1-t)(3-t)}{(3-t^2)(2-t)} \left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r\right)\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right) \\ &\qquad + \left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \left(\frac{t}{3}\right)^r \frac{1}{t}\right)\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right) \\ &\qquad\qquad - \left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r\right)\left(\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{3-2t}{2-t}\right)\Bigg). \end{aligned}$$

The sign is equal to the sign of the second factor, which is a sum of three terms. The sign of the second term $\left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \left(\frac{t}{3}\right)^r \frac{1}{t}\right)\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right)$ is equal to the sign of $\left(\left(\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - (t)^r \frac{1}{t}\right)$. It is negative from $\left(\frac{3-t^2}{t(2-t)}\right)^r \leq \frac{3-t^2}{t(2-t)}$ (because $\frac{3-t^2}{t(2-t)} > 1$), $t^r \geq t$ (because $t < 1$), and $\frac{(1-t)(3-t)}{(2-t)^2} \leq 1$.

The sign of the sum of the first and third terms is negative *iff*

$$\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \frac{(3-t^2)(3-2t)}{(1-t)(3-t)} \leq 0.$$

Now $r \mapsto \left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r$ increases, and $r \mapsto -\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r$ increases too, and so does their sum. At $r = 1$, it is equal to $-\frac{2}{3}\frac{2t^3-2t^2-5t+6}{(1-t)(3-t)}$, which is negative (since $2t^3 - 2t^2 - 5t + 6 > 0$ if $t \geq 0$). This shows that $t \mapsto \left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}$ decreases. It is thus greater than its value at $t = \frac{1}{2}$, so that

$$\left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} \geq \left(\frac{11}{6}\right)^r \frac{\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} = \frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r}. \tag{19}$$

From (18), we obtain

$$B_r(t) \leq B(r),$$

with $B(r) = \frac{1}{\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} - 1}$.

Step 3. In order to get upper bounds of $A(r)$ and $B(r)$, we need accurate upper and lower bounds of expressions of the form $b^r - a^r$, with $b > a > 0$. Notice that $b^r - a^r = \int_a^b \frac{1}{r} x^{r-1} \mathrm{d}x$, and as Figure 8 shows, the convexity of $x \mapsto \frac{1}{r}x^{r-1}$ allows the integral to be bounded by two trapeze areas, such that

$$r(b-a)\left(\frac{a+b}{2}\right)^{r-1} \leq b^r - a^r \leq r(b-a)\frac{a^{r-1} + b^{r-1}}{2}. \tag{20}$$

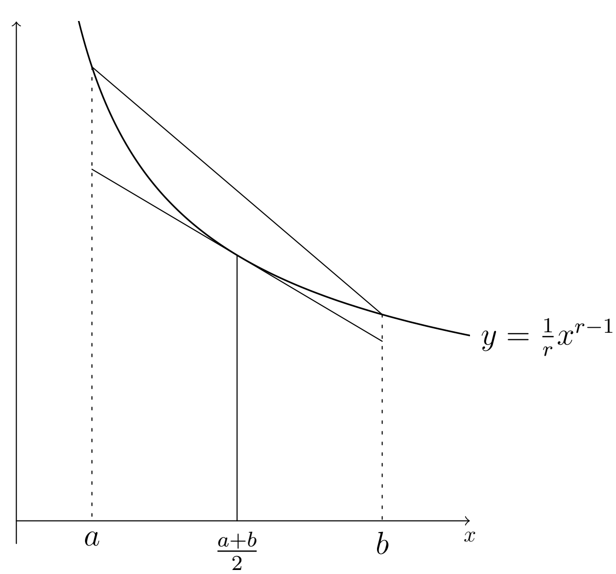

Figure 8: Trapezoidal minoration and majoration of $b^r - a^r$

Step 3A. We need an upper bound for $A(r)$. From (20), we have

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r = \left(\frac{2}{3}\right)^r \left(1^r - \left(\frac{2}{3}\right)^r\right) \leq \left(\frac{2}{3}\right)^r r \left(1 - \frac{2}{3}\right) \frac{1 + \left(\frac{2}{3}\right)^{r-1}}{2}$$

with $\left(\frac{2}{3}\right)^{r-1} \leq \frac{3}{2}$, so that

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \leq \left(\frac{2}{3}\right)^r \frac{5r}{12}.$$

From (20), we also have

$$\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r \geq \frac{8r}{7} \left(\frac{7}{18}\right)^r.$$

Then we obtain

$$A(r) \leq \frac{1}{r} \frac{48}{35} \left(\frac{3}{4}\right)^r \left(\frac{35}{96}\right)^{\frac{1}{r}},$$

which right hand side has a derivative equal to

$$\frac{48}{35} \left(\frac{3}{4}\right)^r \left(\frac{35}{96}\right)^{\frac{1}{r}} \frac{1}{r} \left(\frac{-1}{r} + \ln\left(\frac{3}{4}\right) + \frac{-1}{r^2} \ln\left(\frac{35}{96}\right)\right)$$

which sign is that of $\frac{-1}{r} + \ln\left(\frac{3}{4}\right) + \frac{-1}{r^2}\ln\left(\frac{35}{96}\right) = \frac{1}{r^2}\left(r^2 \ln\left(\frac{3}{4}\right) - r - \ln\left(\frac{35}{96}\right)\right)$ which is positive when $0 < r < \alpha$ with $\alpha = \frac{1 - \sqrt{1 + 4\ln\left(\frac{3}{4}\right)\ln\left(\frac{35}{96}\right)}}{2\ln\left(\frac{3}{4}\right)}$ and negative when $\alpha < r \leq 1$. Therefore, for any $r \in (0, 1]$, we have

$$A(r) \leq \frac{1}{\alpha} \frac{48}{35} \left(\frac{3}{4}\right)^\alpha \left(\frac{35}{96}\right)^{\frac{1}{\alpha}} < 0.39. \tag{21}$$

Step 3B. In order to get an upper bound for $B(r) = \frac{1}{\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} - 1}$, we need to get a lower bound for $\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r}$. From (20), we get

$$\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r \geq r \frac{22}{27} \left(\frac{77}{108}\right)^{r-1},$$

and

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \leq r \frac{1}{9} \left(\left(\frac{2}{3}\right)^{r-1} + \left(\frac{4}{9}\right)^{r-1}\right),$$

so that

$$\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} \geq \frac{22}{3} \frac{1}{\left(\frac{72}{77}\right)^{r-1} + \left(\frac{48}{77}\right)^{r-1}} \geq \frac{22}{3} \frac{1}{\frac{77}{72} + \frac{77}{48}} = \frac{96}{35}$$

(notice that $\frac{x^r}{x} \leq \frac{1}{x}$ if $0 < x \leq 1$). This gives

$$B(r) \leq \frac{1}{\frac{96}{35} - 1} = \frac{35}{61}. \tag{22}$$

Finally, (22) and (21) show that, for each $r \in (0,1]$, and each $t \in [0, \frac{1}{2}]$ such that $t = f_r(t)$, we have the upper bound

$$f_r'(t) \leq 0.39 + \frac{35}{61} < 1$$

as expected. □

B Proof of Proposition 2

Proof. First, we show that $r \mapsto t_r$ increases. It does so *iff* for each $r, r' \in (0,1]$:

$$\text{if } t_{r'} < t_r, \text{ then } r' < r. \tag{23}$$

Let us establish first that for any $r \in (0,1]$ we have

$$t < f_r(t) \text{ for any } t \in [0, t_r) \tag{24}$$

(we also have $t > f_r(t)$ for any $t \in (t_r, \frac{1}{2}]$).

Indeed: $t \mapsto f_r(t) - t$ does not vanish in $[0, t_r)$, since t_r is the unique solution to $t = f_r(t)$ on $(0,1)$, and $f_r(0) - 0 = f_r(0) > 0$.

Now let us consider $r, r' \in (0,1]$, and suppose $t_{r'} \in [0, t_r)$. Then from (24), we have $t_{r'} < f_r(t_{r'})$. Since $t_{r'} = f_{r'}(t_{r'})$, it can be written as:

$$f_{r'}(t_{r'}) < f_r(t_{r'}). \tag{25}$$

To obtain (23), it remains to show that this implies $r' < r$. The implication is a consequence of the increasing $r \mapsto f_r(t)$ which we now show.

For any $0 < x < 1$, the function $r \mapsto x^{\frac{1}{r}}$ is increasing, and therefore it is enough to prove that

$$r \mapsto \frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t})\right)^r - \left(\frac{t}{3}\right)^r} \tag{26}$$

is increasing for any $t \in [0, \frac{1}{2}]$.

Let $t \in (0, \frac{1}{2}]$. For any $r \in (0,1]$, the sign of the derivative of (26) is that of

$$\begin{aligned} g_t(r) = &\left(\left(\frac{2}{3}\right)^r \ln\left(\frac{2}{3}\right) - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \ln\left(\frac{1}{3}\frac{3-2t}{2-t}\right)\right)\left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r\right) \\ &- \left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right)\left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \ln\left(\frac{1}{3}\frac{3-t^2}{2-t}\right) - \left(\frac{t}{3}\right)^r \ln(t)\right). \end{aligned}$$

Now notice that $x^r \ln(x) = \frac{1}{r} x^r \ln(x^r)$ so that $g_r(r)$ can be written

$$\begin{aligned} g_t(r) =& \frac{1}{r}\left(\left(\frac{2}{3}\right)^r \ln\left(\left(\frac{2}{3}\right)^r\right) - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r \ln\left(\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right)\right)\left(\left(\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r\right) \\ &- \frac{1}{r}\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r\right)\left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r \ln\left(\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r\right) - \left(\frac{t}{3}\right)^r \ln\left(\left(\frac{t}{3}\right)^r\right)\right). \end{aligned}$$

Let us set $h(x) = x\ln(x)$, $a = \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r$, $b = \left(\frac{2}{3}\right)^r$, $c = \left(\frac{t}{3}\right)^r$ and $d = \left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r$. Then we obtain

$$g_t(r) = \frac{1}{r}(b-a)(c-d)\left(\frac{h(b)-h(a)}{b-a} - \frac{h(d)-h(c)}{d-c}\right).$$

Therefore the sign of g_r is equal to the sign of $\left(\frac{h(b)-h(a)}{b-a} - \frac{h(d)-h(c)}{d-c}\right)$. This is the difference between the slope of (AB) and the slope of (CD), where A, B, C and D are the points on the graph of h with respective abscissa a, b, c and d (see Figure 9). The convexity of h, and

$$\frac{t}{3} < \frac{1}{3}\frac{3-2t}{2-t} < \frac{1}{3}\frac{3-t^2}{2-t} < \frac{2}{3},$$

so that

$$c < a < d < b,$$

allows us to derive the positivity of $g_t(r)$ from the Three Chords Lemma[19] applied first, to the triangle CAB, where it shows that the slope of (AD) is steeper than the slope of (CD), and second, to the triangle ADB, where it shows that the slope of (AB) is steeper than the slope of (AD).

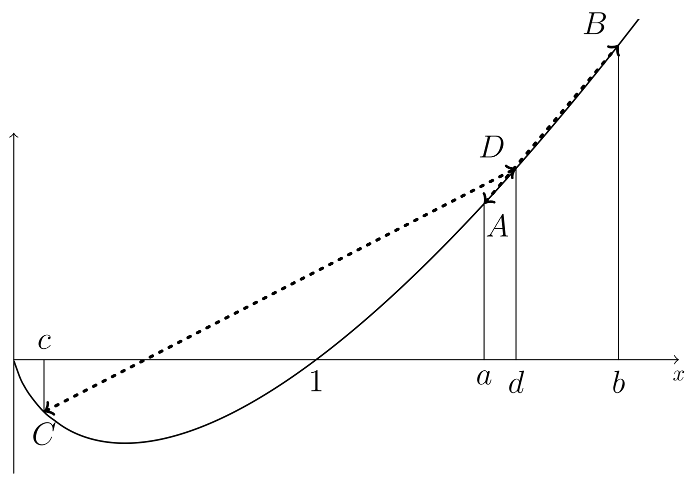

Figure 9: The slopes of (AB) and (CD) with A, B, C, D on $y = x\ln(x)$

[19]The Three Chords Lemma states that if X, Z, Y are three points on the graph of a convex function with abscissa $x_X < x_Z < x_Y$, then the slope of (XZ) will be less steep than the slope of (XY), which will be less steep than the slope of (YZ).

b) The result relies on $\lim_{r\to 0} t_r = 0$, which we now show. More precisely, we have to show that when $r \to 0$, the solution t_r, $0 < t_r < 1$ of

$$t = \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}$$

tends to 0. From l'Hôpital's Rule

$$\lim_{r\to 0} \frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} = \frac{\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3}\frac{3-2t}{2-t}\right)}{\ln\left(\frac{1}{3}\frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)},$$

and since $0 < \frac{\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3}\frac{3-2t}{2-t}\right)}{\ln\left(\frac{1}{3}\frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)} < 1$ for any $t \in (0, \frac{1}{2}]$, we have

$$\lim_{r\to 0} \left(\frac{\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3}\frac{3-2t}{2-t}\right)}{\ln\left(\frac{1}{3}\frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)} \right)^{\frac{1}{r}} = 0.$$

□

C Proof of Proposition 3

Proof. We show that at $t = 0$, $f_r(0) > 0$, and for each $t \in (0, 1]$, there exists a sufficiently small r such that $f_r(t) < t$. Thus, the result is a consequence of Bolzano's Theorem.

For each $\eta \geq 1$, for each $r \in (0, 1]$,

$$f_r(0) = \left(\left(\frac{4}{3}\right)^r - \left(\frac{1}{\eta^2}\right)^r \right)^{\frac{1}{r}} > 0.$$

Now fix $t \in (0, 1]$. Then

$$\lim_{r\to 0} \left(\frac{f_r(t)}{t}\right)^r = \lim_{r\to 0} \frac{\left(\eta^2 \frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{t^r \left(\left(\eta^2 \frac{1}{3}\frac{3-t^2}{2-t}\right)\right)^r - \left(\frac{t}{3}\right)^r \right)} = \lim_{r\to 0} \frac{\left(\eta^2 \frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}{\left(\eta^2 \frac{1}{3}\frac{t(3-t^2)}{2-t}\right)^r - \left(\frac{t^2}{3}\right)^r}$$

$$= \frac{\ln\left(\eta^2 \frac{2}{3}\right) - \ln\left(\frac{1}{3}\frac{3-2t}{2-t}\right)}{\ln\left(\eta^2 \frac{1}{3}\frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)} = \frac{\ln\left(\eta^2 \frac{2(2-t)}{3-2t}\right)}{\ln\left(\eta^2 \frac{3-t^2}{t(2-t)}\right)}.$$

and since for all $t \in (0, 1]$, $\frac{2(2-t)}{3-2t} < \frac{3-t^2}{t(2-t)}$ we obtain $\lim_{r\to 0} \left(\frac{f_r(t)}{t}\right)^r < 1$. So for a sufficiently small r, $f_r(t) < t$. □

D Proof of Proposition 4

Proof. Suppose that the Receiver anticipates a level $\hat{t}$ of asymmetry in the Sender's disclosure rule. Set $e_i(m_j)[\hat{t}]$, $i, j \in \{1,2\}$ the corresponding effort levels. Then the Sender's best response to efforts $e_i(m_j)[\hat{t}]$, $i, j \in \{1,2\}$ is a disclosure rule with a level of asymmetry equal to $\left(\frac{e_2^r(m_2)[\hat{t}]-e_2^r(m_1)[\hat{t}]}{e_1^r(m_1)[\hat{t}]-e_1^r(m_2)[\hat{t}]}\right)^{\frac{1}{r}} = f_r(\hat{t})$. An equilibrium associated with t^* is stable if $|f_r(\hat{t})-t^*| < |\hat{t}-t^*|$. Therefore, stability is measured by $f_r'(t^*)$: if $f_r'(t^*) < 1$, then the equilibrium is stable, and if $f_r'(t^*) > 1$, it is unstable.

Consider the symmetric equilibrium. We have $f_r'(t^*) = f_r'(1) = \frac{\left(\frac{1}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\right)^r} = \frac{2}{2^r-1} > 2$ since $2^r - 1 < 2 - 1 = 1$. Moreover, $f_r'(1)$ increases with r, and $\lim_{r\to 0} f_r'(1) = +\infty$. Therefore the lower r, the more unstable the symmetric equilibrium.

Now consider an asymmetric equilibrium. We already established $f_r'(t_r) < 1$ in Equation (15). □

E Proof of Proposition 5

Proof. We need to compare the expected values $\mathbb{E}[Y_{t^*}]$ of the CES production function computed at $t^* = 1$, i.e. at the symmetric equilibrium, and at $t^* = t_r$, *i.e.* at the asymmetric equilibrium. We have

$$\begin{aligned}
\mathbb{E}[Y_{t^*}] &= \Pr(m = m_1)E[Y_{t^*}(m_1)] + \Pr(m = m_2)E[Y_{t^*}(m_2)] \qquad (27)\\
&= \int_{a_1 \geq t^* a_2} Y_{t^*}(m_1)\,\mathrm{d}a_1\,\mathrm{d}a_2 + \int_{a_1 < t^* a_2} Y_{t^*}(m_2)\,\mathrm{d}a_1\,\mathrm{d}a_2\\
&= \int_{a_2=0}^{1}\int_{a_1=t^*a_2}^{1} \left(\frac{(E_{11}(t^*)a_1)^r + (E_{21}(t^*)a_2)^r}{2}\right)^{\frac{1}{r}} \mathrm{d}a_1\,\mathrm{d}a_2\\
&\quad + \int_{a_2=0}^{1}\int_{a_1=0}^{t^*a_2} \left(\frac{(E_{12}(t^*)a_1)^r + (E_{22}(t^*)a_2)^r}{2}\right)^{\frac{1}{r}} \mathrm{d}a_1\,\mathrm{d}a_2
\end{aligned}$$

with E_{ij}'s given by (8).

If $r = 1$, we find $\mathbb{E}[Y_{t^*=1}] = \frac{5}{18} \cong 0.278$ at the symmetric equilibrium, and, at the asymmetric equilibrium associated with $t^* = t_(1) = \frac{1}{2}$, we find $\mathbb{E}[Y_{t^*=\frac{1}{2}}] = \frac{59}{216} \cong 0.273$. Therefore, the asymmetric equilibrium is *ex-ante* less productive than the symmetric equilibrium.

When r tends to 0, t_r tends to 0 and at $t^* = 0$, we have $E_{11}(0) = \frac{1}{2}$, $E_{21}(0) = \frac{1}{2}$, $E_{12}(0) = 0$, and $E_{22}(0) = \frac{2}{3}$. Moreover, $\left(\frac{(y_1)^r+(y_2)^r}{2}\right)^{\frac{1}{r}}$ tends to $\sqrt{y_1 y_2}$.

Hence when $r \to 0$, at the symmetric equilibrium, we obtain

$$\begin{aligned}\mathbb{E}[Y_{t^*=1}] &= \int_{a_1 \geq a_2} \sqrt{\frac{2}{3}a_1 \frac{1}{3}a_2}\, \mathrm{d}a_1\, \mathrm{d}a_2 + \int_{a_1 < a_2} \sqrt{\frac{1}{3}a_1 \frac{2}{3}a_2}\, \mathrm{d}a_1\, \mathrm{d}a_2 \\ &= \frac{\sqrt{2}}{3} \int_{a_1, a_2} \sqrt{a_1 a_2}\, \mathrm{d}a_1\, \mathrm{d}a_2 = \frac{2\sqrt{2}}{27} \cong 0.21,\end{aligned}$$

and at the asymmetric equilibrium, from $t_r \to 0$, we obtain

$$\mathbb{E}[Y_{t^* \to 0}] = \int_{a_1, a_2} \sqrt{\frac{1}{2}a_1 \frac{1}{2}a_2}\, \mathrm{d}a_1\, \mathrm{d}a_2 = \frac{1}{2} \int_{a_1, a_2} \sqrt{a_1 a_2}\, \mathrm{d}a_1\, \mathrm{d}a_2 = \frac{2}{9} \cong 0.22.$$

Therefore, the asymmetric equilibrium is *ex-ante* more productive than the symmetric equilibrium. □