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We introduce a new methodology that enables the detection onset of convergence towards Nash equilibria, in simple market games with infinite larges strategy spaces. The method works by constraining on a special and finite subset of strategies. We illustrate how the method can be used to …. in a series of experiments

1 Introduction

Quite often, it can be difficult to obtain a proper behavioral prediction of people's action in simple one-shot games, since it is obscured by non-equilibrium behaviors, which can persist even when the games are played repeatedly (*Chong et al. 2016)*. One question is then how one can get a better understanding of the decision making, and eventually dynamics hereof, before a proper equilibrium has set in. In Chong et al. 2016 a generalized cognitive hierarchy was introduced to capture the fact that ….

The introduction of the economist Brian Arthur of his famous El Farol bar game (Arthur 1994) set off a flurry of research

2 Simple multi-period market games: theoretical framework

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The introduction of the economist Brian Arthur of his famous *El Farol* bar game (*Arthur 1994)* set off a flurry of research in simple binary choice decision making models, with the decision of the type “go/stay”, “yes/no”, “buy/sell”, “right/left”, etc. In Arthur’s model a population of N=100 people want to go to the El Farol bar (an existing bar in Sante Fe) to listen to folk music. However the El Farol is quite small, and people are only satisfied if they get seated on one of the 60 seats available. People have to make their decision at the same time, and so if everybody uses the same pure strategy, it will fail: suggesting “go” (assuming an empty bar) everybody will go and the bar be crowded, “stay” (assuming a crowded bar) instead will lead to an empty bar. *Challet and Zhang (1997)* extended the El Farol model in order to describe financial market behavior, now having the binary choice as a simple “buy/sell” decision. More precisely, in the minority game there is an odd number N of players who used different strategies in order to try to always be on the minority side. If a strategy S_i, predicts the majority will buy, then that strategy will recommend to sell, $S_i = -1$. Otherwise, if predicting the majority will sell, the recommendation is to buy, $S_i = 1$. The payoff of strategy i,

$$\pi^{MG}(s_i) = -s_i A, \;\; A = \sum_{j}^{N} s_j$$

with A the order imbalance (i.e. the difference in buy and sell orders). As mentioned in (*Linde et al. 2014*) already the one-shot minority game has quite a large number of Nash equilibria. This happens since any case where exactly (N-1)/2 players choose one side (and (N+1)/2 players the other side) constitutes a Nash equilibrium. The $\frac{N!}{\left(\frac{N+1}{2}\right)!\,\left(\frac{N-1}{2}\right)!}$ Nash equilibria already are quite a large number even for moderate values of N.

In the multi-round minority game, players then use strategies that consider the outcome of not only the last, but the past M market price directions (a positive order imbalance, A,: the market goes up; A negative: the market goes down). Each player holds the same number of S strategies which are assigned randomly at the beginning of the game. A strategy in the multi-round issues a prediction (buy/sell) of the next market move for each of the possible 2^M past price histories. The total number of different strategies is therefore 2^{2^M}. In the multi-round minority-game the players record the cumulative payoff for of each of their S strategies, and use the one which at a given time has the highest cumulative payoff.

As mentioned in *Andersen and Sornette (2003)* one problem with the multi-round minority game describing simple market price dynamics, is that lack of speculative behavior where investors invest to gain a return. It should be noted that the minority dynamics prevents any trend to develop, giving rise to a price dynamics which is mean reverting. In order to capture speculative behavior as seen in bubble/crash phases of financial markets, a modification of the payoff function was suggested in *Andersen and Sornette (2003)*. Denoting the game the $ game (to describe players that speculate), the modified payoff reads:

$$\pi^{\$G}(s_i(t)) \ = \ s_i(t-1)R(t)$$

The payoff favors strategies which are able to predict the price movement over the following time step. Predicting at time t-1 a price increment over the following time step, the strategy i propose to enter a buy position at time t-1, $s_i(t-1) = 1$. If the prediction was successful (a failure) the payoff gained (lost) is the return of the market over that time step.

As mentioned the mere size of the strategy space, 2^{2^M}, complicates considerably a proper understanding of simple market models, like the minority game and the $-game, despite the simplicity of their payoff functions. It is therefore out of question to explore the full strategy space in order to gain insight on how people would react even in simple market games like this. We instead propose to concentrate on a certain subclass of strategies.

Let us call $s_i(t \mid \vec{h}_M(t))$ the action of strategy s_i at time t, *conditioned* on observing a given price history, $\vec{h}_M(t)$, at time t over the last M time steps. $\vec{h}_M(t)$ is a binary string of -1's and +1's describing the last M price movements observed at time t. We now note that some strategies will be *independent* of, $\vec{h}_M(t)$, over the next L time steps. That is, whatever price history over the next t + Q time steps, the strategy $s_i(t + Q)$ will always issue the same prediction *independent* of the price history between t and t+Q. We call such strategies Q time steps *decoupled* (*Andersen and Sornette, 2005*). The most simple example of a strategy that is decoupled, is the strategy that always issues a buy (/sell) action, independent of the past price history $\vec{h}_M(t)$. Such a strategy is trivially an infinite number of time steps decoupled. However the probability that any player will hold this specific strategy is very unlikely, with a probability that goes as $\frac{s}{2^{2^M}}$.

As will be seen in the following, it is advantageous to split the order balance in two, so that it can be written in terms of decoupled and coupled strategies:

$$A(t)^{\vec{h}_M} = A(t)^{\vec{h}_M}_{coupled} + A(t)^{\vec{h}_M}_{decoupled}$$

We have added $\vec{h}_M$ to emphasize that the order imbalance is conditioned on observing the price history $\vec{h}_M$ at time t.

3 Laboratory experiments

We performed a series of 10 experiments at the Laboratory of Experimental Economics in Paris (LEEP). The experiments ran over 60 periods. In each period, the students, received general economic news and could decide whether to buy or sell an asset or simply do nothing. At the end of the 60 periods, the students were paid pro rata according to their performance (for more details about the way the experiments were set up, see “Methods”).

At the beginning of the experiment the students were told that the asset was, at this initial stage, properly priced according to rational expectations[14]. This meant that only information regarding changes in the dividends on the asset or interest rates should have a direct influence on the price of the asset. The information flow consisted of general news from past real records of Bloomberg news items. News was selected in such a way that the general trend over the 60 consecutive periods was neutral. Then, according to rational expectations, there should be no overall price movement of the asset at the end of the 60 time periods. The price was thus expected to oscillate around the fundamental value throughout the experiment.

added $\vec{h}_M$ to emphasize that the order imbalance is conditioned on observing the price history $\vec{h}_M$ at time

4 Results

E1

ΔD	Success rate	Number of events
0.1	1.0	49
0.12	1.0	48
0.14	1.0	47
0.16	1.0	47
0.18	1.0	46
0.2	1.0	46
0.22	1.0	45
0.24	1.0	44
0.26	1.0	44

0.28	1.0	43
0.3	1.0	41
0.32	-	0

E2

ΔD	Success rate	Number of events
0.1	0.72	47
0.12	0.71	41
0.14	0.71	34
0.16	0.72	32
0.18	0.70	30
0.2	0.70	27
0.22	0.68	25
0.24	0.73	22
0.26	0.69	16
0.28	0.62	13
0.3	0.64	11
0.32	0.67	6
0.34	0.75	4
0.36	-	0

E3

ΔD	Success rate	Number of events
0.1	0.59	56
0.12	0.60	50
0.14	0.63	40
0.16	0.58	36
0.18	0.57	30
0.2	0.57	23
0.22	0.67	15
0.24	0.67	3
0.26	-	0

E4

ΔD	Success rate	Number of events
0.1	0.59	51
0.12	0.57	49
0.14	0.57	44

0.16	0.56	39
0.18	0.59	34
0.2	0.65	20
0.22	0.73	15
0.24	0.73	11
0.26	0.70	10
0.28	0.67	9
0.3	0.57	7
0.32	-	0

E5

ΔD	Success rate	Number of events
0.1	0.50	21
0.12	0.41	17
0.14	0.38	13
0.16	0.33	9
0.18	0.25	4
0.2	-	0

E6

ΔD	Success rate	Number of events
0.1	0.56	50
0.12	0.54	41
0.14	0.53	32
0.16	0.57	28
0.18	0.47	17
0.2	0.63	8
0.22	0.50	4
0.24	0.50	2
0.26	-	0

E7

ΔD	Success rate	Number of events
0.1	0.00	4
0.12	0.00	1
0.14	0.00	1
0.16	0.00	1
0.18	0.00	1
0.2	0.00	1

0.22	-	0

E8

ΔD	Success rate	Number of events
0.1	0.50	10
0.12	0.83	6
0.14	1.00	2
0.16	1.00	1
0.18	1.00	1
0.2	1.00	1
0.22	-	0

E9

ΔD	Success rate	Number of events
0.1	0.32	19
0.12	0.22	9
0.14	0.00	4
0.16	0.00	2
0.18	-	0

E10

ΔD	Success rate	Number of events
0.1	0.55	40
0.12	0.43	28
0.14	0.32	19
0.16	0.33	15
0.18	0.38	8
0.2	-	0

5 Finding the Nash equilibrium of the $-Game

The dynamics of the $-Game is driven by a nonlinear feedback mechanism because each agent used his/her *best* strategy (fundamental/technical analysis) at each time step. The sign of the order imbalance, $\sum_{i=1}^{N} a_i^* \left(\vec{h}(t)\right)$, in turn determines the value of the last bit $b(t)$ at time t for the price movement history $\vec{h}(t+1) = (b(t-m+1), b(t-m), \dots, b(t))$. The dynamics of the $-Game can then be expressed in terms of an equation that describes the dynamics of $b(t)$ as:

$$b(t+1) = \square\left[\sum_{i=1}^{N} a_i^*\big(h(t)\big)\right] \tag{1}$$

where $\square$ is a Heaviside function taking the value 1 whenever its argument is larger than 0 and otherwise 0, and $h(t) = \sum_{j=1}^{m} b(t-j+1)2^{j-1}$ is now expressed as a scalar instead of a vector. The nonlinearity of the game can be formally seen from:

$$a_i^*\big(h(t)\big) = a_i^{\left\{j \mid max_{j=1,\dots,s}\left[\prod\{a_i^j(h(t))\}\right]\right\}}\big(h(t)\big) \tag{2}$$

with

$$\prod\{a_i^j\big(h(t)\big)\} = \sum_{k=1}^{t} a_i^j\big(h(k-1)\big) \sum_{i=1}^{N} a_i^*\big(h(k)\big) \tag{3}$$

Inserting the expressions (3) and (2) in expression (1) one obtains an expression that describes the $-Game in terms of just one single equation for $b(t)$ depending on the values of the 5 base parameters variables (m; s; N; $\square$; $D(t)$) and the random variables a_i^j (i.e. their initial random assignments).

We would first like to point out an important difference compared to traditional game theory since in our game the agents have *no* direct information of the action of the other players. The only (indirect) information a given agent have of other agent's action through the aggregate actions of the past, i.e. the past price behavior. Let the action of optimal strategy a_i^* be expressed in terms of the relative payoff, q_i, so as to formulate $\sum_{i=1}^{N} a_i^*\big(h(t)\big)$ as follows

$$\sum_{i=1}^{N} a_i^*\big(h(t)\big) = \sum_{i=1}^{N}\left\{\square\Big(q_i\big(h(t)\big)\Big)\, a_i^2\big(h(t)\big) + \left[1 - \square\Big(q_i\big(h(t)\big)\Big)\, a_i^1\big(h(t)\big)\right]\right\} \tag{4}$$

Inserting (4) into (1) and take the derivative of b in $t+1$

$$\left.\frac{db}{dt}\right|_{t+1} = \delta\left(\sum_{i=1}^{N} a_i^*\big(h(t)\big)\right)\sum_{i=1}^{N}\left\{\delta\left(q_i\big(h(t)\big)\right)\frac{\delta q_i\big(h(t)\big)}{\delta t}\left[a_i^2\big(h(t)\big) - a_i^1\big(h(t)\big)\right]\right.$$

$$\left. + \square\left(q_i\big(h(t)\big)\right)\frac{\delta a_i^2\big(h(t)\big)}{\delta t} + \left[1 - \square\left(q_i\big(h(t)\big)\right)\right]\frac{\delta a_i^1\big(h(t)\big)}{\delta t}\right\} \tag{5}$$

Looking inside the bracket of the sum in (5), it follows that a change in $\sum_{i=1}^{N} a_i^*\big(h(t)\big)$ can occur either because the optimal strategy changes and the two strategies for a given $h(t)$, $a_i^1\big(h(t)\big)$ and $a_i^2\big(h(t)\big)$, differ one each other (first term in the bracket). Furthermore, a change in $\sum_{i=1}^{N} a_i^*\big(h(t)\big)$ can arise also because the optimal strategy changes its prediction for the given $h(t)$ (second and third terms in the bracket).

The change in time of the relative payoff q_i is computed as follows

$$\left.\frac{\delta q_i}{\delta t}\right|_{t} = \left[a_i^2\big(h(t-2)\big)\sum_{i=1}^{N} a_i^*\big(h(t-1)\big)\right] - \left[a_i^1\big(h(t-2)\big)\sum_{i=1}^{N} a_i^*\big(h(t-1)\big)\right] \tag{6}$$

Using $h(t) = \sum_{j=1}^{m} b(t-j+1)2^{j-1}$ and inserting (6) in (5) one obtains:

$$\left.\frac{db}{dt}\right|_{t+1} = \delta\left(\sum_{i=1}^{N} a_i^*\big(h(t)\big)\right)\sum_{i=1}^{N}\left\{\delta\left(q_i\big(h(t)\big)\right)\sum_{i=1}^{N} a_i^*\big(h(t-1)\big)\right.$$

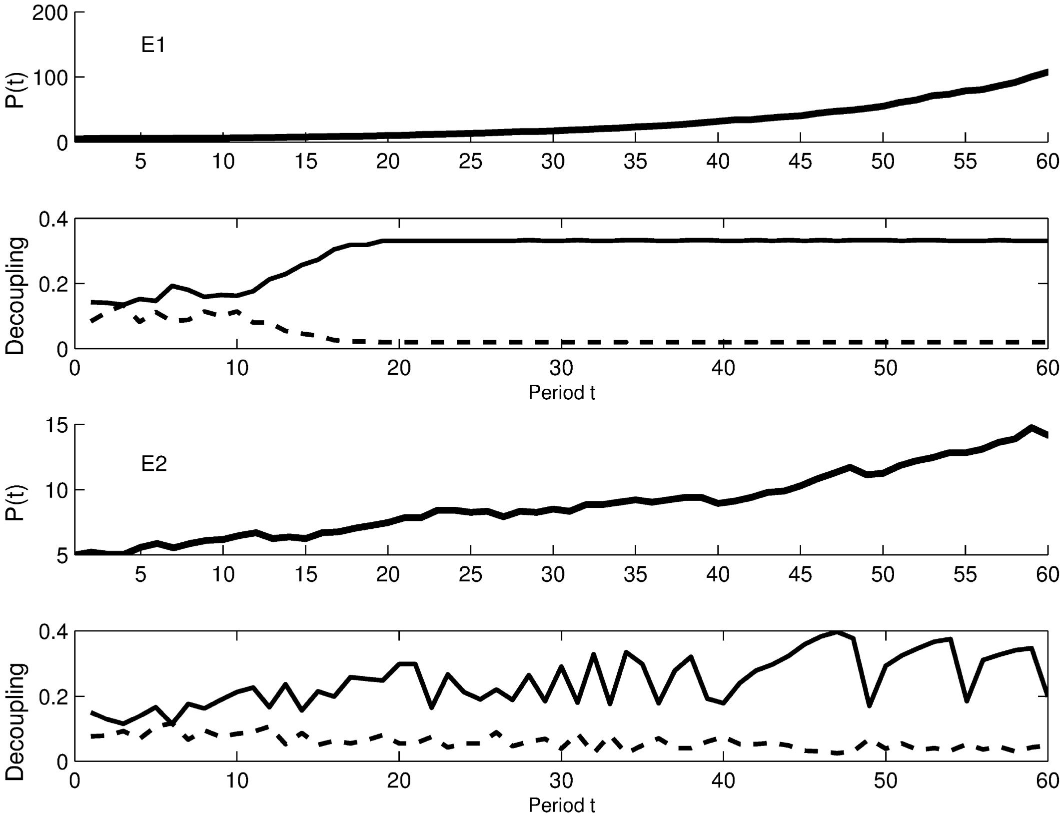
$$\left[\mathrm{a_i^2}\left(\sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{b(t-j-1)2^{j-1}} - \mathrm{a_i^1}\left(\sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{b(t-j-1)2^{j-1}}\right)\right)\right]\times$$

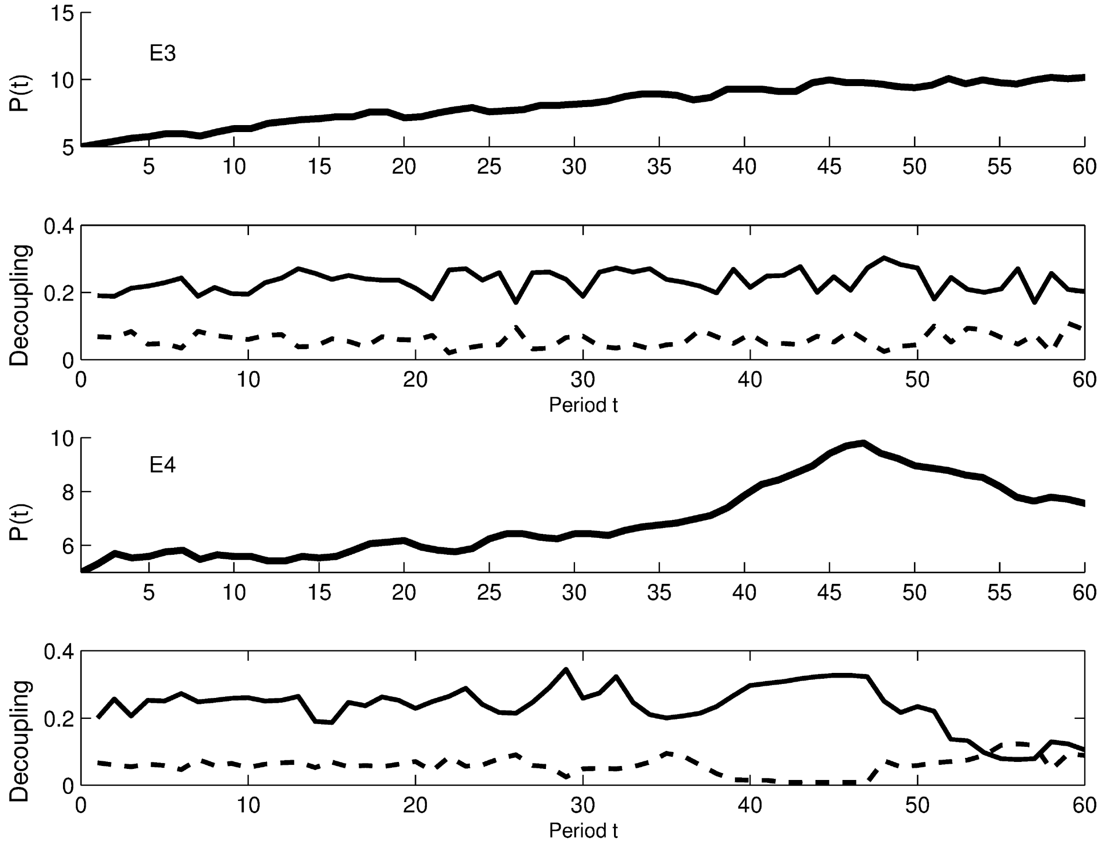
$$\left[a_i^2\left(\sum_{j=1}^{m} b(t-j+1)2^{j-1} - a_i^1\left(\sum_{j=1}^{m} b(t-j+1)2^{j-1}\right)\right)\right]+$$

$$\left.\square\left(q_i\big(h(t)\big)\right)\frac{\delta a_i^2\left(\sum_{j=1}^{m} b(t-j+1)2^{j-1}\right)}{\delta t} + \left[1 - \square\left(q_i\big(h(t)\big)\right)\right]\frac{\delta a_i^1\left(\sum_{j=1}^{m} b(t-j+1)2^{j-1}\right)}{\delta t}\right\} \tag{7}$$

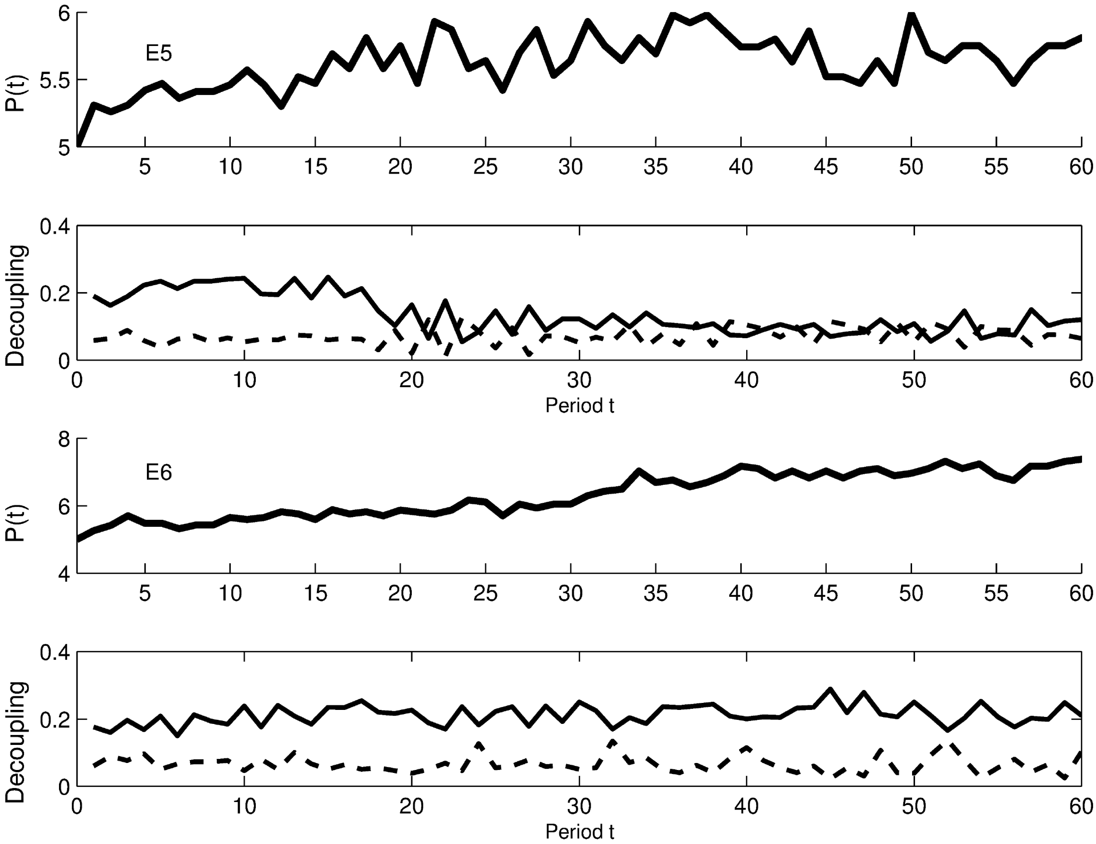
If $\sum_{i=1}^{N} a_i^*\big(h(t-1)\big), \sum_{i=1}^{N} a_i^*\big(h(t-2)\big), \dots, \sum_{i=1}^{N} a_i^*\big(h(t-m)\big)$ have all the same sign, the right-hand-side of (7) becomes 0, thus proving that a constant bit $b(t)$, corresponding to either an exponential increase or decrease in price, is a Nash equilibrium.

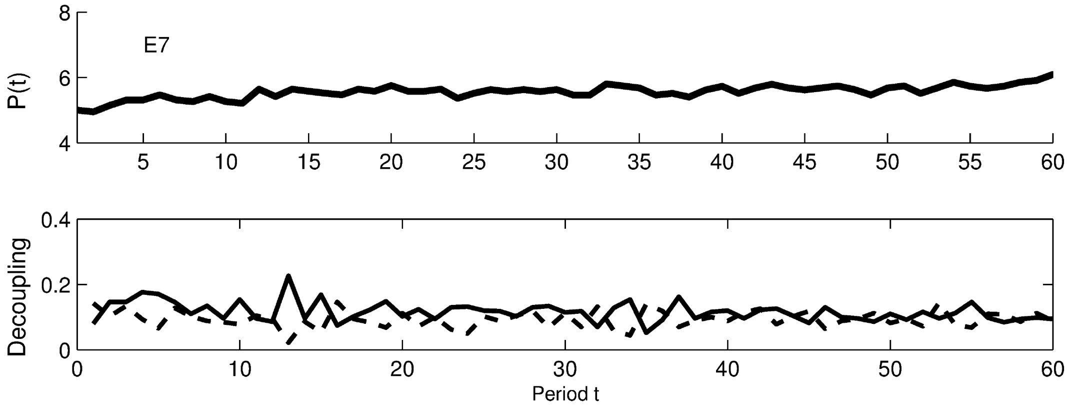
6 Figures of the experiments

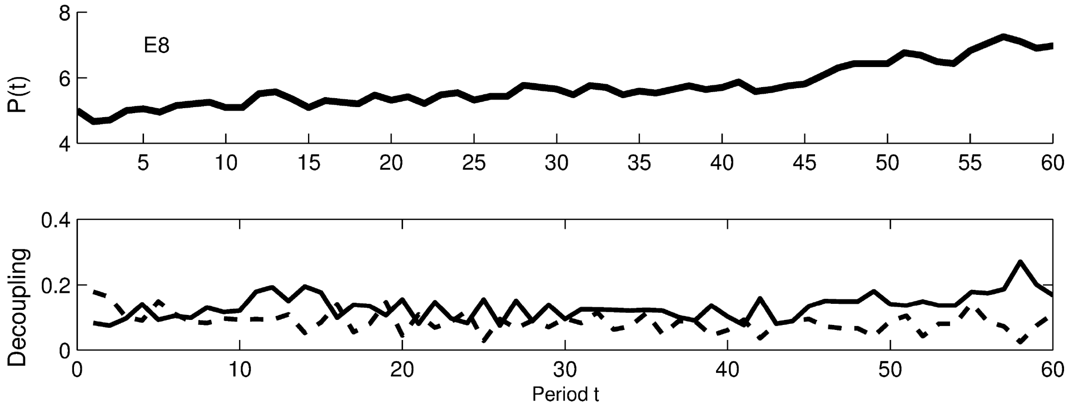

E1
P(t)
200
100
0
5 10 15 20 25 30 35 40 45 50 55 60
Decoupling
0.4
0.2
0
0 10 20 30 40 50 60
Period t
E2
P(t)
15
10
5
5 10 15 20 25 30 35 40 45 50 55 60
Decoupling
0.4
0.2
0
0 10 20 30 40 50 60
Period t

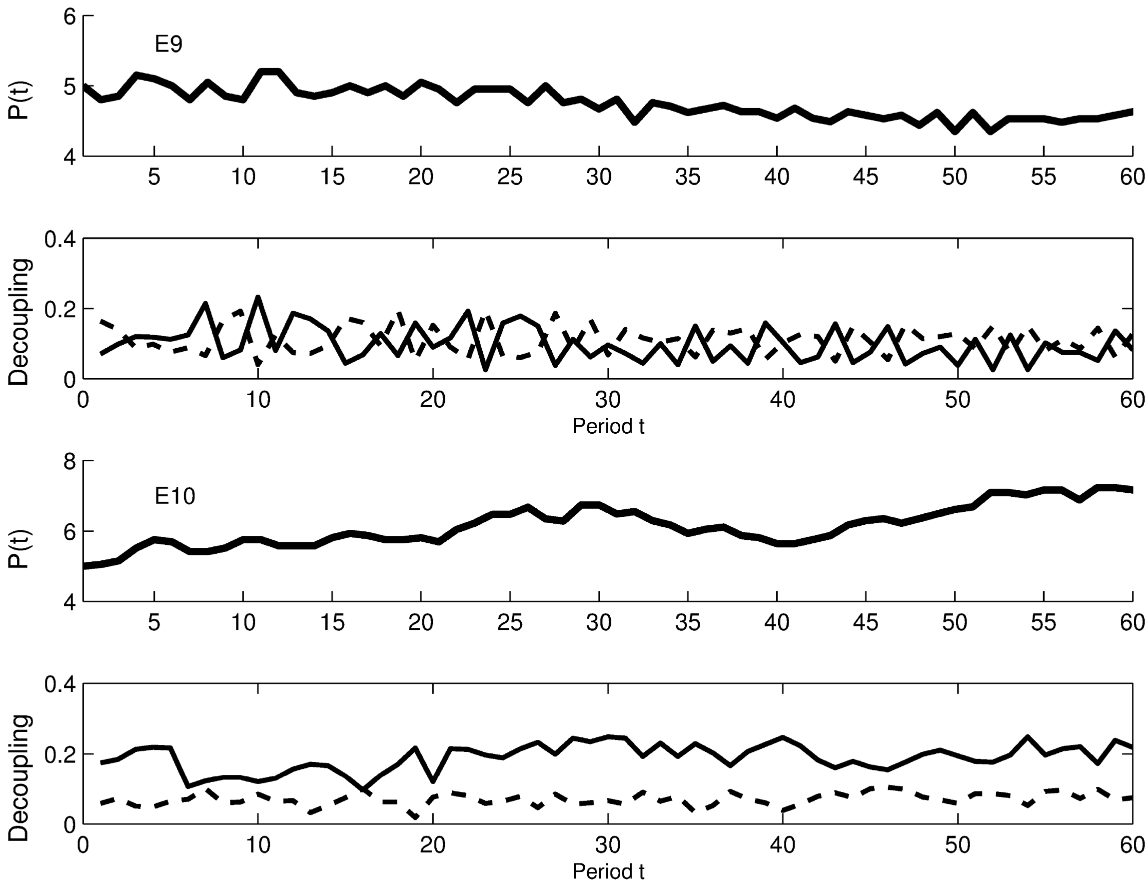


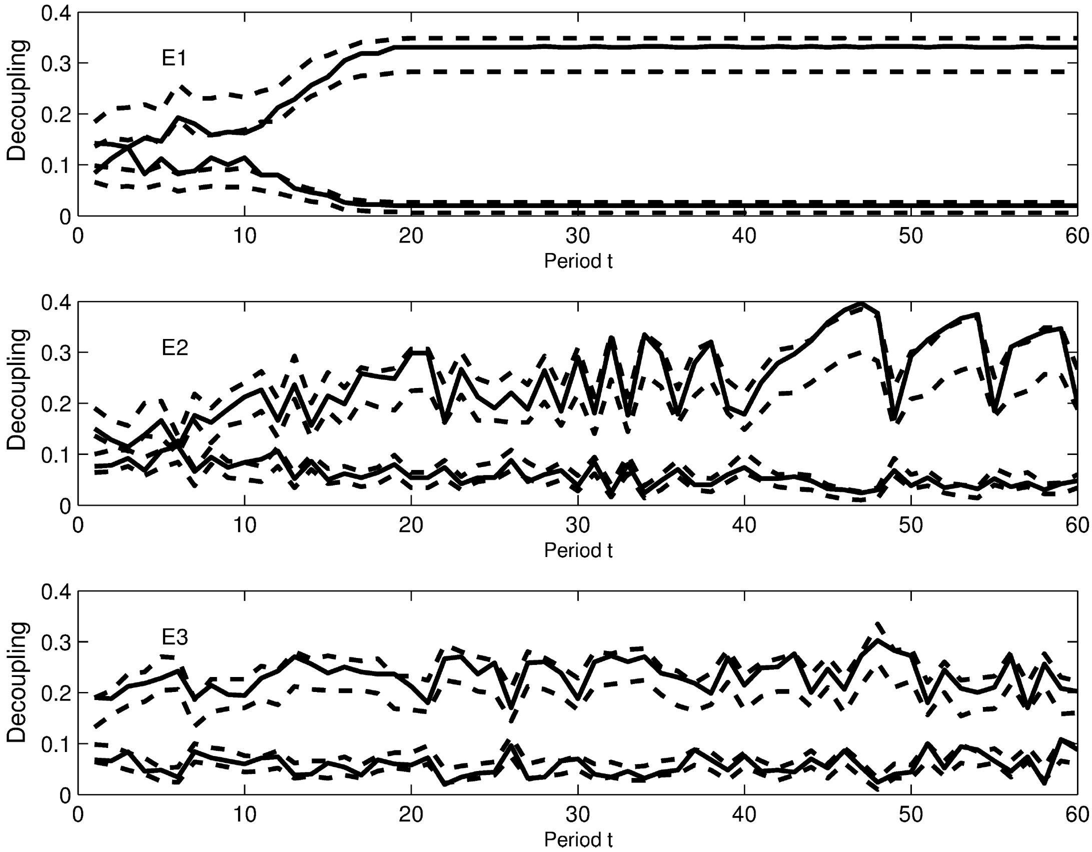
E3
P(t)
Decoupling
Period t
E4
P(t)
Decoupling
Period t


E5
P(t)
Decoupling
Period t
E6
P(t)
Decoupling
Period t


E7
P(t)
Decoupling
Period t


E8
P(t)
Decoupling
Period t


E9
P(t)
Decoupling
Period t
E10
P(t)
Decoupling
Period t



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