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Supply chain coordination under trade credit and retailer effort

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In this paper, we study the role of trade credit in coordinating a Capital Constrained Supply Chain in the presence of retailer Effort (CCSCE), essentially because of the impact of its related default risks on the relationship between the chain’s members. We consider a CCSCE consisting of a supplier and a retailer where the retailer may exert costly promotional efforts to increase the market demand but has limited capital and no access to bank financing due to low credit rating. Conversely, the supplier has adequate funds to offer trade credit to the retailer without borrowing from external channels. We then examine whether the existing coordination contracts can still coordinate the CCSCE under trade credit. Our result shows that these contracts can achieve coordination of the supply chain when the interest rate of trade credit is competitively priced. Nevertheless, this position cannot always be reached. That’s why we propose a generalised contract based on risk compensation to coordinate the CCSCE. Using our proposed coordinating contract, the supplier perfectly coordinates the retailer’s decisions for the largest joint profit, and arbitrarily allocates the maximised joint profit among supply chain members. Finally, the numerical study allows to verify this finding. From managerial insights, our results provide the supply chain managers with novel insights on how to combine trade credit with the existing coordination contracts in order to improve the profitability of the entire supply chain as well as the individual member.

Keywords: supply chain coordination; promotional effort; capital constraint; trade credit; risk compensation

1. Introduction

In the global business environment, competition between individual businesses has shifted to supply chains (Chaharsooghi and Heydari 2010), and the success of a single business increasingly depends on management’s ability to align the interests of all members of the supply chain (SC) (Croxton et al. 2001). As a result, the past decades have seen increasing interest in supply chain management in order to optimise the performance of the entire SC. Supply chain management represents a new way of managing business and relationships among members of the SC. In fact, there always exists some conflicts among the partners in SC since these partners try to maximise their own profits based on their own cost structure, regardless of other SC participants. In the absence of coordination, these conflicts can lead to local optimum solutions which may lower total profit of SC (Corbett, Zhou, and Tang 2004). In contrast, supply chain coordination (SCC) incentives SC members to pursue decisions that are based on the profitability of the entire SC. In this paper, we focus on the SC contract as a critical component in SCC since it provides incentives to induce SC members to behave in ways that are best for the whole SC even as each member attempts to maximise their own profits. Using a Newsvendor framework, several types of SC contracts have been introduced in the literature. These include contracts of buybacks (Pasternack 1985); two part tariffs (Larivi\`ere 1999); quantity flexibility (Ts\`ay 1999); sales rebate (Taylor 2002); quantity discounts (Cachon 2003); revenue-sharing (Cachon 2003; Cachon and Larivi\`ere 2005). These traditional contracts have mainly been applied for coordinating order quantity. From business practices, promotional efforts have become increasingly more common. A retailer can boost the demand for a supplier’s product by applying a promotional effort such as advertising (Somers, Gupta, and Herriott 1990; Yuan, Song, and Yang 2016), corporate social responsibility (Eltantawy, Fox, and Giunipero 2009), post-sales service (Ahmad and Mohsin Butt 2012; Szweczyewski, Goffin, and Anagnostopoulos 2015). However, all of those activities are costly. As a consequence, if the supplier does not provide sufficient incentives, then the retailer will have no motivation to enhance effort level (Krishnan, Kapucinski, and Butz 2004). Therefore, retailer’s promotional effort is another fundamental decision in a SC that must be coordinated. In practice, the suppliers coordinate with its retailers to make decisions on advertising (Gou et al. 2014; Karray and Amin 2015; Lu et al. 2016; Zhao, Zhang, and Xie 2016), service quality (Xiao, Yang, and Shen

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2011; Liu and Xie 2013; Yang and Xiao 2017) and level of social responsibility (Raj, Biswas, and Srivastava 2018). Over the
years, several research efforts have been devoted to dealing with the issues of SCC under promotional efforts (Taylor 2002;
Krishnan, Kapuscinski, and Butz 2004; Xiao, Yang, and Shen 2011; Pang, Chen, and Hu 2014; Karray and Surti 2016).
Recently, Yan and Zoric (2016) provided a comprehensive analysis on this research stream. He classified all coordinating
contracts in prior literature as belonging to five contract families which are defined by the structure of the payment terms
between the parties in contracts.

Notably, the above studies are built on an important assumption that all of the SC members have infinite capital to support
their business decisions. By contrast, in many developing countries that there haven’t had an advanced financial market,
many firms have an insufficient budget and may be unable to obtain sufficient financing to support their operational decisions
due to low credit rating (Caldentey and Haugh 2009). Firms operating under budget (capital) constraints may not be able
to order or produce optimally. Especially, if potential demand is high but capital constraints make them have no choice to
order less. In real-life situations, budget constraint is one reason that retailer’s promotional efforts are unworkable even as
the suppliers totally subsidise the cost of promotion efforts. According to the results of a survey in 2015 by Web Video
Marketing Council 1, 55% of respondents naming limited budget availability as a challenge to conduct video marketing. In
another survey 2, Gleanster Research estimated that about 50% of Co-Op funds go unused every year, and the main reason
is that partners must pay all marketing costs up front while they have limited budgets.

As capital constraints are important factors affecting the success of the SCC, many authors have begun to investigate
coordination for a SC with capital constraints without promotional efforts (Dada and Hu 2008; Kouvelis and Zhao 2015;
Moon, Feng, and Ryu 2015). These papers explicitly did not use trade credit as a coordination mechanism. Trade credit
is widely practiced in SC and is the most important sources of short-term financing for the firms (Atanasova and Wilson
2003). Hill et al. (2017) observed that trade credit constituted 12% of assets while short-term debt constituted 6% of assets.
Under trade credit, the supplier allows the retailer to delay their payments until the products are sold (Tsao 2014; Chuang
and Wu 2017). Using this source of working capital, the retailer can reduce the capital required to increase the order quantity.
Though, the retailer with a lower initial budget would initiate a more aggressive ordering strategy under demand uncertainty
because of limited liability on default risk (the risk that the retailer cannot pay its debt). Therefore, trade credit has been
extensively researched. Some studies such as Tsao (2014), Tsao (2016), Zhan et al. (2017), Zhong et al. (2018) investigated
the impacts of trade credit on SC network design but did not address SCC. In contrast, some papers present trade credit as
a coordination mechanism for the SC with capital constraints without promotional efforts (Sheen and Tsao 2007; Sarmah,
Acharya, and Goyal 2008; Krichen, Laabidi, and Abdelaziz 2011; Lee and Rhee 2011; Teng and Lou 2012; Zhang et al.
2014).

The literature cited above provided a wide variety of coordination mechanisms to deal with SCC issues for the different
decisions impacting on the SC performance including firm’s operational decisions (order quantity), marketing strategy
(promotional efforts) and financial decisions. However, financial and operational decisions of a firm are studied sepa-
ately with its marketing strategy. In fact, decisions regarding order quantity, promotional effort and finance interact with
each other and should be optimised simultaneously. Interaction between marketing strategy with operational and financial
decisions goes in both ways: on one hand, since sales promotion activities are costly, it exacerbates the lack of financial
resources and increases the need for capital financing. On the other hand, these activities lead to higher demand that in turn
increases the order quantity and reduces the default risk under trade credit. Therefore, a special attention to the interface
of marketing strategy, operations and finance can provide useful insight into managers about how to integrate decisions
on SCC.

In particular, this paper is designed to address the following questions: (i) Can the existing coordinating contracts in a
SC with retailer’s promotional efforts without capital constraints (SCE) still coordinate a CCSCE when trade credit and its
default risk are considered?, (ii) If these contracts cannot coordinate such a SC, how the payment terms of trade credit (i.e.
the interest rate of trade credit) and/or the terms of existing coordinating contracts in a SCE should be adjusted to retailers
in order to maintain the coordination? (iii) Can we design a new trade credit contract that coordinate a CCSCE?.

In the present paper, a coordination model has been proposed to answer the above-discussed questions. Our model
considers a two-echelon SC consisting of a supplier (he) and a capital constrained retailer (she) who acts as the follower.
The supplier, as a leader in the Stackelberg game, has adequate funds to provide trade credit to the retailer without borrowing
from external channels (such as banks or other financial institutions). The retailer, on the other hand, faces a shortage of
working capital and has to exert costly efforts to increase market demand for the products but cannot obtain a bank loan
because of low credit rating. With trade credit financing, the retailer can delay her payment and/or borrow fund from the
supplier to support their decisions and goes into default if she fails to repay the full trade credit.

The remainder of the paper is organised as follows: Section 2 explains the notations and assumptions. In Section 3, we
present the model in the case of a centralised SCE. In Section 4, we propose a general contract under trade credit and describe
decentralised CCSCE decision-making problem under this contract. In Section 5, we design the coordinating contracts for
2. Model description and assumptions
We consider a two-echelon SC consisting of a supplier as a Stackelberg leader and a capital constrained retailer who faces a non-negative random demand that is sensitive to retailer promotional effort. We assume the capital constrained retailer has an initial capital $B$ while the supplier has no capital constraints. The supplier produces a single product which the retailer sells to final consumers at a constant retail price $p$. The supplier’s unit production cost is $c$ and he needs to decide a supply contract to achieve the best performance for the entire SC and arbitrarily allocates chain’s profit among members based on their relative bargaining power. The retailer then chooses the quantity ordered $q$ from the supplier and the level of promotional effort $e$ to increase demand. For simplicity, we assume that a stockout does not cause any goodwill loss and the unsold products have zero salvage value. We use a single effort level $e$ to summarise the retailer’s activities in promoting efforts. Let $g(e)$ be the retailer’s cost of effort which we assume to be increasing and convex in $e$ with $g(0) = 0$. Let $x$ be a random variable representing the market demand for products. We assume that demand satisfies the multiplication distribution function (CDF) is from $x$ be a random variable representing the market demand for products. We assume that demand satisfies the multiplication distribution function (CDF) is $F(x)$. This type of effort–demand models has been used in many studies (Gerchak and Parlar 1987; Rao 1990; Taylor 2002).

Let $f(x|e)$ be the PDF and let $F(x|e)$ be the CDF of demand given the effort level $e$. The complementary CDF is $F_0(x|e) = 1 - F(x|e)$. We assume that the demand is stochastically increasing with the retailer’s effort; that is $\frac{dF(x|e)}{de} < 0$. The distribution is the common knowledge of the supplier and retailer. According to the properties of conditional probability, the PDF and the CDF of the market demand is:

$$f(x|e) = \frac{1}{e} f\left(\frac{x}{e}\right) \text{ and } F(x|e) = F\left(\frac{x}{e}\right).$$

3. Centralised SCE model
We first investigate the integrated SC which is considered as a single system operating under a global optimisation strategy. Therefore, we don’t separate the working capital of the retailer from that of the supplier and consider the working capital of the whole chain. As the supplier has no capital constraints, the centralised SC can be operated in that condition. This is a benchmark case with which we compare the case of a coordination scheme in a CCSCE. Let $\xi(q, e)$ be the expected profit of centralised SCE at the end of the selling season (referred to as time $1$).

$$\xi(q, e) = R(q, e) - (qc + g(e))(1 + r_f).$$

The first term $R(q, e)$ is the expectation of sale revenue at time $1$ when the order quantity at time zero is $q$ and retailer’s effort in sale period is $e$, i.e. $R(q, e) = pe \int_0^q F(\xi)d\xi$, and the second term in (2) is the production cost for $q$ products plus the cost of exerting effort level. Here the term $(1 + r_f)$ reflects an adjustment for the time value for money at the risk-free interest rate ($r_f$).

The objective of a centralised SCE is to maximise $\xi(q, e)$. Based on Taylor (2002), the optimal order quantity is $q_c = e_c \hat{q}$ and the optimal effort level $e_c$ satisfies $\frac{d\xi}{de}|_{e_c}(1 + r_f) = p(\hat{q})$ where $\hat{q} = \hat{F}^{-1}[c(1 + r_f)/p]$ and $(\hat{q}) = \int_0^\hat{q} \xi dF(\xi)$. The first-best CSCE’s profit is $\xi = pe_c(\hat{q}) - g(e_c)(1 + r_f)$.

4. Decentralised CCSCE model
4.1. The general contract under trade credit
At the beginning of the selling season (referred to as time zero), the supplier designs a general contract TC (TC hereafter) and announces this to the retailer. It is constructed as $TC[r, w, A]$ where $r$ is the interest rate of trade credit, $w$ is the wholesale cash price for each unit purchased and $A$ denote an additional payment realised at time zero. The additional payment may be constant or depend on the retailer’s order quantity and/or the level of promotional effort that the retailer commits to perform in sale period. Note that a contract with two components of $w$ and $A$ (we call it RE($w$, $A$) for short) represents the coordinating contracts in the SCE. See Yan and Zaric (2016) for detailed discussions of RE($w$, $A$) contracts.

Under TC($r, w, A$), the retailer can delay the payment to finance her inventory and/or borrow money from the supplier in order to pay for the cost of promotion efforts. For each dollar of loan or delayed payment, she has to pay $(1 + r)$ dollars to the supplier at time $1$. In practice, inter-company loans are quite often made between companies in the same group (i.e.
between parent companies and its subsidiaries). In France, the ‘Macron Law’ also allows companies that do not have an ownership relation to grant loans to each other. Loans between companies provide the retailers with the opportunity to receive more financing from the suppliers compared to traditional trade credit. Therefore, the term ‘trade credit’ normally refers to a delay in payment for the order; it may include an inter-company loan under our TC contract.

In particular, when the additional payment is zero and without a loan agreement, the TC\(\{r, w, A\}\) become a two-wholesale price contract (Jing, Chen, and Cai 2012) in which the supplier simultaneously announces two wholesale prices for the retailer to choose: (i) a wholesale cash price \(w\), applicable at time zero if the retailer pays in cash for her entire order and (ii) a postponed wholesale price \(w_1 = w(1 + r)\), applicable at time 1 if the retailer asks for trade credit financing. Equivalently, the supplier may announce a wholesale cash price \(w\) and an associated interest rate \(r\) under trade credit. In practice, the two-wholesale price contract can be part of the dynamic discounting or two-part trade credit policy where the payment is made early in exchange for a lower price (or discount).

Facing a TC\(\{r, w, A\}\), the retailer has three financing choices depending on her initial capital available at time zero: No Credit; Partly Credit; Full Credit. If the retailer has sufficient capital to make any decisions, she pays the total amount of \(wq + A\) in cash to the supplier at time zero (No credit). If the retailer’s capital is insufficient to support her order and cost of promotion efforts, i.e. \(0 < B < wq + A + g(e)\), she makes a choice of Partly Credit by delaying the payment for her order and/or borrowing money from the supplier for paying the cost of promotion efforts. In this case, the delayed payment and loan amount depend on the choice of the retailer in the use of the initial capital to first pay for the cost of promotion efforts or her order. However, the total amount of financing (trade credit amount) is always \(wq + A + g(e) - B\) that less than the total cost of the retailer at time zero. Therefore, the retailer has to pay a total amount of \((wq + A + g(e) - B)(1 + r)\) to the supplier at time 1 because the interest rate under TC contract is the same for every dollar of financing. In addition, the supplier’s net cash flow at time zero generated from trading with the retailer does not depend on the retailer’s choices, i.e. when \(B > g(e)\) the supplier actually gets from the retailer an amount of \(B - g(e)\) in cash and when \(B < g(e)\) the supplier transfers to the retailer an amount of \(g(e) - B\).

In the case that the retailer’s initial capital is zero, she borrows an amount of \(g(e)\) from the supplier and makes a zero payment to the supplier at time zero (Full Credit) but has to repay a total amount of \((wq + A + g(e))(1 + r)\) to the supplier at time 1. This payment consists of two parts: a refund of the capital borrowed from the supplier plus the interest on this amount, i.e. \(g(e)(1 + r)\) and a delayed payment of her entire order \((wq + A)(1 + r)\).

4.2. The retailer’s optimisation

Under TC\(\{r, w, A\}\), the retailer asks the supplier for trade credit financing (delays her payment and/or borrows fund from the supplier) an amount of \(L = \max\{wq + A + g(e) - B, 0\}\) with an interest of \(r\). \((L = 0\) refers to No Credit; \(0 < L < wq + A + g(e)\) refers to Partly Credit and \(L = wq + A + g(e)\) refers to Full Credit). The repayment amount at time 1 is \(L(1 + r)\). Let \(z = z(q, e, r, w, A)\) be the demand threshold for which the revenue at the end of the period is just sufficient to cover the promised payment to the supplier.

\[
z = \frac{L(1 + r)}{p} = \frac{\max\{wq + A + g(e) - B, 0\}(1 + r)}{p}.
\]

(3)

Note that \(z = 0\) if the retailer’s initial capital is sufficient. Also, if \(z > q\), the trade credits are not profitable for the retailer, so she will not adopt the trade credit. Then, when the retailer asks for trade credit, we suppose \(0 < z < q\).

After the demand is realised (at time 1), the retailer needs to pay back \(L(1 + r) = pz\), which represents the full repayment of the trade credit amount plus interest to the supplier. However, if the retailer’s total revenue is not sufficient such that \(p\min\{x, q\} < L(1 + r), i.e. x < z\), the demand falls below \(z\) which means that the sum of the value from sold products is less than that which should be paid by the retailer. The retailer is then forced to declare bankruptcy. In this case, the supplier acquires the total revenue realised. Otherwise, the retailer pays the debt in full. Therefore, the retailer’s expected profit under TC\(\{r, w, A\}\) is:

\[
l(q, e, TC) = pe \int_{z}^{\min\{q, e\}} \bar{F}(\xi) d\xi - B(1 + r_f),
\]

(4)

where \(y = z/e\) is the number of units that must be sold to fully pay off the trade credit undertaken by the retailer for a given retailer’s effort level. We refer to this as the retailer’s default threshold. Hence, the retailer defaults with probability \(\bar{F}(y)\).

After observing the TC\(\{r, w, A\}\), the retailer determines the level of effort as well as the quantity ordered from the supplier and asks for trade credit when necessary to maximise their expected profit by choosing the optimal quantity – effort level pair \((q^*, e^*)\). For a given supply contract TC\(\{w, r, A\}\), it is straightforward to show that if \(\partial^2 A/\partial q^2 > 0\), the retailer’s
4.3. The supplier’s optimisation and coordination

For the supplier, at time 1 he receives a repayment from the retailer that includes two parts: the repayment when the retailer fails to repay the full trade credit which is the retailer’s total realised sales revenue and the repayment when the retailer fully repays the trade credit \( L(1 + r) = pz \) when \( x > z \). Therefore, the supplier’s expected profit in a decentralised CCSCE under TC\((r, w, A)\) can be expressed as:

\[
\Pi_s(q, e, TC) = pe \int_0^\gamma \bar{F}(\xi)d\xi + (B - g(e))(1 + r_f) - qc(1 + r_f).
\]

The first term on the right side of the equation represents the expected total repayment that the supplier gets from the retailer. The second term can be the amount of money that the supplier get from the retailer (when \( B > g(e) \) or the transferred fund from the supplier to the retailer (when \( B < g(e) \) plus the interest at the rate \( r_f \). The third term is the production cost at time 1.

As a leader, the supplier will decide \((w, r, A)\) to maximise its expected profit. For doing this, the supplier first coordinates the SC by inducing the retailer to replicate the optimal outcomes of the centralised SCE in making decisions. After coordinating the SC, the supplier then takes the largest portion of the joint profit by arranging a profit-sharing scheme, which gives the retailer barely enough profit to participate in the coordination (Lee and Rhee 2010). Therefore, the supplier’s problem is to design a TC\((r, w, A)\) contract that satisfies the following two conditions:

- **C1**: Under the TC\((r, w, A)\), the retailer’s optimal decisions in the decentralised CCSCE are the same as in the centralised SCE, i.e., \((q^*, e^*) = (q_r, e_r)\).
- **C2**: Under the TC\((r, w, A)\), each party obtains an arbitrary pre-negotiated portion of coordinated profit based on their relative bargaining power, i.e., \(\Pi_r(q^*, e^*, TC) = \Pi_c\) and \(\Pi_s(q^*, e^*, TC) = (1 - \lambda)\Pi_c\) for any \(0, 1\) denotes the portion of profit sharing.

In the SCC literature, there have been two different approaches in designing of coordinating contracts: The first approach uses first-order conditions (Tsay 2001) and the second one makes the retailer’s profits an affine transformation of the joint SC profit (Cachon 2003; Cachon and Lariviere 2005). We employ both approaches to design the coordinating contract in the next Section.

5. Design of coordinating contracts

5.1. Coordinating contract when the interest rate of trade credit is competitively priced

The perfect competition with a competitively priced loan is a common assumption in capital market ensuring that the expected profit from the loan contract is zero. We argue that the interest rate of trade credit can be competitively priced when the capital market is perfectly competitive. If this position cannot be reached, a bank may obtain a positive profit in a perfect capital market by buying the supplier’s invoices at wholesale cash price. In fact, these invoices are initially issued at a value of postponed wholesale price. This is a common practice in the factoring without recourse. Therefore, the trade credit should be priced so that the supplier’s discounted expected payoff of trade credit financing is equal to the trade credit amount plus the interest on this amount at the risk-free interest rate. Hence, the supplier sets the interest rate of trade credit according to the following equation:

\[
pe \int_0^\gamma \bar{F}(\xi)d\xi = L(1 + r_f).
\]

Let \(\bar{y} = L(1 + r_f)/pe\) be the retailer’s default threshold at the risk-free interest rate, then the above equation can be transformed into:

\[
\int_0^{\bar{y}(1+\bar{r}^2)}/(1+\bar{r}) \bar{F}(\xi)d\xi = \bar{y}.
\]
Equation (6b) permits to determine the competitively priced interest rate, $r^\gamma$, with respect to the retailer’s default threshold at the risk-free interest rate $\gamma$. Let $\Pi_1(q, e, RE)$ be the expected profit functions of the retailer, the supplier in the decentralised SCE under $RE$ respectively. If the interest of trade credit is competitively priced, the retailer’s profit function under $TC[r, w, A]$ becomes $\Pi_1(q, e, TC) = R(q, e) - (wq + A + g(e))(1 + r_f) = \Pi_1(q, e, RE)$ and the supplier’s profit function under $TC[r, w, A]$ will be $\Pi_1(q, e, TC) = (q(w - c) + A)(1 + r_f) = \Pi_1(q, e, RE)$. This observation implies that when the retailer has capital constraints but finances its business with competitively priced trade credit from the supplier, the retailer’s optimal decision is independent of both the interest rate and the initial capital under the trade credit financing, as long as the interest of the trade credit is competitively priced. This also means that the capital constraints have no influence on the retailer’s decisions in a perfect capital market. This result is consistent with the Modigliani and Miller (1958).

If the capital markets are perfect, Modigliani and Miller (1958) prove that managers may consider financial decisions independently from the firm’s operational decisions. The following result comes directly from this observation.

**Theorem 1** If a contract $RE[w, A]$ coordinates the decentralised SCE, the $TC[r, w, A]$ in which the interest rate of trade credit is competitively priced, achieves coordination in the decentralised CCSCE.

**Proof** All proof of the Theorem and the Proposition are put in the Appendix and available upon request from the authors. $\blacksquare$

Theorem 1 indicates that when the interest rate of trade credit is competitively priced, the coordinating contracts in the decentralised CCSCE have similar structural properties to those in the decentralised SCE. This result implies that coordinating contracts for the noncapital constrained SC in the presence of retailer effort continue to be coordinated for the capital constrained SC under trade credit as long as the needed financing is competitively priced.

5.2. Coordinating contracts based on risk compensation

5.2.1. The general conditions of payment terms

In the decentralised CCSCE with trade credit, when payment is delayed, the suppliers undertake the risk that the retailer cannot pay all of the payment at time 1, especially, when the demand at retailer is stochastic. Therefore, the supplier needs to be compensated for risk-taking. Based on Theorem 1, we know that if the supplier is properly compensated for default risk in the form of risk premium (risk premium in this case is the difference between the interest rate of trade credit and the risk-free interest rate, i.e. $r - r_f$), the coordinating contracts in the decentralised CCSCE have similar structural properties to those in the decentralised SCE. Otherwise, if the interest rate of trade credit is not enough to compensate for default risk, i.e. the interest rate of trade credit set by the supplier equals the risk-free interest rate, then, in order to ensure the benefit for the supplier, the additional payments must take the role of compensating the default risk to achieve the coordination.

We provide Theorem 2 to indicate the necessary and sufficient conditions of the additional payment for the existence of a coordinating contract in the decentralised CCSCE.

**Theorem 2** If the following conditions of the additional payment are satisfied, then there exists a contract $TC[r_f, w, A]$ which achieves coordination in the decentralised CCSCE when $A$ depend on $q$ and/or $e$, i.e. $A = A(q,e)$.

\[ R(q, e) - A + pe \int_0^\gamma F(\xi)d\xi \text{ is concave in } q \text{ and } e \]  

(7a)

\[ \frac{\partial A}{\partial q}_{|q,e,\notq,e,\notq,e,\notnull} = \frac{c - w\hat{F}(y_c)}{\hat{F}(y_c)} \]  

(7b)

\[ (1 + r_f)\hat{F}(y_c)\frac{\partial A}{\partial e}_{|q,e,\notq,e,\notq,e,\notnull} = (1 + r_f)F(y_c)\frac{\partial g(e)}{\partial e}_{|q,e,\notnull} - p(y_c) \]  

(7c)

\[ A(q_c, e_c) = (1 - ) \frac{c(1 + r_f) - q_c(w - c)}{1 + r_f} + \frac{pe_c \int_0^{\gamma_c} F(\xi)d\xi}{1 + r_f} \]  

(7d)

where $y_c = (wq_c + A(q_c, e_c) + g(e_c) - B)(1 + r_f)/pe_c$ and, we recall, $(y_c) = \int_0^{\gamma_c} \xi dF(\xi)$. 


\[ \]
Condition (7a) guarantees that the optimal solution \( (q^*, e^*) \) exists and is unique. Condition (7b) incentivises the retailer to order the quantity \( q^* \). Condition (7c) incentivises the retailer to decide on the effort level \( e^* \), and (7d) guarantees that each party obtains an arbitrary pre-negotiated portion of the total system profit. From Theorems 2, if we let \( y_c = 0 \), conditions 7a, b, c, and d can be reduced to the conditions under which a coordinating contract exists in a decentralised SCE.

5.2.2. The specific examples of coordinating contracts

Corollary 1 and 2 give specific examples of contracts satisfying the conditions in Theorem 2.

**Corollary 1** In a decentralised CCSCE where the supplier offers trade credit to the capital constrained retailer and the interest rate of trade credit set by the supplier equals the risk-free interest rate. If RE\([w, A]\) achieves coordination in a decentralised SCE, then, the contracts proposed TC\((r_f, w, A + T)\) achieves coordination in the decentralised CCSCE, where \( T \) is the risk compensation from the retailer to the supplier and has the following form: 
\[
T = \frac{pe^f_0 \int_0^T F(\xi) \, d\xi}{1 + r_f}, \quad \text{where} \quad y = (wq + A + T + g(e) - B)(1 + r_f)/pe.
\]

**Remark 1** The risk compensation \( T \) proposed in Corollary 1 has the following characteristics: (i) \( T = 0 \) when retailer makes a full payment for her order, (ii) when the retailer adopts trade credit, \( T \) depends on the retailer’s initial capital and the retailer’s decision variables (i.e. the order quantity and/or the level of promotional effort). (iii) under TC\((r_f, w, A + T)\), the retailer’s and the supplier’s expected profit functions are linear functions of the expected profit of centralised SCE, i.e., \( \gamma(q, e, TC) = \gamma(q, e, RE) = \gamma(q, e) \) and \( s(q, e, TC) = \gamma(q, e, RE) = \gamma(q, e) \). This ensures that the retailer’s optimal decision is the same as the first-best solution of the centralised SCE and the SC profit can be arbitrarily allocated among members. Therefore, TC\((r_f, w, A + T)\) achieves coordination in a decentralised CCSCE.

The contract proposed in Corollary 1 based on the combination of the risk compensation under trade credit and the coordinating contracts in the decentralised SCE. Under TC\((r_f, w, A + T)\), if at time zero the retailer borrows funds from the supplier and/or asks for a delayed payment, the supplier sets the interest rate equal to the risk-free interest rate (\( r_f \)) to compensate for the time value of money. In addition, to compensate for default risk, the supplier requires the retailer to pay an additional amount equal to risk compensation \( T(1 + r_f) \) at time 1. From the perspective of the retailer, she has three financing choices under TC\((r_f, w, A + T)\) depending on her initial capital at time zero. If the retailer has sufficient capital to make any of decisions, she makes a full payment of \( wq + A \) for her entire order at time zero. If the retailer’s capital is sufficient to support their cost of promotional effort but insufficient to pay for her entire order, the retailer asks the supplier for a delayed payment of \( L = wq + A - (B - g(e)) \leq wq + A \). In this case, the retailer has to pay a total amount of \((L + T)^*(1 + r_f)\) to the supplier at time 1. In the case that the retailer’s initial capital not being sufficient to support their cost of effort, the retailer borrows an amount of \( g(e) - B \) from the supplier and asks the supplier for a delayed payment for her entire order of \( wq + A \). In this case, the retailer has to pay a total amount of \((L + T)^*(1 + r_f)\) to the supplier at time 1.

**Corollary 2** Let \( y = (wq + A + T + g(e) - B)(1 + r_f)/pe \) be the retailer’s default threshold at the risk-free interest rate after adding the risk compensation to the amount of trade credit. Then, the following contract in the form of TC\((r_f, w, A + T)\) coordinates the decentralised CCSCE.

\[
\begin{align*}
\bullet \quad w &= c + \frac{(1-\epsilon)}{2(1 + r_f)} q e, \\
\bullet \quad A &= \frac{c - q e}{2(1 + r_f)} , \\
\bullet \quad T(1 + r_f) &= pe^f_0 \int_0^T F(\xi) \, d\xi .
\end{align*}
\]

The contract proposed in Corollary 2 is a combination of the quantity-discount contract and the risk compensation under trade credit. Under this contract, if the retailer has sufficient capital, she makes a full payment for her order, the supplier then applies a wholesale cash price \( w(q) \) that depends on the order quantity, i.e. \( w(q) = c + \frac{(1-\epsilon)}{2(1 + r_f)} q e + \frac{(1-\epsilon)}{2(1 + r_f)} q e \). The total payment will be \( wq + A = w(q) + q \). This is equivalent to a quantity-discount contract because the wholesale cash price decreases in \( q \). As proved by Yan and Zaric (2016), this contract can achieve coordination when the retailer has sufficient capital to make any of decisions. If the retailer has insufficient capital, she order \( q \) unit and pay \( k \) dollars \((0 \leq k < w(q)q)\) in cash to supplier at time zero \((k < 0 \) would indicate that the retailer borrow \( k \) dollars from the supplier), the amount of trade credit financing will be \( L = w(q)q - k \). In this case, to maintain the coordination, the supplier requires the retailer a
payment at time 1 which consists of two parts: a trade credit amount plus the interest at the risk-free interest rate on this amount, i.e. \( L = L(1 + r_f) \) and a risk compensation \( T = T(1 + r_f) \) which is determined based on the retailer’s initial payment, the order quantity and the level of promotional effort that the retailer commits to perform in sale period.

In Proposition 1 and 2, we provide some characteristics of risk compensation \( T \) when the \( \text{TC}(r_f, w, A + T) \) coordinates the decentralised CCSCE.

**Proposition 1** If the interest rate of trade credit \( r_f \) is competitively priced, the risk compensation \( T \) is related to the trade credit rate \( r_f \) based on Eq. (8) as follows:

\[
(1 + r_f)T = L(r_f - \overline{y}),
\]

where \( L = wq + A + g(e) - B \) is the amount of trade credit and \( \overline{y} = L(1 + r_f)/pe \) is the retailer’s default threshold at the risk-free interest rate before adding the risk compensation to the trade credit amount.

**Proposition 2** If the \( \text{RE}(w, A) \) achieves coordination in a decentralised SCE, then the contracts proposed \( \text{TC}(r_f, w, A + T) \), where \( T \) depends only on the level of effort \( e \) cannot coordinate a decentralised CCSCE.

6. Numerical examples

In this section, we present some examples to obtain further managerial insights based on the theoretical results above. Following Yan et al. (2016), we assume that the random variables \( \xi \) follows an Exponential distribution with a mean of 2.5; the cumulative distribution function is \( F(\xi) = 1 - \exp(-0.4\xi) \) for \( \xi \geq 0 \). We also assume that the retailer can influence the demand by exerting promotional effort with cost of \( g(e) = 0.05e^2 \). We set: the market price at \( p = $6 \), the unit production cost at \( c = $3 \) and the risk free interest rate at \( r_f = 0 \).

Using the above assumptions, in the centralised CSE, the optimal order quantity is \( q_e = 39.88 \), the optimal effort level is \( e_e = 23.014 \) and the expected profit of SC will be \( \Pi_e = $26.482 \).

As we mentioned in Corollary 2, the quantity-discount contracts, which consist of a basic constant price component, \( w = c + \frac{(1 - \lambda)}{2(1 + r_f)}\Pi_e \), and a variable price component (the additional payment), \( A(q) = (1 - \lambda)\frac{\Pi_e}{2(1 + r_f)}q \), can be used to coordinate the decentralised SCE and arbitrarily allocate the total profit between the SC members. We suppose that the retailer and supplier together agree on the portion of the profit allocation \( \lambda \), then the quantity-discount contract has specific parameters depending on the portion of profit allocation and the order quantity \( q \) as follows: \( w = 3 + 0.33201 \) \((1 - \lambda)\) and \( A(q) = 528.0396 \) \((1 - \lambda)/q \). Next, we will show that this contract continues to be coordinating for the decentralised CCSCE by adding the supplier’s decision variables (i.e. the interest rate of trade credit and the risk compensation) under trade credit.

6.1. The retailer has sufficient capital

Table 1 shows that when the retailer has sufficient capital, the retailer’s optimal decisions are \( q = 39.88 \) and \( e = 23.014 \) under the quantity-discount contract. Thus, the retailer’s optimal decisions are also the optimal decisions of whole SC. Accordingly, the retailer’s initial capital needed to make the optimal decisions corresponding to the pre-determined profit allocation is 172.603 \(- 26.481 \), (i.e. $164.66 if \( \lambda = 0.3 \), $162.012 if \( \lambda = 0.4 \) and $159.363 if \( \lambda = 0.5 \)).

6.2. The retailer does not have enough capital but does not use trade credit

Without the consideration of trade credit, if the retailer’s initial capital is smaller than 172.603 \(- 26.481 \), they cannot make their optimal decisions (\( q = 39.88 \) and \( e = 23.014 \)). In column 2 of Table 1, we illustrate this matter for four retailer’s initial capital levels which are $40, $70, $90 and $110 (these capital levels are not enough to support the optimal decisions \( q = 39.88 \) and \( e = 23.014 \)). In this case, the retailer uses up their capital and her optimal decisions are shown in columns 8 and 9 of Table 1, respectively. We find that the order quantity and effort level of retailer are less than the optimal outcomes of the CSCE. Similarly, the retailer’s expected profit and the profit of the SC are smaller than when the retailer’s capital is sufficient. For example, if the retailer’s initial capital (\( B \)) is $90 and the portion of profit sharing (\( \lambda \)) is 0.4, the retailer’s optimal decisions are \( q = 20.395 \) and \( e = 13.357 \), respectively. The retailer’s expected profit is $1.985 (less than $10.593) and the profit of the SC is $21.583 (less than $26.482). This result shows that when the retailer has insufficient capital and without trade credit financing, the quantity-discount contract cannot lead to the maximum profit for the SC and arbitrarily allocate the profit.
Table 1. The retailer’s decision variables for each capital level and profit allocations without trade credit.

<table>
<thead>
<tr>
<th>λ</th>
<th>BS</th>
<th>wS</th>
<th>λ(q)</th>
<th>λ(t)</th>
<th>λ(s)</th>
<th>q</th>
<th>e</th>
</tr>
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<td>0.3</td>
<td>40</td>
<td>3.232</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.4</td>
<td>40</td>
<td>3.199</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.5</td>
<td>40</td>
<td>3.166</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.6</td>
<td>40</td>
<td>3.134</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.7</td>
<td>40</td>
<td>3.103</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

6.3. The retailer uses trade credit

When the retailer adopts the trade credit, the supplier can add additional variables including interest rate of trade credit or risk compensation to the payment terms.

6.3.1. Coordination analysis when the interest rate of trade credit is constant

When the interest rate of trade credit \( r \) is constant, a combination of quantity-discount contract and trade credit may not lead to the maximum profit for the SC. For example, if retailer’s initial capital is $110 and the profit allocation is 0.3, Panel A of Figure 1 shows that the lower the trade credit rate is, the higher the retailer’s profit will be. As the supplier increases the interest rate of trade credit, both the retailer’s order quantity \( q \) and effort level \( e \) decrease in \( r \) (as we showed in Panel B of Figure 1). As a result, the profit of the retailer decreases. For a threshold of between 0.175 and 0.2, the retailers will not use trade credit because the use of trade credit brings in lower profits than using their entire capital. In addition, we see that the profit of the SC is always less than that of the centralised CSE. This also implies that the supplier cannot coordinate SC when the interest rate of trade credit is constant.

6.3.2. Coordination analysis when the interest rate of trade credit is competitively priced

With Theorem 1, the supplier can determine the interest rate of trade credit by using Equation (6b) so that the retailer makes the optimal decisions equal to the optimal solutions in the centralised case and arbitrarily allocates...
In this case, the interest rate of trade credit will depend on the retailer’s default threshold at the risk-free interest rate, $\bar{y}$. By solving Equation (6b), we have $r(\bar{y}) = -1 - \frac{2.53\log[1 - 0.4\lambda]}{0.522}$. Column 4 in Table 2 represents the value of the interest rate that the supplier applies to the trade credit when the retailer replicates the optimal solutions in the centralized case. For example, if $B = 90$, $\lambda = 0.4$, $q^* = q_r = 39.88$ and $e^* = e_r = 23.014$, then $\bar{y} = (3.199 * 39.88 + 7.945 + 0.05 * 23.014^2 - 90)/(6 * 23.014) = 0.522$, the interest rate of trade credit will be $r(\bar{y}) = 0.122$ and the retailer’s default threshold is $y_c = \bar{y}(1 + r(\bar{y})) = 0.522 * (1 + 0.122) = 0.585$ respectively (column 8 of Table 2 represents the value of $y_c$). At these values, the retailer’s expected profit is calculated by Equation (4), namely, $6 * 23.014 \int_{y_c}^{39.88/23.014} \exp[-0.4\lambda]d\xi - 90 = 10.593$ (equivalent to 40% of $\Pi_e$) and the supplier’s expected profit is calculated by Equation (5), namely, $6 * 23.014 \int_{y_c}^{0} \exp[-0.4\lambda]d\xi + 90 - 0.05 * 23.014^2 - 39.88 * 3 = 15.889$ (equivalent to 60% of $\Pi_e$). The profit of the SC will be $26.482$. Thus, the quantity-discount contract achieves SCC in the case where the interest of trade credit is competitively priced.

6.3.3. Coordination analysis based on risk compensation

With Corollary 1, the supplier coordinates the SC by adding risk compensation $T = T(q,e)$ to the additional payment $A(q)$ (the variable price component). The risk compensation is calculated by solving Equation: $T(q,e) = \frac{e^0 r(\bar{y})}{\bar{y}}$. For example, if $B = 90$, $\lambda = 0.5$, the risk compensation depends on the retailer’s decision variables $q$ and $e$ as follows:

$$T(q,e) = -\frac{264.028}{q} - 3.166q - 0.05e^2 + 15e\log\left[-\frac{300q\exp(6/e)}{5280.57 + 63.320q^2 + q(e^2 - 1800 - 300e)}\right],$$

which is applied when the retailer’s default threshold is greater than zero, i.e. $3.166q + 264.019q + 0.05e^2 - 90 > 0$. The total payment under trade credit respectively is

$$A(q,e) = A(q) + T(q,e) = 90 - 3.166q - 0.05e^2 - 15e\log\left[\frac{6 - 17.6q - 0.211q + e - 0.0033e^2}{e}\right].$$

In the coordination state, the value of risk compensation and the total additional payment are $8.070$ and $14.691$, respectively. At these values, the retailer’s expected profit and the supplier’s expected profit will be $13.241$ (equivalent to 50% of $\Pi_e$) and $13.241$ (equivalent to 50% of $\Pi_e$) as calculated by $6 * 23.014 \int_{y_c}^{39.88/23.014} \exp[-0.4\lambda]d\xi - 90$, and $6 * 23.014 \int_{y_c}^{0} \exp[-0.4\lambda]d\xi + 90 - 0.05 * 23.014^2 - 39.88 * 3$, respectively, where $y_c = (3.166 * 39.88 + 14.691 + 0.05 * 23.014^2 - 90)/(6 * 23.0140) = 0.561$. The profit of the SC will be $26.482$. In columns 5 and 6 of Table 2, we present the value of the risk compensation and the total additional payment with respect to $B$ and $\lambda$ when the retailer replicates the optimal decisions of the CSCE. We can also use Proposition 2 to show
that the value of risk compensation in the state of coordination is satisfied $(q_c, e_c) = L_c(r(y) - r_f)/(1 + r_f) = L_c r(y) = (172.603 - 26.481 - B)r(y)$.

7. Conclusions

In this paper, we studied the supply chain contracting and coordination issues under capital constraints and promotional efforts. We consider the setting with a supplier and a newsvendor – retailer in a complex situation: the retailer needs to exert costly promotional efforts to increase the market demand but has an insufficient budget and no access to bank financing while the supplier has adequate funds to offer trade credit to the retailer without borrowing from external sources. With trade credit financing, the retailer can delay her payment and/or borrow fund from the supplier to support their decisions and goes into default if she fails to repay the full trade credit. These settings in this paper represent a more realistic business practices for firms dealing with supply chain coordination (SCC) issues for the different decisions impacting the supply chain performance including not only the operational choices (order quantity) and marketing strategy (promotional efforts) of the firms, but also their financial decisions.

Under these more realistic assumptions, we show that coordinating contracts in prior literature do not fully coordinate the supply chain without trade credit financing. Therefore, we present trade credit as being an instrument for supply chain coordination. However, the existing coordinating contracts are valid under trade credit if the interest of trade credit is competitively priced. In contrast, if this condition is not satisfied, these contracts need adjustment by adding a risk compensation to the payment terms to achieve coordination. Therefore, we proposed a generalised contract based on risk compensation to coordinate the decentralised CCSCE. We have shown that if risk compensation depends only on level of promotional efforts, this contract fails to coordinate the decentralised CCSCE. Finally, we point out that the coordinating contracts of the decentralised CCSCE can exist under some necessary and sufficient conditions of the payment and propose a specific contract that achieves coordination in the decentralised CCSCE based on a combination of quantity-discount contract and a risk compensation under trade credit.

Therefore, we contribute to the literature in many ways. First, our model permits to examine the interaction between marketing strategy with operational and financial decisions in the area of SCC. In other word, we address the SCC issues under the interface of operations, marketing and finance. Second, our results demonstrate the role of trade credit in coordinating a decentralised CCSCE. This result, along with existent studies (e.g. Lee and Rhee 2011; Yan and Zaric 2016), can enhance our understanding of the contracts that coordinate the promotional efforts with retailer’s capital constraint. Third and finally, our proposed coordinating contract can perfectly coordinate the decentralised CCSCE with flexible profit allocation regardless of the initial capital level of the retailer.

From managerial insights, the use of trade credit benefits both the supplier and the retailer. However, in the decision of providing trade credit, it is critical for the suppliers to charge a properly risk compensation to the default risk at the retailer. Therefore, the trade credit terms should be based on the retailer’s initial payment, order quantity and the level of promotional efforts that the retailer commits to perform in sale period. Since the decisions of a firm on financial, operational and marketing strategies are usually decided by different departments within an organisation (Karray and Surti 2016), our results suggest that the managers should integrate information flows between these departments in making decisions.

For future study, our model may be used to examine this coordination during bankruptcy and in the case where the supplier faces also capital constraints. Indeed, we are interested in understanding whether the coordination can exist under these situations and how to modify the existing coordinating contracts to reach this coordination. Another extension is to consider the case of supplier’s efforts. For example, a large brand-name supplier can invest in Corporate Social Responsibility activities to improve consumer perception and increase market demand. Therefore, it might be interesting to examine the SCC issues when both parties exert efforts under capital constraints.

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Disclosure statement

No potential conflict of interest was reported by the authors.

Q8
Notes
3. Law No. 2015-990 of 6 August 2015 on growth, activity and equal economic opportunities.

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