Christopher Cullen, *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems*. Scientific Writings from the Ancient and Medieval World. London; New York: Routledge, 2017, xi, 434 pp.

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Having made his way through most of the rest of what sources we possess for the history of the astral and mathematical sciences of the Han (206 BC–AD 220) that had yet to be translated into a Western language, Christopher Cullen's newest book goes a long way to filling what is, until today, the largest gap: the 'Lü-li zhi' 律曆志, the histories of *lü* 律 tono-metrological standards and *li* 曆 mathematical astronomy included among the *zhi* 志 technical monographs of the Standard Histories.

Cullen's, to be precise, is first of all (Chapters 2–4, pp. 32–355) a translation of the three Han-era *li* procedure texts preserved in as many 'Lü-li zhi': Liu Xin's 劉歆 (c. 50 BC–AD 23) *San tong li* 三統曆 of *circa* AD 10, in *Han shu* 漢書, *j.* 21b; Bin Xin 編訢 and Li Fan's 李梵 *Si fen li* 四分曆, implemented AD 85, in *Hou Han shu* 後漢書, *zhi* 3; and Liu Hong's 劉洪 (c. 130–c. 210) *Qian xiang li* 乾象曆 of *circa* 200, in *Jin shu* 晉書, *j.* 17. A *li* procedure text or 'astronomical system' – often confusingly called a 'calendar' – is a set of numbers (*shu* 數) and algorithms (*shu* 術) used to calculate the time and position of a variety of lunar, solar, and planetary phenomena including – but not limited to – those used to determine the lunisolar civil calendar. Originally independent titles, these procedure texts have come down to us only as preserved in the *lü-li* monographs of the aforementioned state histories, where they are inserted into/after a chronicle of events in the history of *li* alongside other documents such as memorials, edicts, and debate transcripts wherein said *li* is usually the principal subject of discussion. It is to the history portion of the historical monograph that Cullen devotes Chapter 5 (pp. 356–419), translating the Han-era *li* chronicles surrounding the *San tong li* and *Si fen li*, respectively, in the *Han shu* and *Hou Han shu*. In this sense, the author is right to present his work as a sort of period-defined sourcebook, but it is worth

emphasizing that the *li* contents of the *Han shu* and *Hou Han shu* 'Lü-li zhi' *are* translated more or less in their entirety, if not in their original order.

Cullen attributes his work with two aims in the introduction: to make these materials accessible in a Western language to Western historians of astronomy and to serve as a complement to a social, intellectual, and political history written on their basis, namely his companion monograph *Heavenly Numbers: Astronomy and Authority in Early Imperial China* (Oxford: Oxford UP, 2017). As I had yet to receive its companion at the time of writing, I can only speak here for *The Foundations of Celestial Reckoning* as it stands on its own and in relation to the author's earlier writings. I find this book of great potential use to the student of *li*, but before getting to *accessibility*, I want to say a word about his translations' *complementarity*.

Throughout, Cullen's translations are exceedingly well-considered, but the apparatus might feel a little thin were one unfamiliar with his previous/other work and oblivious to how this book is intended to articulate therewith. This is particularly true of the historiographical materials translated in Chapter 5. Cullen introduces these materials with a seven-page summary that nicely captures the essence of what is happening and why, but numerous reference- and jargon-filled passages of the original are passed over without much by way of explanation in the footnotes, e.g. "On the technical aspects of this material, see Cullen (2007a), 75–98" (p. 391n59). In the footnotes, moreover, Cullen refers the reader but seven times to secondary scholarship external to his own, of which one counts but three historians of astronomy and but one Asian-language work. The reason for this, it might have been worth further underscoring, is that Chapter 5 is intended more as an appendix to the author's other writings than as a standalone translation, and by following up his many self-references in the footnotes one will find oneself wanting for very little. There is less intertextuality, in this sense, to Chapters 2–4. Where the author comes to *jintui* 進退 'advance and retardation' (≈ reduction to the equator), however, he does refer the reader elsewhere in his writings rather than explain in significant detail how (and/or why not) to apply the procedure as written (pp. 220–224, 267–268).

As to accessibility, *li* procedure texts pose a challenge for translation and general reading comprehension, in any language, beyond even those of *suan* 算 mathematics. Here, in five points, is what makes these sources particularly difficult: (1) a *li* procedure text is not so much a *collection* of algorithms as it is a *chain* wherein the result of one procedure becomes the operand of the next and/or another some indeterminate number of steps later; (2) a *li* procedure text must frequently switch between cardinal and ordinal numbers without there being a good word to

distinguish 'cardinal' from 'ordinal'; (3) a *li* procedure text relies heavily on rule-of-three unit conversion via *lü* 率, which is a mathematical construct that neither translates nor exists outside of a premodern Chinese context; (4) a *li* procedure text uses ambiguous operational terminology like *chu* 除 'cast out', which can signal the equivalent of division, modulo, *or* subtraction; (5) a *li* procedure text provides the reader with no solutions or worked examples. In discussing how Cullen rises to the challenge of rendering these sources accessible in translation I will refer to these as 'Difficulties 1–5'.

It is in the name of accessibility that most scholars to our day have eschewed the challenge of *translating* these sources by instead distilling their numbers into modern terms and their procedures into symbolic algebra. Suffice it to say that this approach has its problems, and that Cullen is of the camp that one should read the text as written if one wishes to understand it.

To immerse the modern, Western-language reader into this foreign mathematical idiom, Cullen's translation proceeds via a combination of elements. The procedure text is essentially broken down to the level of the sentence, and each sentence of the original is provided a concise, literal translation followed by an explanation, in colloquial language, mixing actors' and observers' categories, words, Arabic numerals, and some symbolic arithmetic. Certain steps in calculation are provided with worked examples for a given year, and where the base text is particularly hard to explain as is he also brings in whatever the commentator or his principal reference, Li Rui 李鋭 (1768–1817), may have to add. The combination of heuristic elements effectively guides the reader to understanding via triangulation, and where Cullen likens his methodology to Nathan Sivin's in *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records* (New York: Springer, 2009), a fruitful comparison might also be made to Liu Hongtao's 劉洪濤 monumental *Gudai lifa jisuanfa* 古代曆法計算法 (Tianjin: Nankai daxue chubanshe, 2003). Each achieves a different balance between these elements – to which we may add diagrams, in the case of Sivin and Liu Hongtao – but all three of them do a commendable job making their respective *li accessible as written*.

Put in different words, one might say that this approach relies on extensive redundancy. Not only does Cullen effectively say the same thing three times between the original, translation, and explanation every line, the explanation, as per Difficulty 1, must constantly remind the reader where he/she is in the chain of algorithms and sea of numbers: "How much are the 'Compatibility Factor' and the 'Day Factor' again?

What is their relation? And what are we multiplying by the one and dividing by the other here to get?" Overall, Cullen strikes an excellent balance between concision, omission, cross-reference, reformulation, and outright repetition in his apparatus, but anyone attempting to, say, read straight through will no doubt find the redundancy a considerable onus weighing on an already difficult text. The trick, Cullen is seemingly the first translator to admit, is that one does not really absorb a *li* procedure text by reading: one learns it by performing the calculations oneself, for which Cullen describes himself as relying upon and recommends the use of spreadsheet automation (p. 29).

To this end – and to Difficulty 5 – the number of worked examples with which Cullen provides the reader is invaluable. He is by no means systematic about giving worked examples in his translation, skipping the entirety of the *Si fen li* and everything having to do with positions, planets, hours, hemerologies, and lunar latitude in the other two, but he gives the reader considerably more to work with in this regard than Sivin or Liu Hongtao. Of course, as generous as he is with worked examples in *The Foundations of Celestial Reckoning*, it used to be that Cullen made his own spreadsheets publically available for download from his Needham Research Institute webpage, and one can only hope for their return or the appearance of some alternative in the near future.

Either way, the clarity with which Cullen's apparatus articulates the *text* and the intended *performance* of the *San tong li*, *Si fen li*, and *Qian xiang li* is likely sufficient for most readers to reproduce the remaining calculations on their own. Particularly noteworthy in this regard is the care with which the author guides the reader through potentially awkward switches between what are effectively cardinal and ordinal numbers (Difficulty 2). *Sufficient* but worth additional reflection is his handling of Difficulties 3 and 4.

As to Difficulty 3, *li* procedure texts rely on unit conversion, which, as in *suan* mathematics, is performed via the *jinyou shu* 今有術 rule of three using ratios formed of *lü* – numbers, as distinct from quantities, that have no meaning except as they relate to one another in proportion. An example of *lü* would be the 'circle circumference *lü*' 圓周率 355 and 'circle diameter *lü*' 圓徑率 113 by which one multiplies and divides, respectively, to find the circumference of a circle from its diameter ($\frac{355}{113} = 3.14159$). *Lü* are not *ratios*, that is to say, but they do combine to form them, the important difference being that two *lü* can easily be reversed ($\frac{113}{355} = 0.31831$), or one swapped for another in a larger set used to convert between any number of things (e.g. grains). Most *li* numbers function as *lü*, and some of them, like those comprising the *Si fen li*'s planetary

resonance periods, are even labeled as such: for Jupiter, its resonance period of 4327 conjunctions : 4725 years is expressed in terms of a 'cycle *lü*' 周率 and a 'solar *lü*' 日率, respectively (p. 187). Cullen does an excellent job explaining the multiplication and division that one is to do with such numbers, as well as the reasons why, but he translates *lü* throughout as 'rate' without a word of explanation as to the difference. This does not make sense in modern terms – the "cycle rate" and "solar rate" *alone* are not *rates* – and modern terms here obscure an emic construct that *does* make sense and that is furthermore revealing of how the authors and users of these texts conceived of the numbers and procedures therein.

As to Difficulty 4, Cullen goes a long way to dispelling the potential confusion surrounding the word *chu* 'casting out' in describing it in the Introduction as covering "[the] two halves of the process called 'Euclidean division'," i.e. "the process that sees one integer, the 'dividend' (say 17) divided by another integer, the 'divisor' (say 5) to yield a 'quotient' (3) and a 'remainder' (2)" (p. 28). The problem is that *chu* can also signal subtraction ($17 - 5 = 12$), and the author seems ill-prepared for this in the body. This usage comes up the most frequently as concerns, first, the subtraction of retrograde motion from the otherwise prograde progress made by a planet from one appearance to the next and, second, the subtraction of the terminal fraction of the circumference of heaven as a celestial body passes the winter solstice. In the first case, Cullen's translation of *chu* is inconsistent, sticking with 'casting out' in the *San tong li* (pp. 74–88), vacillating between that and 'discarding' in the *Si fen li* (pp. 208–218), then switching to 'subtracting' in the *Qian xiang li* (pp. 334–338). Where *chu* is translated 'subtracting' it is not explained why the author is contradicting the definition on p. 28, nor where it is translated otherwise is it explained why the author is *subtracting* in the apparatus. In the second case, he is similarly inconsistent, rendering *chu* 'casting out' in the *Si fen li* (pp. 173, 175, 176, 206) and 'taking off' in the *Qian xiang li* (pp. 256, 258), omitting it completely from his translation on p. 341: "When it goes through the Dipper, take the motion denominator as the rate" 經斗除分，以行母為率.

Cullen stands to lose the reader a little on these points, where they come up, but *The Foundations of Celestial Reckoning* offers the reader an otherwise exemplary entrée into these sources and an invaluable complement to his years of outstanding scholarship based thereupon. I have gone over this book with a fine-tooth comb – with my own notes, spreadsheets, and translations of the same material – and I feel that I can safely say that my main grievance with this book is that I did not possess it alongside Liu Hongtao's when I set about teaching myself the subject

in 2011. *Li* is difficult to learn, and this book – if you do your math! – is one of the best places that I can recommend the reader to begin.