Banks, Sovereign Risk and Unconventional Monetary Policies

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E32, E44, E58, F34

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We develop a two-country model with an explicitly microfounded interbank market and sovereign default risk. Calibrated to the core and the periphery of the Euro Area, the model gives rise to a debt-banks-credit loop that substantially amplifies the effects of financial shocks, especially for the periphery. We use the model to investigate the effects of a stylized public asset purchase program at the steady state and during a crisis. We find that it is more effective in stimulating the economy during a crisis, in particular for the periphery.

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1 Introduction

In this paper, we analyze the interaction between an integrated interbank market and sovereign default risk using a two-country dynamic general equilibrium model, with a focus on the transmission of the recent financial crisis, and unconventional monetary policies in the Euro Area. Our model is rich of financial interactions in the intermediation process, and gives rise to a sovereign risk / interbank market feedback loop. Calibrated to the Euro Area, the latter amplifies the transmission of a large negative capital quality shock with respect to alternative models with financial frictions, especially for the periphery. We then use our model to assess the potential effectiveness of unconventional monetary policies in the form of an sovereign bonds purchase program. The program is more effective when implemented during a crisis, and stimulates macroeconomic conditions of the periphery more strongly. As such, the welfare gains of the program are larger when the latter is implemented during a crisis, and larger for the periphery.

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Interbank markets are at the crossroad of financial and real spheres, as they match creditor and debtor banks. Their dynamics crucially affect the amount of credit in the economy, with effects on investment and GDP. As such, they play a central role in the transmission of monetary policy decisions, as well as in the transmission of potential financial and sovereign debt crises. In particular, the rising interdependence between interbank and sovereign bonds markets was at the heart of ECB's concerns about rising sovereign risk in the Euro Area. It has also been partly exploited by ECB's unconventional monetary policies, to release tensions on both markets at the same time.

To capture this interdependence, we develop a two-country model of a monetary union with sovereign default risk, integrated sovereign and interbank markets and financial intermediaries. We particularly want to analyze the role of banks in the transmission of financial shocks to the economy and in the transmission of unconventional monetary policies. In the model, financial markets interact with the real economy through the balance sheet of banks. Saving banks collect deposits and optimize a portfolio made of domestic and foreign sovereign bonds and interbank loans. Commercial banks use interbank loans to grant loans to capital producers. Both types of banks face agency problems *à la* Gertler and Karadi (2011), that introduce constraints on leverage ratios and lead to a financial accelerator mechanism and endogenous asset spreads. These features generate a strong relation between developments on sovereign bond markets, bank liquidity, and loans, and foster macroeconomic and financial interdependence between both regions.

Our model offers a complex representation of funding in the economy, as we consider a large number of assets in our economy (sovereign bonds, interbank loans, capital) and heterogeneity in the banking system with two types of banks, leading to the existence of an interbank market. In addition, the model features two countries, whose banks interact on an area-wide liquidity (interbank) market and on area-wide sovereign bond markets. This brings our model closer to the situation of banks in the Euro Area. The model also features an *ad-hoc* non-linear risk premium, along the lines of Corsetti, Kuester, Meier and Mueller (2014) for the sovereign risk channel. Sovereign default risk is increasingly and positively related to a country's public debt-to-GDP ratio, and default matters *ex-ante* for the pricing of assets, but not *ex-post*. We differ from Corsetti et al. (2014) however, in that taxes used to stabilize the debt-to-GDP ratio distort labor supply.

First, the model is calibrated to represent the core and the periphery of the Euro Area, and simulated non-linearly under perfect foresight after a large negative capital quality shock that hits symmetrically both regions. The analysis of this artificial crisis shows that our specific assumptions (two types of banks, an interbank market and sovereign default risk) crucially amplify the quantitative responses of key macroeconomic variables. In the model, sovereign risk affects the equilibrium through two interacting channels: the interbank market channel and the labor income tax channel. The first channel relies on saving banks, that allocate their funds between interbank loans and sovereign bonds, giving rise to a flexible no-arbitrage condition that makes sovereign and interbank spreads co-move positively. When sovereign default risk rises because public debt rises in one region, interbank spreads rise in both countries, raising the cost of liquidity and hence the cost of productive capital in both regions. The second channel goes through the labor income tax rate. When sovereign default risk rises, the rollover cost of public debt rises and requires additional increments in the labor income tax rate that further depress the economy. We show that both channels amplify the macroeconomic effects of large negative shocks but that the first channel is clearly dominant compared to the second.

Second, we investigate the macroeconomic effects of a stylized asset purchase program inspired by the January 2015 announcement of the ECB. The program has positive effects on all macroeconomic quantities by significantly reducing spreads on financial markets. Those effects are larger for the periphery. Exploiting the non-linearity of our model simulations, we also find that the program is more effective and produces larger welfare gains when implemented during a crisis rather than at the steady state.

Our paper relates to some of the recent literature on sovereign default, interbank markets or unconventional monetary policies. For instance, important contributions address the relation between sovereign default risk and financial frictions, but do not consider interbank markets.[1] As interbank lending is the main resource for banks' short term funding, we believe it must be considered to understand its role in the transmission of sovereign default risk. Pioneer contributions introduce a micro-founded interbank market in DSGE models to study the effect of Central Bank's interventions on banks.[2] However, they do not address the potential transmission channel of sovereign default risk going through banks' balance sheets. The effects of unconventional monetary policies during the financial crisis and their interactions with the Zero Lower Bound (ZLB) have also been studied but most contributions remain silent about the potential role of interbank markets.[3]

Some recent contributions however relate more closely to our paper by considering an interbank market that interacts in some way with sovereign bond markets. Engler and Grosse-Steffen (2016) study the interbank-sovereign nexus for the case of Greece, considering that banks borrow from the interbank market using risky government bonds as collateral. A sovereign debt crisis erodes the quality the collateral and freezes the interbank market, resulting in an economic downturn. They investigate the effects of unconventional monetary policies (LTROs in their case) and find

[1]See for instance Guerrieri, Iacoviello and Minetti (2013), Mendoza and Yue (2012), Bi (2012), Gertler and Karadi (2011), Gertler and Karadi (2013), van der Kwaak and van Wijnbergen (2014), Bocola (2016) among others.

[2]See Allen, Carletti and Gale (2009), Dib (2010), Gertler and Kiyotaki (2010) or Boissay, Collard and Smets (2016).

[3]See Gertler and Karadi (2011), Gertler and Karadi (2013), Dedola, Karadi and Lombardo (2013), Takamura (2013), Diniz and Guimaraes (2014), Farhi and Tirole (2018) among others.

that they help mitigate the recession. While of great interest, this paper considers a small-open economy while we consider a two-country model, in which cross-country effects might be of importance. Further, sovereign risk is transmitted to the interbank market through the collateral value of sovereign bonds while we consider a transmission through flexible no-arbitrage conditions. Finally, while Engler and Grosse-Steffen (2016) focus on the effects of LTROs, we assess the effects of a stylized asset purchase program inspired by the more recent Asset Purchase Program (APP) launched by the ECB. Lakdawala, Minetti and Olivero (2018) also develop a two-country DSGE model with a micro-founded interbank. As in Engler and Grosse-Steffen (2016), sovereign bonds serve as collateral to interbank loans. Importantly, they find that the "interbank collateral channel" may mitigate the effects of negative capital quality shocks. One potential limitation of this interesting contribution is however that their interbank market is country-specific while we consider an integrated market, allowing for potential cross-country effects and a stronger interdependence through interbank liquidity. In addition, Lakdawala et al. (2018) propose a qualitative analysis of unconventional monetary policies while our paper provides an exercise that is potentially closer to actual programs implemented by the ECB. In any case, we consider our paper as highly complementary to those contributions.

The paper is organized as follows. Sections 2 and 3 respectively describe the model and present the calibration. Section 4 presents the various experiments: a one-time large negative capital quality shock, a sensitivity analysis and the analysis of the macroeconomic effects of a stylized asset purchase program. Section 5 offers concluding remarks.

2 Model

Our model of financial intermediation is an extension of the Gertler and Karadi (2011) model with two regions and two types of banks: saving banks (s) that collect deposits and lend on the interbank market and borrowing banks (b) that borrow on the interbank market and grant loans to capital producers. In addition to interbank loans, saving banks also have access to risky sovereign bonds to allocate their funds, which introduces a tight connection between the developments of sovereign risk and lending on the interbank market. Other agents and features of the model are standard and follow Gertler and Karadi (2011). The model presentation focuses on the core region, being understood that the periphery is characterized by symmetric conditions. Both regions are thus similar in their structures but differ in terms of calibration (size, productivity, taxes, debt levels, etc...).

2.1 Saving Banks

In each region, there is a unit continuum of saving banks. Their problem is solved in a two-step process. First, saving banks choose the optimal size of their total assets subject to the usual incentive compatibility constraint. Second, once the size of total asset is known, saving banks solve for the optimal portfolio that takes the form of a CES aggregate of various assets. As an alternative to our approach, one could have solved for the optimal portfolio directly. However, this would require using zero-order conditions for the optimal portfolio as in Dedola et al. (2013). The latter depend on the business cycle moments of the model, *i.e.* the variance-covariance matrix of asset returns that in turn depend on second-order moments of all other variables. This makes the problem at hand quite difficult for at least two reasons. First, the model is quite complicated and zero-order methods are somehow cumbersome. Second and more importantly, we do not want to initiate the discussion of the sources of business cycles in our model, or in any kind of business cycle moments considerations, as our focus is on the transmission of a large crisis in the Euro Area. Therefore, our solution of a CES bundle of assets is a tractable and a straightforward way of considering a multi-asset model.

Let us start with the problem of choosing the optimal size of the saving bank asset. The balance sheet of the representative saving bank is

a_t = total assets of saving banks	d_t = domestic deposits
	n_t^s = net worth

where a_t is a portfolio of assets that includes interbank lending, domestic and foreign sovereign bonds. The corresponding balance-sheet of equation of saving is

$$a_t = d_t + n_t^s \tag{1}$$

and their net worth evolves according to

$$n_{t+1}^s = r_{t+1}^a a_t - r_t^d d_t + T_t^b \tag{2}$$

where r_t^a is the real composite return on their portfolio a_{t-1} between $t-1$ and t – to be defined later, r_t^d is the real deposit rate and T_t^b is a transfer from the government covering their losses in case of sovereign default.[4] Combining both equations gives the dynamics of the saving bank's net worth

$$n_{t+1}^s = \left(r_{t+1}^a - r_t^d\right) a_t + r_t^d n_t^s + T_t^b \tag{3}$$

The bank maximizes expected net worth given a fixed exit probability $(1-\sigma)$, in which event net worth is rebated to the households, and discounts future outcomes at the stochastic rate $\beta_{t,t+1} = \beta u_{c,t+1}/u_{c,t}$:

$$v_t^s = E_t\left\{\beta_{t,t+1}\left[(1-\sigma)\, n_{t+1}^s + \sigma v_{t+1}^s\right]\right\} \tag{4}$$

In addition, to prevent unlimited expansion of lending due to positive arbitrage opportunities, the representative saving bank may divert a fraction α^s of its assets. This possibility adds the

[4] In our model, as in Corsetti et al. (2014), default only matters *ex-ante* but not *ex-post*.

following incentive constraint on saving banks' activities

$$v_t^s \geq \alpha^s a_t \tag{5}$$

which will be strictly binding in equilibrium. The initial guess for the value function is

$$v_t^s = \gamma_t^a a_t + \gamma_t^s n_t^s \tag{6}$$

which allows to simplify the constraint to

$$\phi_t^s n_t^s \geq a_t \tag{7}$$

where $\phi_t^s = \gamma_t^s / (\alpha^s - \gamma_t^a)$ is the saving bank leverage ratio. This equation shows that banks are constrained in the leverage ratio they can apply. It depends negatively on γ_t^s the shadow value of net worth, as households cut funds supply in expectation of larger fund diversion, and positively on γ_t^a, the shadow value of assets, as larger profits are expected from an additional value of assets, making the constraint looser. Substituting the constraint in the guessed expression for the value function yields $v_t^s = (\gamma_t^a \phi_t^s + \gamma_t^s) n_t^s$ and plugging into the value function after using the accumulation of net worth n_{t+1}^s we get

$$\begin{aligned} v_t^s &= E_t \left\{ \Lambda_{t,t+1}^s n_{t+1}^s \right\} && (8) \\ &= E_t \left\{ \Lambda_{t,t+1}^s \left[\left(r_{t+1}^a - r_t^d \right) a_t + r_t^d n_t^s \right] \right\} && (9) \end{aligned}$$

where $\Lambda_{t,t+1}^s = \beta_{t,t+1} \left[1 - \sigma + \sigma \left(\gamma_{t+1}^a \phi_{t+1}^s + \gamma_{t+1}^s \right) \right]$. This allows to identify the arguments of the value function:[5]

$$\gamma_t^a = E_t \left\{ \Lambda_{t,t+1}^s \left(r_{t+1}^a - r_t^d \right) \right\} \text{ and } \gamma_t^s = E_t \left\{ \Lambda_{t,t+1}^s r_t^d \right\} \tag{12}$$

Once the saving banks have chosen their total level of assets, they solve the portfolio problem. The portfolio a_t is a CES function of interbank loans l_t^s, local bonds b_t and foreign bonds b_t^*, paying respectively the nominal interbank market rate r_t, and the nominal returns on government bonds $r_t^b (1 - \chi_t)$ and $r_t^{b*} (1 - \chi_t^*)$ between period $t-1$ and period t. As the interbank market and the sovereign bond markets are integrated, the nominal rates r_t, r_t^b and r_t^{b*} are common to both countries. Further, in these expressions, χ_t and χ_t^* are the potential hair-cuts applied by governments in case of default risk, to be specified later. The CES specification is rather common

[5]Finally, expressing γ_t^a and γ_t^a recursively using the expression of $\Lambda_{t,t+1}^s$ and the equation for the dynamics of net worth n_t^s:

$$\begin{aligned} \gamma_t^a &= E_t \left\{ \beta_{t,t+1} \left[(1-\sigma) \left(r_{t+1}^a - r_t^d \right) + \sigma \gamma_{t+1}^a \phi_{t+1}^s \left(\left(r_{t+1}^a - r_t^d \right) \phi_t^s + r_t^d \right) / \phi_t^s \right] \right\} && (10) \\ \gamma_t^s &= E_t \left\{ 1 - \sigma + \beta_{t,t+1} \sigma \left(\left(r_{t+1}^a - r_t^d \right) \phi_t^s + r_t^d \right) \gamma_{t+1}^s \right\} && (11) \end{aligned}$$

which are exactly those of Gertler and Karadi (2011).

in the international finance literature, as explained in details by Alpanda and Kabaca (2015).[6] It is a flexible and tractable way of pinning down portfolio shares, to account for potentially imperfect substitutability among assets, and to model the preferences of investors. We define q_t^s, q_t^b and q_t^{b*} as the real prices of assets and q_t^a as the real price of the portfolio. The optimal allocation on the various assets is obtained by minimizing total expenditure

$$q_t^a a_t = q_t^s l_t^s + q_t^b b_t + q_t^{b*} b_t^* \tag{13}$$

for a given amount of asset $\overline{a}$, given the following CES specification

$$a_t = \left(\mu^{1/\varepsilon} \left(l_t^s\right)^{(\varepsilon-1)/\varepsilon} + \eta^{1/\varepsilon} b_t^{(\varepsilon-1)/\varepsilon} + (1-\mu-\eta)^{1/\varepsilon} b_t^{*(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \tag{14}$$

In this equation, μ and η are the steady-state relative weights of interbank loans and domestic sovereign bonds in the portfolio, and ε is the elasticity of substitution between assets. Given that real asset prices are inversely related to their real expected rates of return, the optimal allocation of funds that results from the saving banks choice is thus

$$l_t^s = \mu E_t \left\{ \left(\frac{r_t/\pi_{t+1}}{r_t^a} \right)^{\varepsilon} \right\} a_t \tag{15}$$

$$b_t = \eta E_t \left\{ \left(\frac{r_t^b \left(1-\chi_t\right)/\pi_{t+1}}{r_t^a} \right)^{\varepsilon} \right\} a_t \tag{16}$$

$$b_t^* = (1-\mu-\eta) E_t \left\{ \left(\frac{r_t^{b*} \left(1-\chi_t^*\right)/\pi_{t+1}}{r_t^a} \right)^{\varepsilon} \right\} a_t \tag{17}$$

where π_t is the inflation rate in the core region at time t, and the real composite portfolio return is

$$r_t^a = \left(\mu E_t \left\{ \left(\frac{r_t}{\pi_{t+1}} \right)^{\varepsilon-1} \right\} + \eta E_t \left\{ \left(\frac{r_t^b \left(1-\chi_t\right)}{\pi_{t+1}} \right)^{\varepsilon-1} \right\} + (1-\mu-\eta) E_t \left\{ \left(\frac{r_t^{b*} \left(1-\chi_t^*\right)}{\pi_{t+1}} \right)^{\varepsilon-1} \right\} \right)^{\frac{1}{\varepsilon-1}}$$

2.2 Commercial Banks

In each region, there is also a unit continuum of commercial banks. The representative bank borrows l_t^c from the interbank market, and accumulates net worth. On the asset side, it grants loans to the intermediate goods sector to purchase capital k_t at price q_t. Its balance sheet is thus

$q_t k_t$ = loans to the private sector	l_t^c = borrowing from the interbank market
	n_t^c = net worth

and the balance sheet equation is

$$q_t k_t = l_t^c + n_t^c \tag{18}$$

[6] See also Coeurdacier and Martin (2009) for a similar specification of the asset portfolio.

Net worth evolves according to

$$n_{t+1}^c = r_{t+1}^k q_t k_t - r_{t+1} l_t^c \tag{19}$$

where r_t^k is the return on capital. Combining both equations gives the dynamics of the representative commercial bank's net worth

$$n_{t+1}^c = \left(r_{t+1}^k - r_{t+1}\right) q_t k_t + r_{t+1} n_t^c \tag{20}$$

Commercial banks solve a similar problem as saving banks, so we do not duplicate the derivation of their value function and optimization problem. We define γ_t^k and γ_t^c as the respective weights of their capital stock and net worth in the value function, and α^c as the fraction of their balance sheet they can divert. The incentive constraint binds in equilibrium and writes

$$q_t k_t = \phi_t^c n_t^c \tag{21}$$

where $\phi_t^c = \gamma_t^c / \left(\alpha^c - \gamma_t^k\right)$ and

$$\gamma_t^k = E_t \left\{\Lambda_{t,t+1}^c \left(r_{t+1}^k - r_{t+1}\right)\right\} \text{ and } \gamma_t^c = E_t \left\{\Lambda_{t,t+1}^c r_{t+1}\right\} \tag{22}$$

with $\Lambda_{t,t+1}^c = \beta_{t,t+1}\left[1 - \sigma + \sigma\left(\gamma_{t+1}^k \phi_{t+1}^c + \gamma_{t+1}^c\right)\right]$.

2.3 Intermediate and capital goods producers

Intermediate goods producers use effective capital $u_t k_{t-1}$ in the production process, where u_t is the variable utilization rate. They also hire labor in quantity ℓ_t, that they combine to build the intermediate good, with the following production function

$$y_t^m = \varsigma_t \left(\xi_t u_t k_{t-1}\right)^\iota \ell_t^{1-\iota} \tag{23}$$

and sell intermediate goods at real relative price p_t^m. The installed (*i.e.* period $t-1$) effective capital stock can also be affected by a quality shock ξ_t. The optimizing conditions with respect to labor and utilization respectively give

$$p_t^m \left(1 - \iota\right) y_t^m / \ell_t = w_t \tag{24}$$

$$p_t^m \iota y_t^m / u_t = \delta'\left(u_t\right) \xi_t k_{t-1} \tag{25}$$

where w_t is the real wage and where

$$\delta\left(u_t\right) = \delta + \overline{\delta}\left(u_t^{1+\kappa} - 1\right) / \left(1 + \kappa\right) \tag{26}$$

is the time-varying depreciation rate. The zero-profit condition implies that intermediate goods producers pay the *ex-post* return on capital to the capital goods producers, *i.e.*

$$r_{t+1}^{k} = \left(p_{t+1}^{m}\left(\iota y_{t+1}^{m}/k_t\right) + q_{t+1}\xi_t\left(1-\delta\left(u_{t+1}\right)\right)\right)/q_t \tag{27}$$

Capital goods producers buy the depreciated capital of intermediate goods producers and choose investment to accrue the total amount of available capital based on the evolution of its real price q_t.[7] Their profits write

$$E_t\sum_{s=0}^{\infty}\beta_{t+s,t+s+1}\left(q_{t+s}i_{t+s}\left(1-\left(\varphi^i/2\right)\left(i_{t+s}/i_{t+s-1}-1\right)^2\right)-i_{t+s}\right) \tag{30}$$

and optimization yields

$$q_t - 1 = q_t\varphi^i\left(x_t\left(1+x_t\right)+x_t^2/2\right) - E_t\left(\beta_{t,t+1}q_{t+1}\varphi^i x_{t+1}\left(1+x_{t+1}\right)^2\right) \tag{31}$$

where $x_t = i_t/i_{t-1} - 1$. Given this optimizing condition for investment, the law of capital accumulation gives the dynamics of the capital stock

$$k_t - \left(1-\delta\left(u_t\right)\right)\xi_t k_{t-1} = i_t\left(1-\left(\varphi^i/2\right)x_t^2\right) \tag{32}$$

2.4 Final goods producers

Final goods producers j differentiate the intermediate good y_t^m in imperfectly substitutable varieties. The aggregate bundle of the final good and the corresponding aggregate price level are

$$y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\theta-1}{\theta}}dj\right]^{\frac{\theta}{\theta-1}},\ p_t = \left[\int_0^1 p_t\left(j\right)^{1-\theta}dj\right]^{\frac{1}{1-\theta}} \tag{33}$$

Final goods producers take into account the demand for variety j $y_t\left(j\right) = \left(p_t\left(j\right)/p_t\right)^{-\theta}y_t$ when setting prices subject to Calvo price contracts of average length $1/\left(1-\gamma\right)$ with indexation to past inflation γ^p. The optimal pricing conditions are relatively standard and therefore not reported.

[7] More formally, they maximize

$$E_t\sum_{s=0}^{\infty}\beta_{t+s,t+s+1}\left(q_{t+s}\left(k_{t+s}-\left(1-\delta\left(u_{t+s}\right)\right)\xi_{t+s}k_{t+s-1}\right)-i_{t+s}\right) \tag{28}$$

subject to the law of motion of capital accumulation

$$k_t - \left(1-\delta\left(u_t\right)\right)\xi_t k_{t-1} = i_t\left(1-\left(\varphi^i/2\right)\left(i_t/i_{t-1}-1\right)^2\right). \tag{29}$$

2.5 Households

Households face a simple optimization problem as they choose consumption, labor supply and deposits maximizing lifetime welfare

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s u \left(c_{t+s}, \ell_{t+s} \right) \right\} \tag{34}$$

where $u_{\ell,t} \leq 0$ and $u_{c,t} \geq 0$ are the first-order partial derivatives with respect to hours worked and consumption, subject to the budget constraint

$$d_t + c_t = r_{t-1}^d d_{t-1} + (1 - \tau_t) w_t \ell_t + \Pi_t \tag{35}$$

where d_t denote deposits to saving banks returning r_t^d between t and $t+1$, c_t is consumption, w_t denotes the real wage, τ_t a distortionary tax on labor income, ℓ_t hours worked, and Π_t comprises monopolistic profits from final goods producers, and the net worth rebated by bankrupt banks, net from the starting fund allocated to new banks. First-order conditions give

$$E_t \left(\beta_{t,t+1} r_t^d \right) = 1 \tag{36}$$

$$u_{\ell,t} + (1 - \tau_t) u_{c,t} w_t = 0 \tag{37}$$

2.6 Governments

We adopt the approach of sovereign default from Corsetti et al. (2014). Actual *ex-post* default is neutral while the *ex-ante* probability of default is the key for the pricing of government bonds, which has direct impacts on the interest rates, credit spreads, sustainability of the country's indebtedness, and GDP growth. As in the literature, we assume that the default risk follows a distribution that is non-linearly related to the country's debt-to-GDP ratio.[8] The *ex-ante* probability of default Φ_t, at a certain level of sovereign indebtedness, $by_t = b_t^g / (4y_t)$, will be given by the cumulative distribution function of the beta distribution:

$$\Phi_t = F_{beta} \left(by_t / by_{max}, \alpha_p, \beta_p \right) \tag{38}$$

where by_{max} denotes the upper end of the support for the debt-to-GDP ratio. Actual default occurs with probability Φ_t so that

$$\chi_t = \Delta \text{ if } \mathcal{B}(\Phi_t) = 1 \tag{39}$$

$$\chi_t = 0 \text{ if } \mathcal{B}(\Phi_t) = 0 \tag{40}$$

[8] See for example, Eaton and Gersovitz (1981), Arellano (2008), Bi (2012), and Corsetti et al. (2014).

where $\mathcal{B}(\Phi_t)$ is a Bernoulli. Given these assumptions, the budget constraint of the government writes

$$b_t^g = r_t^b (1 - \chi_t) b_{t-1}^g / \pi_t + g_t - \tau_t w_t \ell_t + T_t^b \tag{41}$$

where b_t^g is real the level of debt, being understood that bonds and returns are nominal. Once again, potential losses from default are fully compensated *ex-post*, so that only *ex-ante* default risk matters. As a consequence

$$T_t^b = r_t^b \chi_t b_{t-1}^g / \pi_t \tag{42}$$

and the consolidated budget constraint writes

$$b_t^g = r_t^b b_{t-1}^g / \pi_t + g_t - \tau_t w_t \ell_t \tag{43}$$

The stability of public debt in the long run is granted by the following tax rule

$$\tau_t - \tau = \rho_\tau (\tau_{t-1} - \tau) + (1 - \rho_\tau) d_b (b_t^g / y_t - b^g / y) \tag{44}$$

Although actual default is not considered in our set-up, sovereign default risk has major real consequences. First, sovereign default risk raises sovereign spreads, and pushes interbank spreads up, which then lowers interbank lending and therefore economic activity. Second, a rise in default risk feeds a rise in public debt that subsequently triggers a rise in the distortionary tax rate. As the latter goes up, hours worked, output, investment, asset prices and inflation collapse. So even in the absence of actual default, sovereign default risk can be a major driver of macroeconomic conditions through these two channels.

2.7 Central bank

The Central Bank controls the common nominal interest rate i_t^n, subject to a constraint on the lower bound of this rate. The relation between the nominal rate and national deposit rates is

$$i_t^n = r_t^d E_t (\pi_{t+1}) = r_t^{d*} E_t \left(\pi_{t+1}^*\right) \tag{45}$$

The Central Bank commits to the following policy rule

$$\log i_t^n = \max \left(0, \rho_i \log i_{t-1}^n + (1 - \rho) (\log i^n + d_\pi \log \pi_t^u + d_y (\log y_t^u - \log \widetilde{y}_t^u))\right) \tag{46}$$

where π_t^u is the union-wide inflation rate and y_t^u the union-wide level of output, $\widetilde{y}_t^u$ being its natural level.[9]

[9] Variations in the union-wide mark-up serve as a proxy for variations in the union-wide output gap.

2.8 Aggregation

Banking sector. At the end of the period, a fraction $1-\sigma$ of each type of bankers becomes households. Dividends are paid to households only when bankers exit. The net worth of continuing bankers is simply carried to the next period, so that aggregate continuing banks net worth evolve according to

$$n_t^{e,s} = \sigma\left(\left[\left(r_t^a - r_{t-1}^d\right)\phi_{t-1}^s + r_{t-1}^d\right]n_{t-1}^s + T_t^b\right) \quad (47)$$

$$n_t^{e,c} = \sigma\left[\left(r_t^k - r_t\right)\phi_{t-1}^c + r_t\right]n_{t-1}^c \quad (48)$$

In addition, households provide a starting net worth to new banks, equal to a fraction $\Upsilon^s/(1-\sigma)$ or $\Upsilon^c/(1-\sigma)$ of the total assets of old exiting bankers, so that the net worth of new banks are

$$n_t^{n,s} = \Upsilon^s a_t \quad (49)$$

$$n_t^{n,c} = \Upsilon^c \xi_t q_t k_{t-1} \quad (50)$$

Overall, aggregate net worth evolve according to

$$n_t^s = n_t^{e,s} + n_t^{n,s} \quad (51)$$

$$n_t^c = n_t^{e,c} + n_t^{n,c} \quad (52)$$

Goods markets. The clearing condition on the intermediate goods market is

$$y_t^m = \int_0^1 y_t(j)\,dj = y_t dp_t \quad (53)$$

where $dp_t = \int_0^1 \left(p_t(j)/p_t\right)^{-\theta} dj$ is the dispersion of prices. On the final goods market, the clearing condition simply writes

$$y_t = c_t + i_t + g_t \quad (54)$$

Financial markets. Given that the interbank market is integrated within the monetary union, the market clearing condition is

$$l_t^s + \varrho l_t^{s*} = l_t^c + \varrho l_t^{c*} \quad (55)$$

where ϱ is the relative size of the foreign economy. This equation determines the nominal interbank market rate r_t.[10] Finally, government bonds markets are also integrated within the

[10]The nominal rate on the interbank market is common but real returns on the interbank market are country-specific given that real interbank rates are computed using the common nominal rate but country-specific inflation rates. A similar structure prevails for sovereign returns.

monetary union, and the corresponding clearing conditions are

$$b_t^g = b_t + \varrho b_{*,t} \tag{56}$$

$$\varrho b_t^{g*} = \varrho b_t^* + b_{*,t}^* \tag{57}$$

where $b_{*,t}$ and $b_{*,t}^*$ are the holdings of local and foreign debt (respectively) from savings banks in the periphery. These two conditions determine the nominal sovereign rates r_t^b and r_t^{b*}.

3 Calibration

We calibrate the model to the Euro Area. The periphery comprises Portugal, Ireland, Italy, Greece and Spain while the core is made of remaining members of the monetary union. The calibration builds on Gertler and Karadi (2011) unless stated otherwise. The time unit is a quarter. The functional form of preferences is

$$u(c_t, n_t) = \log(c_t - hc_{t-1}) - \omega \ell_t^{1+\psi}/1+\psi$$

The discount factor is imposed such that the deposit rate corresponds to its average value of at the end to 2007. Based on ECB data, the later was 4% annually, implying $\beta = 0.9902$. The degree of habits in consumption is $h = 0.815$ and the inverse of the Frisch elasticity on labor supply is $\psi = 3$. This value aims at capturing relatively sluggish labor markets in the Euro Area.

On the production side, the share of effective capital is $\iota = 0.33$, the steady-state depreciation rate is $\delta = 0.018$ (7% annually), and the elasticity of the marginal depreciation rate to utilization is $\kappa = 7.2$.[11] Again, based on ECB data for the rate on loans to non-financial corporations, we impose the steady-state value of r^k in both regions which pins down capital to output ratios. The latter was roughly 5.5% annually at the end of 2007. The investment adjustment cost parameter is $\varphi_i = 1.728$, Calvo parameters are $\gamma = 0.779$ and $\gamma^p = 0.241$ and the steady-state mark-up is 30%, implying $\theta = 4.33$.

On the monetary and fiscal policy side, we follow Corsetti et al. (2014) for the parameters of the default probability function and default size: $\alpha_p = 3.70$, $\beta_p = 0.54$, $by_{max} = 2.56$ and $\Delta = 0.55$. We assume standard Taylor rule parameters, *i.e.* $\rho_i = 0.8$, $d_\pi = 2$ and $d_y = 0.125$. We set the parameter in Equation (44) at $d_b = 0.25$ to ensure the stability of debt to GDP in the medium run. The persistence parameter in Equation (44) is $\rho_\tau = 0.85$.

In the banking sector, as explained in Appendix A, we impose steady-state leverage ratios $\phi^s = \phi^c = 2.5$ both for saving and commercial banks. This value is taken from ECB data for the

[11]Notice that $\bar{\delta}$ is adjusted for the steady-state optimal utilization rate equation to be consistent with the steady-state capital return equation.

aggregate balance sheet of Monetary and Financial Institutions (MFI excluding the Eurosystem). Assets that are not considered in the model are excluded from the data before computation. In addition, we choose not to impose heterogeneity in the banking sector, except for the home bias towards public debt in the portfolios of saving banks. Based on ECB and OECD data for 2007, we calibrate the interbank overnight rate r at 4.25% annually, and sovereign rates r^b and r^{b*} at 4% annually in the core and 4.5% annually in the periphery. In the portfolios of saving banks, we apportion the steady-state holdings of government debt to periphery and core banks following Guerrieri et al. (2013). For the core region, the share of domestic debt held by domestic agents reaches 81%, implying $\eta = 0.81\,(1-\mu)$ and the share of public debt issued in the periphery that is held domestically is 60.5%, implying $\eta = 0.605\,(1-\mu)$. As explained in Appendix A, this calibration strategy implies an adjustment of the share of interbank loans in the portfolio, as well as an adjustment of the elasticity of substitution ε. However, a continuum of (μ, ε) can be chosen although both parameters are not free. This allows us to look at potentially different values of ε, a crucial parameter. In any case our baseline calibration implies $\varepsilon = 1094$ and $\mu = 0.1898$. The former value implies close-to-perfect substitutability among assets held by saving banks while the latter value implies a share of interbank loans of roughly 20%, which is close to observed values. Finally, we set the survival probability of bankers at $\sigma = 0.975$.

Remaining parameters are region-specific and are set based on computations from the data. Using OECD data for 2008, we build subgroup measures of hours worked and find $\ell = 0.2520$ for the core region and $\ell^* = 0.3049$ for the periphery. Proceeding similarly, we impose the share of public expenditure in GDP in each region: $s_g = 0.2080$ for the core region and $s_g^* = 0.1924$ for the periphery. Debt to GDP ratios are also imposed and we assume $b^g/(4y) = 0.6542$ in the core region and $b^{g*}/(4y^*) = 0.7718$ in the periphery.[12] In addition, we impose a higher productivity in the core region, where we assume $\varsigma = 1.2$ while we set $\varsigma^* = 1$ in the periphery. Steady-state labor income tax rates are adjusted to satisfy the budget balance of governments, implying $\tau = 0.4536$ in the core region and $\tau^* = 0.4396$ in the periphery. All variables are considered *per capita* but aggregate variables enter in the debt and interbank market clearing equations so we need to fix the relative size of regions. Based on relative GDPs, we normalize the relative size of the periphery at $\varrho = 0.5959$.

4 Experiments

Various versions of the model are simulated under perfect foresight using a non-linear Newton-type method over 500 periods under various assumptions for shocks.[13] First, we first contrast the dynamics of the model after a capital quality shock with the dynamics produced by a standard

[12] See Appendix B for details.

[13] The algorithm is a built-in routine of Dynare. It is an application of the Newton-Raphson algorithm that takes into consideration the special structure of the Jacobian matrix in dynamic models with forward-looking variables. The details of the algorithm are explained in Juillard (1996)

Table 1: Parameter values

Discount factor, β		0.9902
Habit formation, h		0.815
Inverse of the Frisch elasticity, ψ		3
Steady-state depreciation rate of capital, δ		0.018
Production function, capital parameter, ι		0.33
Steady-state depreciation rate of capital, δ		0.018
Elasticity of the depreciation rate to utilization rate, κ		7.2
Private spreads, r^k/r^d		1.0033
Core sovereign spread, r^b/r^d		1
Peripheric sovereign spread, r^{b*}/r^d		1.0012
Interbank spread, r/r^d		1.0006
Investment adjustment costs, φ_i		1.728
Calvo contracts parameter, γ		0.779
Indexation parameter, γ^p		0.241
Steady-state mark-up, $\theta/(\theta-1)$		1.3
Taylor rule parameter, ρ_i		0.8
Taylor rule parameter, d_π		2
Taylor rule parameter, d_y		0.125
Fiscal rule parameter, d_b		0.25
Tax rule persistence, ρ_τ		0.85
Default probability parameter, α_p		3.70
Default probability parameter, β_p		0.54
Default probability parameter, by_{max}		2.56
Default size, Δ		0.55
Savings banks leverage ratio, ϕ^s		2.5
Comm. banks leverage ratio, ϕ^s		2.5
Share of interbank lending in the portfolio, μ		0.1898
Banker's survival probability, σ		0.975
Elasticity of subs. in the portfolio of saving banks, ε		1094
	Core	Peri.
Fraction of time spent working, ℓ	0.2520	0.3049
Productivity scaling factor, ς	1.2000	1.0000
Government debt to annual GDP, $b^g/(4y)$	0.6542	0.7718
Labor income tax rate, τ	0.4536	0.4396
Government spending to GDP, s_g	0.2080	0.1924
Relative size of the periphery, ϱ	–	0.5959
Share of domestic debt in the portfolio, η	0.6563	0.4902

two-country version of the Gertler and Karadi (2011) model without an interbank set-up. Two version of the Gertler and Karadi (2011) model are considered, one with sovereign risk and one without sovereign risk. This allows for a clear separation of the respective contributions of the interbank and sovereign risk channels. A sensitivity analysis is conducted, with respect to the nature of taxes, and with respect to the elasticity of substitution between assets in the portfolio of saving banks. Second, we evaluate the macroeconomic and welfare effects of a stylized asset purchase program targeted at sovereign bonds. We exploit our non-linear simulations to contrast their effects when implemented at the steady state or conditional on a crisis, generated by a negative capital quality shock that lowers output in both countries roughly as much as during the 2008 financial crisis. We show that the program is potentially more effective in stimulating the economy when implemented during a crisis, and quantify its effects on households' welfare.

4.1 Capital quality shock

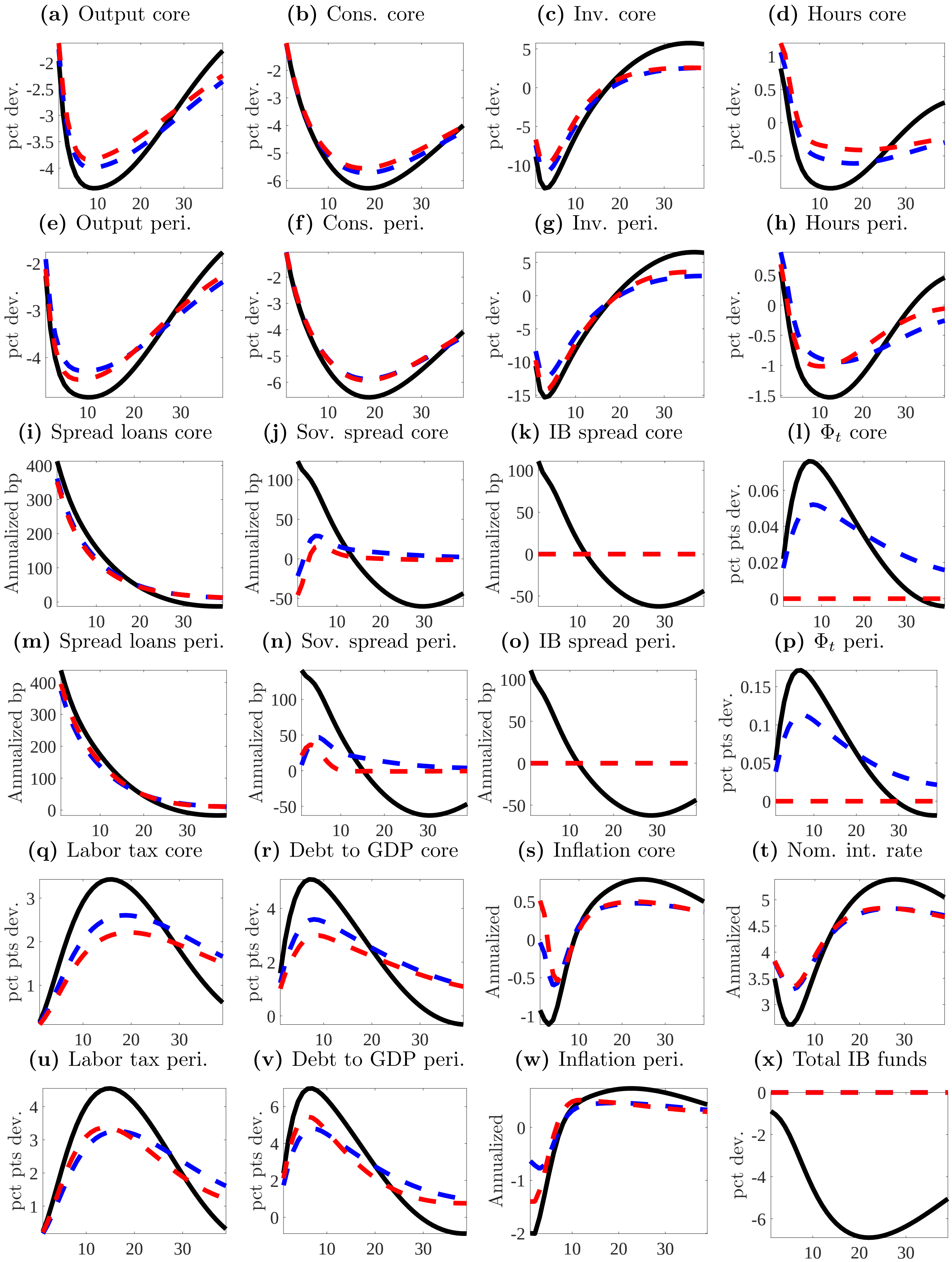
We model an artificial crisis as a negative and unexpected shock to the quality of the effective capital stock ξ_t. More precisely, we assume $\xi_t = (1 - \rho_\xi) + \rho_\xi \xi_{t-1} + s_{\xi,t}$ and feed the model with $s_{\xi,t} = -0.08$ assuming $\rho_\xi = 0.33$. The shock affects the quality of the capital stock of both regions, core and periphery. We compare our model with two model variants: one that neglects the saving banking sector but allows for sovereign default risk, and one that neglects the saving banking sector and abstracts from sovereign risk, to evaluate the relative importance of the interbank channel and the labor income tax channel. In any case, we assume that public spending remain constant $g_t = g$.

Basically, restricted versions of the model amount to neglect the equations that relate to saving banks and consider that commercial banks use deposits directly – instead of interbank loans – to grant loans to capital producers. Consequently, the deposit rate enters in the first-order conditions of commercial banks and replaces the interbank rate. Further, the equilibrium of sovereign bonds markets is modified since savings banks do not buy them anymore. We thus assume that sovereign bonds are held by households and priced through a standard Euler equation. Finally, shutting down the sovereign default risk channel simply amounts to assume $\Phi_t = \Phi$ and $\Delta = 0$.

Figure 1 reports the dynamics of our baseline and restricted models with or without sovereign risk for the both regions. Quantities are reported in percentage deviations from their steady-state values, rates are reported in percents *per annum*, ratios and tax rates in percentage point deviations and spreads in annual basis-points deviations from their steady-state values.

Figure 1 shows that our assumptions of heterogeneity in the banking sector (savings vs. commercial banks) and of sovereign default risk both act as amplifiers of the shock. As the shock generates a large economic downturn characterized by a large fall in GDP, debt to GDP rises, which in turn raises the default probability. Equilibrium on sovereign bonds markets requires

Figure 1: Effects of a negative capital quality shock.



Solid black: baseline. Dotted blue: restricted model (no interbank market) with sovereign risk. Dotted red: restricted model without sovereign risk. Variable Φ_t denotes the *ex-ante* sovereign default probability.

that governments offer larger returns, which raises the interbank rate, and hence the rate at which commercial banks grant loans. The loan rates and the sovereign rates rise more than the deposit and the interbank rates respectively, leading private, interbank and sovereign spreads to increase significantly.

Comparing the dynamics of the model with the restricted model with no interbank market but with sovereign risk shows that the banking structure is extremely important in amplifying the effects of the shock, much more than distortionary tax channel. Indeed, even with sovereign risk, the restricted model does not provide a very large amplification after the shock. In this version of the model, the shock has larger consequences since the rise in debt to GDP produces a rise in the sovereign spread that feeds back in the model only through the channel of higher distortionary taxes. However, this channel appears to be quantitatively small compared to the interbank market channel.

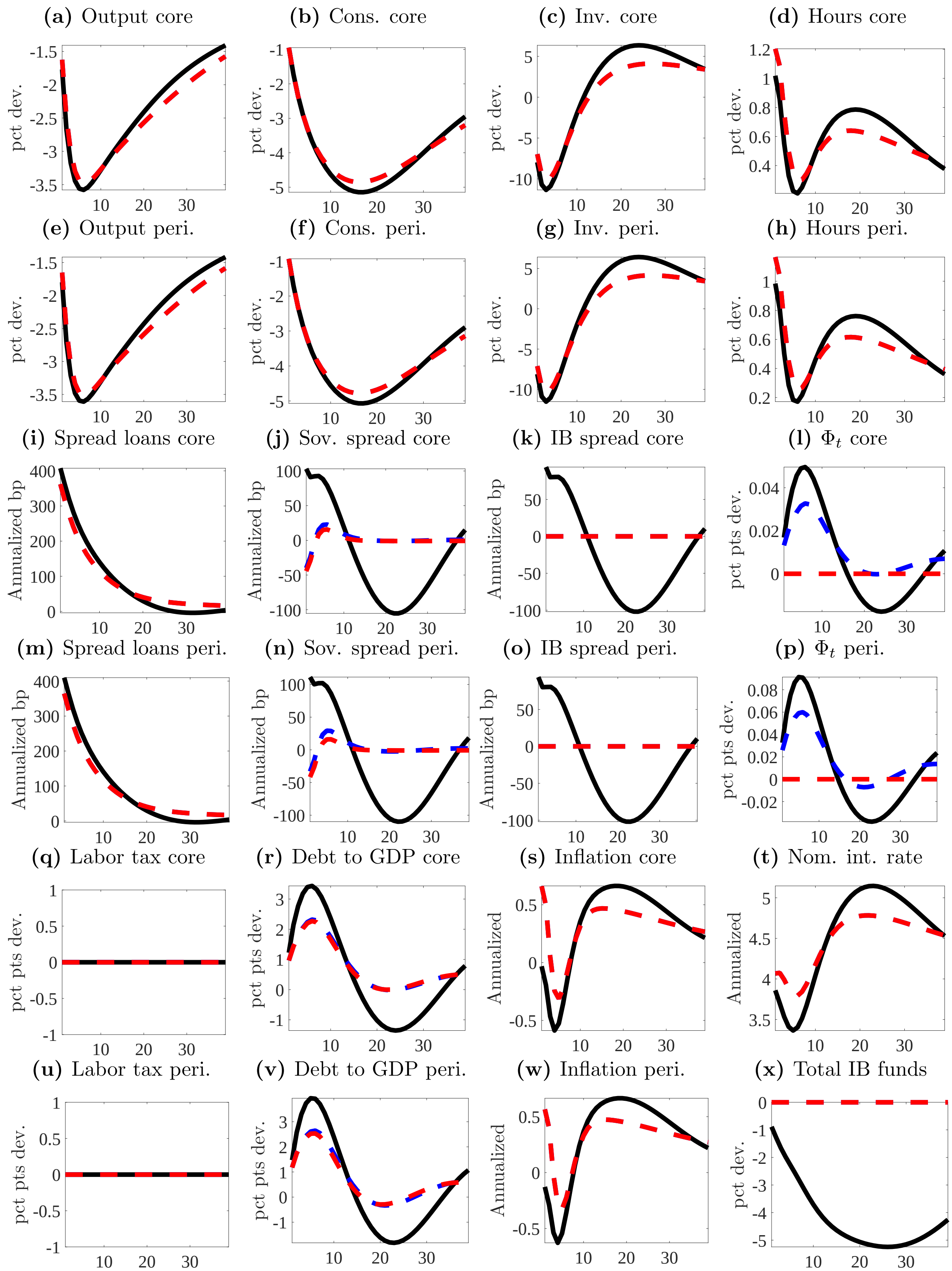
The interbank channel acts in the following way. Our baseline calibration makes sovereign bonds and interbank loans almost perfect substitutes. Hence, a rise in sovereign spreads triggers an equivalently large rise in interbank spreads. Commercial banks face a rise in their funding costs that lowers their demand for interbank liquidity, and is passed to the rates applied on productive loans. Subsequently, the spread on private loans rises much more in our model than in the restricted model, even when the latter embeds sovereign risk.

4.2 Sensitivity

Another way of looking at the relative contribution of the interbank channel is to close the labor income tax channel by assuming that public debt is stabilized using lump-sum taxes instead of distortionary taxes. Figure 2 reports the resulting IRFs. In Corsetti et al. (2014), sovereign risk is passed to private spreads through an imposed spillover function. In our model, it is passed through distortionary taxes and the interbank market. With lump-sum taxes and no interbank market, the impact of sovereign default risk on key macroeconomic variables is null: sovereign risk raises sovereign spreads and magnifies the dynamics of public debts but those do not feedback to the model, as only lump-sum taxes adjust. Considering an interbank market opens up the interbank transmission channel: the rise in sovereign spreads raises interbank spreads, which in turn results in a rise in private credit spreads. Hence, sovereign risk plays some role even when distortionary taxes are absent, but the resulting amplification effect is quantitatively dampened compared to our baseline model with distortionary taxes. In the latter, the distortionary taxes channel and the interbank market channel reinforce each other to produce a significantly larger amplification mechanism.

Finally, transparency on our side requires to point that the strength of the interbank channel is tightly related to the elasticity of substitution among assets in the CES portfolio of saving

Figure 2: Effects of a negative capital quality shock with lump-sum taxes



Black: baseline. Dotted blue: restricted model with sovereign risk. Dotted red: restricted model without sovereign risk. Variable Φ_t denotes the *ex-ante* sovereign default probability.

banks. With a lower elasticity of substitution, as shown in Figure 3, interbank spreads are less synchronized. In other words, when these assets are less substitutable, the interbank market becomes less sensitive to sovereign risk, which downplays the importance of the interbank channel of sovereign risk and therefore weakens the associated amplification mechanism during an economic downturn.

Despite this sensitivity, we consider our benchmark calibration as plausible. First, because it implies a share of interbank loans in the total asset of saving banks that is around 20%, in line with the data. Alternative and lower calibrations of the elasticity of substitution ε produce much larger shares of interbank loans (around 60%) which are counterfactually high. Second, the fact that sovereign bonds are the main source of collateral on the interbank market is consistent with a high degree of substitutability among both assets. Third, the dynamics of inflation rates after a negative capital quality shock implied by lower values of the elasticity of substitution is clearly at odds with the dynamics reported in papers that consider this type of shocks.

4.3 The effects of public asset purchases

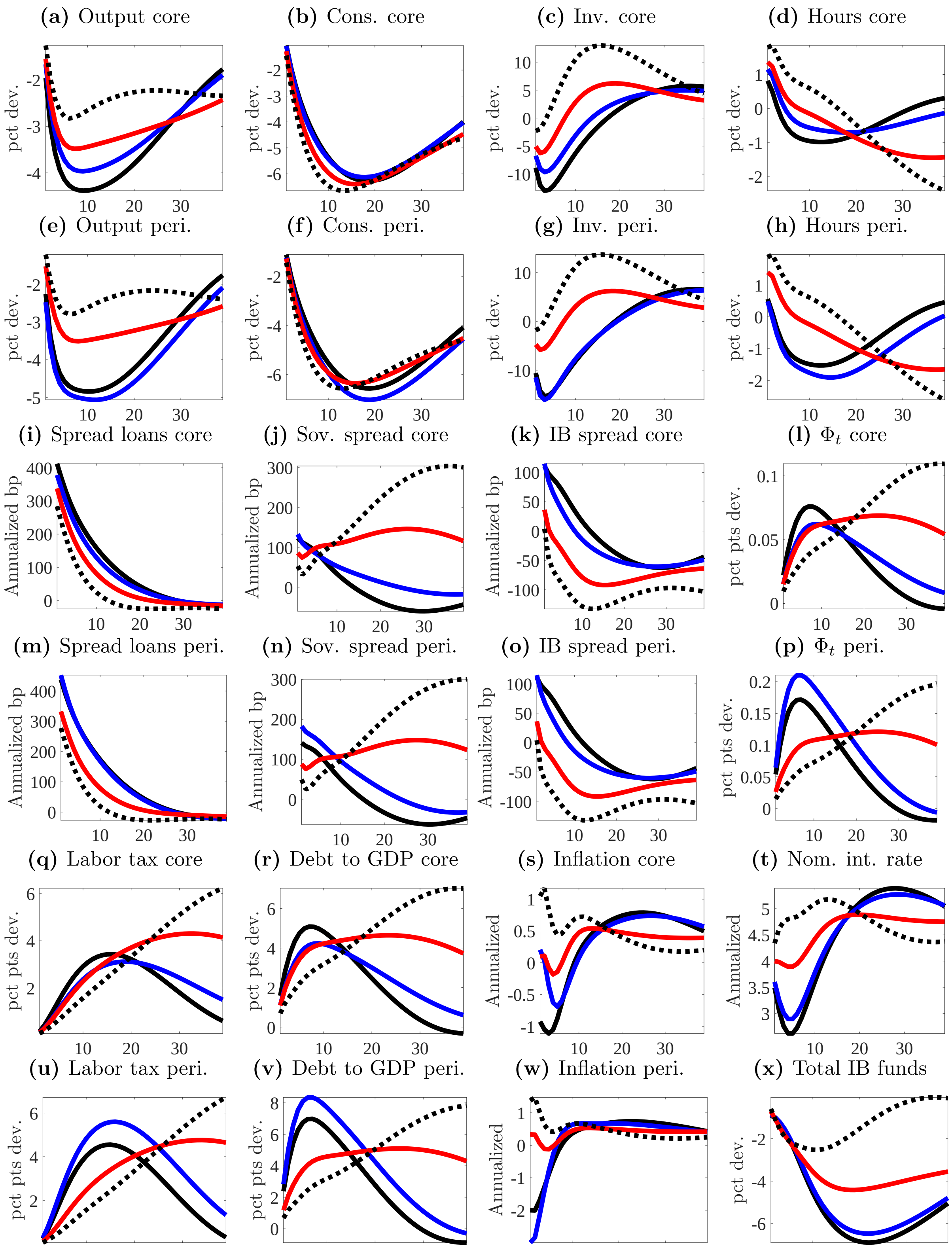
Program and model implementation. We investigate the potential effects of a public asset purchase program, such as the program announced by the ECB in January 2015 (Public Sector Purchase Program, or PSPP). Given our assumption of perfect foresight, we consider only the first announcement of the PSPP, of monthly 60 bn euros purchases of sovereign bonds, from January 2015 to September 2016.[14] Assuming a symmetric geographic implementation, the monthly 60 bn euros purchases convert into 240 bn euros quarterly purchases, that represent roughly 2.3% of each regions' annual GDP, for 7 quarters starting in January 2015.

Since the program was disclosed and announced in January 2015, agents fully expect the flow of purchases – except the initial flow and the announce, which are by definition unexpected – something that our simulations are able to capture. However, purchases concern long-term bonds, implying that the Central Bank's balance sheet remains affected for many periods, until purchased bonds actually mature. Our model is unable to capture this maturity effect, as it features only one-period sovereign bonds. As such, the exercise of simulating the effects of an asset purchase program does not have strong quantitative ambitions. We rather consider it as indicative of the qualitative effects of the program, and as shedding light on a potentially important transmission mechanism.

The program is implemented in the model as follows. Asset purchases are financed by the Central

[14] See https://www.ecb.europa.eu/press/pr/date/2015/html/pr150122_1.en.html. Later on, the January 2015 announcement has been followed by additional announcements by the ECB, that prolonged the program but these could not have been foreseen by agents in January 2015. Those additional announcements are therefore not considered.

Figure 3: Effects of a negative capital quality shock – sensitivity to portfolio parameters



Black: $\varepsilon = 1094$ and $\mu = 0.1898$ (baseline). Blue: $\varepsilon = 50.42$ and $\mu = 0.6133$. Red: $\varepsilon = 15.37$ and $\mu = 0.6284$. Dotted black: $\varepsilon = 8.27$ and $\mu = 0.6315$. Variable Φ_t denotes the *ex-ante* sovereign default probability.

Bank by issuing money.[15] Per-capita sovereign bond purchases Ω_t and Ω_t^* affect the sovereign bond market clearing conditions in the following way:

$$b_t^g = b_t + \varrho b_{*,t} + \Omega_t \tag{58}$$

$$\varrho b_t^{g*} = \varrho b_t^* + b_{*,t}^* + \varrho \Omega_t^* \tag{59}$$

The balance sheet of the Central Bank writes

$$M_t = \Omega_t + \varrho \Omega_t^* \tag{60}$$

and the nominal operational profits from these asset purchases

$$\Pi_{t+1}^{cb} = \left(r_{t+1}^b \left(1-\chi_t\right) - i_t^n\right) \Omega_t + \left(r_{t+1}^b \left(1-\chi_t^*\right) - i_t^n\right) \varrho \Omega_t^* \tag{61}$$

are rebated to the government of each region in proportion of their respective GDPs. Finally, the asset purchases are assumed to entail a small operational (efficiency) cost and good market clearing conditions become

$$y_t = c_t + i_t + g_t + \zeta \Omega_t \tag{62}$$

$$y_t^* = c_t^* + i_t^* + g_t^* + \zeta \Omega_t^* \tag{63}$$

where we assume $\zeta = 0.001$.

Results. Figure 4 reports the effects of the program when implemented at the steady state or conditional on a large negative capital quality shock.[16] In the latter case, Figure 4 reports the net effects of the program, that is, the difference between a crisis *with* the asset purchase program, and a crisis *without* the asset purchase program.[17]

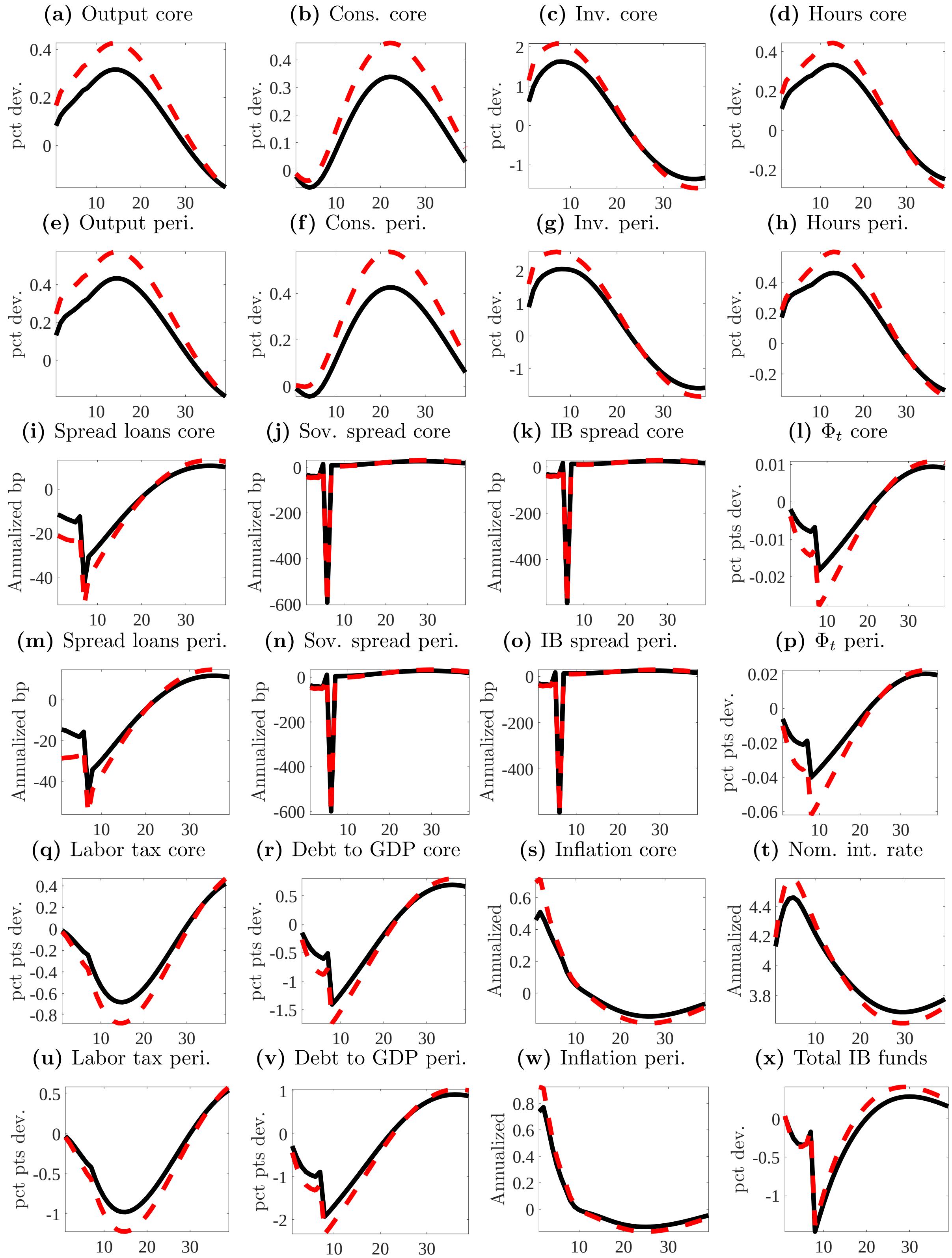
The program works as expected: it boosts output, investment and hours worked. Consumption increases with a lag, given that the increase in investment requires building-up savings in the first quarters. It also lowers spreads on all financial markets and reduces debt-to-GDP ratios. Notice that sovereign and interbank spreads fall brutally when the program is announced, and then return quickly to their steady-state values, while the spreads on corporate loans fall less but more persistently. When implemented at the steady state, the peak effect of the program on output, consumption and hours is around 0.3% in the core region, investment is boosted by more

[15] An alternative could be to issue risk-less bonds paying the deposit rate and bought by households, that would be perfectly substitutable to deposits. In any case, the cost of issuing liquidity for the Central Bank to buy assets is the deposit rate.

[16] The shock has the exact same characteristics (size and persistence) as the shock that generates the IRFs of Figure 1. The shock generates a 4.5% fall in output in the core region and a 5% fall in output in the periphery, a magnitude that is consistent with the drop in GDP observed in 2009.

[17] While a linear solution of the model would make the results independent of the state of the economy, the results will be sensitive to the state of the economy with non-linear simulations.

Figure 4: Effects of the public asset purchase program



Solid black: program implemented at the steady state. Dotted red: Net effects of the program implemented during a crisis.

than 1.5%, and the debt-output ratio falls by slightly less than 1.5 percentage point. The effects for the periphery are qualitatively similar, but quantitatively larger: output, consumption and hours rise by almost 0.4%, investment increases by slightly less than 2%, and the debt-output ratio falls by almost 2 percentage points. The chief reason is that the program is symmetric in size but countries are initially asymmetric in their debt-output ratios, and the periphery features a larger debt-output ratio. Hence, the periphery benefits more significantly from the sovereign bond purchase program: borrowing costs fall more, which boosts private investment and the overall economy more.

Figure 4 also reveals that the state of the economy is important to assess the net effects of the program, as those are found to be substantially larger when the economy is depressed than when the economy is at its steady state. The chief reason can be traced back to the debt-sovereign risk nexus and the level of public debt. A large economic downturn raises public debt and increases sovereign risk in a non-linear way. Precisely because the debt-sovereign nexus is non-linear, the effects of a program that works mainly through alleviating sovereign default risk – the default probability Φ_t falls more in both countries, and even more in the periphery – will be much higher when the public debt-sovereign risk nexus is steeper, that is, during a crisis. Quantitatively speaking, the program is 30 to 40 percent more effective when implemented during a crisis: output, consumption and hours rise by 0.4% in the core region (against 0.3% when the program is implemented at the steady state) and investment by 2% (against 1.5%). For the periphery, the rise in output, consumption and hours is slightly less than 0.6% (against 0.4%), and the rise in investment is more than 2.5% (against 2%). Overall, we conclude that the program is more effective in stimulating the economy when implemented during a crisis.

Welfare effects. What are the welfare effects of this asset purchase program? To address this question, the specific welfare criterion is the constant percentage of consumption that the representative household would be ready to pay that leaves her indifferent between a particular path of the economy and the original path where the economy remains at its initial steady state, namely the value of Θ that solves:

$$\sum_{t=0}^{J} \beta^t u\left(c_t\left(1-\Theta\right), \ell_t\right) = u(c,\ell) \sum_{t=0}^{J} \beta^t. \tag{64}$$

Table 2 below reports the welfare effects of the asset purchase program over different horizons of the transition path: 5 years, 10 years and lifetime (respectively $J = 20$, $J = 40$ and $J = \infty$). In each case, we report the effect of implementing the program at the steady state or conditional on a crisis. In the latter case, the welfare number reported is the difference between the welfare compensation from the crisis with the program and the welfare compensation without the program. Finally, we also report the welfare losses from the crisis, to get a sense of the relative size of the welfare gains from the asset purchase program compared to the total losses from a crisis.

Table 2: The welfare gains/losses (Θ), in percents

	Core			Periphery		
	Asset purchase program					
	$J = 20$	$J = 40$	$J = \infty$	$J = 20$	$J = 40$	$J = \infty$
Steady state	0.0508	0.1090	0.0205	0.0655	0.1390	0.0358
During crisis	0.1363	0.2039	0.0530	0.1978	0.2684	0.0818
	Losses from the crisis					
	−6.3346	−5.6678	−2.6510	−6.3825	−5.7586	−2.6871

First, Table 2 shows that the welfare losses from a crisis that lowers output by 4.5% to 5% are massive. At the horizon of 20 quarters, they reach 6.34% of consumption equivalent in the core region and 6.38% in the periphery. The crisis is quite persistent so its welfare effects at 10 years (40 quarters) are still huge, around 5.7% for both regions. These welfare losses start falling only at much longer horizons and finally reach 2.65% in the core region and 2.68% for the periphery over the full transition path.

When implemented at the steady state, the asset purchase program produces only moderate welfare gains at the 5 years horizon because consumption responds positively with a lag, while hours worked rise immediately, which attenuates the short-run positive effects of the program. Both arguments of the welfare function rise but hours worked rise almost on impact, while consumption rises with a lag due to the rise of private investment produced by the program.[18] Hence, at the 5 years horizon, the asset purchase program implemented at the steady state produces mild welfare gains, reaching 0.05% in the core region, and 0.065% in the periphery. Over the longer run, at 10 years, the positive effects of the program on consumption further raise the associated welfare gains to roughly 0.11% in the core region, and 0.14% in the periphery. Over the full transition path, the asset purchase program produces positive but small welfare gains, of 0.02% and 0.035% respectively for the core and for the periphery.

While these number might seem small, they are in fact quite large compared to the usually reported welfare losses from business cycle fluctuations. In addition, the size of the welfare effects depends on whether the program is announced and implemented at the steady state or during a crisis. In the latter case, the welfare gains from the program are multiplied by 2 or more at all horizons, and more significantly for the periphery. This asymmetry relates to the fact that the crisis that we simulate depresses the economy more deeply in the periphery, as shown by the larger welfare losses from the crisis. Overall, the lifetime welfare effects of the program implemented during a crisis are as large as 0.05% of permanent consumption for the core region, and more than 0.08% for the periphery. These numbers should not be taken

[18]Utility increases with consumption but decrease with hours worked.

too seriously given the above-mentioned caveats. However, there are goods reasons to believe that they are relatively conservative with respect to the effects of the asset purchase programs actually implemented by the ECB. First, a model with long-term bonds would lengthen the effects of the asset purchase program, given that purchased assets remain in the balance sheet of the Central Bank for a much longer period of time. Second, we only focus on the January 2015 announcement and disregard the subsequent announcements. Third, we do not consider the corporate asset purchase program, that is potentially left for future research. All these aspects pertaining to the asset purchase program actually implemented by the ECB would probably magnify the macroeconomic and welfare effects of this type of unconventional monetary policies.

5 Conclusion

This paper builds a two-country model of a monetary union with sovereign default risk, distortionary taxes, two types of banks and an interbank market. These assumptions give rise to a substantial amplification mechanism that relies on the sovereign risk-banks-credit feedback loop. This framework is used to assess the potential macroeconomic and welfare effects of a stylized public asset purchase program. We find that the program has positive macroeconomic and welfare effects. Those are larger when implemented during a crisis than at the steady state, and larger for the periphery.

While the above exercise is illustrative of an important transmission channel of crises and asset purchase programs, the quantitative ambitions of the present paper are limited. Solving the model using global solutions to fully take risk into account, exploring the behavior of the model at the Zero Lower Bound or digging deeper into the relation between corporate and sovereign bonds to assess the effects of corporate asset purchase programs are all interesting potential extensions of the current paper, and are left for future research.

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A Steady state

At the country level, the zero-inflation condition implies that the steady state markup is

$$\mathcal{M} = \frac{\theta}{\theta - 1} = 1/p^m \tag{65}$$

In addition, $\pi = 1$ also implies

$$1 + i = r^d = 1/\beta \tag{66}$$

The price of capital is $q = 1$, investment growth is $x = 0$ and utilization is $u = 1$. We also impose the steady state value of hours worked ℓ and normalize the exogenous variables values to $\varsigma_t = \varsigma$ and $\xi_t = \xi = 1$. We impose r^k and deduce the value of capital to output ratios

$$k/y^m = \frac{\iota\left(1/\mathcal{M}\right)}{r^k - (1-\delta)} \tag{67}$$

From the intermediate goods producers first-order conditions, the following steady-state relation holds between factor prices

$$w = \left(\varsigma\iota^{\iota}\left(1-\iota\right)^{1-\iota}\left(1/\mathcal{M}\right)\left(r^k - (1-\delta)\right)^{\iota}\right)^{\frac{1}{1-\iota}} \tag{68}$$

which determines w. Output $y^m = y$ is then given by

$$y = w\ell/\left(1-\iota\right) \tag{69}$$

k and i by

$$k = \frac{\iota\left(1/\mathcal{M}\right)}{r^k - (1-\delta)} y \tag{70}$$

$$i = \delta k \tag{71}$$

Consumption is given by

$$c = y\left(1 - s_g\right) - \delta k \tag{72}$$

where $s_g = g/y$ is the imposed share of public spending in output. On the government side, we have

$$\Phi = F_{beta}\left(by/by_{max}, \alpha_{b^g}, \beta_{b^g}\right) \tag{73}$$

$$\chi = \Phi\Delta \tag{74}$$

$$\tau = \left(s_g y - (4y.by)\left(1 - r^b\right)\right)/\left(w\ell\right) \tag{75}$$

where by is the debt to annual output ratio and r^b is imposed as a calibration target. Given the utility function considered, $u_\ell = -\omega\ell^{\psi}$, where ψ is the inverse of the Frisch elasticity on labor supply, and $u_c = (1-\beta h)/(c(1-h))$. The labor supply equation

$$\omega \ell^{\psi} = (1-\tau)\, w\,(1-\beta h)/c\,(1-h) \tag{76}$$

is then used to compute the adjusted labor disutility parameter ω that makes hours worked match our target.

As in Gertler and Karadi (2011), we fix the value of leverage ratios, spreads and survival rates of bankers, and adjust relevant parameters. We calibrate the interbank rate r and the sovereign rates r^b and r^{b*} on pre-crisis values. Remember that r^k was imposed as well. These determine r^a and then asset detention l^s, b and b^*. Market clearing conditions on sovereign debt markets give a and a^*, and the weight of interbank loans μ along with the elasticity of substitution ε are then adjusted for the market clearing condition on the interbank market to hold. Once the size of saving banks' balance sheet a is known, their net worth is

$$n^s = a/\phi^s \tag{77}$$

Further,

$$\Lambda^s = \frac{\beta(1-\sigma)}{1-\sigma\beta\left(\left(r^a-r^d\right)\phi^s + r^d\right)} \tag{78}$$

and the diversion and endowment parameters are given by

$$\alpha^s = \Lambda^s \frac{\left(r^a-r^d\right)\phi^s + r^d}{\phi^s} \tag{79}$$

$$\Upsilon^s = \frac{1-\sigma\left[\left(r^a-r^d\right)\phi^s + r^d\right]}{\phi^s} \tag{80}$$

Remember that r^k is imposed in our calibration. The interbank rate r is also imposed. The net worth of commercial banks is given by $n^c = k/\phi^c$. Accordingly, the demand for interbank loans is

$$l^c = k - n^c \tag{81}$$

Further,

$$\Lambda^c = \frac{\beta(1-\sigma)}{1-\beta\sigma\left(\phi^c\left(r^k-r\right)+r\right)}$$

and the diversion and endowment parameters are given by

$$\alpha^c = \Lambda^c\left[\frac{\left(r^k-r\right)\phi^c + r}{\phi^c}\right] \tag{82}$$

$$\Upsilon^c = \frac{1-\sigma\left[\left(r^k-r\right)\phi^c + r\right]}{\phi^c} \tag{83}$$

B Data used for calibration

The calibration matches 2008 measures. Data are taken from the OECD Main Economic Indicators (MEI) database and from the OECD employment and labor market statistics database.

- Hours worked are obtained multiplying hours worked per employee and the total number of employed persons in each region (core and periphery). Taking the sum and dividing by total employment gives an average measure of hours worked in each region, that is finally expressed as a percentage of total time awake.
- Using debt to annual GDP ratios for each country of the Euro Area, we build a measure of public debt to annual GDP in each region (core and periphery).
- Using government expenditure on final goods and GDP measures, we build regional measures of public spending to GDP.