Rational expectations and stochastic systems

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Rational expectations and stochastic systems

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Abstract

TO BE (RE)WRITTEN AT THE END This paper proposes a stochastic model for describing rational expectations. The context is systemic risk, with interconnected components of a unified system. The evolution dynamics leading to the failure of the system is explored either under a theoretical point of view as well as through an extensive scenario analysis.

Keywords: Rational expectations, stochastic system, reliability, systemic risk, evolutionary economics, finance.

1 Introduction

Assessing the failure time of systems with interconnected components is a significant problem in reliability theory and leads to relevant questions in mathematical, statistics and probability modelling.

Many investigations have been carried out for the analysis of reliability systems, and many studies have been oriented to the prediction of the failure times under various assumptions.

There are two different approaches to dealing with the study of reliability systems: a probabilistic approach, analyzing the probability distribution of a system's failure time, and a Bayesian computational approach by estimating the average failure time of a system conditioned by the description of a scenario in which the evolution of the reliability system studied is observed.

Many contributions have appeared in the literature over the years in the context of the first approach in various contexts and applying different methods. Research work on particular reliability systems through the study of the reliability function.

Navarro et al. [1] deepened coherent systems with dependent components. Khaledi and Shaked [2] considered the residual life of systems with equal or different types of component showing a stochastic comparison. A stochastic comparison of systems with component lifetime sharing the same copula was analyzed by Navarro and Spizzichino [3], and subsequently the same type of research was carried out by Di Crescenzo and Pellerey [4] considering, however, systems with components linked via suitable mixtures.

Navarro et al. [5] get ordering properties for coherent systems taking into account the system reliability function as a distorted function of the common component reliability function. Navarro et al. [6] continuing their research on coherent systems with dependent or independent components, they demonstrated sufficient conditions on the components and lifetimes and on the number of components in order to improve the reliability of the whole system according to different stochastic orders.

Gupta et al. [7], always in the stochastic order-based approach, compared the residual lifetime and the inactivity time of failed components of coherent system with the lifetime of a system that has the same structure and the component lifetimes have the same dependence.

Azaron et al. [8] introduced a new approach to determine the reliability function of time-dependent systems with standby redundancy, assuming that not all elements of the system are set to function from the beginning.

Recently, Borgonovo et al. [9], in the context of modern digital system, have exploited the importance of system components in a computationally efficient way in system design proposing a measure for time-independent reliability analysis.

See, also Parsa et al. [10], for a new stochastic order based on the Gini-type index useful as a tool to gain information on the ageing properties of reliability systems thus demonstrating the different characteristics of active or already failed components.

A contribution concerning the mean time to failure and availability of semi-Markov missions is the one written by Çekyay and Özekici [11]. Other recent works, such as Zarezadeh et al. [12], have dealt with the investigation of the joint reliability of two coherent systems with shared components obtaining a pseudo-mixture representation for the joint distribution of the systems failure time.

In literature many researchers have studied K-out-of-N systems in which the failure process of each component depends on its operating environment conditions. See Da Costa et al. [13], who used a martingale approach to reliability theory; Eryilmaz [14] with the deepening of the concept of mean residual life as a fundamental characteristics that has been widely used in dynamic reliability analysis; Wang et al. [15], that considered the reliability estimation problem of weighted k-out-of-n multi-state systems; or Zhang et al. [16] who proposed a model to incorporate the observation information of the environment in the evaluation of the system performances.

Oe et al. [17] used autoregressive models for the prediction of the failure of a stochastic system through four types of performance index of the variations. They detect the failure of a cutting tool of a lathe and predict the width of flank wear.

For the Bayesian approaches to the reliability problem, many researchers have investigated system reliability in the operational research field and have developed a variety of methods in this regard.

Kim et al. [18] developed a method for predicting failures of a partially observable failing system that can be applied to a wide range of deteriorating stochastic systems with multivariate condition monitoring data.

Aktekin and Caglar [19] have studied a software reliability model considering modeling of a multiplicative failure rate whose components are evolving stochastically over testing stages and they discussed its Bayesian estimation.

Van Noortwijk et al. [20] determined a Bayesian failure model for hydraulic structures based on observable deterioration characteristics.

In the context of reliability theory we propose a stochastic model for evaluating the expected time of system failure under a rational expectations perspective.

In our study we also dealt with studying systems with dependent components, deepening the dependency relationships of the life times of the individual components as in the papers of Navarro et al. [1, 5, 6], Khaledi and Shaked [2], Navarro and Spizzichino [3], Gupta et al. [7], Azaron et al., [8] and Oe et al., [17].

However, our research work is part of the second literature group. We are not concerned with studying the reliability function but we use a Bayesian approach to estimate the average failure time of stochastic systems in a context of rational expectations.

There are many contributions that dealt with rational expectations, see for example Muth [24], Blanchard and Kahn [25], Hansen and Singleton [26], Blanchard and Watson [27], Atıcı et al. [28], Becker et al., [29].

Through the rational expectations we can achieved failure times prediction based on the knowledge and conditioning the result on the information that the system with interacting elements with unknown random lifetimes provides, thus obtaining a continuous gain on prediction performance.

We aim to demonstrate how the prediction of failure times improves with the enhance of the information collected, causing the failure of stochastic systems through the failure of the interconnected components.

We started from the difficulty of the methods for predicting the failure of reliability systems based on proper monitoring of the system that fails over time. The goal is the implementation of a procedure that can be useful for any system with interactive components. Examples of practical applications in economics and finance can be banking network, systemic risk of countries, systemic risk of Eurozone, systemic sovereign credit risk, and so on.

The evolution dynamics leading to the failure of the system is explored either under a theoretical point of view as well as through an extensive simulation analysis. A large number of scenarios are then built for simulating the failure of the system. In all the considered scenarios, we are able to identify the connections among the configurations and the lifetime of the system. In so doing, we derive a probability distribution for the random time of failure of the system conditioned to the configurations. Rational expectations are given, in this context, by the expected value of the time in which the system fails under the constraint of the realization of a given configuration.

The method we want to use is that proposed by Andersen and Sornette [21, 22] for the prediction of failure time of the overall system, conditioned on the information revealed by the damage occurred until the present time in which the system is being evaluated (the configurations of the theoretical setting). Their idea was inspired by the method of "reverse tracing of precursors" (RTP), see Keilis-Borol et al. [23], for the earthquake prediction based on seismicity patterns.

The reallocation rule works according to preference relations among the components. Once one of the components fails, its relevance is reallocated to the remaining active components.

The remaining part of the paper is organized as follows. In Section 2 the theoretical model for describing rational expectations is presented. In Section 3 a scenario simulation approach is implemented to validate the effectiveness of the proposed model in Section 2. Section 4 is devoted to results and discussion. We make our conclusions in the last section (Section 5).

2 The model

We consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ containing all the random quantities used throughout the paper. We denote the expected value operator related to the probability measure $\mathbb{P}$ by $\mathbb{E}$.
As said in the Introduction, the scope of the paper is to propose a model on the evolution of the risk of failure of an economic/financial system on the basis of rational expectations. The considered

system is a unified entity composed by individual interconnected components.

We denote the *system* by $\mathbf{S}$, and assume that it is composed by n *components* denoted by $C_1, \ldots, C_n$ and collected in a set $\mathcal{C}$.

The *state* of $\mathbf{S}$ is a binary quantity. If the system is *active* and works, then its state is 1. Otherwise, the state of $\mathbf{S}$ is 0, and the system is said to be *failed.* The state of $\mathbf{S}$ evolves in time, and we denote by $Y(t)$ the state of the system at time $t \geq 0$. At the beginning of the analysis (time $t = 0$) the system is naturally assumed to be in state 1.

Analogously, the state of the j-th component C_j at time t is denoted by $Y_j(t)$, and it takes value 1 when C_j is active and 0 when C_j is failed. At time $t = 0$ we have $Y_j(0) = 1$, for each $j = 1, \ldots, n$.

The value of $Y(t)$ depends on the states of the components of the system at time t. The way in which such a dependence is conceptualized is grounded also on the arguments of the next Section.

2.1 Main assumptions on the system

We now point out three natural assumptions of our model, which are tailored on the empirical evidence on economic/financial systemic risk: first, the different components of the system are assumed to be not homogeneous in terms of their relevance; second, the components of the system are interconnected and exhibit different levels of interconnection; third, relevance and interconnection levels change over time, according to the change of the status of the components of the system.

We enter the details.

For each $j = 1, \ldots, n$ and $t \geq 0$, the *relative importance of the component* C_j over the entire system at time t is measured through $\alpha_j(t)$, where $\alpha_j : [0, +\infty) \to [0, 1]$ and $\sum_{j=1}^{n} \alpha_j(t) = 1$, for each t.

For each $t \geq 0$, we collect the $\alpha(t)$'s in a time-varying vector $\mathbf{a}(t) = (\alpha_j(t))_j$, where

$$\mathbf{a} : [0, +\infty) \to [0, 1]^n \qquad \text{such that} \qquad \mathrm{t} \mapsto \mathbf{a}(\mathrm{t}). \tag{1}$$

If a component is not active at time t, then its relevance for the system is null. Moreover, each active component has positive relative relevance, i.e. the system does not contain irrelevant active components. Formally,

$$\alpha_j(t) = 0 \Leftrightarrow Y_j(t) = 0. \tag{2}$$

Condition (2) is useful, in that it allows to describe the status of the components of the system directly through the α's.

For each $j = 1, \ldots, n$, the relative relevance of C_j changes in correspondence of the variation of the state of one the components of the system. Once a component fails, then it disappears from the economic/financial system – i.e., its relative relevance becomes null – and the relative relevances of the components of the remaining active ones are modified on the basis of a suitably defined *reallocation rule.*

Next example proposes a way to build a reallocation rule.

Example 1. *Consider a system* $\mathbf{S}$ *whose components set is* $\mathcal{C} = \{C_1, C_2, C_3, C_4, C_5\}$*.*
Assume that, at time $t = 0$*, we have* $\alpha_1(0) = 0.1$*,* $\alpha_2(0) = 0.15$*,* $\alpha_3(0) = 0.3$*,* $\alpha_4(0) = 0.2$*,* $\alpha_5(0) = 0.25$*.*

Now, suppose that the first failure of one of the components of the system occurs at time $t = 7$*, when* C_3 *fails. Of course,* $\alpha_j(t) = \alpha_j(0)$*, for each* $t \in [0, 7)$ *and* $j = 1, 2, 3, 4, 5$*. Moreover,* $\alpha_3(7) = 0$*.*

We consider a specific reallocation rule, which states that the relevance is reallocated over the remaining active components proportionally to their α*'s before the failure. This means that*

$$\alpha_1(7) = \frac{0.1}{0.1 + 0.15 + 0.2 + 0.25}, \ \alpha_2(7) = \frac{0.15}{0.1 + 0.15 + 0.2 + 0.25},$$

$$\alpha_3(7) = 0, \ \alpha_4(7) = \frac{0.2}{0.1 + 0.15 + 0.2 + 0.25}, \ \alpha_5(7) = \frac{0.25}{0.1 + 0.15 + 0.2 + 0.25}.$$

In general, if τ_1, τ_2 *are the dates of two consecutive failures, with* $\tau_1 < \tau_2$*, we have*

$$\alpha_j(\tau_2) = \frac{\alpha_j(\tau_1)\mathbf{1}_{\{Y_j(\tau_2)=1\}}}{\sum_{i=1}^{5} \alpha_i(\tau_1)\mathbf{1}_{\{Y_i(\tau_2)=1\}}}, \qquad j = 1, 2, 3, 4, 5.$$

The α*'s are step functions, whose jumps occur in correspondence to the failure of one of the components.*

For what concerns the *interconnections among the components*, we define their time varying relative levels through functions of type $w_{ij} : [0, +\infty) \to [0, 1]$, for each $i, j = 1, \ldots, n$, so that $w_{ij}(t)$ is the relative level of the interconnection between C_i and C_j at time $t \geq 0$. We assume that arcs are oriented, so that in general $w_{ij}(t) \neq w_{ji}(t)$, for each t. Moreover, by construction, $\sum_{i,j=1}^{n} w_{ij}(t) = 1$, for each t. We also assume that self-connections do not exist in our framework, i.e. $w_{ii}(t) = 0$, for each i and t.

For each $t \geq 0$, the $w(t)$'s are collected in a time-varying vector $\mathbf{w}(t) = (w_{ij}(t))_{i,j}$, with

$$\mathbf{w} : [0, +\infty) \to [0, 1]^{n \times n} \qquad \text{such that} \qquad \mathrm{t} \mapsto \mathbf{w}(\mathrm{t}). \tag{3}$$

If C_i is a not active component at time t, then $w_{ij}(t) = w_{ji}(t) = 0$, for each $j = 1, \ldots, n$. This condition simply formalizes that a failed component is disconnected from the system. Such a statement suggests that the failure of a component might generate disconnections among the components of the system.

The behavior of the w's is analogous to that of the α's. Also in this case, the relative levels of interconnections change when one of the components of $\mathbf{S}$ change its state, and there is a reallocation rule for the remaining levels of interconnections.

We synthesize the reallocation rules of the weights on nodes and arcs broadly by $\mathcal{R}$.

Therefore, a natural rewriting of the system $\mathbf{S}$ with components in $\mathcal{C}$ and reallocation rule $\mathcal{R}$ is then

$$\mathbf{S} = \{\mathbf{a}, \mathbf{w}\}. \tag{4}$$

Notice that (4) highlights the observable features of the system with a given set of components and a specific reallocation rule, i.e. the weights on the nodes and on the arcs. Thus, according to (4), we can say that the $\{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}$ is an *observation* of the system, where $\bar{\mathbf{a}} \in [0,1]^n$ and $\bar{\mathbf{w}} \in [0,1]^{n \times n}$.

2.2 The structure of the system

To capture the dependence of the state of $\mathbf{S}$ on the ones of its components, we simply introduce a function $\phi : \{0,1\}^n \to \{0,1\}$

$$Y(t) = \phi(Y_1(t), \dots, Y_n(t)). \tag{5}$$

In reliability theory, ϕ is usually denoted as the *structure function* of the system.

We denote the elements of $\{0,1\}^n$ as configurations of the states of the components of the system or, briefly, *configurations*.

Function ϕ in (5) has the role of clustering the set of configurations in two subsets: the ones leading to the failure (F) of the system and those associated to the not failed (NF) system. Thus, we say that $K_F \subseteq \{0,1\}^n$ is the collection of configurations such that $\phi(x_F) = 0$, for each $x_F \in K_F$ while $K_{NF} \subseteq \{0,1\}^n$ is the collection of configurations such that $\phi(x_{NF}) = 1$, for each $x_{NF} \in K_{NF}$. By definition, $\{K_F, K_{NF}\}$ is a partition of $\{0,1\}^n$.

In order to describe a systemic risk problem, some requirements on ϕ are needed.

First, $(0, \dots, 0) \in K_F$ and $(1, \dots, 1) \in K_{NF}$. This condition means that when all the components of the system are active (not active), then the system is active (not active) as well.

Second, ϕ is non-decreasing with respect to its components. This has an intuitive explanation: the failure of one of the components of the system might worsen the state of the system and cannot improve it.

Third, each component is able to determine the failure of the system. Formally, this condition states that for each $j = 1, \dots, n$ there exists $(y_1, \dots, y_{j-1}, y_{j+1}, \dots, y_n) \in \{0,1\}^{n-1}$ such that $(y_1, \dots, y_{j-1}, 1, y_{j+1}, \dots, y_n) \in K_{NF}$ and $(y_1, \dots, y_{j-1}, 0, y_{j+1}, \dots, y_n) \in K_F$.

2.3 Failure of the system and rational expectations

As said above, time $t = 0$ represents *today* – the starting point of the observation of the evolution of the system –. At time $t = 0$ all the components are active and the system works.

The failure of the system is then a random event, which occurs when the system achieves one of the configurations belonging to K_F.

We define the *system lifetime* as:

$$\mathcal{T} := \inf\{t \geq 0 | \phi(Y_1(t), \dots, Y_n(t)) = 0\}. \tag{6}$$

Analogously, the n-dimensional vector of *components lifetimes* is $\mathbf{X} = (X_1, \ldots, X_n)$, where

$$X_j = \inf\{t > 0 \,|\, Y_j(t) = 0\}. \tag{7}$$

We assume that components exhibit also a "social" behavior in failing. Each X_j is composed by two terms: one of them is idiosyncratic, and captures aspects related to the inner life of C_j; the other one depends on the failures of the other components of the system. We denote the former term by X_j^I and the latter one by X_j^S and write

$$X_j = \min\{X_j^I, X_j^S\}. \tag{8}$$

The quantity X_j^S depends naturally on $\mathbf{a}$ and $\mathbf{w}$, while $\{X_1^I, \ldots, X_n^I\}$ is a set of independent random variables. In general, $\{X_1, \ldots, X_n\}$ are not independent and do not share the same distribution.

We can reasonably assume that the failure of the system coincides with the failure of one its components. However, simultaneous failures of components may occur due to the presence of the terms X^S's in (8).

At each components' failure, the α's and the w's modify and reallocate over the remaining components. Such variations explains the way in which $\mathbf{S}$ fails in accord to the failures of its components.

To fix ideas, we provide an example.

Example 2. *Assume that $\mathcal{C} = \{C_1, C_2, C_3, C_4, C_5\}$ and*

$$\mathbf{a}(0) = (0.1, 0.5, 0.2, 0.1, 0.1), \qquad \mathbf{w}(0) = \begin{pmatrix} 0 & 0 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0.1 & 0.05 \\ 0.1 & 0 & 0.1 & 0 & 0.05 \\ 0 & 0 & 0.05 & 0.05 & 0 \end{pmatrix}$$

Suppose that the reallocation rules $\mathcal{R}$ for relative relevance and interconnection levels are of proportional type, as in Example **??**. *Such reallocations are implemented if the system is not failed.*

Furthermore, suppose that if a given component fails, then the components connected only to it fail as well, independently from their level of interconnections. This law drives the definition of the X^S's.

The idiosyncratic term X^I's are assumed to be driven by a Poisson Process with parameter λ – giving the timing of the failures – jointly with a uniform process over $\mathcal{C}$, independent on the Poisson Process – which identifies the failed component.

Moreover, suppose that the system fails at the first time in which components with aggregated relative relevance greater than 0.4 fail.

Now, suppose that the first failure is observed at time $t = 8$, when C_2 fails. Then, automatically, C_3 fails as well, since it is connected only to C_2. The aggregate relative relevance before the failures is $\alpha_2(8^-) + \alpha_3(8^-) = 0.5 + 0.2 > 0.4$, and the system fails.

A rational expectations approach is used for the computation of the expectation of the random time in which the system fails. Specifically, we will compute the expected value of $\mathcal{T}$ conditioned to the specific values of the weights $\mathbf{a}$ and $\mathbf{w}$.

We denote by RE the rational expectations of the time $\mathcal{T}$ given all the possible observations of the system. Specifically,

$$RE = \left\{ \mathbb{E}\left[\mathcal{T} \,|\, \{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}\right] \,:\, \bar{\mathbf{a}} \in [0,1]^n,\, \bar{\mathbf{w}} \in [0,1]^{n \times n} \right\}. \tag{9}$$

Formula (9) provides the expected value of the lifetime of $\mathbf{S}$ in correspondence of any observation of the system.

3 Scenario simulations

In this section we will adopt a scenario simulations approach for reproducing synthetically a system, with the final aim of writing formula (9) on the basis of the obtained observations $\{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}$.

We set $\mathbb{E}\left[\mathcal{T} \,|\, \{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}\right] = +\infty$ when the observation $\{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}$ does not appear in the set of the simulated observations.

The goal of these extensive scenario simulations is the validation of our stochastic theoretical model proposed to describe rational expectations in a context of systemic risk.

3.1 Overview of the scenario analysis

Our goal is to demonstrate that, through rational expectations, we can achieve a better prediction of the average failure times of a system of dependent components compared to the ex-post average failure times.

To determine a relationship between the avarage errors benchmark (E_B) and the avarage errors in rational expectations prediction ($E_{RE\_average}$), extensive numerical simulations were performed.

We want to know how the rational expectations of the simulated systems change according to two different characteristics of the configurations, the variability and the shape of the distribution, taking into account one aspect of the variability and two aspects of the shape, specifically: (i) the variance of the configurations, to evaluate the variability of the configurations and therefore the dispersion of the weights; (ii) the skewness of the configurations, that measure the lack of symmetry of the weights; (iii) the kurtosis of the configurations, to study if the weights are heavy-tailed or light-tailed relative to a normal distribution.

The simulation procedure will be divided for convenience into four parts.

1. We build a *catalog* of systems (namely $\mathbf{S}^C$'s) and we follow them up to their failure by collecting the relationship that exists between $\{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}$ and $\mathcal{T}$ of the individual systems in this catalog. In this way we are able to construct a synthetic experience on the failure of the system that will be exploited in the Bayesian estimate of the $\mathbb{E}[\mathcal{T}]$ given $\{\bar{\mathbf{a}}, \bar{\mathbf{w}}\}$ that will be illustrated later, in a context of rational expectations.

   $K\_simulations$ is the number of simulations that will be recorded in the catalog ($\mathbf{S}^C$'s). The generic simulated "catalog" system will be denoted by $\mathbf{S}^C_k$, with $k = 1, \ldots, K\_simulations$. Each $((\mathbf{S}^C)_k)$ is composed by a number n of components C_j (also called nodes).

   The allocation of the weights of C_j (α's) at each time (t) is called $configuration$ and it is contained in a vector $\mathbf{a}(t)$ such that $\alpha_j(t)$ is the weight of C_j at time t.

   $\mathcal{T}_k$ is the failure time of $\mathbf{S}^C_k$.

   For each $\mathbf{S}^C_k$ we will record:

   - The α's of each $configuration$ in a 3D matrix $\mathbf{Cat}_\alpha$ that depend on k, t and n (all the weights of the $\mathbf{S}^C$'s);
   - Variance, skewness or kurtosis (depending on the simulated scenario analyzed) calculated on each $configuration$ in a matrix ($\mathbf{Cat_{var}}$, $\mathbf{Cat_{skew}}$, $\mathbf{Cat_{kurt}}$) that depend on k and t. The variance, the skewness and the kurtosis are sets collecting such observed statistical indicators;
   - The failure time $\mathcal{T}_k$ of $\mathbf{S}^C_k$ in a set $\mathcal{T}_{catalog}$ (that containing all the failure times of the $\mathbf{S}^C$'s)

   It is necessary to normalize failure times contained in $\mathcal{T}_{catalog}$, so that comparisons and analysis can be made for the different scenarios. We obtain $\mathcal{T}_{catalog\_norm}$ that depend linearly on $\mathcal{T}_{catalog}$ and with norm 1. Therefore the new time scale is a normalized time scale that marks failure times with respect to 1.

2. At this point, having a catalog available, following the same procedure, we simulate a new series of systems that we will follow until failure, which we will call *"in vivo"* systems (namely $\mathbf{S}^V$'s). These new systems will be necessary for the calculation of rational expectations.

   $X\_simulations$ is the number of simulations for the computation of rational expectations ($\mathbf{S}^V$'s). The generic simulated *"in vivo"* system will be denoted by $\mathbf{S}^V_x$, with $x = 1, \ldots, X\_simulations$.

   For each $\mathbf{S}^V_x$ we will record:

   - Variance, skewness or kurtosis (depending on the simulated scenario analyzed) calculated on each $configuration$ in a matrix ($\mathbf{Cat^{Var}}$, $\mathbf{Cat^{Skew}}$, $\mathbf{Cat^{Kurt}}$) that depend on x and t;

- The failure time $\mathcal{T}_x$ of $\mathbf{S}_x^V$ in a set $\mathcal{T}_{invivo}$ (that containing all the failure times of the $\mathbf{S}^V$'s).

Considering the normalization performed on $\mathcal{T}_{catalog}$ and the identified new time interval, we create $\mathcal{T}_{invivo\_norm}$ that depend linearly on $\mathcal{T}_{invivo}$ and with norm 1. The standardization was carried out based on the failure times recorded in the catalog $\mathcal{T}_{catalog}$. Simulations in which the maximum failure time of the $\mathbf{S}^V$ is greater than the maximum failure time of the $\mathbf{S}^C$ will therefore be excluded.

3. Based on the analyzes carried out for the $\mathbf{S}^C$'s and the values found for the $\mathbf{S}^V$'s, we try to predict the average system failure time by working on variance, skewness and kurtosis. In this phase we compute RE .

   We have two sets of simulations to compare that depend on the weights of the configurations, namely the α's, (on which we calculate variance, skewness and kurotsis), and on the time in which we observe these pairs of systems. Therefore the dynamics of the weights that are associated with each time t becomes fundamental. This connection is made by setting a *Condition* through a threshold (see the third building block of the subparagraph 3.3 and here we introduce the *level of tolerance* depending on the characteristics of the configurations considered:

   - $Tolerance_{var}$ is the threshold of the variance that will allow us to select each $\mathcal{T}_k \in \mathcal{T}_{catalog}$ that satisfies the *Condition* useful for the calculation of rational expectations RE;
   - $Tolerance_{skew}$ is the threshold of the skewness that will allow us to select each $\mathcal{T}_k \in \mathcal{T}_{catalog}$ that satisfies the *Condition* useful for the calculation of rational expectations RE;
   - $Tolerance_{kurt}$ is the threshold of the kurtosis that will allow us to select each $\mathcal{T}_k \in \mathcal{T}_{catalog}$ that satisfies the *Condition* useful for the calculation of rational expectations RE.

4. The last part of the procedure concerns the quantification of the errors of the prediction. We create the avarage errors benchmark E_B. E_B is the mean of the errors between the mean of the failure times of the $\mathbf{S}^C$'s ($\mathcal{T}_{catalog}$) and the failure times of the $\mathbf{S}^V$'s ($\mathcal{T}_{invivo}$). It is constant over time and represents the real failure times without conditionings and without information on the past history of the systems. The benchmark will allow us to compare the errors obtained in the calculation of failure times through the rational expectations approach. In this way we can really understand if the prediction is better, and how it improves over time.

   Then we calculate the avarage errors in rational expectations prediction $E_{RE\_average}$. $E_{RE\_average}$ is the mean error between our rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s. $E_{RE\_average}$ takes into account all the information recorded in the catalog over time.

We need to construct statistics over predictions in order to assess their quality. We decide to focus our attention on fixed percentiles at level p of the distributions of the errors: $p = 10\%$ and $p = 90\%$, i.e. the 10^{th} and the 90^{th} percentile. Let us consider errors in ascending order, so that the 10^{th} percentile represents the smallest errors in our forecast of failure times. On the contrary instead the 90^{th} percentile identifies the biggest errors made in predicting the failure times of stochastic systems. E_{RE_10} and E_{RE_90} are, respectively, the 10^{th} and the 90^{th} of the distribution of the errors between rational expectations predictions and the real failure times contained in $\mathcal{T}_{invivo}$.

3.2 Constitutive assumptions of the reliability systems

We introduce some assumptions that have been included in the implementation of the theoretical model. These may subsequently be modified or removed for future research and implementations. These hypotheses imply a simplification in the application of the model that allow us to realize the simulations avoiding computational complications.

In order to specify the assumptions, consider some variables, not yet introduced, that will be useful in the formalization of the assumptions.

- $n\_failure$ is the number of nodes necessary for the failure of $\mathbf{S}$;
- *n_broken* is the counter of failed nodes;
- r is a random number used to verify the failure condition of each node C_j.

These assumptions are valid both for the failure of the systems recorded in the *catalog*, $\mathbf{S}^C$'s, and for the *"in vivo"*, $\mathbf{S}^V$'s.

Firstly, we define the failure of the system components which is a random event during the lifetime of the system. At any time the failure condition is verified on a randomly extracted node C_j.We check the failure of each component until the failure system rule occurs.

If the weight of the component C_j at time t, $\alpha_j(t)$, is different from 0, C_j has not yet failed. We can therefore verify if $C_j(t)$ will fail. Considering the variable r, which also depends on a random extraction, $\alpha_j(t) = 0$ if $\alpha_j(t) > r$. At each t the failure of C_j is verified and *n_broken* is updated with the new number of failed components. If no node fails the counter remains unchanged.

As a consequence of this assumption, as the weight of the components increases (see reallocation rule), the probability of failure of C_j with a higher α_j value increases.

Secondly, we propose in this first implementation of the model a specific *proportional reallocation rule*, so that the relevance of the failed component is reallocated over the remaining active components proportionally to their α's before the failure.

In reference to the example 1 (see 2.1), considering a **S** composed by *n components*, we suppose that $C_j(t)$ fails, so that $\alpha_j(t) = 0$, and $\alpha_j(t-1)$ will be redistributed among the nodes still active in addition to the $\alpha(t)$'s, proportionally to the α(t-1)'s, remembering that for each $j = 1, \ldots, n$ and $t \geq 0$, $\sum_{j=1}^{n} \alpha_j(t) = 1$, for each t.

Finally, we also consider as a definition of the system failure rule the number of failed components necessary for the system collapse. The failure of **S** will occur when $n\_failure > n\_broken$.By setting the number of failed components ($n\_failure$) necessary for system failure, we will be able to determine the system $\mathcal{T}$.

3.3 Simulation procedure

To facilitate the reader it is now necessary to introduce a general notation which will then be applied depending on the analysis being considered among the 3 performed (variance, skewness and kurtosis).

We define

$$* = \textit{variance+skewness+kurtosis}$$

From now on in the simulation procedure, the label $*$ will be referred to variance, skewness and kurtosis depending on the case analyzed and and the statistical indicator used in the study.

Taking into account the notation mentioned up to this point, we present the following variables:

- $Tolerance_* = Tolerance_{var} + Tolerance_{skew} + Tolerance_{kurt}$;
- $\mathbf{Cat}_* = \mathbf{Cat_{var}} + \mathbf{Cat_{skew}} + \mathbf{Cat_{kurt}}$;
- $\mathbf{Cat}^* = \mathbf{Cat^{Var}} + \mathbf{Cat^{Skew}} + \mathbf{Cat^{Kurt}}$;

For all the other definitions we refer the reader to the subparagraphs 3.1 and 3.2.

The simulation procedure can be divided in four building blocks.

The first building block concerns the creation of the catalog of the simulated systems ($\mathbf{S}^C$'s). We find here the simulation of the failure of a system that will be iterated for a significant number of times up to a maximum of 10^4 realizations to obtain a good estimate of the expectation of the average failure times that will be our prediction tool.

The general iterative method used for creating catalogs is the following($\mathbf{S}^C$'s):

- set $K\_simulations$;
- set $Tolerance_*$;
- for $k = 1, \ldots, K\_simulations$;
  - set n;
  - set $n\_failure$;

- set $t = 0$;
- generate the initial allocation of the weights of C_j (α's) from a sampling exercise from a given distribution with values in (0,1);
- without losing of generality, normalize the α's, so that $\sum_{j=1}^{n} \alpha_j(t) = 1, \forall t$;
- set $n\_broken = 0$;
- while $n\_broken \leq n\_failure$:
  * extract the node j randomly;
  * if $\alpha_j(t) \neq 0$:
    · set $t = t + 1$;
    · calculate the statistical indicator $*$ of $\mathbf{a}(t)$;
    · record the value of $*$ in $\mathbf{Cat}_*$:
    · record the α's in $\mathbf{Cat}_\alpha$:
    · extract r from a given distribution with values in (0,1);
    · if $\alpha_j(t) > r$, C_j fails:
    $n\_broken = n\_broken + 1$;
    $\alpha_j(t) = 0$;
    set the proportional reallocation rule;
- record the failure time in $\mathcal{T}_{catalog}$.

  Each element of $\mathcal{T}_{catalog}$ is associated to a level of statistical indicator in $\mathbf{Cat}_*$: so that the failure time $\mathcal{T}_k$ of $\mathbf{S}_k^C$ is connected to each value of $\mathbf{Cat}_*(k, t)$ of $\mathbf{S}_k^C$. This step is crucial to understand how rational expectations conditioned by variance will be calculated (see the fourth building block).
- without losing of generality, normalize the failure times contained in $\mathcal{T}_{catalog}$ and create $\mathcal{T}_{catalog\_norm}$.

This procedure is replicated $K\_simulations$ times.

The second building block is related to the simulation of the failure of new systems $\mathbf{S}^V$'s.

Replicating the procedure desbribed above (for $x = 1, \dots, X\_simulations$), we therefore simulate the failure of new systems with the aim of obtaining new failure times to find the benchmark that will allow us to understand if it is possible to achieve a good prediction of the failure times of stochastic systems.

- set $X\_simulations$:
- replicate the simulation procedure explained in the first building block for $x = 1, \dots, X\_simulations$;

- the result of this second block will be $\mathbf{Cat}^*$, $\mathcal{T}_{invivo}$ and $\mathcal{T}_{invivo\_norm}$.

The third building block consists in the computation of the RE. We calculate the expected value of the lifetime of the system conditioned to the statistical indicator $*$ of $\mathbf{a}(t)$.

1. Check the tolerance threshold condition.

   We use the $\mathbf{S}^C$'s for the prediction of failure times of the $\mathbf{S}^V$'s, applying a condition considering a tolerance threshold for rational expectations. Specifically, we look at the generic variance level $\bar{*} \in \mathbf{Cat}_*$ such that the following *Condition* holds

$$|\mathbf{Cat}^*(x,t) - \bar{*}| < Tolerance_* \tag{10}$$

2. Take all catalog failure times (from $\mathcal{T}_{catalog}$) of the systems associated to $\bar{*}$, i.e. satisfying $Condition$.

3. Compute RE for failure times of $\mathbf{S}^V$'s through the mean of the failure times of the previous point for each t (see formula (9)).

We have now our prediction obtained in correspondence with the analysis of the variance of the configurations.

The fourth building block consists of the following steps:

1. Create the avarage errors benchmark E_B.

$$E_B = \frac{\sum_{x=1}^{x\_simulations} |\frac{\sum_{k=1}^{k\_simulations} tc_k}{k\_simulations} - Tc_x|}{x\_simulations} \tag{11}$$

2. Create the avarage errors in rational expectations prediction $E_{RE\_average}$.

$$E_{RE\_average} = \tag{12}$$

3. Create the distribution of the percentiles $p = 10\%$ and $p = 90\%$ of the errors in rational expectations prediction. In this way, we have the possibility of quantifying the impact of the error prediction comparing $E_{RE\_average}$ with E_{RE_10} and E_{RE_90}.

4. To understand how rational expectations make it possible to obtain a prediction of failure times using the information stored in the past, it is necessary to analyze the comparison between E_B, $E_{RE\_average}$, E_{RE_10} and E_{RE_90}.

The analysis of several scenario is provided.

3.4 Parameter set

XXXX I HAVE TO INSERT THIS WHOLE SECTION IN A TABLE XXXX

The parameters are set as follow:

- set $n = 10$;
- $n\_failure = \frac{n}{2}$;
- α's are generated from uniform distribution in (0,1) type;
- r is generated from a uniform distribution in (0,1) type;
- The value assigned to *tolerance level* and to the *number of simulations* will depend on the scenario we want to simulate.

We evaluated three different $Tolerance_*$: (i) 0.005; (ii) 0.05; (iii) 0.5.

We also considered two cases taking into account the *number of simulations*: (i) 1000 $K\_simulations$ and 1000 $X\_simulations$; (ii) 10000 $K\_simulations$ and 1000 $X\_simulations$.

We therefore carried out three types of analysis according to three different characteristics of the *alpha*'s distribution: variance, skewness and kurtosis.

We can therefore hypothesize nine different scenarios:

1. $Tolerance_{var} = 0.005$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.
2. $Tolerance_{var} = 0.05$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.
3. $Tolerance_{var} = 0.5$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.
4. $Tolerance_{kurt} = 0.005$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.
5. $Tolerance_{kurt} = 0.05$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.
6. $Tolerance_{kurt} = 0.5$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.
7. $Tolerance_{skew} = 0.005$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.

8. $Tolerance_{skew} = 0.05$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.

9. $Tolerance_{skew} = 0.5$ once with 1000 $K\_simulations$ and 1000 $X\_simulations$ and a second time considering 10000 $K\_simulations$ and 1000 $X\_simulations$.

The following paragraph will show the graphs related to the study of variance. skewness and kurtosis according to the scenario we are simulating.

4 Results and discussion

Here we want to demonstrate how, by conditioning the failure times of simulated stochastic systems to the memory of past events and information stored over time, an improvement in prediction performance is achieved.

INSERT FIGURE 1 AND 2 ABOUT HERE

Figure 1 and figure 2, show the prediction errors (in absolute value) in scenario 1. Red stars line is the $E_{RE\_average}$. Green triangles and blue crosses lines are respectively E_{RE_90} and E_{RE_10}. These prediction errors should be compared with magenta line which represents E_B.

Increasing the number of $k\_simulations$ to 10000, is clear an improvement in the accuracy of the prediciton of failure time and a decrease in all the absolute errors observed at the beginning of the simulations.

The benchmark and the avarage errors start from the same starting point considering that, at $t = 0$ no system has yet failed and we still have no information.

Moreover, we can note a different trend of the E_{RE_90} compared to the $E_{RE\_average}$ and the E_{RE_10}. In fact, we can see that the line of the biggest errors decreases faster toward zero, while in the E_{RE_10} we can see an initial flat behaviour that depends on the fact that when we have little information available, in a big error the gain is more evident than what we will see in a small error. Therefore the initial trend is dominated by the case.

INSERT FIGURE 3 AND 4 ABOUT HERE

INSERT FIGURE 5 AND 6 ABOUT HERE

Extending the discussion to tolerance levels, and how the results change as the variance changes, we can appreciate that when we rise the variance tolerance level, we observe always decreasind behaviour, but more scattered and biasis.

So the error trend is less and less obviously linear, because the incresed tolerance means that we are collecting less experience on the system inself and we are clustering together things that are so more different. So, it can be that you obtain some distorsion from this very good decrease of worst errors to zero.

We can appreciate visually through error analysis the improvement in the accuracy of the predictions over time up to the attainment of prediction errors that tend to zero.

XXXXX OTHER CONSIDERATIONS AND COMMENTS? XXXXX

INSERT FIGURE 7-12 ABOUT HERE

For the anaysis of the kurtosis the behaviour is quite similar.

We can see the gain in the accuracy of the failure times prediction with the increase of the simulations and the decrease of the average errors (as for the percentiles).

There is always a slightly different initial trend between the E_{RE_90} line and the E_{RE_10}. The starting flat behavior of the smallest errors is more lasting and shows how at the beginning, in the absence of information, any improvement is very random and the use of rational expectations is not as effective as a prediction tool.

Also regarding the rise in the tolerance level of kurtosis we can note an increasingly less linear trend and a greater probability of the occurrence of distortions in our analysis.

INSERT FIGURE 13-18 ABOUT HERE

XXXXX NO DISCUSSION RESULTS ON THE SKEWNESS. PRACTICALLY THE TENTH PERCENTILE IS ALWAYS ZERO. AND THIS THING SEEMS VERY STRANGE. XXXXX

5 Conclusions

By using rational expectations and therefore exploiting the information obtained over time and conditioning results to the knowledge we have cataloged in the past, it is possible to obtain a prediction of failure times of stochastic systems which improves with the increase in recorded information and with the time that tends to the time of system failure.

In fact, we have succeeded in demonstrating how the line of average errors obtained by comparing rational expectations for the prediction of the average failure times of stochastic systems with the real failure times of these systems (and consequently the lines of the 90th and 10th percentile examined) tends to zero with a gain in prediction over time.

So, by increasing the use of stored information, a decline in the uncertainty of when the system will fail is obtained.

The graphs confirm the significant gain in the prediction accuracy taking into account indicators such as variance, kurtosis and skewness that synthetize the systems.

In conclusion we have proposed a model for describing RE in a context of systemic risk, and we have validated the effectiveness of our model through extensive scenario simulations.

In future papers will be to develop further implementations of the model by increasing the complexity of the simulation procedure considering a distance measure to verify which combination of parameters will allow us to get the lowest error; considering instead of the Uniform distribution in (0,1) type, the Beta distribution for its properties, when the parameters alpha and beta change, to describe different situations with a only one functional form; considering, in addition to variance, kurtosis and skweness, the Gini Index for future analysis; replacing the proportional reallocation rule with other rules: for example a uniform reallocation rule or a reallocation rule threshold based.

It is also our intention to apply the simulation procedure to real data to model any system with interconnected components. Numerical experiments in the economic-financial field will follow.

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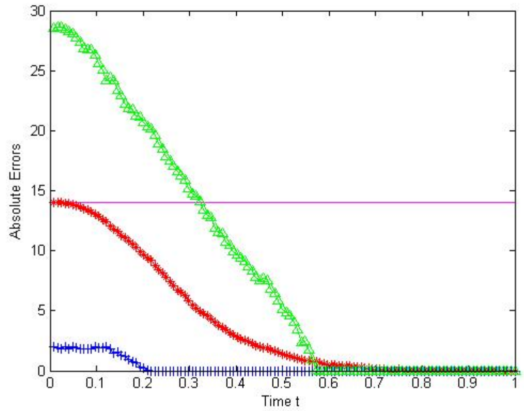

Figure 1: Prediction errors (in absolute value) in scenario 1 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{var} = 0,005$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

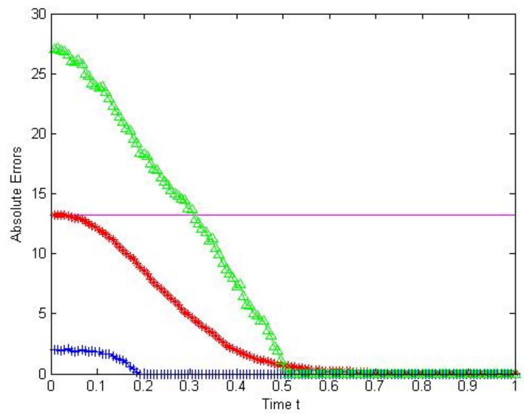

Figure 2: Prediction errors (in absolute value) in scenario 1 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{var} = 0,005$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

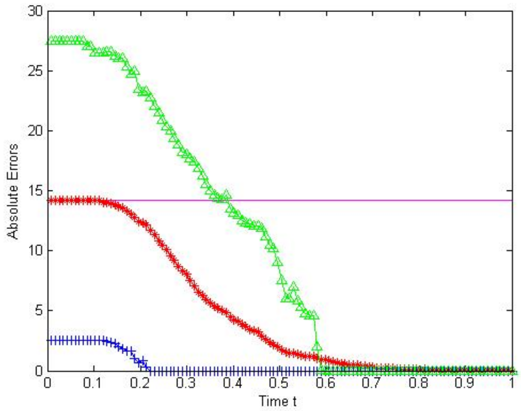

Figure 3: Prediction errors (in absolute value) in scenario 2 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{var} = 0,05$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

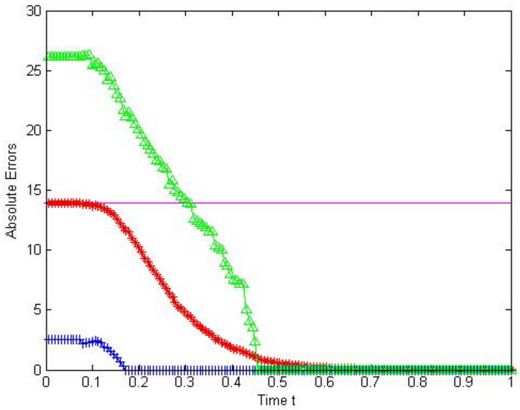

Figure 4: Prediction errors (in absolute value) in scenario 2 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{var} = 0,05$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

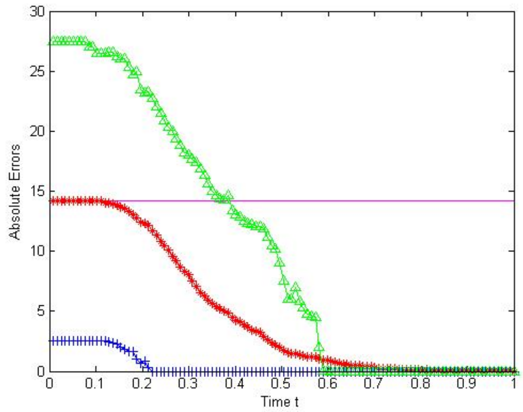

Figure 5: Prediction errors (in absolute value) in scenario 3 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{var} = 0,5$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

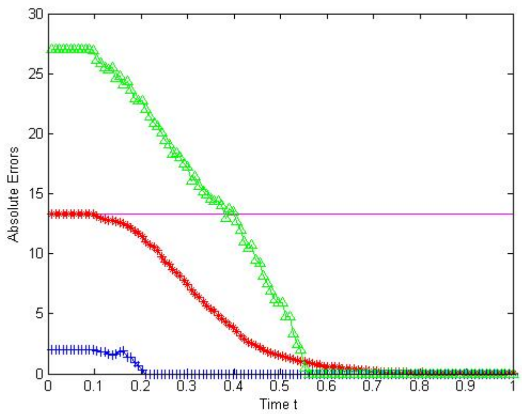

Figure 6: Prediction errors (in absolute value) in scenario 3 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{var} = 0,5$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

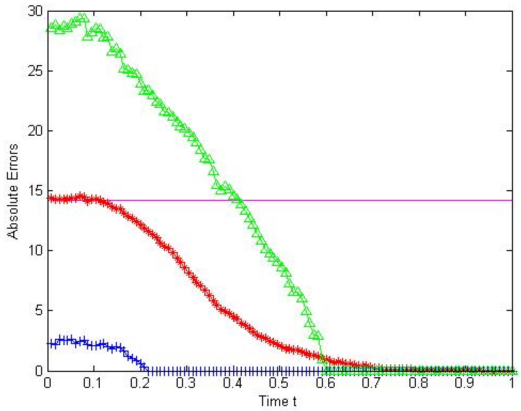

Figure 7: Prediction errors (in absolute value) in scenario 4 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{kurt} = 0,005$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

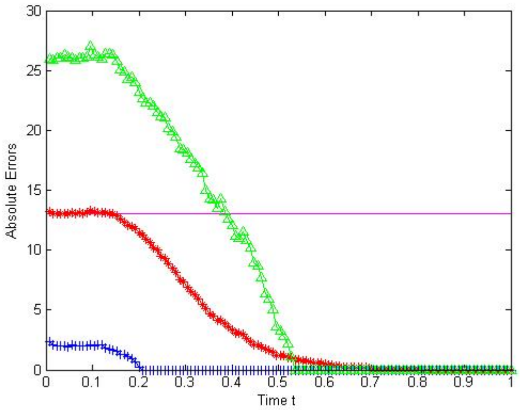

Figure 8: Prediction errors (in absolute value) in scenario 4 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{kurt} = 0,005$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

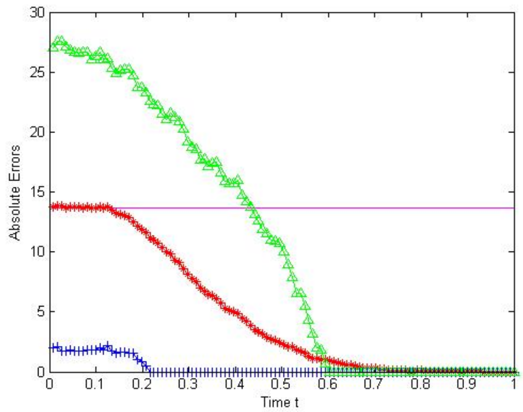

Figure 9: Prediction errors (in absolute value) in scenario 5 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{kurt} = 0,05$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

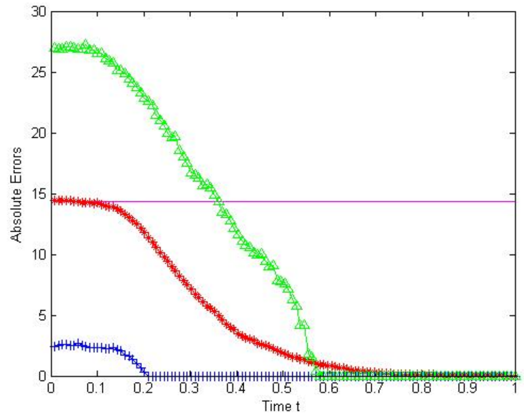

Figure 10: Prediction errors (in absolute value) in scenario 5 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{kurt} = 0,05$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

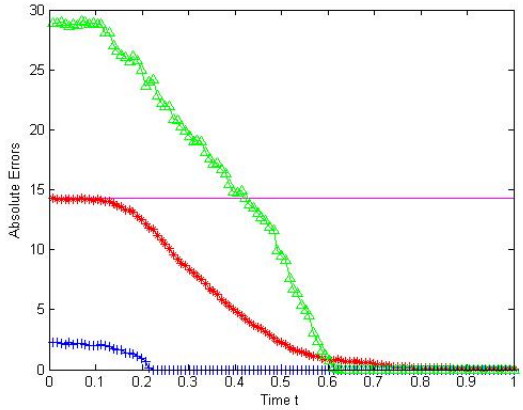

Figure 11: Prediction errors (in absolute value) in scenario 6 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{kurt} = 0,5$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

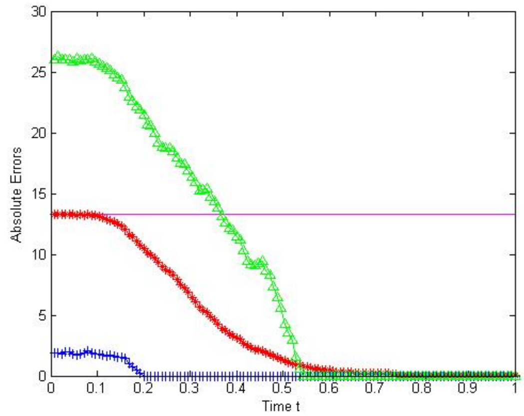

Figure 12: Prediction errors (in absolute value) in scenario 6 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{kurt} = 0,5$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

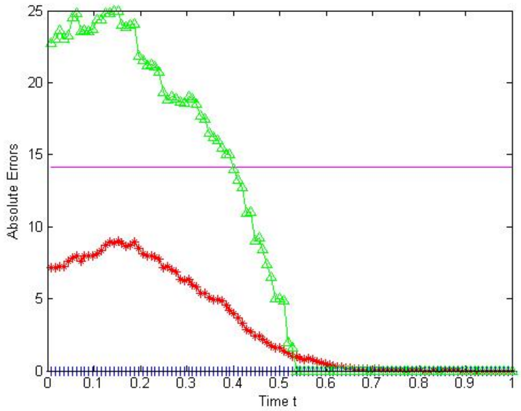

Figure 13: Prediction errors (in absolute value) in scenario 7 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{skew} = 0,005$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

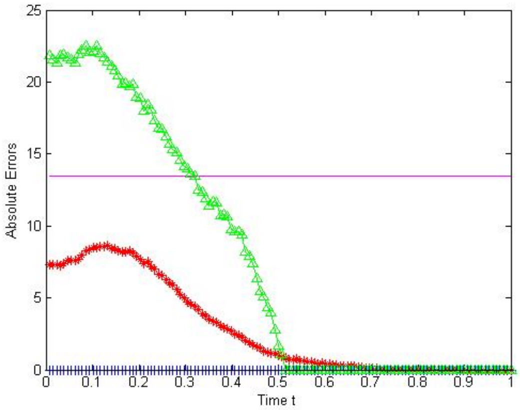

Figure 14: Prediction errors (in absolute value) in scenario 7 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{skew} = 0,005$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

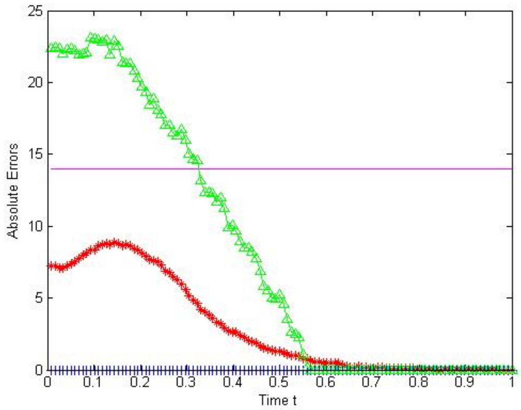

Figure 15: Prediction errors (in absolute value) in scenario 8 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{skew} = 0,05$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

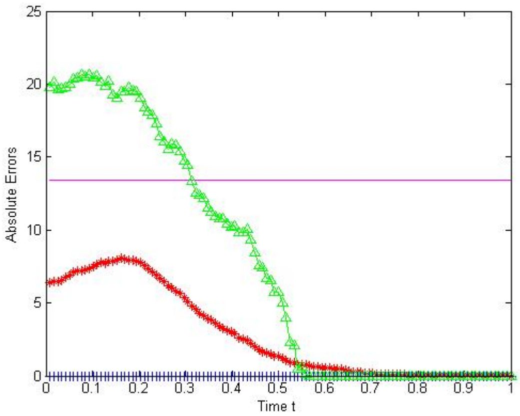

Figure 16: Prediction errors (in absolute value) in scenario 8 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{skew} = 0,05$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

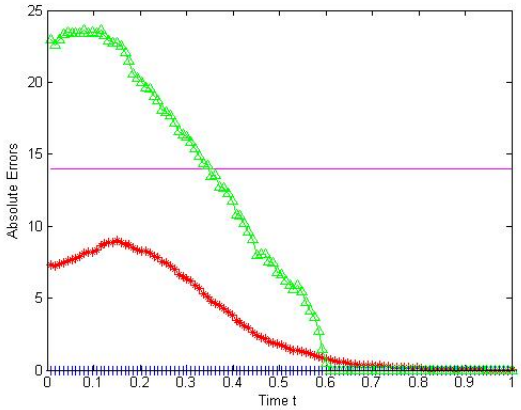

Figure 17: Prediction errors (in absolute value) in scenario 9 with 1000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{skew} = 0,5$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.

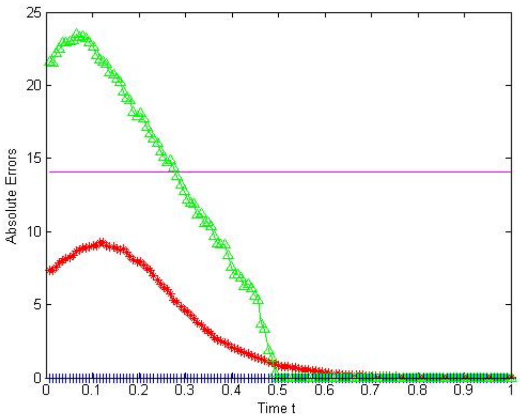

Figure 18: Prediction errors (in absolute value) in scenario 9 with 10000 $K\_simulations$ and 1000 $X\_simulations$ and $Tolerance_{skew} = 0,5$. Red stars line is the avarage of the errors between rational expectations predictions and the real failure times of the $\mathbf{S}^V$'s defined by XXXXXXX. Green triangles and blue crosses lines are respectively E_{RE_90} defined by XXXXXX and E_{RE_10} defined by XXXXXX. These prediction errors should be compared with magenta line which represents E_B defined by XXXXXX.