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The Market Size Effect in Endogenous Growth Reconsidered

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Abstract

This paper aims at disentangling two effects of the labor supply size on long-run growth that are traditionally undistinguishable under preference homotheticity: a scale effect, and a market size effect. To reach this goal, we present two horizontal-innovation models of endogenous growth with non-homothetic preferences. We demonstrate in particular that in such set-ups, keeping the economy's total effective labor supply constant, a "richer" country (i.e., with higher labor productivity and a smaller labor force) grows faster than a "poorer" country (i.e., with lower labor productivity and a larger labor force), leading the two countries to diverge.

Keywords: Divergence, Horizontal Innovation, Knowledge Spillover, Endogenous Growth, Balanced Growth Path, Variable price elasticity, Endogenous Markup, Nonhomotheticity, Direct Explicit Additivity, Indirect Explicit Additivity

JEL classification: O11, O31, O33

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1 Introduction

An important feature of the first generation of endogenous technological progress models is the presence of a scale effect: the larger is the country's population size (and hence the labor supply) L, the greater is the economy's growth rate. The existence and the extent of this scale effect is linked to the properties of the knowledge-creation function, namely the way the flow of knowledge (and hence growth) depends on the amount of R&D expenditures (among other parameters): in the canonical R&D-driven growth models (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), the assumption of increasing returns to knowledge leads the economy's growth rate to be proportional to the country's total labor endowment. This property has been empirically challenged early on (Jones, 1995b), and several modelling alternatives have been developed over the years so as to discuss the relevance and robustness of the scale effect.[1] In all the models provided so far though, whether the total amount of labor units invested in R&D are concentrated in a small number of highly educated workers or spread out across a much larger number of less educated workers has no impact whatsoever on a country's growth rate; only the total amount of effective labor supply within the economy matters.

The aim of this paper is to allow for the investigation of the differential effects of labor *productivity* and the *size* of the labor force on long-run growth. More precisely, we argue that altering the canonical endogenous growth model's demand side by allowing for variable mark-ups through the assumption of non-homothetic preferences makes it possible to disentangle two distinct effects that are usually completely interchangeable under the standard homotheticity assumption: the *scale effect* operating on the supply side, and the *market size effect* stemming from the demand side. As pointed out above, the well-known scale effect is directly linked to the properties of the knowledge-production function, and systematically enhances growth in the case of a larger effective labor supply. The market size effect on the other hand originates from the properties of the economy's demand system. In frameworks allowing for variable mark-ups, this effect operates in different directions depending on whether a larger labor endowment is due to a greater number of less educated workers or to a higher level of qualification of a few highly educated workers, hence either reinforcing or dampening the scale effect, and leading to drastically different predictions regarding the evolution of a country's growth rate.

The introduction of variable mark-ups is obtained by departing from the standard use of homothetic CES preferences, which have so far been ubiquitous in the R&D-driven growth literature. While the use of preference specifications allowing for variable price elasticity in dynamic models had so far been hindered by its perceived incompatibility with the balanced growth property,[2] we provide two variations of a generic expanding-variety

[1] Semi-endogenous growth models (Jones, 1995a; Kortum, 1997; Segerstrom, 1998) posit decreasing returns to knowledge in order to avoid the scale effect, while second-generation endogenous growth models (Peretto, 1998; Dinopoulos and Thompson, 1998; Young, 1998; Howitt, 1999; Peretto and Smulders, 2002) take into account the product proliferation effect stemming from an increase in the population size: the development of new product lines fragments the economy into submarkets whose size does not increase with population.

[2] "[The constancy of mark-ups] is a very special feature. [...] Most industrial organization models imply that markups over marginal cost decline in the number of competing products [...]. While plausible, this property makes endogenous growth models less tractable, because in many classes of models, endogenous technological change corresponds to a steady increase in the number of products. If markups decline toward zero as the number of products increases, the process of innovation would ultimately stop, and thus sustained economic growth would be impossible." (Acemoglu, Introduction to Modern Economic Growth, 2008, p.428)

growth model who overcome this apparent impossibility: each of the provided frameworks features a different class of additive preferences (directly explicitly additive - DEA - and indirectly explicitly additive - IEA), both ensuring variable price elasticity while also guaranteeing the preservation of the balanced growth property. Indeed, in each model, mark-ups depend on a single endogenous variable that remains constant along the BGP, given the assumptions made on the nature of the knowledge spillovers. The two classes of preferences, DEA and IEA, do not overlap, except that both contain CES preferences as a limit case; yet, we are able to show that the growth rate and the markup rate along the balanced growth path respond to changes in the parameters in remarkably similar ways in the two models. In particular, we demonstrate that under empirically relevant conditions - the demand for each differentiated product becomes more elastic as one moves up the demand curve - and controlling for the aggregate supply of effective labor, a richer country (i.e., with higher labor productivity and a smaller labor force) grows faster than a poorer country (i.e., with lower labor productivity and a larger labor force).

The intuition regarding this result is best understood when considering a country's total labor endowment L as the product of the *size* of the labor force N and an individual *labor productivity* parameter h: $L = hN$. In both our endogenous growth frameworks, the knowledge spillovers (i.e. increasing returns of the knowledge-creation function) lead to the traditional scale effect: a larger labor supply L leads to a higher growth rate. Whether this increase in L stems from an increase in h or in N however matters for the direction of the market-size effect, which operates on the demand side and stems from the use of variable elasticity preference classes. A larger population size N entails a decrease in firms' endogenous mark-ups, which automatically leads to an more-than-proportional increase in the number of consumption good units being produced, finally ending up in a proportionally lower share of labor being devoted to R&D: the market size effect on growth is negative, dampening the positive scale effect. On the other hand, a higher level of qualification h (hence a higher income per capita) leads to higher mark-ups and a proportionally higher share of labor devoted to R&D: the market size effect on growth is positive, strenghtening the scale effect.

It is worth noting that the properties of the two developed frameworks entail several further contributions to the existing literature. First, the similarity of comparative static results between the two models challenge the properties of directly/indirectly explicitly additive preferences identified so far in the IO literature. Indeed, both Bertoletti and Etro (2016) as well as Parenti et al. (2017) have emphasized that in *static* set-ups, labor productivity (resp. labor force/population size) have no impact on mark-ups when preferences are directly explicitly additive (resp. indirectly explicitly additive). In our dynamic framework, the neutrality of mark-ups with respect to a given structural variable (labor productivity in the case of DEA, labor force/population size in the case of IEA) disappears. Moving from a static to a dynamic set-up hence provides a unifying perspective where the predictions of the two preference classes converge. Second, as already outlined in Boucekkine et al. (2017), the introduction of endogenous mark-ups challenges the systematically positive relationship that existed between firm's market power and growth in the R&D-driven models using CES preferences. Indeed, in both frameworks presented here, a variation in the production cost and in labor productivity generates a positive correlation between the growth rate and the markup rate under empirically relevant conditions, while a variation in the discount rate, in the innovation cost, and in the size of the labor force generates a negative correlation between the growth rate and the markup rate.

The rest of the paper is organised as follows. So as to facilitate comparisons, section 2 presents the benchmark case of CES preferences, which nests two canonical models of balanced growth, Grossman and Helpman (1993) and Gancia and Zilibotti (2005) - those two canonical models differing in their specification of the knowledge spillovers that make growth sustainable. Section 3 presents the balanced growth model featuring DEA preferences with the Gancia-Zilibotti specification of knowledge spillovers. Section 4 presents succintly the balanced growth model featuring IEA preferences with the Grossman-Helpman specification of knowledge spillovers; it can be viewed as an extension of Boucekkine et al. (2017), and constitutes a robustness check of the results obtained under DEA preferences. Section 5 discusses the models' predictions and properties, and section 6 finally concludes.

2 The CES benchmark

We start with a benchmark Balanced Growth model featuring classic CES preferences. In doing so, we slightly extend the models of Grossman and Helpman (1993) and Gancia and Zilibotti (2005) in preparation for our use of more general preference specifications.

2.1 Households' intratemporal problem

We consider an economy populated by N identical households, each supplying inelastically h units of labor measured in efficiency units, which we shall call labor productivity. Hence, the total labor supply measured in efficiency units is given by $L = hN$.

At time t, each household earns an income equal to $w_t h$, and spends

$$E_t = \int_0^{V_t} p_t(\omega)x_t(\omega)d\omega \tag{1}$$

where V_t is the breadth of the product range at time t, $x_t(\omega)$ and $p_t(\omega)$ denoting respectively the individual consumption and the price of a single variety ω. Intertemporal preferences take the following form:

$$\mathcal{U}_t = \int_t^{\infty} \log\left(U(\mathbf{x_s})\right) e^{-\rho(s-t)}ds$$

where $\mathbf{x}_s = x_s(\omega); \omega \in [0, V_s]$ is the consumption profile. In this section, we assume that the intratemporal utility function is a CES with the elasticity of substitution, $\sigma > 1$:

$$U(\mathbf{x}_s) := \int_0^{V_s} x_s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \tag{2}$$

Maximising (2) subject to (1) with a given expenditure E_t and a given price profile $\mathbf{p}_s = \{p_s(\omega); \omega \in [0, V_s]\}$ yields the standard individual demand for each variety under CES preferences:

$$x_s(\omega) = \frac{p_s(\omega)^{-\sigma}}{\int_0^{V_s} p_s(\omega')^{1-\sigma} d\omega'} E_s \tag{3}$$

By inserting this demand into $U(\mathbf{x}_s)$, one could easily verify that maximising U_t is equivalent to maximizing

$$\mathcal{V}_t = \int_t^{\infty} \log\left(v(\mathbf{p}_s, E_s)\right) e^{-\rho(s-t)} ds$$

where

$$v(\mathbf{p}_s, E_s) := \int_0^{V_s} \left(\frac{p_s(\omega)}{E_s}\right)^{1-\sigma} d\omega \tag{4}$$

is the intratemporal indirect utility function.

2.2 Firms' intratemporal problem

We now move to describing the firm problem. Producing one unit of each existing variety requires ψ_t efficiency units of labor. Firms compete monopolistically and choose $p_t(\omega)$ using the following maximisation problem:

$$\max \pi_t(\omega) \equiv (p_t(\omega) - w_t\psi_t)q_t(\omega)$$

with $q_t(\omega) = Nx_t(\omega)$ being the overall demand for a particular variety.

With $\sigma > 1$, it is well known that the firm problem admits a unique solution so that firms all adopt the same following pricing rule:

$$p_t(\omega) \equiv p_t = \frac{\sigma}{\sigma - 1}\psi_t w_t = M\psi_t w_t,$$

where $M := \frac{\sigma}{\sigma-1}$ is the markup rate, constant under the CES assumption.

Because firms all set the same price, firm symmetry entails that all varieties are produced and consumed by the same amount, and all firms earn the same level of profits:

$$q_t(\omega) \equiv q_t;\ x_t(\omega) \equiv x_t;\ \pi_t(\omega) \equiv \pi_t$$

From the mark-up rule above, the profit share in total expenditures can be written as follows:

$$\frac{\pi_t V_t}{NE_t} = \frac{(p_t - w_t\psi_t)q_t V_t}{p_t q_t V_t} = \frac{p_t - w_t\psi_t}{p_t} = \frac{1}{\sigma} \tag{5}$$

Likewise, denoting the total number of efficiency units of labor employed in the production of existing varieties by $L_{Et} := \psi_t q_t V_t = \psi_t N x_t V_t$, the share of manufacturing labor in total expenditures can be expressed as:

$$\frac{w_t L_{Et}}{N E_t} = \frac{\sigma - 1}{\sigma} \equiv \frac{1}{M} \tag{6}$$

2.3 R&D and ressource constraints

Because firms are all symmetric, the market value of each firm is the same and equal to $B_t \equiv \int_t^{\infty} \pi_s e^{-(R_s - R_t)} ds$ where $R_s \equiv \int_0^s r_\tau d\tau$ is the cumulative interest rate and r_τ is the instantaneous one. Log-differentiating this expression of B_t w.r.t. time, we obtain:

$$\frac{\dot{B}_t + \pi_t}{B_t} = r_t \tag{7}$$

Innovating and creating a new variety requires F_t efficiency units of labor:

$$F_t \dot{V}_t = L_{Rt} \tag{8}$$

where L_{Rt} is the number of units of labor being employed in the R&D sector at time t. There is free entry in the R&D sector, so that net returns from R&D $B_t \dot{V}_t - w_t L_{Rt}$ must be equal to zero. This implies that, whenever the R&D sector is active (i.e. $F_t \dot{V}_t = L_{Rt} > 0$), the cost of creating a variety (the R&D cost) and the value of creating a variety B_t must be equalized:

$$w_t F_t = B_t \tag{9}$$

Finally, equilibrium on the labor market implies:

$$hN = L = L_{Rt} + L_{Et} = F_t \dot{V}_t + \psi_t N x_t V_t \tag{10}$$

2.4 Intertemporal problem

We now turn to the intertemporal maximisation problem of the household. First, we derive the intertemporal budget constraint. Each household holds $1/N$ fraction of the shares of the profit-making firms, hence their asset holding is $a_s = \frac{B_s V_s}{N}$. At time s, a household earns the wage income hw_s, the profit income $\frac{\pi_s V_s}{N}$, spends E_s and purchases an amount of assets equal to $\frac{B_s \dot{V}_s}{N}$. The flow budget constraint is hence of the form:

$$\frac{B_s \dot{V}_s}{N} + E_s = w_s h + \frac{\pi_s V_s}{N}$$

Adding the capital gains $\frac{\dot{B}_s V_s}{N}$ on both sides, using (7) and the fact that $a_t = \frac{B_t V_t}{N}$, it is possible to rewrite the above expression as:

$$\dot{a}_s + E_s = w_s h + r_s a_s$$

from which we obtain the intertemporal budget constraint as follows:

$$a_t + \int_t^{\infty} (w_s h - E_s) e^{-(R_s - R_t)} ds = \lim_{s \to \infty} a_s e^{-R_s} \geq 0 \tag{11}$$

Subject to this intertemporal budget constraint, households choose an expenditure path so as to maximise the following intertemporal utility:

$$\mathcal{V}_t = \int_t^{\infty} \log \left(V_s \left(\frac{p_s}{E_s} \right)^{1-\sigma} \right) e^{-\rho(s-t)} ds$$

This leads to the standard Euler equation:

$$\frac{\dot{E}_t}{E_t} = r_t - \rho \tag{12}$$

2.5 Properties of the Balanced Growth Path

We define the balanced growth path as an intertemporal equilibrium path satisfying

i) The growth rate of the number of varieties $g_t \equiv \frac{\dot{V}_t}{V_t}$ is constant and positive.

ii) The allocation of labor between the manufacturing and R&D sectors is also constant: $L_{Et} = L_E^*$ and $L_{Rt} = L_R^*$.

iii) The markup rate is constant: $M := \frac{\sigma}{\sigma - 1}$, satisfied automatically with CES preferences.

To guarantee the existence of such a BGP, we follow Grossman and Helpman (1993) and Gancia and Zilibotti (2005) by assuming that knowledge spillovers from past R&D experiences reduce the cost of R&D as follows:

$$F_t = \frac{F}{V_t} \tag{13}$$

which implies that $L_{Rt} = F_t \dot{V}_t = F g_t$. Two remarks are in order. First, this last equality exemplifies how the increasing returns to knowledge assumption is at the heart of the well-known scale effect property: the growth rate of an economy g_t is proportional to the amount of efficient labor invested in R&D L_{Rt}. Second, although Grossman and Helpman (1993) assume that knowledge spillovers are limited to R&D by postulating that the productivity in the manufacturing sector $\psi_t = \psi$ and Gancia and Zilibotti (2005) assume that they benefit both R&D and manufacturing by postulating $\psi_t = \frac{\psi}{V_t}$, we intentionally leave ψ_t unspecified for reasons that will become clear later.

We now derive the law of motion for this economy under the assumption that $L_{Rt} = F g_t > 0$. Using (7) and the Euler equation (12), we obtain $\frac{\dot{E}_t}{E_t} = \frac{\dot{B}_t + \pi_t}{B_t} - \rho$; using (5), (9) and (13), $\frac{\dot{E}_t}{E_t}$ then becomes:

$$\frac{\dot{E}_t}{E_t} = \frac{\dot{w}_t}{w_t} - g_t + \frac{N}{\sigma F} \frac{E_t}{w_t} - \rho$$

Defining the real expenditure as $e_t := \frac{E_t}{w_t}$, the above expression can be reformulated as follows:

$$\frac{\dot{e}_t}{e_t} = \frac{N}{\sigma F} e_t - g_t - \rho \tag{14}$$

while the he labor market equilibrium condition (10) becomes:

$$L = L_{Rt} + L_{Et} = F g_t + L_{Et} = F g_t + \left(1 - \frac{1}{\sigma}\right) N e_t \tag{15}$$

Finally, combining (14) and (15), we obtain the following law of motion for e_t:

$$\frac{\dot{e}_t}{e_t} = \frac{N}{F} e_t - \frac{L}{F} - \rho \tag{16}$$

In equilibrium, e_t immediately jumps to its steady-state value $e^* = \frac{L+\rho F}{N}$, from which we can express the labor allocation between sectors L_E^* and L_R^* along the BGP:

$$L_E^* = \left(1 - \frac{1}{\sigma}\right)(L + \rho F); \quad L_R^* = \frac{L}{\sigma} - \left(1 - \frac{1}{\sigma}\right) \rho F > 0$$

which finally entails the following expression for the economy's growth rate:

$$g^* = \frac{L}{\sigma F} - \left(1 - \frac{1}{\sigma}\right) \rho > 0 \tag{17}$$

where $g^* > 0$ is ensured by assuming

$$\frac{L}{\rho F} + 1 > \sigma > 1$$

or, equivalently,

$$1 + \frac{\rho F}{L} < \frac{\sigma}{\sigma - 1} \equiv M.$$

The benchmark case of CES preferences exhibits two special features that are worth mentioning. First, the labor productivity parameter h and the size of the labor force N enter in the law of motion for e_t and in the expressions for e^*, $L_R^* = L - L_E^*$ and g^* only through their product $L = hN$: in other words, only the total effective labor supply matters for an economy's long run growth rate. This property is due to the homotheticity of preferences, and as we will see below, will not hold any more once we move to non-homothetic preference systems. Second, because the mark-up depends solely on the exogenous preference parameter σ, the share of aggregate spending accruing to firm profits does not depend on the productivity in the manufacturing sector. As a consequence, ψ_t neither impacts the long-run growth rate nor the labor allocation under CES preferences, which is why we left it unspecified above.[3]

[3]Note however that ψ_t affects the time path of x_t, because $L_E^* = N\psi_t x_t V_t$. For example, if we assume $\psi_t = \psi$ as in Grossman and Helpman (1993) then x_t must shrink at the rate equal to g^*. Instead, if we assume $\psi_t = \psi / V_t$ as in Gancia and Zilibotti (2005) then x_t remains constant.

Again, this feature will disappear as we depart from the CES assumption.

3 Directly explicitly additive (DEA) preferences

In this section, we now depart from the assumption of CES preferences and consider the broader class of DEA preferences, that admit CES as a limit case.

3.1 Households and firms' intratemporal problem

The intratemporal preferences are said to be directly explicitly additive (DEA) if the direct utility function is explicitly additive:

$$U(\mathbf{x_s}) := \int_0^{V_s} u\left(x_s(\omega)\right) d\omega$$

where $u(.)$ is increasing, concave and thrice differentiable. The inverse demand for each variety can then be derived as follows:

$$p_t(\omega) = \frac{E_t u'\left(x_t(\omega)\right)}{\int_0^{V_t} x_t(\omega')u'\left(x_t(\omega')\right) d\omega'}$$

while the firm's pricing rule can now be written as:

$$p_t(\omega)\left(1 - \frac{1}{\eta(x_t(\omega))}\right) = w_t\psi_t$$

where $\eta(x) = -\frac{u'(x)}{xu''(x)} > 0$ is the price-elasticity, with the existence of a solution ensured by the condition that $\eta(x) > 1$ holds for some range of x. In what follows, we further assume that

$$\eta'(x) < 0 \tag{18}$$

so that the demand for each variety becomes more elastic as one moves up along the demand curve. Under monopolistic competition, imposing this condition is equivalent to assuming an incomplete pass-through of a cost shock, a property that has been extensively supported by recent empirical works (Loecker and Goldberg, 2014; Mayer et al., 2016). Because this assumption implies that the marginal revenue (i.e. the LHS of the firm's pricing rule) is strictly decreasing in $x_t(\omega)$, all firms choose the same $x_t(\omega) \equiv x_t$ and hence the same price at equilibrium:

$$p_t(\omega) \equiv p_t = \frac{\eta(x_t)}{\eta(x_t) - 1} w_t\psi_t = M(x_t)w_t\psi_t \tag{19}$$

where $M(x_t) := \frac{\eta(x_t)}{\eta(x_t)-1}$ is the markup, which is now endogenous and increasing in x_t.

Following the same logic as for the CES case, the profit share equation reads as

$$\frac{\pi_t V_t}{NE_t} = \frac{1}{\eta(x_t)} \tag{20}$$

while the share of manufacturing labor in total expenditures (6) becomes

$$\frac{w_t L_{Et}}{NE_t} = \frac{\eta(x_t) - 1}{\eta(x_t)} \equiv \frac{1}{M(x_t)} \tag{21}$$

3.2 Intertemporal problem

Households' intertemporal utility can now be expressed as

$$\mathcal{V}_t = \int_t^{\infty} \log\left(V_s u\left(\frac{E_s}{V_s p_s}\right)\right) e^{-\rho(s-t)} ds$$

Households maximize their utility subject to the intertemporal budget constraint (11), which leads to an augmented Euler equation, featuring an additional term with respect to (12):

$$\frac{\dot{E}_t}{E_t} = r_t - \rho + \frac{\dot{\nu}(x_t)}{\nu(x_t)} \tag{22}$$

with $\nu(x_t) = \frac{u'(x_t)x_t}{u(x_t)}$.

3.3 Balanced Growth Path

Following the same definition for the BGP as before, we notice that the constant markup rate immediately entails that $x_t = x^*$ is constant. Furthermore, this and the constant labor allocation $L_{Et} = L_E^*$ implies that $\psi_t V_t = \frac{L_{Et}}{Nx_t} = \frac{L_E^*}{Nx^*}$. Thus, in order to ensure the existence of a balanced growth path, it is now necessary to assume along Gancia and Zilibotti (2005) that knowledge spillovers from past R&D experiences increase manufacturing productivity: $\psi_t = \psi / V_t$.[4]

Again, we derive here the law of motion for the economy. Following the same steps as in the CES case, but noticing that the Euler equation now takes the augmented form given by (22), (14) can now be expressed as

$$\frac{\dot{e}_t}{e_t} = -g_t + \frac{Ne_t}{\eta(x_t)F} - \rho + \frac{\dot{\nu}(x_t)}{\nu(x_t)}$$

while equation (15) now becomes

$$L = L_{Rt} + L_{Et} = Fg_t + \frac{Ne_t}{M(x_t)}$$

[4]This assumption has also been made in other R&D-driven models of endogenous growth such as Foellmi and Zweimuller (2006).

Combining these two expressions, we obtain the following expression for (16):

$$\frac{\dot{e}_t}{e_t} = \frac{Ne_t}{F} - \frac{L}{F} + \frac{\dot{\nu}(x_t)}{\nu(x_t)}$$

Finally, because $L_{Et} = \psi N x_t$ and (21) implies $e_t = \psi M(x_t)x_t$, the above equation can be written as the law of motion of x_t:

$$\left(\frac{M'(x_t)x_t}{M(x_t)} - \frac{\nu'(x_t)x_t}{\nu(x_t)} + 1\right)\frac{\dot{x}_t}{x_t} = \frac{N}{F}\psi M(x_t)x_t - \frac{L}{F} - \rho = \frac{N}{F}\left(\psi M(x_t)x_t - h\right) - \rho \tag{23}$$

In equilibrium, x_t jumping to its steady state value implies that $e_t = \psi M(x_t)x_t$ does the same:

$$e^* = \psi M(x^*)x^* = \frac{L + \rho F}{N} \tag{24}$$

from which we can infer again the labor allocation between sectors L_E^* and L_R^* along the BGP:

$$L_E^* = \left(1 - \frac{1}{\eta(x^*)}\right)(L + \rho F)\,; \quad L_R^* = \frac{L}{\eta(x^*)} - \left(1 - \frac{1}{\eta(x^*)}\right)\rho F$$

as well as the economy's growth rate g^*:

$$g^* = \frac{L}{\eta(x^*)F} - \left(1 - \frac{1}{\eta(x^*)}\right)\rho > 0$$

where $g^* > 0 \Leftrightarrow L_E^* = \psi N x^* < L \Leftrightarrow x^* < h/\psi$ is ensured by the following condition:

$$\frac{L}{\rho F} + 1 > \eta\left(h/\psi\right) > 1,$$

or equivalently

$$1 + \frac{\rho F}{L} < \frac{\eta\left(h/\psi\right)}{\eta\left(h/\psi\right) - 1} := M\left(\frac{h}{\psi}\right)$$

Last, we see directly from the law of motion (23) that if $\frac{\partial \ln \nu}{\partial \ln x_t} < 0$, then the BGP is the only equilibrium. Parenti (2018) shows that if $\nu(.)$ is monotonic in x_t and if $u(0) = 0$ then it is implied by condition (18).

3.4 Comparative statics along the BGP

Using (21) again leads to a generalization of (17):

$$g^* = \frac{L}{\eta(x^*)F} - \left(1 - \frac{1}{\eta(x^*)}\right)\rho$$

Proposition 1 - Comparative statics under DEA preferences

Under DEA preferences, we have the following comparative statics:

i) An increase in the consumers' discount rate ρ increases the consumption per variety x^* and markups $M(x^*)$, increases the labor allocated to the manufacturing sector L_E^* and decreases the economy's growth rate g^*;

ii) An increase in the R&D costs F increases x^* and $M(x^*)$, increases L_E^* and decreases g^*;

iii) An increase in the economy's labor force size N decreases x^* and $M(x^*)$, increases L_E^* and increases g^*;

iv) An increase the workers' labor productivity h increases x^* and $M(x^*)$, increases L_E^* and increases g^*;

v) An increase in the production costs ψ decreases x^* and $M(x^*)$, increases L_E^* and decreases g^*.

Proof: We first slightly reformulate condition (24)

$$M(x^*)x^* = \frac{hN + \rho F}{\psi N} \tag{25}$$

Under (18), the individual consumption of a given variety x^* responds less than proportionately to a change in the LHS of (25). An increase in ρ, F or h then lead the RHS of (25) to increase, leading to a corresponding increase in x^* and the markup $M(x^*)$ so as to re-establish the equality. This, in turn, raises labor allocated to the manufacturing sector $L_E^* = \psi N x^*$. Labor allocated to the R&D sector $L_R^* = hN - L_E^*$ then necessarily decreases following an increase in ρ and F, leading to a decrease in the long-run growth rate $g^* = L_R^*/F$. On the other hand, in the case of an increase in h, the less than proportional variation of x^* implies that the decrease in L_E^* is more than compensated by the overall increase in total labor supply hN, thereby leading to an increase in L_R^* and g^*.

An increase in ψ or N lead to a decrease in the LHS of (25), decreasing thereby x^* and the markup $M(x^*)$. Because x^* responds less than proportionately to a variation in the LHS of (25) though, $L_E^* = \psi N x^*$ nevertheless increases following both shocks. Labor allocated to the R&D sector $L_R^* = hN - L_E^*$ then necessarily decreases following an increase in ψ, leading to a decrease in g^*; on the other hand, both L_R^* and g^* increase following an increase in N, since the increase in L_E^* is more than compensated by the increase in overall labor supply hN. This ends the proof. □

Labor productivity, the size of labor force and long-run growth

We now focus on the obtained comparative statics regarding variations in the labor productivity parameter h and the labor force size N. As outlined above, an increase in both parameters leads to an increase in the long-run growth g^*: indeed, higher values of N and h both increase the aggregate supply of effective labor L, which leads to in turn to an increase in the labor being allocated to both the manufacturing sector L_E^* *and* the R&D sector L_R^*. This positive impact of the labor supply on long-run growth is what has been identified by the literature as the well-known scale effect.

On the other hand, the two parameters have opposite effects on endogenous markups $M(x^*)$. On the one hand, an increase in the population size N leads to a decrease in markups, which amplifies the increase in L_E^* and leads to a less-than-proportional increase in L_R^*: what can de denoted as a market size effect on growth is hence negative in this case, dampening the impact of an increased effective labor supply L on long-run growth. On the other hand, an increase in h leads to an increase in markups: in this case, the increase in firms' market power lessens the increase in L_E^*, with this positive market size effect leading to an amplification of the impact of an increase in L on long-run growth g^*.

An immediate consequence of those results is that the **concentration** of the effective labor supply has an impact on the long-run growth rate: two countries with the same overall labor supply L will have different growth rates depending on their per-capita labor efficiency. More precisely, controlling for the aggregate supply of effective labor L, a richer country (i.e., with higher labor productivity h and a smaller labor force N) grows faster than a poorer country (i.e., with lower labor productivity and a larger labor force). Such a result yields drastic implications when it comes to the convergence debate, emphasizing the importance of initial conditions. In our set-up, initial inequalities across countries would only be amplified in the absence of adequate policies.

4 Indirect explicitly additive (IEA) preferences

In section 3, we considered the implications of departing from CES within DEA non-homothetic preferences. Given the ubiquity of the homotheticity assumption in the existing literature, it is however legitimate to question the robustness of the obtained results. In this section, we hence demonstrate that the same comparative statics can be obtained with an alternative class of preferences, IEA, which also nests CES preferences as a special case; we do so by briefly presenting an extension of Boucekkine et al. (2017) in which we model labor productivity h and manufacturing costs ψ_t as separate parameters to explore their implications for long-run growth.

It is important to note that the two considered classes of preferences, IEA and DEA, are quite different from each other. First, only CES preferences belong to the two classes. Second, in a static model, IEA and DEA preferences yield strikingly different comparative statics with respect to h and N (Parenti et al., 2017). Yet it will turn out, perhaps surprisingly, that the two classes of preferences generate the remarkable similar comparative statics results.

4.1 Households and intratemporal problem

The intratemporal preferences are said to be indirectly explicitly additive (IEA) if its indirect utility function is explicitly additive as:

$$v(\mathbf{p}_s, E_s) := \int_0^{V_s} v\left(\frac{p_s(\omega)}{E_s}\right) d\omega$$

where $v(.)$ is decreasing, convex and thrice differentiable. Using the Roy's identity,the demand for each variety can be derived as shown:

$$x_t(\omega) = \frac{v'\left(\frac{p_t(\omega)}{E_t}\right)}{\int_0^{V_t} \frac{p_s(\omega')}{E_s} v'\left(\frac{p_s(\omega')}{E_s}\right) d\omega'}$$

The firm's pricing rule can now be written as:

$$p_t(\omega)\left(1-\frac{1}{\sigma(p_t(\omega)/E_t)}\right)=w_t\psi_t$$

where $\sigma(z)=-\frac{zv''(z)}{v'(z)}>0$ is the price-elasticity, with the existence of a solution ensured by the condition that $\sigma(z)>1$ holds for some range of z. In what follows, we further assume that

$$\sigma'(z)>0 \tag{26}$$

so that the demand for each variety becomes more elastic as one moves up along the demand curve. Again, under monopolistic competition this is equivalent to assuming an incomplete pass-through of a cost shock, a feature that has been extensively supported by recent empirical work (see Loecker and Goldberg (2014) for a review). Because this assumption implies that marginal revenue is strictly increasing in $p_t(\omega)$, all the firms choose the same price in equilibrium:

$$p_t(\omega)\equiv p_t=\frac{\sigma(p_t/E_t)}{\sigma(p_t/E_t)-1}\psi_t w_t=\frac{\sigma(1/X_t)}{\sigma(1/X_t)-1}=M(X_t)w_t\psi_t \tag{27}$$

where $X_t=x_tV_t=E_t/p_t$ is the total units of varieties consumed by each household, and $M(X_t):=\frac{\sigma(1/X_t)}{\sigma(1/X_t)-1}$ is the markup, which is now endogenous and increasing in X_t.

Following the same logic as for the CES case, the profit share equation (5) now reads as

$$\frac{\pi_t V_t}{NE_t}=\frac{1}{\sigma(1/X_t)} \tag{28}$$

while the share of manufacturing labor in total expenditures (6) becomes

$$\frac{w_t L_{Et}}{NE_t}=\frac{\sigma(1/X_t)-1}{\sigma(1/X_t)}\equiv\frac{1}{M(X_t)} \tag{29}$$

4.2 Intertemporal problem

Households intertemporal utility can now be expressed as

$$\mathcal{V}_t=\int_t^\infty \log\left(V_s v\left(\frac{p_s}{E_s}\right)\right)e^{-\rho(s-t)}ds$$

Households maximize their utility subject to the intertemporal budget constraint (11) which leads to an augmented Euler equation, featuring an additional term with respect to (12):

$$\frac{\dot{E}_t}{E_t}=r_t-\rho+\frac{\dot{\nu}(X_t)}{\nu(X_t)} \tag{30}$$

with $\nu(X_t) = -\frac{v'(1/X_t)}{X_t v(1/X_t)}$. As will become clear later, the variations of $\nu(.)$ matter to guarantee that the BGP is a unique equilibrium. We thus postpone the discussion of its properties and its relation to (26).

4.3 Balanced Growth Path

Following the same line of reasoning as before, the constant markup rate immediately entails $X_t = X^*$ to be constant. Furthermore, this and the constant labor allocation $L_{Et} = L_E^*$ implies that $\psi_t = \frac{L_{Et}}{NX_t} = \frac{L_E^*}{NX^*}$. Thus, it is now necessary to assume, as done by Grossman and Helpman (1993), that knowledge spillovers from past R&D experiences benefit only R&D, and not manufacturing, $\psi_t = \psi$, in order to ensure the existence of a balanced growth path.

Following the same steps as in the DEA case, it is possible to obtain the following law of motion for X_t:

$$\left(\frac{M'(X_t)X_t}{M(X_t)} - \frac{\nu'(X_t)X_t}{\nu(X_t)} + 1\right)\frac{\dot{X}_t}{X_t} = \frac{N}{F}\psi M(X_t)X_t - \frac{L}{F} - \rho = \frac{N}{F}\left(\psi M(X_t)X_t - h\right) - \rho \tag{31}$$

It is noteworthy that (31) and (23) are similar when replacing overall consumption X_t with individual consumption x_t. In equilibrium X_t, and hence $e_t = \psi M(X_t)X_t$ immediately jumps to the steady state value, given by:

$$e^* = \psi M(X^*)X^* = \frac{L + \rho F}{N} \tag{32}$$

from which we obtain again the labor allocation between sectors L_E^* and L_R^* along the BGP:

$$L_E^* = \left(1 - \frac{1}{\sigma(1/X^*)}\right)(L + \rho F)\,; \quad L_R^* = \frac{L}{\sigma(1/X^*)} - \left(1 - \frac{1}{\sigma(1/X^*)}\right)\rho F$$

and the economy's growth rate g^*:

$$g^* = \frac{L}{\sigma(1/X^*)F} - \left(1 - \frac{1}{\sigma(1/X^*)}\right)\rho > 0$$

where $g^* > 0 \Leftrightarrow L_E^* = \psi N X^* < L \Leftrightarrow X^* < h/\psi$ is ensured by the following condition:

$$\frac{L}{\rho F} + 1 > \sigma\left(\psi/h\right) > 1,$$

or equivalently

$$1 + \frac{\rho F}{L} < \frac{\sigma\left(\psi/h\right)}{\sigma\left(\psi/h\right) - 1} := M\left(\frac{h}{\psi}\right)$$

Last, we see directly from the law of motion (31) that if $\frac{\partial \ln \nu}{\partial \ln X_t} < 0$, then the BGP is the only equilibrium. We show in the appendix that under an additional assumption this is actually guaranteed by condition (26).

4.4 Comparative statics along the BGP

Using (29) leads to a generalization of (17):

$$g^* = \frac{L}{\sigma(1/X^*)F} - \left(1 - \frac{1}{\sigma(1/X^*)}\right)\rho$$

Proposition 2 - Comparative statics under IEA preferences

Under IEA preferences, we have the following comparative statics:

i) An increase in the consumers' discount rate ρ increases overall consumption X^* and markups $M(X^*)$, increases the labor allocated to the manufacturing sector L_E^* and decreases the economy's growth rate g^*;

ii) An increase in the R&D costs F increases X^* and $M(X^*)$, increases L_E^* and decreases g^*;

iii) An increase in the economy's labor force size N decreases X^* and $M(X^*)$, increases L_E^* and increases g^*;

iv) An increase the workers' labor productivity h increases X^* and $M(X^*)$, increases L_E^* and increases g^*;

v) An increase in the production costs ψ decreases X^* and $M(X^*)$, increases L_E^* and decreases g^*.

The proof as well as the intuitions are identical in every respect to those preseneted under DEA preferences. The main result obtained under DEA preferences is therefore robust to the case of IEA preferences: a country with less workers (and therefore a higher effective labor supply per capita) will have a lower growth rate than a country with more workers.

Those comparative statics also imply that the result of Boucekkine et al. (2017) of a negative correlation between mark-ups and growth has to be nuanced in our more flexible framework. While g^* and $M(X^*)$ are indeed found to vary in opposite directions following shocks on the discount rate ρ, the R&D costs F and the labor force size N, they move in the same direction following shocks on production costs ψ and workers' labor productivity h. This means that firms' market power may amplify or dampen the impact of different model parameters on economic growth.

5 Discussion

As extensively commented in the two previous sections, the main contribution of the two proposed frameworks if to allow for the disentanglement of two effects whose impact on long-run growth could not be differentiated under homothetic preferences, namely the scale effect and the market size effect. It however turns out that allowing for non-homothetic preference systems in endogenous growth models yields several other notable implications, that we will now discuss in the present section.

5.1 Unification of the IEA and DEA cases in our dynamic framework

Beyond their implications for long-run growth, our Propositions 1 and 2 challenge the properties of directly/indirectly explicitly additive preferences identified so far by the literature. Indeed, in previous contributions, Bertoletti and Etro (2016) and Parenti et al. (2017) have established that in static set-ups, a given structural parameter is systematically orthogonal to markups. More precisely, under IEA preferences, the labor force size has no impact on markups (Parenti et al., 2017), while labor productivity is found to increase markups. Under DEA preferences on the other hand, labor productivity has no impact on markups (Bertoletti and Etro, 2016), while the size of the labor force is found to be pro-competitive (i.e. to decrease markups). In our dynamic framework, the neutrality of mark-ups with respect to a given structural variable (labor productivity in the case of DEA, labor force size in the case of IEA) disappears, and the markup variations found in static frameworks (positive impact of labor productivity, negative impact of the labor force size) are confirmed. Moving from a static to a dynamic set-up hence provides a unifying perspective, allowing for the predictions of the two preference classes to converge.

5.2 Market power and growth under endogenous mark-ups

Propositions 1 and 2 also demonstrate that as already outlined in Boucekkine et al. (2017), the introduction of endogenous mark-ups challenges the systematically positive relationship that existed between firm's market power and growth in the R&D-driven models using CES preferences. Indeed, both under DEA and IEA preference systems, a variation in the discount rate ρ, in the innovation cost F, and in the size of the labor force N generates a negative correlation between the growth rate and the markup rate (since both vary in opposite directions when considering comparative statics w.r.t. those parameters). This result hints at a more complex relationship between product market competition (PMC) and long-run growth than what had been identified by the existing literature, which has so far modeled variations in the degree of PMC as exogenous shocks on consumers' preference specifications/the economy's technology (Aghion et al., 2005, 2009; Acemoglu and Akcigit, 2012).

5.3 Welfare implications

As seen above, the properties of the two models regarding long-run growth and markup variations are strongly similar. In the IEA case however, the sole variable displaying a positive rate of growth is the number of varities V_t, while consumers' overall consumption level X^* remains constant along the BGP. In the DEA case on the other hand, Gancia-Zilibotti knowledge spillovers in the manufacturing sector ensure that both V_t and X_t grow at a positive and constant rate g^* along the BGP, ensuring the constancy of the per-variety consumption x^*.

It is therefore finally legitimate to wonder whether these two models have different welfare implications. Perhaps surprisingly, we show below that they don't.

In the IEA case, the intertemporal indirect utility along the BGP reads as

$$\mathcal{V}_t = \int_t^{\infty} \log\left(V_s \upsilon\left(1/X^*\right)\right) e^{-\rho(s-t)} ds \equiv \frac{1}{\rho}\left(\log \upsilon\left(\frac{1}{X^*}\right) + g^* \cdot t\right)$$

while in the DEA case, the intertemporal direct utility is given by

$$\mathcal{U}_t = \frac{1}{\rho}\left(\log u\left(x^*\right) + g^* \cdot t\right)$$

In the previous sections we have shown that the comparative statics on the growth rate g are the same under IEA and DEA. Furthermore, we have already argued that the most plausible case for the preference for variety is that it decreases under IEA (resp. DEA) preferences with overall (resp. individual) consumption i.e. $\nu' < 0$. It is therefore readily verified that in both models, the welfare implication of our comparative statics are the same. Formally, the impact of a variable y in both models reads as:

$$\rho\frac{d\mathcal{V}_t}{dy} = \frac{1}{X^*}\frac{dX^*}{dy}\nu(X^*) + \frac{dg^*}{dy}t; \quad \rho\frac{d\mathcal{U}_t}{dy} = \frac{1}{x^*}\frac{dx^*}{dy}\nu(x^*) + \frac{dg^*}{dy}t$$

6 Conclusion

In this paper we provided two variations of a generic expanding-variety growth model that allow for variable mark-ups through the use of non-homothetic preference systems; those set-ups enabled us to distinguish two separate effects of the labor supply size on long-run growth: the scale effect operating on the supply side, and the market size effect stemming from the properties of the demand system. Traditionally interchangeable under the preference homotheticity assumption, those two effects may operate in different directions when allowing for endogenous firm mark-ups, leading to an impact of the *concentration* of the effective labor supply on the long-run growth rate that had so far been overlooked in existing endogenous growth models.

We demonstrated in particular that two countries with the same overall labor supply L will have different growth rates depending on their per-capita labor efficiency. More precisely, controlling for the aggregate supply of effective labor L, a richer country (i.e., with higher labor productivity h and a smaller labor force N) grows faster than a poorer country (i.e., with lower labor productivity and a larger labor force). Such a result yields drastic implications when it comes to the convergence debate, emphasizing the importance of initial conditions.

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