A Theory of Reverse Retirement

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Abstract

The retirement system is usually regarded as giving a fair reward for a long life of labor. However, the fairness of that system can be questioned, on the grounds that only workers who have a sufficiently long life benefit from that reward, but not workers who die prematurely. In order to reexamine the fairness of retirement systems under unequal lifetime, this paper compares standard retirement (i.e. individuals work before being retired) with - purely hypothetical - reverse retirement (i.e. individuals are retired before working). We first show that, whereas reverse retirement cannot be a social optimum under the utilitarian criterion (unlike standard retirement), reverse retirement can be optimal under the *ex post* egalitarian criterion (giving priority to the worst-off in realized terms). From an *ex post* egalitarian perspective, reverse retirement dominates standard retirement in economies with high life expectancy and a flat age-productivity profile, whereas the opposite holds in less developed economies.

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1 Introduction

The historical roots of retirement systems are old. These date back, in England, to the Poor Laws (1599), which included specific dispositions for elderly individuals unable to work. In France, an *édit royal* of 1604 required the exploitants of a mine to dedicate 1/30th of their output to minors in need. Although those systems implied some form of solidarity to the old, these differed from modern pension systems on the grounds that these were far from universal: Poor Laws were implemented at the parish level, whereas the French *édit royal* concerned only the mining industry.[1] Universal pension systems are more recent: Bismarck's old-age insurance system in Germany dates back to 1889, while Beveridge's pension system in the U.K. dates back to 1942.

Retirement systems were introduced not only because of an insurance motive (protecting individuals against the risk of being poor at the old age), but, also, for redistributive reasons (see Cremer and Pestieau 2011). Retirement systems allow, in theory, for both vertical redistribution (from - potentially richer - active young individuals towards inactive old individuals) and horizontal redistribution (from richer to poorer retirees thanks to non-proportional replacement rates). Distributive aspects of retirement systems are widely studied by economists, in particular when dealing with the design of fair pension systems (see Schokkaert and Van Parijs 2003, Schokkaert et al 2017).

While studying the fairness of retirement systems is already complex in hypothetical societies with equal lifetimes, things become even more complex when introducing longevity inequalities. Actually, individuals are highly unequal in front of death, and a significant proportion of the workforce dies *before* reaching the retirement age. For instance, in France, about 10 % of men and 4 % of women die before reaching the age of 60 (based on the 2014 lifetable).[2] Those people obviously cannot enjoy retirement. Thus, although the retirement system is usually presented as giving a fair reward for a long life of labor, the fairness of that system can be questioned, on the grounds that only workers who have a sufficiently long life can benefit from that reward, whereas those who die prematurely are not rewarded.

The goal of this paper is to reexamine the fairness of retirement systems in an economy with unequal lifetimes. When considering issues of fairness, an important question that arises is whether there is an ethical support for compensating disadvantaged individuals. In the context of unequal lifetimes, the disadvantaged is, under general conditions, the short-lived, and longevity inequalities are largely due to circumstances, i.e. factors on which individuals have little control (such as genetic background or environmental quality).[3] Hence, if one adopts the Principle of Compensation (Fleurbaey and Maniquet,

[1] Another *édit royal* published in 1673 created a pension for officers of the *Marine Royale*, while pensions were introduced for soldiers and civil servants in, respectively, 1831 and 1853 (see Lavigne 2013).

[2] Sources: The Human Mortality Database.

[3] On the impact of genetic background on longevity inequalities, see Christensen et al (2006).

2004, Fleurbaey 2008), according to which inequalities in well-being that are due to circumstances should be abolished by governments, there is a strong ethical support for compensating individuals for unequal lifetimes.

While there is an ethical support for compensating the short-lived, it is not trivial to see how governments could proceed to achieve such a compensation. In a recent paper, Fleurbaey et al (2016) proposed to reexamine how varying the age at retirement could achieve such a compensation. Those authors proposed to study the issue of the optimal retirement age, while adopting an *ex post* egalitarian social welfare function, which gives absolute priority to the worst-off in realized terms (who is, in general, the short-lived). Fleurbaey et al (2016) showed that the compensation of the unlucky short-lived pushes towards postponing retirement, in comparison with the utilitarian social optimum. The underlying intuition is that postponing retirement allows to transfer more resources towards the young age, and, hence, to increase the well-being of unlucky young individuals who will turn out, *ex post*, to be short-lived.

Although the study of Fleurbaey et al (2016) casts some light on how taking care about the compensation of the short-lived can affect the optimal age at retirement, it remained based on the standard view on retirement. Actually, Fleurbaey et al (2016) assumed the usual human life cycle, where individuals work at the young age, and become retiree as they reach some (older) age. But is this standard retirement system the only possible one? Can we think about alternative retirement systems that would be more fair with respect to the unlucky short-lived?

In order to reexamine the fairness of retirement systems under unequal lifetimes, this paper proposes to go beyond the standard representation of retirement systems, in which individuals are first workers at the young age, and, then, if they survive to sufficiently high ages, retirees. We propose to consider also, in this paper, the possibility of what we call a "reverse" retirement system, where individuals would - unlike in existing societies - be first retirees at the young age, and, then, workers at the old age. This paper examines the conditions under which such a - purely hypothetical - reverse retirement system dominates, from an *ex post* egalitarian perspective, the standard retirement system.

At this early stage of our explorations, it should be stressed that such a reverse retirement system does not exist in actual economies, and is thus a pure theoretical abstraction. One can regard reverse retirement as a form of "utopia", in the same way as standard retirement was also an utopia during the longest part of History (as stressed above).[4] Note, however, that, in the common language, the term "reverse retirement" exists, and refers to the behavior of a minority of retirees who go back to work. In some sense, the reverse retirement system that we consider here is a generalization of this behavior to the whole

[4] Although reverse retirement does not exist in actual economies, it is sometimes mentioned as a fanciful utopia, for instance by humorists. An example is the reform of reversing retirement introduced in the hypothetical country "Groland" of the French humorists of the Canal + TV channel. This TV show presents, as a parody, elderly workers in bad health with low productivity, who are serving young people enjoying leisure. This parody illustrates how individual perceptions about age can interplay with beliefs about desirable social architecture.

society.[5] We propose here to compare existing standard retirement systems with the - purely hypothetical - reverse retirement system.

At first glance, one may believe that, if one cares about the unlucky short-lived, the reverse retirement system must dominate the standard retirement system, on the grounds that, under the reverse system, the young can consume without working. However, this belief may be misleading, because a life of leisure at the young age does not necessarily make the unlucky short-lived better off than under the standard retirement system. The resource constraint matters, and the welfare comparison between the two systems depends also on what the young will be able to consume under a reverse retirement system, that is, *in fine*, on what old workers produce. Therefore further examination is required to identify the conditions on technology and preferences under which reverse retirement dominates - or is dominated by - standard retirement.

For that purpose, we develop a three-period overlapping generations model (OLG) with risky lifetime. We consider a production process involving physical capital as well as young and/or old labor. Assuming a perfect substitutability between young and old labor, as well as a decreasing productivity age-profile, we compare the standard retirement system with the reverse retirement system.

Anticipating our results, we first show that, whereas reverse retirement can never be a social optimum under the utilitarian social criterion (unlike standard retirement), reverse retirement can be part of the social optimum under the *ex post* egalitarian social criterion (giving priority to the short-lived). Then, comparing standard and reverse retirement systems from the *ex post* egalitarian perspective, we show that, whereas standard retirement dominates reverse retirement in less developed economies (characterized by low expectancy and a steep age-productivity profile), reverse retirement dominates in advanced economies (with high life expectancy and a flatter age-productivity profile).

The underlying intuition is that, in less developed economies, life expectancy is lower, so that, under a reverse retirement system, the ratio active/inactive population would be too low, which would strongly reduce the consumption possibilities at the young age, and, hence, would make the short-lived worse off than under the standard retirement system. Similarly, when the production process is highly physical (leading to a steep age-productivity gradient), reverse retirement would strongly reduce consumption of the young. On the contrary, in advanced economies, life expectancy is higher, leading to a higher ratio active/inactive population under reverse retirement, and productivity is less related to the age (since production is less physical), so that reverse retirement dominates standard retirement. Thus demographic and technological changes during the last century may contribute to make reverse retirement more desirable, from an *ex post* egalitarian perspective, than standard retirement.

The paper is organized as follows. The model is presented in Section 2. Section 3 examines whether standard or reverse retirement can constitute a social optimum under either a utilitarian or an *ex post* egalitarian social welfare

[5] Here again, there is an obvious paralel with standard retirement, which was introduced initially for some jobs, before being generalized to the whole society in the 20th century.

criterion. Section 4 then studies the conditions under which reverse retirement dominates standard retirement from an *ex post* egalitarian perspective. Some problems, including slower learning by doing, moral hazard, migration of the old, or rise of the (dependent) very old, are examined in Section 5. Section 6 concludes.

2 The model

Let us consider a three-period overlapping generation (OLG) economy.[6] Time is discrete and goes from 0 to $+\infty$. Each time period has a unitary length. Fertility is at the replacement level (one child per individual), and each cohort has a size $N > 0$.[7]

Period 1 is childhood, during which no decision is made. Period 2 is young adulthood, during which individuals have one child, consume, work during a period ℓ_t (i.e. $0 \leq \ell_t \leq 1$), and save some resources for old days. Period 3 is old adulthood. This is reached with a probability π, with $0 < \pi < 1$. In period 3, individuals consume, and work during a period $\tilde{\ell}_{t+1}$ (i.e. $0 \leq \tilde{\ell}_{t+1} \leq 1$).[8]

Production Production takes place by using physical capital and labor, according to the following production function:

$$Y_t = F\left(K_t, L_t\right) \tag{1}$$

where $F(\cdot)$ is increasing and concave in its arguments, capital K_t and labor L_t, and exhibits constant returns to scale.

Capital fully depreciates after one period of use.

Moreover, we assume that there is perfect substitutability between labor at the young age and labor at the old age:

$$L_t = aN\ell_t + b\pi N\tilde{\ell}_t \tag{2}$$

where $a > 0$ and $b > 0$ capture, respectively, the productivity of labor at the young age and at the old age. Given that the empirical literature on the link between age and productivity provides mixed results, we will assume, without loss of generality, that young workers are at least as productive as old workers, that is, that $a \geq b$.[9]

[6] Note that adding a fourth period of life where individuals would necessarily be too old to work would not modify our results, but would increase the number of egalitarian constraints considered, and, hence, would lengthen the presentation of our results. See Section 5 on this.

[7] We do not consider here the issue of optimal fertility. See Pestieau and Ponthiere (2017) on optimal fertility under age-dependent labor productivity.

[8] It should be stressed here that this paper focuses on the selection of optimal *extensive* margins of labor (i.e. duration of periods of labor), whereas Leroux and Ponthiere (2018) focused on *intensive* margins only (i.e. number of hours worked per period of labor).

[9] On the link between age and producivity, empirical studies provide mixed results. Haegeland et Klette (1999) show that older workers are more productive than younger workers, whereas Crepon et al (2003) show that productivity exhibits an inverted U shaped curve with the age. Aubert and Crépon (2007) and Gobel et Zwick (2009) find that productivity grows with age until age 45, and then stabilizes.

Preferences In young adulthood, well-being U_t^y is equal to:

$$U_t^y = u(c_t) - v\ell_t \tag{3}$$

where c_t is consumption in young adulthood, $u(\cdot)$ is increasing and concave, and $v > 0$ captures the disutility of working. As usual, we assume that there exists a level of consumption $\bar{c} > 0$ such that $u(\bar{c}) = 0$.

A the old age, individual derive also some utility from consumption, and some disutility from labor, but functional forms are allowed to differ. At old adulthood, well-being U_t^o is equal to:

$$U_t^o = \tilde{u}(d_t) - \tilde{v}\tilde{\ell}_t \tag{4}$$

where d_t is consumption at old age, $\tilde{u}(\cdot)$ is increasing and concave. We suppose that there exists a level of consumption $\tilde{c} > 0$ such that $\tilde{u}(\tilde{c}) = 0$. The disutility of labor at the old age is captured by $\tilde{v}$. Without loss of generality, we assume that, for a given quantity of labor, the disutility from working at the old age is larger than the disutility from working at the young age, so that $\tilde{v} > v$.

3 Retirement as a social optimum

In our model, standard retirement and reverse retirement coincide with two corner solutions. Under standard retirement, we have $\ell_t = 1$ (i.e. the entire young age is worked) and $\tilde{\ell}_t = 0$ (i.e. the entire old age is not worked). On the contrary, under reverse retirement, we have $\ell_t = 0$ (i.e. the entire young age is not worked) and $\tilde{\ell}_t = 1$ (i.e. the entire old age is worked).

When considering the comparison of those two retirement systems, a first question that arises consists of exploring under which conditions on structural parameters those two corner solutions $(1,0)$ and $(0,1)$ can constitute a social optimum. For that purpose, we consider here two distinct social welfare criteria: utilitarianism and *ex post* egalitarianism.

The utilitarian social criterion is standard in the literature since the pioneer works of Bentham (1789) and Mill (1863), and has become a kind of benchmark normative criterion in public economics. However, when considering environments where individuals have unequal lifetimes, the utilitarian criterion yields somewhat counterintuitive results, as shown in Fleurbaey et al (2014). In a few words, utilitarianism tends to redistribute resources from short-lived towards long-lived agents, against any intuition for compensation.[10]

This motivates the use of an alternative social criterion, that gives more weight to the short-lived. Actually, the *ex post* egalitarian social welfare function (see Fleurbaey et al 2014, 2016) gives absolute priority to the worst off individual in realized terms (rather than in expected terms). Within our model, the worst off in realized terms is, under general conditions, the short-lived.

[10] The reason is that short-lived individuals have, under general assumptions on preferences, a lower capacity to convert resources into well-being.

Hence that social criterion amounts to give priority to the interests of short-lived individuals, and will assess the retirement systems in the light of their capacity to make the short-lived better off.

The ethical justification for relying on the *ex post* egalitarian social welfare function in the present context is that longevity inequalities are here circumstances on which individuals have no control. Hence, on the basis of the Principle of Compensation (Fleurbaey and Maniquet 2004, Fleurbaey 2008), those arbitrary inequalities due to circumstances should be compensated by governments.

Let us now examine whether standard and reverse retirement can be part of the social optimum, either under the utilitarian social welfare criterion or under the *ex post* egalitarian social welfare criterion.

Under the utilitarian social welfare criterion, the social planner chooses consumptions c, d and working periods ℓ, $\tilde{\ell}$ as well as the capital stock K that maximizes the sum of individual utilities at the stationary equilibrium, subject to the resource constraint of the economy. That problem can be written as:[11]

$$\begin{array}{rrl} \max\limits_{c,d,\ell,\tilde{\ell},K} & & N\left[u\left(c\right)-v\ell+\pi\left[\tilde{u}(d)-\tilde{v}\tilde{\ell}\right]\right] \\ \text{s.t.} & & F\left(K,aN\ell+b\pi N\tilde{\ell}\right)=Nc+\pi Nd+K \\ \text{s.t.} & \ell\geq & 0 \text{ and } 1-\ell\geq 0 \\ \text{s.t.} & \tilde{\ell}\geq & 0 \text{ and } 1-\tilde{\ell}\geq 0 \end{array}$$

Under the *ex post* egalitarian social welfare criterion, the social planner chooses consumptions c, d and working periods ℓ, $\tilde{\ell}$ as well as the capital stock K that maximizes the realized lifetime well-being of the worst off living at the stationary equilibrium, subject to the resource constraint of the economy. That problem can be written as:

$$\begin{array}{rrl} \max\limits_{c,d,\ell,\tilde{\ell},K}\min & & \left\{u\left(c\right)-v\ell,u\left(c\right)-v\ell+\tilde{u}(d)-\tilde{v}\tilde{\ell}\right\} \\ \text{s.t.} & & F\left(K,aN\ell+b\pi N\tilde{\ell}\right)=Nc+\pi Nd+K \\ \text{s.t.} & \ell\geq & 0 \text{ and } 1-\ell\geq 0 \\ \text{s.t.} & \tilde{\ell}\geq & 0 \text{ and } 1-\tilde{\ell}\geq 0 \end{array}$$

Those two social planning problems are fully solved in the Appendix. The following proposition summarizes our main result concerning the comparison of standard and reverse retirement systems.

Proposition 1 *Assume a lower productivity for old-age labor than for young-age labor (i.e. $a > b$) and a higher disutility for old-age labor than for young-age labor (i.e. $\tilde{v} > v$).*

- *Under the utilitarian social criterion, standard retirement can be a social optimum, whereas reverse retirement is never a social optimum.*

[11] Given that we are here at the stationary equilibrium, we abstract from time indexes.

- *Under the ex post egalitarian social criterion, both standard retirement and reverse retirement can be a social optimum.*

Proof. See the Appendix. ■

Under the utilitarian social welfare criterion, standard retirement (i.e. $\ell = 1$ and $\tilde{\ell} = 0$) can, under some conditions, be part of the utilitarian social optimum, especially when there is low old-age labor productivity and high old-age labor disutility. However, reverse retirement is never optimal under a utilitarian social welfare criterion. The underlying intuition is that, at zero labor for the young, making a young work brings, at the margin, more output and has a lower disutility than making an old work a bit more. Thus it has to be that the young should work positive amount (i.e. $\ell > 0$) at the utilitarian optimum.

However, under the *ex post* egalitarian social welfare criterion, reverse retirement can, in some cases, be socially optimal, even when old workers are less productive than the young and suffer from a higher disutility of labor than the young. The underlying intuition is that it is still the case that making an old work brings, at the margin, less output and creates more disutility, but that can nonetheless be optimal under some conditions, since the *ex post* egalitarian optimum focuses only on the well-being of the worst-off, who is, in general, the short-lived, so that the social criterion focuses only, at the end of the day, on the temporal welfare of the young.

In sum, whereas standard retirement can, under some conditions, be optimal under the utilitarian and the *ex post* egalitarian criteria, reverse retirement finds no ethical support under utilitarianism, but can find some support only under *ex post* egalitarianism. While that result informs us about the possible optimality of those two retirement systems, Proposition 1 is silent on the conditions under which one system can dominate the other. The next section identifies conditions on structural parameters under which reverse retirement dominates standard retirement from an *ex post* egalitarian perspective.

4 Reverse *versus* standard retirement

This section proposes to compare the standard retirement system with the reverse retirement system under the *ex post* egalitarian social criterion, and to derive conditions under which one system dominates the other. Obviously, it is clear from the social planning problem studied in the previous section that, in many cases, neither standard retirement ($\ell = 1$ and $\tilde{\ell} = 0$) nor reverse retirement ($\ell = 0$ and $\tilde{\ell} = 1$) are optimal, since in many cases the social optimum is an interior solution with $0 < \ell < 1$ and $0 < \tilde{\ell} < 1$, that is, a solution where individuals work only a subperiod of the young age and a subperiod of the old age. However, given that in real economies there are strong organizational constraints limiting the possibility of those interior solutions, it makes sense to focus only on standard and reverse retirement, and to examine under which conditions one system dominates the other.[12]

Under the *ex post* egalitarian social criterion, the best retirement system is the one that maximizes the realized lifetime well-being of the worst-off subject to the associated resource constraint prevailing at the stationary equilibrium. Obviously, the resource constraint depends on which age group takes part to the production process. The problem faced by the *ex post* egalitarian social planner can be written as:

$$\max_{c,d,K} \min \left\{ \begin{array}{l} \{u(c) - v + \tilde{u}(d), u(c) - v\} \text{ under standard retirement} \\ \{u(c) + \tilde{u}(d) - \tilde{v}, u(c)\} \text{ under reverse retirement} \end{array} \right\}$$

$$\text{s.t.} \quad \left\{ \begin{array}{l} F(K, aN) = Nc + \pi Nd + K \text{ under standard retirement} \\ F(K, b\pi N) = Nc + \pi Nd + K \text{ under reverse retirement} \end{array} \right\}$$

In order to solve that social planning problem, we need to solve the two subproblems one by one, and, then, compare the situation of the worst-off in each case, that is, under each retirement system.

For that purpose, we will, to obtain analytical results, assume that the production process follows a Cobb-Douglas technology, that is:

$$F(K, L) = AK^{\alpha} L^{1-\alpha} \tag{5}$$

where A is a total productivity parameter, and $0 < \alpha < 1$ is the elasticity of output with respect to physical capital.

Standard retirement This problem can be rewritten as:

$$\begin{array}{ll} \max\limits_{c,d,K} & u(c) - v \\ \text{s.t.} & AK^{\alpha}(aN)^{1-\alpha} = Nc + \pi Nd + K \\ \text{s.t.} & \tilde{u}(d) = 0 \end{array}$$

where the last constraint guarantees that the short-lived is as well off as the long-lived (i.e. the egalitarian constraint).

FOCs yield:

$$\begin{aligned} u'(c) &= \lambda N = \tilde{u}'(d)\frac{\mu}{\pi} \\ A\alpha K^{\alpha-1}(aN)^{1-\alpha} &= 1 \end{aligned}$$

where λ and μ are the Lagrange multipliers associated, respectively, to the resource constraint and the egalitarian constraint.

The last expression describes the Golden Rule capital level (see Phelps 1961). That Golden Rule also prevails under the *ex post* egalitarian optimum. Using that FOC for optimal capital, we obtain: $K = \left[A\alpha (aN)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$. Moreover,

[12] In technical terms, this section develops a second-best analysis focusing only on two possible retirement systems - standard and reverse - among many other systems.

from the egalitarian constraint, we obtain $d = \tilde{c}$. Hence, from the resource constraint, we have:

$$c = A^{\frac{1}{1-\alpha}} a \left[[\alpha]^{\frac{\alpha}{1-\alpha}} - [\alpha]^{\frac{1}{1-\alpha}} \right] - \pi \tilde{c} \tag{6}$$

Hence

$$U^{SL} = u\left(a\Lambda - \pi\tilde{c}\right) - v \tag{7}$$

where $\Lambda \equiv A^{\frac{1}{1-\alpha}} \left[[\alpha]^{\frac{\alpha}{1-\alpha}} - [\alpha]^{\frac{1}{1-\alpha}} \right]$

Reverse retirement The social planning problem is:

$$\begin{array}{ll} \max\limits_{c,d,K} & u(c) \\ \text{s.t.} & AK^{\alpha}\left(b\pi N\right)^{1-\alpha} = Nc + \pi N d + K \\ \text{s.t.} & \tilde{u}(d) = \tilde{v} \end{array}$$

where the last condition is the egalitarian constraint, which takes here a different form from above. Since the surviving old work, their consumption must be increased so as to compensate them for their labor. Otherwise long-lived individuals would be worse off than short-lived individuals.

FOCs are:

$$\begin{aligned} u'(c) &= \lambda N = \tilde{u}'(d)\frac{\mu}{\pi} \\ A\alpha K^{\alpha-1}\left(b\pi N\right)^{1-\alpha} &= 1 \end{aligned}$$

where λ and μ are the Lagrange multipliers associated, respectively, to the resource constraint and the egalitarian constraint.

From the FOC for capital, we obtain: $K = \left[A\alpha\left(b\pi N\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$. We also have, from the egalitarian constraint, $d = \tilde{u}^{-1}\left(\tilde{v}\right)$. Hence, from the resource constraint, we obtain

$$c = A^{\frac{1}{1-\alpha}} b\pi \left[[\alpha]^{\frac{\alpha}{1-\alpha}} - [\alpha]^{\frac{1}{1-\alpha}} \right] - \pi \tilde{u}^{-1}\left(\tilde{v}\right) \tag{8}$$

Hence the utility of the short lived is here

$$U^{SL} = u\left(b\pi\Lambda - \pi\tilde{u}^{-1}\left(\tilde{v}\right)\right) \tag{9}$$

Comparison of the two systems Collecting the results from the two subproblems, we are now able to examine the conditions on the structural parameters of the economy under which one retirement system dominates the other. Indeed, we know that the lifetime well-being of the worst-off is equal, under the two distinct retirement systems, to:

$$U^{SL} = \begin{cases} u\left(a\Lambda - \pi\tilde{c}\right) - v \text{ under a standard retirement system} \\ u\left(b\pi\Lambda - \pi\tilde{u}^{-1}\left(\tilde{v}\right)\right) \text{ under a reverse retirement system} \end{cases}$$

Note that, under our assumptions, consumption is unambiguously larger under the standard retirement system than under the reverse retirement system. However, given that the young have, under the reverse retirement system, no disutility of labor, the comparison of those systems in terms of welfare is ambiguous, and depends on the structural parameters of the economy. Our results are summarized in Proposition 2.

Proposition 2

- *If* $\max\left\{u\left(a\Lambda - \pi\tilde{c}\right) - v, u\left(\Lambda b\pi - \pi\tilde{u}^{-1}\left(\tilde{v}\right)\right)\right\} = u\left(a\Lambda - \pi\tilde{c}\right) - v$*, then the standard retirement system yields the largest lifetime well-being for the worst-off;*
- *If* $\max\left\{u\left(a\Lambda - \pi\tilde{c}\right) - v, u\left(\Lambda b\pi - \pi\tilde{u}^{-1}\left(\tilde{v}\right)\right)\right\} = u\left(\Lambda b\pi - \pi\tilde{u}^{-1}\left(\tilde{v}\right)\right)$*, then the reverse retirement system yields the largest lifetime well-being for the worst-off.*

Proof. See above. ■

Proposition 2 states that, depending on the levels of the structural parameters, the lifetime well-being of the worst-off is larger either under a standard retirement system or under a reverse retirement system. Whether one system dominates the other depends on the structural parameters, which include production technology parameters Λ (including A and α), a and b, as well as preference parameters v, $\tilde{v}$ and $\tilde{c}$, utility functions $u(\cdot)$ and $\tilde{u}(\cdot)$, as well as a demographic parameter, π.

In the following, we focus on how some parameters affect the social desirability of a particular retirement system.

Proposition 3

- *The welfare gap between reverse retirement and standard retirement is decreasing in the labor productivity of young workers* a *and increasing in the labor productivity of old workers* b*.*
- *When* $\Lambda b > \tilde{u}^{-1}\left(\tilde{v}\right)$*, the welfare gap between reverse retirement and standard retirement is increasing in life expectancy* $2 + \pi$*.*
- *The welfare gap between reverse retirement and standard retirement is increasing in labor disutility at the young age* v *and decreasing in labor disutility at old age* $\tilde{v}$*.*

Proof. The welfare gap is equal to:

$$gap \equiv u\left(\Lambda b\pi - \pi\tilde{u}^{-1}\left(\tilde{v}\right)\right) - u\left(a\Lambda - \pi\tilde{c}\right) + v$$

It is straightforward to see that this gap is decreasing in a and $\tilde{v}$, and increasing in b and v. Moreover, when $\Lambda b > \tilde{u}^{-1}\left(\tilde{v}\right)$, we have that the gap is increasing in π. ■

Thus, when considering traditional societies with highly physical production activities, it is the case that the age of workers affects labor productivity substantially, leading to $a >> b$. In that case, from the perspective of the lifetime well-being of unlucky short-lived persons, the standard retirement system dominates the reverse retirement system. Note, however, that, in modern economies, labor is no longer as physically demanding as before, so that the gap between the labor productivity of young workers and old workers is reduced. Hence, Proposition 3 suggests here that in a modern economy the reverse retirement system is, *ceteris paribus*, more likely to dominate the standard retirement system.

The second part of Proposition 3 states that, provided old-age labor is sufficiently productive (i.e. $\Lambda b > \tilde{u}^{-1}(\tilde{v})$), standard retirement tends, in poor economies with low life expectancy, to dominate reverse retirement, whereas the opposite is true in advanced economies with a higher life expectancy.

Regarding preferences, we obtain that the welfare gap between the reverse retirement system and the standard pension system is increasing with labor disutility at the young age, and decreasing with labor disutility at the old age. Thus, in traditional economies with highly physical activities, the disutility of old age labor is especially large, which makes standard retirement systems more desirable. On the contrary, in modern economies with lower old-age disutility of labor, the reverse retirement system dominates *ceteris paribus*.

In order to further explore the role of preferences, let us impose an explicit functional form on utility functions $u(\cdot)$ and $\tilde{u}(\cdot)$:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} - \beta \text{ and } \tilde{u}(d) = \frac{d^{1-\gamma}}{1-\gamma} - \delta$$

Under those functional forms, we have that $\tilde{u}^{-1}(\tilde{v}) = [(\tilde{v}+\delta)(1-\gamma)]^{\frac{1}{1-\gamma}}$ and $\tilde{c} = [\delta(1-\gamma)]^{\frac{1}{1-\gamma}}$, so that the utility of the short-lived becomes:

$$U^{SL} = \begin{cases} \frac{\left(a\Lambda - \pi[\delta(1-\gamma)]^{\frac{1}{1-\gamma}}\right)^{1-\sigma}}{1-\sigma} - \beta - v \text{ under standard retirement} \\ \frac{\left(b\pi\Lambda - \pi[(\tilde{v}+\delta)(1-\gamma)]^{\frac{1}{1-\gamma}}\right)^{1-\sigma}}{1-\sigma} - \beta \text{ under reverse retirement} \end{cases}$$

Proposition 4 summarizes some results obtained under those particular functional forms for utility.

Proposition 4 *The reverse retirement system dominates the standard retirement system from an ex post egalitarian perspective iff:*

$$v > \frac{\left(a\Lambda - \pi\left[\delta(1-\gamma)\right]^{\frac{1}{1-\gamma}}\right)^{1-\sigma} - \left(b\pi\Lambda - \pi\left[(\tilde{v}+\delta)(1-\gamma)\right]^{\frac{1}{1-\gamma}}\right)^{1-\sigma}}{1-\sigma}$$

Proof. See above. ■

Proposition 4 states that a necessary and sufficient condition for the dominance of the reverse retirement system over the standard retirement system is that the disutility of young age labor exceeds some limit value, which is a function of other preference parameters, as well as of production and demographic parameters. Actually, that limit value coincides with the utility gain, in terms of utility from consumption at the young age, of the additional consumption possibilities obtained by making the young rather than the old work.

In sum, this section suggests that, when adopting an *ex post* egalitarian perspective (thus giving priority to the worst off), it appears that, in traditional economies with physical production activities (leading to high productivity and disutility differentials between young labor and old labor) and low life expectancy, standard retirement dominates, in welfare terms, reverse retirement. This result may seem counterintuitive, since one may expect at first glance that making the old work is always better from the perspective of the short-lived. But the underlying intuition is that, in that case, the labor of the old is so unproductive and requires such a high compensation in terms of old-age consumption that it strongly restricts the consumption possibilities of the young under a reverse retirement system. However, when considering a modern economy, with less physical production activities (leading to a lower productivity and disutility gap between young labor and old labor) and with a higher life expectancy, the reverse retirement system dominates, from an *ex post* egalitarian perspective, the standard retirement system.

5 Discussions

Whereas the previous section shows that, from an *ex post* egalitarian perspective, there can be, in an advanced economy, some support for a shift from standard retirement to reverse retirement, one may raise some criticisms against this argument. This section explores some criticisms, which allow us also to examine the robustness of the above argument.

5.1 Productivity and learning by doing

A first line of criticism concerns the impact of reverse retirement on labor productivity, on the potential for economic growth, and, hence, for consumption possibilities. In particular, one may argue that postponing the entrance on the labor market may prevent individuals from acquiring a strong experience in the firm. Since Adam Smith (1776)'s pin factory example, it has been argued by economists that a major source of productivity growth lies in workers' repetition of actions. By repeating their actions, workers become more and more productive. This is close to the idea of "learning by doing" by Arrow (1962). In the light of this, adopting a reverse retirement system may prevent the economy from enjoying a substantial learning by doing, and, hence, may make significant productivity gains vanish. Hence, in the model, the absence or minoring of learning by doing would make the parameter b low under a reverse retirement system, in comparison to its level when individuals work at the young age.

Although that criticism is relevant for the issue at stake, it should be stressed that this argument is far from decisive. That argument is true only to the extent that learning by doing can only be made at the young age. Actually, in our model, the period of labor has the same duration whatever one considers the standard retirement system or the reverse retirement system. Thus, from the perspective of allowing for repeated activity and learning about the production process, the reverse retirement system does not seem to prevent those processes, and the associated productivity gains.

But even if one assumes that "learning by doing" is more difficult for older workers than for younger workers, this assumption can be taken into account by assuming $a >> b$ in our model. However, as shown above, assuming a strong productivity gap is not sufficient to rule out reverse retirement. The fact that old workers are less productive than young workers is not a sufficient condition for making standard retirement dominate reverse retirement.

5.2 Moral hazard

Another line of attack against the reverse retirement system may consist of claiming that, if that system were implemented in real economies, where there exist strong asymmetries of information in the production process (i.e. unobservable effort levels), then old workers would not make efforts, and this would strongly reduce the social desirability of reverse retirement.

To discuss that criticism, let us now suppose that the contribution of labor to output depends on (i) the worker's efforts; (ii) a random component. For simplicity, there are only two levels of effort: 1 (high) and 0 (low). If the effort is 1, the output of young labor equals a with probability 1/2 and $\tilde{a} < a$ with probability 1/2.[13] Under a 0 effort, the output of young worker equals $\tilde{a}$.[14] For the old, the output under a high effort equals b with probability 1/2 and $\tilde{b} < b$ with probability 1/2, whereas, under low effort, the output equals $\tilde{b}$. We assume also that there is, in case of low effort, a probability of detection $\phi < 1$, leading to a social stigma $z > 0$ for a young worker, and $\tilde{z} > 0$ for an old worker.

Under standard retirement, one needs to add to the social planning problem the following constraint, which states that, under the prevailing allocation of resources, a young individual is better off in expected terms when making the high effort level than when making the low effort level:

$$u(c) - v + \pi u(d) \geq \phi [u(c) - z + \pi u(d)] + (1 - \phi) [u(c) + \pi u(d)]$$

This condition can be simplified to:

$$\phi z \geq v \tag{10}$$

Young individuals choose the high effort provided the expected social stigma from being identified as defector exceeds the disutility of labor.

Concerning labor at old age under a reverse retirement system, the constraint stating that the old is, in expected terms, better off making a high effort, is:

$$\phi \tilde{z} \geq \tilde{v} \tag{11}$$

[13] In that case, the disutility of labor is set to v.

[14] In the case of a low effort, the disutility of labor is set to 0.

Given that old-age labor exhibits a higher disutility than young-age labor, i.e. $\tilde{v} > v$, that constraint is, under an equal social stigma $z = \tilde{z}$, stronger than under standard retirement. Thus a reverse retirement system would require, due to a higher disutility of old-age labor, stronger monitoring of efforts than a standard retirement system. However, if the social stigma is increasing with the age ($\tilde{z} > z$), that is, if there is more tolerance for the young's defect than for the old's defect, this conclusion may be reversed, and a reverse retirement system might then require less monitoring than a standard retirement system.

In the light of this, and assuming that monitoring has a cost in terms of resources $C(\phi)$, it appears that, when $\frac{\tilde{v}}{\tilde{z}} > \frac{v}{z}$, introducing asymmetric information reduces young-age consumption more under the reverse retirement system than under the standard retirement system, making the former system less desirable. On the contrary, when $\frac{\tilde{v}}{\tilde{z}} < \frac{v}{z}$ (larger social stigma for the old's defection), then asymmetric information increases the support for reverse retirement.

In sum, introducing asymmetric information in the production process would have ambiguous effects on the social desirability of the standard and reverse retirement systems, depending on the extent to which the social stigma varies with the age of the defector.

5.3 Migrations

One may argue against the reverse retirement system that it is not realistic, since this would lead old workers to migrate out of the country, in order to avoid having to work at the old age.[15]

That argument can be addressed formally by introducing a migration rate $\tilde{m} > 0$ for the old. According to that alternative assumption, the labor force would be, in case of a reverse retirement system, equal to:

$$L = (1 - \tilde{m})\pi N b \tag{12}$$

whereas the number of retirees in economies with a standard retirement system would be augmented in the same proportion, i.e., by a factor $1 + \tilde{m}$.

Under the possibility to migrate out of the economy before reaching the old age, there is no absolute support for the standard retirement system. On the contrary, there is only a modification of the conditions under which the reverse retirement system dominates, as shown below.[16]

Proposition 5 *Assume that a proportion $\tilde{m}$ of the surviving old migrates from an economy with reverse retirement towards an economy with standard retirement.*

[15] This argument exhibits some limitations. If one country started a reverse retirement system, then, under this assumption of free mobility of individuals, lots of *young* workers from other countries with standard retirement would leave their country, and join the economy with reverse pensions. Hence, the other countries in the world would be *de facto* forced to adopt a reverse pension system as well, that is, to make the old go back to work. This would then limitate the extent of migrations across countries.

[16] We assume here a migration of old individuals from economies with a reverse retirement system to an economy with a standard retirement system.

- *If* $\max\left\{\begin{array}{l} u\left(a\Lambda-\pi\left(1+\tilde{m}\right)\tilde{c}\right)-v, \\ u\left(\Lambda b\pi(1-\tilde{m})-\pi(1-\tilde{m})\tilde{u}^{-1}\left(\tilde{v}\right)\right) \end{array}\right\}=u\left(a\Lambda-\pi\left(1+\tilde{m}\right)\tilde{c}\right)-v$*, then the standard retirement system yields the largest lifetime well-being for the worst-off;*
- *If* $\max\left\{\begin{array}{l} u\left(a\Lambda-\pi\left(1+\tilde{m}\right)\tilde{c}\right)-v, \\ u\left(\Lambda b\pi(1-\tilde{m})-\pi(1-\tilde{m})\tilde{u}^{-1}\left(\tilde{v}\right)\right) \end{array}\right\}=u\left(\Lambda b(1-\tilde{m})\pi-\pi(1-\tilde{m})\tilde{u}^{-1}\left(\tilde{v}\right)\right)$*, then the reverse retirement system yields the largest lifetime well-being for the worst-off.*

Proof. The proof is similar to the one of Proposition 2. ■

In the light of Proposition 5, it appears that introducing a reasonable migration rate for old workers has a limited impact on the conditions under which the reverse retirement system dominates the standard retirement system. The reason is twofold. First, under a reverse pension system, migrations of the old affect not only the output, but, also, the resources dedicated to the consumption of individuals at the old age. Second, once migrations are allowed, this will bring extra retirees in economies with standard pension systems, which will reduce consumption possibilities there, which tends to reduce the relative social desirability of that system.

Having stressed this, it remains true that, for extremely high migration rates (i.e. $\tilde{m}\simeq 1$), it is as if life expectancy were very low (i.e. $\pi\simeq 0$), so that we know from above that in that case the standard retirement system dominates reverse retirement. However, assuming so high migration rates is highly unrealistic, since pension systems are not key drivers of international labor mobility.

All in all, the migration argument is not decisive against the relative desirability, from an *ex post* egalitarian perspective, of reverse retirement with respect to standard retirement systems.[17]

5.4 The dependent elderly

An important limitation of our analysis is that we consider here a highly stylized, three-period, representation of the human lifecycle. Our model thus focuses on the first three ages of life (childhood, young adulthood and old adulthood), without considering the very old age, at which individuals are too old to remain autonomous, and thus need to receive long term care.

In order to deal with that criticism, a simple way to proceed is to add a fourth period in our OLG model, and to see how this would affect the comparison of standard and reverse retirement. For that purpose, let us now suppose that individuals in the third period of their life can reach the very old age with probability $p>0$, and do not reach it with probability $1-p$.

Since very old individuals are unable to work, making them work would only bring disutility without increasing consumption possibilities, so that it follows

[17] Note that, in a fully fledged model of migrations without any migration cost, in equilibrium the two systems should bring the same expected lifetime utility, and, hence, given the ex post egalitarian nature of the associated allocations, the same utility for the short-lived.

trivially that it is socially optimal to make them retire.[18] Thus, the addition of a period of very old age leads trivially to the result that the end of life should be spent as a retiree. Note, however, that this trivial result does not inform us on whether it is optimal to make individuals work in the second period of life and be retiree in the third period (standard retirement), or whether it is optimal to start with a period of retirement and leave labor only for the third period (reverse retirement). It only tells us that the reversion of retirement can only be *partial*, since the very old age is necessarily lived as being retired.

The problem faced by an *ex post* egalitarian social planner choosing between standard and (partial) reverse retirement becomes now:

$$\max_{c,d,b,K} \min \left\{ \begin{array}{l} \{u(c)-v+\tilde{u}(d)+\hat{u}(b), u(c)-v+\tilde{u}(d), u(c)-v\} \text{ (standard retirement)} \\ \{u(c)+\tilde{u}(d)-\tilde{v}+\hat{u}(b), u(c)+\tilde{u}(d)-\tilde{v}, u(c)\} \text{ (partial reverse retirement)} \end{array} \right\}$$

$$\text{s.t.} \quad \left\{ \begin{array}{l} F(K, aN) = Nc + \pi Nd + \pi pNb + K \text{ (standard retirement)} \\ F(K, b\pi N) = Nc + \pi Nd + \pi pNb + K \text{ (partial reverse retirement)} \end{array} \right\}$$

where $\hat{u}(b)$ denotes the utility of the individual at the very old age, while b is the consumption at very old age.

Solving the two subproblems yields:

$$U^{SL} = \left\{ \begin{array}{l} u\left(a\Lambda - \pi\tilde{c} - \pi p\hat{c}\right) - v \text{ under standard retirement} \\ u\left(b\pi\Lambda - \pi\tilde{u}^{-1}\left(\tilde{v}\right) - \pi p\hat{c}\right) \text{ under (partial) reverse retirement} \end{array} \right.$$

where $\hat{c}$ is such that $\hat{u}(\hat{c}) = 0$.

Whether standard retirement dominates or is dominated by (partial) reverse retirement still depends, as above, on the structural parameters of our economy, in terms of production and preferences. It should be stressed, however, that a new demographic parameter appears here: the (conditional) probability of survival to the very old age p.

Proposition 6 *The welfare gap between (partial) reverse retirement and standard retirement is decreasing in the (conditional) probability of survival to the very old age p.*

Proof. The welfare gap from shifting from standard to (partial) reverse retirement is:

$$gap \equiv u\left(\Lambda b\pi - \pi\tilde{u}^{-1}\left(\tilde{v}\right) - \pi p\hat{c}\right) - u\left(a\Lambda - \pi\tilde{c} - \pi p\hat{c}\right) + v$$

The effect of a rise in the proportion of very old persons on the gap is thus:

$$\frac{dgap}{dp} = \pi\hat{c}\left[-u'\left(\Lambda b\pi - \pi\tilde{u}^{-1}\left(\tilde{v}\right) - \pi p\hat{c}\right) + u'\left(a\Lambda - \pi\tilde{c} - \pi p\hat{c}\right)\right] < 0$$

[18] Indeed, denoting by $\hat{\ell}$ the labor of the very old, the production function is now:

$$F(K, a\ell + b\pi\tilde{\ell} + \xi\pi p\hat{\ell})$$

If $\xi = 0$, making the very old work would bring disutility $\hat{v}\hat{\ell}$ without any additional output, which cannot be socially optimal.

due to the concavity of $u(\cdot)$. ■

Proposition 6 states that a higher proportion of very old, inactive persons, reduces the welfare gain from shifting from traditional retirement towards reverse retirement. The underlying intuition is that adding a very old population unable to work puts even more pressure on consumption possibilities, which are reduced under reverse retirement.

Thus introducing the very old affects the comparison of standard retirement with reverse retirement, by making the latter less desirable. However, one should be cautious when interpreting Proposition 6. Clearly, the extent to which an increase in the proportion of the very old reduces the welfare gain from shifting towards reverse retirement depends on the concavity of $u(\cdot)$, and, hence, depends strongly on whether the economy is highly productive or not. In a less developed economy, consumption takes a low level, at which the function $u(\cdot)$ is quite steep, so that a change in the proportion of the very old will strongly reduce the welfare gain from shifting to reverse retirement. On the contrary, in an advanced economy, consumption takes a high level, so that the variation takes place in an interval of values for consumption for which $u(\cdot)$ is less steep, leading to a small variation in the welfare gain from shifting to reverse retirement.

6 Concluding remarks

Under unequal lifetimes, the standard retirement system, in which individuals work a long life before enjoying retirement, does not look fair, since it implies that some unlucky individuals work and die before enjoying retirement. But would the - purely hypothetical - reverse retirement system (in which individuals are first retiree and then work) be more fair to the unlucky short-lived?

Our analysis shows that the social desirability of reverse retirement depends on the underlying ethical foundations. Under the utilitarian social welfare function, reverse retirement cannot be a social optimum (unlike standard retirement). However, if one adopts the *ex post* egalitarian criterion (giving priority to the worst-off, who is, in general, the short-lived), then, it can be the case, under some conditions, that reverse retirement is a social optimum.

But even if one adopts the *ex post* egalitarian criterion, it is not necessarily the case that reverse retirement dominates standard retirement, that is, that it increases the lifetime well-being of the unlucky short-lived. Actually, in less developed countries (with a low life expectancy and a steep age-productivity profile), the standard retirement system dominates the reverse retirement system, even from the perspective of the well-being of the unlucky short-lived. On the contrary, in advanced economies, where production does not require high physical efforts (leading to a lower age productivity gap), and where survival conditions are favorable, reverse retirement dominates standard retirement. Therefore the *ex post* egalitarian argument supporting a shift from standard to reverse retirement holds only for sufficiently advanced economies.

We then examined the robustness of that argument to alternative settings, and we showed that limited learning by doing, moral hazard problems and migration of the old could, to some extent, weaken the welfare gain of shifting from standard to reverse retirement. Moreover, extending the framework to introduce dependent elderly individuals shows that the rise of long term care affect our results in two distinct ways: first, by reducing the desirability of shifting to reverse retirement; second, by making the reversion of retirement only partial (old workers going back to retirement once they become dependent).

It is important to stress here that the *ex post* egalitarian argument supporting a reverse retirement system is distinct from other possible arguments. A first alternative argument could be based on education and human capital accumulation. Actually, reverse retirement could stimulate investment in education, which would favor economic growth. Our argument differs from this, since it involves neither education choices, nor assumptions on the return of education. Another alternative argument would consist in claiming that reverse retirement would allow young people to work for pro-social NGOs in a benevolent way, in the same way as retirees give their time to NGOs nowadays. But this differs from our argument, which does not require any pro-social sector.

It is also important to distinguish reverse retirement from a basic income system (possibly limited to the young). Reverse retirement differs from basic income on at least two grounds. First, whereas the basic income is universal, and covers all ages, reverse retirement would only concern the young. Second, given that the purpose is, from an *ex post* egalitarian perspective, to favor the unlucky short-lived as much as possible, the young would, under reverse retirement, receive more than a basic allowance.

To conclude, it is worth underlining some limitations of the present study. First, this study considered only social planning problems faced by benevolent planners, without taking into account that, in real societies, what determines policies is political competition. Given that prematurely dead individuals do not vote, it is unlikely that a reform serving their interests could be voted. Thus political constraints are an important dimension of the picture that is not taken into account here. Second, this paper does not study the practical implementation of a reverse retirement system in an intergenerational context. Note, however, that it is generally the case that individuals, due to a too short time horizon, save too little with respect to what leads to the Golden Rule capital level. Thus the resolution of that underaccumulation problem involves taxing consumption to favor accumulation, whatever the retirement system in presence. Third, this paper does not examine the problems raised by the transition from standard retirement to reverse retirement. Although the transition could be slow (i.e. progressive postponement of entry in and exit from the labor market), the shift from standard to reverse retirement may face strong resistance among the population, which is used to regard retirement as a "reward" for labor (and thus that should come *after* labor) and not as something that should come *before* labor. Note, however, that, during the last centuries, there were already sizeable postponements of entry on the labor market (end of child labor in industrialized economies) as well as significant postponements of exit from the labor market. Thus reverse retirement may not be as utopian as it may look at first glance.

7 References

Arrow, K. (1962). The economic implications of learning-by-doing. *Review of Economic Studies*, 29 (3): 155-173.

Aubert, P., Crepon, B. (2007). Are older workers less productive ? Firm level evidence on age-productivity and age-wage profiles, mimeo.

Bentham, J. (1789). *An Introduction to the Principles of Morals and Legislation*. London.

Christensen, K., Johnson, T., Vaupel, J. (2006). The quest for genetic determinants of human longevity: challenges and insights. *Nature Review Genetics*, 7, 436-448.

Cremer, H., Pestieau, P. (2011). Myopia, redistribution and pensions. *European Economic Review*, 55(2): 165-175.

Crepon, B., Deniau N., Perez-Duarte, S. (2003). Wages, productivity and worker characteristics: a French perspective. CREST INSEE Working Papers.

Fleurbaey, M. (2008). *Fairness, Responsibility and Welfare*. Oxford University Press.

Fleurbaey, M., Leroux, M.L., Pestieau, P., Ponthiere, G. (2016). Fair retirement under risky lifetime. *International Economic Review*, 57 (1): 177-210.

Fleurbaey, M., Leroux, M.L., Ponthiere, G. (2014). Compensating the dead. *Journal of Mathematical Economics*, 51 (C): 28-41.

Fleurbaey, M., Maniquet, F. (2004). Compensation and responsibility. In: K. J. Arrow & A. K. Sen & K. Suzumura (ed.), *Handbook of Social Choice and Welfare*, edition 1, volume 2, chapter 22: 507-604.

Gobel, C., Zwick, T. (2009). Age and productivity – evidence from linked employer employee data, working paper.

Human Mortality Data Base. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org

Lavigne, A. (2013). *Economie des retraites*. La Découverte, Paris.

Leroux, M.L., Ponthiere, G. (2018). Working time regulation, unequal lifetimes and fairness. *Social Choice and Welfare*, forthcoming.

Mill, J.S. (1863). *Utilitarianism*. London.

Pestieau, P., Ponthiere, G. (2017). Optimal fertility under age-dependent labor productivity. *Journal of Population Economics*, 30 (2): 621-646.

Phelps, E. (1961). The golden rule of accumulation: a fable for growthmen. *American Economic Review*, 51 (4): 638-643.

Schokkaert, E., Devolder, P., Hindriks, J., Vandenbroucke, F (2017). Towards an equitable and sustainable points system. CORE Discussion Papers 2017-006.

Schokkaert, E., Van Parijs, P. (2003). Social justice and the reform of Europe's pension systems. *Journal of European Social Policy*, 13 (3): 245-63.

Smith, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations*. London.

8 Appendix

8.1 Proof of Proposition 1

The utilitarian social planning problem can be rewritten by means of the following Lagrangian:

$$\begin{aligned}\max_{c,d,\ell,\tilde{\ell},K} \quad & N\left[u\left(c\right)-v\ell+\pi\left[\tilde{u}(d)-\tilde{v}\tilde{\ell}\right]\right] \\ & +\lambda\left[\left[F\left(K,aN\ell+b\pi N\tilde{\ell}\right)-Nc-\pi Nd-K\right]\right] \\ & +\rho\ell+\varsigma(1-\ell)+\varphi\tilde{\ell}+\psi(1-\tilde{\ell})\end{aligned}$$

where $\lambda,\rho,\varsigma,\varphi,\psi$ are Lagrange multipliers.

First-order conditions (FOCs) are:

$$\begin{aligned}u'(c) &= \lambda \\ \tilde{u}'(d) &= \lambda \\ -Nv+\lambda F_L\left(K,aN\ell+b\pi N\tilde{\ell}\right)aN+\rho-\varsigma &= 0 \\ -\pi N\tilde{v}+\lambda F_L\left(K,aN\ell+b\pi N\tilde{\ell}\right)b\pi N+\varphi-\psi &= 0 \\ F_K\left(K,aN\ell+b\pi N\tilde{\ell}\right) &= 1\end{aligned}$$

as well as conditions

$$\begin{aligned}\rho &\geq 0,\ell\geq 0 \\ \varsigma &\geq 0,1-\ell\geq 0 \\ \varphi &\geq 0,\tilde{\ell}\geq 0 \\ \psi &\geq 0,1-\tilde{\ell}\geq 0\end{aligned}$$

with complementary slackness.

Under standard retirement ($\ell=1$, $\tilde{\ell}=0$), we have $\rho=0$ and $\varsigma\geq 0$ and $\varphi\geq 0$ and $\psi=0$ so that

$$\begin{aligned}Nv+\varsigma &= u'(c)F_L\left(K,aN\right)aN \\ \pi N\tilde{v}-\varphi &= u'(c)F_L\left(K,aN\right)b\pi N\end{aligned}$$

Those two conditions can be satisfied when

$$Nv+\varsigma=\left(\pi N\tilde{v}-\varphi\right)\frac{a}{b\pi}$$

which can arise.

Under reverse retirement ($\ell=0$, $\tilde{\ell}=1$), we have $\rho\geq 0$ and $\varsigma=0$ and $\varphi=0$ and $\psi\geq 0$ so that

$$\begin{aligned}Nv-\rho &= u'(c)F_L\left(K,b\pi N\right)aN \\ \pi N\tilde{v}+\psi &= u'(c)F_L\left(K,b\pi N\right)b\pi N\end{aligned}$$

Those two conditions are satisfied when

$$Nv - \rho = (\pi N\tilde{v} + \psi)\frac{a}{b\pi} = \left(N\tilde{v} + \frac{\psi}{\pi}\right)\frac{a}{b}$$

Given that $a > b$ and $v < \tilde{v}$, that condition is never satisfied. Thus reverse retirement cannot be a social optimum under the utilitarian social welfare criterion.

The *ex post* egalitarian social planning problem can be rewritten by means of the following Lagrangian:

$$\begin{aligned}
&\max_{c,d,\ell,\tilde{\ell},K} u(c) - v\ell \\
&+\lambda\left[F\left(K, aN\ell + b\pi N\tilde{\ell}\right) - Nc - \pi Nd - K\right] \\
&+\mu\left[\tilde{u}(d) - \tilde{v}\tilde{\ell}\right] + \rho\ell + \varsigma(1-\ell) + \varphi\tilde{\ell} + \psi(1-\tilde{\ell})
\end{aligned}$$

FOCs yield:

$$\begin{aligned}
u'(c) &= \lambda N \\
\mu\tilde{u}'(d) &= \lambda N\pi \\
v - \rho + \varsigma &= \lambda F_L\left(K, aN\ell + b\pi N\tilde{\ell}\right)aN \\
\tilde{v}\mu - \varphi + \psi &= \lambda F_L\left(K, aN\ell + b\pi N\tilde{\ell}\right)b\pi N \\
F_K\left(K, aN\ell + b\pi N\tilde{\ell}\right) &= 1
\end{aligned}$$

as well as conditions

$$\begin{aligned}
\mu &\geq 0, u(d) - \tilde{v}\tilde{\ell} \geq 0 \\
\rho &\geq 0, \ell \geq 0 \\
\varsigma &\geq 0, 1 - \ell \geq 0 \\
\varphi &\geq 0, \tilde{\ell} \geq 0 \\
\psi &\geq 0, 1 - \tilde{\ell} \geq 0
\end{aligned}$$

with complementary slackness.

Under standard retirement ($\ell = 1$, $\tilde{\ell} = 0$), we have $\rho = 0$ and $\varsigma \geq 0$ and $\varphi \geq 0$ and $\psi = 0$ so that

$$\begin{aligned}
v + \varsigma &= \lambda F_L(K, aN)\, aN \\
\tilde{v}\mu - \varphi &= \lambda F_L(K, aN)\, b\pi N
\end{aligned}$$

Those two conditions can be satisfied when

$$v + \varsigma = (\tilde{v}\mu - \varphi)\frac{a}{b\pi}$$

which can arise.

Under reverse retirement ($\ell = 0$, $\tilde{\ell} = 1$), we have $\rho \geq 0$ and $\varsigma = 0$ and $\varphi = 0$ and $\psi \geq 0$ so that

$$\begin{aligned} v - \rho &= u'(c) F_L(K, b\pi N) aN \\ \mu\tilde{v} + \psi &= u'(c) F_L(K, b\pi N) b\pi N \end{aligned}$$

Those two conditions are satisfied when

$$v - \rho = (\mu\tilde{v} + \psi) \frac{a}{b\pi}$$

Although $\frac{a}{b\pi} > 1$ and $\tilde{v} > v$, this equality can arise provided the shadow value of relaxing the egalitarian constraint μ is sufficiently low.