Spatial patterns of urban systems and multifractality

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1 Introduction

Sets of cities belonging to a large geographic region or to the same country have many comparable properties. All of them have a very strong hierarchical structure of city sizes including a few number of very large metropolises, a larger number of medium sized cities and a very large number of small towns. They also have a rather regular geographical organization of towns and cities of different size, according to patterns which have been formalised geometrically in the spatial models of central place theory by W. Christaller (1933). Moreover, large sets of connected cities have remarkable similarities in their evolution, due to the permanent competition between them for the creation and capture of all kinds of innovations. That is why B. Berty (1964) suggested to consider "cities as systems within systems of cities". We think that systems of cities are open systems, which can be conceptualised in a relevant way within the framework of self-organization theory.

The hierarchical structure is the main common property of systems of cities. The statistical distribution of city sizes is extremely skewed, and this shape remains stable over time. It has received several formalisations. The best known is the famous "rank-size rule" of Zipf (1949), which linked the population n_k of a city to its rank k (ranking in decreasing order) in the following way:

$$n_k = n_1 k^{-q}, \tag{1}$$

where n_1 is the size of the largest city and q is a parameter with a value not too different from one.

This formula has been sometimes misleading, but it is actually nothing else than a Pareto distribution, giving the number k of cities with a population larger or equal to n_k:

$$k = n_1^p n_k^{-p} \tag{2}$$

where the parameter $p = 1/q$.

Several types of highly skewed distributions can actually fit the empirical distribution of city sizes with a rather good quality of adjustment (Quandt, 1964). A model can then be chosen among them according to theoretical considerations. Such a model should for instance be consistent with the dynamic properties of urban systems, or with the behaviour of individuals migrating from one city to another. A convenient model should also allow to simulate the genesis and evolution of various urban systems, in order to compare the range of simulated situations with empirical observations.

Despite they do have very general common properties, urban systems are also characterized by their sensitivity to historical conditions which sharped them differently, for very long time periods. For instance, on figure 1, the spatial patterns of urban systems are compared throughout the European Community. Cities are represented by circles proportional to their size and the regions around them are classified according to the size of the main market areas and density of cities. Four main types of spatial organisation can be recognised: the "Parisian" model has a very dominant capital excluding other cities in a large radius; it appears also around cities like London, or Madrid, or München. The "Rhenian" model is characteristic of the Rhine valley. In this model the cities are less different in size, they are closer to each other and share the urban functions less unequally. Such a spatial pattern also can be found in the Midlands of Britain, or in northern Italy, or in eastern France. The "peripherical" model is like the Parisian one on a reduced scale and with lower population density, whereas the "intermediary" one is similar to the Rhenian, with a little more contrasts in size and functional levels among cities.

Such a variety of spatial organisations is reflected in the statistical distribution of city sizes. Whatever the model which is chosen, a graphic plot on a double logarithmic scale is a convenient way for describing and comparing the distributions. Figure 2 illustrates the types of distributions of four countries, as presented by F. Moriconi-Ebrard (1994). The first two curves for Germany show how important the choice of a proper delimitation of urban areas is: the spatial entities which are considered have to be defined consistently and in a comparable way from one country to another, before adjusting any model. The results would be quite different if one would use the figures about the "Gemeinden" as published by the German census, or if the whole contiguous urban area around the main urban center is considered. This was done by F. Moriconi-Ebrard in the Geopolis data base and it allows for international comparisons. However, the statistical distribution of city sizes for Germany is the more regular among the four presented, there are large differences in sizes among cities but no one is dominating more than expected by the "rank size rule" model. This is in conformity with the Rhenian urban spatial pattern. On the contrary, all other curves exhibit a strong discontinuity, either between the first city and the second one like in France (the population of Paris is seven times the one of Lyon, the second city), or between the second and the third one like in Spain, where Madrid and Barcelona dominate the urban hierarchy, or between the third and the fourth like in Italy, where the three largest cities (Milano, Roma, Napoli) dominate the rest of the urban system.

The statistical pattern of city size distribution reflects the spatial pattern of urban systems, which is obviously linked with specific features of the historical genesis of each national territory: the Centralism of the French political and administrative system since the XIIIth century at least is a well known particularity and the Parisian primacy is the result of it, whereas the late unification of German states into a single country preserved the changes of several cities in the competition until more recent time. The three headed system of Italy is resulting of a even longer history of territorial integration.

One may wonder which are the processes of creating and maintaining such structures strongly organised in the same way but slightly different in their particular shape and very stable over very long periods of time (centuries). What is the explanation for the preeminence of large cities? As the general structure is very similar in countries of various physical, historical and cultural conditions, could this behaviour be simulated by a pure random process? Could the specificities of observed urban systems be explained by stochastic variations around a single process?

2 The Dynamics of Urban Pattern Formation

Several reliable empirical studies are now available for understanding the evolution of the size and spacing of towns. They used comparative data bases, either national (like Madden, 1955, Robson, 1973, Pumain, 1982, Guérin-Pace, 1993), or international for historical times (De Vries, 1984, Bairoch, Batou, Chévre, 1988), or for contemporary periods (Cattan et al., 1994, Moriconi- Ebrard, 1994). All studies showed a kind of meta-stability of the hierarchical differentiation and spatial configuration of systems of towns and cities. That means that the ranking of the towns and cities belonging to the same country remain about the same for very long time periods, the shape of the size distribution of cities keep its distinctive features, and the relative size of neighbouring cities in average is only very slowly changing. Such a metastability occured despite a very general process of urbanization including population growth (2% per year during the XIXth century in Britain for instance), very significant increases in productivity and income levels, and dramatic qualitative changes in economic and social structures as well as in urbanism and housing.

Could such a dynamic equilibrium be explained by a stochastic process of growth and change within the system? We shall examine first a simple model as suggested by Gibrat (1931) before describing a more recent synergetic model (Haag and Max (1994), Haag (1994)).

2.1 The Gibrat Model

Gibrat was a statistician who found several applications of the lognormal distribution, which was suitable for the description of settlement sizes according to him. The number $f(n_k)$ of settlements of size n_k can be written:

$$f(n_k) = (1/\sigma\sqrt{2\pi}) \exp\left[-(\log n_k - n)^2/2\sigma^2\right] \tag{3}$$

where n is the mean of the logarithms of size and σ their standard deviation.

Such a lognormal distribution is the result of a stochastic repartition of growth within the settlement system, given the following rules:

- time is subdivided in short intervals
- during each interval the population of a settlement is increased (or decreased) by a quantity which is proportional to its size at the beginning of the interval ("law of proportional effect"),
- the growth is small compared to the settlement size
- the growth rates (relative growth) are independent of city sizes
- the growth rates are randomly independant variables from one time interval to the other.

When a convenient data base is provided with population of settlements known at various successive census dates, it is easy to test such a model. This was done by Robson (1973) on towns and cities of England and Wales for the XIXth century, and by Pumain (1982) and Guérin-Pace (1993) on French settlements betwen 1831 and 1990. The conclusions are about the same: Gibrat's law holds on the whole, since in average urban growth is proportional to city size. On short time intervals, the relative growth rates are very different from one town to the other and from one time period to the next for the same town. Those local fluctuations in growth are reflecting a continuous process of adaptation of towns and cities to their changing socio-economic environment in a very competitive context of urban actors for capturing the benefits of innovations, since towns and cities belonging to the same system are very well connected through various communication links, information circulates very fast through the entire system, and processes of imitation and challenge produce a process of spatial diffusion of innovation which has become faster and faster over time. Such a highly connected system of cities also explains why in average they all grow at about the same rate over long time periods.

However, there are two rather systematic deviations to the stochastic growth process: first, there is a general trend for larger cities to have larger growth rates. This is illustrated by the correlation between growth rates and city size (figure 3 above), which is almost never negative but always slightly positive, and even larger with the logarithm of size. Second, the growth rates are not always independant from one period to the following one, but they may be correlated (figure 3, below).

Such deviations from a pure stochastic process can be easily explained from urban theory and empirical observations. Large cities are more likely to create innovations, or to adapt faster to most innovations. The general process of innovation diffusion in an urban system usually follows an hierarchic way, starting in the largest settlements and filtering down the urban hierarchy of sizes. This can explain why in average they grow faster than the other, because they capture a large share of the initial advantage each time an innovation occurs. Urban growth or decline also may persist over longer periods of time in the same cities because of the specialisation process. This happens when a city maintains its advance in the adaptation process to a given innovation, or if there are only a few possible locations for some innovation to be exploited (for instance in the case of local resources like mining or a natural amenity for tourism). This can lead to a specialisation of the town in the activity in question. The process has an influence on the relative position of the town within the urban system, since the growth cycle which affects the city is linked to the growth cycle of the production. In general, small towns are more sensible in their evolution to such cycles of growth and decline, whereas the effect of the cyclic variations in a single activity are less visible in the evolution of large cities where compensations between growing and declining activities are more likely to occur.

There is another reason why historically an urban system should evolve with large cities growing faster than the other. This is a well–known geographical process called "the space-time contraction". When new transportation means are created, they increase the speed of travelling. Large cities are then expanding their market areas at the expenses of smaller towns, which decline, at least in relative terms. This has been observed everywhere. One should then be able to include in a dynamic model of urban systems the evolutional properties which make them adaptive, to a changing socio–economic and geographic environment. The main specificity of the dynamics of the urban systems compared to a stochastic process is the trend for a faster growth of large cities.

One has then to find a model which keeps some of the interesting features of the Gibrat's model, including the stochastic aspect and the general growing trend, but one should express in a more explicit way how the growth process is linked with the interactions between cities. The "law of proportional effect" has actually two components: the natural increase of urban population is multiplicative (biological growth, births and deaths), it may be considered in a first approximation to be the same rate from one city to another, the second component is the net migration balance. As migrations are also (roughly) proportional to city size, according to all migration theory (for instance in gravity models), one has to simulate a dissymmetry in migration according to the attractivity of cities, in order to generate a differenciated pattern of growth through the migration process. The synergetic migration model as elaborated by G. Haag and W.Weidlich (1984) can fulfil such a purpose.

2.2 The Weidlich–Haag–Model

Let the urban system be composed of $i = 1, 2, \ldots, L$ settlements and the hinterland h. The population configuration of the urban system is given by:

$$\vec{n} = \{n_1, \ldots, n_i, \ldots, n_L\}, \tag{4}$$

where n_i is the population size of city $i = 1, 2, \dots, L$, and N, where

$$\sum_{i=1}^{L} n_i = N, \tag{5}$$

is the total urban population. We assume a hierarchical ordering of the settlements such that n_1 is the population of the largest place, n_i is the population of the i-th ranked place, and n_L corresponds to the smallest urban place. L is the maximal rank considered.

The dynamics of $\vec{n}(t)$ is caused by decisions of individuals to change their location from city j to city i and by birth/death-processes.

Each of the n_j residents of settlement j may change to settlement i with a "mean individual" transition rate $p_{ij}(\vec{n})$ and thus gives rise to the population configuration transition on the macro-level

$$w_{ij}(\vec{n}) = n_j p_{ij}(\vec{n}) = n_j \nu_{ij}(t) \exp[u_i(n_i) - u_j(n_j)], \tag{6}$$

where $\nu_{ij}(t) = \nu_{ji}(t)$ is a symmetric mobility matrix measuring the intensity of interaction between city i and city j. The $\nu_{ij}(t)$ are not only depending on the geographical distance but also on other effects like social proximity and learning processes due to previous migration, and $[u_i(n_i) - u_j(n_j)]$ is the difference between the attractivities of the origin, $u_j(\vec{n})$, and the destination, $u_i(\vec{n})$.

The mean value equation of the (Weidlich–Haag–model, 1984) then reads:

$$\begin{aligned} \frac{d\bar{n}_k(t)}{dt} &= \sum_{l=1}^{L} \bar{n}_l \nu_{kl} \exp(u_k - u_l) - \sum_{l=1}^{L} \bar{n}_k \nu_{lk} \exp(u_l - u_k) \\ &+ \rho_k(t)\bar{n}_k + I_k(t), \end{aligned} \tag{7}$$

for $k = 1, 2, \dots, L$, and where $I_k(t)$ is the net immigration rate, $\rho_k(t)$ the net growth rate of settlement k, respectively. The change in population size of city k is due to inter-settlement migrations as well as to interactions with the hinterland, the environment and birth/death events.

Evidently, (7) is a set of L coupled nonlinear equations of motion describing the spatial organization of a system of settlements at the macro-level. Theoretical details of this model with an application to the French system of cities (78 cities) were discussed previously (Pumain, Haag, 1991; Haag, 1991; Haag et al, 1992).

Four reasonable assumptions (figure 4) are used in the following analysis and simulations to test their influence on the skewness of the rank–size–distribution:

- **assumption A:**
  The attractivities $u_j(\vec{n})$ are expanded in a truncated Taylor series up to second order:

  $$u_j(\vec{n}) = \delta_j(t) + \tilde{\kappa}_j n_j(t) - \tilde{\sigma}_j n_j^2(t). \tag{8}$$

  This corresponds to an "optimal" settlement size $n_j^{opt} = \tilde{\kappa}_j / 2\tilde{\sigma}_j$, which maximizes the settlement attractivity.

- **assumption B:**
  The attractivities $u_j(\vec{n})$ depend in a logarithmic manner on its population size $n_j(t)$:

  $$u_j(\vec{n}) = \delta_j(t) + \tilde{\kappa}_s \log(n_j(t)) \tag{9}$$

  Here, a convex decreasing marginal settlement attracitivity is assumed.

- **assumption C:**
  The attractivities $u_j(\vec{n})$ are independent on its population size:

  $$u_j(\vec{n}) = \delta_j(t) \tag{10}$$

  In this case, the attractivity of a settlemet is completely determined by size independent effects.

- **assumption D:**
  The attractivities $u_j(\vec{n})$ dependent only on the rank of the city:

  $$u_j(\vec{n}) = \tilde{\kappa}_r \log(j) \tag{11}$$

The propensity of the population to agglomerate (synergy effect) is represented via the agglomeration parameters $\tilde{\kappa}, \tilde{\kappa}_s, \tilde{\kappa}_r$, respectively. In assumtion A, a possible saturation effect of city size owing to negative externalities is taken into account via the saturation parameter $\tilde{\sigma} > 0$.

3 Equation of Motion for the Rank–Size–Coefficients

The total urban population is decreasing or increasing in time, depending on the evolution of $N(t)$. Despite of dramatic growth in the urban population (Pumain, 1991) and of the number and size of settlements, the shape of the distribution of settlement sizes remains remarkable stable (Figure ??). The general spatial pattern and the preserved size distribution of settlements over time suggest that the dynamics of a system of settlements at the most aggregate level could be formalized by a time–dependent Pareto–distribution:

$$\bar{n}_k(t) = \bar{n}_1(t) k^{-q(t)}, \tag{12}$$

where $\bar{n}_k(t)$ is the (mean) population of the k-th ranked place, $\bar{n}_1(t)$ is the (mean) population of the largest place, and $q(t)$ the Pareto–coefficient.

This power–law has shown to be an acceptable approximation to empirical observations in different countries. However, the parameters of the Pareto–distribution (Pareto–coefficient $q(t)$, number of cities considered L, urban population of the largest place $\bar{n}_1(t)$) change in time. Therefore, it is reasonable to assume that variations of those parameters will depend on the evolution and detailed spatial structure of the system of settlements. In other words the "scale symmetry" (Pareto–coefficient), involved in the dynamics of the nested system of settlements, will be a function of the inherent parameters of the spatial system:

$$q = q(\kappa, \nu_0, f_{ij}, \delta_i, N, L, \ldots). \tag{13}$$

It is one aim of this paper to investigate under what conditions the Pareto–distribution is most likely to be found. However, in general, deviations of this power–law, in other words multifractality, must be expected.

Multifractal measures are related to the study of a distribution of physical or other quantities on a geometric support (Feder, 1991). The idea is that a multifractal may be represented in terms of intertwined fractal subsets having different scaling exponents (Mandelbrot, Evertsz, 1991). At least at the beginning of the selforganization process, urban systems must be seen as a multifractal phenomenon. In other words, many time–dependent parameters (fractal exponents) are needed in order to describe the shape and the evolution of the distribution of city sizes in a regional set of cities.

Therefore, a nonlinear transformation is introduced (Haag and Max, 1994):

$$\bar{n}_k(t) = \bar{n}_1(t)k^{-q_k(t)}, \quad \text{or} \quad q_k(t) = \frac{\ln\left(\frac{\bar{n}_1(t)}{\bar{n}_k(t)}\right)}{\ln k} \tag{14}$$

where the $q_k(t)$ are denoted as rank–size–coefficients. Without any loss of generality, $q_1(t) = 0$ can be assumed.

Any arbitrary initial distribution of cities, represented by $q_k(t = 0)$, can be used as starting point of the selforganization process.

Inserting the transformation (14) into (7) one obtains after some straightforward manipulations (Haag and Max, 1994) the equation of motion for the rank–size–coefficients:

$$\begin{aligned} \frac{dq_k}{dt} &= \frac{1}{\ln k}\sum_{l=1}^{L} l^{-q_l}\left[\nu_{1l}\exp(u_1 - u_l) - k^{q_k}\nu_{kl}\exp(u_k - u_l)\right] \\ &- \frac{1}{\ln k}\sum_{l=1}^{L}\left[\nu_{l1}\exp[-(u_1 - u_l)] - \nu_{lk}\exp[-(u_k - u_l)]\right] \\ &+ \frac{(\rho_1 - \rho_k)}{\ln k} + (I_1 - k^{q_k}I_k)\frac{C_L(\vec{q})}{N\ln k} \end{aligned} \tag{15}$$

for $k = 2, 3, \ldots, L$, where

$$C_L(\vec{q}) = \sum_{k=1}^{L} k^{-q_k}. \tag{16}$$

Therefore, this dynamic theory of urban settlements connects the slow dynamics of the rank–size–distribution with the inter–urban migratory dynamics of the population.

Equation (15) becomes fully explicit by insertion of the analytical form (assumption A to D) of the attractivities $u_k(n_k)$. For this aim, the size–dependency is replaced via (14):

- **assumption A:**

$$u_k = u_1 - \kappa(1 - k^{-q_k}) + \sigma(1 - k^{-2q_k}) + (\delta_k - \delta_1) \tag{17}$$

- **assumption B:**

$$u_k = u_1 - \kappa_s q_k \ln k + (\delta_k - \delta_1) \tag{18}$$

- **assumption C:**

$$u_k = u_1 + (\delta_k - \delta_1) \tag{19}$$

- **assumption D:**

$$u_k = u_1 - \kappa_r \ln k \tag{20}$$

where the parameters $(\kappa, \sigma, \kappa_s, \kappa_r)$ have been scaled appropriatly. The level of attractivity of a settlement u_k is now quantitatively linked to the rank of this settlement via assumptions A – D. Of course, regional differences in the preference parameters δ_k may shift the attractivity levels of the settlements. Therefore, the ranking of the attractivities u_k may deviate from the size–ranking of the settlements, especially in the higher ranks.

The Pareto–distribution would be a stable attractor of the spatial system of settlements if and only if

$$\lim_{t \to \infty} q_k(t) = q, \quad \forall_{k=2,\ldots,L}. \tag{21}$$

However, this seems to be a rather restrictive condition.

4 Simulation of the Settlement Hierarchy

In the two following simulations an almost homogeneous initial distribution $q_k(t = 0) \approx 0$ has been chosen. Furthermore, the system of settlements has been treated as closed $(\rho_k(t) = I_k(t) = 0)$. Therefore, the settlement hierarchy is the result of the underlying dynamic selforganization process.

In figure 5 assumption A, is used in the numerical simulation of (15). Under those conditions the urban system remains close to the initially chosen homogeneous distribution for a prolonged time period ($\approx$ 300 units in time). When the hierarchy starts to develop the evolution accelerates dramatically and within the next 40 time units as an intermediate state, a Pareto–like distribution appears which finally ends up in a distribution of a small number of dominating cities (3 metropolitan areas in this example) and a set of almost equally populated small settlements. The reason for this strong bifurcation behaviour of settlements with respect to their population is caused by the shape of the attractivity function (polynomial of second order) as favouring an optimal city size.

In figure 6 simulation with a logarithmic dependence of the settlement attractivity on rank–size, under assumption D, is carried out. The hierarchical ordering process starts immediately. Within a much shorter time interval of approximately 100–200 units the Pareto–distribution is approached. The slope of the distribution is in this case completely determined by the agglomeration parameter κ_r. Independent of the initial conditions $q_k(t=0) = q_0$, the dynamics leads to a unique stationary Pareto–coefficient $q(t) = \kappa_r$. In this case the Pareto–distribution is the result of a dynamic selforganization process with the Pareto–coefficient $q(t)$ as a stable attractor. In other words, the slope of the distribution is completely determined by the parameters of the migratory dynamics and therefore linked via the "individual" choice processes to the micro–level.

5 Conclusions and Outlook

The hierarchical ordering process of settlements can be understood as a selforganization process. The dependence of the settlement attractivity on its rank determines the hierarchical ordering of the system in the long–run.

The agglomeration parameter is the most important parameter for the occurrence of any hierarchical organization in case of assumption A and D. Possible damping or saturation effects are also of importance, especially for the shape of the distribution, according to assumption A.

The size independent parameter δ_j of a settlement influences the choice process to migrate as well. Those settlement specific parameters may be reasonable for certain changes in the rank of a settlement in the course of time and differences in the magnitudes of inter–urban flows.

However, the great variety of observed rank–size–distributions require some modifications of our theory:

- The age–dependence of the migration flows must be taken into account. Therefore, the group of young people is going to concentrate within the most attractive cities and will decrease in the less attractive ones (figure 7).
- Since the fertility rate is strongly age–dependent, the relative growth rate will also depend on the demographic structure of the cities.

Those both effects can be taken into account via the introduction of appropriate age groups. The population configuration of the urban system has therefore to be extended:

$$\vec{n^\mu} = \{n_1^\mu, \ldots, n_i^\mu, \ldots, n_L^\mu\}, \tag{22}$$

where n_i^μ is the population size of city $i = 1, 2, \ldots, L$, of age group μ. Of course, it is crucial to introduct an age dependent relative groth rate $\rho_k^\mu(t)$ as well, yielding to a coupling between demographic growth and inter urban migration.

References

Aharony A., 1991: "Fractal Growth", in Bunde A., Havlin S. (Ed.): "Fractals and Disordered Systems", (Springer, Heidelberg)

Allen P., Sanglier M., 1979: "A Dynamic Model of Growth in a Central Place System", *Geographical Analysis*, **11**, 256–272

Auerbach F., 1913: "Das Gesetz der Bevölkerungskonzentration", *Petermanns Mitteilungen*, 59 (1)

Bailey N.T.J., 1964: "The Elements of Stochastic Processes", (Wiley and Sons)

Beckmann M.J., 1958: "City Hierarchies and the Distribution of City Size", *Economic Development and Cultural Change*, **6**, 243–248

Berry B.J.L., Garrison W.L., 1958: "Alternate Explanations of Urban Rank–Size Relationships", *Annals of the Association of American Geographers*, **1**, 83–91

Bunde A., Havlin S., 1991: "Fractals and Disordered Systems", (Springer, Berlin)

Camagni R., Diappi L., Leonardi G., 1986: "Urban Growth and Decline in Hierarchical Systems. A Supply-oriented Dynamic Approach", *Regional Science and Urban Economics*, **16**, 145–160

Cattan N., Pumain D., Rozenblat C., Saint–Julien T., 1994: "Le systeme des villes europeenncs", (Anthropos, coll. Villes, Paris)

Chapman G.P., 1970: "The Application of Information Theory to the Analysis of Population Distribution in Space", *Economic Geography*, **2**, 317–331

Curry L., 1964: "The Random Spatial Economy", *Annals of the Association of American Geographers*, 138–146

Dendrinos D., Haag G., 1984: "Towards a Stochastic Theory of Location: Empirical Evidence", *Geographical Analysis*, **16**, 287–300

Dendrinos D., Mullally H., 1985: "Urban Evolution: Studies in the Mathematical Ecology of Cities", (Oxford University Press, Oxford)

Dendrinos D., Sonis M., 1990: "Signatures of Chaos: Rules in Sequences of Spatial Stock Size Distribution for Discrete Relative Dynamics", *Socio-Spatial Dynamics*, **1**, No. 1: 57–73

Feder J., 1988: "Fractals", (Plenum Press, New York)

Frankhauser P., 1991: "Beschreibung der Evolution urbaner Systeme mit der Mastergleichung", Dissertation, Universität Stuttgart

Gibrart R., 1931: "Les inégalités économiques", (Sirey, Paris)

Guerin-Pace F., 1993: "Deux sicles de croissance urbaine", (Anthropos, coll. Villes)

Haag G., 1989: "Dynamic Decision Theory: Application to Urban and Regional Topics", (Kluwer Academic Publishers, Dordrecht)

Haag G., 1991: "Towards an Integrated Urban Model", in D. Pumain (Ed.), " Spatial Analysis and Population Dynamics", (John Libbey, Eurotext)

Haag G., 1994: "The Rank-Size Distribution of Settlements as a Dynamic Multifractal Phenomenon", *Chaos, Solitons and Fractals*, **4**, No. 4: 519–534

Haag G., Dendrinos D., 1983: "Towards a Stochastic Theory of Location: A Nonlinear Migration Process", *Geographical Analysis*, **15**, 269–286

Haag G., Max H., 1993: "Rank–Size–Distribution of Settlement Systems: A Stable Attractor in Urban Growth", to be published

Haag G., Munz M., Pumain D., Sanders L., Saint–Julien Th., 1992: "Interurban Migration and the Dynamics of a System of Cities: 1. The Stochastic Framework with an Application to the French Urban System", *Environment and Planning*, **24**, 181–198

Haag G., Weidlich W., 1984: "A Stochastic Theory of Interregional Migration", *Geographical Analysis*, **16**, 331–357

Haran E.G.P., Vining D.R., 1973: "On the Implications of a Stationary Urban Population for the Size Distribution of Cities", *Geographical Analysis*, **4**, 194–200

Mandelbrot B.B., Evertsz C.J.G., 1991: " Exactly Self-Similar Left-Sided Multifractals", in Bunde A., Havlin S. (Ed.): "Fractals and Disordered Systems", (Springer, Heidelberg)

Mandelbrot B.B., 1991: "Die fraktale Geometrie der Natur", (Birkhäuser Verlag, Berlin)

Moriconi–Ebrard F., 1994: "L úrbanisation du monde", (Anthropos, coll. Villes, Paris)

Pumain D., 1982: "La Dynamique des Villes", (Economica, Paris)

Pumain D., 1991: "City Size Dynamics in Urban Systems", Proceedings of the Conference on Dynamic Modelling and Human Systems, Cambridge, Department of Archeology

Pumain D., 1993: "Geography, Physics and Synergetics", in Haag G, Mueller U.,Troitzsch K.G. (Eds.): "Economic Evolution and Demographic Change", *Lecture Notes in Economics and Mathematical Systems*, **395**, 157–175

Pumain D., Haag G., 1991: "Urban and Regional Dynamics – Towards an Integrated Approach", *Environment an Planning A*, **23**, 1301–1313

Pumain D., Sanders L., Saint–Julien Th., 1989: " Villes et Auto–Organisation", (Economica, Paris)

Quandt R.E., 1964: "Statistical Discrimination Among Alternate Hypothesis and Some Economic Regularities", *Journal of Regional Science*, **2**, 1–23

Robson B., 1973: "Urban Growth, An Approach", (Methuen, Paris)

Roehner B., Wiese K.E., 1982: "A Dynamic Generalization of Zipf's Rank–Size Rule", *Environment and Planning*, **14**, 1449–1467

Sanders, L., 1992: "Systeme de Villes et Synergetique", (Anthropos, Paris)

Simon H., 1955: "On a Class of Skew Distributions", *Biometrica*, 425–440

Steindl J., 1968: "Size Distributions in Economics", in Silks (Ed.) "International Encyclopedia of the Social Science", **14**, (Macmillan, New York)

Weidlich W., Haag G., 1983: "Concepts and Models of a Quantitative Sociology", *Springer Series in Synergetics*, **14**, (Springer, Heidelberg)

Wilson A.G., 1981 (a): "Catastrophe Theory and Bifurcations: Applications to Urban and Regional Systems", (Croom Helm, London)

Wilson A.G., 1981 (b): "The Evolution of Spatiale Structure" in R.J. Bennett (Ed.), "European Progress in Spatial Analysis", (Pion Books, London)

Zipf G.K., 1949: "Human Behavior and the principle of Least Effort", (Addison-Wesley Press)

Figure 1 **Diversity of spatial patterns of urban systems**

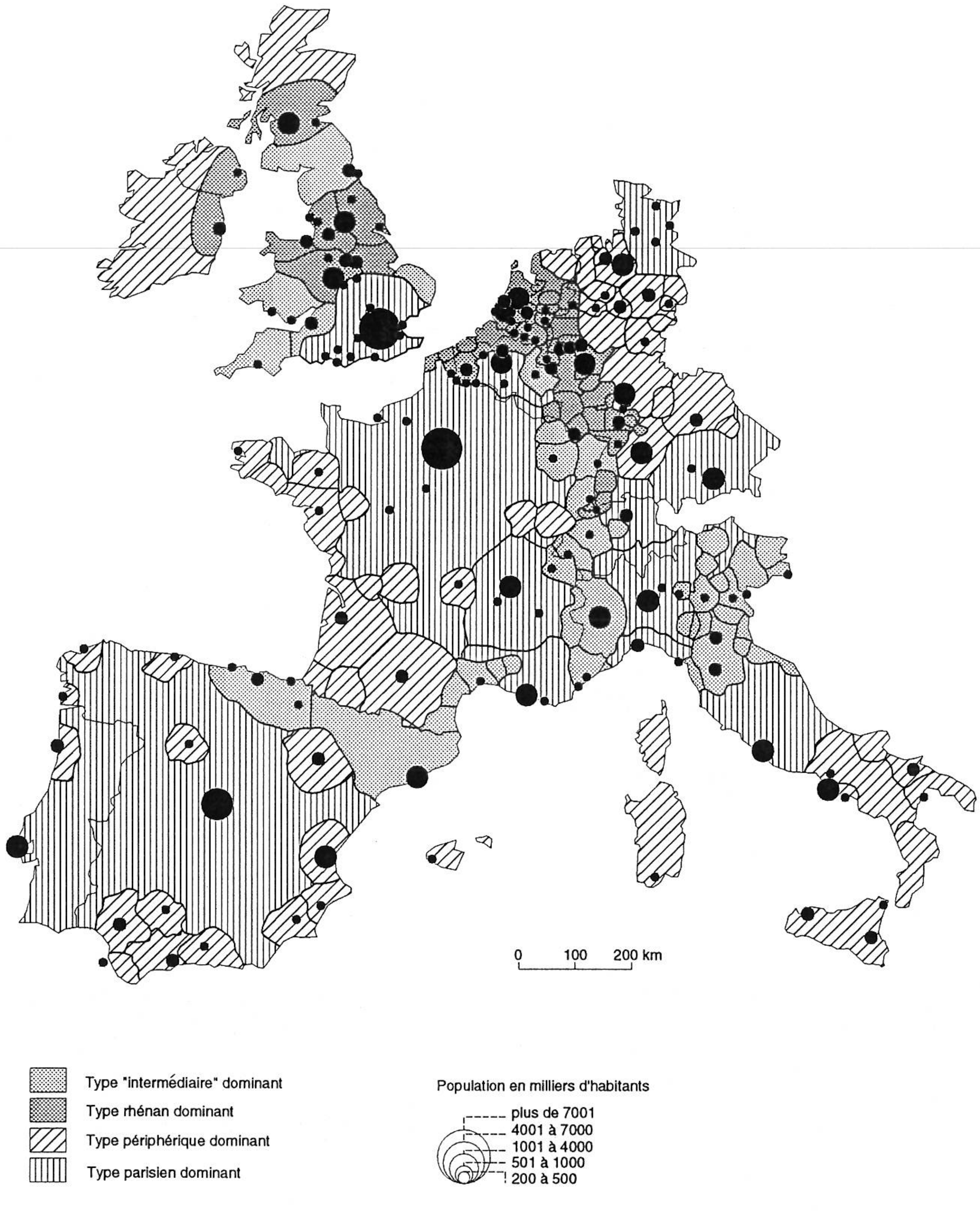
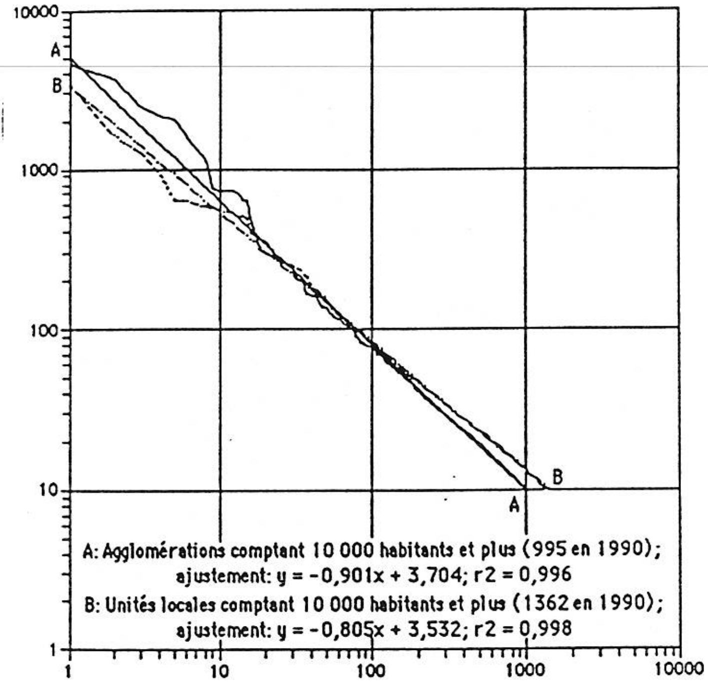


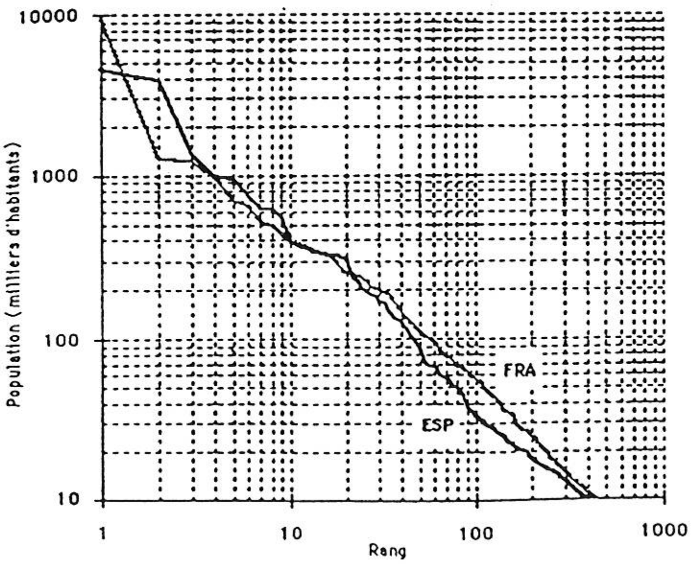
Figure 2 **Diversity of city size distributions**

GERMANY (1990)

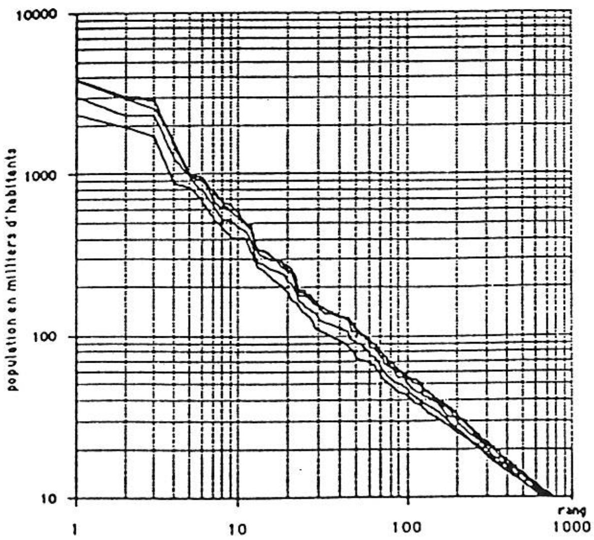


(above: urban areas; below: Gemeinden)

SPAIN and FRANCE (1990)



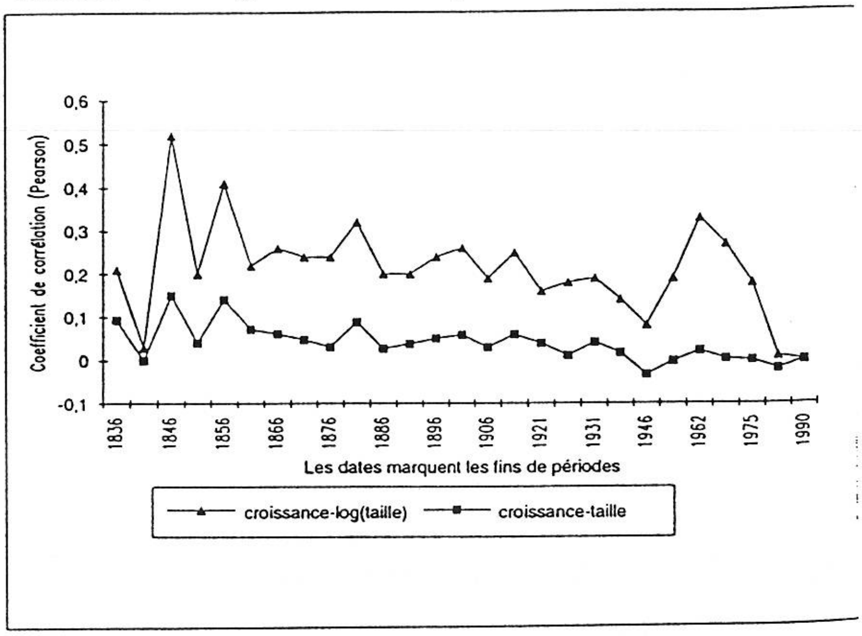
ITALY (1951, 1961, 1971, 1981)



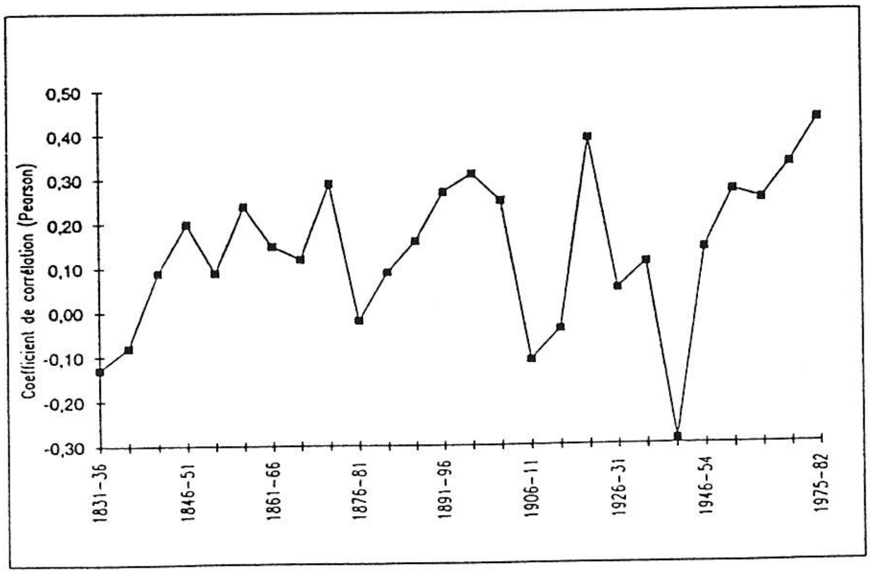
Source: F. Moriconi-Ebrard, 1994

Figure 3 Systematic deviations from Gibrat's stochastic growth process

Correlation between growth rates and city size



Correlation between successive growth rates



Source: F. Guérin-Pace, 1993

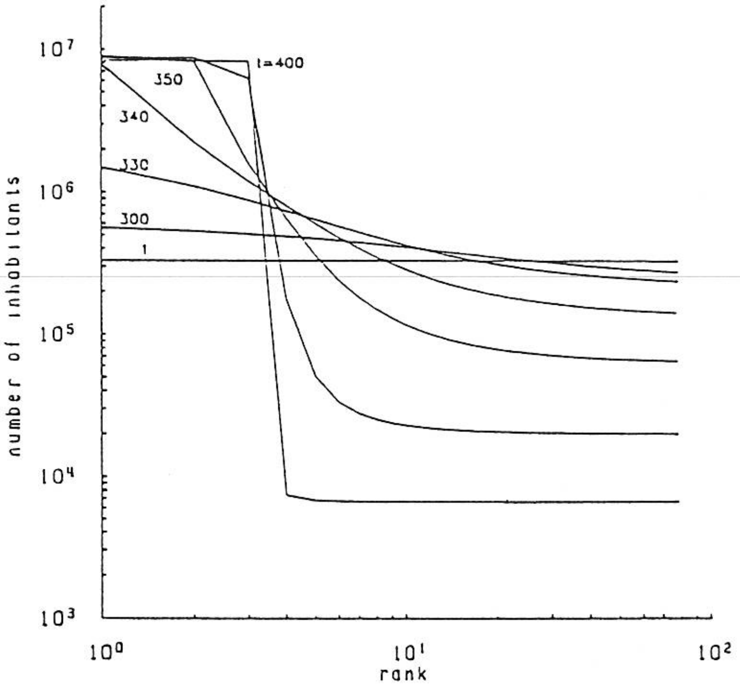


Figure 5 **Simulation of an urban system using assumption A**

$\kappa = 0.596$; $\sigma = 0.188$; $\nu = 0.001$; $N = 25.6 * 10^6$; $L = 78$.

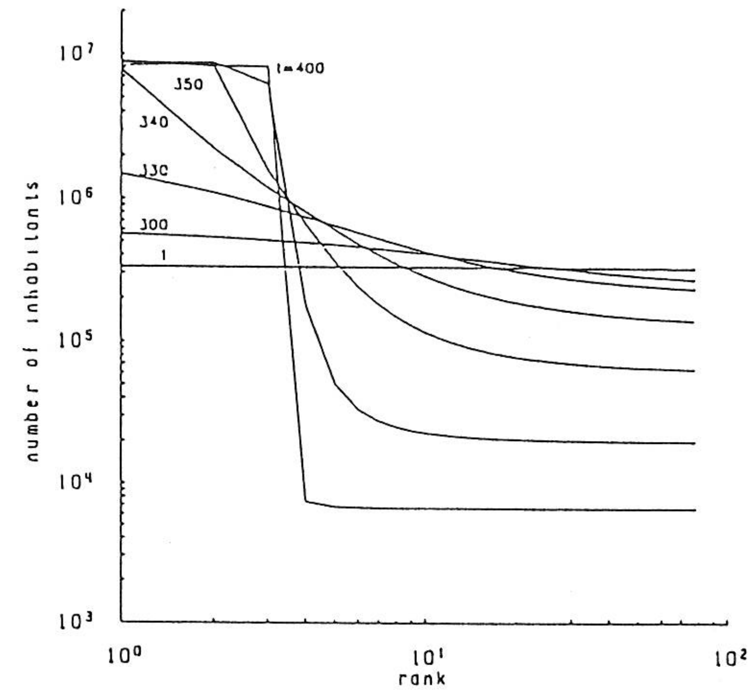


Figure 6 **Simulation of an urban system using assumption** $\mathcal{B}$

$\kappa = 0.596$; $\sigma = 0.188$; $\nu = 0.001$; $N = 25.6 * 10^6$; $L = 78$.

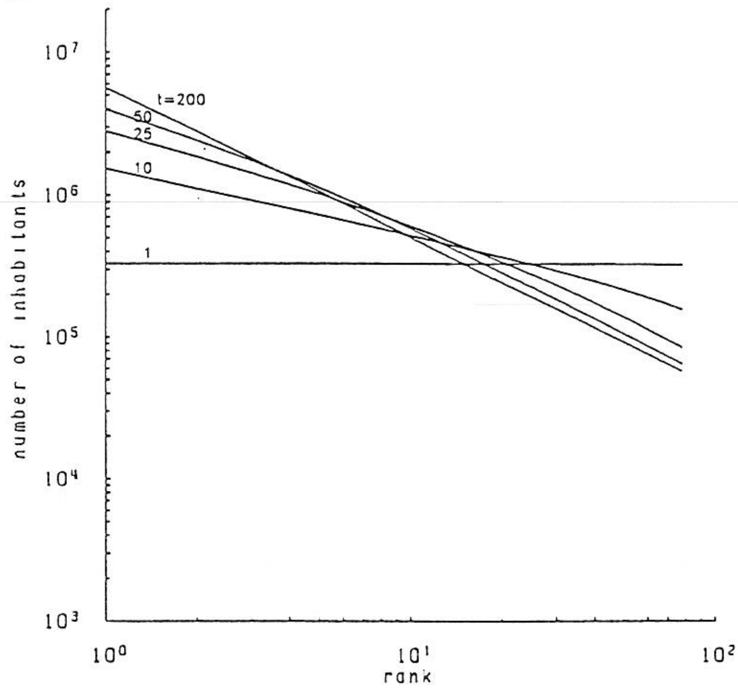


Figure 6 **Simulation of an urban system using assumption** B

$\kappa = 0.596$; $\sigma = 0.188$; $\nu = 0.001$; $N = 25.6 * 10^6$; $L = 78$.

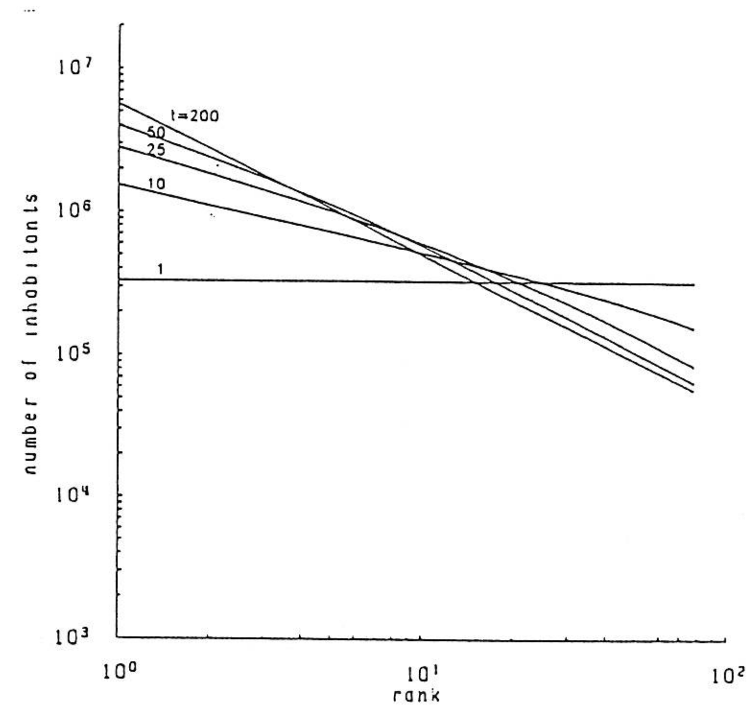
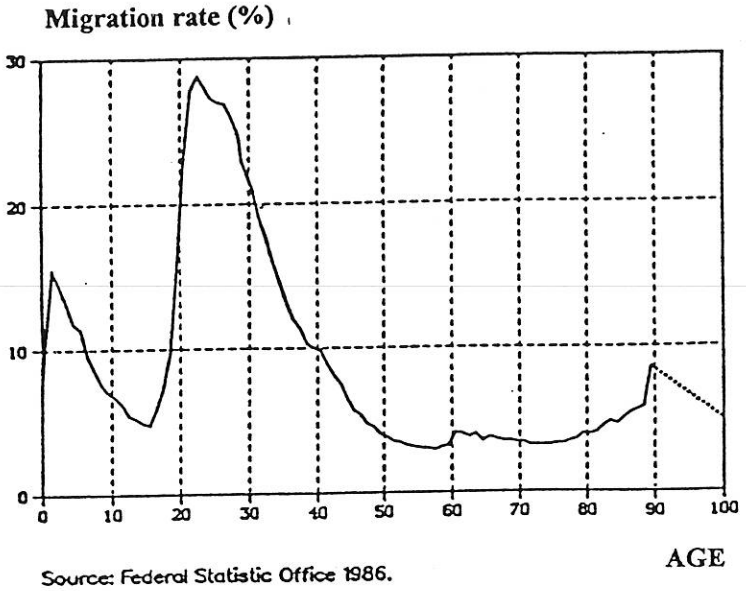


Figure 6 **Simulation of an urban system using assumption** *B*

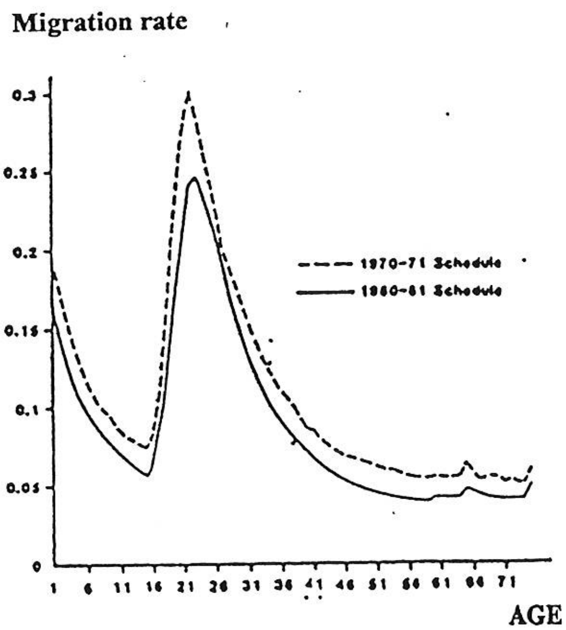
$\kappa = 0.596$; $\sigma = 0.188$; $\nu = 0.001$; $N = 25.6 * 10^6$; $L = 78$.

Figure 7 Age dependency of mobility

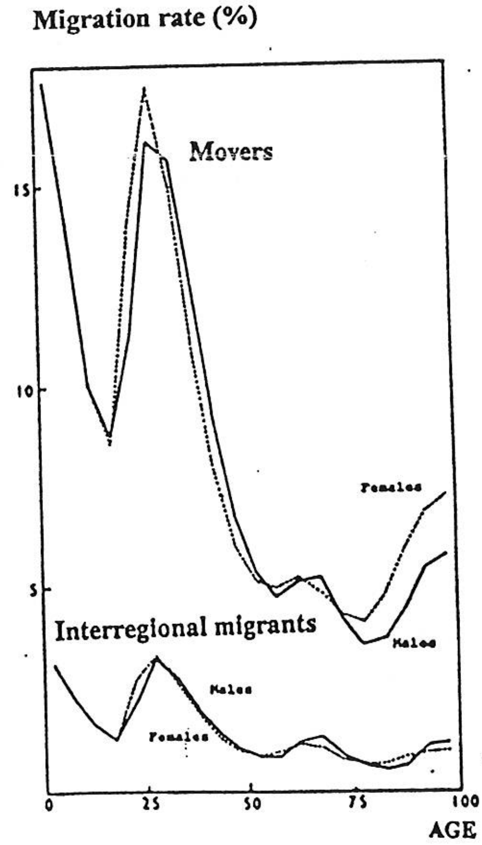


Source: Federal Statistic Office 1986.

Germany: Friedrich, 1990



Great Britain: Stilwell et al. 1989



France: Courgeau, 1990