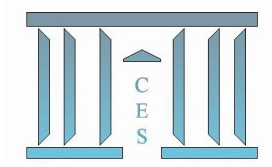
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# Income-Factor Polarization: A Methodological Approach

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# Income-Factor Polarization

# A Methodological Approach

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**Abstract**

We here propose a novel method for the analysis of the distribution of two sources of income across the population. The polarization curve for each income source is defined and compared to the *Zero-Polarization Curve*. The latter provides a benchmark of zero inequality in income composition. We then illustrate the method via an empirical application to the case of Italy.

**Highlights**

- We present a method to assess income-source polarization across the population.
- The *Polarization Curve for Income Source* is introduced.
- Both the concept of zero-polarization and the *Zero Polarization Curve* are defined.
- We provide an application of the method for the case of Italy.

**JEL Classification:** C430, E250.

**Keywords:** Income Distribution, Lorenz Curve, Polarization curve, Income-Source Polarization.

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# 1 Introduction

There are many different ways of interpreting the concept of polarization in the economic literature. Duclos and Taptué (2015) distinguish five types: income polarization, income bipolarization, social polarization, socioeconomic polarization, and multidimensional polarization. However, only little attention has been paid to income-source polarization, which is the subject of our contribution here. The analysis of income-source polarization could provide us with a direct link between the income distribution and political interests, inasmuch as "*factor shares influence collective bargaining*" (Atkinson, 2009). For example, if the richest 20% of the population receive only capital income, they will likely be opposed to a capital-tax increase. In what follows, we propose a method to investigate income-source polarization by making use of polarization curves. We state that two income sources, notably profits and wages, are not polarized when each individual has the same shares of profits and wages as in the overall population. The *Polarization Curve for Income Source* is defined and compared to the *Zero-Polarization Curve*. The latter characterizes a population in which income factors are evenly distributed with respect to both the overall level of income inequality and total factor shares in the economy. An empirical application of the method to the case of Italy is then proposed.

# 2 The Lorenz Curve for Income Source

Suppose we have a fixed population of $n$ individuals, each endowed with income $Y_i$ with $i = 1, \ldots, n$. We define each individual's income share as $y_i = \frac{Y_i}{Y}$, where $Y = \sum_{i=1}^{n} Y_i$ is the total income of the population. Total income is divided into two sources, profits ($\Pi$) and wages ($W$), so that $Y = \Pi + W$ and hence $y = 1 = \pi + w$, where $\pi = \frac{\Pi}{Y}$ and $w = \frac{W}{Y}$ are the profit and wage shares in income respectively. Consider the following decomposition of individual $i$'s income:

$$y_i = \alpha_i \pi + \beta_i w \tag{1}$$

where $\alpha_i = \frac{\Pi_i}{\Pi}$ and $\beta_i = \frac{W_i}{W}$ are the relative shares of profits and wages of individual $i$, such that $\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i = 1$, and $\Pi_i$ and $W_i$ represent $i$'s total amount of profits and wages. Assume that $y_i \leq y_{i+1}\ \forall i = 1, \ldots, n-1$ and $y_0 = 0$, so that individuals are indexed by their income rank. We define $p = \frac{i}{n}$ as the proportion of the population with income less than or equal to $y_p$, so that $p \in \mathbb{Q} \coloneqq [0, 1]$. Let $\mathscr{L}(\mathbf{y}, p) = \sum_{j=1}^{i} y_j$, with $i = 1, \ldots, n$, be the *Lorenz Curve for Income* corresponding to the distribution $\mathbf{y}$.[1] We define the *Polarization Curve for Profits*, $\mathscr{L}(\boldsymbol{\pi}, p)$, corresponding to the distribution $\boldsymbol{\pi}$, as follows:

$$\mathscr{L}(\boldsymbol{\pi}, p) = \pi \sum_{j=1}^{i} \alpha_j \quad \forall i = 1, \ldots, n \tag{2}$$

Similarly, the *Polarization Curve for Wages*, $\mathscr{L}(\mathbf{w}, p)$, corresponding to the distribution $\mathbf{w}$, is:

$$\mathscr{L}(\mathbf{w}, p) = w \sum_{j=1}^{i} \beta_j \quad \forall i = 1, \ldots, n \tag{3}$$

The two curves describe the cumulative distribution of profits and wages across the population with individuals being *indexed by their income rank*. It is hence possible that an individual with a higher profit share be ranked below someone with a lower profit share, if the income of the latter is above that of the former (formally, we can find a pair $(i, j)$ s.t. $\alpha_i > \alpha_j$ and $y_i < y_j$). Additionally, note that when $i \longrightarrow n$ (or $p \longrightarrow 1$) then $\mathscr{L}(\boldsymbol{\pi}, p) \longrightarrow \pi$ and $\mathscr{L}(\mathbf{w}, p) \longrightarrow w$, where $\pi, w \leq y$. According to the previous decomposition of individual income, we can write the following:

$$\mathscr{L}(\mathbf{y}, p) = \mathscr{L}(\boldsymbol{\pi}, p) + \mathscr{L}(\mathbf{w}, p) \ \ \forall i = 1, \ldots, n \tag{4}$$

The Lorenz curve for income $\mathscr{L}(\mathbf{y}, p)$, for every $p$, can therefore be decomposed into the sum of the two previously-defined polarization curves. At this point, we can approximate the Gini coefficient, $\mathscr{G}$, as follows:

$$\mathscr{G} = 1 - \frac{1}{n} \left( \sum_{i=1}^{n} \left[ \mathscr{L}\left(\boldsymbol{\pi}, \frac{i}{n}\right) + \mathscr{L}\left(\boldsymbol{\pi}, \frac{i-1}{n}\right) + \mathscr{L}\left(w, \frac{i}{n}\right) + \mathscr{L}\left(w, \frac{i-1}{n}\right) \right] \right) \tag{5}$$

Figure 1 plots an example of $\mathscr{L}(\mathbf{y}, p)$ (the blue curve) and $\mathscr{L}(\boldsymbol{\pi}, p)$ (the red curve) for a population of size $n = 10$. Total income is equally split between profits and wages, hence

[1] I am defining the Lorenz curve here as in Shorrocks (1983).

$\pi = w = \frac{1}{2}$ (see Table A for the numerical details).

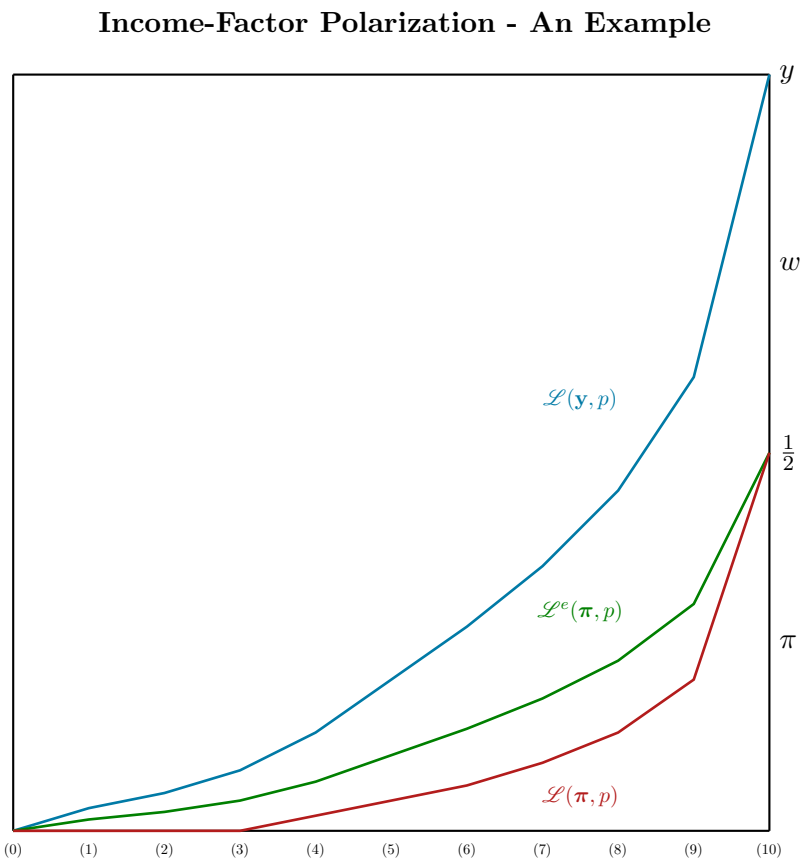



Figure 1: A graphical representation of the *Polarization Curve for Profits* $\mathscr{L}(\boldsymbol{\pi}, p)$, the *Lorenz Curve for Income* $\mathscr{L}(\mathbf{y}, p)$ and the *Zero-Polarization Curve* $\mathscr{L}^e(\boldsymbol{\pi}, p)$ with 10 individuals (or groups) and equal sources of income in the economy ($\pi = w = \frac{1}{2}$).

## 3 The Zero-Polarization Curve

In this section we introduce in a formal setting the concept of the *zero polarization of income sources*. As stated in the introduction, we say that two income sources are not polarized across a population when each individual has the same population shares of profits and wages. Formally, we have zero polarization of income sources when $\frac{W_i}{\Pi_i} = \frac{w}{\pi}\ \forall i$, or, equivalently, when $\alpha_i = \beta_i\ \forall i$.[2] Note that the previous definition is not related to the concept of income inequality: the population can exhibit zero polarization of income sources even with positive income inequality. At this stage of the analysis we define

[2] As $\frac{W_i}{\Pi_i} = \frac{w}{\pi} \iff \frac{W_i}{w} = \frac{\Pi_i}{\pi} \iff Y \times \frac{W_i}{W} = Y \times \frac{\Pi_i}{\Pi} \iff \alpha_i = \beta_i$.

the *Zero-Polarization Curve*, $\mathscr{L}^e(\boldsymbol{\kappa}, p)$, corresponding to the distribution $\boldsymbol{\kappa}$, which is the polarization curve for the income share when the income sources are not polarized as:

$$\mathscr{L}^e(\boldsymbol{\kappa}, p) = \kappa \sum_{j=1}^{i} y_j \quad \forall i = 1, \ldots, n \tag{6}$$

with $\kappa = \pi, w$. The choice of $\kappa$ depends on the particular source we analyze. If we were interested in the distribution of profits in the population, we would compare the *actual* polarization curve for profits with the polarization curve for profits in the case of zero polarization, $\mathscr{L}^e(\boldsymbol{\pi}, p)$. It should be noted that the zero-polarization curve is a *scaled* version of the Lorenz curve for income: we can write $\mathscr{L}^e(\boldsymbol{\kappa}, p) = \kappa \mathscr{L}(\mathbf{y}, p)\ \forall i$. Let us now consider the following relationship:

$$\mathscr{L}(\kappa, p) = \mathscr{L}^e(\kappa, p) + \mathscr{R}(\kappa, p) \quad \forall i = 1, \ldots, n \tag{7}$$

where $\mathscr{R}(\boldsymbol{\kappa}, p)$ is the *Residual-Polarization Curve* corresponding to the distribution $\boldsymbol{\kappa}$. When $\mathscr{L}(\boldsymbol{\kappa}, p)$ is above $\mathscr{L}^e(\boldsymbol{\kappa}, p)$ over all of the domain (i.e. $\mathscr{L}(\boldsymbol{\kappa}, p) \succeq_L \mathscr{L}^e(\boldsymbol{\kappa}, p)$, so that $\mathscr{L}(\boldsymbol{\kappa}, p)$ second-order dominates $\mathscr{L}^e(\boldsymbol{\kappa}, p)$ [3]) then $\sum_{i=1}^{n} \mathscr{R}(\boldsymbol{\kappa}, p) > 0$ and source $\kappa$ is mainly concentrated at the bottom of the distribution; on the contrary, when $\mathscr{L}(\boldsymbol{\kappa}, p)$ is below $\mathscr{L}^e(\boldsymbol{\kappa}, p)$ over all of the domain then $\sum_{i=1}^{n} \mathscr{R}(\boldsymbol{\kappa}, p) < 0$ and the opposite situation holds (i.e. $\mathscr{L}^e(\boldsymbol{\kappa}, p)$ second-order dominates $\mathscr{L}(\boldsymbol{\kappa}, p)$). In the case of zero polarization of income sources, Equation 5 becomes:

$$\mathscr{G} = 1 - \frac{1}{n} \left( \sum_{i=1}^{n} \left( \sum_{j=1}^{i} \beta_j + \sum_{j=1}^{i-1} \beta_j \right) \right) \tag{8}$$

which is also equivalent to:

$$\mathscr{G} = 1 - \frac{1}{n} \left( \sum_{i=1}^{n} \left( \sum_{j=1}^{i} \alpha_j + \sum_{j=1}^{i-1} \alpha_j \right) \right) \tag{9}$$

The Gini coefficient in this particular case can thus be written as a function of individuals' relative shares of any one income source.

[3] The symbol $\succeq_L$ stands for *Lorenz-dominates*: see Cowell (2000) for further details.

# 4 Empirical Application

In this section we apply the method described above to 2014 data from the Survey on Household Income and Wealth carried out by the Bank of Italy.[4] Capital income is defined as the sum of real-estate income, which includes actual rents and imputed rents, net self-employment income and income from financial assets, which latter is the sum of interest on deposits, interest on government securities, income from other securities and interest payments.

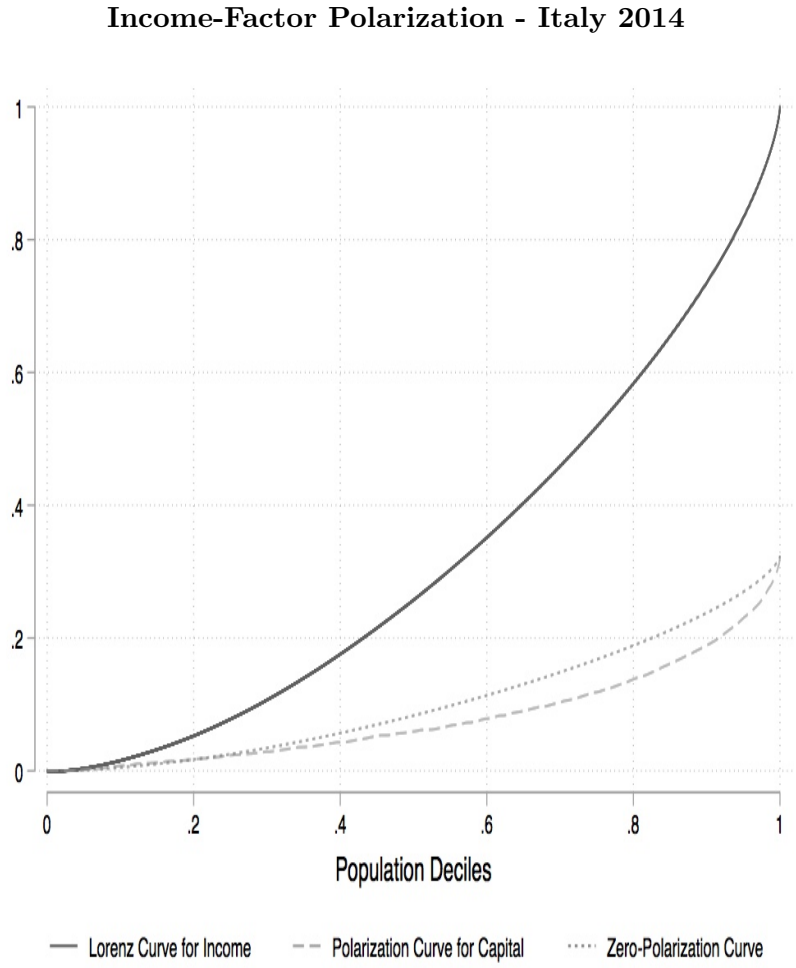



Figure 2: The polarization curve for profits (dashed line), the zero-polarization curve (dotted line) and the Lorenz curve for income (continuous line) for Italy in 2014 are presented using data from the 2014 *Survey on Household Income and Wealth* carried out by the Bank of Italy.

[4]Banca d'Italia, Indagine sui bilanci delle famiglie italiane, Indagine sul 2014.

Labor income is the sum of payroll income and pensions and net transfers. The former includes net wages, salaries and fringe benefits, while the latter covers pensions, arrears, financial assistance, scholarships, and alimony and gifts. For the sake of simplicity, net self-employment income is not considered in the analysis. Figure 2 depicts the Lorentz curve for income $\mathscr{L}(\mathbf{y}, p)$ (the unbroken line), the polarization curve for profits $\mathscr{L}(\boldsymbol{\pi}, p)$ (the dashed line) and the zero-polarization curve $\mathscr{L}^e(\boldsymbol{\pi}, p)$ (the dotted line). As can be seen from the graph, the red and green lines are distant from each other, meaning that income sources are polarized over the distribution of income. As the polarization curve for profits lies below the zero-polarization curve for almost for all population deciles, capital income is more concentrated in the top of the income distribution (thus, $\sum_{i=1}^{n} \mathscr{R}(\boldsymbol{\pi}, \frac{i}{n}) < 0$), and so wages are concentrated at the bottom. However, this result should be taken with caution. The amount of capital income may be underestimated in the survey, and so therefor may be the concentration of capital at the top of the distribution.

## 5 Conclusion

We here proposed a method to tackle the issue of income-source polarization. The polarization curve for profits and wages are introduced, together with the zero-polarization curve. This latter is constructed using a particular definition of the zero polarization of income sources across the population. We then used the 2014 Survey on Household Income and Wealth from the Bank of Italy to provide an empirical illustration of the method proposed.

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# A Curve Estimates

| *Decile* | $\mathscr{L}_\pi$ | $\mathscr{L}_y$ | $\mathscr{L}_e$ |
|---|---|---|---|
| (1) | 0% | 3% | 1.5% |
| (2) | 0% | 5% | 2.5% |
| (3) | 0% | 8% | 4% |
| (4) | 2% | 13% | 6.5% |
| (5) | 4% | 20% | 10% |
| (6) | 6% | 27% | 13.5% |
| (7) | 9% | 35% | 17.5% |
| (8) | 13% | 45% | 22.5% |
| (9) | 20% | 60% | 30% |
| (10) | 50% | 100% | 50% |