Conditionals and Legal Reasoning. Elements of a Logic of Law

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Abstract: The main aim of this paper is to study the notion of conditional right by means of constructive type theory (CTT) which provides the means to develop a system of contentual inferences rather than of syntactic derivations. Moreover, in line with Armgardt, we will first study the general notion of dependence as triggered by hypotheticals and then the logical structure of dependence specific to conditional right. I will develop this idea in a dialogical framework where the distinction between play-object and strategy-object leads to the further distinction between two basic kinds of pieces of evidence and where meanings is constituted by the interaction of obligations and entitlements.

The present paper is based on Rahman (2015). However, though the underlying CTT-analysis is the same, the dialogical reconstruction makes use of a new way of linking dialogical logic and CTT.

Introduction:

Sébastien Magnier (2013) provides a remarkable analysis of the notion of conditional right[2] that he generalizes for the logical study of legal norms. The main idea of Magnier, motivated by the exhaustive precedent textual and systematic work of Matthias Armgardt (2001, 2008, 2010)[3] and the subsequent studies carried out by Alexandre Thiercelin (2001, 2008, 2010) [4], involves Leibniz' notion of *certification,* which has a central role in the famous *De conditionibus*. According to Magnier, the *certification* of the antecedent of a sentence expressing a conditional right such as in *If a ship arrives, Primus must pay 100 dinar to Secundus,* is linked to an epistemic understanding of evidence: in our example, the *certification of the arrival of a ship* amounts to *there is public evidence for the arrival of a ship* and this amounts to *being in possession of the knowledge required to produce a piece of evidence for the arrival of a ship*. Moreover, inspired by Kelsen's conception of legal norms Magnier generalizes his own approach and this leads him to both reject a material-implication-approach[5] and to reconstruct conditional right and legal norms in the frame of a dialogical formulation of dynamic epistemic logic that includes sentences where a *public announcement operator* occur. In other words, Magnier's contribution consists in a shift of perspective on the semantics of truth-dependence underlying the meaning of conditional rights: the main idea is to identify the epistemic dynamics involved in the fulfilment of the

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[2] The terminology used in the present paper follows the one of Leibniz rather than the one in use in nowadays Law-contexts.
[3] The work of Matthias Armgardt launched and influenced a host of new researches on the bearings of Leibniz's approach to nowadays studies in legal rationality.
[4] In fact, Thiercelin researches have been triggered by the work of Armgardt.
[5] In fact Magnier (2013, pp. 151-157, 261-292) rejects other forms of implication readings too including strict implication or connexive implication.

condition as constituting the core of the meaning of dependence specific to the notion of conditional right. He implements this shift by the means of a dynamic epistemic logic called *Public Announcement Logic* (PAL).

The dialogical frame furnishes a further development of this dynamic by furnishing a dynamic theory of meaning In a nutshell: the meaning of *If a ship arrives, Primus must pay 100 dinar to Secundus* boils down to fixing the conditions of a legal debate where *Secundus* claims the 100 dinar, given that the arrival of a ship has been *certified* (i.e. given that *it is known that a ship arrived*, or given that *there is evidence of the arrival of a ship*) - rather than rendering this meaning by the means of a model-theoretic semantics. More generally, the meaning of the notions of conditional right and legal norm is fixed by means of identifying the main logical features of those argumentative interactions that are deployed in legal trials. This leads Magnier to design specific logical language games (dialogues) that yield a theory of meaning rooted in the legal practice itself.

We certainly endorse the idea that a theory of meaning involving legal reasoning should be based on an argumentative-based-semantics, that an epistemic approach to the notion legal evidence should have a central role in a theory of legal reasoning, and that implication is not really at stake in the logical analysis of conditional right. However, we think that the role of evidence should be pushed forward and developed to a general epistemic theory of meaning where evidence is understood as an object that makes a proposition true. More precisely, we think that we should explore the possibilities of placing the piece of evidence that grounds a proposition (the object that makes the proposition true) at the object language level and not via the formal semantics of an operator that introduces that evidence via the metalogical definitions of a formal (model-theoretical) semantics. That a proposition is true is supported by a piece of evidence, but this piece of evidence must be placed in the object language if that language is purported to have content. This move seems to be particularly important in the context of legal trials where acceptance or rejection of legal evidence is as much as part of the debate as the main thesis is. More generally the notion of legal evidence should be linked to the meaning of a proposition not only of an operator occurring within a proposition.

The main aim of the present paper is to study the notion of conditional right by means of constructive type theory (CTT) according to which propositions are sets, and proofs are elements. That a proposition is true means the set has at least one element. The analysis of legal norms should follow as a generalization the details of which are not the subject of the present paper. In such a frame the logical structure of sentences expressing conditional rights is analyzed as corresponding to the one of hypotheticals - rather than as implications. The proof-objects that make the implications of the hypothetical true are *pieces of evidence dependent upon the evidence for the condition (*i.e. *dependent upon the evidence for the head of the hypothetical*). Herewith we follow Thiercelin's (2009, 2010) interpretation that makes of the notion of dependence the most salient logical characteristic of Leibniz' approach to conditional right. Moreover, in line with Armgardt (2001, pp. 220-25) we will study the general notion of dependence as triggered by hypotheticals and then the logical structure of dependence specific to conditional right. However, on my view, the dependence of the conditioned to the condition is defined on the pieces of evidence that support the truth of the hypothetical rather than in the propositions that constitute it. According to this analysis, the famous example for a conditional right:

If a ship arrives, then Primus must pay 100 dinar to Secundus

has the form of the hypothetical

Primus must pay 100 dinar to Secundus, provided there is some evidence x for the arrival of a ship

And this means

The evidence *p* for a payment-obligation that instantiates the proposition *Primus must pay 100 dinar to Secundus* is dependent on some evidence *x* for a ship arrival

Furthermore, the *general logical structure* of the underlying notion of dependence yields:

$p(x) : P\ (x : S)$

where, *x* is a yet not known element of the set of arrivals *S* (i.e. $x : S$), and
where the evidence for a payment-obligation (the piece of writing that establishes the conditional right) is dependent on the arrival *x* of a ship, i.e., the evidence for payment-obligation is represented by the a function $p(x)$.

In this setting, when there is knowledge of some ship arrival *s*; the variable will be substituted by *s*.

Still, the logical structure $p(x) : P\ (x : S)$ represents the more general case of dependence triggered by an underlying hypothetical form and that is common to all those right-entitlements that are dependent upon a proviso clause – such as the *requirements clause* of statutory right-entitlements or the *condition clause* of conditional right-entitlements. Moreover, a further deeper analysis requires an existential quantification embedded in a hypothetical of the sort:

If (∃w : S)Arrive(w) ***true****, then Pay (100 dinar, primus, Secundus)* ***true***[6]

Even this deeper analysis does not seem to fully capture the *future contingency* of those conditions that build conditional rights. Nevertheless, this formalization $p(x) : P\ (x : S)$ already provides a general formal approach to the notion of dependence that, as pointed out by Armgardt (2001, pp. 221-25) seems to be in line with Leibniz's (A VI; we, pp. 235) own approach to the generalization of right-entitlements by means of hypotheticals.

In relation to the specificity of conditional right, Leibniz, himself defended on one hand a biconditional reading of the notion of dependence[7] and on the other hand the uncertainty about the fulfilment of the condition at the moment of the formulation of a (legally valid) concrete case of conditional right-entitlement. [8]
If we consider explicitly the underlying epistemic and temporal structure in the way that Granström (2011, pp. 167-170) tackles (in the CTT-frame) the issue on future contingents, a biconditional formalization specific to *Leibniz's notion of* condition-dependence is possible.[9]

[6] This has been suggested by Göran Sundholm in a personal email.
[7] The biconditional reading relates to the link between condition and conditioned – Leibniz calls this feature of the conditional right *convertibility*. It is not clear if, on Leibniz's view, the biconditional reading only applies to condtional right.
[8] This seems to be rooted in actual legal practice: If the condition *A* is not satisfied, the benefactor is not entitled to *B*. The actuality of this feature of the Leibnizian approach to the notion of conditional right has been defended by nowadays scholars of Law theory such as Koch / Rüßmann (1982, p. 47) and more thoroughly by Armgardt (2001, 2008, 2010).
[9] Cf. Rahman/ Granstöm forthcoming.

As a matter of fact, Aristotle's chapter of the *Perihermeneias* on the sea-battle naturally leads to Leibniz's example of the ship. Roughly, the idea behind is that both implications hold:

> *If a ship arrives then, Primus must pay 100 dinar to Secundus, (provided (S or not S) and assuming that the arrival of a ship proves the disjunction).*
>
> *If Primus must pay 100 dinar to Secundus, (provided (S or not S) and assuming that the arrival of a ship proves the disjunction), then a ship arrival is the case.*

However, it seems that a general approach does not require biconditionality after all – at least not in its full extension. In relation to the link between condition and conditioned it only requires an hypothetical conjunction constituted by the following implications:

> *If the condition C is fulfilled then the beneficiary is entitled to the right at stake, assuming that an evidence for C solves the uncertainty (C or not C) underlying the conditional right.*
>
> *If the condition not C is fulfilled then the beneficiary is not entitled to the right at stake, assuming that an evidence for not C solves the uncertainty (C or not C) underlying the conditional right.*

Furthermore, we will develop this idea in a dialogical frame where the distinction between *local reason* and *strategy-object* (*or proof-object*) leads to the further distinction between two basic kinds of pieces of evidence such that strategy-evidences are made of play-evidences. The proposed approach includes the study of *formation rules* that model the argumentation on the acceptance of a piece of evidence.

we do not claim having captured all the complex issues related to the notion of legal evidence but the aim is to give more precision to the logical and semantic place it should have in legal reasoning in general and in conditional right in particular.

The present work has been structured in two main parts and two longer chapters that present the main traits of the formal background of parts we and II.

> In the first part: **Leibniz's Logical Analysis of the Notion of Conditional Right and Beyond** we propose a study of the notion of conditional right by means of the CTT approach to hypotheticals based on Leibniz's logical analysis.
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> In the second part: **Dialogical Logic and Conditional Right** we will develop the analysis of the precedent chapter in an appropriate dialogical frame. By means of this we adopt Magnier's idea that the pragmatist-semantics of dialogical logic can capture (some of) the properties that Leibniz ascribes to a notion of conditional right rooted in actual legal practice.
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> **The Formal Background we**. Herewith we will establish a link between Dialogical Logic and Constructive Type Theory by discussing (briefly) the theory of meaning underlying both approaches. A theory of meaning where sign and object are to be

thought as constituting an unity and where, accordingly, the object (piece of evidence) that makes a proposition true and the proposition itself are both explicitly introduced in the object language.

The **Formal Background II: Standard Dialogical Logic** develops Dialogical Logic for CTT.

1. Leibniz's Logical Analysis of the Notion of Conditional Right and Beyond

At the early age of 19 years Leibniz's launched a logical analysis of the notion of conditional right that provided insights which still inspire nowadays researches in the field of legal reasoning. The main aim of the present chapter is to delve into those insights in order to gather new perspectives for the nowadays understanding of the logical structures underlying the legal meaning of the notion of conditional right. Herewith we follow on one hand Magnier's (2013) remarkable study that elucidates the role of epistemic evidence in Leibniz's analysis and on the other Thiercelin's (2008, 2009, 2010) interpretation – based on the precedent textual and systematic work by Matthias Armgardt (2001, 2008, 2010) - that makes of the notion of dependence the most salient logical characteristic of Leibniz' approach to conditional right. However, we will depart from Armgardt's, Thiercelin's and Magnier's approach to the extent that we will develop my proposal in the frame of a CTT-formulation of hypotheticals.

1.1 Leibniz' Logical Analysis of Conditional Right

During the period 1664-1669 the young Gottfried Wilhelm Leibniz (1646-1716) studied the theory of law with the prolific creativity that made him famous. It is during this period that Leibniz developed his theory of conditional right in two main texts that furnished the content of two academic dissertations:

(1) The *Disputatio Juridica (prior) De Conditionibus* (A VI we, pp. 97-150) that was defended in July and August 1665. At that time Leibniz was a 19-year-old student, bearing the title of Master of Philosophy since February 1663 thanks to his *Disputatio Metaphysica De Principio Individui* defended in December 1662.

(2) The *Disputatio Juridica (posterior)De Conditionibus* (A VI we, pp. 97-150)) which is part of Leibniz's *Specimina Juris* (1667-1669),

A modified version is given in The *Specimen Certitudinis Seu Demonstrationum In Jure, Exhibitum In Doctrina Conditionum* (A VI we, pp. 367-430) which is part of Leibniz's *Specimina Juris* (1667-1669), a compilation and reformulation of three of his already held disputations: the *Disputatio Inauguralis De Casibus Perplexis In Jure*, that granted Leibniz the doctoral degree in November 1666. The prior and posterior disputations constitute the main source of the present discussion.

1.1.1 Suspension as Dependence

As pointed out by Thiercelin (2008, 2009, 2010) the main point of the work on conditional right of the young Leibniz is to provide a logical analysis of the juridical modality called *suspension* (the terminology stems from the Roman jurists), which should stress the specificity of conditional right in relation to other conditional propositions such as those of geometry or those expressing causal necessity. The novelty of Leibniz's approach, developed as an answer to puzzles raised by the Roman jurists[10], is to understand the modality of suspension as affecting the condition of a conditional proposition. According to Leibniz, the notion of condition (and its modality) relevant for the study of conditional right should be studied in the context of its role of affecting the truth of *proposition* that expresses some legal

[10] Pol Boucher (2008) discusses thoroughly the Roman sources of Leibniz' own developments.

right provided by an individual agent (the benefactor or *arbiter*) in favour of a second individual (the beneficiary). The effect of the (suspensive) condition δ of a conditional right is that its beneficiary is entitled to a certain right if the condition δ (fixed by the benefactor) is fulfiled.

The approach underlying Leibniz's proposal is that the role of the notion of condition specific to conditional right is that of introducing a dependence-relation such that the *truth of the proposition* that expresses the conditioned is said to be dependent upon the *truth of the condition*. That is, on Leibniz's view, what, from a logical point of view, *suspension* is about: the truth of a proposition is *dependent* on the truth of a given condition fixed by the arbiter (benefactor).

Thus, Leibniz's analysis of the notion of conditional right is based on a logical study of propositions and herewith, as thoroughly discussed by Armgardt (2001; 2008, 2010), the ancient links between logic and law are implemented in a novel way.[11] In fact, Leibniz (A, VI, we, p. 101) searches for a logical system that makes legal reasoning *almost as certain as that of mathematical demonstrations*. Now, the logical form of a proposition the closest to this analysis is that of conditional sentences that express some specific type of hypotheticals. Hypotheticals that formalize a conditional right are constituted by an antecedent that Leibniz (A, VI, we, p. 235) calls the *fact* and a consequent that he calls *jus*. Thus, Leibniz' main claim is that hypotheticals such as

> *If a ship arrives from Asia, then Primus must pay 100 dinar to Secundus*

provide an appropriate approach to the meaning of juridical formulations such as

> *Secundus right to receive 100 dinar from Secundus is suspended until a ship arrives from Asia*".

However, as already mentioned, Leibniz would like to distinguish those hypotheticals that formalize conditional rights – he calls them *moral conditionals* - from other forms of hypotheticals that share some logical and semantics properties with moral conditionals.[12] In order to do so Leibniz fixes logical, epistemic and pragmatic properties that should characterize moral conditionals: suspension has also epistemic and pragmatic features specific to the legal meaning of moral conditionals. Let us discuss briefly each of these separately, though, as we will see below, these levels are interwoven in crucial ways.

1.1.1a Truth-Dependence and Convertibility

On Leibniz's view the main logical property of moral conditionals is, as already mentioned above, that of the *dependence* of the truth of the **jus** (the consequent of the hypothetical) on the **fact** (the antecedent of the hypothetical). Now this, dependence has been introduced by the will of the *arbiter* (the benefactor) and has thus also a pragmatic (Leibniz calls it *moral*) feature. The pragmatic outcome of the creation of such a form of dependence is that if the condition is not fulfilled there is no ground for the legal claim.[13]

Now, since, according to Leibniz's view, the logical understanding of dependence amounts to the truth-dependence of the **jus** on the **fact**, Leibniz drives the conclusion that the logical

[11] See also Schepers (1975).

[12] For a thorough discussion on Leibniz's view on the links between moral conditionals and hypotheticals of geometry and causal necessity see Vargas (2008).

[13] Cf. Leibniz's (A, VI, we, p. 375). For a discussion on this issue see Thiercelin (2010, p 207).

structure of moral conditionals is such that if the antecedent of the hypothetical is false so is also its consequence. However, since he also takes contraposition to be part of the axioms that characterize moral conditionals, formally speaking, the notion of dependence leads him to concede that moral conditions are – from the pure logical form – biconditionals. Leibniz ((A, VI, we, p. 375)) summarizes this by saying that condition and conditioned of moral conditionals are *convertible*.[14] Our author certainly sees that convertibility, from the pure logical viewpoint, might blur the crucial difference between condition (**fact**) and conditioned (**jus**). Leibniz's (A, VI, we, p. 112) strategy out of the dilemma is to point out that, though, formally speaking antecedent and consequent of moral conditionals are convertible, from a legal point of view the pair **fact-jus** is analogue to the pair **cause-effect** and though in hypotheticals expressing a relation of cause-effect antecedent and consequent *are accomplished together*, the *condition starts to exist first*. Notice that, for its own, the analogy threatens to undermine the claim that convertibility is the specific property of moral condition. A way to further develop Leibniz's response is to stress that, according to this approach, the logical convertibility of moral conditions is the effect of a specific act of will that provides the hypothetical with legal meaning. Thus, if we were to adopt this viewpoint, we should claim that what makes of some hypotheticals moral conditions is that the legal meaning of the underlying conditional right grounds the dependence between condition and conditioned. Hence the difference between hypotheticals that express a **cause-effect** and hypotheticals that express conditional right is to be found in the meaning on the basis of which the respective dependencies are defined: while **cause-effect** dependence is defined on some notion of natural necessity **fact-jus** dependence is defined on the will of the of arbiter in such a way that the dependence of the **jus** upon the **fact** is the result of legal acts acknowledged as such by competent authorities. Thus, in general, it is not the dependence itself but it is the meaning of the notion of dependence involved what distinguishes moral conditionals from other hypotheticals. Following such a path requires a thorough description of how meaning triggers the targeted notion of dependence. Armgardt (2001, pp. 362-363) points out that Leibniz's logic of legal reasoning underlies an (incipient) *Conceptography*. Unfortunately, Leibniz does not develop– at least not explicitly – the link between the notion of dependence and the logic of concepts. Nevertheless, it might argued that Leibniz's argument on the interrelation between condition and conditioned mentioned above delivers the elements for linking the logical structure of conditional right with the logic of concepts intrinsic to this notion.

Be that as it may, Pol Boucher (2008) seems to think that though in the context of legal reasoning Leibniz continues to be a rationalist who looks for general patterns of inference, he adopts here some sort of Gricean procedure. According to this interpretation dependence is a logical property –manifested by convertibility – but the difference between condition and conditioned is a presupposition of legal practice. Actually, it looks that this is in agreement with the standard legal practice even nowadays. This practice seems to furnish the basis of Koch / Rüßmann (1982, p. 47) defence of the biconditional reading of the dependence of the **jus** upon the **fact** and of Armgardt's (2001) further careful study Actually, as mentioned in the introduction and as we will discuss in 2.2 below, a sophisticated form of Lebiniz's take on biconditionality can be worked out that articulates the interaction between meaning and logic features discussed afore. This seems to relate to Marcelo Dascal's (2008a) findings of a *soft-rationality* in Leibniz work. According to my view on soft-rationality, Dascal's interpretation suggests that in the context of legal reasoning it is crucial to see that the notion of rationality behind this kind of reasoning is the result of the interaction between meaning and logic features, or more generally between syntax, semantic, pragmatic and epistemic features – and

[14] For a further discussion on the criticism of the biconditional rendering of the suspensive modality see Magnier (2013, pp. 155-156).

this connects with Armgardt's remark on the interaction between legal reasoning and Conceptography in Leibniz's work on the logic of Law.

Alexandre Thiercelin (2008, 2009, 2010) proposes to tackle the issue on convertibility by means of connexive logic. The proposal is sensible since connexive logic has its roots in the Stoic tradition that was certainly known by Leibniz. The axiom of connexive logic relevant to the formalization of moral conditionals is

$(A \Rightarrow B) \Rightarrow \neg(\neg A \Rightarrow B)$ (where "⇒"is the connexive conditional)[15]

This is quite close to the convertibility without falling into the total convertibility of condition and conditioned. Indeed the point is that the moral condition *If A, then B* (connexively) implies that is not the case that if the condition is not fulfiled the *jus* is true. Moreover, the moral conditional *If A, then B* also implies that it is not the case that if the condition is fulfiled the *jus* is false:

$(A \Rightarrow B) \Rightarrow \neg(A \Rightarrow \neg B)$ (where "⇒"is the connexive conditional)

The problem with this conditional is that its semantics departures importantly from standard classical logic, it is not even a conservative extension of it.[16] Furthermore, as signalized by Thiercelin (2010, 208-211) and criticized by Magnier (2013, pp. 157-159) the epistemic component has to be added in some ad hoc manner.

Let us now study some of the epistemic features of moral conditions

1.1.1b Suspension and its Epistemic Nature

Suspension has a crucial epistemic feature, very well known in Roman Law, explicitly discussed by Leibniz and has become part of the definition of conditional right in nowadays legal systems, namely: the legal validity of a concrete case of conditional right-entitlement requires a situation where the fulfilment of the condition is not yet known at the moment of its formulation. In fact, as stressed by nearly all the scholars on Leibniz's work on the logic of law, one of the mains original contributions of Leibniz is to have linked this epistemic feature with a conditional logical structure. Some Roman jurists, for example, connected the non-fulfilment of the condition with existential issues that should account for this non-fulfilment. Leibniz's logical solution is clear and simple: suspension amounts to the dependence of the truth of the conditioned upon the truth of the condition combined with the uncertainty about the truth value of the condition. This assumes that though the hypothetical expressing a conditional right might be true, the truth value of the condition might not be yet known to be true. The lifting of the suspension amounts to what Leibniz (A VI, we, 424) calls the *certification* of the fulfilment of the condition (that is, the production of an evidence for the

[15] Rahman/Rückert (2001, Rahman/Redmond 2008, pp. 20-57) provided a dialogical semantics for it based on the idea that this conditional is a particular kind of strict implication (defined in S4) where the head is satisfiable and the negation of the tail is also satisfiable. See also Pizzi/Williamson (1997), Priest (1999), Wansing (2005 and 2006).

[16] Notice that according to Rahman/Rückert semantics though that $A \Rightarrow A$ is connexively valid the implication $(A \Rightarrow A) \Rightarrow (A \Rightarrow A)$ is not. For further non-classical features of their semantics see Rahman/Rückert (2001, pp. 106-108; 120-121).

fulfilment of the condition) Moreover, as we will discuss in 2.2 Leibniz's solution can be linked to the temporal structure underlying assertions on future contingents.

Magnier (2013a, pp. 141-187, 2013b,) proposal is based on a shift of perspective on the truth-dependence underlying conditional rights: the main idea is to identify the epistemic dynamics involved in the fulfilment of the condition as constituting the core of the meaning of dependence specific to the notion of conditional right. Thus, truth-dependency is, on Magnier's view, a consequence of the epistemic nature of suspension. Magnier implements this shift by means of the use of *Public Announcement Logic* (PAL). Indeed, the PAL-approach to conditional right allows Magnier to describe how the initial model (defined for a PAL-sentence expressing a given conditional right) that might include scenarios where the condition is fulfiled and scenarios where it is not, changes when it is known that the condition is fulfiled (: the initial model shrinks to a model that only contains scenarios that fulfil the condition)[17]. Furthermore, Magnier's work does not only provide a new formal frame for capturing the dynamics inherent to logical structure of the notion of conditional right, it also highlights, so far as we know for the first time, another aspect of the epistemic nature of the condition of conditional right, namely that the *certification* of the fulfilment of the condition must be object of public knowledge.[18] This aspect of the epistemic nature of the condition has been inspired by Kelsen and the PAL-reconstruction allows Magnier to express in the same frame all the epistemic features, namely:

(1) the uncertainty about the fulfilment of the condition at the moment of the formulation of a concrete case of (legally valid) conditional right-entitlement.
(2) the epistemic dynamics triggered after the fulfilment of the condition. The dynamics also cares of the temporal dimension on the notion of suspension signalized by Armgardt (2001, pp. 349-351).
(3) the requirement of a public knowledge about the evidence for such a fulfilment.

Magnier's approach also can also deal with the truth dependence in way that involves some subtle distinctions between a *false announcement* in relation to the fulfilment of the condition and the assertion that the condition is false. In fact, in the PAL-framework, there is no way to express a false announcement at the object language level. Leibniz's certification of the condition corresponds to asserting it, such that if we certify that *A is false* we assert that *non-A is true.* And this is certainly different from performing a false announcement. If there is a false announcement, the epistemic updating process gets, so to say, aborted and hence the truth-value of the whole PAL-sentence cannot be established. From the view point of the legal practice a false announcement corresponds to making it public that the condition has been fulfiled while it is not, and hence, presumably, either we are driven back to the initial situation where the condition has not been yet fulfiled or the whole obligation expressed by the conditional right is declared to be null and void.[19]. Certainly, this is different from certifying

[17] $[\varphi]\psi$ *is true at the evaluation world* **s** *iff* φ *is true at* **s** *implies that* ψ *is true at the reduced model* $\mathbf{M}|\varphi$ *– where the reduced model* $\mathbf{M}|\varphi$ *is the result of removing from* **M** *all the worlds where* φ *is false:* $\mathbf{M}, s \models [\varphi]\psi$ iff $\mathbf{M}, s \models \varphi$ *implies* $\mathbf{M}|\varphi, s \models \psi$.

[18] Actually; this epistemic requirement for the fulfilment of the condition has been already pointed out by Thiercelin (2009b, p. 141), however it has not been incorporated in the logical analysis of the conditional before de work of Magnier.

[19] The case of the cancellation corresponds to the one of ¨PAL, since, actually, false announces cannot be performed: the system aborts. The first case, where a false announcement drives us back to the initial models, corresponds to the *Total Public Announcement Logic* (TPAL) of Steiner / Staud (2007), where false announcements do not change the initial model at all. Indeed, take it that it has been announced that *A* is true. Then, if we come to know that that *A* was not true after all, the original PAL process stops there since the model

the falsity of the condition: in this case it is the truth-value of the tail (Magnier calls it the *post-condition*) of the PAL-sentence that will not follow. The dialogical game of the certification of the falsity of the condition shows that the Proponent will win, but he will win his thesis about the truth of the whole PAL-sentence without engaging at all on any assertion involving the post-condition. Independently of the distinctions discussed afore, in such a frame, if the PAL-sentence is true then it cannot happen that the post-condition B will be evaluated as true though the condition A cannot be announced (because A is false). Putting all together the PAL-approach to conditional right yields the following rendering of the truth-dependence:

The condition is true iff PAL-sentence expressing the conditional right is true

Still there are some arguments coming from legal practice for the biconditional reading of dependence. The point is that if the condition is false, then the claim for the right involved in the conditional right at stake will be rejected. If we follow Magnier's approach the analysis will yield the following: since in this frame it cannot happen that the post-condition can be evaluated as true without the condition being true, and the jury must evaluate the post-condition as true in order to ascribe the right claimed by the benefactor. Hence, in the case that the condition is false, the jury will not be able to evaluate the post condition as true (there will be no grounds to support the post-condition) and reject the claim. However, as pointed out before, the falsity of the condition A does not entail that $[A]B$ is false. This corresponds to some cases of legal practice where though the beneficiary might not be entitled to the claimed right, this does not mean that the conditional right is not legally valid.

Perhaps, if we would like to continue the PAL-path to conditional right after all, we might formulate its logical form as the conjunction

$$[S]\boldsymbol{O}P \wedge [\neg S]\neg\boldsymbol{O}P$$

or

$$[S]\boldsymbol{OK}P \wedge [\neg S]\boldsymbol{K}\neg\boldsymbol{O}P$$

(or some other combination of modal operators in the tail of the PAL-sentence)

Notice that $[S]\boldsymbol{O}P$ and $[\neg S]\neg\boldsymbol{O}$ are not contradictory: the submodel for the left PAL-sentence contains worlds were S is true, the submodel for the right PAL-sentence excludes those that worlds where S is true, so, after the update, both submodels will contain different worlds in an in no of them we have S and not S .

If we would further on incorporate the uncertainty underlying the notion of conditional right the following formulation seems to be appealing:

$$((\boldsymbol{K}S \vee \boldsymbol{K}\neg S) \rightarrow [S]\boldsymbol{O}P) \wedge ((\boldsymbol{K}S \vee \boldsymbol{K}\neg S) \rightarrow [\neg S]\neg\boldsymbol{O}P)$$

or

$$((\boldsymbol{K}S \vee \boldsymbol{K}\neg S) \rightarrow [S]\boldsymbol{KO}P) \wedge ((\boldsymbol{K}S \vee \boldsymbol{K}\neg S) \rightarrow [\neg S]\boldsymbol{K}\neg\boldsymbol{O}P)$$

has been shrunk and now there is no way to come back. However, one can think of a system, such as TPAL, that allows once we come to know that A is not true, to came back to the model before it was shrunk, since the grounds adduced for reducing it in the first place, are not available any more. This might also relate to Armgardt's recent work (2013= on the defeasibility of the grounds adduced for establishment of a fact (in our case the defeasibility of the grounds adduced for the fulfilment of condition).

Now, besides the logical and epistemic features underlying the structure of notion of conditional right there are also pragmatic aspects that contribute to the, so to say, moral aspect of the suspensive modality.

1.1.1c Suspension and its Pragmatics

Conditional rights are structures with legal content. It is the content that interacts with some features of the underlying logical and epistemic structure. As discussed above, epistemic features are essential to the (legal) definition of conditional right. But this content and the validity of concrete conditional right-entitlement is also determined by pragmatic features that qualify conditional right as conditional and not as some other kind of right-entitlement under assumption. The most decisive of pragmatic features is the one that determines that the attribution of a conditional right to some beneficiary is due to the sole will of a benefactor (and not of the legislator). This assumes that the arbiter should be factually and legally able to ascribe the conditional right at stake and that condition and conditioned meet specific legal requirements. Sometimes the underlying legal meaning of the conditional right hinges on the envisaged target of the arbiter in relation to the fulfilment of the condition: the ultimate goal of engaging in a particular conditional right might be directly dependent on the arbiter's interest of motivating the beneficiary to fulfil a given condition. The pragmatic features also interact with the logical structure of the conditional right-entitlement attributed to a given benefactor. For example, legal systems will rule out impossible or unlawful conditions and similarly for the **jus**-part of a conditional right. Hence, it seems sensible to require that condition and the conditioned are logically and factually possible: it does not make any legal sense to formulate a conditional right involving a logical truth or a logical falsity.
Thiercelin (2011, p. 213) remarks that scholars in the field do not seem to have paid very much attention to these practical aspects of the notion of conditional right, despite the fact that Leibniz (A VI, we, 409, 422) himself makes a careful study of the cases of what Roman jurists called *ridiculous* conditions. On my view, the point here is once more the interaction of content with logical structure; however, as mentioned in A.1, the standard model theoretical approach to semantics places this interaction at the metalogical level. A clear example of this metalogical viewpoint is the semantics of PAL deployed by Magnier, where the whole epistemic dynamics manifests itself in the formal semantics of the model, and the latter is metalogically defined. What we need is a language where we can check at the object language level the meaning of a given expression and furthermore that a given proposition is true. This is linked with the formation plays mentioned above: before we check the truth of a given proposition we need to check its meaning, its legal meaning, on the background of the knowledge of the legal system. Let us explore this line of thought.

1.2 Hypotheticals and Conditional Right

The following approach is based on Leibniz's idea that the most salient characteristic of the logical structure of the notion of conditional right is the truth dependence of the conditioned upon the condition. Moreover, we will adopt Magnier's epistemic shift, though we will propose a new shift that takes us from the epistemic nature of the propositions to the objects (the pieces of evidence) that ground the knowledge required by the notion of suspension.

1.2.1 Hypotheticals and the General Form of Dependence

In the CTT-frame it is possible to express at the object-language level that *A is true*, namely, by means of the assertion *d: A* (*there is a piece of evidence d for A* or *there is a proof-object d for A).*[20] Therefore, in this frame the dependence of the truth of *B* upon the truth of *A* amounts to the dependence of the proof-object of the former to the proof-object of the latter. The dependence of the proof object of *B* upon the proof-object of *A* is expressed by means of the function *b(x)* (from *A* to *B*), where *x* is a proof-object of *A* and where the function *b(x)* itself constitutes the dependent proof-object of *B*. As discussed in AI dependent-proof objects provide proof-objects for hypotheticals, for instance:

b(x) : B (x : A),

that reads, *b(x)* is a (dependent) proof object of *B* provided *x* is a proof object of *A*.

In our context, proof-objects, in principle[21], correspond to pieces of evidence. Thus, the dependence of the truth of the **jus** *B* upon the truth of the condition *A* boils down to the fact that the piece of evidence for *B* is the function *b(x)*.

It follows from this analysis that notion of dependence relevant for Leibniz' famous example for a conditional right:

If a ship arrives, then Primus must pay 100 dinar to Secundus

Can be expressed by means of the hypothetical

Primus must pay 100 dinar to Secundus, provided there is some evidence x for the arrival of a ship

And this means

The evidence *p* for a payment-obligation that instantiates the proposition *Primus must pay 100 dinar to Secundus* is dependent on some evidence *x* for a ship arrival

This would naturally lead to render the underlying logical structure with help of a hypothetical, which roughly amounts to

p(x) : P (x : S)

where, *x* is a yet not known element of the set of arrivals *S* (i.e. *x : S*), and
where the evidence for a payment-obligation (the piece of writing that establishes the conditional right) is dependent on the arrival *x* of a ship, i.e., the evidence for payment-obligation is represented by the a function *p(x)*

A deeper – though not definitive yet -rendering of the logical structure is the following

If (∃w : S)Arrive(w) ***true****, then Pay (100 dinar, primus, Secundus)* ***true***[22]

[20] See appendix AI.

[21] In chapter 2, we will distinguish between local reasons and strategy-objects, while the latter correspond to proof-objects, pieces of evidence or evidences might be either play- or strategy-objects.

[22] This analysis has been suggested by Göran Sundholm in a personal email.

and this then demands a hypothetical proof

$b(x) : Pay\ (100\ dinar, Primus, Secundus)$

under the hypothesis that

$x : (\exists w : S)Arrive(w)$

Even this deeper analysis does not fully capture either the *future contingency* or the *convertibility* of those conditions that build conditional rights. However, as we mentioned in the introduction, in the context of law, generally speaking, this logical structure is shared by all other forms of right-entitlement with proviso clauses such as statutory right-entitlements under *requirements*. According to this analysis all of them share some form of hypothetical structure the meaning of which is provided by dependent proof objects. Further distinctions are necessary in order to distinguish between them. For instance, while requirements for statutory right-entitlements do not demand uncertainty about the satisfaction of these requirements, conditions of conditional right-entitlements, as discussed above, do. Furthermore both have a different origin: while requirements are fixed by the legislator, conditions are fixed by the sole will of the arbiter.

Nevertheless, the study of the general form is desirable from both the logical and the legal point of view. Furthermore, as discussed by Armgardt (2001, pp. 221-25) Leibniz (A VI; we, pp. 235) himself pointed out that hypotheticals provide the general logical form of those right-entitlements where the proviso clause (such a conditions or requirements) correspond to the *antecedent* of the hypothetical and the *consequent* to its **jus**.

According to this analysis, a general and basic form of right-entitlement with a proviso clause that provides the conditions/requirements under which the proposition is made true. This seems to coincide with Leibniz's and nowadays legal terminology, where a right is granted on the occurrence of *fact*. Thus, in line with this analysis, the logical structure of such kind of right-entitlements *is not* that of a proposition but that of a hypothetical that binds assertions (or judgments)[23], in such a way that the assertion of the **jus** is made dependent on the assertion of the condition. For instance:

The point of the formalization is that we can formulate explicitly at the object language level that the pieces of evidence for the fulfilment of the antecedent of the hypothetical are not yet known, namely, by the use of variables. It is the variables for pieces of evidence that make of right-entitlements with proviso clause hypotheticals. More precisely, in the context of CTT, the variable in an hypothetical such as $p(x) : P\ (x : S)$ stand for *an unknown element of S*, that can be instantiated by some *s* when the required knowledge is available.[24] Thus, in this frame, instantiating the *unknown* element *x* by some *s known* to be a fixed (but arbitrary) element of *S* is what the Leibniz' notion of *lifting the suspension* is about. In analogy to nowadays terminology of epistemic logic in the style of Hintikka (1962) where we say that a judgment of the form

$x : S$

expresses *belief* rather than *knowledge* and that

[23] Recall that a judgement or assertion expresses that a proposition is true, the assertion *A is true* introduces an epistemic feature: it is known that *A* is the case. Furthermore, judgements can also involve sets: *1 is an element of the set of Natural numbers* (in the CTT-notation: $1 : N$).

[24] Cf. Granström (2011, pp. 110-112). In fact, the whole chapter V of Granström (2011) contains a thorough discussion of the issue.

$s : S$

represents the passage from belief to knowledge, we say in our context of discussion that this also might represent the passage from a right-entitlement under *the hypothesis (or belief) S to be case (i.e. x : S)* to *a right-entitlement grounded by the knowledge that the condition/requirement S has been satisfied (i.e., s : S)*. In fact for this passage to count as passage to knowledge, it is not only necessary to have $s : S$, but it is also necessary that it is known that the piece of evidence s (a concrete ship arrival) is the piece of evidence of the adequate sort.[25] In other words, we also need to have the definition

$x=s : S$

This definition of x can be called an *anchoring* of the hypothesis (belief) S in the *actual* world.[26] Thus, the result of this anchoring-process yields

$p(x=s) : P\ (s : S)$

If there are more than one hypotheses (including interdependences – temporal or otherwise - between them: a requirement for statutory right-entitlements can be dependent on other requirements, similar holds for conditions of conditional right-entitlements[27]), it is not required that all the variables will be substituted at once. It is possible to think of a gradual reduction of uncertainty by gradual introduction of definitions of the variables – in the case of temporal interdependences the graduality of the fulfilment is determined by a fixed order. A general formulation of this kind of passage[28] is the following, where Γ and Δ are hypotheses that represent some kind of proviso (such as conditions or requirements) for right-entitlements:

$\Gamma = (x_1 : A_1, \ldots x_n : A_n)$ becomes
$\Delta = (\Gamma, x_k = a : A_k)$

So that in the new hypotheses every occurrence of x_k is substituted by a. The new hypothesis Δ is obtained from Γ by removing the hypothesis $x_k : A_k$ by $a(x_1 \ldots x_n)$. Thus, as required, this operation reduces the uncertainty within the original hypothesis.

Let us now start to study the path that goes from the general to the specific.

1.2.2 Granting Statutory and Conditional Rights

The main features that distinguishes statutory right from conditional right are

(a) the uncertainty concerning the fulfilment of the condition of conditional rights – that does not apply to the requirements-proviso of statutory rights;

[25] Cf. Ranta (1985, pp. 151-154).
[26] Cf. Ranta (1985, p. 152).
[27] Armgardt (2001, p. 256) studies different kind of interdependences between conditions discussed by Leibniz (A VI, we pp. 387-388) including temporally ordered conjunctions and subsidiary ones such as *If it is known that the condition A cannot be fulfiled then the condition B should be fulfiled*. Armgardt (2001) makes ample use here and in other parts of his book of temporal indexes.
[28] This passage is known in CTT as *definitional extension of hypotheses or contexts*.

(b) the (ontological) type of the individual that grants the correspondent right. While the individual that grants a conditional right is a person (natural or legal), the individual that grants a statutory right is a legislator.

Let us turn our attention to (b) - we already discussed (a) above and we will study it further on in the next section. The CTT-frame has the means to make explicit at the object language level, the ontological type of the individual that grants a certain right. These means are related to the formation (and Predicator rules). Recall that the CTT-frame claims that syntactic and meaning traits are to be processed at the same time and both of them occur at the object language level. For instance, before proving the logical validity of sentence, it is required to display its content, and the latter amounts to ascribe to each part of the sentence the adequate type – and when appropriate, to identify the canonical elements of the correspondent sets. In our case, let us have the following hypothetical:

> $b(x): B\ (x : A)$ – recall that $b(x)$ is the dependent object that constitutes one of the piece of evidences for the hypothetical

For the sake of simplicity let us for the moment ignore the inner (existentially quantified) structure of *A*. Let us further assume that the piece of evidence $b(c)$[29] – the contract – expresses a statutory right R^S:

> $R^S(b(c))$ *true*

This presupposes that $R^S(y)$ is a proposition, provided $y : B$, that $b(c) : B$, and that $b(c)$ is the result of a substitution in the function $b(x)$ from *A* to *B*. In other words, $(b(x) : B\ (x : A)$ and $c : A$) Moreover, the explicit presentation of the presuppositions involved requires displaying the putative pieces of evidence that might count as acceptable – this might also involve describing the *canonical elements* of the sets involved.

Following a simplified form of legal terminology we say that $b(c)$ is a *statutory right* iff it is granted ($G(y, w, z)$) to person *y* (natural or juridical) by a *legislator* *z*:[30]

> $R^S(b(c))$ *iff* $G(l, b(c), p\ )$ *true*

Where, $G\ y, w, z)$ is a proposition provided $y : legislator, w : B, z : person$) and it is *known* that $l : legislator$, $b(c) : B$ and that $p : person.$

This, in turn, presupposes that the instance *x* of *A* and *w of B* are *neither illegal nor against boni mores* ($\neg M(w)$). Since *w* is some function $b(x)$ defined on *A* it seems to be sufficient to require this restriction on the function only:

> $\neg M(b(c))$ *true*

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[29] For the sake of clarity of exposition we do not quantify on the function dependent objects $b(x)$, however, a full development of the definitions of statutory and conditional right should quantify universally over them.

[30] In fact, as pointed out by Armgardt in a personal email, we need the following parameters: Who grants What, Whom, When, and based on Which legal norm. we did not add all parameters in order not to avoid a heavy notation. Let me mention that in the CTT-frame the introduction of temporal indexes in the object language is pretty straight-forward – see Ranta (1994, pp. 101-124).

Where *M(w)* is a proposition, provided *w* : *B* propositions, and it is known that *b(c) : B*.

In fact, from the point of view of law, granting happens independently of knowing if the proviso has been satisfied. Thus, we need to go deeper into the structure of the proviso in order to achieve generality:

B ***true*** $(\exists v : V)\ A(v))$ ***true***
b(x) : B, provided
$x : (\exists v : V)\ A(v))$

This yields the general form of a grant

$R^S(b(x))$ *iff* $G(l, b(x), p)$ *true*

Thus the following should be included to the list of presuppositions:

b is a function from $x : (\exists v : V)\ A(v))$ to *B.*

The explicit presentation of presuppositions by means of formation rules seems to be very natural to a legal trial. This is one of the main motivations of the use of a dialogical frame

Similarly, let us now say that *b(c)* expresses an instance of a *conditional right* iff it is granted (*G(...)*) to a person *y* (natural or juridical) by a *person z.* Indeed the difference between statutory and conditional right is, in this respect, the type of the individual that provides the grant: a *person* in the latter case and a *legislator* in the former.

A last tricky point, concerns the *closing* of the engagement expressed by the statutory/conditional right once the proviso has been fulfiled by *one* instance. we will come back to this issue in chapter 2. Let us study first the logical form of conditional rights.

1.2.3 The Specificity of Conditional Rights: Uncertainty With and Without Biconditionals[31]

One of the most difficult issues on the logical structure of conditional right relates to the fact that the obligation expressed by the **jus** is made dependent on the occurrence of a future, uncertain *event*. In other words on the occurrence of a future contingent event – the example of the ship recalls almost explicitly Aristotle's' see-battle case. On the other hand, as discussed above, Leibniz's approach and nowadays legal practice seems to lead to the idea that convertibility is at the core of the logical form of conditional rights: In the context, once more of ship- example: If we know that a ship does not come, it looks as if should be infer that it is not the case that *Primus* must pay. The point is here to find a formalization that makes explicit these two crucial features of conditional rights.

However it seems that, though the logical form of conditional right requires that if it is known that the condition will ever be fulfiled, then the right-entitlement should fail; it *does not*

[31] The following developments are based on Rahman/Granström (2013), forthcoming.

require that if the beneficiary is entitled to the right involved, the condition has been fulfiled. In fact in 1.2.3b we claim that biconditionality is not necessary after all.

1.2.3a The Logical Form of Leibniz's Approach: Uncertainty and the Biconditional

In order to implement this double task, we will supply the head of the hypothetical with a richer structure than the one discussed above. More specifically, we take it that the head of hypotheticals underlying conditional right have the form of a constructivist disjunction. That is, a disjunction, such that proof-object of it, amounts to indicating explicitly which of both obtains. Thus the head looks like:

$$x : S \vee \neg S$$

(there is some piece of evidence *x* for the disjunction *a ship arrival is the case or not)*

Since we are in the frame of a constructive disjunction, its truth requires that we know which of both obtains: in our case we need to know that that ship did arrive. Let express this with

$$(\exists y : S)\ (y = x)$$

(there is some ship arrival and this ship arrival constitutes the evidence for the disjunction – i.e. it is equal to the evidence x for the disjunction $S \vee \neg S$)

However, the above notation does not explicitly express that *y* constitutes an evidence for the left part of the disjunction. *It is precisely the left part of the disjunction that represents the condition required by the conditional right of our example*. In order to do so we need to make use of the function *left(y)*. This yield:

$$(\exists y : S)\ (left(y) = x)$$

Clearly, the arrival of a ship (which constitutes the fulfilment of the condition) implies that *Primus* must pay. That is,

$$a(x) : (\exists y : S)\ (left(y) = x) \rightarrow P\ (x : S \vee \neg S)$$

Moreover, since we postulate that *Primus* must pay if the disjunction is true, and since we established that the disjunction is true if a ship arrives (and not the contrary), it follows that a payment obligation dependent of the disjunction, implies the arrival of a ship:

$$b(x) : P\ (x : S \vee \neg S) \rightarrow (\exists y : S)\ (left(y) = x)$$

If will pull all together we obtain what it could be taken as the logical form underlying *Leibniz's notion of conditional obligation:*

$$(\exists y : S)\ (left(y) = x) \leftrightarrow P\ (x : S \vee \neg S)$$

The proof object of which is an object dependent upon *x*:

$$d(x) : (\exists y : S)\ (left(y) = x) \leftrightarrow P\ (x : S \vee \neg S)$$

However, *x* is the evidence for the fulfilment of the condition. Thus

$$d(left(y)) : (\exists y : S)\ (left(y) = x) \leftrightarrow P\ (x : S \vee \neg S)$$

Still; this seems to express the biconditionality of the condition in relation to the whole hypothetical, can we also prove the *convertibility* of condition and conditioned? Indeed, this is the case, as we show in the following paragraph:

Let us show that if the condition is false, also the conditioned. That is, let us show that if $\neg S$ *true*, it is also the case that $\neg P$ *true*. Let assume that we have a dependent-proof object for the biconditional:

$$d(x) : (\exists y : S)\ (left(y) = x) \leftrightarrow P\ (x : S \vee \neg S)$$

If $\neg S$ *true*, then the evidence x for the disjunction consists in some z : $\neg S$, such that *x* is *right(z).* Thus, by substitution we have:

$$d(right(z)) : (\exists y : S)\ (left(y) = right(z)) \leftrightarrow P\ (x : S \vee \neg S)$$

The left part of the biconditional is clearly false, and thus also right part, and hence $\neg P$ *true*. Thus, if $\neg S$ *true*, it is also the case that $\neg P$ *true*, as required.

1.2.3b Uncertainty and Convertibility without Biconditional

Assume that, there have been two conditional agreements. One makes the obligation to pay 100 dinar dependent upon the arrival of a ship and the other, dependent on another condition, say, a caravan from Asia arrives. In such a case the biconditional seems to be a too strong requirement, since, it might be that we know that no ship will ever arrive, but this does not mean, that *Secundus* has not payment obligation. Leibniz (A VI, we, p. 388) discussion on disjunctive conditions might provide a kind of solution: from this viewpoint the logical structure of the condition is in fact a disjunction and this, as pointed out by Armgardt (2001, pp; 267-269) can be embedded in a biconditional structure. The problem with this solution is that every conditional right-agreement must be completed with, perhaps not yet known, disjunctive elements such as *If a ship or a caravan or ... or ... arrives, then P*. Moreover, if there are more agreements, it is not fixed, if we have to add the conditions of the different agreements as disjunctions or conjunctions or if we have to interpret the different agreements as different conditional-right-entitlements – in the latter case, *Secundus* might be entitled to 200 dinar: 100 when a ship arrives, other 100 when a caravan does. Nowadays lawyers tend to follow Leibniz, who was an active lawyer, and switch to a pragmatic strategy. Confronted with such cases, the court decides on view of the best possible interpretation of the benefactor's will.

A different possibility is to give up the (full) biconditional structure, and propose the following hypothetical conjunction.

$$d(x) : (((\exists y : S)\ (left(y) = x)) \rightarrow P) \wedge ((\exists z : \neg S)\ (right(z) = x)) \rightarrow \neg P))\ (x : S \vee \neg S)$$

That reads:

*If there some evidence for a ship arrival and this arrival solves the uncertainty (S or not S) underlying the conditional right, (i.e., if the ship arrival provides an evidence for the **left side** of the disjunction) then the beneficiary is entitled to the right at stake.*

*If there is some evidence for no ship arrival and this solves the uncertainty (S or not S) underlying the conditional right, (i.e., if the evidence for no ship arrival provides an evidence for the **right side** of the disjunction) then the beneficiary is not entitled to the right at stake.*

In fact, this (hypothetical) conjunction of implications seems to be the most suitable formalization of the logical and epistemic structure underlying the notion of conditional right.

Indeed, such a conjunction ensures that *if it is known that a ship will ever arrive there is no obligation to pay*, but what will be blocked is that *If Primus is obliged to pay, then a ship did arrive* – clearly, *Primus*, might be obliged to pay because a caravan arrived even if a ship did not. This approach leaves it still open if *Primus* must pay twice 100 dinar if both, a caravan and a ship arrived. However, it is precisely the blocked implication that does render justice to the possibility to interpret two agreements (with the same **jus** but with two different conditions) as two different conditional entitlements or not, without assuming some tacit or retroactive enrichments of the logical form involved. Certainly, by contraposition on the implication of the right side of the conjunction we obtain that $\neg\neg P \rightarrow \neg\neg S$. However this is not a full biconditional: it only says that if it is impossible that a payment obligation is not due, then it is impossible that a ship-arrival did not happen.

1.2.3d Interpreting Conditions

In legal practice, interpretation is crucial. Not only in order to apply the general norm to a particular case (see end of 1.2.1) but also in order to complete or further elucidate terms of a given formulation. In the case of conditional right, the interpretation processes based on the logical structure of the underlying hypothetical are of three kinds:

(1) application to a particular case
(2) extension of the set of conditions, interdependent or not (this includes interdependencies induced by temporal order
(3) making precise the structure of the evidence required to fulfil the condition

The first two cases have been already discussed at the end of 1.2.1. Thus let us turn our attention to the third case. Assume that the formulation of the conditional right has been left pretty vague in relation to the condition to the fulfilled in order to obtain *B*, to take a simple case:

$$d : B\ (u : A \vee \neg A)$$

The interpretation process will consist here in an extension to the new context for example:

$$B\ (x : \neg A)$$

By means of a mapping, such that:

$$B\ (u = right(x) : A \vee \neg A\ )$$

This provides the necessary information to formulate precisely the logical form of the conditional right:

$$d(u) : (((\exists y : A)\ (left(y) = x)) \rightarrow \neg B) \wedge ((\exists z : \neg A)\ (right(z) = x)) \rightarrow B))\ (x : A \vee \neg A)$$

In fact the third case generalizes the others. These kind of interpretation-processes consist in a mapping that extends the original context into a new one with less uncertainty. [32]

2 Dialogues, Play-objects, and the Dynamics of Conditional Right-Entitlements

Besides the general aim of developing a pragmatist semantics for legal reasoning rooted in its specific argumentative practices the dialogical setting provides insights in the dynamics underlying the meaning of the notion of conditional right. Indeed, the language games typical of dialogical logic, the dialogues, distinguishes the *play level* from the *strategy level*,[33] and this distinction, as discussed below, allows to study the dynamics of a trial involving a particular instance of conditional right.

During a trial, there are purely logical moves and others that are not. In relation to the latter, there are moves concerning the legal validity of the original conditional right-entitlement, for instance the validity of a given contract involves questions of content. There might be also moves that question some pieces of evidence, and finally there might also be moves concerning the closing of a trial in front of the presented evidence.

The following sections contain a development of each of these points; however they will be preceded by a general presentation of dialogical local reasons for conditional right. Notice that the dialogical plays are to work as language games, and not descriptions of actual practices. In other words they are purported to be, in Wittgenstein's words, measurement rods, constructions by means of which understanding and insight might be gathered – it is remarkable that, in order to explain this point, Wittgenstein brought forward the use of the reconstruction of a fact in a trial.

2.1 Dialogues with Local Reasons as Language Games for Trials on Conditional Right

The following form of play is the simplest one and only involves the entitlement claim of the *ship example* once the legality of the contract and the piece of evidence for the

[32] Cf. Granström (2011, chapter V).

[33] See appendix II.

fulfilment of the condition (the ship-arrival *s*) have been accepted.[34] Moreover, the premise will not be the one with the conditional form but the one that does not assume convertibility. However, in the context of how the play described below is being developed, the difference between the conjunctive and the biconditional form is irrelevant. Indeed, in the play to be developed below the Proponent chooses the side of the conjunction that involves the implication from the non-negative condition to the conditioned, and this is implication occurs in both, Leibniz's biconditional sentence and in the one without it. Thus, the premises are:

(I) $d(x) : (((\exists y : S)\ (left(y) =_S x)) \rightarrow P) \wedge ((\exists z : \neg S)\ (right(z) =_S x)) \rightarrow \neg P))\ (x : S \vee \neg S)$

(II) $s : S$.

The proponent, *Secundus*, claims, that, grounded on I and on the piece of evidence *s* of a ship arrival S (II), he is entitled to the payment *P* of 100 dinar by *Primus*. Thus, the thesis is:

$$p : P$$

Notation:
Recall that *S* stands for the set of ship-arrivals; *P* for *Primus must pay 100 dinar to Secundus*, *s* for the arrival of the ship *s*, and *p* a concrete payment obligation, grounded on the ship arrival *s*.

In Dialogical Logic *left(y)* is written *L(y)*. In order to differentiate between the left side of a disjunction and a conjunction exponentials will be added similar applies to *right(y)*. Accordingly the main sentence is rewritten as:

$$d(x) : (((\exists y : S)\ (L^{\vee}(y) =_S x)) \rightarrow P) \wedge ((\exists z : \neg S)\ (R^{\vee}(z) =_S x)) \rightarrow \neg P))\ (x : S \vee \neg S)$$

Convention: For the sake of legibility the development of the present play only records only one challenge on the local reason, namely the one of the thesis.

	O			P	
I II	$d(x) : (((\exists y : S)\ (L^{\vee}(y) =_S x)) \rightarrow P) \wedge ((\exists z : \neg S)\ (R^{\vee}(z) =_S x)) \rightarrow \neg P))\ (x : S \vee \neg S)$ $s : S$			$!\, P$	0.0
0.1	$n{:=}1$			$m{:=}2$	0.2
0.3	$?reason$	0.0		$p : P$	0.4
1	$p = ?$	0		$R^{\rightarrow}(a_1) = p\ : P$ **[LAST MOVE: P WINS]**	30
7	$d(s) : (\exists y : S)\ (L^{\vee}(y) = s) \rightarrow\ P \wedge$ $((\exists z : \neg S)\ (R^{\vee}(z) =_S x \rightarrow \neg P)$		I	$q : S \vee \neg S$	2
3	$?\ ^{\vee}$	2		$L^{\vee}(q) : S$	4
5	$L^{\vee}(q) = ?$	4		$s : S$	6
9	$a : (\exists y : S)\ (L^{\vee}(y) =_S L^{\vee}(s)) \rightarrow\ P$		7	$\ldots / d(s) = ?$	8
11	$L^{\wedge}(a) : (\exists y : S)\ (L^{\vee}(y) =_S L^{\vee}(s)) \rightarrow\ P$		9	$?_L$	10

[34] The formation play will not be developed here (see AII).

13	$a_1: (\exists y : S) (L^{\vee}(y) = s) \to P$		11	$\ldots / L^{\wedge}(a) = ?$	12
27	$R^{\to}(a_1) : P$		13	$L^{\to}(a_1) : (\exists y : S) (L^{\vee}(y) =_S s)$	14
15	$\ldots / L^{\to}(a_1) = ?$			$a_2 : (\exists y : S) (L^{\vee}(y) =_S s)$	16
17	$?_L{}^{\exists}$	16		$L^{\exists}(a_2) : S$	18
19	$\ldots / L^{\to}(a_2) = ?$			$s : S$	20
21	$?_R{}^{\exists}$	16		$R^{\exists}(a_2)) : L^{\vee}(s) =_S s$	22
23	$--- / L^{\vee}(s) ?$	22		$R^{\exists}(a_2)) : s=_S s$	24
25	$--- / R^{\exists}(a_2)) = ?$	18		**refl**$(A, a) : s=_S s$	26
29	$p : P$		27	$R^{\to}(a_1) = ?$	28

The reader might wish to check that if the premise is that a ship arrival is not the case, a dual play can be developed in favour of the benefactor: the point is now to make use of the right side of the initial conjunction (premise I).

Notice that this is not still a winnings strategy. In order to do so, we should show that the series of moves of this play is one the terminal series that will always lead to a win.[35] Now; certainly during a play, like in the practice of legal trials, "silly" or "logically not optimal" moves are always possible: At move 20 for example the Proponent might have had chosen *b* as a substitution for *y:* that is, for whatever reasons, the Proponent, might have brought forward a ship arrival different of *s* as piece of evidence This, is logically a weak move since both the antagonists have already agreed that *s* has been *certified* (to make use of the words of Leibniz). New pieces of evidence introduced during the play, are totally procedural and might be contested. This brings us to the next section.

2.2 Formation Plays and on How to Challenge Pieces of evidence

In relation to the specific content of a given instance of a conditional right, it definitely concerns the development of *formation* plays where the legality of the terms constituting the contract can be questioned. The development of this kind of formation plays involves displaying not only the elements of its logical structure but also the elements required by its legal validity, such as: who granted the conditional right to whom, when, and if it is clear that it fulfils the requisite of not being either illegal or against *boni mores* (see 1.2.2) Now, it might be the case that the elements mentioned above might be introduced *during* a trial, and they might be contested on the spot. However although a strategic view point requires an overview of all the possible plays[36]: this does not seem to be either necessary or desirable.
In fact, Magnier (2013a), discusses in several parts of his book in the context of a dialogical reconstruction of conditional right the case where a proposition has been certified and where it has been introduced so to say *for the sake of the discussion*. Accordingly he distinguishes between two types of plays: those that are formal (purely procedural) and those that are not material. If we apply this very useful distinction for the local reasons that furnish the pieces of evidence for elementary posits, we distinguish between *procedural* and *not procedural pieces* of evidence. The latter amounts assuming that one player introduces a new piece of evidence that was not discussed or agreed at the start of the play. In such a case, a new formation play

[35] For the notion of strategy see II.6 (it contains the definitions for standard dialogical logic) and II.3 (for the definition in the context of CTT)
[36] See appendix II.

might start asking for its *exact typing*, that is, for the description of the type it belongs too – roughly, for the description of the fact or proposition it is purported to support. In general, we might indeed distinguish between both kinds of evidence. If the typing is correct and legally valid, the antagonist might accept it, for the sake of the discussion. However the antagonist might reject it and demand an examination of the new piece of evidence. If the result of such an examination is not clearly cut it might lead to the introduction of *presumptions* with formulations such as: *it is presumed, in the absence of evidences for the contrary, that the piece of evidence a furnished a suitable local reason for B*. Moreover, perhaps in practice the force and suitability of every piece of evidence is the result of a presumption. In such a case the difference between purely procedural and not procedural might lose its clear edges and we might require some kind of non-monotonic approach – if we are willing to go down that path. But the place to develop such a kind of approaches should start at the level of the formation plays. Certainly, formation plays and their correspondent main plays are intimately linked: if a piece of evidence might be contested in the middle of a play, the following logical moves might require some revision. This is where the recent works of Armgardt (2013a, 2013b) on Presumptions, of Gabbay/Woods (2012b) on the relevance of pieces of evidence, and of Magnier (2013a) on burden of the proof can be linked with the frame developed in the present work. However, once more, this is facilitated by the play level, where pieces of evidence might not be considered indisputable facts, and therefore might be rejected. we cannot develop here this link and this is part of future work, where the recent work of Giuseppe Primiero on Belief-Revision in the context of CTT seems to furnish a useful tool, but let me point out that formation plays involved might involve a rich structure: it might involve issues on legality – see 1.2.2. The present dialogical approach to CTT provides the semantics place where such content-based challenges can be examined and developed.

2.3 Dialogues and The Closing of Conditional Right-Entitlements

Let us assume that one piece of evidence has been rejected – as in the precedent paragraph, and that in front of the production of a new piece of evidence a new trial starts again. However, once the condition has been fulfiled and the claim has been judged as legally valid the process is closed. No new piece of evidence will – under normal circumstances – entitle once more the beneficiary to a new claim in the context of the same conditional right contract at stake. For instance, coming back once more to the ship: If a ship arrived and it has been decided that the 100 dinar must be paid to *Secundus*, in general, a new arrival will not entitle *Secundus* to new 100 dinar: The win of a play by the beneficiary, who made use, during the play, of a particular instance of the condition (a particular ship arrival), may not win again, with a new arrival, under the sole ground that the condition is existentially quantified.

Also in this case, it is the play level that provides the right insight. Plays might be classified by their *repetition rank.* As presented in II.6, repetition ranks fix how many times a player can defend or challenge the same formula. Let us assume that the Proponent agreed that, for an existentially quantified condition one instance is sufficient. To make it more concrete, assume that the benefactor, *Primus*, agreed that one ship arrival, is sufficient for the fulfilment of the condition that entitles *Secundus* to a payment obligation. Within such a repetition rank (namely 1), there might be a logically infinite number of different plays, each of them satisfying the condition with a different ship arrival. But the point is that the closing of a trial on such a right-entitlement is defined on a play!

Closing of a trial is modelled by the end of a given play within the context of a fixed rank. Moreover, if the play is lost by the benefactor-party because, for example, the purported piece of evidence was not such, this does not preclude that another trial can be run when a new piece of evidence is brought forward.

3 Conclusion

The central problem with the use of subjectively grounded opinions is its impotence when drawn into disagreements. Nothing is advanced in our disagreement by my putting forward opinions that you reject.[37] This certainly applies to legal debates and it seems to be at the root of Leibniz's interest in the logic of Law and his efforts towards a unification of the fields involved. As pointed out in several occasions by John Woods one of the best unification results of interdisciplinary work is one in which the offspring does not owe its identity to the one of its parents, though it certainly carries the marks of them: *It produces results that are very much worth having and which neither parent is able to deliver by its own*[38].
The dialogic approach is born from the idea that the rational way of overcoming disagreements is by the means of an interactive understanding of reasoning and meaning. Meaning is, according to the dialogical approach, constituted by and within interaction. Now, in relation to the interdisciplinary amity of law and dialogical constructive logic, it is perhaps still too early to make a definitive assessment in relation to the achievement of the kind of unification described by Woods. However, it seems save to be fairly optimistic. The very point is that if the field of legal reasoning should have an own identity, this must be based on an approach to meaning that provides insights into the structure of legal argumentative practices. The Dialogical approach to Constructive Type Logic furnishes a setting where the legal content shapes the resulting formal system in some specific ways. The proposed unifying dialogical setting harbours further features that constitute also new challenges: namely the study of direct and indirect evidence and the epistemic dynamics specific to the legal notion of evidence: this strongly suggests that the work on presumptions mentioned above is one of the tasks to be tackled next.

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[37] Cf. Woods (2003, pp. 325-326)
[38] Gabbay /Woods (2012, p. 196)

APPENDIX: LOCAL REASONS AND DIALOGUES FOR IMMANENT REASONING[39]

Introductory remarks on the choice of CTT

Recent developments in dialogical logic show that the Constructive Type Theory approach to meaning is very natural to the game-theoretical approaches in which (standard) metalogical features are explicitly displayed at the object language-level.[40] This vindicates, albeit in quite a different fashion, Hintikka's plea for the fruitfulness of game-theoretical semantics in the context of epistemic approaches to logic, semantics, and the foundations of mathematics.[41]

From the dialogical point of view, the actions—such as choices—that the particle rules associate with the use of logical constants are crucial elements of their full-fledged (local) meaning: if meaning is conceived as constituted during interaction, then all of the actions involved in the constitution of the meaning of an expression should be made explicit; that is, they should all be part of the object-language.

This perspective roots itself in Wittgenstein's remark according to which one cannot position oneself outside language in order to determine the meaning of something and how it is *linked to syntax*; in other words, language is unavoidable: this is his *Unhintergehbarkeit der Sprache*, one of Wittgenstein's tenets that Hintikka explicitly rejects.[42] According to this perspective of Wittgensteins, language-games are supposed to accomplish the task of studying language from a perspective that acknowledges its *internalized feature*. This is what underlies the approach to meaning and syntax of the dialogical framework in which all the speech-acts that are relevant for rendering the meaning and the "formation" of an expression are made explicit. In this respect, the metalogical perspective which is so crucial for model-theoretic conceptions of meaning does not provide a way out. It is in such a context that Lorenz writes:

> *Also propositions of the metalanguage require the understanding of propositions, […] and thus cannot in a sensible way have this same understanding as their proper object. The thesis that a property of a propositional sentence must always be internal, therefore amounts to articulating the insight that in propositions about a propositional sentence this same propositional sentence does not express a meaningful proposition anymore, since in this case it is not the propositional sentence that is asserted but something about it.*
>
> *Thus, if the original assertion (i.e., the proposition of the ground-level) should not be abrogated, then this same proposition should not be the object of a metaproposition […].*[43]
>
> *While originally the semantics developed by the picture theory of language aimed at determining unambiguously the rules of "logical syntax" (i.e. the logical form of linguistic expressions) and thus to justify them […]—now language use itself, without the mediation of theoretic constructions, merely via "language games", should be sufficient to introduce the talk about "meanings" in such a way that they supplement the syntactic rules for the use of ordinary language expressions (superficial grammar) with semantic rules that capture the understanding of these expressions (deep grammar).*[44]

Similar criticism to the metalogical approach to meaning has been raised by Göran Sundholm (1997; 2001) who points out that the standard model-theoretical semantic turns semantics into a meta-mathematical formal object in which syntax is linked to meaning by the assignation of truth values to uninterpreted strings of signs (formulae). Language does not express *content* anymore, but it is rather conceived as a system of signs that speak *about* the world—provided a suitable metalogical link between the signs and the world has been fixed. Moreover, Sundholm (2016) shows that the cases of quantifier-dependences motivating Hintikka's IF-logic can be rendered in the CTT framework. What we will here add to Sundholm's observation is that even the interactive features of these dependences can be given a CTT formulation, provided the latter is developed within a dialogical setting.

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[39] The appendix is based in the book in preparation *Immannet Reasoning*, by Rahman/Klev/McConaughey/Clerbout (2018). .

[40] See for instance Clerbout/Rahman (2015), Dango (2016), Rahman/Clerboout/Jovanovic (2015).

[41] Cf. Hintikka (1973).

[42] Hintikka (1996) shares this rejection with all those who endorse model-theoretical approaches to meaning.

[43] (Lorenz, 1970, p. 75), translated from the German by Shahid Rahman.

[44] (Lorenz, 1970, p. 109), translated from the German by Shahid Rahman.

Ranta (1988) was the first to link game-theoretical approaches with CTT. Ranta took Hintikka's (1973) Game-Theoretical Semantics (GTS) as a case study, though his point does not depend on that particular framework: in game-based approaches, a proposition is a set of winning strategies for the player stating the proposition.[45] In game-based approaches, the notion of truth is at the level of such winning strategies. Ranta's idea should therefore in principle allow us to apply, safely and directly, instances of game-based methods taken from CTT to the pragmatist approach of the dialogical framework.

From the perspective of a general game-theoretical approach to meaning however, reducing a proposition to a set of winning strategies is quite unsatisfactory. This is particularly clear in the dialogical approach in which different levels of meaning are carefully distinguished: there is indeed the level of strategies, but there is also the level of plays in the analysis of meaning which can be further analysed into local, global and material levels. The constitutive role of the play level for developing a meaning explanation has been stressed by Kuno Lorenz in his (2001) paper:

> *Fully spelled out it means that for an entity to be a proposition there must exist a dialogue game associated with this entity, i.e., the proposition A, such that an individual play of the game where A occupies the initial position, i.e., a dialogue D(A) about A, reaches a final position with either win or loss after a finite number of moves according to definite rules: the dialogue game is defined as a finitary open two-person zero-sum game. Thus, propositions will in general be dialogue-definite, and only in special cases be either proof-definite or refutation-definite or even both which implies their being value-definite.*
>
> *Within this game-theoretic framework […] truth of A is defined as existence of a winning strategy for A in a dialogue game about A; falsehood of A respectively as existence of a winning strategy against A.*[46]

Given the distinction between the play level and the strategy level, and deploying within the dialogical framework the CTT-explicitation program, it seems natural to distinguish between *local* reasons and *strategic* reasons: only the latter correspond to the notion of *proof-object* in CTT and to the notion of *strategic-object* of Ranta. In order to develop such a project we enrich the language of the dialogical framework with statements of the form "$p : A$". In such expressions, what stands on the left-hand side of the colon (here p) is what we call a *local reason*; what stands on the right-hand side of the colon (here A) is a proposition (or set).[47]

The *local* meaning of such statements results from the rules describing how to compose (*synthesis*) within a play the suitable local reasons for the proposition A and how to separate (*analysis*) a complex local reason into the elements required by the composition rules for A. The synthesis and analysis processes of A are built on the formation rules for A.

The rock-bottom of the dialogical approach is still the play level-notion of dialogue definiteness of the proposition. Namely:

- For an expression to count as a proposition A there must exist an individual play about **X** ! A, such that **X** is committed to bring forward a local reason to back that proposition, and the play reaches a final position with either win o loss after a finite number of moves according to definite local and structural rules.

In this section we will spell out all the relevant rules for the dialogical framework incorporating features of Constructive Type Theory—that is, a dialogical framework making the players' reasons for asserting a proposition explicit. The rules can be divided, just as in the standard framework, into rules determining local meaning and rules determining global meaning. These include:

1. Concerning *local meaning* (section 0):
    a. formation rules (p. 28);
    b. rules for the synthesis of local reasons (p. 30); and
    c. rules for the analysis of local reasons (p. 31).
2. Concerning *global meaning*, we have the following (structural) rules (section 0):
    a. rules for the resolution of instructions;
    b. rules for the substitution of instructions (p. 34);
    c. equality rules determined by the application of the Socratic rules (p. 34); and
    d. rules for the transmission of equality (p. **Erreur ! Signet non défini.**).

[45] That player can be called Player 1, Myself or Proponent.
[46] (Lorenz, 2001, p. 258).
[47] See (Rahman, Redmond, & Clerbout, 2017), (Clerbout & Rahman, 2015) (Rahman & Clerbout, 2013; 2015).

Local meaning in dialogues of immanent reasoning

The formation rules

Formation rules for logical constants and falsum

The formation rules for *logical constants* and for *falsum* are given in the following table. Notice that a statement '$\perp$: **prop**' cannot be challenged; this is the dialogical account for falsum '$\perp$' being by definition a proposition.

Table 1: Formation rules

	Move	Challenge	Defence
Conjunction	**X** $A \wedge B: \boldsymbol{prop}$	**Y** ? $F_{\wedge 1}$ or **Y** ? $F_{\wedge 2}$	**X** $A: \boldsymbol{prop}$ (resp.) **X** $B: \boldsymbol{prop}$
Disjunction	**X** $A \vee B: \boldsymbol{prop}$	**Y** ? $F_{\vee 1}$ or **Y** ? $F_{\vee 2}$	**X** $A: \boldsymbol{prop}$ (resp.) **X** $B: \boldsymbol{prop}$
Implication	**X** $A \supset B: \boldsymbol{prop}$	**Y** ? $F_{\supset 1}$ or **Y** ? $F_{\supset 2}$	**X** $A: \boldsymbol{prop}$ (resp.) **X** $B: \boldsymbol{prop}$
Universal quantification	**X** $(\forall x: A)B(x): \boldsymbol{prop}$	**Y** ? $F_{\forall 1}$ or **Y** ? $F_{\forall 2}$	**X** $A: \boldsymbol{set}$ (resp.) **X** $B(x): \boldsymbol{prop}[x: A]$
Existential quantification	**X** $(\exists x: A)B(x): \boldsymbol{prop}$	**Y** ? $F_{\exists 1}$ or **Y** ? $F_{\exists 2}$	**X** $A: \boldsymbol{set}$ (resp.) **X** $B(x): \boldsymbol{prop}[x: A]$
Subset separation	**X** $\{x : A \mid B(x)\}: \boldsymbol{prop}$	**Y** ? F_1 or **Y** ? F_2	**X** $A: \boldsymbol{set}$ (resp.) **X** $B(x): \boldsymbol{prop}[x: A]$
Falsum	**X** $\perp: \boldsymbol{prop}$	—	—

The substitution rule within dependent statements

The following rule is not really a formation-rule but is very useful while applying formation rules where one statement is dependent upon the other such as $B(x): \boldsymbol{prop}[x: A]$.[48]

Table 2: Substitution rule within dependent statements (subst-D)

	Move	Challenge	Defence
Subst-D	**X** $\pi(x_1, \dots, x_n)[x_i: A_i]$	**Y** $\tau_1: A_1, \dots, \tau_n: A_n$	**X** $\pi(\tau_1, \dots, \tau_n)$

In the formulation of this rule, "π" is a statement and "τ_i" is a local reason of the form either $a_i: A_i$ or $x_i : A_i$.

[48] This rule is an expression at the level of plays of the rule for the substitution of variables in a hypothetical judgement. See (Martin-Löf, 1984, pp. 9-11).

A particular case of the application of Subst-D is when the challenger simply chooses the same local reasons as those occurring in the concession of the initial statement. This is particularly useful in the case of formation plays:

Example of a formation-play

Here is an example of a formation play with some explanation. The standard development rules are enough to understand the following plays.

In this example, the Opponent provides initial concession before the Proponent states his thesis. Thus the Proponent's thesis is

$$(\forall x : A)(B(x) \supset C(x)) : \mathbf{prop}$$

given these three provisos that appear as initial concessions by the Opponent:

$$A : \mathbf{set},$$
$$B(x) : \mathbf{prop}\ [x : A]$$
$$\text{and } C(x) : \mathbf{prop}\ [x : A],$$

This yields the following play:

<u>Play 1: formation-play with initial concessions: first decision-option of **O**</u>

	O			**P**	
0.1	A : **set**				
0.2	$B(x)$: **prop** $[x : A]$				
0.3	$C(x)$: **prop** $[x : A]$			$(\forall x : A)\ B(x)\ \supset\ C(x)$: **prop**	0
1	$m := 1$			$n := 2$	2
3	$?\,F_{\forall 1}$	0		A : **set**	4

P wins.

Explanation:

- 0.1 to 0.3: **O** concedes that A is a **set** and that $B(x)$ and $C(x)$ are propositions provided x is an element of A.
- Move 0: **P** states that the main sentence, universally quantified, is a proposition (under the concessions made by **O**).
- Moves 1 and 2: the players choose their repetition ranks.
- Move 3: **O** challenges the thesis by asking the left-hand part as specified by the formation rule for universal quantification.
- Move 4: **P** responds by stating that A is a **set**. This has already been granted with the concession 0.1 so even if **O** were to challenge this statement the Proponent could refer to her initial concession.

This dialogue obviously does not cover all the aspects related to the formation of

$$(\forall x : A)\ B(x)\ \supset\ C(x) : \mathbf{prop}.$$

Notice however that the formation rules allow an alternative move for the Opponent's move 3,[49] so that **P** has another possible course of action, dealt with in the following play.

<u>Play 2: formation-play with initial concessions: second decision-option of **O**</u>

O		**P**

[49] As a matter of fact, increasing her repetition rank would allow **O** to play the two alternatives for move 3 within a single play. But increasing the Opponent's rank usually yields redundancies (Clerbout, 2014a; 2014b) making things harder to understand for readers not familiar with the dialogical approach; hence our choice to divide the example into different simple plays.

0.1 0.2 0.3	A : **set** $B(x)$: **prop** [$x : A$] $C(x)$: **prop** [$x : A$]			$(\forall x : A)\, B(x) \supset C(x)$: **prop**	0
1	$m := 1$			$n := 2$	2
3	? $F_{\forall 2}$	0		$B(x) \supset C(x)$: **prop** [$x : A$]	4
5	$x : A$	4		$B(x) \supset C(x)$: **prop**	6
7	? $F_{\supset 1}$	6		$B(x)$: **prop**	10
9	$B(x)$: **prop**		0.2	$x : A$	8

P wins.

Explanation:

The second play starts like the first one until move 2. Then:

- Move 3: this time **O** challenges the thesis by asking for the right-hand part.
- Move 4: **P** responds, stating that $B(x) \supset C(x)$ is a proposition, provided that x : A.
- Move 5: **O** challenges the preceding move by granting the proviso and asking **P** to respond (this kind of move is governed by a Subst-D rule).
- Move 6: **P** responds by stating that B(x) ⊃ C(x) is a proposition.
- Move 7: **O** challenges move 6 by asking the left-hand part, as specified by the formation rule for material implication.
  To defend against this challenge, **P** needs to make an elementary move. But since **O** has not played it yet, **P** cannot defend it at this point. Thus:
- Move 8: **P** launches a counterattack against initial concession 0.2 by granting the proviso $x : A$ (that has already been conceded by **O** in move 5), making use of the same kind of statement-substitution (Subst-D) rule deployed in move 5.
- Move 9: **O** answers to move 8 and states that $B(x)$ is a proposition.
- Move 10: **P** can now defend the challenge initiated with move 7 and win this dialogue.

Once again, there is another possible choice for the Opponent because of her move 7: she could ask the right-hand part. This would yield a dialogue similar to the one above except that the last moves would be about $C(x)$ instead of $B(x)$.

Concluding on the formation-play example:

By displaying these various possibilities for the Opponent, we have entered the strategic level. This is the level at which the question of the good formation of the thesis gets a definitive answer, depending on whether the Proponent can always win—that is, whether he has a winning strategy. The basic notions related to this level of strategies are to be found in our presentation of standard dialogical logic.

The rules for local reasons: synthesis and analysis

Now that the dialogical account of formation rules has been clarified, we may further develop our analysis of plays by introducing local reasons. Let us do so by providing the rules that prescribe the synthesis and analysis of local reasons. For more details on each rule, see section **Erreur ! Source du renvoi introuvable.**.

Table 3: synthesis rules for local reasons

	Move	Challenge	Defence
Conjunction	**X** ! $A \wedge B$	**Y** ? $L^{\wedge}$ or **Y** ? $R^{\wedge}$	**X** $p_1 : A$ (resp.) **X** $p_2 : B$

Existential quantification	**X** ! $(\exists x : A)B(x)$	**Y** ? $L^{\exists}$ or **Y** ? $R^{\exists}$	**X** $p_1 : A$ (resp.) **X** $p_2 : B(p_1)$
Subset separation	**X** ! $\{x : A \mid B(x)\}$	**Y** ? L or **Y** ? R	**X** $p_1 : A$ (resp.) **X** $p_2 : B(p_1)$
Disjunction	**X** ! $A \vee B$	**Y** ?$^{\vee}$	**X** $p_1 : A$ or **X** $p_2 : B$
Implication	**X** ! $A \supset B$	**Y** $p_1 : A$	**X** $p_2 : B$
Universal quantification	**X** ! $(\forall x : A)B(x)$	**Y** $p_1 : A$	**X** $p_2 : B(p_1)$
Negation	**X** ! $\neg A$ Also expressed as **X** ! $A \supset \bot$	**Y** $p_1 : A$	**X** : $\bot$ (**X** gives up[50]).

Table 4: analysis rules for local reasons

	Move	Challenge	Defence
Conjunction	**X** $p : A \wedge B$	**Y** ? $L^{\wedge}$ or **Y** ? $R^{\wedge}$	**X** $L^{\wedge}(p)^X : A$ (resp.) **X** $R^{\wedge}(p)^X : B$
Existential quantification	**X** $p : (\exists x : A)B(x)$	**Y** ? $L^{\exists}$ or **Y** ? $R^{\exists}$	**X** $L^{\exists}(p)^X : A$ (resp.) **X** $R^{\exists}(p)^X : B(L^{\exists}(p)^X)$
Subset separation	**X** $p : \{x : A \mid B(x)\}$	**Y** ? L or **Y** ? R	**X** $L^{\{\ldots\}}(p)^X : A$ (resp.) **X** $R^{\wedge}(p)^X : B(L^{\{\ldots\}}(p)^X)$
Disjunction	**X** $p : A \vee B$	**Y** ?$^{\vee}$	**X** $L^{\vee}(p)^X : A$ or **X** $R^{\vee}(p)^X : B$
Implication	**X** $p : A \supset B$	**Y** $L^{\supset}(p)^Y : A$	**X** $R^{\supset}(p)^X : B$
Universal quantification	**X** $p : (\forall x : A)B(x)$	**Y** $L^{\forall}(p)^Y : A$	**X** $R^{\forall}(p)^X : B(L^{\forall}(p)^Y)$
Negation	**X** $p : \neg A$ Also expressed as **X** $p : A \supset \bot$	**Y** $L^{\neg}(p)^Y : A$ **Y** $L^{\supset}(p)^Y : A$	**X** $R^{\neg}(p)^X : \bot$ **X** $R^{\supset}(p)^X : \bot$

[50] The reading of stating bottom as giving up stems from (Keiff, 2007).

Slim instructions: dealing with cases of anaphora

One of the most salient features of the CTT framework is that it contains the means to deal with cases of anaphora,[51]

Notice that in the formalization of traditional syllogistic form *Barbara*, the projection **fst**(z) can be seen as the tail of the anaphora whose head is z:

$$\frac{\begin{array}{ll}(\forall z : (\exists x : D)A)B[\mathbf{fst}(z)] \text{ true} & \textit{premise 1} \\ (\forall z : (\exists x : D)B)C[\mathbf{fst}(z)] \text{ true} & \textit{premise 2}\end{array}}{(\forall z : (\exists x : D)A)C[\mathbf{fst}(z)] \text{ true} \qquad \textit{conclusion}}$$

In dialogues for immanent reasoning, when a local reason has been made explicit, this kind of anaphoric expression is formalized through *instructions*, which provides a further reason for introducing them. For example if a is the local reason for the first premise we have

$$\mathbf{P}\ p : (\forall z : (\exists x : D)A(x))B(L^{\exists}(L^{\forall}(p)^{\mathbf{O}}))$$

However, since the thesis of a play does not bear an explicit local reason (we use the exclamation mark to indicate there is an implicit one), it is possible for a statement to be bereft of an explicit local reason. When there is no explicit local reason for a statement using anaphora, we cannot bind the instruction $L^{\forall}(p)^{\mathbf{O}}$ to a local reason p. We thus have something like this, with a blank space instead of the anaphoric local reason:

$$\mathbf{P}\ !\ (\forall z : (\exists x : D)A(x))B(L^{\exists}(L^{\forall}(\ \ )^{\mathbf{O}}))$$

But this blank stage can be circumvented: the challenge on the universal quantifier will yield the required local reason: **O** will provide $a{:}\,(\exists x{:}\,D)A(x)$, which is the local reason for z. We can therefore bind the instruction on the missing local reason with the corresponding variable—z in this case—and write

$$\mathbf{P}!\ (\forall z : (\exists x : D)A(x))B(L^{\exists}(L^{\forall}(z)^{\mathbf{O}}))$$

We call this kind of instruction, *slim instructions*. For the substitution of slim instructions the following two cases are to be distinguished**:**

Substitution of Slim Instructions 1

Given some slim instruction such as $L^{\forall}(z)^{\mathbf{Y}}$, once the quantifier $(\forall z{:}\,A)B(\ldots)$ has been challenged by the statement $a : A$, the occurrence of $L^{\forall}(z)^{\mathbf{Y}}$ can be substituted by a. The same applies to other instructions.

In our example we obtain:

$$\begin{array}{c}\mathbf{P}!\ (\forall z : (\exists x : D)A(x))B(L^{\exists}(L^{\forall}(z)^{\mathbf{O}})) \\ \mathbf{O}\ a : (\exists x : D)A(x) \\ \mathbf{P}\ b : B(L^{\exists}(L^{\forall}(z)^{\mathbf{O}})) \\ \mathbf{O}\ ?\ a / L^{\forall}(z)^{\mathbf{O}} \\ \mathbf{P}\ b : B(L^{\exists}(a)) \\ \ldots\end{array}$$

Substitution of Slim Instructions 2

Given some slim instruction such as $L^{\forall}(z)^{\mathbf{Y}}$, once the instruction $L^{\forall}(c)$—resulting from an attack on the universal $\forall z : \varphi$—has been resolved with $a : \varphi$, then any occurrence of $L^{\forall}(z)^{\mathbf{Y}}$ can be substituted by a. The same applies to other instructions.

Global Meaning in dialogues for immanent reasoning

We here provide the structural rules for dialogues for immanent reasoning, which determine the global meaning in such a framework. They are for the most part similar in principle to the precedent logical framework for dialogues; the rules concerning instructions are an addition for dialogues for immanent reasoning.

[51] See Sundholm (1986, pp. 501-503) and Ranta (1994, pp. 77-99).

Structural Rules

SR0: Starting rule

The start of a *formal dialogue of immanent reasoning* is a move where **P** states the *thesis.* The thesis can be stated under the condition that **O** commits herself to certain other statements called *initial concessions*; in this case the thesis has the form ! A [B_1, …, B_n], where A is a statement with implicit local reason and $B_1, \dots, B_n$ are statements with or without implicit local reasons.

A dialogue with a thesis proposed under some conditions starts if and only if **O** accepts these conditions. **O** accepts the conditions by stating the *initial concessions* in moves numbered 0.1, … 0.*n* before choosing the repetition ranks.

After having stated the thesis (and the initial concessions, if any), each player chooses in turn a positive integer called the *repetition rank* which determines the upper boundary for the number of attacks and of defences each player can make in reaction to each move during the play.

SR1: Development rule

The Development rule depends on what kind of logic is chosen: if the game uses intuitionistic logic, then it is SR1i that should be used; but if classical logic is used, then SR1c must be used.

SR1i: Intuitionistic Development rule, or Last Duty First

Players play one move alternately. Any move after the choice of repetition ranks is either an attack or a defence according to the rules of formation, of synthesis, and of analysis, and in accordance with the rest of the structural rules.

If the logical constant occurring in the thesis is not recorded by the table for local meaning, then either it must be introduced by a nominal definition, or the table for local meaning needs to be enriched with the new expression.[52]

Players can answer only against the *last non-answered* challenge by the adversary.

Note: This structural rule is known as the Last Duty First condition, and makes dialogical games suitable for intuitionistic logic, hence the name of this rule.

SR1c: Classical Development rule

Players play one move alternately. Any move after the choice of repetition ranks is either an attack or a defence according to the rules of formation, of synthesis, and of analysis, and in accordance with the rest of the structural rules.

If the logical constant occurring in the thesis is not recorded by the table for local meaning, then either it must be introduced by a nominal definition, or the table for local meaning needs to be enriched with the new expression.

Note: The structural rules with SR1c (and not SR1i) produce strategies for classical logic. The point is that since players can answer to a list of challenges in any order (which is not the case with the intuitionistic rule), it might happen that the two options of a ***P****-defence occur in the same play—this is closely related to the classical development rule in sequent calculus allowing more than one formula at the right of the sequent.*

SR2: Formation rules for formal dialogues

A formation-play starts by challenging the thesis with the formation request **O** $?_{\mathbf{prop}}$; **P** must answer by stating that his thesis is a proposition. The game then proceeds by applying the formation rules up to the elementary constituents of **prop**/**set**.

After that the Opponent is free to use the other particle rules insofar as the other structural rules allow it.

Note: The constituents of the thesis will therefore not be specified before the play but as a result of the structure of the moves (according to the rules recorded by the rules for local meaning).

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[52] If the logical constant occurring in the thesis is not recorded by the table for local meaning, then either it must be introduced by a nominal definition based on some logical constant already present in the local rules, or the table for local meaning needs to be enriched with the new expression.

SR3: Resolution of instructions

1. A player may ask his adversary to carry out the prescribed instruction and thus bring forward a suitable local reason in defence of the proposition at stake. Once the defender has replaced the instruction with the required local reason we say that the instruction has been resolved.
2. The player index of an instruction determines which of the two players has the right to choose the local reason that will resolve the instruction.
   a. If the instruction $\mathcal{I}$ for the logical constant $\mathcal{K}$ has the form $\mathcal{I}^{\mathcal{K}}(p)^{\mathbf{X}}$ and it is **Y** who requests the resolution, then the request has the form **Y** ?…/ $\mathcal{I}^{\mathcal{K}}(p)^{\mathbf{X}}$, and it is **X** who chooses the local reason.
   b. If the instruction $\mathcal{I}$ for the logic constant $\mathcal{K}$ has the form $\mathcal{I}^{\mathcal{K}}(p)^{\mathbf{Y}}$ and it is player **Y** who requests the resolution, then the request has the form **Y** p_i / $\mathcal{I}^{\mathcal{K}}(p)^{\mathbf{Y}}$, and it is **Y** who chooses the local reason.
3. In the case of a sequence of instructions of the form $\pi[\mathcal{I}_i(...(\mathcal{I}_k(p))...)]$, the instructions are resolved from the inside ($\mathcal{I}_k(p)$) to the outside ($\mathcal{I}_i$).

This rule also applies to functions.

SR4: Substitution of instructions

Once the local reason b has been used to resolve the instruction $\mathcal{I}^{\mathcal{K}}(p)^{\mathbf{X}}$, and if the same instruction occurs again, players have the right to require that the instruction be resolved with b. The substitution request has the form $?b/\mathcal{I}_k(p)^{\mathbf{X}}$. Players cannot choose a different substitution term (in our example, not even **X**, once the instruction has been resolved).

This rule also applies to functions.

SR5: Socratic rule and definitional equality

The following points are all parts of the Socratic rule, they all apply.

SR5.1: Restriction of P statements

P cannot make an elementary statement if **O** has not stated it before, except in the thesis.

An elementary statement is either an elementary proposition with implicit local reason, or an elementary proposition and its local reason (not an instruction).

SR5.2: Challenging elementary statements in formal dialogues

Challenges of elementary statements with implicit local reasons take the form:

$$\boldsymbol{X} \,!\, A$$
$$\boldsymbol{Y} \,?_{reason}$$
$$\boldsymbol{X}\, a : A$$

Where A is an elementary proposition and a is a local reason.

P cannot challenge **O**’s elementary statements, except if **O** provides an elementary initial concession with implicit local reason, in which case **P** can ask for a local reason, or in the context of transmission of equality.

SR5.3: Definitional equality

O may challenge elementary **P**-statements, challenge answered by stating a definitional equality, expressing the equality between a local reason introduced by **O** and an instruction also introduced by **O**.

These rules do not cover cases of transmission of equality. The Socratic rule also applies to the resolution or substitution of functions, even if the formulation mentions only instructions.

We distinguish reflexive and non-reflexive cases of :

SR5.3.1: Non-reflexive cases of the Socratic rule

We are in the presence of a *non-reflexive case* of the Socratic rule when **P** responds to the challenge with the indication that **O** gave the same local reason for the same proposition when she had to resolve or substitute instruction $\mathcal{I}$.

Here are the different challenges and defences determining the meaning of the three following moves:

Table 5: Non-reflexive cases of the Socratic rule

	Move	Challenge	Defence
SR5.3.1a	**P** $a : A$	**O** $? = a$	**P** $\mathcal{I} = a : A$
SR5.3.1b	**P** $a : A(b)$	**O** $? = b^{A(b)}$	**P** $\mathcal{I} = b : D$
SR5.3.1c	**P** $\mathcal{I} = b : D$ (this statement stems from **SR5.3.1b)**	**O** $? \ldots = A(b)$	**P** $A(\mathcal{I}) = A(b)$: ***prop***

Presuppositions:

(i) The response prescribed by SR5.3.1a presupposes that **O** has stated A or $a = b : A$ as the result of the resolution or substitution of instruction $\mathcal{I}$ occurring in $\mathcal{I} : A$ or in $\mathcal{I} = b : A$.

(ii) The response prescribed by SR5.3.1b presupposes that **O** has stated A and $b : D$ as the result of the resolution or substitution of instruction $\mathcal{I}$ occurring in $a : A(\mathcal{I})$.

(iii) SR5.3.1c assumes that **P** $\mathcal{I} = b : D$ is the result of the application of SR5.3.1b. The further challenge seeks to verify that the replacement of the instruction produces an equality in **prop**, that is, that the replacement of the instruction with a local reason yields an equal proposition to the one in which the instruction was not yet replaced. The answer prescribed by this rule presupposes that **O** has already stated $A(b)$: **prop** (or more trivially $A(\mathcal{I}) = A(b)$: **prop**).

The **P-**statements obtained after defending elementary **P-**statements cannot be attacked again with the Socratic rule (with the exception of SR5.3.1c), nor with a rule of resolution or substitution of instructions.

SR5.3.2: Reflexive cases of the Socratic rule

We are in the presence of a *reflexive case* of the Socratic rule when **P** responds to the challenge with the indication that **O** adduced the same local reason for the same proposition, though that local reason in the statement of **O** is not the result of any resolution or substitution.

The attacks have the same form as those prescribed by SR5.3.1. Responses that yield reflexivity presuppose that **O** has previously stated the same statement or even the same equality.

The response obtained cannot be attacked again with the Socratic rule.

SR6: Transmission of definitional equality

As can be expected, definitional equality is transmitted by reflexivity, symmetry[53], and transitivity. Definitional equalities however can also be used in order to carry out a substitution within dependent statements—they can in fact be seen as a special form of application of the substitution rule for dependent statement Subst-D presented in the first section for local meaning, with the formation rules (0, p. 28).

The identity-predicate Id

The dialogical meaning explanation of the identity predicate **Id**(x, y, z) – where x is a **set** (or a **prop**) and y and z are local reasons in support of A – is based on the following: **X**'s statement **Id**(A, a, b) presupposes that $a : A$ and $b : A$, and expresses the claim that " a and b are identical reasons for supporting A. The presupposition yields already its formation rule, the second requires a formulation of the Socratic Rule specific to the identity predicate. Let us start with the formation:

Formation of Id

Statement	Challenge	Defence
X ! **Id**(A, a_i, a_j) : **prop**	**Y** $?_{F1}$ **Id**	**X** ! A : **set**
	Y$?_{F2}$ **Id**	**X** ! $ai : A$
	Y$?_{F3}$ **Id**	**X** ! $aj : A$

[53] Symmetry used here is not the same notion as the symmetry of section **Erreur ! Source du renvoi introuvable.**.

Socratic Rules for Id

Opponent’s statements of identity can only be challenged by means of the rule of global analysis or by Leibniz-substitution rule

The following rules apply to statements of the form **Id**(*A*, *a*, *a*) and the more general statement of identity **Id**(*A*, *a*, *b*). Let us start with the reflexive case.

SR-Id.1 Socratic Rules for Id(*A*, *a*, *a*)

If the Proponent states **P** !**Id**(*A*, *a*, *a*), then he must bring forward the definitional equality that conditions statements of propositional intensional identity (see chapter II.8). Furthermore, the statement **P** !**Id**(*A*, *a*, *a*) commits the proponent to make explicit the local reason behind his statement, namely, the local reason **refl**(*A*, *a*) **specific** of **Id**-statements, the only internal structure of which is its dependence on *a*. Thus; the dialogical meaning of the instruction **refl**(*A*, *a*) amounts to prescribing the definitional equality *a* = **refl**(*A*, *a*) : *A* as defence to the challenge **O** ? =**refl**(*A*, *a*). The following two tables display the rules that implement those prescriptions.

Socratic Rule for the Global Synthesis
of the local reason for P ! Id(*A*, *a*, *a*)

Statement	Challenge	Defence
P ! **Id**(*A*, *a*, *a*)	**O** ? $_{reason\textbf{\textit{Id}}}$	**P** **refl**(*A*, *a*) : **Id**(*A*, *a*, *a*)
P **refl**(*A*, *a*) : **Id**(*A*, *a*, *a*)	**O** ? = **refl**(*A*, *a*)	**P** *a* = *a* : *A*

(This rule presupposes that the well-formation of **Id**(***A***, *a*, *a*) has been established)

The following rule is just applying the general Socratic Rule for local reasons to the specific case of **refl**(*A*, *a*) and shows that the local reason **refl**(*A*, *a*) is in fact equal to *a*.

Socratic Rule
for the challenge upon P’s use of refl(*A*, *a*)

Statement	Challenge	Defence
P **refl**(*A*, *a*) : **Id**(*A*, *a*, *a*)	**O** ? =**refl**(*A*, *a*)	**P** *a* = **refl**(*A*, *a*) : *A*

Since in the dialogues of immanent reasoning it is the Opponent who is given the authority to set the local reasons for the relevant sets, **P** can always trigger from **O** the identity statement **O** *p* : **Id**(*A*, *a*, *a*) for any statement **O** *a* : *A* has brought forward during a play. This leads to the next table that *constitutes one of the exceptions* to the interdiction on challenges on **O**'s elementary statements

Socratic Rule
for triggering the reflexivity move O ! Id(*A*, *a*, *a*)

Statement	Challenge	Defence
O *a* : *A*	**P** ?$^{\textbf{Id-}\textbf{\textit{a}}}$	**O** **refl**(*A*, *a*) : **Id**(*A*, *a*, *a*)

Remarks

Notice that it looks as if **P** will not need to use this rule since according to the rule for the synthesis of the local reason for an Identify statement by **P**, he can always state **Id**(*A*, *a*, *a*), provided **O** stated *a* : *A*.

However, in some case, such as when carrying out a substitution based on identity, **P** might need **O** to make an explicit statement of identity suitable for applying that substitution-law.
This rule

The next rule prescribes how to analyse some local reason *p* brought forward by **O** in order to support the statement **Id(*A, a, a*)**

Analysis I
The Global Analysis of O *p* : Id(*A, a, a*)

Statement	Challenge	Defence
O *p* : **Id**(*A*, *a*, *a*)	**P** ? $_{\mathbf{Id}}$= *p*	**O** *p* = **refl**(*A*, *a*) : **Id**(*A*, *a*, *a*)

The second rule for analysis involves statements of the form **Id**(*A*, *a*, *b*), so we need to general rules for statements that are not restricted to reflexivity. In fact the rules for **Id**(*A*, *a*, *b*) can be obtained by re-writing the precedent rules – with the exception of the rule that triggers statements of reflexivity by **O**.
We will not write the rules for **Id**(*A*, *a*, *b*) down but let us stress two important points

(1) the unicity of the local reason **refl**(*A*, *a*).
(2) the non-inversibily of the intensional predicate of identity in relation to judgmental equality.

(1) In relation to the first remark, the point is that the local reason produced by a process of synthesis for any identity statement is always **refl**(*A*, *a*). In other words, the local reason prescribed by the procedures of synthesis involving the statement ! **Id**(*A*, *a*, *a*) and the statement **Id**(*A*, *a*, *a*) , is the *same one,* namely **refl**(*A*, *a*).

(2) In relation to our second point, It is important to remember that the **global synthesis** rule refers to the commitments undertaken by **P** when he affirms the identity between *a* and *b*. Such commitment amount to i) providing a local-reason for such identity ii) stating $a = b : A$.
On the contrary the rule of **global analysis of an identity statement by O** prescribes what **P** may require from **O**'s statement. In that case, **P** cannot force **O** to state $a = b : A$ only because she stated **Id**(*A*, *a, b*).. This is only possible with the so-called extensional version of propositional identity (see II.8 above and thorough discussion in Nordström et al., 1990, pp. 57-61,). The dialogical view of non-reversibility here is that the r*ule of synthesis set the conditions **P** must fulfil* when he states and identity, not *what follows from his statement of identity*:

Id is transmitted by the rules of reflexivity, symmetry, transitivity and by the **substitution of identicals.**

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