Mapping Regional Traditions in Chinese Astronomy and Mathematics, 311–618 CE

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Abstract The period of disunion from 311 to 589 CE saw the territories of the former Han Empire (206 BCE–220 CE) carved up into as many as twelve contemporaneous states ruled by a tumultuous succession of some forty different bloodlines, the majority of which were 'barbarian' in origin. As it happens, this was also one of the most fruitful periods in the history of Chinese-language astronomy and mathematics. Experts were divided, working on the same problems in rival capitals, increasingly disconnected in written and oral tradition except as punctuated by violent redistributions of human and material resources by invading armies. If ever there were a place and time to go looking for 'different mathematical cultures' in early imperial China (Chemla 2009; 2016; 2017a; 2018; Zhu Yiwen 2016), these 278 years are it. Catering to this particular mission of the SAW Project, this paper will break the history of astronomy and mathematics in this period into that of four distinct regional networks, between which we can effectively divide more than a dozen received texts and what we know of many more that have not survived in full. Grounding our sources in their immediate geopolitical and interpersonal context, this paper will argue that the dividing lines between regional traditions is often stronger than those between genres of mathematics within *li* 曆 and *suan* 筭.

Question and Methodology

Given the topic and the context of this conference, I thought it appropriate to dedicate my paper to the following question: Were it my goal to find and prove the existence of different mathematical cultures in early imperial China, how would I do that? The question is essentially Karine's, and the answer is inspired from Christine and John's work on regional sources in Cuneiform, so allow me to begin with a brief review of the literature.

In her first article on the topic, Chemla (2009) introduces the concept of 'mathematical culture' as a category by which to group and juxtapose sources for *suan* attesting to 'different ways of doing mathematics'. As a category, a 'culture' is something that at once *transcends* the individual author and individual work and *divides* what our historical subjects indiscriminately label *suan*. As an *observer's category*, moreover, it is for the modern author to define, which Chemla does by

[*] I thank the organisers of this conference for their kind invitation, and I would also like to emphasise that this PDF is a *speech* presenting the *preliminary results* of a work in progress, so please forgive me if it is light on citations and heavy on mistakes.

choosing nine criteria constituent of a 'mathematical culture' and, thus, sufficient to distinguish two texts self-identifying as *suan* as products of 'different cultures':[1]

> Au nombre des composantes qui me semblent pertinentes pour saisir la spécificité d'une culture et qui requièrent par suite toute notre attention, j'identifie les éléments suivants : [1] les *Problèmes*, c'est-à-dire les situations et les valeurs numériques que leurs énoncés impliquent ; [2] les *Nombres*, qui font l'objet de « procédures » ; [3] les *Procédures*, que, dans une terminologie technique moderne, on désigne du terme d'« algorithmes » ; [4] les Types de texte, comme nous l'avons déjà évoqué avec les « Classiques » et les « commentaires » ; [5] les *Instruments de calcul* ; [6] les *Figures* ; [7] les *Démonstrations* ; [8] les *Valeurs épistémologiques* ; [9] les *Types d'institutions* et de *groupes sociaux*, de *profils individuels*, etc. (Chemla 2009: 108).

Of these, Karine immediately eliminates '[9] institutions, social groups, and individual profiles' from consideration, citing that:

> Pour ce qui est de cette dernière dimension, les documents nous font défaut pour permettre un examen détaillé du contexte et de l'organisation sociale propre à l'activité mathématique en Chine ancienne, et nous ne pourrons donc pas approfondir cet aspect pourtant essentiel de toute culture au sens où je l'entends (ibid.).

In this, her 2009 paper, and a host of talks and articles to follow (Chemla 2016; 2017a; 2017b; 2018), Karine then proceeds to group and juxtapose two sets of sources within *suan* mathematics according to their differences and commonalities vis-à-vis the first eight criteria on her list. In a similar vein, Zhu Yiwen (2016) and Chemla and Zhu (forthcoming) expand the scope of analysis to include *suan* mathematics as appear in Kong Yingda (574–648 CE) and Jia Gongyan's (fl. 637 CE) subcommentaries to the Confucian Classics. The results of this effort are summarised in Table 1.

Karine and Yiwen can obviously speak for their own results, so let me stop at that and outline how I would work within this framework to meet the same objectives given my own particular interests, training, and experience. In a nutshell, there are four directions in which I would take this in terms of sources and methodology.

First, Karine and Yiwen's work on this topic has focused exclusively on *suan*, which is but one branch of the astral and mathematical sciences such as our historical subjects saw fit to divide the tree of knowledge and organized contemporary libraries (see Fig. 1). I would look at *all the branches.* There are three reasons for this. One, to different degrees, all of these other fields involve mathematics. Two, the best place to find evidence of 'different mathematical cultures' is probably *across* actor-defined categories of mathematics (conversely, any such differences

[1] Note that I will be placing scare quotes around the word 'culture' and the idea of 'different mathematical cultures' throughout to remind the reader that these are observer's categories originating, in this study, from the definitions given them in Chemla (2009). I do this so as to avoid equivocation and, also, so as to avoid slipping into speaking of them as actors' categories, as sometimes happens even to the best of us: 'cela signifie que les acteurs du VII^{e} siècle les considéraient de fait comme associés à une même culture mathématique' (Chemla 2016: 18); 'I interpret the fact that these canons… were taught in the same curriculum at the time, as evidence that for 7th century actors they could be considered as related to the same mathematical culture' (Chemla 2018: 5).

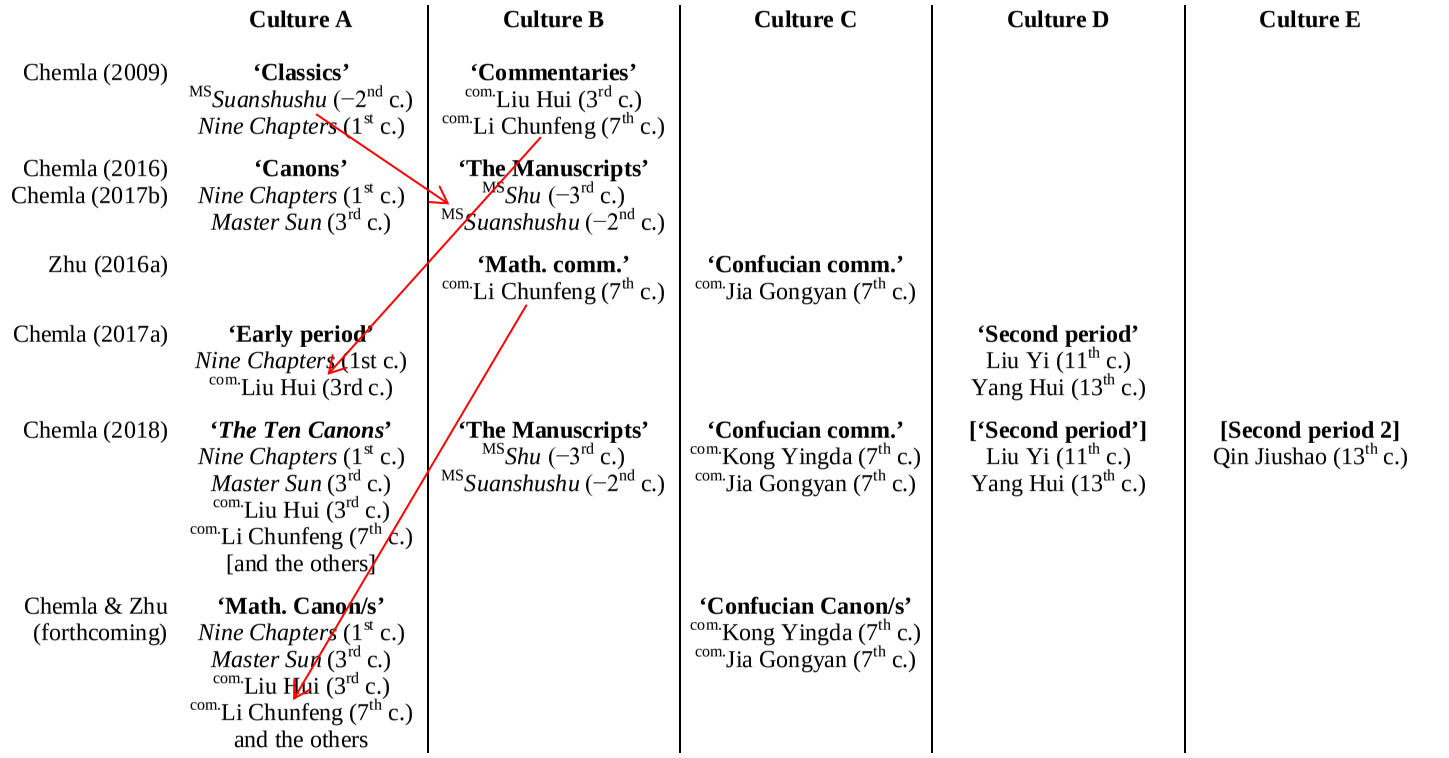

Table 1 ‘Different mathematical cultures’ as delineated by Karine Chemla and Zhu Yiwen. Note that Chemla transitions from ‘classic’ to ‘canon’ in her translation of *jing* 經 after 2009, which she uses to designate both a single classical work and collection formed thereof. As concerns the red arrows, one notes that Chemla (2009) focuses ‘aux *Neuf Chapitres*, en mentionnant parfois le *Livre de procédures mathématiques* [*Suanshushu*], qui à mon sens adhérait à la même culture’ (108) as distinct from ‘la pratique de démonstration dont témoignent les commentaires [of Liu Hui and Li Chunfeng] et la culture à laquelle elle adhère’ (127), which Chemla and Zhu later reclassify via the same criteria.

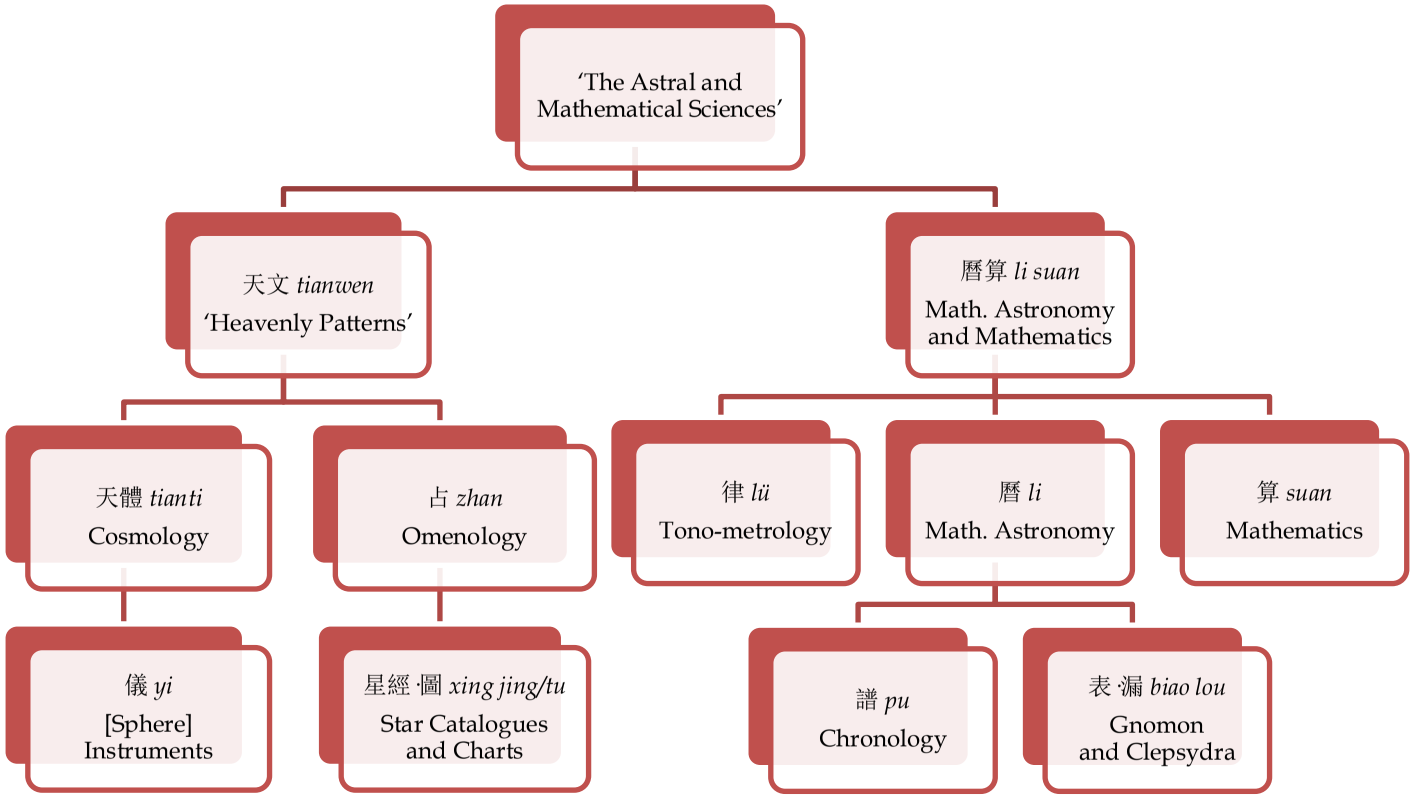

Fig. 1 Fields and subfields of 'the astral and mathematical sciences' as organised in imperial library catalogues, historical monographs (*zhi* 志), etc., first century BCE to seventh century CE.

would provide us a control against which to assess differences *within* actor-defined categories). Third, these fields are interrelated, and the majority of people we know to have been active in *one* were active in *several*.

Second, in Table 2 you will the titles filed under the classifications 'Heavenly Patterns' (Tianwen) and 'Astronomy and Numbers' (Li shu) in the catalogue of the Sui (581–618 CE) imperial library. In blue are the works of *suan* mathematics that have come down to us through the written tradition, such as feature in Karine and Yiwen's work on the subject of 'mathematical cultures'. As you can see, these six sources constitute but a fraction of the literary output in these fields up to the seventh century CE. Now, if we extend our analysis to include the *other* astral and mathematical sciences, in green, then we have more than four times as many to work with in this particular sample. Much more importantly, if we include *lost works* for which we have authors and/or dates, in orange, then we are looking at a completely different scale of analysis. Now, it is true that we cannot apply Karine's first eight criteria to an analysis of their *contents*, but many of these authors have *biographies*, have authored other, extant works, and feature here and there in historical writing. And what we are looking at in Table 2 is just *one catalogue*: if you collate lost titles across *all* early imperial sources, you get *hundreds* of works with authors and/or dates.

Third, if we are looking for 'different mathematical cultures', and if we can localise hundreds of authored works in time and space, then it strikes me that we had best choose a *really messy period* to look at – one where continental China is divided into multiple warring courts. The messiest in this regard is probably the period from 317 to 618 CE, known as the 'Five Barbarians and Sixteen Kingdoms' and the 'Northern and Southern Dynasties' (see Fig. 2). Think about it: if there were expert communities working on these subjects in isolation from one another in a half dozen different cities over these three hundred years, half of them governed and heavily populated by non-Chinese, then it is here where one would most expect to find some cultural differences in the way that people are doing mathematics.

Fourth is *people*. It's true that we cannot always go into the biographical detail that we would like in ancient history, but we have more documentary evidence for institutions, social groups, and individual profiles in Chinese sources than probably any other written tradition prior to modern times. Modern scholars have made particularly fruitful use of these materials as concerns 'astronomy' (*tianwen* and *li*) and *lü* tono-metrology.[2] Admittedly, we do know significantly less about *suan* mathematics in this regard: many of the authors of extant works are basically unknown, and, sadly, *suan* is one of the two branches of the astral and mathematical sciences not to have histories written about it from the first century BCE on. Still, even in focusing on only the 'great men', histories like Wu Wenjun et al. (1998–2004) find plenty to say. An excellent example of how much is there and what can be done with it is Christopher Cullen's chapter in the 2009 *Oxford Handbook of the History of Mathematics*, entitled 'People and Numbers'. Stepping out of the typical focus on

[2] On the history of astronomical institutions covering the period in question, see Deane (1989) and Chen & Zhang (2008). On social groups, see for example Cullen (2007) and other politics-focused studies like Yabuuti (1969), Hasebe (1991), Kawahara (1996), Chen Meidong (2007), Cullen (2017a), or Morgan (2017a). On individual profiles, see De Crespigny (1976), Chen Meidong (2003), Chen Jiujin (2008), Xu Jie (1999), Xu Xingwu (2005), and Lien (2011). In tono-metrology, see Goodman (2010) and Hegesh (2018: 120–169).

天文 ‘Heavenly Patterns’		
周髀一卷趙嬰注。	天文十二卷史崇注。	荊州占二十卷宋通直郎劉巖撰。
周髀一卷甄鸞重述。	天文十二次圖一卷	翼氏占風一卷
周髀圖一卷	婆羅門天文經二十一卷婆羅門捨仙人	日月暈三卷梁日月暈圖二卷。
靈憲一卷張衡撰。	婆羅門竭伽仙人天文說三十卷	孝經內記二卷
渾天象注一卷吳散騎常侍王蕃撰。	婆羅門天文一卷	京氏釋五星災異傳一卷
渾天義二卷	陳卓四方宿占一卷	京氏日占圖三卷
渾天圖一卷石氏	黃帝五星占一卷	夏氏日旁氣一卷許氏撰。
渾天圖一卷	五星占一卷丁巡撰。	日食雨候占一卷。
渾天圖記一卷	五星占一卷	魏氏日旁氣圖一卷
昕天論一卷，姚信撰；	五星占一卷陳卓撰。	日旁雲氣圖五卷
安天論六卷，虞喜撰；	星占二十八卷孫僧化等撰。	天文占雲氣圖一卷
圖天圖一卷，	星占一卷	雜望氣經八卷，
原天論一卷，	石氏星經七卷，陳卓記；	候氣占一卷，
神光內抄一卷。	石氏星官十九卷，	章賢十二時雲氣圖二卷。
定天論三卷	星經七卷，郭歷撰。	天文洪範日月變一卷
天儀說要一卷陶弘景撰。	天官星占十卷陳卓撰。	洪範占二卷
玄圖一卷	天官星占二十卷，吳襲撰。	洪範五行星曆四卷。
石氏星簿經讚一卷	星占八卷	黃道暑景占一卷
星經二卷	有星占十八卷。	暑景記二卷。
甘氏四七法一卷	中星經簿十五卷	日行黃道圖一卷
巫咸五星占一卷	星官簿贊十三卷，	日月交會圖鄭玄注一卷，
天儀說要一卷陶弘景撰。	星書三十四卷，	日月本次位圖二卷。
錄軌象以頌其章一卷內有圖。	雜家星占六卷，	月暈占一卷
天文要集四十卷晉太史令韓楊撰。	論星一卷	日月食暈占四卷
天文集占十卷晉太史令陳卓定。	著明集十卷	日食占一卷
天文要集四卷	雜星圖五卷	日月薄蝕圖一卷
天文要集三卷	天文外官占八卷	日變異食占一卷
天文集占十卷。	雜星占七卷	日月暈珥雲氣圖占一卷
石氏天文占八卷。	雜星占十卷	君失政大雲雨日月占二卷。
甘氏天文占八卷。	海中星占一卷	二十八宿十二次一卷
天文占六卷李暹撰。	論星一卷。	二十八宿分野圖一卷
天文占一卷	星圖海中占一卷	五緯合雜一卷
天文占氣書一卷	解天命星宿要決一卷	五星合雜說一卷
天文集要鈔二卷	摩登伽經說星圖一卷	垂象志一百四十八卷
天文書二卷	星圖二卷	太史注記六卷
雜天文書二十五卷。	星書圖七卷。	靈臺祕苑一百一十五卷太季才撰。
雜天文慣占一卷	彗星占一卷	
天文橫圖一卷高文洪撰。	妖星流星形名占一卷	
天文集占圖十一卷	太白占一卷	
天文五行圖十二卷，	流星占一卷	
天文雜占十六卷，亡。	石氏星占一卷吳襲撰。	
天文錄三十卷梁奉朝請祖暅撰。	候雲氣一卷	
天文志十二卷吳震撰。	星官次占一卷	
天文志雜占一卷吳震撰。	彗孛占一卷	
天文雜占十五卷，亡。	二十八宿二百八十三官圖一卷	

曆數 ‘Astronomy and Numbers’		
四分曆三卷	新造曆法一卷	雜曆二卷
三統曆法三卷，劉歆撰	開皇甲子元曆一卷	雜曆術一卷
趙爽居四分曆一卷	曆術一卷華州刺史張賓撰。	三基推法一卷。
魏甲子元三統曆一卷	七曜本起三卷後魏甄叔遵撰。	太史注記六卷
姜氏三紀曆一卷	七曜小甲子元曆一卷	太史記注六卷
曆序一卷姜氏撰。	七曜曆術一卷	見行曆一卷
乾象曆三卷吳太子太傅闞澤撰。	七曜曆法四卷。	八家曆一卷
乾象曆五卷，漢會稽都尉劉洪等注；	七曜要術一卷	漏刻經一卷何承天撰。
闞澤注五卷，	七曜曆法一卷	霍融、何承天、楊偉等撰三卷
乾象五星幻術一卷。	推七曜曆一卷	漏刻經一卷祖暅撰。
曆術一卷吳太史令吳範撰。	五星曆術一卷	漏刻經一卷梁中書舍人朱史撰。
景初曆三卷晉楊偉撰。	天圖曆術一卷	漏刻經一卷梁代撰。
景初曆術二卷，	陳永定七曜曆四卷	天監五年修漏刻事一卷
景初曆法三卷，	陳天嘉七曜曆七卷	漏刻經一卷陳太史令宋景撰。
景初曆略要二卷。	陳天康二年七曜曆一卷	雜漏刻法十一卷皇甫洪澤撰。
景初王辰元曆一卷楊冲撰。	陳光大元年七曜曆二卷	晷漏經一卷
正曆四卷晉太常劉智撰。	陳光大二年七曜曆一卷	九章術義序一卷
河西甲寅元曆一卷凉太史趙歐撰。	陳太建年七曜曆十三卷	九章算術十卷劉徽撰。
甲寅元曆序一卷趙歐撰。	陳至德年七曜曆二卷	九章算術二卷徐岳、甄鸞重述。
宋元嘉曆二卷何承天撰。	陳禎明年七曜曆二卷	九章算術一卷李遵義疏。
元嘉曆統二卷，	開皇七曜年曆一卷	九九算術二卷楊淑撰。
元嘉中論曆事六卷，	仁壽二年七曜曆一卷	九章別術二卷
元嘉曆疏一卷，	七曜曆經四卷張賓撰。	九章算經二十九卷徐岳、甄鸞等撰。
元嘉二十六年度日景數一卷	春秋去交分曆一卷	九章算經二卷徐岳注。
曆術一卷何承天撰。	曆日義說一卷	九章六曹算經一卷
駁日食法三卷，何承天撰；	律曆注解一卷	九章重差圖一卷劉徽撰。
論頻月合朔法五卷，	龍曆草一卷	九章推圖經法一卷張峻撰。
雜曆七卷，	推漢書律曆志術一卷	綴術六卷
曆法集十卷，	推曆法一卷崔陽居撰。	孫子算經二卷
曆術十卷；	曆疑質論序二卷	趙爽算經一卷
京氏要集曆術四卷，姜岌撰。	興和曆疏二卷	夏侯陽算經二卷
曆術一卷崔浩撰。	七曜曆數算經一卷趙歐撰。	張丘建算經二卷
神龜王子元曆一卷後魏蠕軍將軍祖瑩撰。	算元嘉曆術一卷	五經算術錄遺一卷
魏後元年甲子曆一卷	七曜曆疏一卷李業興撰。	五經算術一卷
王子元曆一卷後魏校書郎李業興撰。	七曜義疏一卷李業興撰。	算經異義一卷張積撰。
甲寅元曆序一卷趙歐撰。	七曜術算二卷甄鸞撰。	張去斤算疏一卷
魏武定曆一卷	七曜曆疏五卷太史令張胄玄撰。	算法一卷
癸甲子元曆一卷宋氏撰。	陰陽曆術一卷趙歐撰。	黃鍾算法三十八卷
宋景業曆一卷景業，後齊散騎常侍。	朔氣長曆二卷，皇甫謐撰；	算律呂法一卷
周天和年曆一卷甄鸞撰。	曆章句二卷，	眾家算陰陽法一卷
甲子元曆一卷李業興撰。	月令七十二候一卷，	婆羅門算法三卷
周大象年曆一卷王琛撰。	三五曆說圖一卷。亡。	婆羅門陰陽算曆一卷
曆術一卷王琛撰。	雜注一卷	婆羅門算經三卷
王辰元曆一卷	曆注一卷	
甲午紀曆術一卷	曆記一卷	

Table 2 Works in the astral and mathematical sciences as recorded in the catalogue of the Sui (581–618 CE) imperial library. Blue: extant works of *suan* mathematics. Green: other works extant in whole or in important part through citation. Orange: works with author and/or date.

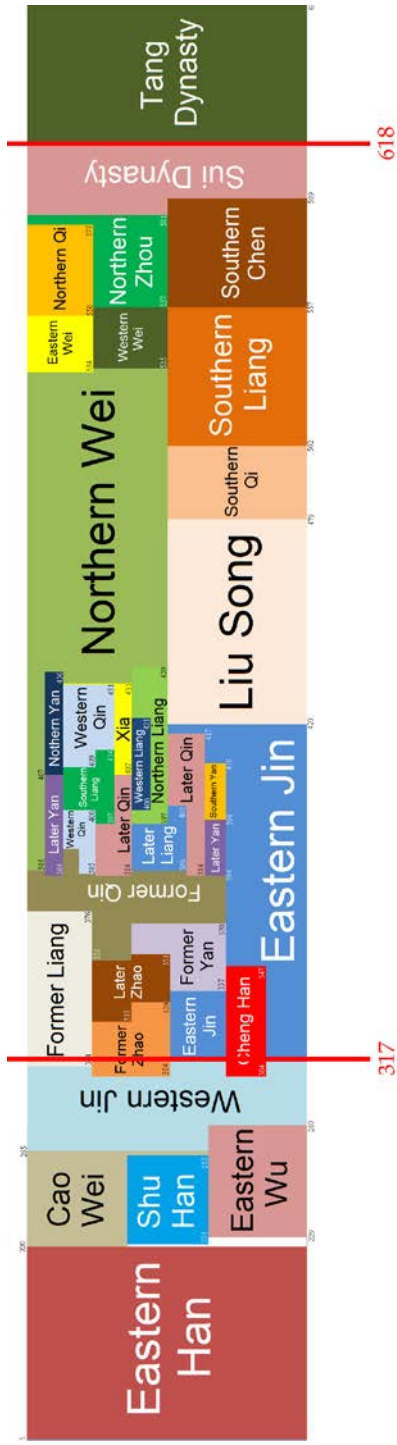

Fig. 2 The 'Messy Period'. Image modified from Alvin Lee (CC BY).

big names and the authors of 'mathematical classics' (*suanjing*), Cullen (2009) makes a list of people mentioned as being 'good at', learning, or teaching *suan* mathematics under the Han (206 BCE–220 CE). The list comes out to twenty-two people, and he outlines the role that mathematics played in their education, associations, writing, public service, and career trajectories.[3]

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Cullen has already done this for the Han, and it is easy enough to reproduce, so, out of curiosity, I went ahead and did the same thing for 'the Messy Period' (317–618 CE) over the course of about ten days.

More specifically, I ran an exhaustive database search of all received texts in the lay and Buddhist canon available through CBETA, the Chinese Text Project, Kanripo, and Scripta Sinica looking for: (1) authors and co-authors of written works – extant *or* lost – in any of the astral and mathematical sciences (hereafter AMS); (2) authors of *unwritten work* in observation, experiment, testing, instrument-building, or theory in the AMS; (3) meaningful contributors to public debate in the AMS; (4) people identified as having *learned*, *taught*, or been '*good at*' either a subject or specific work within the AMS; (5) people holding specialist posts in the AMS, e.g. professor of tono-metrology or student of mathematics (below).

Once I knew *who* I was looking for, I then ran a full database search on each of their names, gathering every single occurrence thereof in the historical record into individual files. Having thus circumscribed everything that there is to know about a given person, I then sorted all appearances chronologically and, *where available*, synthesised this with the individual's personal timeline as outlined in his official biography. By way of caveat, I got *way too many results*, so I decided to eliminate *zhan* omenology from consideration, which basically cut things in half. Also, in the case of particularly well-known figures with ample biographies, I did not always bother supplementing the latter with all further historical mentions of their names if the hit count was above fifty.

In the end, looking at practitioners of only the most mathematical of the astral and mathematical sciences over the three centuries from 317 to 618 CE, I got 224 names, 131 of which have biographies; 227 works for which we have authors and/or dates; and a total of about a thousand pages of primary sources, single spaced, in Word, of purely biographical information. What kind of information?

[3] Another excellent example, for that matter, is the survey of individual profiles involved in the redaction and reception history of the *Nine Chapters on Mathematical Procedures* in Chemla & Guo (2004: 54–70), which cites the lengthy, extant biographies of the mathematicians and/as mathematical astronomers Zhang Cang 張蒼 (253–152 BCE), Zhang Heng 張衡 (78–139), Zheng Xuan 鄭玄 (127–200), Wang Can 王粲 (177–217), Wang Fan 王蕃 (228–266), Zu Chongzhi 祖沖之 (429–500), Xindu Fang 信都芳 (d. 543/550), Yin Shao 殷紹 (fl. 458), Gu Yue 顧越 (480–569), Liu Zhuo 劉焯 (544–610), and Li Chunfeng 李淳風 (602–670). For me, it is difficult to reconcile all the people and biographical sources referenced in Chemla & Guo (2004: 54–70) with the claim, five years later, that 'les documents nous font défaut pour permettre un examen détaillé du contexte et de l'organisation sociale propre à l'activité mathématique en Chine ancienne' as concerns 'les *Types d'institutions* et de *groupes sociaux*, de *profils individuels*, etc.' (Chemla 2009: 108), given that the former was of crucial help in compiling the sources discussed in this talk.

Well, where we have a biography, in particular, we often get a person's dates, place of birth, education, friends, family members, and associates, movements, posts and noble titles held, competencies, hobbies, research activities, and publications, and, sometimes, his height.

I have never worked with this many sources all at once, particularly not for a forty-minute talk, so the only thing that I could think to do was model my approach on how Christine and John have treated the regional division and circulation of written knowledge in Babylon, Nippur, and Uruk.[4] More specifically, I will divide people and works into regional networks – networks localised in time and space, oriented around the capital of a given territory, and whose members and works are clearly in interaction. I will give you a geographical and chronological overview of where and when these networks were active, the conditions of their isolation, and the ways in which they communicated and were consolidated. Lastly, in the Appendix, I've given you a selective bibliography of the period as divided into six major regional networks for you to consult as we go along. This presentation is going to be rough and rapid, but my goal is simply to give you an idea of what we *do know* about institutions, social groups, and individual profiles in the astral and mathematical sciences and to get us thinking about how we might integrate these sources into a discussion of 'mathematical cultures'.

The Fall of Luoyang and the Rise of Hexi, 311–439 CE

Alright, let's get into a little history. In 280 CE, after the disintegration of the Han Empire (206 BCE–220 CE) and the War of the Three Kingdoms (220–280), Emperor Wu of the Western Jin (r. 266–290) manages to reunify the former territories of the Han. Unfortunately, his death ten years later leads to the War of the Eight Princes (291–306). This is a civil war between brothers fighting for control of the regency using mercenary nomad armies, and, after killing each one another for thirteen years in a Chinese civil war, the nomad mercenaries ultimately decide to band together and to turn against their masters. By 317, the nomad coalition manages to split the empire in two and push the Chinese out of their homeland in the Yellow and Wei River Valleys, resulting in one of the greatest exoduses in human history. The Jin (266–420) court moves south with waves of refugees to the untamed Yangtze River Valley, it loses the Sichuan Basin to a millenarian Taoist uprising, and it maintains only brief, nominal control over the Hexi Corridor.

For the next hundred and twenty years, the north is divided by continuous warfare between non-Chinese forces, and the Chinese in the South are desperately busy defending themselves from further incursions amidst difficulties integrating populations, customs, and power structures. North and South, these hundred and twenty years see almost *no activity* in the astral and mathematical sciences excepting the practice of military omenology[5]. Rather, the only place that is reliably producing mathematical writings from this point on is the Hexi Corridor.

[4] E.g. Proust (2007) and Steele (2016).

[5] By my current count, there are a total of nine figures potentially active in these fields in the South, 317–437. Five of these are people who lived through the events of 317, and whose undated involvement in the astral and mathematical sciences may well fall prior to this date. The most historically significant of these are the hermits Yu Xi 虞喜 (fl. 307–346) and Ge

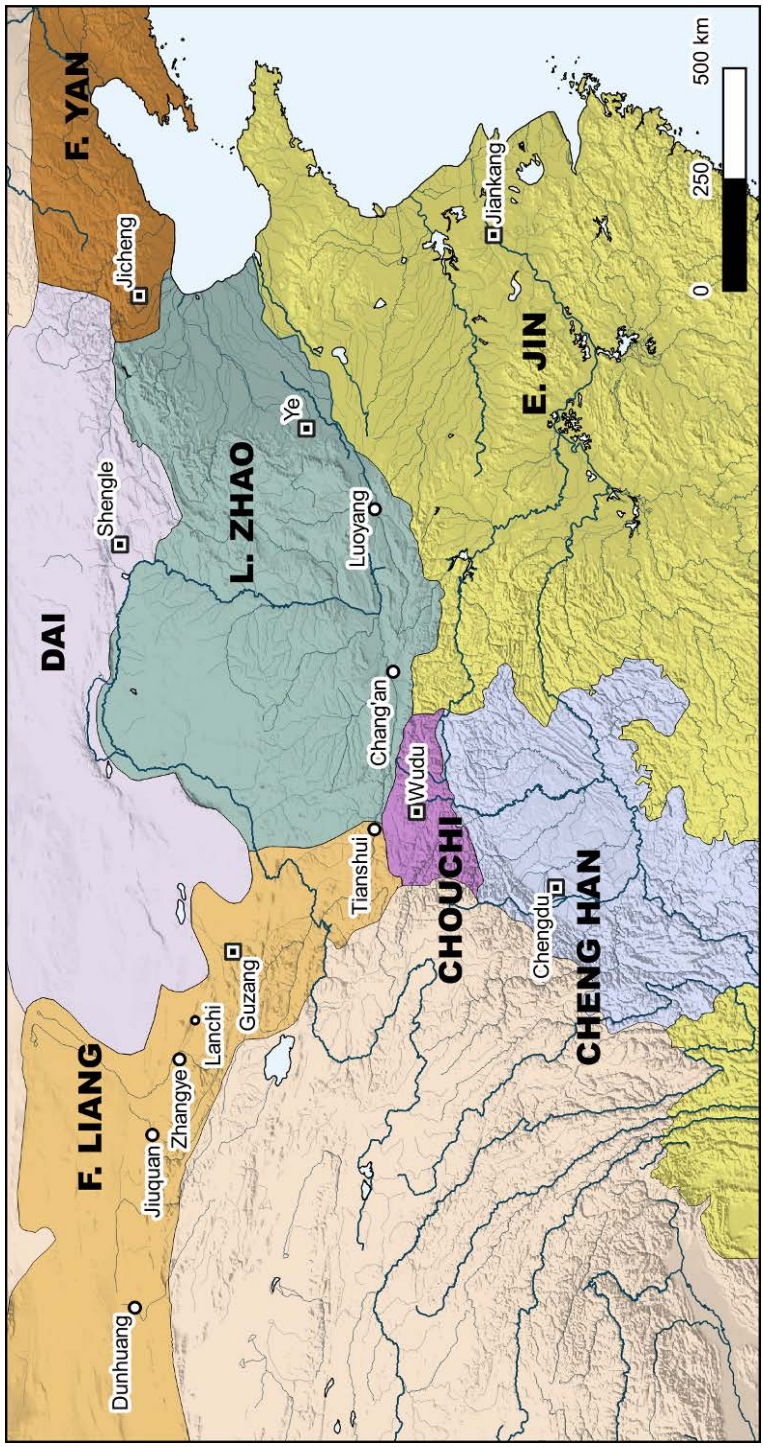

Fig. 3 Political divisions in 338 CE, highlighting key commanderies and districts along the Hexi Corridor (F. Liang).

It is to the Hexi Corridor that we owe the first extant commentary to the *Gnomon of Zhou* (*Zhoubi*), written by Zhao Ying (AKA Zhao Shuang), who appears as a regional administrator in Lanchi in 322.[6] Sixty years later, we have the celebrated astronomer Jiang Ji (fl. 384), who serves the Former Qin (351–394) court at Chang'an during its brief reunification of the North, but who originates from the once Former Liang (320–376) city of Tianshui.[7] And thirty years after that, we have Zhao Fei of Dunhuang (fl. 412), the celebrated mathematician and astronomer royale of the Northern Liang (397–439) court at Guzang (see Appendix for a bibliography of the Hexi Corridor).[8]

These are the three most important mathematicians and astronomers active in this hundred-and-twenty-year span; they all come out of the same region, centred on Dunhuang and Guzang, and, more than that, their work shares a number of peculiar traits in common. Namely, what we know from fragmentary sources indicates that Zhao Fei was working in the tradition of the ancient 'quarter-remainder astronomy' (*sifen li*), which he brought up to date and modified with his own innovations. One tell-tale sign is his use of the *bu* 'obscuration' among his astronomical resonance periods, as recorded in *Kaiyuan zhanjing*, 105.5b, and, as Li Chunfeng (602–670) tells us in his history of the period:

> Coming to the Western Liang, they also worked on the obscuration divisor, but the traces of the matter contradict one another, leaving us unable to record it in detail.[9]

This is peculiar, because the 'quarter-remainder' framework had been abandoned everywhere else in the empire something like a century and a half earlier, the last mention of which is by Du Yu (222–285):

Hong 葛洪 (283–343). Of the other four, there are two monks noted for their skill in *suan* mathematics – Huiyuan 慧遠 (334–416) and Dao'an 道安 (d. 385) – and the other two are of somewhat ambiguous status in the history of mathematical astronomy. First, there is Wang Shuozhi 王朔之, who wrote the quickly forgotten *Universal Astronomy* (*Tongli* 通曆) in 352 CE. Second, there is Xu Guang 徐廣 (352–425), He Chengtian's 何承天 (370–447) uncle, whose work measuring gnomon shadows He Chengtian cites as having informed his movement of the winter solstice sun in 443.

[6] On the *Gnomon of Zhou* and Zhao Ying's commentary, see Cullen (1995; 1996). On Zhao Ying 趙嬰 (AKA Zhao Shuang 趙爽) and the *Gnomon of Zhou*'s connection with the North-West, see Morgan (2018).

[7] On Jiang Ji 姜岌, see Chen Meidong (2003: 252–254) and the entry 'Jiang Ji' in Martin & Chaussende (2019).

[8] On Zhao Fei 趙斐, see Chen Meidong (2003: 254–255) and Wu Wenjun (1998–2004: vol. 4, 106–107).

[9] 至於西涼，亦為蔀法，事迹糾紛，未能詳記, *Sui shu*, 17.416. Divided into Former (320–376 CE), Later (386–403 CE), Southern (397–414 CE), Northern (397–439 CE; 442–460 CE), and Western (400–421 CE) branches, the Liang 涼 was a distant north-western state based in the Hexi or Gansu Corridor. All of these are 'Western' as viewed from the Central States, so it is safe to assume that Li Chunfeng might not be distinguishing between them. The 'obscuration divisor' (*bufa* 蔀法) is a resonance period first used in the Han Quarter-remainder *li* (*Sifen li*) of 85 CE and the quarter-remainder-style *li* treated in the astronomical portion of the *Gnomon of Zhou*, on which see Cullen (1996: esp. 25; 2017b: esp. 153–155).

> Following my discourse on mathematical astronomy, coming to the mid-Xianning period (275–280), expert *suan* mathematicians Li Xiu and Bu Xian made a procedure [text] in accordance with the essence of [my] discourse, calling it the *Supernal Measure Astronomy* and submitting it to the [Western Jin] court. Their procedures accorded with the quarter-remainder numbers for solar motion while slightly increasing [those for] lunar motion, and in accordance the idea of 'a change in constitution every three hundred years', they calculated from two respective [system] origins…[10]

One notes that this is precisely the tradition in which Zhao Ying was also working, both in his commentary to the *Gnomon of Zhou* and, in all likelihood, in the '*Hermit Zhao's Quarter Remainder Astronomy*, 1 roll,' recorded in the Sui imperial library.[11] Moreover, one notes that Jiang Ji likewise experimented with multiple origins a generation or so after the Southern hermit Yu Xi (fl. 307–346) came up with the solution to the precession of the equinoxes that would eventually emerge the historical winner: the 'year difference' (*suicha*).[12]

Another couple of decades later, in 437, the now Xiongnu emperor of the Northern Liang initiates a book exchange with the Chinese emperor of the Liu Song (420–479). He sends the Chinese emperor 19 books in 154 volumes, the majority of which are about the history of the region or written by authors from Dunhuang, and, in return, he requests the court diaries of the previous dynasties to the east. Included in the books sent from Guzang to Jiankang were the *Gnomon of Zhou*, Zhao Fei's biography, and one of Zhao Fei's latest astronomical procedure texts:

> In [437 CE], [Emperor Juqu] Maoqian [of the Northern Liang] (r. 433–439 CE) presented a memorial and a tribute of regional items. Also offered as tribute were the

[10] 余為曆論之後，至咸寧中，善算者李修、卜顯，依論體為術，名乾度曆，表上朝廷。其術合日行四分數而微增月行，用三百歲改憲之意，二元相推, *Jin shu*, 18.564. Prior to this, according to Li Chunfeng, 'The Liu Clan in Shu (221–263) still used the Han *Quarter-remainder Astronomy*' 其劉氏在蜀，仍漢四分曆 (*Jin shu*, 17.503), Liu Hong's 劉洪 (fl. 167–206) *Supernal Icon Astronomy* (*Qianxiang li* 乾象曆) having become 'the exemplar of calculation for the subsequent age' 洪術為後代推步之師表 (ibid.) everywhere else. For a study and translation of the *Quarter-remainder* and *Supernal Icon*, see Cullen (2017b).

[11] 趙隱居四分曆一卷, *Sui shu*, 34.1022.

[12] An 'origin' (*yuan* 元) is a hypothetical zero-point in the far past at which all calendrical and astronomical phenomena are aligned. According to Li Chunfeng's description in *Jin shu*, 18.570, Jiang Ji used a 'a simplified method for the planets that began with first [morning] visibility and that was not tied to the origin itself. As such, his [normal] calculations ran until the beginning of [the lower?] origin, and he extended his simplified method for modern use. He bent [the rules] in search of something that would work, and it was [indeed] suitable for each [period, ancient and modern], thus did the author set out his two methods separately' 五星約法，據出見以為正，不繫於元本。然則算步究於元初，約法施於今用，曲求其趣，則各有宜，故作者兩設其法也. Already mentioned in Note 5, Yu Xi is one of the more important figures in the astral and mathematical sciences active in the Chinese south over the period 317–437 CE. Yu Xi was a hermit and a classical commentator whose two main claims to fame in the history of science were a work on cosmology roundly declaimed as nonsense from the fifth century on (*Antian lun* 安天論) and the proposition that a difference between the length of the tropical and sidereal year might explain the difference between those stars reported as culminating at dawn and dusk at the solstices and equinoxes in the ancient classics compared to those of the fourth century CE. On Yu Xi, see Chen Meidong (2003: 241–242, 251) and the entry 'Yu Xi' in Martin & Chaussende (2019).

> *Zhoushengzi* in 13 rolls, *On Seasonal Duties* in 12 rolls, *A Comprehensive Summary of the Three Kingdoms* in 20 rolls, *Questions on Customs* in 11 rolls, *Treatise on the Thirteen Provinces* in 10 rolls, *Literary Examinations* in 6 rolls, *Biography of the Four Classifications [of Writing]* in 4 rolls, *Veritable Records of Dunhuang* in 10 rolls, *The History of Liang* in 10 rolls, *Biography of Han Imperial Virtue* in 25 rolls, *Lost Laws* in 7 rolls, *Wei Arguments* (?) in 9 rolls, *The Collected Works of Xie Ai* in 8 rolls, *Ancient and Modern Characters* in 2 rolls, *The Mulberrymound Sensei* in 3 rolls, ***The Gnomon of Zhou* in 1 roll**, *Annals of the Three Unities of the Sequence of Emperors and Kings* (?) in 1 roll, the ***Biography of Zhao Fei*** and [his] ***Jiayin-origin li*** in 1 roll and *Praises of Master Kong* in 1 roll, for a total of 154 rolls. [Emperor Juqu] Maoqian also requested the court diaries of the Jin (265–420 CE) and Zhao (318–351 CE) and a various selection of several dozen other books, which Emperor Taizu [of the Liu-Song] (r. 424–453 CE) granted him.[13]

Two years after that, in 439, the Northern Wei (386–535) then conquer the region, reunifying the North and moving all human and library resources out of Guzang to their own capital at Pingcheng. At Pingcheng, these newly acquired resources are placed under the supervision of the scholar Cui Hao of Qinghe (d. 450), who is charged with their *integration*, notably in the framework of collective projects to produce compendia and a unified history of the previous period.[14]

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For a hundred and twenty years up to this point, the North and South have essentially gone dark as concerns mathematics and mathematical astronomy. Within a span of two years, however, both get an injection of books from the Hexi Corridor, and, *immediately*, there is an explosion of activity in Pingcheng (N) and Jiankang (S).

In **the North**, the first new author in these fields in more than a century is the same Cui Hao, who immediately begins writing in mathematical astronomy and attempts his own reform. Instead, the court simply opts for a wholesale adoption of Zhao Fei's recently acquired work:

> When [Emperor] Shizu (r. 408–452) stepped up to reign, he immediately pacified the various [northern barbarians] and then ordered the Duke of Dong Commandery and former minister of works, Cui Hao, to comb through and synthesise their numbers. Hao was broadly steeped and deeply conversant [in all things, so] he reworked [the/their] astronomical procedures and authored *On Five Agents [Metaphysics]*. … Even so, Hao and company's research was incomplete and inaccurate, [so] when

[13] 【元嘉】十四年，茂虔奉表獻方物，并獻『周生子』十三卷，『時務論』十二卷，『三國總略』二十卷，『俗問』十一卷，『十三州志』十卷，『文檢』六卷，『四科傳』四卷，『燉煌實錄』十卷，『涼書』十卷，『漢皇德傳』二十五卷，『亡典』七卷，『魏駮』九卷，『謝艾集』八卷，『古今字』二卷，『（乘）〔桑□丘先生』三卷，『周髀』一卷，『皇帝王歷三合紀』一卷，『趙畋傳』并『甲寅元歷』一卷，『孔子讚』一卷，合一百五十四卷。茂虔又求晉、趙起居注諸雜書數十件，太祖賜之, *Song shu*, 96.2416. As presented from the Northern Liang perspective, see *Shiliuguo chunqiu*, 95.4a–b.

[14] On the 437 book exchange, the position of Dunhuang as a bastion of Han culture, and the role of Cui Hao in integrating the human and library resources of Guzang with those in Pingcheng, see Trombert (2005).

> Gaozong (440–465) took the throne, [the court] then [decided to] use Zhao Fei of Dunhuang's *Jiayin*$_{51}$ [origin] astronomy.[15]

What is more, two further commentaries to the *Gnomon of Zhou* come out of the North over the following period, including Zhen Luan's subcommentary to Zhao Ying's commentary (below).[16] And everything that survives of northern mathematical astronomy over the same period points to the continued use of Zhao Fei's modified 'quarter-remainder' framework as well as, if any, the 'double origin' solution to the precession of the equinoxes.[17]

In **the South**, by contrast, experts ignore the *Gnomon of Zhou*, and they reject the 'double origin' in favour of Yu Xi's 'year-difference' (*suicha*). That said, they do adapt one of Zhao Fei's tell-tale innovations to their own framework for astronomical resonance periods: the idea of 'breaking the rule' to move from an intercalary period of 235 months:19 years to one of 4836 months:391 years. In the South, no one talks about Zhao Fei, and credit for his new 4836-month-to-391-year 'rule' eventually goes to Zu Chongzhi (429–500), the first Southern astronomer to publish on the matter.[18]

Sadly, **the Hexi corridor** stops producing experts and writings in these fields after its conquest and intellectual pillage in 439, and, for the *next hundred years*, we can focus our attention North and South.

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[15] 世祖應期，輯寧諸夏，乃命故司徒、東郡公崔浩錯綜其數。浩博涉淵通，更修曆術，兼著五行論。是時故司空、咸陽公高允該覽羣籍，贊明五緯，并述洪範。然浩等考察未及周密，高宗踐祚，乃用敦煌趙歇甲寅之曆，然其星度，稍為差遠, *Wei shu*, 107A.2660. This would appear to correspond to Cui Hao's *Wuyin yuan li* 五寅元曆 of 440/451 CE mentioned in ibid., 107A.2659. In addition, *Sui shu*, 34.1023, records a '*Astronomical Procedures*, 1 roll, written by Cui Hao' 曆術一卷崔浩撰; *Jiu Tang shu*, 47.2038, records a '*Lecture Notes on Mathematical Astronomy*, 1 roll, written by Cui Hao' 曆疏一卷崔浩撰; and *Xin Tang shu*, 59.1547, records a '*Cui Hao's Tono-metrological and Astronomical Proceduces*, 1 roll' 崔浩律曆術一卷. Note that prior to Cui Hao's involvement with these fields, there was also the iron armillary sphere built at Pingcheng in 398 by Xie Lan 斛蘭 and Chao Chong 晁崇 of Xiangping, the former of whom has a Xianbei surname, and the latter of which previously served the Later Yan (384–409) as astronomer royale in the far North-East prior to its conquest by the Northern Wei.

[16] The other is Xindu Fang's 信都芳 (d. 543/550) *Zhoubi sishu zhu* 周髀四術注, cited in *Sui shu*, 19.526.

[17] More specifically, the North sees the continued use of the *bu* 'obscuration' up into Zhang Bin 張賓 (fl. 568–584) et al.'s *Kaihuang li* 開皇曆 of 584 CE (*Sui shu*, 17.421). One sees Northern Astronomers 'break the rule' 破章 (of 235 months:19 years) as early as Li Yexing 李業興 (484–549 CE) et al.'s *Zhengguang li* 正光曆 of 518/520 CE. And in place of the [Southern] *suicha* 歲差 'year difference', one sees Northern astronomers like Zhang Hong 張洪 (fl. 500–515) produce titles like the *Jiawu-Jiaxu Double-Origin Astronomy* (*Jiawu, Jiaxu eryuan li* 甲午、甲戌二元曆) reminiscent of Li Xiu, Bu Xian, Jiang Ji's solutions.

[18] See for example *Chouren zhuan huibian*, 'Fanli' 凡例, p. 3. For the first signs of an awareness of the inadequacy of an intercalary 'rule' of 235 months to 19 years in the work of fellow Southerner He Chengtian, in 443, see Liu Hongtao (2003: 241, 286).

The Turko-Mongol (Xianbei) Northern Wei, 439–535 CE

Of the two, the situation in the Turko-Mongol (Xianbei) Northern Wei (386–535) is by far the more interesting, so let's start there. Furthermore, to keep on point, let's frame our approach to the practice of mathematics and mathematical astronomy in this region in specific terms of institutions, social groups, and individual profiles.

As to **institutions**, the Northern Wei inherited those of the Han as passed down through the Cao-Wei (220–265) and Jin (265–420). First, there was a bureau for everything having to do with astronomy: the Office of the Grand Clerk (*taishi ling*) or 'Clerk's Office' (*shiguan*), a subsidiary of the Ministry of Rights (*taichang*). Tono-metrology (*lü*) was more or less the official responsibility of the Office of the Grand Musician (*taiyue ling*), also under the Ministry of Rights, as well as the Imperial Library (*bishu sheng*), between which tono-metrological standards were stored. We know that *suan* mathematics and *li* mathematical astronomy were sometimes taught alongside the classics at the Imperial Academy (*taixue*), another subsidiary of the Ministry of Rites.[19] Lastly, we also know that *suan* mathematics was practiced in the National Treasury, local Accounts Sections, Granaries, and other accounting offices here and there. The workings and organisation of these institutions is well-documented and well-understood amongst historians of China, but this is not entirely relevant for our purposes here. I say this because the majority of writing, projects, and innovations in these fields come from people working other jobs in the civil service, so we can think of these institutions as *resource centres* rather than, say, *secret laboratories* holding a monopoly on a given field of study.[20]

It is worth noting that what is new about the *Northern Wei* in this regard is that, within a few decades of its reunification of the North and absorption of the intellectual tradition of the Hexi Corridor, the court launches a series of major legal and administrative reforms. One of these reforms is the addition of students of mathematics (*suansheng*) to the Imperial Secretariat and Buddhist Temples and the appointment of professors (*boshi*) in the astral sciences and tono-metrology.[21] This is an important turning point, because the study of these subjects had never really been institutionalised in the same way as classical studies, and this change in educa-

[19] Such, for example, was the education that Zheng Xuan 鄭玄 (127–200) received at the Imperial Academy (Morgan forthcoming). In a similar vein, one notes that the mathematician-astronomer polymaths Liu Zhuo 劉焯 (544–610) and Liu Xuan 劉炫 (*c*.546–*c*.613) were disciples of Xiong Ansheng 熊安生 (*c*.499–578), a prominent academician (*boshi*) of the Northern Qi and Zhou specialising in the ritual classics (*Zhou shu*, 45.813). Xiong Ansheng, for his part, was a student of the wandering classical scholar Xu Zunming 徐遵明 (475–529), among whose other students one finds the important Northern/Eastern Wei astronomer Li Yexing 李業興 (484–549), father of astronomer-mathematician Li Chongzu 李崇祖 (fl. 539–550).

[20] For a history of institutions in the imperial bureaucracy, see Balazs (1968), Bielenstein (1980), and Hucker (1985); as concerns the astral sciences specifically, see Deane (1989) and Chen & Zhang (2008). For studies focusing on the elites working inside, outside, and across these institutions in the astral and mathematical sciences, see Cullen (2009; 2017a), Goodman (2010), and Morgan (2015; 2017a). Equally important are studies of the vast amount of administrative accounting documents that have been excavated from the early imperial period [citation].

[21] 律博士……尚書算生……太史博士……諸寺算生, *Wei shu*, 113.2976 ff.

tion policy will go on to have long-term effects on the social organisation of mathematical astronomy and the formation of a mathematical canon, schools, and exams under the Tang (618–907).[22] Anyways, the effects take a couple of centuries to sink in, so let's move on to **social groups** as reconstructed from **individual profiles**.

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In Table 3 you will find a list of the seventeen people on record as active in *suan* mathematics in Northern courts from 439 to 581 CE. We know all of these peoples' dates and places of birth, and fourteen of seventeen have biographies. Each of these men has an interesting personal history worthy of exploring in its own right, but in thinking about them as *a group*, there are two points to take away from this table. First, these men neatly break down into several distinct social classes: most importantly, there are Buddhist monks and there are laymen, and amongst the laymen we have a mix of noblemen and/as civil servants of varying ranks. Second, of our seventeen mathematicians, we know eleven to have been active in other branches of the astral and mathematical sciences, namely *li* mathematical astronomy, *zhan* omenology, *lü* tono-metrology, and *tianti* cosmology.

This second point goes to support my assertion in the introduction (following Cullen, Wu Wenjun et al., etc.): if we are going to talk about 'mathematical cultures', we cannot isolate practitioners of *suan* mathematics from those of these other fields, because, often, *they are the same people*. And this is extremely good news for the history of mathematics, because what it means is that all the histories, biographies, and modern, people-oriented scholarship in the astral sciences and tono-metrology is of direct relevance to thinking about social groups and individual profiles in the history of mathematics.[23]

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In this region, the simple fact that we cannot disentangle practitioners of *li* mathematical astronomy from practitioners of *suan* mathematics more than quintuples the number of individual profiles that must be taken into consideration in a discussion of 'mathematical cultures'. If we narrow or focus back down to the ninety-five years from 439 to 535 CE and include everyone in the North on record as active in these fields, then we are dealing with a total of eighty-one people. So, the eighty-one individuals on this slide are all active in *different*, overlapping constellations of the *eight*, *non-divinatory subfields* of the astral and mathematical sciences as oriented around the *same capital city* in the *same ninety-five years*, but can we treat them as *a community*? Yes; yes we can.

Twenty-four of these individuals belong to the same nine clans – e.g. Lu Bian 盧辯 (fl. 534–560), Lu Daoqian 盧道虔 (d. 534), Lu Daoyue 盧道約 (486–543), and Lu Yuanming 盧元明 (fl. 528–539) – and we can assume that they received similar family educations therein. Twenty-three of these individuals worked on the

[22] As concerns the astral sciences, one notes a strong shift in in-house training, bans on private practice, and the rise of clans at the astronomical bureau over the following centuries (Morgan 2016; 2017b). As concerns mathematics, see Keller & Volkov (2016: 58–63).

[23] See the studies cited in Note 2.

	Name	Dates	Cmd of Birth	Bio	Math	MA	Om	TM	Cos
1	Diao Chong 刁沖	fl. 515–520	Bohai 勃海	x	x		x		
2	Fan Shao 范紹	d. 528	Dunhuang 敦煌	x	x				
3	Gao Qianzhi 高謙之	486–527	Bohai 勃海	x	x				
4	Gao Yun 高允	390–487	Bohai 勃海	x	x	x	x		
5	Li Mushu 李穆叔	fl. 535/577	Zhao 趙郡	-	x		x		
6	Liu Xuan 劉炫	*c*.546–*c*.613	Hejian 河間	x	x	x	x		
7	Liu Zhuo 劉焯	554–610	Hejian 河間	x	x	x		x	x
8	Lu Daoqian 盧道虔	d. 534	Fanyang 范陽	x	x	x			
9	卍Ratnamati 勒那漫提	fl. 508	India 天竺	x	x		x		
10	卍Shi Fashang 釋法上	495–580	Linlü 林慮	x	x				
11	卍Shi Sengfan 釋僧範	476–555	Zhao 趙郡	x	x	x			
12	Xiao Hui 蕭撝	515–573	Lanling 蘭陵	x	x				
13	Xiao Ji 蕭吉	d. 606	Linxiang 臨湘	x	x			x	
14	Xindu Fang 信都芳	d. 543/550	Hejian 河間	x	x	x		x	
15	Yin Shao 殷紹	fl. 458	Changle 長樂	x	x				
16	Zhang Qiujian 張丘建	fl. 431/484	Qinghe 清河	-	x				
17	Zhen Luan 甄鸞	fl. 535–570	Julu 鉅鹿	-	x	x			

Table 3 *Suan* mathematicians in the North, 439–581 CE. The symbol 卍 indicates a Buddhist monk. Abbreviations: Math[ematics (*suan*)], M[athematical] A[stronomy (*li*)], Om[enology (*zhan*)], T[ono-]M[etrology (*lü*), Cos[mology (*tianti*)].

same astronomical reform project in 503–518, and if you take the period's other large-scale collaborative projects into consideration, they end up linking to all nine clans and just about every other individual on this list. And this is not to mention relations of professional association or master-disciple lineages, which are there, but which I have not yet had the time to map.[24]

What I *have* had time to map is where these people grew up – the fifty-nine of them, that is, for whom we have a place of birth (see Fig. 4). The red dots here are scaled by the number, so the big dot on the new capital at Luoyang represents twelve people, and the smallest dots represent one.

Admittedly, the people who grew up in these places usually bounced around between their hometown, the capital, and regional postings, but there are three rather important pieces of information that we can extract from a global view of where these people are from. First, as stated earlier, the Hexi Corridor basically dries up of mathematical talent after 439 (with the one exception of Fan Shao, on Table 3). Second, *no one* is from the South, so it seems fair to treat what's happening in the North in isolation, as we did with the Hexi Corridor in the century prior. Third, one notes that the vast majority of talent comes out of the North China Plain. And this is important, because this is the area in which these experts were educated in their childhood; it is the area to which many of them would normally return in retirement to teach; and it is an area that will keep popping up as we go along. For the sake of concision, I will refer to this area as the 'mathematical breadbasket'.

***

One of the guys to come out of this area is **Zhang Qiujian**. Zhang Qiujian is one of the two mathematicians that *doesn't* have a biography, but he is one whose work was canonised in the *Ten Mathematical Classics* (*Suanjing shishu*) of 656 CE and transmitted into modern times.

We know where this man is from, because he signs his preface 'Zhang Qiujian of Qinghe' (below). We also know that he was writing in the North sometime between 431 and 484 CE, because one of the problems in his eponymous mathematical classic involves a Northern Wei tax relief policy that was in effect between these dates.[25] To make matters interesting, Zhang Qiujian's is one of the extant works of *suan* that Karine and Yiwen have yet to really focus on in their research on distinguishing 'different mathematical cultures', so it is something of a blank slate as concerns *cultural categorisation*, if you will.

Now, the first thing that *I* would do as concerns the identifying the 'mathematical culture' that produced this text is to situate the text within everything we know about the institutions, social groups, and individual profiles in this specific milieu, then look for connections with what is happening elsewhere in the astral and mathematical sciences *in this region, in these ninety-five years*. For example, we know that Zhang Qiujian comes out of the educational breadbasket of Northern mathematics and mathematical astronomy. We know that the *Mathematical Classic of Zhang Qiujian* was written in a time of massive institutional reforms, including the

[24] For an example of the sort of academic ties that can be traced between members of the Northern mathematical community, see Note 19.

[25] Ji Zhigang (1999: 109–110).

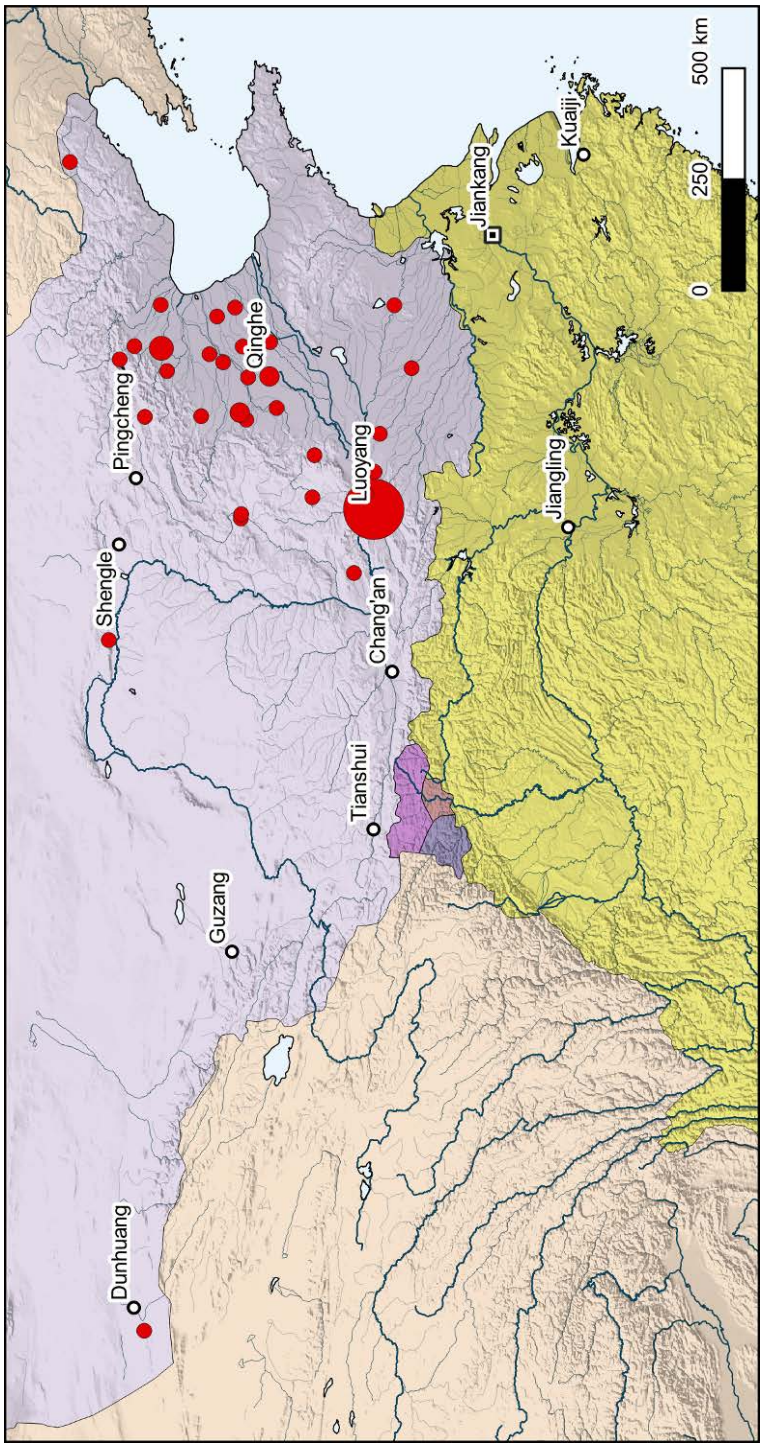

Fig. 4 Hometowns of mathematicians and mathematical astronomers active in the Northern Wei, 439–535 CE (largest dot = 12 people)

establishment of the first official professorships and government students in the astral and mathematical sciences. Furthermore, at the end of his preface is a rather unambiguous statement of purpose:

> Thus, for the sake of later students who are fond of study but do not possess the means to arrive [at an understanding of the works of Xiaohou Yang and Master Sun], I have provided here a big-picture outline, trying not to make their methods any more complicated or taxing than they already are in the hope of making them easier to understand. Zhang Qiujian of Qinghe respectfully [signs this] preface.[26]

It seems safe to conclude from all this that Zhang Qiujian is writing a textbook for this new age.

Now, Zhang Qiujian describes himself as 'outlining' and 'simplifying' the works of Xiahou Yang of Pinglu (n.d., North) and Master Sun (280/317 CE, South), both of whose 'mathematical classics' were canonised alongside his own two centuries later, and he also reworks eighteen problems from the *Nine Chapters on Mathematical Procedures* (*c*.5/*c*.93 CE). In sum, Zhang Qiujian lives in a very specific milieu in the fifth-century North, *but* he's also plugged into a centuries-old written tradition extending north and south. The author understands the more 'complicated' mathematics therein, *but* he has chosen to write a simple, lower-level introduction for 'students' (*sheng*). Moreover, *this particular book* is dedicated to *suan* mathematics, *but* the author is living in a context where most people who study mathematics also seem to study some aspect of astronomy.

Given these sorts of considerations, I am at a loss as to how draw the sort of neat lines of 'different mathematical cultures' around or through Zhang Qiujian's ancient sources and immediate context such as we see in Table 1, particularly when they are not lines that our subjects draw themselves. In the comments, I would be curious to hear your ideas on how this could be done and, more importantly, how it would help us better understand this text or the others, listed in the Appendix, produced in the same milieu. For now, let's leave that an open question and move on down to the South.

The Chinese South, 437–589 CE

Excluding omenology, there are all of sixty people on record as active in the astral and mathematical sciences in the Southern courts at Jiankang and Jiangling over the 272 years from 317 to 589 CE. By means of comparison, that is one born every *four and a half years* in the South compared to one born every *fourteen months* in the Northern Wei. Moreover, most of the activity and surviving works are concentrated in the three decades immediately following the sino-Xiongnu book exchange, when we get the important work of He Chengtian (370–447), Qian Lezhi (fl. 436–443), and Zu Chongzhi (429–500) in mathematics, astronomy, and instrumentation as well as the testing, debate, and reforms around that.[27] After this, the South progressively loses momentum in terms of creative output. This, for example, is all that Li

[26] 余為後生好學有無由以至者，故舉其大槩，而為之法不復煩重，庶其易曉云耳。清河張邱建謹序, *Zhang Qiujian suanjing*, 'Xu', p.4b.

[27] On the handful of people potentially active in the South between 317 and 437, see Note 5.

Chunfeng has to say about the Chen dynasty (557–589) in his history of *li* mathematical astronomy for this period:

> The Chen Clan followed the Liang (502–557), using Zu Chongzhi's astronomy without further innovation or reform.[28]

There is, frankly, not much more than that to say about any other branch of the astral and mathematical sciences in the sixth-century South, excepting omenology and the 'steady drip', if you will, of writings about the clepsydra.

As to **institutions**, we are, to the best of my knowledge, dealing with the same basic set-up as inherited by the Northern Wei, the difference being that there is no concerted state involvement with mathematical education and research like in the North. Anyways, most of our fifty-five practitioners were not employed in specialist, technical posts.

Moving to **social groups**, one notes that, both *proportionally* and in *absolute terms*, there are a lot more people involved in *suan* mathematics in the South: out the grand total of fifty-six practitioners of the eight branches of the astral and mathematical sciences considered, there are twenty-three, listed in Table 4, that are explicitly involved in *suan*. That makes about 38 per cent, compared to only about 21 per cent in the North. There are also a lot more monks. This might just be a question of preservation bias, but what matters is that eighteen of these guys have biographies, so, once again, there is plenty to be said about **individual profiles**.

Considered in aggregate, we see a similar level of polymathy in these individual profiles: about *a third* of our Southern mathematicians were also involved in *lü* tono-metrology or some aspect of astronomy, so, *once again*, it is hard to justify treating this list as a closed group independent of practitioners in these other fields. Zooming out to the *sixty individuals* active in *all* the astral and mathematical sciences (excluding omenology), let us ask, once again, if the list we see on this slide constitutes *a community*. The answer, in this case, is 'yes and no'.

Eight of these men are monks, whose recorded lives rarely intersect with the lay experts, and another three are hermits. Of the remaining forty-nine, fourteen belong to the same five clans, and, *thus*, family traditions, e.g. Yu Shen 庾詵 (fl. 502), Yu Manqian 庾曼倩 (6th cent.), and Yu Jicai 庾季才 (d. 603). For the most part, however, there's not much tying these individuals together beyond their involvement in the same fields. There are two reasons for this. First, there was very little creative output in these fields in the South, and what little there was is sporadic and disconnected, at least as it is recorded in bibliographies, histories, and biographies. Second, where we *do* have historical accounts of innovation, testing, debate, and reform in the South, there is nothing like the number of participants and co-authors that you see in the North. This makes it hard to speak of these sixty individuals as members of one tight-knit community, but maybe that *in itself* is this community's defining characteristic: that the astral and mathematical sciences were more solitary, individualist, and text-oriented affairs in the South.

The map in Fig. 5 tells a similar story. Of forty-three provenanced practitioners, only nineteen are from the heart of the South, in Yellow. The other half are almost all from the same areas in the North where the Northern Wei was getting all of its mathematicians – territories that the South progressively lost over the course of

[28] 陳氏因梁，亦用祖冲之曆，更無所創改, *Sui shu*, 17.417.

	Name	Dates	Cmd of Birth	Bio	Math	MA	Om	TM	Cha.	Inst.	Cos
1	Fu Manrong 伏曼容	421–502	Pingchang 平昌	x	x						
2	Geng Xun 耿詢	fl. 589–611	Danyang 丹陽	x	x	x	x		x	x	
3	Gu Yue 顧越	480–569	Wu Cmd. 吳郡	x	x	x		x			
4	Guan Kangzhi 關康之	415–477	Hedong 河東	-	x						
5	卍Guṇabhadra 求那跋陀羅	394–468	Cent. India 中天竺	x	x		x				
6	Jiang Zhan 江湛	407–453	Jiyang 濟陽	x	x						
7	Kong Shanshi 孔山士	d. 450	Kuaiji 會稽	x	x						
8	Mao Qicheng 毛栖誠	fl. *c.*502	Xingyang 滎陽	-	x			x			
9	Pi Yanzong 皮延宗	fl. 443	-	-	x	x					x
10	卍Shi Daoan 釋道安	d. 385	Changshan 常山	x	x						
11	卍Shi Huijun 釋慧頵	564–637	Qinghe 清河	x	x						
12	卍Shi Huiyuan 釋慧遠	334–416	Yanmen 雁門	x	x						
13	卍Shi Senghan 釋僧含	fl. 430	-	x	x		x				
14	卍Shi Tanguang 釋曇光	fl. 465–472	Kuaiji 會稽	x	x						
15	Xiao Hui 蕭撝	515–573	Lanling 蘭陵	x	x						
16	Xiao Ji 蕭吉	d. 606	Linxiang 臨湘	x	x			x			
17	Yu Changsun 虞長孫	5th cent.	-	-	x						
18	Yu Manqian 庾曼倩	6th cent.	Xinye 新野	x	x						
19	Yu Shen 庾詵	fl. 502	Xinye 新野	x	x						
20	Zhang Zuan 張纘	499–549	-	x	x						
21	卍Zhi Sengna 支僧納	fl. 430	-	-	x						
22	Zhu Yi 朱异	483–549	Wu Cmd. 吳郡	x	x						
23	Zu Chongzhi 祖沖之	429–500	Fanyang 范陽	x	x	x		x			

Table 4 *Suan* mathematicians in the South, 317–589 CE. The symbol 卍 indicates a Buddhist monk. Abbreviations: Math[ematics (*suan*)], M[athematical] A[stronomy (*li*)], Om[enology (*zhan*)], T[ono-]M[etrology (*lü*), Cha[rts] (*tu*), Inst[ruments], and Cos[mology (*tianti*)].

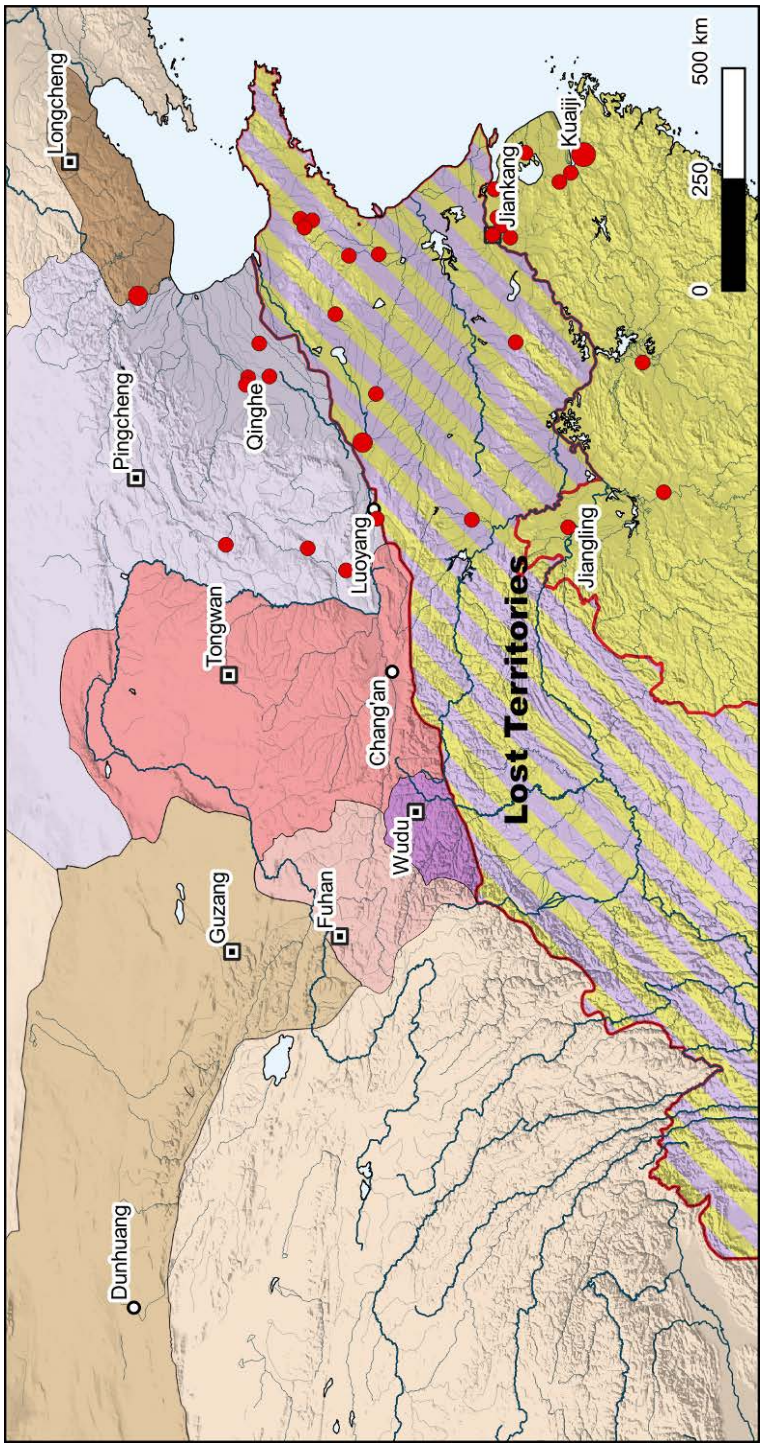

Fig. 5 Hometowns of mathematicians and mathematical astronomers active in the South, 317–589 CE (largest dot = 4 people).

these three-hundred years. How, then, did these people end up in the South? Some were refugees that came in the early 300s. Some were born in territories that were subsequently lost. Some were monks, who travelled freely across political divides. Others, like the Zu family, are a bit of a mystery.[29]

Now, this is highly speculative, but I find myself wondering two things about the geographic distribution we see in Fig. 5. First, if there's not much of a *community* in Southern mathematics, might it be because we are dealing with people who grew up in very different places, speaking different dialects, a good number of whom were refugees in a society with serious problems of integration? Second, if both the North and South depended primarily on the North China Plain for mathematicians and mathematical astronomers, might not the control of this region essentially decide which was winning the space race, so to speak? We will come back to this in a minute, but first, let's get on with history.

East and West, 534–577 CE

So, following the injection of human and library resources from Hexi Corridor, there is an explosion of activity in the astral and mathematical sciences, North and South, starting in the 430s. Sixty years later, in 493, the Northern Wei move their capital from Pingcheng to Luoyang, and from this point forward the North's creative output in astronomy and mathematics really begins to take off, while, in the South, things begin to taper off.

At the same time, in the 520s, a number of major rebellions break out along the Northern Wei's northern frontier and spread to the interior of the empire. The empress dowager poisons the eighteen-year-old emperor, and this starts a civil war among the generals, who divide into two camps around two different heirs. In 534–535, the empire then splits in two, each side establishing a new capital away from enemy lines, one in Chang'an, the other in Ye. In total, this East–West division lasts forty-three years from 534 to 577.

Everything is split in two, including the community of mathematicians and mathematical astronomers, which continues to grow in numbers and productivity despite the deteriorating political conditions of their work. Now, I have already shown you *lists and lists* of individual practitioners, and I have highlighted just how many of them have biographies and outlined the sort of information that those biographies contain, but we haven't really focused on an ***individual profile*** as such, so let's take a look at that of a *suan* mathematician active during the East–West schism.

[29] Zu Chongzhi's 祖沖之 (429–500) biography in *Nan shi*, 72.1773, introduces as 'a man of Tiao [District], Fanyang [Commandery]' 范陽遒人也, near modern-day Beijing, whose family served the Southern court at Jiankang going back to his father Shuozhi 朔之, his grandfather Chang 昌, and his great grandfather Taizhi 台之. His biography in *Nan Qi shu*, 52.903, identifies him as 'a man of Ji [District], Fanyang [Commandery]' 范陽薊人 (modern Beijing). Neither explain whether the family was moving back and forth across enemy lines or if Tiao/Ji District was simply his distant ancestral hometown. Chongzhi's son Geng 晅 (*fl.* 504–525 CE) is introduced as 'a man of South of the Yangtze' 江南人 in *Bei shi*, 89.2933, but his *grandson* Hao 皓 (d. 550) is identified as 'Zu Hao of Fanyang' 范陽祖皓 in *Liang shu*, 29.429, and the parallel passage in *Nan shi*, 52.1319.

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We are going to look at **Zhen Luan (fl. 535–570)**. Zhen Luan is one of the most prolific figures in the history of *suan* mathematics, and *five* of his works on this subject survive: two books and two commentaries canonised in the *Ten Mathematical Classics*, and a third commentary on a *non-canonical* text. What is more, Zhen Luan is one of the few mathematicians who *hasn't* left us with a biography, so in listening to what we can reconstruct of *his* life and works, I invite you to imagine how much there is to say about all the guys who have biographies![30]

Zhen Luan was born in Wuji District, Zhongshan Commandery, near modern-day Beijing. In his youth, he was fascinated with Religious Daoism, and the age of twenty he went to join a temple. To his surprise, the induction ceremony was an orgy, at which, in this words:

> They were swapping husbands and exchanging wives, their only priority desire; their fathers and brothers standing before them, ignorant of the shame.[31]

Zhen Luan leaves, devoting his life instead to classical exegesis, tono-metrology, mathematics, and mathematical astronomy – the 'exact sciences' of his day.

In 535, he somehow shows up at the Southern court of the Liang studying tono-metrological instruments.[32] Later, he pops back up in the North, serving the Northern Qi (550–577) as Governor of Changshan, a couple hours by horse from where he grew up.[33] Then, a while later, he pops up under the Northern Zhou (557–581) as Commandant of the Metropolitan Region – the mayor, essentially, of *their capital* in the West.[34] Within the space of thirty years, he has managed to serve *all three rival courts*, two of them in *very high positions*, so there is clearly a story here, and if we had his biography we might know what that story is…

Anyways, while serving the Northern Zhou, Zhen Luan writes a wide array of books and commentaries on mathematics, tono-metrology, and mathematical astronomy, and, in 566, he submits an astronomical procedure text to the throne, which is subsequently adopted for the calculation of official calendars and ephemerides – one of the greatest honours that an astronomer could hope to achieve.[35] A short while later, he is made the Governor of Hanzhong Commandery,[36] and, in 569,

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[30] The following account of the life and times of Zhen Luan 甄鸾 is reconstructed from the aforementioned database work and, much more importantly, personal details revealed in his *Laughing at the Dao* (*Xiao Dao lun* 笑道論) as translated and studied in Kohn (2008). See also Wu Wenjun et al. (1998–2004: vol. 4, 169–172), Chen Meidong (2003: 305), and the entry 'Zhen Luan' in Martin & Chaussende (2019).

[31] 教夫易婦惟色為初，父兄立前不知羞恥, *Xiao Dao lun*, cited in *Guang hong mingji*, T52, no. 2103, p. 152b1.

[32] *Xiahou Yang suanjing*, 1.5a.

[33] 'Guangzhou duji Zhen Gong bei' 廣州都督甄公碑, cited in *Wenyuan yinghua*, 913.9a.

[34] *Xiao Dao lun*, cited in *Guang hong mingji*, T52, no. 2103, p. 144a22.

[35] *Sui shu*, 17.416, 17.418–419.

[36] *Zhoubi suanjing*, 1.1a, 2.1a.

the emperor invites him to produce a, *quote*, 'detailed examination of the two religions, Buddhism and Daoism'.[37]

Buddhism was made the state religion at the founding of the Northern Zhou, but the current emperor was paranoid, and a group of Daoist priests managed to exploit his paranoia to ordain him and turn him against the Buddhist clergy. One day, the emperor shows up to court in Daoist robes, and he orders a hundred monks to the palace for a seven-day prayer session, *which he infiltrates, dressed as a monk*. He discovers the monks *praying* – proof positive that he had been discovered, and that the monks were trying to *deceive him*. Foiled, he orders representatives of Buddhism, Daoism, and Confucianism to present their faiths at a conference of some two thousand attendees. After hearing their respective presentations, the emperor proceeds to *rank them* – Daoism first, Buddhism last – and ask the participants what they think of that. What the emperor *wanted* was justification for a purge, but the conference didn't break out into the fight for which he was hoping. Foiled once again, he organised a *second* and a *third conference*, changing the topic to the 'right and wrong' of each religion, but, unfortunately, the debate was too peaceful and evenly-matched to justify bloodshed.

At the end of his rope, the emperor thought, *essentially*, 'Who better than the most respected astronomer and mathematician in the empire to give a definitive answer to the question of which is the one true religion?' Unfortunately for everyone involved, the emperor seems to have been ignorant of Zhen Luan's personal history with Daoism, and Zhen Luan took this task *very, very seriously*.[38]

The following year, in 570, Zhen Luan submitted his verdict in the form of a treatise in three volumes entitled *Laughing at the Dao* (*Xiao Dao lun*), in which he takes out his life-long hatred of the emperor's preferred religion through rigorous *text-critical*, *mathematical*, and *astronomical* proof that the Daoist canon contradicts reality, reason, and itself to the point where, as the title suggests, you can only laugh at anyone who takes it seriously. The emperor had it burned, and this is the last we see of Zhen Luan in the historical record; luckily for us, however, *Laughing at the Dao* made its way in the Buddhist canon.

Zhen Luan was succeeded by his son Zhen Zu.

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Zhen Luan's oeuvre is spread out over several bibliographic categories, covering mathematics (*suan*), mathematical astronomy (*li*), chronology (*pu*), and religion:

Mathematics *(suan):*

1. *Commentary to the Mathematical Classic of Master Sun* 孫子算經注, 3 rolls.
2. *Commentary to the Mathematical Classic of Xiahou Yang* 夏侯陽算經注, 3/1 roll.
3. *Commentary to the Mathematical Classic of Zhang Qiujian* 張丘建算經注, 1 roll.
4. *Commentary to Records of Remnants of Numbers and Procedures by Xu Yue* 徐岳數術記遺注, 1 roll.
5. *Commentary to Three Grade Numbers by Dong Quan* 董泉三等數注, 1 roll.
6. *Gnomon of Zhou Renarrated* 周髀重述 (subcomm. to comm. by Zhao Ying), 1 roll.

[37] 奉勅令詳佛道二教, *Xiao Dao lun*, cited in *Guang hong mingji*, T52, no. 2103, p. 143c21.

[38] This is my somewhat dramatised version of events as presented in in Kohn (2008).

7. *Mathematical Classic of the Five Bureaux* 五曹算經, 5/3 rolls.
8. *Mathematical Procedures of the Five Classics* 五經算術, 1 roll.
9. *Mathematical Procedures of Zhen Luan* 甄鸞算術.
10. *Nine Chapters on Mathematical Procedures Renarrated* 九章算術重述 (subcomm. to comm. by Xu Yue), 9/1 rolls.

Mathematical Astronomy *(li):*

1. *Astronomical Procedures* 曆術, 1 roll.
2. *Astronomy of [the Era of] Celestial Harmony* 天和曆, 1 roll, 566 CE.
3. *Seven Luminaries Astronomical Calculations* 七曜曆算, 2 rolls.
4. *Seven Luminaries Arising Astronomy* 七曜本起曆, 5/3/2 rolls.
5. *Seven Luminaries Procedures and Calculations* 七曜術算, 2 rolls.

Chronology (*pu*):

1. *Chronology of Emperors and Kings* 帝王世錄, 1 roll.

Religion:

1. *Laughing at the Dao* 笑道論, 570 CE.

Now, were we to take just Zhen Luan's mathematical writings and limit ourselves to Chemla's (2009) first eight criteria of 'problems', 'numbers', 'procedures', 'text types', 'illustrations', 'proofs', and 'epistemic values', there is a case to be made, by *reductio ad absurdum*, that every single one represents a 'different mathematical culture'. Namely, Zhen Luan writes treatises, commentaries, *and* subcommentaries, each *text type* usually determining questions of *proofs* and *epistemic values*; all of his writings have different *problems*, *numbers*, and *procedures*; some of it's canonical, some of it's not; some have *illustrations*, others don't; some of his writings in *suan* go into tono-metrology, mathematical astronomy, and Confucian commentary; and one of his commentaries uses *Buddhist* terminology and sources whilst discussing some sort of *abacus*.

If we look at the author's *individual profile* (criterion 9c), on the other hand, then it's hard not to treat this corpus as a coherent whole. It's the *same man* who wrote all these things, after all, and, *thematically*, what we're looking at is the product of one man's lifelong problem with authority – his righteous mission to question authority and establish *truth* from the ground up using the most rigorous and credible bodies of knowledge at his disposal.

Here again, I do not really know what to do with an observer's category like 'culture': Where do we situate 'culture' between this author and his works? Is everything that is the product of the *same author* necessarily the product of the *same culture*? If it is by these nine criteria that we are going to divide his writings into 'different mathematical cultures', then how do we incorporate and weigh *individual profiles* against, say, *text-types* and *numbers*? If we *can* divide one man's works into 'different mathematical cultures', then how many would be reasonable in this case? *Two? Four? Seventeen?* And if we divide the seventeen works of one author into however many 'different cultures', how does that help us better understand the works *or* the author either individually or as a whole? My personal inclination would be to focus instead on discrete people, texts, and actor's categories, but let's zoom back out to the level of social groups.

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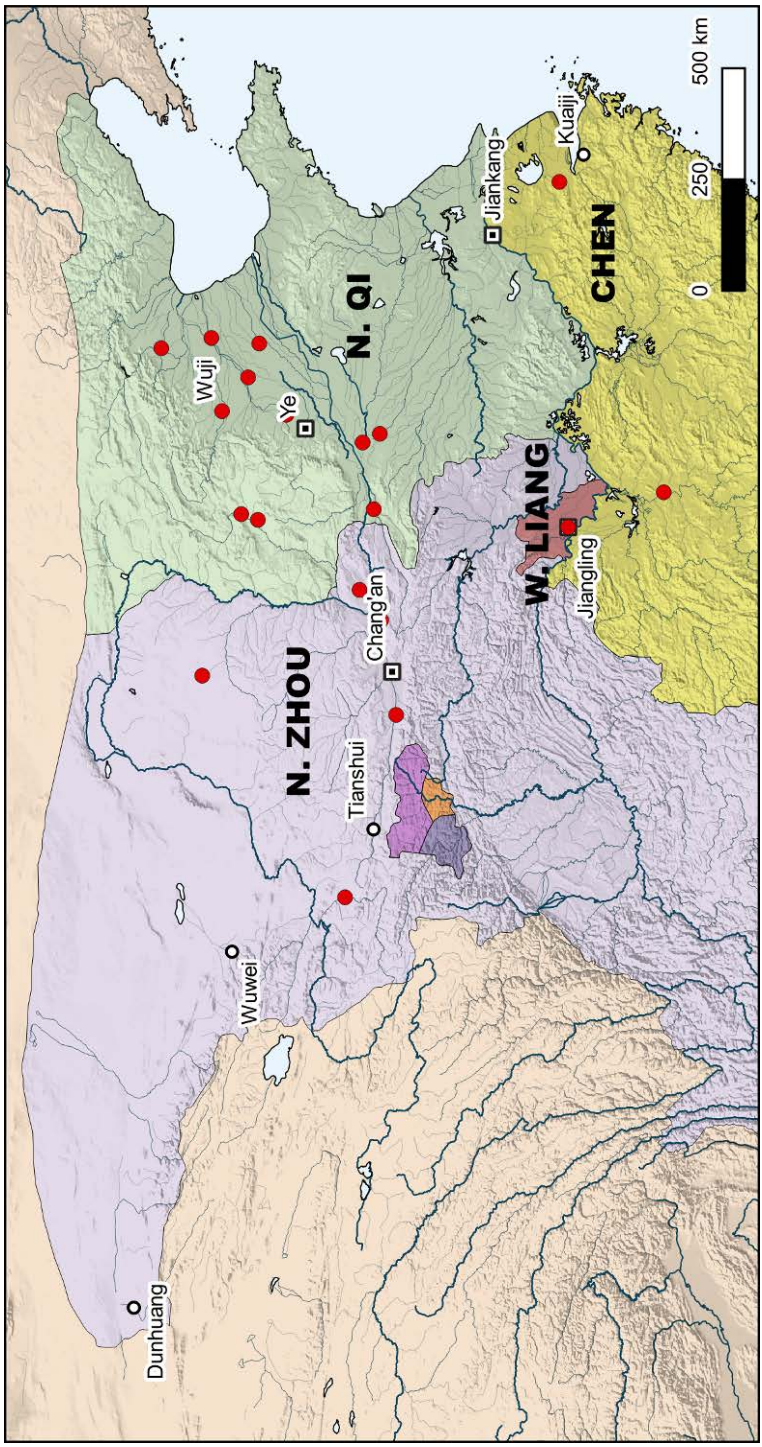

Fig. 6 Hometowns of mathematicians and mathematical astronomers active in Chang'an, 535–581 CE (largest dot = 2 people).

In Fig. 6, I have mapped out the hometowns of the astronomers and mathematicians active in Chang'an, in the West. As you can see, Zhen Luan comes out of the same area as most of the experts in this period, and, like most of those active in Chang'an, he moved from East to West during the schism. In addition to everyone from the North China Plain that we would expect to see in Chang'an, however, for the first time in more than *two hundred years* we finally get a handful of experts from the *South*, and what they're doing in Chang'an is of considerable interest for the topic of 'mathematical cultures'.

I'm running low on time, so let's just say that, in 548, the war between East and West in the North spills into the South, dividing the *South* into *East* and *West* and transforming a *rebellion* into a *proxy war* that forces the Southern court at Jiankang to move west to Jiangling. Once in Jiangling, the Western Wei invade, taking all of the new capital's resources to Chang'an, installing a puppet emperor in Jiangling, and swallowing up all the territory to the South-West. In the middle of all this the generals in the North overthrow their own respective puppet emperors, giving us the Zhou (NW), Qi (NE), Liang (SW), and Chen (SE).

The emperor of Liang had the foresight to order the imperial library burned at the outset of the siege, but the Northern army made away with an impressive array of resources:

> They captured more than 100,000 men and women and emptied the imperial warehouses of their treasures, obtaining the Song (Jiankang, 420–479) armillary sphere(s), the Liang (Jiankang/Jiangling, 502–557) sundials and bronze gnomon… as offerings [to their emperor], the soldiers keeping nothing for themselves.[39]

In addition, the army captured a number of Southern astronomers and mathematicians, such as Yu Jicai (above). This, from Yu Jicai's biography, is how he experienced the event:

> Suddenly, Jiangling was completely destroyed. As soon as Emperor Wen of Zhou (r. 507–556) saw him, he gave him the most lavish of welcomes, ordering him to assist in the direction of the Astronomical Bureau, saying 'My good minister had best practice the utmost honesty in service to Us, for you shall be rewarded in kind with wealth and noble [title]'. At the outset, the [Liang] territory was completely lost, and uniformed officers [of the court] mostly disappeared as lowly [slaves]. Jicai [sold] everything that was gifted to him to purchase [the liberty] of his friends and family. [The emperor] asked, 'How could you?', to which Jicai [responded], 'The [Liang] capital is completely defeated, its [former] lord having been a true criminal, but what fault have the government officials merits their having been taken as slaves? *Honestly*, I feel pity for them, and this is my only reason for buying their freedom.' [The emperor] came to his senses… ordering the release of several thousand Liang captives that had been enslaved. In [560], [Yu Jicai] was added to the ranks of the Unicorn Toe scholars with Wang Bao and Yu Xin.[40]

[39] 虜其男女十餘萬人，收其府庫珍寶，得宋渾天儀、梁日晷、銅表、魏相風烏、銅蟠螭趺、大玉徑四尺圍七尺及諸輿輦法物以獻，軍無私焉, *Zhou shu*, 25.248.

[40] 俄而江陵覆滅。周文帝一見，深加優禮，令參掌太史，曰：「卿宜盡誠事孤，當以富貴相答。」初，荊土覆亡，衣冠士人，多沒為賤。季才散所賜物，購求親故。周文問：「何能若此？」季才曰：「郢都覆敗，君信有罪，搢紳何咎，皆為賤隸？誠竊哀

Indeed, once rendered to Chang'an by the occupying army, other lesser Southern experts like Ming Kerang (525–594), Shen Chong (500–583), and Xiao Ji (d. 606) were all given respectable government positions and ordered to join collaborative research projects with their northern analogues. And while most descriptions of this affair tend to focus on individual people and individual resources, in this one passage from the *Book of Sui* we have the second closest description of something like 'different mathematical cultures' that we see in this entire period:

> One of the first edicts of [559 CE] was to order officials to construct a Zhou astronomy. At that, Dew Gate Scholar Gentleman Ming Kerang, Unicorn Toe Scholar Yu Jicai, and a number of hemerologists picked through Zu Geng's old court opinion(s) and synthesised the procedures of North and South. From this point onward, one saw a lot of absurdities.[41]

Again, Ming Kerang and Yu Jicai were new arrivals to Chang'an, and the late Zu Geng – son of Zu Chongzhi, father of Zu Hao (d. 550) – was a member of one of the most important Southern families in the history of astronomy and mathematics.

***

To recap, the Western Wei (535–557) court at Chang'an brings most of its experts from the east in 535; there is a handful from the west; and in 555, it imports a third set of *Southern* experts and tries to get everyone working together for the sake of 'synthesis', much like Cui Hao's mission vis-à-vis the Hexi Corridor a century prior.

Compare this, however, to where Eastern court at Ye is getting their astronomers and mathematicians in Fig. 7. It probably comes as no surprise by this point that all but one come from within a 400 km radius of the capital, situated as it is in the middle of the 'mathematical breadbasket'. *Also*, of the four capitals in this forty-three-year period, it is at Ye were we get a complete revolution in *li* mathematical astronomy. In short, there is a court doctor named Zhang Zixin of Qinghe (d. 577) who disappears to an island to conduct observations, working out the inequality of solar and planetary motion, etc. Zhang comes back, he takes students from the same region – Zhang Mengbin of Guangping and Liu Xiaosun of Guangping (d. 594/597) – and his students start fighting for recognition in court astronomy in 576.

The next year, however, Ye falls to the Northern Zhou, and two years after that, the Zhou emperors are replaced by the Sui (581–618) in a coup d'état. Chang'an is renamed Daxing, and, amid the chaos, the expert community at Ye remerges there to continue the fight. This is the *fourth* such community that the city has absorbed in forty-three years, and eight years later, in 589, the Sui conquer the Chen, adding all of *its* human and material resources to the mix in Daxing.

In all, the Sui court at Daxing captures six leading experts from the Chen court at Jiankang: Cai Ziyuan, Geng Xun of Danyang (Table 4), Mao Shuang of Yangwu, Yu Puming, Yuan Chong of Danyang (544–618), and Zhou Fen (fl. 583–589). As with those captured at Jiangling thirty-four years earlier, many of whom were still

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之，故贖購耳。」周文乃悟曰：「微君，遂失天下之望。」因出令，免梁俘為奴婢者數千口。武成二年，與王褒、庾信同補麟趾學士, *Bei shi*, 89.2947–2948.

[41] 至周明帝武成元年，始詔有司造周曆。於是露門學士明克讓、麟趾學士庾季才，及諸日者，採祖暅舊議，通簡南北之術。自斯已後，頗覩其謬, *Sui shu*, 17.418–419.

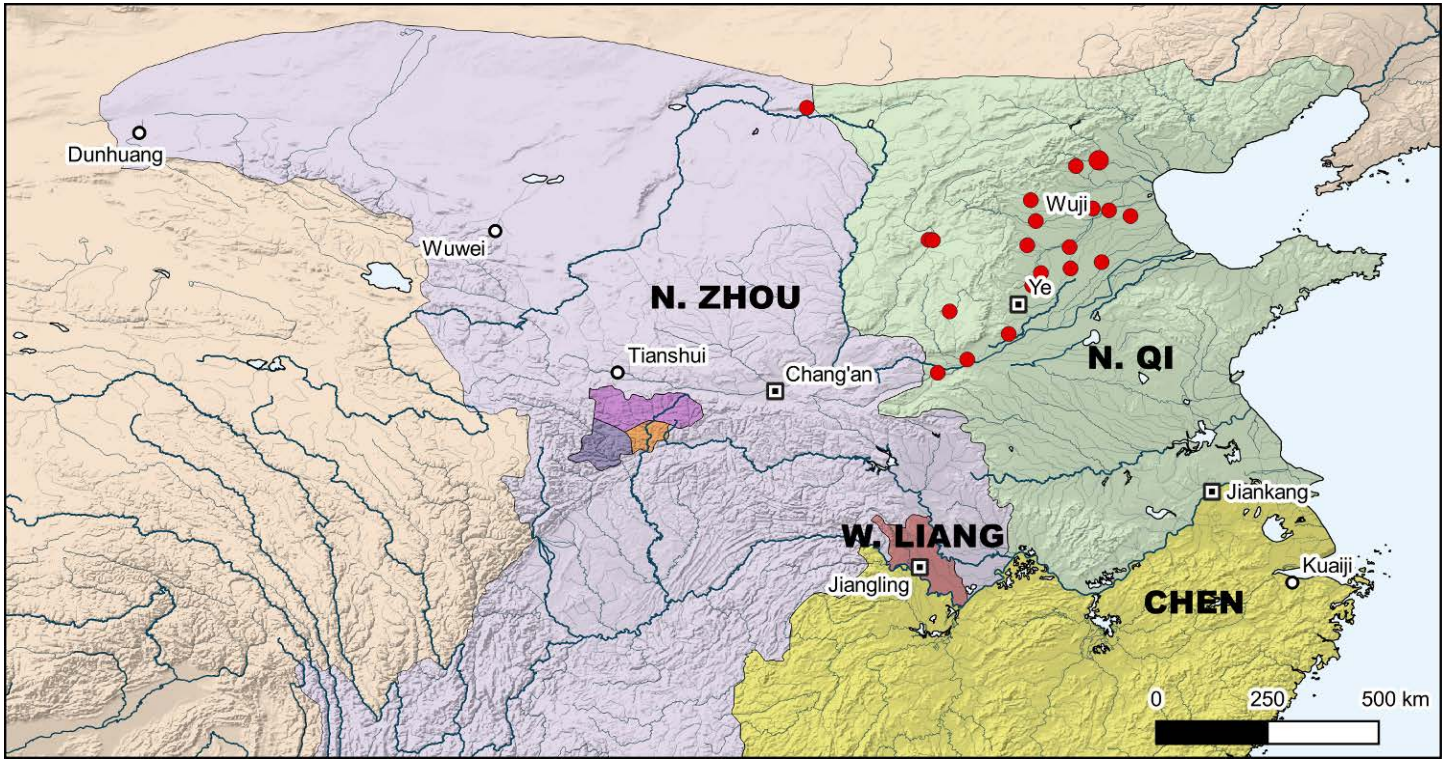

Fig. 7 Hometowns of mathematicians and mathematical astronomers active in Ye, 534–577 CE (largest dot = 3 people).

alive, these people are given respectable government positions and engaged in collective projects in the name of 'synthesis'. Unlike the previous block quote, however, descriptions of this work focus on individuals, works, and dynasties rather than cultural (?) blocks like 'North and South'. Consider for example the collective project in 'heavenly patterns' involving Zhou Fen and Yuan Chong (captured 589 in Jiankang) and Yu Jicai (captured 555 in Jiangling):

> When the High Ancestor [of the Sui] (r. 581–604) pacified the Chen (557–589), he obtained the [constellation] expert **Zhou Fen**, and he also obtained the Song (420–479) armillary sphere(s).
>
> [The emperor] thereupon ordered **Yu Jicai** and company to study and collate the old official [star] charts of the [Northern courts of] Zhou and Qi and [the Southern courts of] Liang and Chen as well as the private charts of [the Southerner] Zu Geng and [the Northerner] Sun Senghua (fl. 530/532), to cut their sizes, correct their [accuracy], and to create an umbrella diagram on the basis of the Three Experts' star positions. … it would become the orthodox model.
>
> [The emperor] made [Zhou] Fen the astronomer royale. Fen was broadly studied in the classics and documents and assiduous at teaching and learning, and from this point forward the Astronomical Bureau's students in observation finally began to know [their constellations]. Emperor Yang (r. 605–617) furthermore assigned forty palace [wo]men to go [take up posts in] the Astronomical Bureau and, in a separate edict, ordered **Yuan Chong** to teach them star and vapour [omens]. Those who finished their studies were inducted into the interior of the palace for consultation about omens and their verifications.[42]

The same can be said of how Mao Shuang, Cai Ziyuan, and Yu Puming are placed under the leadership of the Northerner, Niu Hong of Chungu (545–610), to integrate newly acquired materials into the court's tono-metrology:

> During the pacification of Right Bank (the South), [the Sui] obtained [a set of] ten and two pitch pipes from the [defeated] Chen Clan, all of which they gave over to [Niu] Hong. [Also,] they dispatched sound and pitch pipe experts such as Chen-dynasty Grand Administrator of Shanyang **Mao Shuang**, Prefect Grand Musician **Cai Ziyuan**, and **Yu Puming** to watch the nodal *qi* and make a pitch pipe register [with the Chen tubes]. At the time, [Mao] Shuang was old in years, and he presented himself to the High Ancestor (r. 581–604 CE) in white clothing; he was granted [the office of] Inspector of Huaizhou, but he declined it and did not go to his post. Thereupon, [the court] dispatched Harmonics Gentleman Zu Xiaosun to go to him to learn his methods. [Niu] Hong furthermore took the [Chen] tubes back, and blew them to fix the tones. Once the sub-celestial realm had been unified, the instruments and items of other dynasties were all collected at the Music Bureau, and sound and pitch pipe experts debated and subjected them to much investigation in order to fix the bells and pitch pipes. They further constructed musical instruments in order to cover

[42] 高祖平陳，得善天官者周墳，并得宋氏渾儀之器。乃命庾季才等，參校周、齊、梁、陳及祖暅、孫僧化官私舊圖，刊其大小，正彼疎密，依準三家星位，以為蓋圖……將為正範。以墳為太史令。墳博考經書，勤於教習，自此太史觀生，始能識天官。煬帝又遣宮人四十人，就太史局，別詔袁充，教以星氣，業成者進內，以參占驗云, *Sui shu*, 19.504–505.

(accompany by music) the fourteen August Xia [Emperors'] ritual songs. When the High Ancestor and the worthies at court heard this [music], they said…[43]

To put this in perspective, I invite you to compare these descriptions of how people and resources from the Chinese South were integrated with those of Chang'an to the way that contemporary sources speak about the astronomy and mathematics coming out of India.

In the catalogue of the Sui imperial library, we see that, even in translation, such works are given a unified label to set them apart from their (unlabelled) Chinese analogues:

HEAVENLY PATTERNS 天文…

婆羅門天文經二十一卷婆羅門捨仙人所說。
Brahmin Heavenly Patterns Classic, 21 rolls, spoken by the Immortal Brahmin Tyāga.
婆羅門竭伽仙人天文說三十卷
The Immortal Brahmin Gārgya's Sayings on Heavenly Patterns, 30 rolls.
婆羅門天文一卷
Brahmin Heavenly Patterns, 1 roll. …

ASTRONOMY AND NUMBERS 曆數 …

婆羅門算法三卷
Brahmin Mathematical Methods, 3 rolls.
婆羅門陰陽算曆一卷
Brahmin Yin-Yang Astronomical Calculations, 1 roll.
婆羅門算經三卷
Brahmin Mathematical Classic, 3 rolls.[44]

And in the following bibliographic notice to a lost work of 'comparative mathematics', we see a culturally sensitive and presumably *bilingual* translator of Buddhist sütras speak of India (Tianzhu) and China (Huaxia) as equally monolithic blocks:

A Comprehensive Comparison of Foreign and Domestic Mathematical Methods, 1 roll.

The [above] title is in one roll, written by sūtra translation scholar Liu Ping of Jingyang (fl. 581/618). [Liu] Ping devoted his work to the study of numbers and procedures, domestic and foreign. He would always compare the major differences in procedures and methods in the calculation of numbers in the sūtra translation of previous ages, and it is for this reason that he elaborated upon this [in this book]. His prefatory abstract reads:

'The arts of the world are sometimes shallow and sometimes profound, and the education of man is sometimes loose and sometimes tight. Thus, if you are looking [to

[43] 遇平江右，得陳氏律管十有二枚，並以付弘。遣曉音律者陳山陽太守毛爽及太樂令蔡子元、于普明等，以候節氣，作律譜。時爽年老，以白衣見高祖，授淮州刺史，辭不赴官。因遣協律郎祖孝孫，就其受法。弘又取此管，吹而定聲。既天下一統，異代器物，皆集樂府，曉音律者，頗議考覈，以定鍾律。更造樂器，以被皇夏十四曲，高祖與朝賢聽之，曰……, *Sui shu*, 16.391.

[44] *Sui shu*, 34.1019, 34.1026.

> understand] the use of calculation/calculation rods, then it is all about the Great Expansion [method of milfoil-stick combinatorics in the *Book of Changes*]; if you do not think about [such things], then it renders confusion as to the other three corners [of the metaphorical square one is trying to understand from one corner]. This being so, within Chinese (Huaxia) numerical methods there is a difference of three ranks [of numbers], while those elaborated in India (Tianzhu) are, for whatever reason, without examples of [such?] heterodoxy. This being so, the first interpretations of the sūtras all called the Great Thousand (i.e., the Trichiliocosm or World of 1000^3 Worlds) "a hundred *yi*" (*yi* = 10^5 or 10^8), and they say that one *yojana* was forty *li*. However, according to all possible calculations, the sums do not match. In my humble opinion, I suspect that on the day of the translation of the [oral] transmission, the two sides heard different sounds, and that in the moment of dictation, we got a mistake as a result. Thus do I record [here] the computational methods of all the sūtras as compared against [those of] the Chinese (Huaxia). …'[45]

Curiously absent from our sources is any description of how the expert community of Ye was welcomed into that of Chang'an/Daxing. If you read Li Chunfeng's history of the period, the politics of astronomy at the Sui capital are extraordinarily brutal, particularly for those seeking recognition for Zhang Zixin's game-changing discoveries and their own improvements upon his work. As Li Liang (forthcoming) writes about in vivid detail, reformists like Liu Xiaosun, Zhang Zhouxuan (d. 605/617), and Liu Zhuo (544–610) are attacked from *every side* by *multiple cliques*, while the emperor's favourites win the day with completely out-dated methods. As it turns out, the reformists are all new subjects from the North China Plain, and if you look at the *people* in each clique and the *ideas* that they espouse, the politics break down fairly neatly along regional lines.

Conclusion

Alright, so that was an overview of what I found over the course of a couple weeks spent thinking about what I would do if it were my goal to find and prove the existence of different mathematical cultures in early imperial China. In summary, I took Karine's 2009 framework for distinguishing mathematical cultures, and I expanded the scope of analysis to include (1) *all* the interrelated astral and mathematical sciences, excepting omenology, (2) the authors of *lost* and *extant* texts alike, (3) and sources for institutions, social groups, and individual profiles, (4) all while narrowing the chronological focus to a single three-hundred-year period most likely to give us what we're looking for.

[45] 外內傍通比校數法一卷。右一部一卷。翻經學士涇陽劉憑撰。憑內外學數術偏工。每以前代翻經算數比校術法頗有不同。故為斯演。其序略云。世之道藝有淺有深。人之稟學有疎有密。故尋算之用也則兼該大衍。其不思也則致惑三隅。然華夏數法自有三等之差。天竺所陳何無異端之例。然先譯經並以大千稱為百億。言一由旬為四十里。依諸算計悉不相合。竊疑翻傳之日彼此異音。指麾之際於斯取失。故錄眾經算數之法。與華夏相參, *Lidai Sanbao ji*, T49, no. 2034, p. 12.107a5-17. In a similar vein, see the dialogue between He Chengtian and the monk Huiyan 慧嚴 in 435 CE concerning 'what astronomy they use in Buddhist country/ies' 佛國將用何曆 as studied in Jiang & Niu (2001: 168–176) and Zheng & Jiang (2007).

What I've found, if anything, is that there is an enormous wealth of documentary evidence for institutions, social groups, and individual profiles in these fields, and that this embarrassment of riches allows us to break down the history of this period into along a variety of axes. With 224 practitioners, 131 biographies, 227 authored works, and about a thousand pages of biographical information on the experts of this three-hundred-year period, we can divide people and works into regional networks active in different territories centred around different cities at different times. Within these networks, we can connect individual people and works along lines of geographical origin, professional associations, collective projects, research tradition, and family and educational lineage. Moreover, having roughly mapped these networks, we can plot how people and works move *between them*, how the same ideas develop in different directions in two different cities, and how one network *joins with* and is eventually *absorbed into* another.

All of those relations I have just described correspond to concrete actors' categories by which our historical subjects divide up their own socio-political world, and, and though I set out to find 'different cultures', I am frankly more confused than ever as to how this observer's category is meant to articulate with the spider-web of tangible, historical relations recognised by our subjects. I have a couple of ideas, but, more than anything, I wish to leave you with some questions.

First of all, I think we need to move institutions, social groups, and individual profiles to the very top of the list of things to take into consideration in delineating 'mathematical cultures'. By any standard anthropological definition, 'culture' is about human beings and their interactions, and in the case of this three-hundred-year period, at least, we have a lot more sources about the *lives* of mathematicians and mathematical astronomers than we do the professional literature they were reading and writing. I think, therefore, that these are worth putting front and centre.

Second, I think it's a good idea to start from how our historical subjects themselves describe 'different ways of doing mathematics' and to work outward from there. Surely, there must be something to be learned about 'culture' in how our sources describe 'Brahmin mathematics' or the integration of regional networks. It's true that our sources very rarely discuss differences on any level higher than the individual or the individual lineage as concerns *Chinese-language materials*, but the two region-based counterexamples I found are something.

Lastly, if we're going to break up the history of mathematics over this period into 'different cultures', I would say that the best candidates are probably the six distinct regional networks outlined here. If we go with that, then I would say that our first step should be a comparison of how criteria of problems, numbers, epistemic values, etc., vary *within* and *across* the regional bibliographies provided in the Appendix.

Anyways, the goal of this talk is not so much to provide answers as it is to raise questions to stimulate discussion in the time that we have left, so, in that spirit, I return first to those already formulated in the course of this presentation:

- With the *Mathematical Classic of Zhang Qiujian*, we have a book that was written in a very specific milieu in the fifth-century North *but* that reworks a centuries-old written tradition spanning North and South. The author understands 'complicated' mathematics', *but* he chooses to write something simpler, for students. *This book* is about *suan*, *but* it's written in a milieu where *suan* is often studied alongside mathematical astronomy, for one. Given all this, to

what one 'culture' can we say that the *Mathematical Classic of Zhang Qiujian* belongs?

- With Zhen Luan, we have one man active at three courts who writes seventeen works in multiple genres in four different fields, several of which cross genres and fields, and all of which are different in terms of problems, numbers, procedures, text types, instruments, illustrations, proofs, and epistemic values. Into how many 'cultures' is it reasonable to divide his bibliography?
- In the same vein, how do we articulate such matters of a text's *contents* with everything we know about its *context* in terms of institutions, social groups, individual profiles, regional networks, places of origin, professional associations, collaborations, research traditions, and family and academic lineages?

And now several more questions in parting:

- If we choose to label the six regional networks outlined in the Appendix as 'different mathematical cultures', then how must we adapt our criteria for identifying 'cultures' to match their respective contents and differences?
- What does 'culture' bring us that tangible actors' categories like transmission lineage (*chuan/jia*) or court-based (*chao*) regional networks do not? And what role should actors' categories have in a discussion of 'culture'?
- What do we do with the fact that, every time two regional traditions come into contact in this period, the inevitable response is to 'synthesise', integrate, and immediately erase any differences between them? If this is the trend, how could 'different mathematical cultures' survive outside of isolation?
- What do we do with the fact that our sources firmly juxtapose 'Indian' and 'Chinese' mathematics as monolithic blocks?
- Lastly, and perhaps most importantly, I would love to hear some suggestions about how to better mine the wealth of biographical information we have at our disposal in the history of mathematics and astronomy in early imperial China. I'm speaking with some people about databases, and I've started learning some software for geographical and social networks analysis, but I don't yet have a clear idea how best to organise and navigate this tangled social world.

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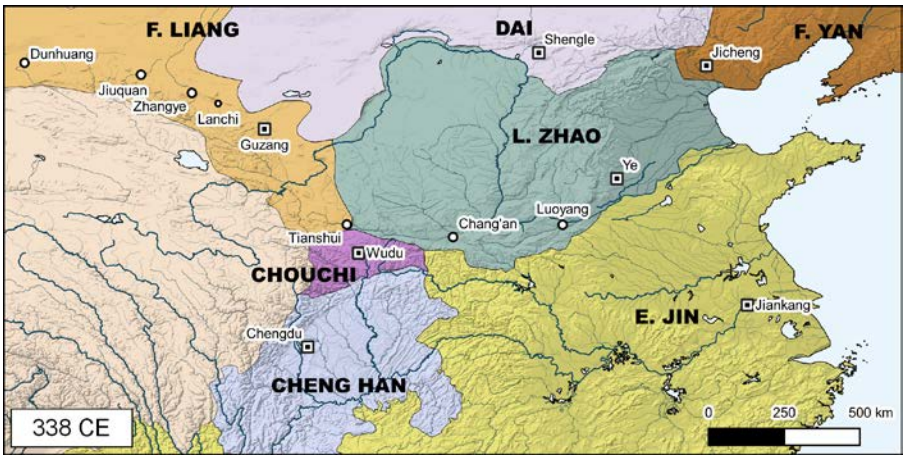
Appendix:
Bibliography of the Exact Sciences by Region, 317–618 CE



Hexi Corridor, 317–439 CE

Jiang Ji 姜岌 (fl. 384) of Tianshui, *Astronomical Procedures for Mr Jing's Essential Collection* 京氏要集曆術, 4 rolls.
———, *On Sphere Heaven [Cosmology]* 渾天論.[extant]
———, *Triple Epoch Jiazi$_{01}$-Origin Astronomy* 三紀甲子元曆, 1 roll, 384 CE.[part.]
———, *[Triple Epoch Jiazi$_{01}$-Origin] Astronomy Preface* 曆序, 1 roll, 384 CE.[extant]
Zhao Fei 趙斐 (fl. 412) of Dunhuang, *Astronomical Procedures for [Lunar Latitude]* 陰陽曆術, 1 roll.
———, *Hexi [Corridor] Jiayin$_{51}$-Origin Astronomy* 河西甲寅元曆, 1 roll.
———, *Hexi [Corridor] Jiayin$_{51}$-Origin Astronomy Preface* 河西甲寅元曆序, 1 roll.
———, *Hexi [Corridor] Renchen$_{29}$-Origin Astronomy* 河西壬辰元曆, 1 roll.
———, *Mathematical Classic of the Astronomical Numbers of the Seven Luminaries* 七曜曆數算經, 1 roll.
———, *Mathematical Classic of Zhao Fei* 趙斐算經, 1 roll.
Zhao the Hermit 趙隱居 (Zhao Ying?), *Quarter-Remainder Astronomy* 四分曆, 1 roll.
Zhao Ying 趙嬰 / Shuang 爽 (fl. 322), Chief of Lanchi, *Commentary to the Gnomon of Zhou* 周髀注, 1 roll.[extant]

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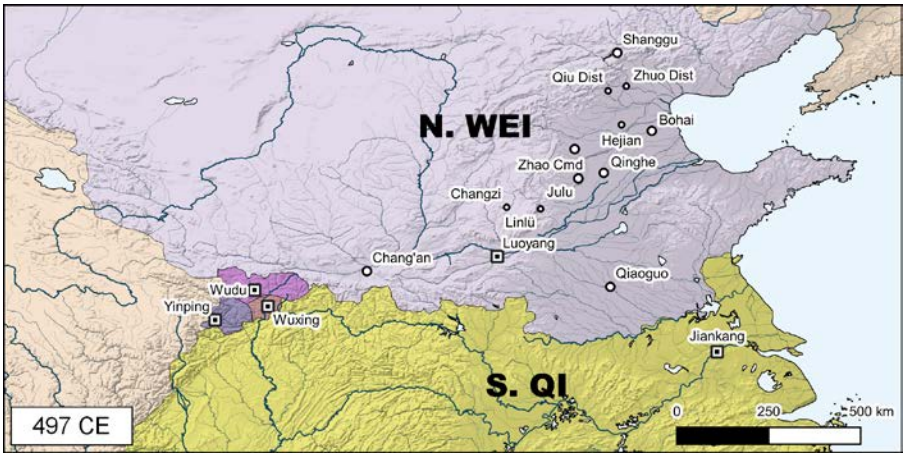

Pingcheng/Luoyang, 439–535 CE

Cui Hao 崔浩 (d. 450) of Qinghe,[bio] *Astronomical Lecture Notes* 曆疏, 1 roll.
———, *Astronomical Procedures* 曆術, 1 roll.
———, *Metrological and Astronomical Procedures* 律曆術, 1 roll.
———, *Five Yin$_{B03}$-Origin Astronomy* 五寅元曆, *c*.440 CE.
Gao Yun 高允 (390–487) of Bohai,[bio1];[bio2] *Mathematical Procedures* 算術, 3 rolls.
Gongsun Chong 公孫崇 (fl. 471–508) of Qiaoguo and Zhao Fansheng 趙樊生 (fl. 499–500), *Luminous Enlightenment Astronomy* 景明曆, 500 CE.
Li Mi 李謐 (484–515) of Zhao Cmd.,[bio1];[bio2] [a private astronomy 私曆], *c*.512 CE.
Li Yexing 李業興 (484–549) of Changzi,[bio1];[bio2] *Lecture notes on Seven Luminaries Astronomy* 七曜曆疏, 1 roll.
———, *Lecture Notes on the Meaning of the Seven Luminaries* 七曜義疏, 1 roll.
———, *Wuzi$_{25}$-Origin Astronomy* 戊子元曆, *c*.512 CE.
Long Yidi 龍宜弟 (fl. *c*.471), *Astronomy of [the Era of] Protracted Ascendency* 延興曆, *c*.471 CE.
Sun Senghua 孫僧化 (fl. 531), *Astronomy of [the Era of] Perpetual Peace* 永安曆, 1 roll, 528 CE.
Xindu Fang 信都芳 (d. 543/550) of Hejian,[bio] *Instrument Standards* 器準, 3 rolls.
———, *Commentary to the Four Procedures of the Gnomon of Zhou* 周髀四術注, 1 roll.
Yuan Yanming 元延明 (484–530) [bio] and Xindu Fang 信都芳 (d. 543/550) of Hejian,[bio1];[bio2] *Summary of the Ancestry of the Five Classics* 五經宗略 (mathematics), 23/40 rolls.
Zhang Hong 張洪 (fl. 500–515) of Luoyang, *Jiawu$_{31}$-Jiaxu$_{11}$ Double Origin Astronomy* 甲午甲戌二元曆, *c*.508 CE.
———, *Jiazi$_{01}$-Jihai$_{36}$ Double Origin Astronomy* 甲子己亥二元曆, *c*.512 CE.
Zhang Hong 張洪 (fl. 500–515) of Luoyang, Zhang Longxiang 張龍祥 (fl. 508–532) of Shanggu, Li Yexing 李業興 (484–549) of Changzi,[bio1];[bio2] Lu Daoqian 盧道虔 (d. 534) of Zhuo Dist,[bio1];[bio2] Wei Hongxian 衛洪顯 (fl. 518), Hu Rong 胡榮 (fl. 499–539), Tong Daorong 統道融 (fl. 518) of Linlü,[bio] Fan Zhongsun 樊仲遵 (fl. 518) of Luoyang, and Zhang Sengyu 張僧豫 (fl. 518) of Julu, *Astronomy of [the Era of] Orthodox Glory* 正光曆 (AKA *Astronomy of [the Era of] the Divine Tortoise* 神龜曆), 1 roll, 517 CE.[extant]
Zhang Longxiang 張龍祥 (fl. 508–532) of Shanggu, *Jiazi$_{01}$-Origin Astronomy* 甲子元曆, *c*.512 CE.

Zhang Mingyu 張明豫 (fl. 487–515) of Shanggu, *Astronomy of [the Era of] Great Harmony* 太和曆, 487 CE.
Zhang Qiujian 張丘建 of Qinghe , *Mathematical Classic of Zhang Qiujian* 張丘建算經, 431/484 CE.[extant]
Zu Ying 祖瑩 (fl. 512–534) of Qiu Dist,[bio1];[bio2] *Divine Tortoise Renzi$_{49}$-Origin Astronomy* 神龜壬子元曆, 1 roll, 517 CE.

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South, 317–589 CE

Ge Hong 葛洪 (283–343) of Jurong,[bio] *Discourse on Cosmology.[extant]
He Chengtian 何承天 (370–447) of Tan Dist,[bio] *Astronomy of [the Era of] Epochal Excellence* 元嘉曆 (*gengchen*$_{17}$ origin), 2 rolls, 444 CE.[extant]
———, *Astronomical Procedures* 曆術, 1 roll.
———, *Clepsydra Classic* 刻漏經, 1 roll.
———, *Methods for the Verification of Solar Eclipse* 驗日食法, 3 rolls.
———, 'Monograph on [the History of] Tono-metrology and Mathematical Astronomy' 律曆志, in Xu Yuan 徐爰 (394–475), *History of the Jin* 晉書.
Song Jing 宋景 (fl. 557/589), *Clepsydra Classic* 刻漏經, 1 roll.
Tao Hongjing 陶弘景 (456–536) of Moling,[bio] *The Essentials of Astronomical Instruments Explained* 天儀說要, 1 roll.
Wang Shuozhi 王朔之 of Langya, *Universal Astronomy* 通曆, 352 CE.[part.]
Wu Boshan 吳伯善 (fl. 557/589), *Seven Luminary Astronomy of the Chen* 陳七曜曆, 5 rolls.
Xiao Yan 蕭衍 (464–549) of Zhongdu Hamlet (AKA Emperor Wu of Liang), 'Lecture [on Cosmology] at the Hall of Eternal Spring' 長春殿講義.[extant]
———, *Companion to Bells and Pitchpipes* 鍾律緯.
Xu Guang 徐廣 (352–425) of Gumu,[bio1];[bio2] *Astronomy of the Seven Luminaries of the Past* 既往七曜曆, 371 CE.
Yu Kuo 虞劇, *Astronomy of [the Era of] Great Uniformity* 大同曆 (*jiazi*$_{01}$ origin), 1 roll, 544 CE.[part.]
Yu Shen 庾詵 (fl. 502) of Xinye,[bio1];[bio2] *Sequence (Chronology) of Emperors* 帝歷, 20 rolls.
Yu Xi 虞喜 (fl. 307–346) of Kuaiji,[bio] *On Secure Heaven [Cosmology]* 安天論, 6/1 rolls.
Zhang Kang 張亢 (fl. 317) of Anping,[bio] *Guide to Laying out Astronomy* 述曆贊, 1 roll.
Zhu Shi 朱史 (fl. 557–560), *Clepsydra Classic* 刻漏經, 1 roll.

———, *On Fixed Heaven [Cosmology]* 定天論, 3 rolls.
Zu Chongzhi 祖冲之 (429–500) of Ji Dist,[bio] *Astronomy of [the Era of] Great Enlightenment* 大明曆 (*jiazi*$_{01}$ origin), 1 roll, 465 CE.[extant]
———, *Clepsydra Classic* 漏刻經.
———, *[Mathematical] Procedures for Stitching* (?) 綴術, 6 rolls.
Zu Geng 祖暅 (fl. 504–525) of Ji Dist (?),[bio] *Clepsydra Classic* 漏刻經, 1 roll.
———, *Court Opinion on Geometrical Cosmology.[extant]
———, *Lü Ratios and Procedures for Weighing the Heaviness of Objects* 稱物重率術, 2 rolls.
———, *Record of Steelyards* 權衡記, 2 rolls.

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Chang'an, 535–579 CE

Li Yexing 李業興 (484–549) of Changzi,[bio1];[bio2] *Nine Palaces Moving Chess Piece Astronomy* 九宮行棊曆, 549 CE.
———, *Yellow Emperor Astronomy, Xinmao*$_{28}$ *[Origin]* 黃帝辛卯, 1 roll.
———, *Yin [Dynasty] Astronomy, Jiayin*$_{51}$ *[Origin]* 殷曆甲寅, 1 roll.
Li Yexing 李業興 (484–549) of Changzi,[bio1];[bio2] Wang Chun 王春 of Hedong, He Guixing 和貴興, Lu Daoyue 盧道約 (486–543) of Zhuo Dist,[bio1];[bio2] Li Xie 李諧 (496–544),[bio] Pei Xianbo 裴獻伯 (fl. 539–548), Wen Zisheng 溫子昇 (495–547) of Taiyuan,[bio1];[bio2] Lu Cao 陸操 (fl. 538–550) of Dai Cmd.,[bio] Lu Yuanming 盧元明 (fl. 528–539) of Zhuo Dist,[bio] Li Tonggui 李同軌 (d. 547/550) of Gaoyi,[bio1];[bio2] Xing Ziming 邢子明 (fl. 528–539) of Hejian, Yuwen Zhongzhi 宇文忠之 (fl. 534–543) of Luoyang,[bio1];[bio2] Yuan Zhongjun 元仲悛, Du Bi 杜弼 (491–559) of Quyang,[bio1];[bio2] Li Puji 李溥濟, Xin Shu 辛術 (d. 581) of Longxi,[bio] Yuan Changhe 元長和 of Luoyang, Hu Shirong 胡世榮 (fl. 499–539), Zhao Hongqing 趙洪慶 (fl. 499–539), Hu Fatong 胡法通 (fl. 499–539), Zhang Zhe 張喆, Cao Weizu 曹魏祖 (fl. 539–546), Guo Qing 郭慶, Hu Zhonghe 胡仲和, et al., *Astronomy of [the Era of] Ascendant Harmony* 興和曆 (*jiazi*$_{01}$ *origin*), 1 roll, 539 CE.[extant]
Ma Xian 馬顯 (fl. 579–581), *Astronomy of [the Era of] the Great Emblem* 大象曆 (*bingyin*$_{03}$ origin), 1 roll, 579 CE.[part.]

Ming Kerang 明克讓 (524–593) of Ge Dist,[bio1];[bio2] and Yu Jicai 庾季才 (d. 603) of Jiangling,[bio1];[bio2] *Zhou Astronomy* 周曆, 559 CE.
Wang Chen 王琛 (fl. 579), *Astronomical Procedures* 曆術, 1 roll.
———, *Astronomy of [the Era of] the Great Emblem* 大象曆, 2 rolls, 579 CE.
Xindu Fang 信都芳 (d. 543/550) of Hejian,[bio1];[bio2] *Astronomy of the Numinous Constitution* 靈憲曆, 543/550 CE.
Zhen Luan 甄鸞 (fl. 535–570) of Wuji Dist, *Astronomical Procedures* 曆術, 1 roll.
———, *Astronomy of [the Era of] Celestial Harmony* 天和曆 (*jiayin*$_{51}$ origin), 1 roll, 566 CE.[part.]
———, *Commentary to the Mathematical Classic of Master Sun* 孫子算經注, 3 rolls.
———, *Commentary to the Mathematical Classic of Xiahou Yang* 夏侯陽算經注, 3/1 roll.
———, *Commentary to the Mathematical Classic of Zhang Qiujian* 張丘建算經注, 1 roll.[extant]
———, *Commentary to Records of Remnants of Numbers and Procedures by Xu Yue (fl. 226)* 徐岳數術記遺注, 1 roll.[extant]
———, *Commentary to Three Grade Numbers by Dong Quan* 董泉三等數注, 1 roll.
———, *Gnomon of Zhou Renarrated* 周髀重述 (w/ comm. by Zhao Ying/Shuang, above), 1 roll.[extant]
———, *Mathematical Classic of the Five Bureaux* 五曹算經, 5/3 rolls.[extant]
———, *Mathematical Procedures of the Five Classics* 五經算術, 1 roll.[extant]
———, *Mathematical Procedures of Zhen Luan* 甄鸞算術.
———, *Nine Chapters on Mathematical Procedures Renarrated* 九章算術重述 (w/ comm. by Xu Yue), 9/1 rolls.
———, *Seven Luminaries Astronomical Calculations* 七曜曆算, 2 rolls.
———, *Seven Luminaries Arising Astronomy* 七曜本起曆, 5/3/2 rolls.
———, *Seven Luminaries Procedures and Calculations* 七曜術算, 2 rolls.

Ye, 534–577 CE

Liu Xiaosun 劉孝孫 (d. 594/597) of Guangping, *Jiazi*$_{01}$*-Origin Astronomy* 甲子元曆, 576 CE.[part.]
Song Jingye 宋景業 (fl. 543–550) of Guangzong,[bio1];[bio2] *Astronomy of [the Era of] Celestial Preservation* 天保曆 (*jiazi*$_{01}$ origin), 1 roll, 550 CE.[part.]
Wei Shou 魏收 (507–572) of Quyang,[bio1] 'Monograph on [the History of] Tono-metrology and Mathematical Astronomy' 律曆志, in adem., *History of the [Northern] Wei* 魏書, 4 rolls, 551 CE[extant].
Zhang Mengbin 張孟賓 (fl. 576–579) of Guangping, **Zhang Mengbin's Astronomy* 張孟賓曆, 576 CE[part.].
Zheng Yuanwei 鄭元偉 (fl. 576–581) and Dong Jun 董峻 (fl. 576–579), *Astronomy of [the Era of] Military Pacification* 武平曆 (*jiayin*$_{51}$ origin), 576 CE.[part.]

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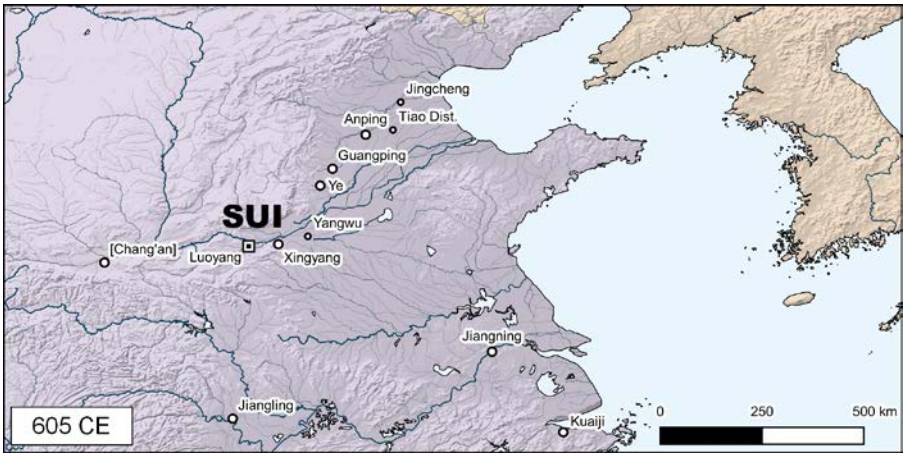

Sui (Daxing [Chang'an], Luoyang), 581–618 CE

Li Delin 李德林 (532–592) of Anping,[bio1];[bio2] *Astronomy of [the Era of] Opening Sovereignty* 隋開皇曆, 1 roll.

Lin Xiaogong 臨孝恭 (fl. 581/618) of Chang'an,[bio1];[bio2] *Classic of the Bronze Earthquake Instrument* 地動銅儀經, 1 roll.

———, *Diagrams of Strange [Metrological] Implements* 欹器圖, 3 rolls.

Liu Ping 劉憑 (fl. 581/618), *A Comprehensive Comparison of Foreign and Domestic Numeration* 外內傍通比校數法, 1 roll.

Liu Xiaosun 劉孝孫 (d. 594/597) of Guangping, *Astronomy of [the Era of] Opening Sovereignty* 隋開皇曆, 1 roll.

———, *Miscellaneous Procedures for the Seven Luminaries* 七曜雜術, 2 rolls.

———, *Tuning to Correct Pitch* 正聲調, 1 roll.

Liu Xuan 劉炫 (*c*.546–*c*.613) of Jingcheng,[bio] *Mathematical Procedures* 算術, 1 roll.

Liu You 劉祐 (fl. 581) of Xingyang,[bio1];[bio2] *Methods for the Handy Tables of the Four Seasons* 四時立成法, 1 roll.

———, *Miscellaneous Mathematical Text of the Nine Chapters* 九章雜算文, 2 rolls.

———, *Monograph on Safe Astronomy (?)* 安曆志, 12 rolls.

———, *Procedure Text(s) for Tono-Metro. and Mathematical Astronomy* 律曆術文, 1 roll.

Liu Zhuo 劉焯 (544–610) of Hejian,[bio] *Astron. Sovereign Pole* 皇極曆, 1 roll, *c*.605 CE.[extant]

———, *Astronomical Writings* 曆書, 10 rolls.

Mao Shuang 毛爽 (fl. 589) of Yangwu, *Tono-Metrological Register* 律譜.

Yin Gongzheng 尹公正 and Ma Xian 馬顯 (fl. 579–581), *Clepsydra Classic* 漏刻經.

Zhang Bin 張賓 (fl. 568–584), *Astronomical Procedures* 曆術, 1 roll.

———, *Classic of Seven Luminaries Astronomy* 七曜曆經, 4 rolls.

Zhang Bin 張賓 (fl. 568–584), Liu Hui 劉暉 (fl. 581–594), Dong Lin 董琳, Liu You 劉祐 (fl. 581) of Xingyang,[bio1];[bio2] Ma Xian 馬顯 (fl. 579–581), Zheng Yuanwei 鄭元偉 (fl. 576–581), Ren Yue 任悅 (fl. 581–597), Zhang Che 張徹, Zhang Yingzhi 張膺之, Heng Hongjian 衡洪建, Su Xiang 粟相, Guo Di 郭翟 (fl. 581–597), Liu Yi 劉宜 (fl. 581–594), Zhang Qianxu 張乾敍, Wang Junrui 王君瑞, Xun Longbo 荀隆伯 et al., *Jiazi$_{01}$-origin Astronomy of [the Era of] Opening Sovereignty* 開皇甲子元曆, 1 roll, 581 CE.[part.]

Zhang Zhouxuan 張冑玄 (d. 605/617) of Tiao Dist,[bio] *Astronomy of [the Era of] the Great Patrimony of the Sui* 隋大業曆 (*jiazi*$_{01}$ origin), 1 roll, 597 CE.[extant]

———, *Lecture Notes on Seven Luminaries Astronomy* 七曜曆疏, 5/3 rolls.

———, *Mysterious Astronomical Procedures* 玄曆術, 1 roll.