The Fiscal Theory of the Price Level in non-Ricardian Economy

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WP 1824 – October 2018

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Working Paper

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*We would like to thank Guido Ascari, Hafedh Bouakez, Giancarlo Corsetti, Aurélien Eyquem and Eric Leeper for very helpful comments and suggestions. This paper has circulated under many titles: "A simple public debt-deflation theory: Leeper revisited" and "Public debt and deflation". The usual disclaimers apply.

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1 Introduction

The study of the interaction between monetary and fiscal policies has been the object of vigorous interest since the seminal works of Sargent and Wallace (1981), Aiyagari and Gertler (1985), then with the studies of Leeper (1991), Sims (1994), Woodford (1994, 2003) and Cochrane (2005), among others, around the controversial Fiscal Theory of the Price Level (FTPL, hereafter).[1] The main contribution of this literature is an explicit specification of the conditions under which fiscal and monetary policies interact to guarantee the determination of the general price level. As opposite to the monetary view, the FTPL puts the light on the fiscal policy as an important determinant, or even as the sole determinant of the general price level as pointed out by Sims (2013) in his Presidential Address at the American Economic Association.

In its simplest form, the FTPL assumes that the government commits to a fixed and exogenous present value of primary surpluses, i.e. active fiscal policy in the terms of Leeper (1991). The price level is determined to equate the real government debt to the present value of primary surpluses. This can be achieved when the monetary rule responsiveness is less than one-for-one to inflation, i.e. a passive monetary policy in the terms of Leeper (1991).

The FTPL relies on the presence of primary surpluses (or deficits) to hold.[2] If the present value of primary surpluses is zero then there is no outstanding nominal debt, and this relationship cannot determine the equilibrium price level. We show that this condition is not necessary in a non-Ricardian economy. The FTPL still hold even when the present value of exogenous primary surpluses is null. A bubble-like component of government debt appears leading to the determinacy of the price level by the fiscal policy. The reason is that the government can play "Ponzi games" even if the transversality condition of each agent is satisfied.[3]

We consider an overlapping generations of infinitely-lived dynasties model with simple fiscal and monetary policies, where the ZLB on the nominal interest rate is taken into account. More generally, this paper analyses the determination of the general price level in a non-Ricardian framework where interest rates may get stuck at the zero lower bound (hereafter, ZLB), or more precisely, at the effective lower bound.[4]

Before, getting more into the details of our results let us recall the main findings of Leeper (1991). The interaction between simple monetary and fiscal policies yields four configurations depending

[1] See, for example, Buiter (1999), Niepelt (2004) and Daniel (2007) for critics on the FTPL.

[2] Bassetto and Cui (2018) show that, in a low interest rates environment, for the FTPL to remain valid, there is a case where the government must run recurring deficits.

[3] For example, Tirole (1985) shows that in an OLG model a satisfied transversality condition for agents does not prevent the government from playing a Ponzi game.

[4] In what follows, we will use the more common terminology, i.e. the ZLB. But, we have in mind the possibility that the lower bound may be different from zero. Recently, some central banks experienced indeed negative interest rates.

on the policy parameters set by monetary and fiscal authorities. A determinate equilibrium then requires one active and one passive policies: active monetary-passive fiscal and passive monetary-active fiscal regimes. In the first regime, the monetary authority reacts aggressively towards inflation by responding more than one-for-one to the deviations of inflation from the target while the fiscal authority takes the price level as given and restricts itself to a fiscal policy consistent with a sustainable debt burden. In the second regime, taxes set by the fiscal authority are not sufficient to cover interest payments on its debt and expenditures—constraining thus the monetary authority—forcing the price level to adjust to reach a stable steady-state dynamics. This regime describes the FTPL view where fiscal policy plays an important role in the price level determinacy.

Our framework is characterized by a double nonlinearity, i.e., wealth effects and ZLB, leading to the coexistence of the four types of regimes described above but for one set of policy parameters. This means that in our framework the determinacy region is no longer specified by the policy parameter space. In a dynamic monetary general equilibrium model, the implications of the ZLB on nominal interest rates on the determinacy of the price level was first pointed out by Benhabib et al. (2001a,b). They argue that the presence of the ZLB on nominal interest rates results in multiple equilibria. This nonlinearity generates a second steady state locally indeterminate, describing a liquidity trap situation. Consequently, the global determinacy is no more guaranteed. In our paper, wealth effects, induced by the non Ricardian structure of the economy, add another source of nonlinearity to the framework describes in Benhabib et al. (2001a,b), amplifying thus the global indeterminacy by the appearance of two additional steady states. Interestingly, one of these two additional steady states is locally determinate and have the same characteristics of the FTPL.

It tuns out that monetary policy can switch from locally active to locally passive depending on the bindingness of the ZLB on nominal interest rates. Similarly, fiscal policy can switch from locally passive to locally active depending on the level of government debt. The steady-state level of government debt, in our model, is relevant due to the presence of wealth effects. We call the equilibrium where the ZLB is binding and the steady state level of government debt is high, *Debt-Liquidity trap equilibrium*.[5] We show that, around this equilibrium, the economy behaves as in the FTPL framework. In fact, we demonstrate that, around the Debt-Liquidity trap equilibrium, the government debt valuation equation of Cochrane (2005) has a bubble/Ponzi game component, in addition to the present value of primary surpluses. As the economy converges to a higher steady-state level of government debt, the amount of taxes generated by the simple

[5]In this equilibrium, the monetary policy is locally passive because the nominal interest rate reaches its lower level and the fiscal policy is active because the steady state level of government debt is high.

fiscal rule becomes insufficient to guarantee the convergence of real government debt for a given price level. Consequently, the price level adjusts to achieve this goal. But, while the presence of the primary surpluses (or deficits) is essential to the FTPL, the Debt-Liquidity trap equilibrium still exist and locally determinate even with zero-primary surpluses.

The paper proceeds as follows. In section 2, we build the model of a non Ricardian economy with a simple monetary and fiscal rules. In Section 3, we analyze the dynamics of the model and focus on the local dynamic characteristics of each equilibrium. Section 4 is devoted to explaining the main mechanism at work and shows why the FTPL can still work with zero-primary surpluses. Section 5 summarizes our main results and suggests futures directions for analysis.

2 The model

We use an extended version of Weil (1987, 1989) overlapping-generations model.[6] The economy consists of many infinitely-lived families of agents. Each period new and identical infinitely-lived families appear in the economy without initial wealth. In period t, the economy is populated by a large number N_t of agents. Each period a new dynasty appears consisting of $(N_t - N_{t-1}) = nN_{t-1}$ agents, where $n \geq 0$ represents at the same time the population growth rate and the birth rate. This parameter can also be interpreted as a measure of the heterogeneity of the population.[7] Our model embodies the Ricardian case which is reached when $n = 0$.

2.1 Households

Each household belonging to the dynasty $j \leq t$ has preferences defined over consumption described by the utility function,

$$\sum_{s=t}^{\infty} \beta^{s-t} \ln c_{j,t}, \tag{1}$$

where $\beta \in [0, 1]$ represents a subjective discount factor, and $c_{j,t}$ denotes the consumption of the household j in period $t \geq j$. At the beginning of period t, the household $j < t$ receives a *per capita* endowment of $y_{j,t}$ units of consumption good and pays taxes to the government of $\tau_{j,t}$ units of consumption good. Let $0 < y_{j,t} < \infty, \forall t$. The household j have access to the bond market. Let $B_{j,t+1}$ be the units of nominal bond demanded by the household j at time t, $Q_{t,t+1}$ be the price of the bond in nominal terms, and P_t is the price of consumption good, then time t

[6]The Weil's model is useful because it embodies the Ricardian framework of the FTPL allowing thus for comparison to the non Ricardian framework.

[7]As explained by **?**, n is a measure of the economic disconnectedness of the population.

budget constraint of the household j takes the following form

$$P_t c_{j,t} + Q_{t,t+1} B_{j,t+1} \leq B_{j,t} + P_t \left(y_{j,t} - \tau_{j,t} \right). \tag{2}$$

We assume that each family cannot roll over its debt forever;

$$\lim_{T \to +\infty} Q_{t,T} B_{j,T+1} = 0, \tag{3}$$

where $Q_{t,T}$ denotes a discount factor given by $Q_{t,T} = Q_{t,t+1} \times Q_{t+1,t+2} \times .... \times Q_{T-1,T}$ and $Q_{t,t} = 1$.[8]

The representative household of generation j maximizes his intertemporal utility (1) subject to the budget constraint (2). This optimization problem leads to the following individual Euler equation summarizing the intertemporal arbitrage between present and future consumptions,

$$\beta \frac{c_{j,t}}{c_{j,t+1}} = q_t \frac{P_{t+1}}{P_t}. \tag{4}$$

Let $b_{j,t} \equiv B_{j,t}/P_{j,t}$ and $R_t \equiv (q_t P_{t+1}/P_t)^{-1}$ denote the real bond and the gross real interest rate, respectively. The real terms budget constraint of the household j is therefore given by

$$c_{j,t} + \frac{b_{j,t+1}}{R_t} = b_{j,t} + y_{j,t} - \tau_{j,t}. \tag{5}$$

2.2 Aggregation

Noting that the generation j is composed of $N_j - N_{j-1}$ agents, *per capita* aggregate consumption is given by

$$c_t = \sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_t} c_{j,t}. \tag{6}$$

Suppose that the endowment and the taxes are age-independent. Then their *per capital* aggregate values are equal to their individual values, that is $\tau_t = \tau_{j,t}$, $\forall j$ and $y_t = y_{j,t}$, $\forall j$. Consequently, period t,*per capita* aggregate value of the real bond is given by

[8] In OLG models, transversality condition is not needed with finitely lived agents to the existence of bubbles. However, Weil (1989), shows that in an OLG model with infinite horizon, the presence of transversality condition does not rule out the existence of bubbles.

$$\begin{aligned}\sum_{j\leq t}\frac{(N_j-N_{j-1})}{N_t}b_{j,t+1} &= \sum_{j\leq t+1}\frac{(N_j-N_{j-1})}{N_{t+1}}b_{j,t+1}-\frac{N_{t+1}-N_t}{N_t}b_{t+1,t+1}\\ &= \frac{N_{t+1}}{N_t}\sum_{j\leq t+1}\frac{(N_j-N_{j-1})}{N_{t+1}}b_{j,t+1}\\ &= (1+n)\,b_{t+1},\end{aligned} \tag{7}$$

since $d_{t+1,t+1}=0$, as the dynasty $j=t+1$ having non-financial wealth in period $t+1$.
Using this result, aggregating equations (4) and (5), after some rearrangements,[9] we get

$$c_t=\beta^{-1}\frac{c_{t+1}}{R_t}+n\left(\beta^{-1}-1\right)\frac{b_{t+1}}{R_t}. \tag{8}$$

This equation is the aggregate Euler equation that differs from the individual Euler condition (4), as long as the population growth rate is different from zero.[10] In fact, in period $t+1$, the aggregate consumption of agents alive; $c_{t+1}=\sum_{j\leq t+1}(N_j-N_{j-1})\,c_{j,t+1}/N_{t+1}$, is lower than the average consumption of currently alive agents; $\sum_{j\leq t}(N_j-N_{j-1})\,c_{j,t+1}/N_t$. This is because the aggregate consumption includes the low level of consumption of new agents who arrive in the economy without financial wealth. A real wealth effect appears as long as $n\neq 0$, which is a characteristic of a non Ricardian economy. An increase in the beginning-of-period financial wealth in $t+1$ benefits only to currently alive consumers in period t, and thus it cannot be proportionally distributed amongst present and future aggregate consumptions.

2.3 Monetary and Fiscal Authorities

Each period t, the government levies lump-sum taxes T_t and consumes G_t. Let B_t be promised nominal debt repayments by the government due at the beginning of period t and $Q_{t,t+1}$ the price of newly issued debt at time t that matures at $t+1$. The government instantaneous budget constraint in nominal terms is therefore given by

$$Q_{t,t+1}B_{t+1}+P_tT_t=B_t+P_tG_t, \tag{9}$$

with $T_t=\sum_{j\leq t}(N_j-N_{j-1})\,\tau_{j,t}=N_t\tau_t$.

[9] See appendix A for more details

[10] Recall that in Weil's model the population growth rate could not be negative because the model excludes the death.

Fiscal Rule Suppose that in order to determine the amount of the *per capita* lump-sum taxes, the fiscal policy follows this simple rule

$$\tau_t = z_t y_t + \theta \left(\frac{B_t}{N_t P_t} - \bar{b} \right), \tag{10}$$

The first term on the right-hand side of Eq. (10), $z_t y_t$, represents the part of taxes proportional to the endowment.[11] The second component reflects the fact that the government debt is partially backed by direct taxes, where $0 \leq \theta \leq 1$. Suppose that the government expenditures are proportional to the endowment, that is

$$G_t = N_t g_t y_t, \tag{11}$$

where g_t is determined by the fiscal authority. Inserting (10) and (11) into the budget constraint (9) and dividing both sides by P_t and N_t, the dynamic of the *per capital* real government debt is therefore given by

$$\frac{b_{t+1}}{R_t} = \frac{1}{1+n} \left[(1-\theta)\, b_t + (g_t - z_t)\, y_t + \theta \bar{b} \right], \tag{12}$$

where $b_t \equiv B_t / P_t N_t$. For a simplification purpose, suppose that in the long run the fiscal authority imposes the condition $g = z$, in order to guarantee that the primary deficit can equal zero when the debt is entirely paid back.

Monetary Rule Using the assumption introduced by Leeper (1991) and then generalized and popularized by Taylor (1993), the monetary policy adjusts in a systematic manner its nominal interest rate in response to the deviation (or the ratio) of inflation from its long-run target, $\bar{\Pi}$. In order to take into account a lower bound constraint on the nominal interest rates, we specify the following non linear monetary rule

$$1 + i_t = \max \left\{ \bar{R}_t \bar{\Pi} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi} ; 1 + \underline{i} \right\}, \tag{13}$$

where i_t, $\underline{i}$, and $\bar{R}$ are nominal interest rate in period t, the effective lower bound on the nominal interest rate and the gross real interest rate target, respectively. The monetary rule given by (13), satisfies the Taylor Principle, with $\phi > 1$. From now on, suppose $\underline{i} = 0$.[12]

[11] z is not a proportional tax rate—taxes are lump sum in our economy—but a part of the fiscal rule that is proportional to the endowment.

[12] $\underline{i}$ can be different from zero. $\underline{i}$ is called indeed effective lower bound rather than zero lower bound because many central banks are targeting slightly negative interest rates. What matters is not that the interest rate gets stuck at zero but that it becomes unresponsive to shocks.

2.4 Market Clearing and Steady State

The system of equilibrium equations for this economy takes the following form

$$\begin{aligned} c_t &= \beta^{-1}\frac{c_{t+1}}{R_t} + n\left(\beta^{-1}-1\right)\frac{b_{t+1}}{R_t}, &(14)\\ c_t &= (1-g_t)\, y_t, &(15)\\ b_{t+1} &= \frac{R_t}{1+n}\left[(1-\theta)\, b_t + (g_t - z_t)\, y_t\right], &(16)\\ \Pi_{t+1} &= \max\left\{\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}; \frac{1}{R_t}\right\}, &(17)\end{aligned}$$

where Eq. (14) stands for the non-Ricardian Euler equation, Eq. (15) describes the good market clearing condition, Eq. (16) represents the simplified law of motion of the real public debt, and Eq. (17) comes from the combination of the Fisher equation; $1 + i_t = R_t\Pi_{t+1}$, and the Taylor rule equation (13) and describes the dynamic of the inflation gross rate.

Using (15) into (14), dropping all time indices, assuming $g = z$, and rearranging equations (14)-(17), lead to the following set of steady-state equations

$$\left[R - \beta^{-1}\right](1-g)\, y = n\left(\beta^{-1}-1\right) b, \tag{18}$$

$$\left[\frac{1+n}{R} - (1-\theta)\right] b = 0 \tag{19}$$

and

$$R\Pi = \max\left\{R\bar{\Pi}\left(\frac{\Pi}{\bar{\Pi}}\right)^{\phi}; 1\right\}. \tag{20}$$

A deterministic steady state equilibrium is a vector (b, Π, R) verifying the above system of equations; (18) to (20).

Proposition 1. *Two different sources of non-linearity can be identified:* i) *the existence of a lower bound on the nominal interest rate and* ii) *the existence of wealth effects with a simple fiscal rule, result in the emergence of four steady states.*

Notice that the first two equations, (18) and (19), are independent of Π. The system is then dichotomous and allows to find (R, b) independently of the monetary policy.[13] For a given value of R, equation (20) allows to find the steady state value(s) of Π according to the target, $\bar{R}$, which can be chosen to be equal to the actual steady state value of R.[14]

[13] Notice that this long run dichotomy is not a fundamental characteristic of such a model. It is the consequence of several simplifying assumptions used: flexible prices and the simple monetary and fiscal rules.

[14] Assuming $\bar{R}_t = R_t$ implies that the long term inflation rate depends on the equilibrium value of the real interest rate, except in the liquidity trap. So, the inflation target $\bar{\Pi}$ can be reached for R^* and for R^{**}.

As shown by Benhabib et al. (2001a,b), when the zero lower bound is taken into account multiple steady state equilibria may exist. It is notably the case when the monetary policy is active in the sense of Leeper (1991) around the inflation target $\bar{\Pi}$. Eq. (20) generates two steady state levels for the gross inflation rate: $\bar{\Pi}$ and β^{-1}.

Now we turn back to the real part of the deterministic steady state. Equation (19) admits two obvious solutions, $b^* = 0$ and $R^{**} = \frac{1+n}{(1-\theta)}$, corresponding to two stationary equilibrium vectors of the variables b and R.

The first solution corresponds to a zero public debt, $b^* = 0$, and the gross real interest rate equals the inverse of the discount factor, $R^* = \beta^{-1}$. We call this solution the *autarkic equilibrium.*

The second solution, $R^{**} = (1+n)/(1-\theta)$, allows to compute the equilibrium value of the real public debt given by

$$b^{**} = \frac{(1-g)}{n(1-\beta)}(\beta R^{**} - 1)\,y, \qquad \text{for } n > 0. \tag{21}$$

We call this solution the *debt equilibrium.*

Comparing autarkic and debt equilibria, we obtain the following proposition.

Proposition 2. *The real value of the per capita public debt is positive in a debt equilibrium if and only if the associated real interest rate is greater than the autarkic real interest rate, i.e.:*

$$R^{**} = \frac{1+n}{1-\theta} \geq R^* \Longleftrightarrow b^{**} \geq 0. \tag{22}$$

It is straightforward to notice that in order to guarantee a positive level for real public debt at the debt equilibrium, $b^{**} > 0$, the following condition must hold

$$\beta R^{**} - 1 > 0. \tag{23}$$

Recall, that $R^* = \beta^{-1}$ in the autarkic equilibrium, so accordingly, $R^{**} > R^*$. Equivalently, in order to guarantee that $b^{**} > 0$, condition (23) can be replaced by

$$1 + n > \beta(1-\theta),$$

which is always satisfied, in our model, due to our assumption $n \geq 0$.

To sum up, our economy potentially admits four steady state equilibria; the Autarkic-Inflation Targeted (AIT) equilibrium, $b = 0$ and $\Pi = \bar{\Pi}$, the Autarkic-Liquidity Trap (ALT) equilibrium, $b = 0$ and $\Pi = 1/R^*$, the Debt-Inflation Targeted (DIT) equilibrium, $b = b^{**}$ and $\Pi = \bar{\Pi}$, and the Debt-Liquidity Trap (DLT) equilibrium, ($b = b^{**}$ and $\Pi = 1/R^{**}$). These equilibria are represented on Figure 1 below.

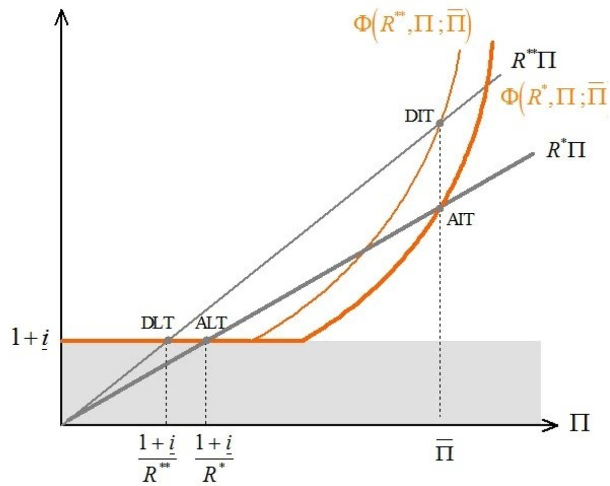


Figure 1: Multiple steady states.

3 The Dynamics

In this section, we show that the four steady state equilibria found correspond to—and have, locally, the same dynamics properties as—those analyzed by Leeper (1991). In addition, we show that unlike the case of the Ricardian economy considered by Leeper, these four equilibria coexist for a unique set of the fundamental parameters. We start by setting the parameter values to fit the benchmark case of active monetary policy and passive fiscal policy described in Leeper (1991) and used in standard dynamic general equilibrium models, i.e., $\phi > 1$ and $1 - (1+n)\,\beta < \theta < 1$. Using (15) into (14), and assuming $y_t = y\ \forall t$, and $g_t = g\ \forall t$, lead to the following IS equation of this non-Ricardian endowment economy,

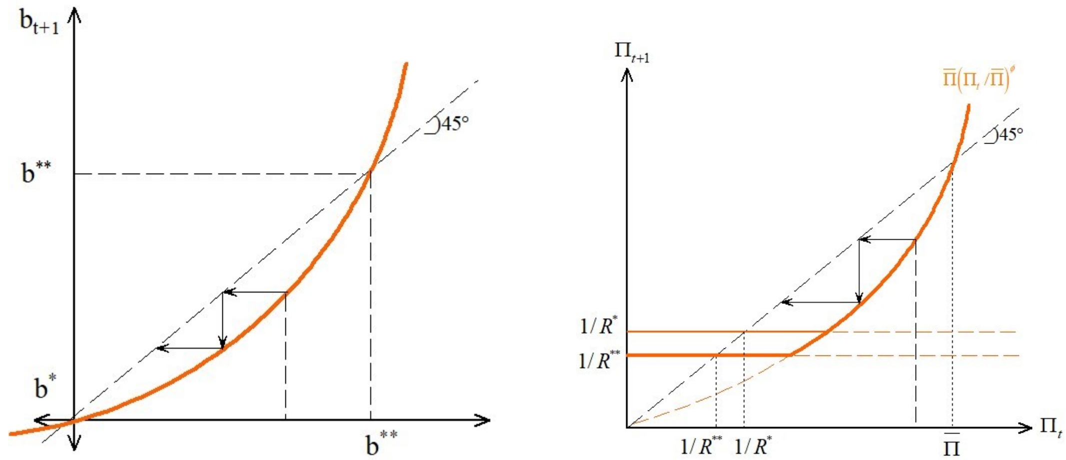
$$R_t = R^* + \frac{n\left(\beta^{-1} - 1\right)}{(1-g)\,y} b_{t+1}. \tag{24}$$

Using (24) into (16) and assuming that $z_t = g_t\ \forall t$, the law of motion for the real public debt is given by

$$b_{t+1} = \frac{R^* b_t}{R^{**} - \frac{n(\beta^{-1}-1)}{(1-g)y} b_t}, \tag{25}$$

where, we recall that $R^{**} \equiv \frac{1+n}{1-\theta}$, which verifies $R^{**} > \beta^{-1}$. Figure 2a describes the dynamics of the real public debt.

Notice that the dynamic of the gross inflation rate given by (14) is autonomous as long as the ZLB constraint is not binding. Nevertheless, assuming that the value of the real public debt is between b^* and b^{**}, the corresponding real interest rate lies between its two steady state values R^* and R^{**}. Consequently the inflation dynamics can be represented on the heuristical Figure 2b.



(a) The dynamics of b

(b) The dynamics of Π

Figure 2: Multiple Equilibria

3.1 Local determinacy conditions

Let $\hat{u}_t = u_t - u$, be a variable in difference, where u represents the variable u_t evaluated in one of the stationary equilibria. We linearize the equations (24), (25) and (14) around a given stationary equilibrium using $\hat{u}_t$.[15] The following two-dimensional dynamic system comes out,

$$\hat{b}_{t+1} = \frac{R^* R^{**}}{\left(R^{**} - \frac{n(\beta^{-1}-1)}{(1-g)y} b\right)^2} \hat{b}_t, \tag{26}$$

$$\hat{\pi}_{t+1} = \begin{cases} \phi \hat{\pi}_t, & \text{if } \pi = \bar{\pi} \\ -\frac{n(\beta^{-1}-1)}{R^2(1-g)y} \hat{b}_{t+1}, & \text{if } \pi = \beta \end{cases}, \tag{27}$$

The vector $\hat{Y}_t$ is composed of the two variables $\hat{b}_t$ and $\hat{\pi}_t$, both potentially non predetermined but linked to one another by the same unique jump variable P_t. Recall indeed that $b_t = B_t/P_t N_t$ and $\Pi_t = P_t/P_{t-1}$. It is therefore necessary, in order to apply Blanchard and Kahn (1980) conditions, to consider one of the two variables ($\hat{\pi}_t$ or $\hat{b}_t$) as predetermined and the other one ($\hat{b}_t$ or $\hat{\pi}_t$) as non predetermined. We can then state the following proposition.

Proposition 3. *Under the assumption $R^{**} > \beta^{-1}$*

[15] We use a variable in difference rather than in ratio, because one of the variables, b_t, could equal zero in the long run.

i) *the Autarkic-Inflation Targeted (AIT) equilibrium is locally determinate;*

ii) *the Autarkic-Liquidity Trap (ALT) equilibrium is locally indeterminate;*

iii) *the Debt-Inflation Targeted (DIT) equilibrium is locally overdeterminate;*

iv) *the Debt-Liquidity Trap (DLT) equilibrium is locally determinate.*

Proof. See appendix B. ∎

Based on proposition 3, the four potential stationary equilibria of our economy have, locally, the same dynamic properties of the four equilibria described by Leeper (1991) associated different configurations of fiscal and monetary policies. But unlike in Leeper (1991), our economy allows the coexistence of these four configurations for a given set of parameters.

3.2 Leeper revisited: active *vs* passive policy rules

The intuition of Proposition 3 is the following. Recall that for Leeper a fiscal policy is said to be active when the fiscal authority pays attention to the primary deficits regardless of the debt stabilization objective. On the other hand, an active monetary policy is defined as an interest rate rule that is sufficiently responsive to the inflation rate; it respects the Taylor principle. Within the framework considered by Leeper, monetary and fiscal policies that are simultaneously passive lead to indeterminacy while active policies lead to overdeterminacy (instability). Only the configurations where one of the two policies is active and the other passive provide the determinacy—*i.e.* the local uniqueness—of the equilibrium.

Plus, as it was noted by Benhabib et al. (2001b), a monetary policy could be active around the intended equilibrium but is necessarily passive in the liquidity trap. Accordingly our liquidity trap equilibria are both characterized by a locally passive monetary policy, while equilibria associated with the inflation target, $\bar{\Pi}$, are characterized by a locally active monetary policy, in particular $\phi > 1$.

In a Ricardian economy, the Barro equivalence Barro (1974) isolates the real interest rate from the real public debt level which is not the case in our non-Ricardian economy. The presence of wealth effects results indeed in the dependence of the real interest rate on the public debt level. In this case, a passive fiscal policy in the sense of Leeper—sufficiently responsive to low levels of debt and real interest rate—is not able to offset the increased debt burden associated with a high level of real interest rate. Around this last kind of equilibria, the fiscal policy can be defined as locally active. This finding has already been reported by Cushing (1999) and Benassy (2000)

in a case where the monetary authorities peg the nominal interest rate. In the same vein, Leith and von Thadden (2008) show that, in a non-Ricardian economy, the determinacy conditions of the unique equilibrium they consider depend crucially on the level of real public debt.
In our model, As we have already noted, the two steady state equilibria that are locally determinate—*i.e.* associated with saddle (or locally unique), trajectories—are the Autarkic-Inflation Targeted equilibrium and the Debt-Liquidity Trap equilibrium, respectively. What are the local characteristics of monetary and fiscal policy around these two equilibria?
In the Autarkic-Inflation Targeted equilibrium, since the Taylor principle is verified around this equilibrium, the associated monetary policy is said to be active. On the other hand, we can easily show that the fiscal policy is locally passive. To do so, computing the derivative of equation (25) and expressing it when $b_t = b^* = 0$, we get $\partial b_{t+1}/\partial b_t = R^*/R^{**} < 1$, and the real public debt converges to 0, its long run value.
Because the nominal interest rate is stuck at its effective lower level, the monetary policy is forced to be passive in the Debt-Liquidity Trap equilibrium. On the other hand, the fiscal policy is locally active because the too low value of θ does not permit to compensate for the public debt burden associated with a high real interest rate. More precisely, when $b_t = b^{**}$, we have $\partial b_{t+1}/\partial b_t = R^{**}/R^* > 1$. This case corresponds to the original situation pointed out by Leeper (1991) that gave rise to the Fiscal Theory of the Price Level.

4 Extension of the FTPL in an OLG framework: the bubble-like component of the public debt

Can we really understand the nature of the Debt-Liquidity Trap (DLT) equilibrium in light of the FTPL? In order to clarify the difference between a FTPL-type equilibrium and the DLT equilibrium, we can use the "stock analogy" introduced by Cochrane (2005). Let us rewrite the *per capita* government budget constraint (16) as a valuation equation

$$b_t = (z_t - g_t)\, y + \theta b_t + \frac{1+n}{R_t} b_{t+1}, \tag{28}$$

where $b_t = B_t/N_t P_t$ is considered as a jump variable, as long as the price level is not constrained by the conduct of the monetary policy. By iterating forward this equation, one obtains

$$\frac{B_t}{N_t P_t} = \sum_{s=0}^{+\infty} \frac{(z_{t+s} - g)\, y + \theta b_{t+s}}{r_{t,t+s}} + \lim_{s\to+\infty} \frac{b_{t+s+1}}{r_{t,t+s+1}} \tag{29}$$

with $r_{t,t+s} = R_t \times R_{t+1} \times .... \times R_{t+s-1}/\left(1+n\right)^s$, and $r_{t,t} = 1$.

The traditional FTPL situation can be reproduced in our framework by analyzing the limit case of a Ricardian economy, *i.e.* $n = 0$, $R_t = \beta^{-1}\ \forall t$, and where the fiscal policy is active in the sense of Leeper, that is $\theta < 1 - \beta$.[16] We easily verify that the last term on the right hand side of (29) is zero when the household's transversality condition holds. In this case, if

$$\sum_{s=0}^{+\infty} \beta^s \left(z_{t+s} - g\right) y > 0,$$

then the price level is determined such that the value of real government debt is "a claim to government primary surpluses, just as private stock is valued as a claim to corporate profits" (Cochrane (2005)). Notice that, when $\theta = 0$, the future primary surpluses become exogenous and the price level is directly determined by

$$P_t = \frac{B_t}{N_t \sum_{s=0}^{+\infty} \beta^s \left(z_{t+s} - g\right) y}.$$

Consider now the non Ricardian economy at the DLT equilibrium and let us start with the simplified case $\theta = 0$.[17] Using $R_t = 1 + n\ \forall t$ and $r_{t,t+s} = \left(R^{**}/1+n\right)^s = 1$, equation (29) can be rewritten

$$\frac{B_t}{N_t P_t} = \sum_{s=0}^{+\infty} \left(z_{t+s} - g\right) y + \lim_{s \to +\infty} b_{t+s+1}$$

with $\lim_{s\to+\infty} b_{t+s+1} = b^{**}$.

By pushing further the stock analogy of Cochrane, the real public debt value can be split into two separate parts. The first one is the present value of future primary surpluses. It can be considered as the fundamental value of the real public debt. The second one, $\lim_{s\to+\infty} b_{t+s+1} = b^{**}$, can be viewed as a bubble component. In this case, the valuation equation (29) determines a finite price level, even when the fundamental part is null, that is $z_t - g = 0\ \forall t$. We then obtain

$$P_t = \frac{B_t}{N_t b^{**}}. \tag{30}$$

In other words, when the exogenous component of the primary public surplus is zero and when $\theta = 0$, the right-hand term of the valuation equation is only constituted by the bubble, *i.e.* the unbaked part of the public debt.[18] The OLG structure of our model permits the existence of an equilibrium—the Debt-Liquidity Trap equilibrium—where this bubble component is positive

[16] Note that this correspond to the case where our assumption $R^{**} > \beta^{-1}$ is not verified and $b^{**} = b^* = 0$.

[17] Henceforth, our assumption $R^{**} > \beta^{-1}$ is again supposed to be verified.

[18] Note that this equilibrium requires that the Government plays a Ponzi game by rolling over its debt. Nevertheless, if we follow the FTPL and consider the price level as the inverse of the price of the nominal short term government debt, the equilibrium value of the real debt can be viewed as a pure bubble.

at the steady state. A well-known condition for the existence of a bubble in an OLG model is that the growth rate is superior to the autarkic interest rate.[19] This condition is equivalent to assumption $R^{**} > R^{*}$, which is written as $1 + n > \beta^{-1}$, when $\theta = 0$.

Finally, consider the case where $\theta > 0$, but assume again $z_t - g_t = 0\ \forall t$. Using $R_t = (1+n)/(1-\theta) = R^{**}\ \forall t$, and $r_{t,t+s} = (R^{**}/1+n)^s = (1-\theta)^{-s}$, the valuation equation (29) takes now the following form

$$\frac{B_t}{N_t P_t} = \sum_{s=0}^{+\infty} \theta (1-\theta)^s b_{t+s} + \lim_{s\to+\infty} (1-\theta)^{s+1} b_{t+s+1}.$$

If $\lim_{s\to+\infty} b_{t+s+1}$ is finite and equals b^{**}, the "pure bubble" component—$\lim_{s\to+\infty} (1-\theta)^{s+1} b_{t+s+1}$— equals zero as $0 < 1-\theta < 1$. But, interestingly, the fundamental part of the valuation equation determines the same solution as in (30) for the equilibrium price level. To see why, take a simpler version of the valuation equation (28), where $z_t = g_t$ and $R_t = R^{**}$, that is,

$$\begin{aligned} b_t &= \theta b_t + (1-\theta) b_{t+1} \\ &= b_{t+1} \\ &= \lim_{s\to+\infty} b_{t+s+1} \end{aligned}$$

with $\lim_{s\to+\infty} b_{t+s+1} = b^{**}$. In other words, the equilibrium value of the real public debt is not a "pure bubble" but has the same characteristics. Particularly, the steady state solutions for R^{**} and b^{**} are both growing and continuous functions of θ. On one hand, a positive value of θ then plays the same role as a negative growth rate of money supply in a two-period OLG model.[20] On the other hand, the positive link between θ and b is a characteristic of a FTPL equilibrium when part of the primary surplus is function of the level of the real public debt.[21] The sole difference is that the standard FTPL needs the existence of a positive exogenous primary surplus that is not necessary in our economy, this role being played by the bubble-like component of the public deb

[19] See Samuelson (1958), Gale (1973) and Tirole (1985). In a standard OLG model of an exchange economy, this condition is fulfilled in the case of a Samuelsonian economy.

[20] The real gross interest rate is equal to $(1+n)/(1+\mu)$, where μ is the growth rate of money creation in a Samuelson-type OLG model.

[21] With $\theta > 0$, the FTPL-equilibrium value of the real public debt is given by:

$$b_t = \sum_{s=0}^{+\infty} \left(\frac{\beta}{1-\theta}\right)^s \left(\frac{z_{t+s} - g}{1-\theta}\right) y$$

5 Conclusion

This paper shows that combining a non-Ricardian structure of the economy with the ZLB on nominal interest rates produces a very rich global dynamics. Multiple steady state equilibria with different local dynamic properties. We show that these equilibria have the same local dynamic characteristics of the four regimes described by Leeper (1991). More interestingly, we show that one of these equilibrium—Debt-Liquidity trap equilibrium—has the same mechanism at work as in the FTPL. Around this equilibrium, we demonstrate that the government debt valuation equation of Cochrane (2005) has a bubble/Ponzi game component, in addition to the present value of primary surpluses. This implies that even with zero-primary surpluses, the FTPL still work.

In our economy, the presence of multiple steady-state equilibria raises the issue of the selection criteria and gives an important role to self-fulfilling expectations. We believe that our model offers a useful framework to explain switching steady states based on agents' expectation change. For example, assume that a severe exogenous shock moves the economy away from the "targeted" steady-state. Does the economy go back to the targeted steady state or jump on one of the other trajectories? Recall, that in our model there exist two 'locally unique' trajectories. Each of them leads toward one of the two locally determinate equilibria. Can agent expectations coordinate to bring the economy to the Debt-Liquidity trap equilibrium? These are open questions which, we hope will be addressed in future works. Replacing the hypothesis of rational expectations by a learning process could be a promising approach. In addition, an other development of our work would consist in incorporating explicitly the default risk. It is well-known, since Uribe (2006), that this can weaken the uniqueness of the price level. On the other hand, assuming a default strategy followed by the government, should restore the uniqueness of the price level while reducing the associated deflation.

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A Appendix A: Non-Ricardian Euler Equation

This appendix details the steps involved to obtain Eq. (**??**). Let $h_{j,t}$ denotes the household j's human wealth corresponding to the discounted value of future labor incomes net of taxes;

$$h_{j,t} \equiv \frac{1}{P_t}{}_{s=t}^{+\infty} Q_{t,s}\left[P_s\left(y_{j,s} - \tau_{j,s}\right)\right]. \tag{31}$$

Iterating the household j budget constraint (2) forward, and using (3), leads to the following household j's intertemporal budget constraint

$$B_{j,t} ={}_{s=t}^{+\infty} Q_{t,s}\left[P_s c_{j,s} - P_s\left(y_{j,s} - \tau_{j,s}\right)\right]. \tag{32}$$

Now, iterating (4) forward, gives the following expression for future consumption at time $s \geq t$

$$c_{j,s} = \beta^{s-t} c_{j,t} \frac{P_t}{Q_{t,s} P_s}. \tag{33}$$

Using Eq. (33) into Eq. (32) expressed in real terms, gives

$$c_{j,t} = (1-\beta)\left[b_{j,t} + h_{j,t}\right]. \tag{34}$$

Eq. (34) describes the optimal consumption of agent j which is a constant fraction of his total wealth; nonhuman plus human wealth. Recall, that in our model nonhuman wealth is age independent. Now, we iterate Eq. (34) one step ahead, and then we replace $c_{j,t+1}$ by it expression given by Eq. (4);

$$c_{j,t} = \beta^{-1}(1-\beta)\left[\frac{b_{j,t+1} + h_{j,t+1}}{R_t}\right] \tag{35}$$

Applying the definitions of the aggregate consumption and the aggregate real debt, given by (6) and (7), respectively, to Eq. (35), leads to the following aggregate consumption

$$c_t = \beta^{-1}(1-\beta)\left[\frac{(1+n)\, b_{t+1} + h_{t+1}}{R_t}\right] \tag{36}$$

Finally, we iterate Eq. (34) one period forward, then we aggregate it over all agents and using it into Eq. (36), give the following non-Ricardian Euler equation

B Appendix B: Local Dynamic

Proposition 2. Under the assumption $n \geq 0$, $\phi > 1$, and $1 - (1+n)\beta < \theta < 1$,

i) *the Autarkic-Inflation Targeted (AIT) equilibrium is locally determinate;*

ii) *the Autarkic-Liquidity Trap (ALT) equilibrium is locally indeterminate;*

iii) *the Debt-Inflation Targeted (DIT) equilibrium is locally overdeterminate;*

iv) *the Debt-Liquidity Trap (DLT) equilibrium is locally determinate.*

Proof.
Denoting $\hat{Y}_t = \left[\begin{array}{cc} \hat{b}_t & \hat{\pi}_t \end{array} \right]'$, the vector of the endogenous variables, using the long run equations (18) to (20) and neglecting the shocks $\hat{g}_t$ and $\hat{z}_t$, the equations (17), (24), and (25) could be combined in order to get the following state-space form

$$E_t \hat{Y}_{t+1} = J(b, \pi, R) \cdot \hat{Y}_t, \tag{37}$$

where the Jacobian matrix $J(\cdot)$ is given by

$$J(\cdot) = \begin{cases} \begin{pmatrix} \frac{R^* R^{**}}{\left(R^{**} - \frac{n(\beta^{-1}-1)}{(1-g)y} b\right)^2} & 0 \\ 0 & \phi \end{pmatrix}, & \text{if } \pi = \bar{\pi}, \\ \begin{pmatrix} \frac{R^* R^{**}}{\left(R^{**} - \frac{n(\beta^{-1}-1)}{(1-g)y} b\right)^2} & 0 \\ -\frac{n(\beta^{-1}-1)}{R^2(1-g)y} & 0 \end{pmatrix}, & \text{if } \pi = \beta. \end{cases} \tag{38}$$

Autarkic Equilibria

In an autarkic steady state equilibrium, the real debt equals zero, $b^* = 0$. The matrix J reduces to

$$J(\cdot) = \begin{cases} \begin{pmatrix} \frac{R^*}{R^{**}} & 0 \\ 0 & \phi \end{pmatrix}, & \text{if } \pi = \bar{\pi}, \\ \begin{pmatrix} \frac{R^*}{R^{**}} & 0 \\ -\frac{n(1-\beta)\beta}{(1-g)y} & 0 \end{pmatrix}, & \text{if } \pi = \beta. \end{cases} \tag{39}$$

It is straightforward to determine the eigenvalues associated with the matrix J given by (39). In the Autarkic-Inflation Targeted (AIT) equilibrium, the absolute values for the two eigenvalues are given by $\frac{R^*}{R^{**}}$ and ϕ. The first one is less than one, $R^{**} > R^*$, and the second one greater than one, $\phi > 1$. As a consequence, the Autarkic-Inflation Targeted (AIT) equilibrium is locally determinate. On the other hand, the Autarkic-Liquidity Trap equilibrium is locally indeterminate. The two eigenvalues, in this case, are $\frac{R^*}{R^{**}}$ and 0. The absolute value for these two eigenvalues are indeed both less than one.

Debt Equilibria

In an autarkic steady state equilibrium, the real debt equals $b^{**} = \frac{(1-g)}{n(1-\beta)} \left(\beta R^{**} - 1\right) y$ and $R = R^{**}$. The matrix J reduces to

$$J(\cdot) = \begin{cases} \begin{pmatrix} \frac{R^{**}}{R^{*}} & 0 \\ 0 & \phi \end{pmatrix}, & \text{if } \pi = \bar{\pi}, \\ \begin{pmatrix} \frac{R^{**}}{R^{*}} & 0 \\ -\frac{n\left(\beta^{-1}-1\right)}{R^{**2}(1-g)y} & 0 \end{pmatrix}, & \text{if } \pi = \beta. \end{cases} . \tag{40}$$

Also in this case, the determination of the eigenvalues associated with matrix J, (40) is straightforward. In the Debt-Inflation Targeted equilibrium the two eigenvalues are both greater than one and given by $\frac{R^{**}}{R^{*}} > 1$ and $\phi >$. The Debt-Inflation Targeted equilibrium is therefore locally overdeterminate. However, the Debt-Liquidity Trap equilibrium is locally determinate. Because in this case, one of the two eigenvalues becomes less than one. The two eigenvalues are $\frac{R^{**}}{R^{*}} > 1$ and 0.‖