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Development, Fertility and Childbearing Age: A Unified Growth Theory*

Hippolyte d'Albis[†] Angela Greulich[‡] Gregory Ponthiere[§]

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Abstract

During the last century, fertility has exhibited, in industrialized economies, two distinct trends: the cohort total fertility rate follows a decreasing pattern, while the cohort average age at motherhood exhibits a U-shaped pattern. This paper proposes a Unified Growth Theory aimed at rationalizing those two demographic stylized facts. We develop a three-period OLG model with two periods of fertility, and show how a traditional economy, where individuals do not invest in education, and where income rises push towards advancing births, can progressively converge towards a modern economy, where individuals invest in education, and where income rises encourage postponing births. Our findings are illustrated numerically by replicating the dynamics of the *quantum* and the *tempo* of births for cohorts 1906-1975 of the Human Fertility Database.

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1 Introduction

In the 20th century, growth theorists paid particular attention to interactions between, on the one hand, the production of goods, and, on the other hand, fertility behavior, that is, the production of men. When studying those interactions, they have mainly focused on one aspect of fertility: the number or *quantum* of births. From that perspective, the key stylized fact to be explained is the declining trend in fertility.[1] That decline is illustrated on Figure 1, which shows the cohort total fertility rate (hereafter, TFR) for cohorts of women aged 40 in industrialized countries. That fertility decline was explained through various channels, such as the rise in the opportunity costs of children (Barro and Becker 1989), a shift from investment in children's quantity towards children's quality caused by lower infant mortality (Erhlich and Lui 1991, Doepke 2005, Bhattacharya and Chakraborty 2012), a rise in the return to education (Galor and Weil 2000), a rise in women's relative wages (Galor and Weil 1996), and the rise of contraception (Bhattacharya and Chakraborty 2017, Strulik 2017).

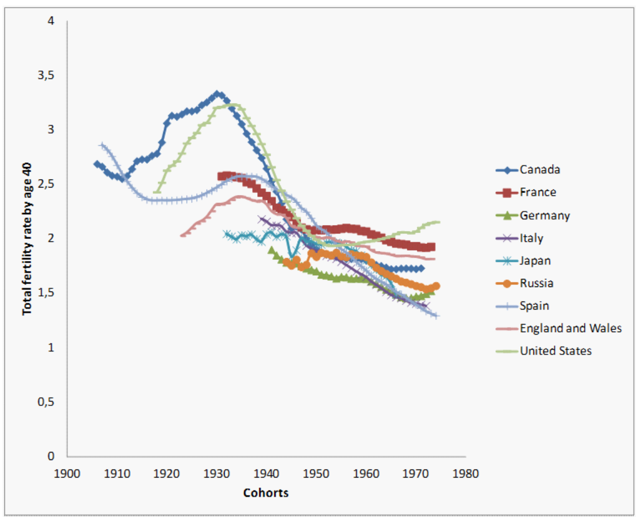

Figure 1: Cohort total fertility rate by age 40 (source: Human Fertility Database)

Although those models cast substantial light on interactions between fertility and development, their exclusive emphasis on the *quantum* of births leaves aside another important aspect of fertility, which has a strong impact on economic development: the timing or *tempo* of births. Studying the *tempo* of births - and not only the *quantum* of births - matters for understanding long-run economic development, because of two distinct reasons.

[1]Note that, although the long-run trend of the TFR is decreasing, the TFR can nonetheless exhibit significant short-run fluctuations around that trend, as shown on Figure 1.

First, theoretical papers, such as Happel et al (1984), Cigno and Ermisch (1989) and Gustafsson (2001), studied, at the microeconomic level, the mechanisms by which the birth timing decision is related to education and labor supply decisions. A lower wage early in the career reduces the opportunity cost of an early birth, and pushes towards more early births (substitution effect), but limits also the purchasing power, which pushes towards fewer early births (income effect). Moreover, investing in education raises the purchasing power in the future (which pushes towards more late births), but, at the same time, raises also the opportunity cost of future children (which pushes towards fewer late births). Those strong interactions between birth timing, education and labor decisions at the temporary equilibrium motivate the study, in a dynamic model, of how development affects - and is affected by - birth timing.

Second, there is also strong empirical evidence supporting the existence of complex, multiple interactions between the *tempo* of births and various economic variables, with causal relations going in both directions. Demographic studies show that the *tempo* of births is strongly correlated with the education level, which affects the human capital accumulation process (see Smallwood, 2002, Lappegard and Ronsen 2005, Robert-Bobée et al 2006). Moreover, several works, such as Schultz (1985), Heckman and Walker (1990) and Tasiran (1995), show that a rise in women's wages tends to favor a postponement of births.[2] There is also evidence that the wage level is affected by the timing of births (see Miller 1989, Joshi 1990, 1998, Dankmeyer 1996).

The timing of births has varied significantly during the 20th century, as illustrated on Figure 2, which shows the cohort mean age at motherhood by age 40 (hereafter, MAM).[3] Whereas patterns differ across countries, Figure 2 reveals an important stylized fact: the MAM exhibits, across cohorts, a U-shaped pattern. The MAM has been first decreasing for cohorts born before 1940/1950, and, then, has been increasing for later cohorts.[4]

The U-shaped pattern for the MAM raises several questions. A first question concerns the economic causes and consequences of that non-monotonic pattern. How can one explain that economic development is associated first with advancing births, and, then, with postponing births? How can one relate this stylized fact with income and substitution effects, and with the education decision? Another key question concerns the relation between the dynamics of the *quantum* of births (Figure 1) and the *tempo* of births (Figure 2). Why is it the case that, at a time of strong fertility decline, cohorts tended to advance births, and, then, tended to postpone births once total fertility was stabilized?

[2] Those studies focused on Sweden. Similar results were shown for Japan (Ermish and Ogawa 1994), for Canada (Merrigan and Saint-Pierre 1998), and for the UK (Joshi 2002).

[3] The cohort mean age at motherhood by age 40 is computed for *all* births combined (and not only for first births). Consequently, this measure depends on the *quantum* of births (women having more children being likely to be older, on average, when giving birth to a child). Our paper focuses precisely on this relation between the *tempo* of births and the *quantum* of births.

[4] Note that the timing of the reversal varies across countries. For instance, in Russia, the reversal of the mean age at motherhood occurred for cohorts born in the late 1960s.

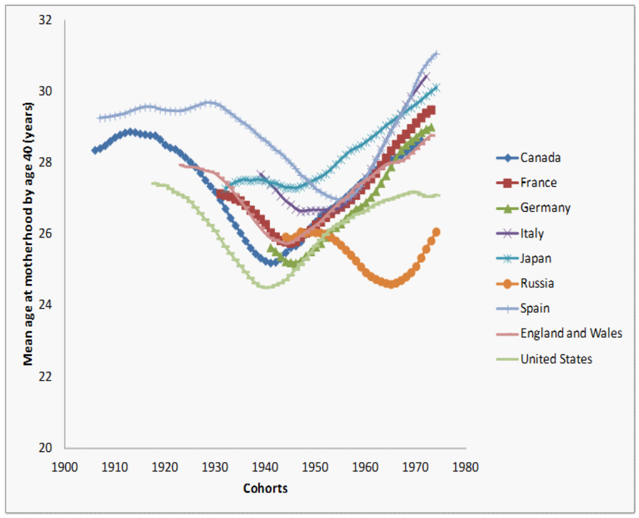

Figure 2: Cohort mean age at motherhood by age 40
(Source: Human Fertility Database)

The goal of this paper is precisely to cast some light on the relation between the *quantum* of births, the *tempo* of births, and economic development. For that purpose, we propose to adopt a Unified Growth Theory approach. As pioneered by Galor (see Galor and Moav, 2002, Galor, 2010), Unified Growth Theory pays a particular attention to the relation between quantitative changes (i.e. changes in numbers) and qualitative changes (i.e. changes in the form of relations between variables).[5] Qualitative changes are studied by means of regime shifts, which are achieved as the economy develops, and which cause major changes in the relations between fundamental variables. The fact, shown on Figure 2, that development was first associated with an advancement of births, and, then, with a postponement of births, can be regarded as a major qualitative change. Our paper aims at developing a single unified framework of analysis to understand the relation between development and birth timing, and, as such, can be regarded as a contribution to Unified Growth Theory.[6]

For that purpose, this paper develops a three-period overlapping generations (OLG) model with lifecycle fertility, that is, with two fertility periods (instead of one as usually assumed). In our model, individuals decide not only the *quantum* of births, but, also, how they allocate those births along their lifecycle, that is, the *tempo* of births. In order to study the interactions between birth timing

[5] Recent works in that approach include de la Croix and Licandro (2013), de la Croix and Mariani (2015), Lindner and Strulik (2015) and Strulik (2016).

[6] As argued by Galor (2011, p. 141), Unified Growth Theory refers to a class of economic theories that "capture the entire growth process within a single unified framework of analysis". Our model, which provides a single unified framework to analyze the relation between fertility and development, is in line with that class of theories, even though it does not consider all facets of development, but only its relations with the *quantum* and the *tempo* of births.

and education, we also assume that individuals choose how much they invest in their education when being young, which will affect their future productivity.

Anticipating our results, we show, using a general model with additively separable preferences, that there exist conditions on preferences such that, depending on the prevailing level of human capital, the temporary equilibrium takes three distinct forms, which correspond to three distinct regimes. In each of those regimes, a rise in human capital pushes towards fewer births. However, those regimes differ regarding the relation between human capital and the timing of births. In the first regime, which prevails for low levels of human capital, individuals do not invest in education, and rises in human capital push them towards advancing births (a decline of the MAM). In the second regime, which is achieved once human capital crosses a first threshold, individuals start investing in education, but education remains so low that human capital accumulation still makes individuals advance births. Then, once human capital is sufficiently high, and reaches a second threshold, the economy enters a third regime, where human capital accumulation leads to postponing births (a rise in the MAM).

Whereas the last, modern regime (with declining TFR and births postponement) was studied in the literature (see Iyigun 2000), a merit of this paper is to highlight the existence of two anterior regimes, where the decline of the TFR is associated not with the postponement of births, but with the advancement of births. In particular, the second regime, which exhibits increasing education and births advancement, has received little attention so far, but plays a key role in the transition from a traditional economy with a high TFR and a decreasing MAM to a modern economy with a low TFR and an increasing MAM.

The conditions on preferences leading to the existence of those regimes have three main aspects, which admit intuitive economic interpretation. First, those conditions guarantee that, as human capital accumulates, the substitution effect dominates the income effect for both early and late fertility rates (leading to a fall of TFR). Second, the conditions on preferences are such that, beyond some threshold for human capital, the level of education becomes positive and increasing with human capital. Third, standard assumptions on preferences imply also that the level of education tends to weaken the strength of the substitution effect with respect to the income effect for late births only. This latter property explains that, once education reaches a sufficiently high level, the tendency to advance births as human capital accumulates (which prevails for low levels of human capital) is replaced by a tendency to postpone births, leading to the U-shaped curve for the MAM.

Our dynamic lifecycle fertility model can thus rationalize both the decrease in fertility and the U-shaped pattern of the mean age at motherhood. That rationalization of the non-monotonic relation between development and birth timing is achieved by means of regime shifts as the economy develops, without having to rely on exogenous shocks. Besides this analytical finding, we also explore the capacity of our model to replicate numerically the dynamics of the *quantum* and the *tempo* of births. Using data for 28 countries from the Human Fertility Database (cohorts 1906-1975), we show that our model can, with some degree of approximation, fit the patterns of the TFR and the MAM.

Our paper is related to several branches of the literature. First of all, it complements microeconomic studies of birth timing, such as Happel et al (1984), Cigno and Ermisch (1989) and Gustafsson (2001), which examine birth timing decisions in a static setting, whereas we propose to draw the corollaries of those decisions for long-run dynamics. Second, we also complement the literature focusing on the relation between birth timing and long-run development. In a pioneer paper, Iyigun (2000) showed, by means of a 3-period OLG model with two fertility periods, that human capital accumulation leads to the postponement of births. While Iyigun's paper rationalizes the increasing part of the U-shaped curve for the MAM, our paper provides a rationalization for the entire U-shaped curve, including its decreasing segment.[7] Our paper complements also other papers, such as, in continuous time, d'Albis et al (2010), and, in discrete time, Momota and Horii (2013) and Pestieau and Ponthiere (2014, 2015). Those papers examined the relation between, on the one hand, physical capital accumulation, and, on the other hand, the *quantum* and *tempo* of births.[8] Whereas those papers paid attention to the existence and stability of a stationary equilibrium under several fertility periods, our paper adopts, on the contrary, a Unified Growth Theory approach, where regime shifts are used to rationalize the non-monotonic pattern exhibited by the *tempo* of births.

The rest of the paper is organized as follows. Section 2 has a closer look at the data, and examines the statistical significance and the robustness of the stylized facts concerning the *quantum* and the *tempo* of births, as well as the relation between fertility behavior and education. The model is presented in Section 3. Section 4 characterizes the temporary equilibrium, and examines the distinct regimes. Long-run dynamics is studied in Section 5. An analytical example is developed in Section 6. Section 7 illustrates our findings numerically, by focusing on cohorts of the Human Fertility Database. Section 8 concludes.

2 Stylized facts

Before considering how a theory of the *quantum* and the *tempo* of births can be developed to rationalize the two stylized facts mentioned in Section 1 (the declining long-run trend in cohort TFR and the U-shaped trend for cohort MAM), it is useful to have a closer look at the data, in order to examine the statistical significance and robustness of those stylized facts, and, also, in order to explore the relation between those stylized facts and education data.

Regarding the issue of statistical significance and robustness, we run simple statistical regressions using data from the Human Fertility Database. The data provides measures of cohort fertility by age 40 (TFR) and cohort mean

[7] Another related paper is Sommer (2016), which studies the relation between the *quantum* and the *tempo* of births under risk (in earnings and fertility). More recently, de la Croix and Pommeret (2017) study the optimal timing of births under fertility risks. Unlike those papers, our study involves no risk, but adopts a Unified Growth Theory approach with multiple regimes.

[8] The interactions between the *quantum* of births and physical capital accumulation are studied under more general preferences in Momota (2016).

age at childbirth by age 40 (MAM) for 28 countries, for cohorts 1906 to 1975 (unbalanced panel). Table A1 in the Appendix provides an overview of covered countries and cohorts for the variables used in this section. Regressions are all run with country-fixed effects. These models include country-specific dummies which implicitly control for country-specific variables that are constant over time. Our fixed-effects models therefore focus on within-country variations.

The data confirms, first of all, the existence of a long-run declining trend in cohort fertility at age 40 (cohort TFR).[9] Concerning the *tempo* of births, our regressions confirm the existence of a statistically significant U-shaped relation between the birth year of cohort and the cohort MAM.[10] The estimated U-shaped relation is shown on Figure 3, with a 95 % confidence interval.

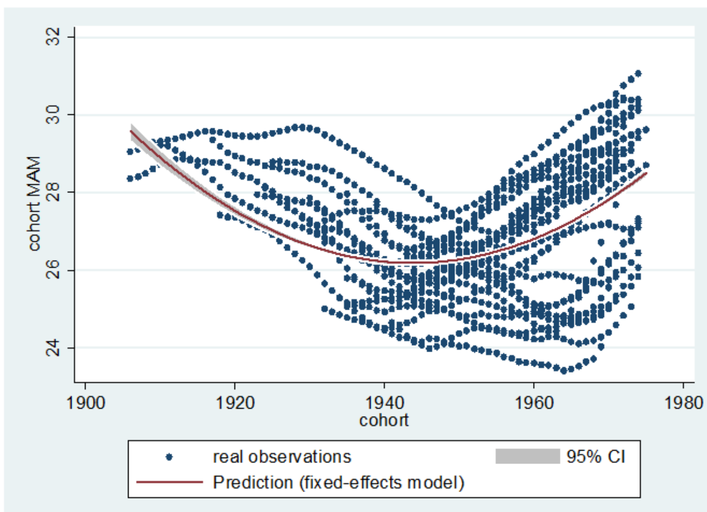

Figure 3: MAM by age 40 by cohorts (source Human Fertility Database).

The estimated U-shaped pattern is somewhat flatter than the one that is actually observed for most countries. This is partly due to a difference in the timing of the reversal. Some countries, such as France, exhibit a reversal for cohorts born in the 1940s, whereas in other countries, such as Russia, the reversal took place for cohorts born only in the 1960s. In addition, the estimated curve is flattened by the fact some countries join the panel later than others. It is likely that for these countries, we do not observe the full declining branch and/or the

[9] See, in the Appendix, Table A2 for the regression and Figure A1 for the graphical representation of the evolution of cohort TFR across cohorts. For most countries, the overall trend exhibited by the TFR is a decline over the observed cohorts. Two major exceptions are the United States and Canada, for which cohort TFR increases for cohorts 1910 to 1930.

[10] See Table A3 in the Appendix. Note that a cubic specification has also been tested for the cohort MAM, but was found to be non-significant (results available on request).

full re-increasing branch. However, despite the fact that the estimated U-shaped curve is flatter than the one for each country, it remains nonetheless statistically significant. Thus the stylized fact of a U-shaped cohort MAM is a robust empirical fact, which is not specific to only some of the observed countries.[11]

Given that education will play a key role in our theory of the *quantum* and the *tempo* of births, it is also worth considering data on education, and their relation with the TFR and the MAM. As cohort data on education is not available for most cohorts of interest, we use, as a second-best option, periodic data and assign the period measures to our cohorts 1906-1975. We use estimates on education attainment for the female population aged 15-24, which come from the Lee and Lee (2016) Long-Run Education Dataset, and cover years 1870 to 2010 (5-year intervals). In order to assign the periodic observations of the average years of total schooling (primary, secondary and tertiary) of a female population aged 15-24 to our birth cohorts of 1906 to 1974, we allow a 20-year delay between the periodic observations of education and our cohorts.[12] As shown in the Appendix, female educational attainment has been growing over the cohorts born during the 20th century for all countries.[13]

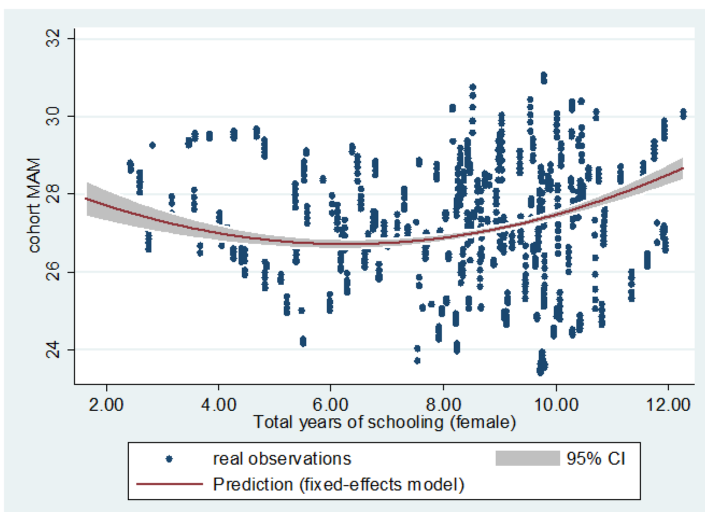

Figure 4: Cohort MAM by education (sources: Human Fertility Database and Lee-Lee Long-Run Education Dataset).

[11]Note that, as discussed in the Appendix, the estimated path for the mean age at first motherhood (MA1M) over cohorts exhibits also a U-shaped pattern, but the declining branch is smaller than the rising branch, which is most likely due to the fact that observations of MA1M are not available for cohorts born earlier than 1929 for the majority of countries.

[12]See the Appendix for the extrapolation of cohort educational attainments from period educational attainments observations.

[13]Figure A2 in the Appendix shows the countries' average years of female schooling assigned to the cohorts 1906-1974 as well as the estimated within-country path.

When relating education attainment with fertility behavior, a first observation is that, for the cohorts under study, there has been both a decline in the *quantum* of births, as well as a rise in education over cohorts (Figures A1 and A2 in the Appendix). Regarding the relation between cohort education and the *tempo* of births, one would, at first glance, expect that the rise in education is associated with a postponement of births. However, as shown in Figure 4, which contrasts the country-observations to the estimated within-country path, the relation between cohort education (average years of schooling: primary, secondary and tertiary) and the cohort MAM is non monotonous, and exhibits a U-shaped form. This U-shaped relation between education and MAM is robust to the definition of the education variable, and remains statistically significant even when education is restricted to only secondary and tertiary education.[14]

Figure 4, which plots real country observations against the regression result of Model 1 in Table A4 in the Appendix, can be interpreted as follows. Contrary to what one may expect at first glance, the relation between education and the *tempo* of births is not always increasing for the observed cohorts. Actually, there exists a threshold level in education such that, for education levels below the threshold, an increase in education leads to a reduction in the MAM (advancement of births), whereas, for education levels above the threshold, an increase in education leads to a rise in the MAM (postponement of births).

The goal of the next sections is to develop a simple theoretical model rationalizing the declining trend in the TFR, the U-shaped pattern for the MAM, as well as the U-shaped relation between education and the MAM.

3 The model

Let us consider a three-period OLG model with lifecycle fertility. Time goes from 0 to $+\infty$. Each period has a unitary length. Period 1 is childhood, during which the child is raised by the parent, and does not make any decision. Period 2 is early adulthood, during which individuals work, consume, have n_t children and invest in education. Period 3 is mature adulthood, during which individuals work, consume, and can complete their fertility by having m_{t+1} children. Figure 5 shows the formal structure of the model.

[14] See Figure A3 the Appendix.

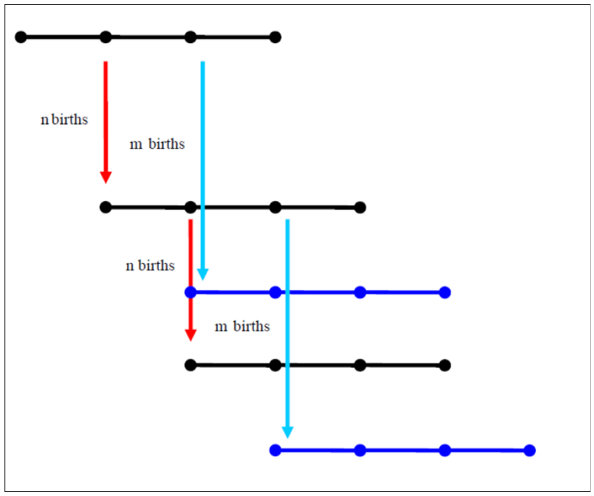

Figure 5: The lifecycle fertility model.

Production Production involves labor and human capital. The output of an agent at time t, denoted by y_t, is equal to:

$$y_t = h_t \ell_t \tag{1}$$

where h_t is the human capital of the agent at time t, while ℓ_t is the labor time.

Human capital accumulation When becoming a young adult at time t, each agent is endowed with a human capital level $h_t > 0$.[15] h_t determines the marginal productivity of labor when being a young adult.

The young adult can invest in education, in such a way as to increase his human capital stock at the next period. Human capital accumulates according to the law:

$$h_{t+1} = (v + e_t)\, h_t \tag{2}$$

where e_t denotes the level of effort/investment in education, while v is an accumulation parameter, which determines the rate at which human capital accumulates in the absence of education (i.e. when $e_t = 0$).

Throughout this paper, we will suppose that $v > 1$, that is, that even in the absence of education, individuals can become more productive over time. Human skills can improve despite the absence of education, because either of a

[15] This uniformity of endowments across all individuals born at the same point in time (independently from the age of their parent) allows us to keep a representative agent structure, unlike in Pestieau and Ponthiere (2016), where early and late children, who differ in terms of time constraints, are studied as two distinct populations.

standard learning by doing mechanism, or of an exogenous technological progress raising the productivity of labor.[16]

We assume here that education takes the form of a non-monetary, non-temporal, physical effort, which is a source of disutility. This non-standard formulation allows us to provide an analytically tractable framework by reducing the number of possible regimes (and sequences of regimes) that can emerge as human capital accumulates (see *infra*).[17]

Budget constraints It is assumed that raising a child has a time cost $q \in]0,1[$. That cost is supposed to be the same for early and late children. Assuming that there is no saving, so that each agent consumes what he produces at each period of life, the budget constraint at early adulthood is:

$$c_t = h_t\left(1 - qn_t\right) \tag{3}$$

where c_t denotes consumption at early adulthood for a young adult at time t.

The budget constraint at mature adulthood is:

$$d_{t+1} = h_{t+1}\left(1 - qm_{t+1}\right) \tag{4}$$

where d_{t+1} denotes consumption at mature adulthood for a mature adult at time $t+1$.

Assuming that there is no possibility for individuals to save or to borrow resources is in line with Unified Growth Theory (Galor 2011). Whereas that assumption simplifies the picture, it should be stressed that education and savings are quite redundant in the present context, since these are two ways to transfer resources over time. As far as fertility choices are concerned, education and savings are also redundant: by increasing the hourly wage in period 3, human capital and physical capital accumulations are two ways to increase the future purchasing power of individuals (which push, through an income effect, towards more late births), and, also, two ways to increase the opportunity cost of having late births (which push, through a substitution effect, towards fewer late births). Given this redundancy, and the necessity to keep the model parsimonious, this framework will abstract from savings choices.[18]

[16] The assumption $v > 0$ is widespread in the literature (see de la Croix and Doepke 2003 and Moav 2005). Assuming $v > 1$ is stronger but that assumption is close to what Kremer (1993) assumed in his seminal paper, where the variable reflecting knowledge or labor productivity grows at a positive rate as long as the population size is positive.

[17] Since the marginal return of education is h_t, the marginal utility gain from investing in education at $e_t = 0$ is increasing in h_t. Our assumption of a physical education effort, by implying that the marginal disutility of education is constant with h_t, guarantees the existence of a single threshold for h_t below which $e_t = 0$ and above which $e_t > 0$ (see *infra*). If, on the contrary, education effort were time, the marginal disutility of investing in education at $e_t = 0$ would be increasing in h_t, so that there could be several thresholds for h_t defining a succession of regimes with zero or positive education. Our assumption prevents that this counterintuitive case appears.

[18] The reasons why we focus on education choices rather than on savings choices are twofold. First, demographic studies emphasized the key role played by education choices for the timing

Preferences Individuals derive utility from consumption and from having children. They also derive disutility from investing efforts in education. Individuals are endowed with time-additive preferences having the form:[19]

$$U(c_t, n_t, e_t) + V(d_{t+1}, m_{t+1}) \tag{5}$$

where $U(\cdot)$ and $V(\cdot)$ are additively separable in their components, increasing and strictly concave with respect to $(c_t, n_t, d_{t+1}, m_{t+1})$. Moreover, we assume that $U(\cdot)$ is strictly decreasing and strictly convex with respect to e_t.

With respect to fertility, an important feature of that utility function is that it exhibits limited substitutability between early births and late births. The main justification for this feature is that, in the same way as there is no perfect substitutability for the allocation of consumption across time periods, there is no perfect substitutability for the allocation of births across time periods.

Regarding preferences over early children and late children, it should be stressed that our utility function treats all children as sources of temporal welfare at the period of their birth. Alternatively, one may want to treat children as durable goods, and make third-period welfare depend not only on late children, but, also, on early children. Whereas treating children as durable goods is a potentially fruitful approach, we prefer here to rely on a more standard formulation, to avoid that our results are driven by a too asymmetric treatment of early children and late children at the level of parental preferences.[20]

4 The temporary equilibrium

At the beginning of early adulthood, individuals choose consumptions c_t and d_{t+1}, the education effort e_t, the number of early children n_t, and the number of late children m_{t+1}, in such a way as to maximize their lifetime welfare, subject to budget constraints.

Throughout this paper, we will, without loss of generality, look at a solution that is interior for $(c_t, n_t, d_{t+1}, m_{t+1})$. However, we will allow the education effort e_t to be a corner solution.

The problem faced by young adults can be written as:

$$\max_{(c_t, n_t, e_t, d_{t+1}, m_{t+1})} U(c_t, n_t, e_t) + V(d_{t+1}, m_{t+1}),$$
$$s.t. \left| \begin{array}{l} c_t = h_t (1 - q n_t), \\ d_{t+1} = h_t (\nu + e_t)(1 - q m_{t+1}), \\ e_t \geq 0. \end{array} \right.$$

of births (see Smallwood 2002, Lappegard and Ronsen 2005 and Robert-Bobée et al 2006). This makes the education variable more relevant than savings for the study of births timing. Second, works by Momota and Horii (2013) and Pestieau and Ponthiere (2014, 2015) examined the relation between physical capital accumulation and the timing of births.

[19] Being time-additive, those preferences guarantee that fertility choices are time consistent.

[20] Note also that, in our representation of preferences, it is still possible to account indirectly for the longer period of coexistence of parents with early children than with late children, simply by assigning a larger weight to early births than to late births.

The first-order conditions (FOCs) for, respectively, optimal interior levels of early fertility n_t, education e_t and late fertility m_{t+1}, are:

$$
\begin{aligned}
-U_{c_t} h_t q + U_{n_t} &= 0, & (6)\\
U_{e_t} + V_{d_{t+1}} h_t \left(1 - q m_{t+1}\right) + \lambda_t &= 0, & (7)\\
-V_{d_{t+1}} h_t \left(\nu + e_t\right) q + V_{m_{t+1}} &= 0, & (8)
\end{aligned}
$$

where λ_t is the multiplier associated to the inequality constraint on e_t. The complementary slackness condition is therefore:

$$
e_t \lambda_t = 0. \tag{9}
$$

When considering those FOCs, we see that the prevailing stock of human capital h_t constitutes a major determinant of individual choices in terms of fertility (number and timing of births). A rise in h_t leads to both a rise in the opportunity cost of having children (substitution effect), which pushes towards fewer births, and to a rise of the purchasing power of the individual (income effect), leading to more births. The strength of those various income and substitution effects, by affecting the early and late fertility rates n_t and m_{t+1}, determines the reactivity of the TFR (equal to $n_t + m_{t+1}$) and of the MAM (equal to $\frac{n_t+2m_{t+1}}{n_t+m_{t+1}}$) to variations in human capital h_t.

Since h_t is the unique latent variable in our model, studying the dynamics of the TFR and the MAM is equivalent to examining how these variables react to variations in h_t. In the rest of this section, we derive sufficient conditions on preferences for the existence of the following three regimes, whose occurrence depends on the prevailing level of human capital h_t:

1. Regime I: $e_t = 0$ and the TFR and the MAM decrease in h_t;
2. Regime II: $e_t > 0$ and the TFR and the MAM decrease in h_t;
3. Regime III: $e_t > 0$, the TFR decreases in h_t and the MAM increases with h_t.

In order to derive conditions on preferences that are sufficient for the existence of those three regimes, this section proceeds in four successive steps, each step of the proof being associated with a lemma:

- Lemma 1 states conditions under which n_t and m_{t+1} are decreasing in h_t and MAM decreases with h_t when $e_t = 0$.
- Lemma 2 states conditions under which n_t and m_{t+1} are decreasing in h_t and e_t increases with h_t when $e_t > 0$.
- Lemma 3 states conditions under which there exists a threshold $\bar{h}$ below which $e_t = 0$ and above which $e_t > 0$.
- Lemma 4 states conditions under which there exists a second threshold $\tilde{h} > \bar{h}$ below which the MAM decreases with h_t and above which the MAM increases with h_t.

Taken together, the conditions on preferences stated in those four lemmas suffice to prove the existence of Regimes I, II and III. Given that our general framework does not rely on particular functional forms for $U(\cdot)$ and $V(\cdot)$, those conditions are quite general, and these point to key features of preferences that lead to rationalizing both the decline of TFR and the U-shaped MAM. Another advantage of our general model lies in the fact that it allows us to provide economic interpretations of those conditions on preferences in terms of income and substitution effects for early and late births. By doing so, our general framework contributes to generalize the microeconomic analysis of fertility choices, which focussed only on the microeconomic determinants of the *quantum* of births, without examining the determinants of the *tempo* of births.[21]

Regarding notations, we will, throughout this section, abstract from time indexes, as they are not necessary here: the solution of the individual's program is a set of functions of h that we denote as follows: n^*, e^* and m^*. Analyzing the responses of those variables with respect to h is equivalent to analyzing their behavior with respect to time, as h is the only latent variable in the model.

For that purpose, let us define the elasticities of early fertility n^* and late fertility m^*, computed at the optimum, with respect to h, as:

$$\varepsilon_{nh}^* := \frac{\partial n^*}{\partial h}\frac{h}{n^*} \text{ and } \varepsilon_{mh}^* := \frac{\partial m^*}{\partial h}\frac{h}{m^*}. \tag{10}$$

Moreover, it is convenient to define the following elasticities:

$$\varepsilon_c^* := -\frac{U_{cc}c^*}{U_c},\ \varepsilon_n^* := -\frac{U_{nn}n^*}{U_n},\ \varepsilon_e^* := -\frac{U_{ee}e^*}{U_e},\ \varepsilon_d^* := -\frac{V_{dd}d^*}{V_d},\ \varepsilon_m^* := -\frac{V_{mm}m^*}{V_m}, \tag{11}$$

which are evaluated at the optimum and are all positive.

Let us first assume the existence of Regime I (i.e. $e^* = 0$), and consider the reactions of n^* and m^* to human capital conditionally on being in Regime I.[22]

Lemma 1 *Suppose that* $e^* = 0$*. The optimal responses of early and late fertility rates to human capital satisfy:*

$$\varepsilon_{nh}^* = -\frac{1-\varepsilon_c^*}{\left(\varepsilon_c^*\frac{qn^*}{1-qn^*}+\varepsilon_n^*\right)} \quad \textit{and} \quad \varepsilon_{mh}^* = -\frac{1-\varepsilon_d^*}{\left(\varepsilon_d^*\frac{qm^*}{1-qm^*}+\varepsilon_m^*\right)}. \tag{12}$$

Thus, the TFR decreases with h *if* $\varepsilon_c^* < 1$ *and* $\varepsilon_d^* < 1$*. Moreover, the MAM is decreasing with* h *if:*

$$\frac{1-\varepsilon_c^*}{\varepsilon_c^*\frac{qn^*}{1-qn^*}+\varepsilon_n^*} < \frac{1-\varepsilon_d^*}{\varepsilon_d^*\frac{qm^*}{1-qm^*}+\varepsilon_m^*}. \tag{13}$$

[21] One drawback is that those general conditions may look quite abstract, so that it is not easy to see how restrictive those conditions are. However, we will show, when examining an analytical example in Section 6, that those general conditions have a simple counterpart once particular functional forms are imposed on $U(\cdot)$ and $V(\cdot)$.

[22] The existence of Regime I for some levels of human capital is proved in Lemma 3.

Proof. See the Appendix. ■

As stated above, a rise in the human capital stock implies a rise in the opportunity cost of having children, which pushes towards fewer births, as well as a rise in the purchasing power of individuals, which favors more births. The conditions $\varepsilon_c^* < 1$ and $\varepsilon_d^* < 1$ stated in Lemma 1 imply that, for both early births and late births, the substitution effect dominates the income effect, implying that n^* and m^* are decreasing with human capital. As a consequence, those conditions on preferences imply that, as human capital accumulates, the TFR, equal to $n^* + m^*$, declines. This result is in line with the fertility transition.

Regarding the timing of births, whether the MAM is increasing or decreasing in human capital depends on the relative strengths of the responses of n^* and m^* to variations of human capital. We have that the MAM is decreasing in human capital (i.e. an advancement of births) when the rate at which n^* falls is smaller, in absolute value, than the rate at which m^* falls. Expression (13) states the condition on preferences under which, in the absence of education, the rate at which the early fertility rate falls when human capital accumulates is, in absolute value, lower than the rate at which the late fertility rate falls, leading to the advancement of births.[23]

Let us now assume the existence of regimes characterized by a non-binding investment in education (i.e. $e^* > 0$), and characterize the responses of fertility rates and education to human capital in those regimes.[24]

Lemma 2 *Suppose that $e^* > 0$. The optimal responses of early fertility to human capital is the same as in (12), and thus n^* decreases with h if $\varepsilon_c^* < 1$. The optimal response of late fertility to human capital satisfies:*

$$\varepsilon_{mh}^* = -\frac{1-\varepsilon_d^*}{\left(\frac{qm^*}{1-qm^*}\varepsilon_d^* + \varepsilon_m^*\right) + \left(\frac{qm^*}{1-qm^*} + \varepsilon_m^*\right)\frac{\left(1-\varepsilon_d^*\right)}{\left(\varepsilon_e^*\frac{\nu+e^*}{e^*}-1\right)}}. \tag{14}$$

Moreover, m^* decreases with h and e^* increases with h if $\varepsilon_d^* < 1$ and:

$$\varepsilon_e^*\frac{\nu+e^*}{e^*} < \varepsilon_d^* - \frac{(1-\varepsilon_d^*)^2}{\varepsilon_d^* + \varepsilon_m^*\frac{1-qm^*}{qm^*}}. \tag{15}$$

Proof. See the Appendix. ■

Given our assumption that the cost of education is a utility cost, early fertility n^* does not depend on education. As a consequence, the response of early fertility to variations in human capital is the same expression as in Regime I (when education equals zero), implying that the condition guaranteeing that n^* is decreasing in human capital is the same as in Lemma 1 (i.e. $\varepsilon_c^* < 1$).

On the contrary, the late fertility rate m^* is impacted by education, and the response of late fertility to human capital ε_{mh}^* is also impacted, in a way that

[23] That condition, expressed in terms of elasticities, may seem quite abstract, so that it is difficult, at first glance, to see how restrictive it is. We show in Section 6 that this condition takes a simple form under standard functional forms for $U(\cdot)$ and $V(\cdot)$.

[24] See Lemma 3 on the existence of those alternative regimes.

depends on the level of education, and on how education affects the relative sizes of the substitution and income effects. When the preferences satisfy $\varepsilon_d^* < 1$ and (15), it is still the case that the substitution effect dominates the income effect, and that m^* decreases with h. Those conditions also guarantee that education is increasing with human capital, in line with empirical evidence.

We now turn to the existence of Regime I.

Lemma 3 *There exists a unique threshold $\bar{h} > 0$ such that $e^* = 0$ for all $h \leq \bar{h}$ and $e^* > 0$ for all $h > \bar{h}$ if:*

$$\lim_{d \to 0} V_d d < -\lim_{e \to 0} U_e < \lim_{d \to +\infty} V_d d. \tag{16}$$

and if conditions $\varepsilon_d^* < 1$ and (15) are satisfied.

Proof. See the Appendix. ■

Lemma 3 states conditions on preferences that are sufficient for the existence of a single threshold in human capital $\bar{h} > 0$ such that for any level of human capital below $\bar{h}$, education is equal to zero, whereas for any level of h above $\bar{h}$, education is strictly positive. The conditions on preferences include boundary conditions on utility functions, plus some conditions guaranteeing that education increases with human capital (which imply the uniqueness of the threshold $\bar{h}$).

The reason why $e^* = 0$ when $h < \bar{h}$ lies in the fact that the marginal utility gain from investing in education is, given the low human capital stock, too low with respect to the marginal disutility of education efforts, which makes investment in education not worthy. However, once $h > \bar{h}$, it becomes desirable to invest in education, leading to $e^* > 0$. Thus Lemma 3 shows that the economy is in Regime I if $h \leq \bar{h}$, and does not lie in Regime I when $h > \bar{h}$.

Finally, Lemma 4 provides conditions on preferences under which there exists a unique level of human capital below which the MAM is decreasing in human capital, and beyond which the MAM is increasing with human capital, implying thus the existence of Regime III.

Lemma 4 *There exists a threshold $\tilde{h} > \bar{h}$ such that the MAM is decreasing with h for $h \leq \tilde{h}$ and is increasing with h for $h > \tilde{h}$ if condition (13) is satisfied, if*

$$\lim_{e \to +\infty} \frac{U_{ee}}{U_e} (\nu + e) = -1, \tag{17}$$

and if $\lim_{d \to +\infty} V_d d$ *is not too large. Moreover, $\tilde{h}$ is unique if:*

$$\left. \frac{\partial \left(|\varepsilon_{nh}^*| - |\varepsilon_{mh}^*| \right)}{\partial h} \right|_{h=\tilde{h}} > 0. \tag{18}$$

Proof. See the Appendix. ■

Condition (17) guarantees that, when education is sufficiently large, it is necessarily the case that the elasticity of early births to human capital exceeds, in absolute value, the elasticity of late births to human capital, so that $|\varepsilon_{nh}^*| >$

$|\varepsilon^*_{mh}|$. Jointly with the condition on the $\lim_{d\to+\infty} V_d d$, it implies that there exists at least one threshold $\tilde{h}$ at which $|\varepsilon^*_{nh}| = |\varepsilon^*_{mh}|$, so that the derivative of the MAM with respect to h equals 0. The uniqueness of that threshold is guaranteed by condition (18). The intuition behind that condition goes as follows. We know, under the conditions of Lemma 1, that the MAM is decreasing with h for a low h, that is, that $|\varepsilon^*_{nh}| < |\varepsilon^*_{mh}|$. We also know that, when h tends to $+\infty$, the MAM is increasing with h, that is, $|\varepsilon^*_{nh}| > |\varepsilon^*_{mh}|$. Hence a necessary and sufficient condition for uniqueness of the threshold $\tilde{h}$ at which $|\varepsilon^*_{nh}| = |\varepsilon^*_{mh}|$ is that the derivative of $(|\varepsilon^*_{nh}| - |\varepsilon^*_{mh}|)$ is positive at any such $\tilde{h}$. This condition is given by (18).

In economic terms, the conditions in Lemma 4 can be interpreted as follows. From Lemma 1, we know that, under our conditions on preferences, the MAM is decreasing with human capital when education equals zero. Moreover, from Lemma 2, our conditions on preferences imply that education, once positive, is increasing with h. The conditions stated in Lemma 4 can be interpreted as conditions that imply that there exists a unique threshold in terms of education, below which the MAM is decreasing with h (i.e. advancement of births), and above which the MAM becomes increasing with h (i.e. postponement of births). This reversal of the sign of the impact of human capital on the MAM suggests that, under our assumptions, education tends to weaken the relative strength of the substitution effect with respect to the income effect for late births, but not for early births (since the response of early births to h is not affected by the level of education).

The conditions stated in Lemma 4 guarantee that Regime III prevails when $h > \tilde{h} > \bar{h}$. Given that we know, from Lemma 3, that Regime I prevails when $h < \bar{h} < \tilde{h}$, we can deduce from Lemmas 3 and 4 that there must exist an intermediate interval for human capital $(\bar{h}, \tilde{h}]$ where Regime II prevails. Indeed, in that interval, we know from Lemma 3 that education is strictly positive, and from Lemma 4 that the MAM is decreasing with h. These are the two features of Regime II. Our results are summarized in Proposition 1.

Proposition 1 *Let us assume that $\varepsilon^*_c < 1$ and $\varepsilon^*_d < 1$ and that conditions (13), (15), (16), (17) and (18) are satisfied. Then there exist three regimes. The human capital in Regime I is lower than that of Regime II, which is lower than that of Regime III. The three regimes are characterized as follows:*

- *In Regime I ($h \leq \bar{h}$), there is no education and the TFR and the MAM decrease with human capital.*
- *In Regime II ($\bar{h} < h \leq \tilde{h}$), education is positive and increases with human capital, while the TFR and the MAM decrease with human capital.*
- *In Regime III ($\tilde{h} \leq h$), education is positive and increases with human capital, the TFR decreases with human capital, while the MAM increases with human capital.*

Proof. The proof follows from Lemmas 1 to 4. ■

Proposition 1 states conditions on preferences such that, depending on the prevailing level of the human capital stock, the economy is either in Regime I, or in Regime II, or in Regime III. Those conditions, which are presented in the previous lemmas, have three main aspects, which admit intuitive economic interpretation. First, those conditions guarantee that, as human capital accumulates, the substitution effect dominates the income effect for both early and late fertility rates (leading to a fall of the TFR). Second, the conditions are such that the level of education becomes positive beyond some threshold of human capital, and increasing with human capital. Third, our assumptions on preferences imply also that the level of education tends to weaken the strength of the substitution effect with respect to the income effect for late births only (and not for early births). This explains that, once education reaches a sufficiently high level, the tendency to advance births as human capital accumulates (which prevails for low levels of human capital) is replaced by a tendency to postpone births, leading to the U-shaped curve for the MAM.

Each of the three regimes described in Proposition 1 can be regarded as a distinct stage of development, with particular relations between human capital, the *quantum* of births (i.e. the TFR) and the *tempo* of births (i.e. the MAM).

Under Regime I, the response of early fertility to human capital accumulation is, in absolute value, lower than the one of late fertility. Thus, the MAM decreases with human capital, leading to an advancement of births. In Regime II, where education is positive, the response of late fertility is still higher than the one of early fertility, so that the MAM continues to decrease. But when education reaches a sufficiently high level, the economy enters Regime III, and the response of late fertility becomes lower, in absolute value, than the one of early fertility, so that the MAM starts increasing with human capital, leading to a postponement of births.

Thus, when comparing Regime III with Regimes I and II, there is an important qualitative change. Whereas the rise in income used to push towards advancing births in Regimes I and II, this is no longer the case in Regime III, where the rise in income pushes towards postponing births. This third regime coincides with what could be called the modern regime, where the decline in fertility is associated with births postponement.

5 Economic and demographic dynamics

Having shown that the temporary equilibrium can take three distinct forms, depending on the prevailing level of human capital, let us now describe how the economy evolves over time, that is, how the economy shifts from one regime to another as human capital accumulates.

For that purpose, let us assume that the economy starts at an initial human capital level $h_0 < \bar{h}$. Let us also make all assumptions on preferences made in Proposition 1. Thus, using Proposition 1, the economy lies initially in Regime I. Given $v > 1$, human capital grows in Regime I, so that it will, sooner or later, cross the threshold $\bar{h}$, leading the economy to Regime II, where education becomes positive. Note that, in Regime II, the growth rate of human capital is positively correlated with education, which is itself increasing with human capital. Given those relations, the growth rate of human capital is larger in Regime II than in Regime I, and the human capital stock will, sooner or later, cross the second threshold, i.e. $\tilde{h} > \bar{h}$, which coincides with entry in Regime III. Hence, starting from an initial human capital $h_0 < \bar{h}$, the economy will move from Regime I to II and from Regime II to III, and then stay in Regime III.

This transition from Regime I to Regime II, and from Regime II to Regime III has some consequences in demographic terms. From Proposition 1, we can see that this implies that the TFR exhibits a declining trend, which is in line with the fertility transition. Moreover, we also know from Proposition 1 that the shift from Regimes I and II to Regime III leads to a reversal of the relationship between human capital and the MAM. Whereas economic development was associated with the advancement of births (i.e. a decreasing MAM) in Regimes I and II, this is not the case in Regime III, where development leads to postponing births (i.e. an increasing MAM). Thus, as the economy develops, the MAM exhibits the U-shaped curve, in line with the stylized fact identified above.

Proposition 2 summarizes our results concerning long-run dynamics.

Proposition 2 *Assume an initial human capital $h_0 < \bar{h}$. Assume also the conditions made in Proposition 1.*

- *The economy is initially in Regime I, and tends, as human capital accumulates, to shift from Regime I to Regime II, and, then, from Regime II to Regime III.*
- *The growth rate of human capital is lower in Regime I than in Regime II, and lower in Regime II and in Regime III.*
- *Over time, the TFR and MAM variables exhibit the following patterns:*
  - *The TFR exhibits a decreasing pattern;*
  - *The MAM exhibits a U-shaped pattern.*

Proof. Proposition 2 is a corollary of the accumulation law $h_{t+1} = (v + e_t)h_t$ and of Proposition 1. ■

Proposition 2 states that our model can rationalize the observed patterns for the *quantum* and the *tempo* of births. That rationalization of the patterns for the TFR and the MAM does not rely on any external, exogenous shock. In line with Unified Growth Theory (Galor 2011), the patterns for the TFR and the MAM are induced by the dynamics of a latent variable (here human capital). As that latent variable crosses some thresholds, the economy enters into different regimes, which can be regarded as distinct stages of development, characterized by different relations between economic and demographic variables.

Our rationalization of the U-shaped pattern for MAM is particularly in line with Unified Growth Theory, since there is here a qualitative change, a reversal of the relation between development and the timing of births: below some level of development, income rises push towards advancing births, whereas beyond some level of development, income rises push towards postponing births. That qualitative change is, in our model, rationalized by a shift from Regime II to Regime III as human capital accumulates.

Note that the ability of our model to rationalize the observed patterns for the TFR and the MAM relies on some conditions on preferences (see Proposition 1). Not all kinds of preferences are compatible with a decline in TFR and a U-shaped curve for MAM. Those trends can be rationalized when preferences are such that the substitution effect dominates the income effect for the two fertility rates (implying a decline in the TFR), and when the emergence of education weakens the strength of the substitution effect relative to the income effect for late births (but not for early births), implying that the rate at which late fertility declines when human capital accumulates becomes, for a sufficiently high level of education, lower than the rate at which early fertility reacts.

Given the general forms for $U(\cdot)$ and $V(\cdot)$, the conditions stated in Proposition 1 are not easy to interpret, since these concern elasticities that are defined in terms of the variables prevailing at the temporary equilibrium. Hence it is not simple to see whether these conditions are satisfied under standard functional forms for $U(\cdot)$ and $V(\cdot)$. In order to answer that question, the next section develops an analytical example based on specific forms for $U(\cdot)$ and $V(\cdot)$. This example will illustrate the scope of parametric restrictions on utility functions that can rationalize the declining TFR and the U-shaped pattern for MAM.

6 An analytical example

This section develops an analytical example of the general model presented in the previous sections. The goal of that example is not only to examine whether standard functional forms for $U(\cdot)$ and $V(\cdot)$ can satisfy the conditions stated in Proposition 1, but, also, to provide explicit solutions for the patterns of all variables, including the TFR and the MAM.

Preferences Let us now impose the following functional forms for the utility function:

$$U(c_t, n_t, e_t) \ : \ = \alpha \log(c_t + \delta) - \sigma \log(e_t + \eta) + \gamma \log(n_t) \quad (19)$$

$$V(d_{t+1}, m_{t+1}) \ : \ = \beta \log(d_{t+1} + \varepsilon) + \rho \log(m_{t+1}) \quad (20)$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\delta > 0$, $\sigma > 0$, $\varepsilon > 0$, $\eta > 0$ and $\rho > 0$ are preference parameters. α (resp. β) captures the weight assigned to consumption early (resp. late) in life. σ captures the disutility of education efforts. γ (resp. ρ) captures the taste for early (resp. late) fertility. The parameters δ, η and ε allow us to have a sufficiently general structure for preferences.

Temporary equilibrium Substituting for the budget constraints, the problem of the individual can be written as follows:

$$\begin{aligned}\max_{e_t, n_t, m_{t+1}} \quad & \alpha \log\left(h_t(1-n_t q)+\delta\right)-\sigma \log\left(e_t+\eta\right) \\ & +\beta \log((v+e_t)h_t(1-m_{t+1}q)+\varepsilon)+\gamma \log(n_t)+\rho \log(m_{t+1})\end{aligned}$$

The FOCs for, respectively, optimal interior levels of early fertility n_t, education e_t and late fertility m_{t+1}, are:

$$\frac{\alpha h_t q}{h_t(1-n_t q)+\delta} = \frac{\gamma}{n_t} \tag{21}$$

$$\frac{\sigma}{(e_t+\eta)} = \frac{\beta h_t(1-m_{t+1}q)}{(v+e_t)h_t(1-m_{t+1}q)+\varepsilon} \tag{22}$$

$$\frac{\beta(v+e_t)h_t q}{(v+e_t)h_t(1-m_{t+1}q)+\varepsilon} = \frac{\rho}{m_{t+1}} \tag{23}$$

The first FOC, which characterizes the optimal early fertility level, equalizes the marginal utility gain from early fertility (RHS) with the marginal utility loss from early fertility (LHS). Note that, since $\delta > 0$, the marginal utility loss of early births depends on the prevailing level of human capital. This would not be the case under $\delta = 0$, since in that case the income and substitution effects would cancel each other, making early fertility independent of human capital.

The second FOC states that the optimal education equalizes the marginal disutility of education effort (LHS) and the marginal utility gain from extra consumption at mature adulthood thanks to education (RHS). Under $\varepsilon = 0$, the optimal education would be independent from late fertility (and from h_t).

Finally, the third FOC characterizes the optimal late fertility level. It equalizes the marginal utility gain from late births (RHS) with the marginal utility loss from late births (LHS). Under $\varepsilon = 0$, the marginal utility loss from late births would be invariant to human capital, which would make m_{t+1} independent from h_t. On the contrary, under $\varepsilon > 0$, the LHS depends on the human capital level, which affects both the purchasing power at mature adulthood and the opportunity cost of late births.

Whereas the 3 FOCs characterize a temporary equilibrium with interior levels for early fertility n_t, education e_t and late fertility m_{t+1}, such an interior temporary equilibrium is not necessarily reached, depending on the level of human capital at time t.

Proposition 3 states conditions on the structural parameters under which the conditions stated in Lemmas 1 to 4 are valid, so that Proposition 1 holds and the economy exhibits the three regimes studied in the previous sections.

Proposition 3 *Define* $\bar{h} \equiv \frac{\varepsilon(\sigma v+\rho\eta)}{v(\beta\eta-\sigma v)}$, $\Delta(h_t) \equiv [h_t\Omega+\varphi]^2+4h_t\omega v(\eta\beta-v\sigma)(h_t-\bar{h})$, $\varphi \equiv \varepsilon(\sigma+\rho)$ *and* $e(h_t) \equiv \frac{-[h_t\Omega+\varphi]+\sqrt[2]{\Delta(h_t)}}{2h_t\omega}$.

- *The utility function* $\alpha \log(c_t+\delta) - \sigma \log(e_t+\eta) + \gamma \log(n_t) + \beta \log(d_{t+1}+\varepsilon) + \rho \log(m_{t+1})$ *satisfies the conditions stated in Lemmas 1 to 4 iff the parameters satisfy:*

$$\begin{aligned} \varepsilon &> \delta v \text{ and } \eta\beta > v\sigma, \\ h_t \Delta'(h_t) &> 2\Delta(h_t) - 2\varphi \sqrt[2]{\Delta(h_t)}, \\ \eta &> v \text{ and } \beta \text{ is not too high} \end{aligned}$$

  as well as:

$$\frac{\delta}{\left(\tilde{h}+\delta\right)^2} + \frac{\left[e''(\tilde{h})\tilde{h} + e'(\tilde{h})\right]\left(v+e(\tilde{h})\right) - \tilde{h}\left(e'(\tilde{h})\right)^2}{\frac{1}{\varepsilon}\left[v+e(\tilde{h})\right]^2 \rho\left(\left(v+e(\tilde{h})\right)\tilde{h}+\varepsilon\right)} > \frac{\delta}{\tilde{h}+\delta}\frac{e'(\tilde{h})\tilde{h}+v+e(\tilde{h})}{\left(v+e(\tilde{h})\right)\tilde{h}+\varepsilon}$$

  where $\tilde{h}$ *is the solution to:* $e(h_t) + (h_t+\delta)\, e'(h_t) = \frac{\delta}{\varepsilon}(v+e(h_t))^2 - v$.

- *The three regimes can be characterized as follows:*
  - *In Regime I* ($h_t \leq \bar{h}$), $e_t = 0$; $n_t = \frac{\gamma(h_t+\delta)}{h_t q(\alpha+\gamma)}$; $m_{t+1} = \frac{\rho(vh_t+\varepsilon)}{vh_t q(\beta+\rho)}$, *as well as* $\frac{\partial TFR_t}{\partial h_t} < 0$ *and* $\frac{\partial MAM_t}{\partial h_t} < 0$.
  - *In Regime II* ($\bar{h} < h_t \leq \tilde{h}$), $e_t = e(h_t) > 0$; $n_t = \frac{\gamma(h_t+\delta)}{h_t q(\alpha+\gamma)}$; $m_{t+1} = \frac{\rho((v+e(h_t))h_t+\varepsilon)}{(v+e(h_t))h_t q(\beta+\rho)}$, *as well as* $\frac{\partial TFR_t}{\partial h_t} < 0$ *and* $\frac{\partial MAM_t}{\partial h_t} < 0$.
  - *In Regime III* ($\tilde{h} < h_t$), $e_t = e(h_t) > 0$; $n_t = \frac{\gamma(h_t+\delta)}{h_t q(\alpha+\gamma)}$; $m_{t+1} = \frac{\rho((v+e(h_t))h_t+\varepsilon)}{(v+e(h_t))h_t q(\beta+\rho)}$, *as well as* $\frac{\partial TFR_t}{\partial h_t} < 0$ *and* $\frac{\partial MAM_t}{\partial h_t} > 0$.

Proof. See the Appendix. ■

Proposition 3 states that, under the particular functional forms for $U(\cdot)$ and $V(\cdot)$, there exist conditions on the structural parameters of our economy such that the conditions stated in Lemmas 1 to 4 are satisfied, so that Proposition 1 holds. As a consequence of Proposition 1, the economy exhibits, under those conditions, the three regimes studied in the previous sections: Regime I where education equals zero, and where both the TFR and the MAM decrease when human capital accumulates; Regime II, where education is positive and increasing in h_t, but where the TFR and the MAM are still decreasing in h_t; and, finally, Regime III, where education is still increasing in h_t, and where the TFR is decreasing in h_t, whereas the MAM is increasing in h_t.

Long-run dynamics The development process follows the same pattern as in Section 5. Assuming an initial human capital $h_0 < \bar{h}$, as well as $h_0 > \frac{\gamma\delta}{\alpha}$, $\frac{\varepsilon\rho}{\beta v}$ (to guarantee non-negative consumptions at $t=0$), the economy lies initially in Regime I, and, given $v > 1$, human capital will grow and will sooner or later reach the threshold $\bar{h}$, and thus enter Regime II. Given that education becomes positive in Regime II, this reinforces the accumulation of human capital, which will at one point cross the second threshold $\tilde{h}$, implying entry into Regime III.

The monotonicity of education in human capital guarantees that the economy will then remain in Regime III.

Regarding demographic variables, the economy exhibits all along a decreasing TFR, while the MAM, which is decreasing in h_t in Regimes I and II, becomes increasing in h_t in Regime III, and, hence, exhibits a U-shaped form.

While those results are already obtained in the general model, an important contribution of this analytical example is that this allows us to compute the long-run levels of the TFR and the MAM. These are obtained by taking the limit of n_t and m_{t+1} when human capital tends to infinity.[25] We obtain:

$$\lim_{h_t \to \infty} n_t = \frac{\gamma}{q\left(\alpha+\gamma\right)} > 0 \text{ and } \lim_{h_t \to \infty} m_{t+1} = \frac{\rho}{q\left(\beta+\rho\right)} > 0$$

Thus the TFR and MAM converge asymptotically towards, respectively:

$$\begin{aligned} TFR_{\infty} &= \frac{\gamma}{q\left(\alpha+\gamma\right)} + \frac{\rho}{q\left(\beta+\rho\right)} \\ MAM_{\infty} &= \frac{\gamma\left(\beta+\rho\right)+2\rho\left(\alpha+\gamma\right)}{\gamma\left(\beta+\rho\right)+\left(\alpha+\gamma\right)\rho} \end{aligned}$$

Thus, whereas human capital keeps on increasing over time without limit, the *quantum* and the *tempo* of births tend to stabilize over time. The asymptotic TFR depends on the preference parameters α, β, γ and ρ, as well as on the time cost of children q. Regarding the timing of births, the asymptotic MAM does not depend on the time cost of children (which is the same for early and late births), but only depends on preference parameters α, β, γ and ρ.

7 Numerical illustration

The previous Sections show that our model can, qualitatively, explain or rationalize the global patterns exhibited by the *quantum* and the *tempo* of births. One may want to go further in the replications, and wonder to what extent it is possible, by calibrating our model, to reproduce the TFR and MAM patterns for a real-world economy. This is the task of this Section.

For that purpose, this Section takes, as a reference, the estimated trends for the cohort TFR and the cohort MAM based on the Human Fertility Database (Section 2), and examines the extent to which the analytical model of Section 6 can fit those patterns. Note that, in our model, periods are of about 18 years (which implies having early children at age 18 and late children at age 36). Hence, since the database includes cohorts born from 1906 onwards, our

[25] Notice also that the education converges asymptotically towards:

$$\lim_{h \to \infty} e(h_t) = v + \frac{-\Omega}{2\omega} + \sqrt[2]{\frac{2\Omega^2 + 8\omega v\left(\eta\beta - v\sigma\right)}{8\omega^2}} > 0$$

numerical analysis will try to replicate TFR and MAM for the cohorts born in 1906, 1924, 1942, 1960 and 1978.[26]

This numerical exercise requires to calibrate the structural parameters of our model. Regarding the parameter q, which captures the time cost of children, we rely on the *American Time Use Survey*, which shows that parents spend on average 9.83 % of their time with their children. We thus assume $q = 0.10$. Regarding preference parameters, we first normalize α to 1. The parameter of time preferences β is calibrated while assuming a quarterly discount factor equal to 0.99, which implies that $\beta = (0.99)^{4\times 18} = 0.484$. Regarding the calibration of ρ (taste for late births), we assume, as a proxy, that time preferences are the same for consumption goods and for fertility, so that $\rho = \beta\gamma$.

Under those assumptions, we are left with 5 preference parameters to calibrate, i.e. γ, δ, ε, η and σ. We also need to set a value for the initial human capital h_0, and to calibrate v, which determines the speed at which the economy goes through the 3 regimes. Together with β, the parameters γ, δ, ε, η, σ and v determine the first threshold in human capital $\bar{h} \equiv \frac{\varepsilon(\sigma v+\beta\gamma\eta)}{v(\beta\eta-\sigma v)}$, below which education is equal to zero (Regime I), and above which education becomes positive (Regime II). Assuming that the cohort born in 1906 lies in the Regime I, whereas the cohort born in 1924 invests in education (and is thus in Regime II), h_0, β, γ, δ, ε, η, σ and v must satisfy the condition: $h_0 \leq \frac{\varepsilon(\sigma v+\beta\gamma\eta)}{v(\beta\eta-\sigma v)} < vh_0$. Whereas several combinations of values for those parameters satisfy that condition, we selected the values that provide the best fit for the MAM of the first two cohorts (1906 and 1924). This is achieved when we set $h_0 = 0.001$, $v = 2.8$, $\gamma = 0.015$, $\delta = 0.0035$, $\varepsilon = 0.02$, $\eta = 18$ and $\sigma = 0.7$.[27] Figure 6 below compares the observed cohort MAM pattern (in continuous trait) with the cohort MAM pattern simulated from the model (thick discontinuous trait).

[26] This last observation is not available yet on a cohort basis. This is approximated by taking the last cohort of the sample (i.e. 1974).

[27] Note that those values satisfy the conditions stated in Proposition 3 (and, hence, the general conditions stated in Lemmas 1 to 4), as well as the non-negativity constraints for c and d, that is, $h_0 > \frac{\gamma\delta}{\alpha}, \frac{\varepsilon\rho}{\beta v}$.

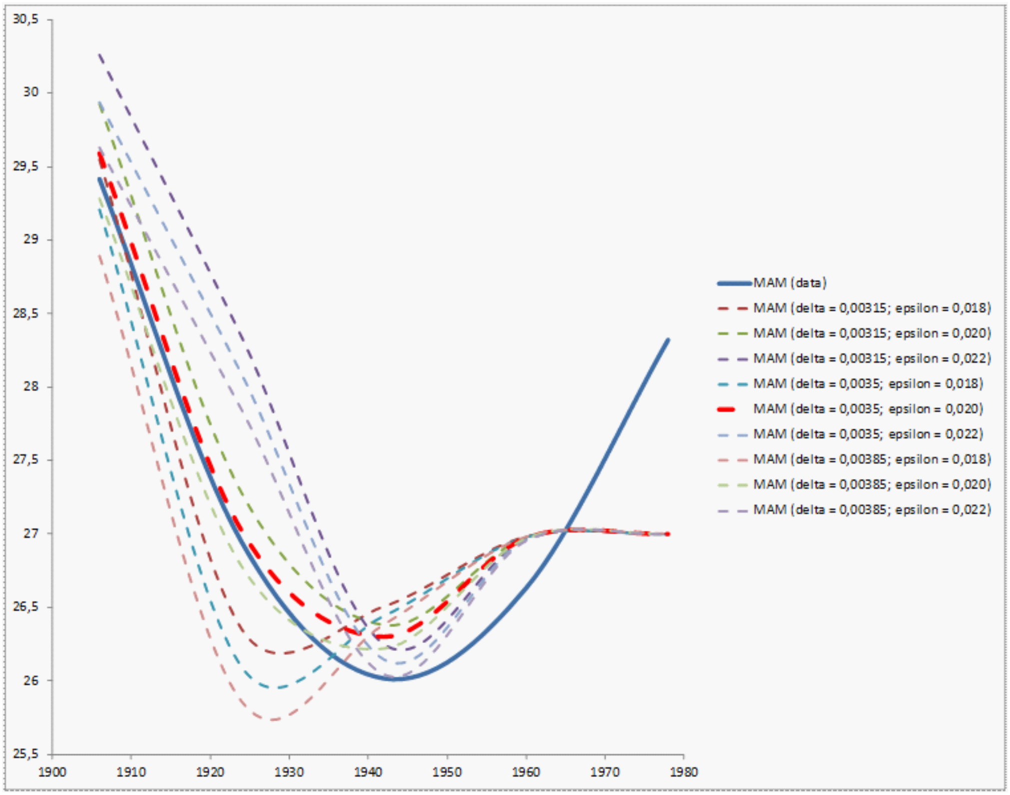

Figure 6: Cohort MAM: data and model.

As shown on Figure 6, this calibration allows us to replicate, with some degree of accuracy, the estimated U-shaped pattern for the MAM for cohorts born during the 20th century. The replication of this pattern is achieved without using any exogenous shock, but by means of regime shifts. The economy starts in Regime I for cohorts born in 1906. In that regime, the rise in incomes generates an advancement of births, that is, a fall of the MAM. Then, the economy enters Regime II for the cohort 1924. During that regime, there is also an advancement of births, but at a slower rate. Finally, the economy enters Regime III for the cohort 1942, for which the rise in income is associated not with an advancement, but with a postponement of births (i.e. a rise in the MAM).

To explore the robustness of our simulations, Figure 6 also shows, in light discontinuous traits, the simulated MAM patterns obtained when we allow for a +/- 10 percent variation for the parameters δ and ε around their calibrated value, everything else remaining unchanged. We can see that, in any case, the MAM still exhibits a U-shaped pattern. It is also worth noticing that all those scenarios lead to the same long-run level for MAM, which does not vary with δ and ε. The only differences across those scenarios concern: (1) the precise level of the turning point (minimum MAM) and (2) the precise temporal location of that turning point. But the overall U-shaped form of MAM is a robust result, and does not constitute a knife edge solution.

In the light of this, it appears that our Unified Growth model can rationalize the dynamics of the *tempo* of births as a succession of regime shifts. However, it is also worth noticing some limitations of our numerical analysis.

First, although the model can replicate the U-shaped pattern of the MAM, it exhibits nonetheless, from a quantitative perspective, some limitations in being able to reproduce the very strong growth in the MAM for cohorts born after 1960. Figure 6 shows that the model can only explain about one third of the total variation in the MAM that occurred for the cohorts born after 1960. This suggests that other factors - possibly cultural, institutional or medical - may have also affected the magnitude of the recent births postponement.

Second, while our calibrated model replicates a declining trend in the TFR, it tends to exaggerate the size of that decline. Whereas the TFR has exhibited a relatively slow decline (equal, on a yearly average, to about 0.6 %), our calibrated model leads to a stronger decline of the TFR (equal, on a yearly average, to about 2.5 %). This is due to the fact that the TFR has exhibited relatively slow changes, in comparison with the rapid variations in the MAM. We are thus here in presence of two distinct temporalities for changes in the *quantum* and in the *tempo* of births. Given that our Unified Growth model has a unique latent variable, it can hardly replicate accurately both the pattern for the *quantum* and for the *tempo* of births, since the former requires a slow growth of the latent variable, whereas the latter requires a quick growth of the latent variable.[28]

8 Conclusions

The goal of this paper was to study the relationship between economic development, the *quantum* of births and the *tempo* of births. Our purpose was to build a model that can rationalize the patterns of the TFR and the MAM. In particular, our goal was to build a theory explaining that, as the economy develops, there is first an advancement of births, and, then, a postponement of births.

We developed a 3-period OLG model with lifecycle fertility, where individuals with additively-separable preferences choose their education, the number and timing of births. Adopting a Unified Growth approach, we assumed that human capital accumulation drives the latent dynamics that affects individual decisions, and we examined how, as human capital accumulates, individuals modify their fertility choices (number and timing of births).

Our analysis shows that there exist conditions on preferences such that, depending on the prevailing level of human capital, an economy can be in three

[28] One solution consists in relaxing the assumption of a constant time cost of children q. Allowing for time-varying time cost of children q_t can allow to rationalize both the pattern for MAM and for TFR, provided one assumes that the time cost of children has fallen over time. Note also that, unlike the TFR, the MAM does not depend on q, so that the MAM curve would be left unaffected by introducing time-varying cost of children q_t.

distinct regimes. In Regime I, there is no education and fertility is high. In that traditional regime, a rise in income pushes towards advancing births. Then, once human capital reaches some threshold, the economy enters Regime II, where individuals start investing in education and fertility is lower. But it is still the case that, as income grows, births are being advanced. However, once the human capital stock reaches a second threshold, the economy enters Regime III, where higher incomes lead now to postponing births.

The conditions on preferences implying those regimes admit simple economic interpretations. Those conditions imply that (i) the substitution effect dominates the income effect for both early and late fertility (leading to a decrease of TFR as human capital accumulates); (ii) the level of education becomes positive beyond some threshold of human capital, and increasing with it; (iii) the level of education weakens the substitution effect with respect to the income effect for late births (but not for early births). Taken together, those conditions rationalize the U-shaped pattern for the MAM: they explain why, at low levels of education, human capital accumulation leads to advancing births, whereas, beyond some threshold for education, it leads to postponing births.

The identification of three distinct regimes casts significant light on the relation between economic development and fertility behaviors. First of all, our model can explain why advanced industrialized economies exhibit, since the 1970s, both low fertility and births postponement. This coincides with Regime III in our model. More importantly, whereas most studies focused on the postponement of births since the 1970s, our framework allows us also to bring new light on what happened during the first part of the 20th century, an epoch where economic development was associated with a decline of fertility and an advancement of births. This coincides with Regimes I and II in our model.

Finally, using data for 28 countries from the Human Fertility Database, we examined the capacity of our model to fit the estimated fertility patterns. We showed that our model can approximately replicate the observed U-shaped pattern of the MAM. Note, however, that the model can replicate only one third of the substantial rise in the MAM observed for cohorts born after 1960. This suggests that other factors have been at work in the postponement of births.

This leads us to mentioning some limitations of our analysis. First, our model focuses on the impact of education on childbearing age. This leaves aside several other explanations of the MAM pattern, such as variations in infant mortality, advances in procreation techniques, and changes in the age at marriage.[29] However, it should be stressed that those alternative explanations are all closely related to the accumulation of human capital, which drives the latent dynamics in our model. A second limitation is that our lifecycle fertility model cannot rationalize other stylized facts (e.g. age at the first/last birth), whose rationalization would require a broader model. Hence, much work remains to be done to develop a more general theory of lifecycle fertility.

[29] Note that the age at marriage exhibits also a U-shaped form (Iyigun and Lafortune 2016).

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10 Appendix

10.1 Material for Section 2

	Covered cohorts for MAM and TFR	Covered cohorts for MA1M	Covered cohorts for female education
Austria	1936-1974	1969-1974	1936-1974
Belarus	1949-1974	1949-1974	NA
Bulgaria	1932-1969	1932-1969	1932-1969
Canada	1906-1971	1929-1971	1906-1971
Czech Republic	1935-1974	1935-1974	1935-1974
England and Wales	1923-1973	NA	NA
Estonia	1944-1974	1944-1974	NA
Finland	1924-1974	1967-1975	1924-1974
France	1931-1975	NA	1931-1975
Germany East	1941-1973	1941-1950	NA
Germany West	1941-1973	NA	NA
Hungary	1935-1974	1937-1974	1935-1974
Iceland	1945-1972	NA	1945-1972
Italy	1939-1972	NA	1939-1972
Japan	1932-1974	1953-1974	1932-1974
Lithuania	1944-1974	1955-1974	NA
Netherlands	1935-1974	1935-1974	1935-1974
Northern Ireland	1959-1973	NA	NA
Norway	1952-1974	1952-1974	1952-1974
Portugal	1925-1975	1944-1975	1925-1975
Russia	1944-1974	1944-1974	NA
Scotland	1930-1973	NA	NA
Slovakia	1935-1969	1935-1969	NA
Spain	1907-1974	1960-1974	1907-1974
Sweden	1906-1974	1955-1974	1906-1974
Switzerland	1917-1974	NA	1917-1974
Ukraine	1944-1973	1944-1973	NA
United States	1918-1974	1944-1973	1918-1974

Table A1: An overview of the data

10.1.1 Regressions for TFR by cohorts

	Endogenous variable: cohort TFR
Exogenous variables:	
Cohort	0.481***
	(6.23)
Cohort^2	-0.000127***
	(-6.43)
Constant	-451.2***
	(-6.01)
Country-Fixed Effects	Yes
Number of observations:	1151
Number of countries:	28
Covered cohorts (unbalanced):	1906-1975
R^2 within:	0.5799
t statistics in parentheses	
* p<0.05, ** p<0.01, *** p<0.001	

Table A2: Regression for TFR by cohorts.

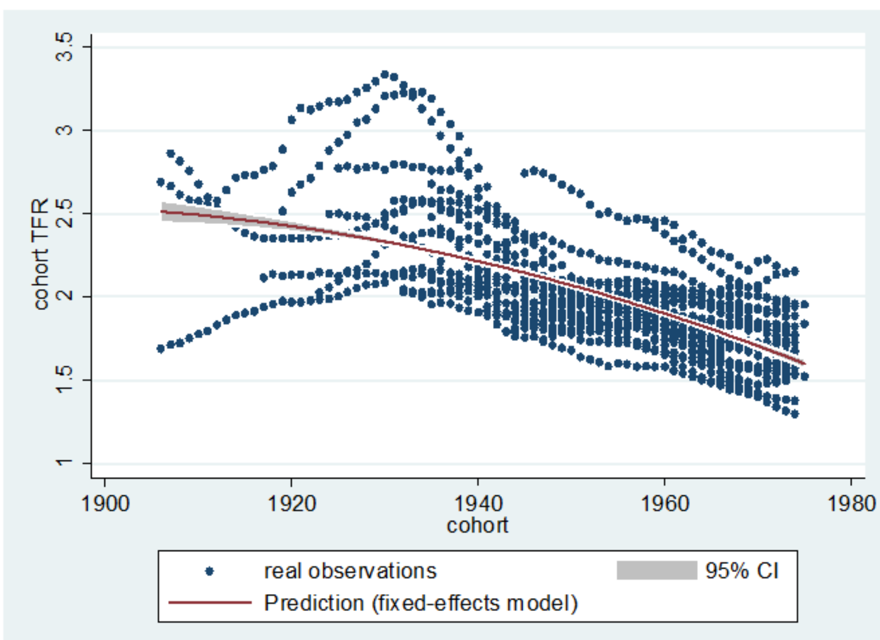

Figure A1: TFR by cohorts (28 countries, covered cohorts: 1906-1975).

10.1.2 Regressions for MAM by cohorts

Regressions of Model 1, 2 and 5 in Table A3 include cohort MAM as endogenous variable, whereas Models 3 and 4 include the cohort mean age at first motherhood (cohort MA1M) as endogenous variable. Models 1 and 3 test a linear pattern of cohort MAM and cohort MA1M over cohorts, while Models 2 and 4 test a quadratic specification.

Model 2 has a much higher goodness of fit than Model 1, which suggests a non-linear rather than a linear pattern of cohort MAM over cohorts. The significantly positive coefficient of the square of 'cohort' points to a minimum in the estimated correlation pattern, i.e. a U-shaped, pattern of cohort MAM over cohorts within the observed countries.

A comparison of Models 3 and 4 confirms that the U-shaped pattern also holds when we focus on cohort mean age at first motherhood (cohort MA1M). This suggests that the convexity of cohort MAM over cohorts is not due to a change in cohort TFR. Model 5 confirms this by showing that the convexity of cohort MAM stays significant when controlling for cohort TFR.[30]

	Endogenous variable:				
	cohort MAM	cohort MAM	cohort MA1M	cohort MA1M	cohort MAM
	Model 1	Model 2	Model 3	Model 4	Model 5
Exogenous variables:					
Cohort	0.0161***	-9.242***	0.0421***	-5.136***	-8.990***
	(7.93)	(-31.02)	(12.82)	(-5.14)	(-27.40)
Cohort^2		0.00238***		0.00132***	0.00231***
		(31.08)		(5.19)	(27.39)
Cohort TFR					-1.831**
					(-2.61)
(Cohort TFR)^2					0.450**
					(3.10)
Constant	-4.494	9008.6***	-58.39***	5004.0***	8763.7***
	(-1.14)	(31.06)	(-9.08)	(5.13)	(27.52)
Country-Fixed Effects	Yes	Yes	Yes	Yes	Yes
Number of observations:	1150	1150	539	539	1150
Number of countries:	28	28	20	20	28
Covered cohorts (unbalanced):	1906-1975	1906-1975	1929-1975	1929-1975	1906-1975
R^2 within:	0.0531	0.4915	0.2409	0.2784	0.4991
t statistics in parentheses					
* p<0.05, ** p<0.01, *** p<0.001					

Table A3: Regression for MAM by cohorts.

[30] Model 5 also suggests a U-shaped relation between cohort TFR and cohort MAM: declining cohort TFR first decreases cohort MAM, and then increases cohort MAM beyond a certain threshold. We find that the same shape for the pattern between cohort MA1M and cohort TFR (results available on request).

10.1.3 Education data

Education data come from the Lee and Lee Long-Run Education Dataset (Lee and Lee 2016), and cover years 1870 to 2010 (5-year intervals). In order to assign periodic observations of the average years of total schooling (primary, secondary and tertiary) of a female population aged 15-24 to our birth cohorts of 1906 to 1974, we allow a 20-year delay between the periodic observations of education and our cohorts.[31]

Moreover, since educational data is only available at 5-year intervals, we also assign the education estimate for the year 1930 to the four cohorts around 1910, i.e. to the cohorts 1908, 1909, 1911 and 1912. This means that for these cohorts, we assign the periodic estimate of education attainment for the female population aged 15-24 at ages 22, 21, 19 and 18. Robustness checks are made by using data with 5-year cohorts only (see Model 2 of Table A4 below).

Figure A2 shows educational attainment by cohorts for 17 countries (see Table A for the list of countries and covered cohorts). Note that the fact that we assigned each period measure to five successive cohorts (due to limited data availability) explains why the calculated educational attainment increases step-wise over cohorts.

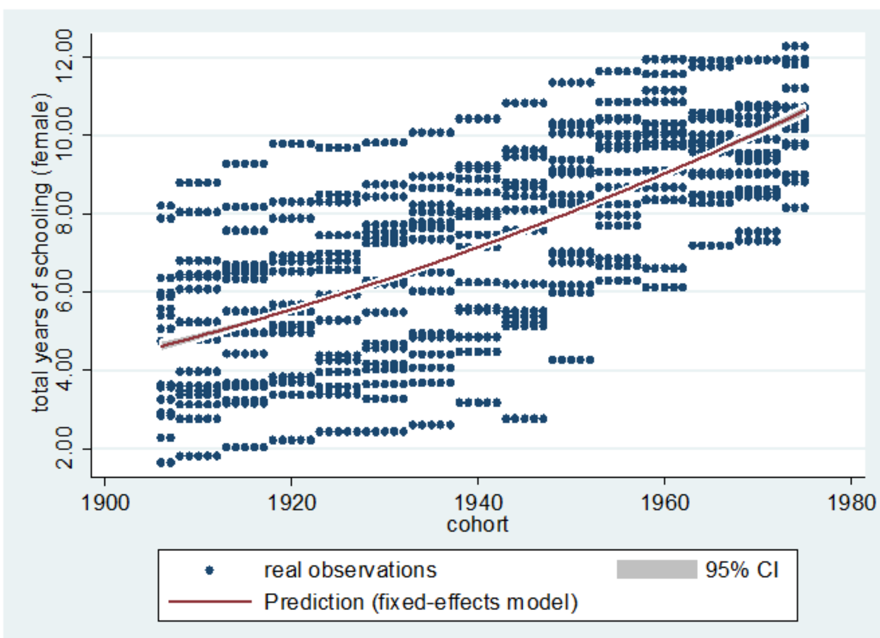

Figure A2: Educational attainment by cohorts (17 countries, covered cohorts: 1906-1975).

[31] For example, for a cohort born in 1910, we observe the education attainment of the female population aged 15-24 in the year 1930, i.e. when our cohort of interest is 20 years old.

10.1.4 Regressions for MAM by education

In order to estimate the relation between the *tempo* of births and educational attainments, we run several regressions (Table A4). Model 1 in Table A4 shows regression results which confirm a statistically significant minimum in the relation between female education and cohort MAM. Model 2 and 3 show some robustness checks: Model 2 is based on 5-year cohorts only, while Model 3 is based on a slightly different variable of female education: We restrict here education to secondary and tertiary education. Both models confirm that the U-shaped relation between cohort education and cohort MAM is robust.

	Endogenous variable:			
	cohort MAM Model 1	cohort MAM Model 2	cohort MAM Model 3	cohort MAM Model 4
Exogenous variables:				
female years of total schooling	-0.684***	-0.666**		-0.892***
	(-6.52)	(-2.71)		(-8.99)
(female years of total schooling)^2	0.0545***	0.0504**		0.0569***
	(7.83)	(3.08)		(8.80)
female years of secondary / tertiary schooling			-0.470***	
			(-5.02)	
(female years of secondary / tertiary schooling)^2			0.128***	
			(8.20)	
cohort TFR				-10.32***
				(-11.44)
(cohort TFR)^2				2.110***
				(11.16)
Constant	28.86***	28.95***	27.22***	42.43***
	(75.21)	(32.29)	(218.17)	(33.95)
Country-Fixed Effects	Yes	Yes	Yes	Yes
Number of observations:	790	152	790	790
Number of countries:	17	17	17	17
Covered cohorts (unbalanced):	1906-1975	1910-1970 (5-year intervals)	1906-1975	1906-1975
R^2 within:	0.1060	0.0790	0.1648	0.2364
t statistics in parentheses				
* p<0.05, ** p<0.01, *** p<0.001				

Table A4: Regressions for MAM by educational attainment.

Figure A3 compares the estimated results of Model 3 to real country observations. Model 4 finally confirms that the U-shaped relation between education and cohort MAM also holds when controlled for cohort TFR.

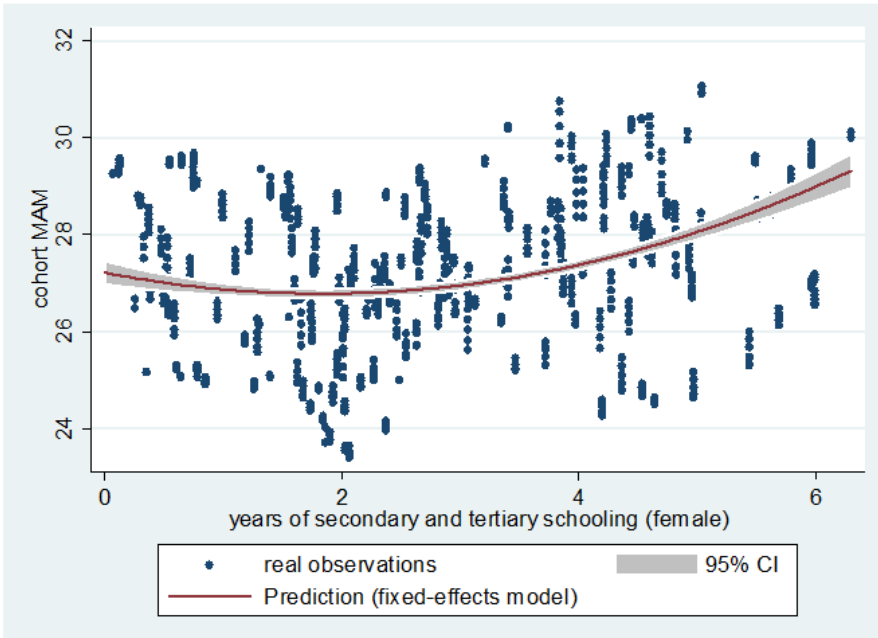

Figure A3: MAM by educational attainments (only secundary and tertiary education) (17 countries, covered cohorts: 1906-1975).

10.2 Proof of Lemma 1

Assuming that $c > 0$ and $d > 0$, first-order conditions of problem with respect to n, e, and m are:

$$-U_c hq + U_n = 0, \tag{24}$$
$$U_e + V_d h (1 - qm) + \lambda = 0, \tag{25}$$
$$-V_d h (\nu + e) q + V_m = 0, \tag{26}$$

where λ is the multiplier associated to the inequality constraint on e; the complementary slackness condition is therefore:

$$e\lambda = 0. \tag{27}$$

The optimal early fertility, n^*, is defined by (24) while the optimal education and late fertility, e^* and m^*, are solutions of system (25)-(26). Using (24) as an implicit function and applying the implicit function theorem, we obtain:

$$\frac{dn}{dh} = q \frac{U_c + U_{cc} h (1 - qn)}{U_{cc} (hq)^2 + U_{nn}}. \tag{28}$$

Rearranging terms with the elasticities defined in (11) gives the expression of ε_{nh}^{*} that is provided in (12). For $e^{*}=0$, the optimal m^{*} is given by (26), which can be used as an implicit function to obtain:

$$\frac{dm}{dh}=(\nu+e)\,q\frac{V_{d}+V_{dd}d}{V_{dd}\left[h\left(\nu+e\right)q\right]^{2}+V_{mm}}, \tag{29}$$

and, rearranging further, the expression of ε_{mh}^{*} that is provided in (12).

Let us now observe that the mean age at motherhood (MAM) decreases with the ratio n/m. Indeed, we have:

$$MAM:=\frac{n\times 1+m\times 2}{n+m}=1+\frac{1}{1+\frac{n}{m}}. \tag{30}$$

Condition (13) characterizes both the evolution of n/m with respect to h and the evolution of the MAM with respect to h.$\square$

10.3 Proof of Lemma 2

As a preliminary, let us use (26) as an implicit function to define $m:=f\left(e,h\right)$ and compute the following derivatives:

$$f_{e}^{\prime}=-\frac{m}{(\nu+e)}\frac{1-\varepsilon_{d}}{\varepsilon_{d}\frac{qm}{1-qm}+\varepsilon_{m}}\text{ and }f_{h}^{\prime}=-\frac{m}{h}\frac{1-\varepsilon_{d}}{\varepsilon_{d}\frac{qm}{1-qm}+\varepsilon_{m}}. \tag{31}$$

By replacing m with $f\left(e,h\right)$ in (25), we define $F\left(e,h\right):=U_{e}+V_{d}h\left(1-qf\left(e,h\right)\right).$ The assumption $e>0$ implies that $F\left(e,h\right)=0$. Using (31), we have

$$F_{h}^{\prime}=V_{d}\left(1-\varepsilon_{d}\right)\left[\left(1-qm\right)+qm\frac{1-\varepsilon_{d}}{\varepsilon_{d}\frac{qm}{1-qm}+\varepsilon_{m}}\right], \tag{32}$$

which is positive if $\varepsilon_{d}<1$. Moreover,

$$F_{e}^{\prime}=\frac{hV_{d}\left(1-qm\right)}{(\nu+e)}\left(\varepsilon_{e}\frac{(\nu+e)}{e}-\varepsilon_{d}+\frac{\left(1-\varepsilon_{d}\right)^{2}}{\varepsilon_{d}+\varepsilon_{m}\frac{1-qm}{qm}}\right), \tag{33}$$

which is negative if the condition (15) is satisfied. Those two conditions then imply $de/dh>0$.

Let us now turn to m. Its derivative with respect to h satisfies:

$$\frac{dm}{dh}=-f_{e}^{\prime}\frac{F_{h}^{\prime}}{F_{e}^{\prime}}+f_{h}^{\prime}, \tag{34}$$

which using (31), (32) and (33) can be rewritten as:

$$\frac{dm}{dh}=\frac{m}{h}\frac{1-\varepsilon_{e}\frac{(\nu+e)}{e}}{\varepsilon_{e}\frac{(\nu+e)}{e}-\varepsilon_{d}+\frac{(1-\varepsilon_{d})^{2}}{\varepsilon_{d}+\varepsilon_{m}\frac{1-qm}{qm}}}\frac{1-\varepsilon_{d}}{\varepsilon_{d}\frac{qm}{1-qm}+\varepsilon_{m}}, \tag{35}$$

which is negative if $\varepsilon_d < 1$ and if condition (15) is satisfied (as those two inequalities imply $1 - \varepsilon_e \frac{(\nu+e)}{e} > 0$). Let us now recall that $\varepsilon^*_{mh} := \frac{\partial m^*}{\partial h}\frac{h}{m^*}$, which using (35) can be rewritten as:

$$\varepsilon^*_{mh} = -\frac{\varepsilon_e \frac{(\nu+e)}{e} - 1}{\left(\varepsilon_e \frac{(\nu+e)}{e} - 1\right) + (1-\varepsilon_d) + \frac{(1-\varepsilon_d)^2}{\varepsilon_d + \varepsilon_m \frac{1-qm}{qm}}} \frac{1-\varepsilon_d}{\varepsilon_d \frac{qm}{1-qm} + \varepsilon_m}, \tag{36}$$

and can be finally rewritten as (14). $\square$

10.4 Proof of Lemma 3

Equation (26) indicates that $\lambda > 0$ provided that $F(e,h) < 0$. Since condition (15) implies that $F'_e < 0$, the existence of $\bar{h}$ is obtained if:

$$\lim_{h\to 0} F(0,h) < 0 < \lim_{h\to+\infty} F(0,h). \tag{37}$$

We notice that:

$$\lim_{h\to 0} F(0,h) = \lim_{e\to 0} U_e + \lim_{d\to 0} V_d d \text{ and } \lim_{h\to+\infty} F(0,h) = \lim_{e\to 0} U_e + \lim_{d\to+\infty} V_d d, \tag{38}$$

and thus conclude that conditions (16) are sufficient. Then, uniqueness of $\bar{h}$ is obtained if $F'_h > 0$, which is ensured by condition $\varepsilon_d < 1$.$\square$

10.5 Proof of Lemma 4

Let us first observe that, for any $h > \bar{h}$, the MAM is increasing with h if and only if $\varepsilon^*_{nh} \leq \varepsilon^*_{mh}$, which can be rewritten using (12) and (14) as:

$$\frac{1-\varepsilon^*_c}{\left(\varepsilon^*_c \frac{qn^*}{1-qn^*} + \varepsilon^*_n\right)} - \frac{1-\varepsilon^*_d}{\left(\varepsilon^*_d \frac{qm^*}{1-qm^*} + \varepsilon^*_m\right) + \left(\frac{qm^*}{1-qm^*} + \varepsilon^*_m\right)\frac{\left(1-\varepsilon^*_d\right)}{\left(\varepsilon^*_e \frac{\nu+e^*}{e^*} - 1\right)}} \geq 0. \tag{39}$$

Our proof provides first existence conditions for $\tilde{h}$ and then a uniqueness condition. For existence, we first observe that there exists $\mu > 0$ such that the sign of the derivative of the MAM with respect to h is negative for $\bar{h}$ and $\bar{h} + \mu$. By continuity, and provided that (13) is satisfied, we have:

$$\frac{1-\varepsilon^*_c}{\left(\varepsilon^*_c \frac{qn^*}{1-qn^*} + \varepsilon^*_n\right)} - \frac{1-\varepsilon^*_d}{\left(\varepsilon^*_d \frac{qm^*}{1-qm^*} + \varepsilon^*_m\right)} < 0, \tag{40}$$

for h close to $\bar{h}$. Since it has been shown in the proof of Lemma 2 that $\varepsilon^*_e \frac{\nu+e^*}{e^*} < 1$, we therefore conclude that:

$$\frac{1-\varepsilon^*_c}{\left(\varepsilon^*_c \frac{qn^*}{1-qn^*} + \varepsilon^*_n\right)} - \frac{1-\varepsilon^*_d}{\left(\varepsilon^*_d \frac{qm^*}{1-qm^*} + \varepsilon^*_m\right) + \left(\frac{qm^*}{1-qm^*} + \varepsilon^*_m\right)\frac{\left(1-\varepsilon^*_d\right)}{\left(\varepsilon^*_e \frac{\nu+e^*}{e^*} - 1\right)}} < 0 \tag{41}$$

for $h = \bar{h} + \mu$. Hence, the MAM decreases with h for $h = \bar{h} + \mu$. Let us provide conditions such that the MAM increases with h for $h \to +\infty$. If (i) $\lim_{h \to +\infty} e^*$ is sufficiently high and (ii) $\lim_{e \to +\infty} \varepsilon_e \frac{\nu + e}{e} = 1$, we obtain:

$$\lim_{h \to +\infty} \frac{1 - \varepsilon_d^*}{\left(\varepsilon_d^* \frac{qm^*}{1-qm^*} + \varepsilon_m^*\right) + \left(\frac{qm^*}{1-qm^*} + \varepsilon_m^*\right) \frac{\left(1-\varepsilon_d^*\right)}{\left(\varepsilon_e^* \frac{\nu+e^*}{e^*} - 1\right)}} = 0, \tag{42}$$

which is sufficient for inequality (39) to be satisfied. Condition (i) depends on the limit behavior of e^*, which could be either infinite (and then the condition is satisfied) or finite. In the latter case, one shall consider the solution of system (24)-(25)-(26) for $h \to +\infty$. Provided $\lim_{c \to +\infty} U_c c = A \ll +\infty$ and $\lim_{d \to +\infty} V_d d = B \ll +\infty$, we can define limit values for the endogenous variables, denoted $(\bar{n}, \bar{m}, \bar{e})$, which satisfy: $\bar{n} = U_n^{-1}(A)$, $\bar{m} = V_m^{-1}(B)$ and $\bar{e} = -U_e^{-1}(B)$. The condition is therefore to have a B that is not too high. Using the definition of ε_e condition (ii) can be rewritten as (17). Regarding the uniqueness of the threshold, this requires that the derivative of $(|\varepsilon_{nh}^*| - |\varepsilon_{mh}^*|)$ with respect to h at $h = \tilde{h}$ has the same sign for any threshold $\tilde{h}$. Given that this difference is negative for low h (advancement of births) and positive for high h (postponement of births), the sign of that derivative must be positive. $\square$

10.6 Proof of Proposition 3

The proof proceeds as follows. We show that, if the parameters satisfy the conditions stated in Proposition 3, then the conditions imposed in Lemmas 1 to 4 are verified, so that Proposition 1 (existence of the three regimes) holds for the analytical model presented in Section 6.

Let us first show that, if $\delta > 0$, $\varepsilon > 0$ (which is assumed at the very outset of Section 6) and $\varepsilon > \delta v$, the conditions stated in Lemma 1 are satisfied.

To see this, let us assume that $e_t = 0$. Note that, from the FOC for education, we have that $e_t > 0$ if and only if $h_t > \bar{h} \equiv \frac{\varepsilon(\sigma v + \rho\eta)}{v(\beta\eta - \sigma v)}$. This condition would be trivially satisfied for any $h_t > 0$ when $\beta\eta - \sigma v < 0$. Hence, to allow for the case where $e_t = 0$ for some $h_t > 0$, we need to impose $\beta\eta > \sigma v$.

When $e_t = 0$, we have $n_t = \frac{\gamma(h_t + \delta)}{h_t q(\alpha + \gamma)}$ and $m_{t+1} = \frac{\rho(v h_t + \varepsilon)}{v h_t q(\beta + \rho)}$. Hence:

$$\varepsilon_{nh}^* = \frac{\partial n_t^*}{\partial h_t} \frac{h_t}{n_t^*} = \frac{-\delta}{h_t + \delta} \text{ and } \varepsilon_{mh}^* = \frac{\partial m_{t+1}^*}{\partial h_t} \frac{h_t}{m_{t+1}^*} = \frac{-\varepsilon}{v h_t + \varepsilon}.$$

Lemma 1 states conditions under which $\varepsilon_{nh}^* < 0$, $\varepsilon_{mh}^* < 0$ and $|\varepsilon_{nh}^*| < |\varepsilon_{mh}^*|$. The first two inequalities hold when $\delta > 0$ and $\varepsilon > 0$. The inequality $|\varepsilon_{nh}^*| < |\varepsilon_{mh}^*|$, which leads to a MAM decreasing with h_t, holds when $\frac{\delta}{h_t + \delta} < \frac{\varepsilon}{v h_t + \varepsilon}$, that is, when $\varepsilon > \delta v$.

Let us now turn to Lemma 2, and show that, if $\delta, \varepsilon > 0$, as well as $h_t \Delta'(h_t) > 2\Delta(h_t) - 2\varphi \sqrt[2]{\Delta(h_t)}$, then the conditions stated in Lemma 2 are satisfied.

Under Lemma 2, we have $e_t > 0$. We still need $\varepsilon_{nh}^* < 0$, $\varepsilon_{mh}^* < 0$ and $\varepsilon_{eh}^* > 0$. The condition for $\varepsilon_{nh}^* < 0$ remains valid when $\delta > 0$.

Note that, in Lemma 2, the conditions imply that $1-\varepsilon_e\frac{(\nu+e_t)}{e_t}>0$. In our analytical example, that condition can be written as: $1-\frac{e_t}{e_t+\eta}\frac{\nu+e_t}{e_t}>0$, which is true if and only if

$$\eta > v$$

Note also that, from the FOC for education, we have: $e(h_t)=\frac{-[h_t\Omega+\varphi]+\sqrt[2]{\Delta(h_t)}}{2h_t\omega}$ where $\Delta(h_t)>0$ because $\beta\eta>\sigma v$.

Hence we have:

$$\varepsilon^*_{eh}=\frac{\partial e^*_t}{\partial h_t}\frac{h_t}{e^*_t}>0 \iff h_t\Delta'(h_t)>2\Delta(h_t)-2\varphi\sqrt[2]{\Delta(h_t)}$$

Under $e_t>0$, we now have: $m_{t+1}=\frac{\rho((v+e(h_t))h_t+\varepsilon)}{(v+e(h_t))h_t q(\beta+\rho)}$. Hence:

$$\varepsilon^*_{mh}=\frac{\partial m^*_{t+1}}{\partial h_t}\frac{h_t}{m^*_{t+1}}=\frac{-\varepsilon\left[\frac{(e'(h_t))h_t}{v+e(h_t)}+1\right]}{\rho\left((v+e(h_t))\,h_t+\varepsilon\right)}$$

We need $\varepsilon^*_{mh}<0$ when $e'(h_t)>0$. Note that $h_t\Delta'(h_t)>2\Delta(h_t)-2\varphi\sqrt[2]{\Delta(h_t)}$ implies $e'(h_t)>0$. Hence, under that condition, $\varepsilon^*_{mh}<0$ holds when $\varepsilon>0$ (equivalent to condition in Lemma 1 for m_{t+1}: $\varepsilon_d<1$).

Concerning Lemma 3, the condition $\lim_{d_{t+1}\to 0}V_d d_{t+1}<-\lim_{e_t\to 0}U_e<\lim_{d_{t+1}\to+\infty}V_d d_{t+1}$ can be written here as:

$$0<\frac{\sigma}{\eta}<\beta$$

Given $\sigma,\eta,\beta>0$, this amounts to assume: $\beta\eta>\sigma$. This condition is not stated in Proposition 3, since it is implied by the condition $\beta\eta>\sigma v$.

Finally, concerning Lemma 4, four conditions need to be satisfied. Condition (13) is already satisfied from our assumptions relative to Lemma 2. The remaining three conditions are:

$$\lim_{e\to+\infty}\frac{U_{ee}}{U_e}(\nu+e)=-1,\ \left.\frac{\partial\left(\varepsilon^*_{nh}-\varepsilon^*_{mh}\right)}{\partial h_t}\right|_{h_t=\tilde{h}}>0 \text{ and } \lim_{d\to+\infty}V_d d_{t+1} \text{ not too large}$$

In our analytical example, we have $U_e=\frac{-\sigma}{e_t+\eta}$ and $U_{ee}=\frac{\sigma}{(e_t+\eta)^2}$, so that the first condition can be rewritten as:

$$\lim_{e\to+\infty}-\frac{(\nu+e)}{(\eta+e)}=-\frac{\left(\frac{\nu}{e}+1\right)}{\left(\frac{\eta}{e}+1\right)}=-1$$

which is trivially satisfied.

The second condition (uniqueness of the threshold) can be rewritten as:

$$\frac{\delta}{\left(\tilde{h}+\delta\right)^2}+\frac{\left[e''(\tilde{h})\tilde{h}+e'(\tilde{h})\right]\left(v+e(\tilde{h})\right)-\tilde{h}\left(e'(\tilde{h})\right)^2}{\frac{1}{\varepsilon}\left[v+e(\tilde{h})\right]^2\rho\left(\left(v+e(\tilde{h})\right)\tilde{h}+\varepsilon\right)}>\frac{\delta}{\tilde{h}+\delta}\frac{e'(\tilde{h})\tilde{h}+v+e(\tilde{h})}{\left(v+e(\tilde{h})\right)\tilde{h}+\varepsilon}$$

where $\tilde{h}$ is the solution to: $e(h_t) + (h_t + \delta)\, e'(h_t) = \frac{\delta}{\varepsilon}(v + e(h_t))^2 - v$, at which $\frac{\partial MAM}{\partial h_t} = 0$.

The third condition amounts to assume that β is not too high (i.e. there exists necessarily a finite real number such that β is inferior to that number).

Thus, if the conditions stated in Proposition 3 are satisfied, we know from Proposition 1 that there exist three regimes in our economy, whose properties are presented in Proposition 1.$\square$